Ultrafast Laser Physics

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Chapter 5: Relaxation oscillations in lasers
Diode-pumped solid-state lasers

Longitudinal pumping

Transversal pumping
Diode-pumped solid-state lasers
Diode-pumped solid-state lasers

Three-level system

Yb-doped

Four-level system

Nd-doped
Ti:sapphire
Laser rate equation

- ideal four-level system
- single mode laser
- homogeneous broadened gain material

\[ N \text{ inversion } N = N_2 - N_1, \quad N_1 \approx 0 \]

\[ n \text{ photon number (intracavity)} \]

\[ W^{stim} \equiv W_{12} = W_{21} = Kn \]

\[ KN \text{ spontaneous emission rate} \]

\[ KnN \text{ stimulated emission rate} \]

\[ \gamma_c \text{ cavity photon decay rate} \]

\[ \gamma_L \text{ spontaneous decay rate of level 2} \]
Below threshold, i.e. $r < 1$

\[
\begin{align*}
\Leftrightarrow \quad n_s &= \frac{r}{1-r} \\
N_s &\approx rN_{th}
\end{align*}
\]

Above threshold, i.e. $r > 1$

\[
\begin{align*}
\Leftrightarrow \quad n_s &\approx \frac{\tau_c}{\tau_L} N_{th} (r-1) = \frac{\gamma_L}{K} (r-1) \\
N_s &\approx N_{th} = \frac{\gamma_c}{K}
\end{align*}
\]
Linearized rate equations

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]
\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

\[ \frac{dn_1(t)}{dt} = \gamma_L (r - 1) N_1(t) \]

\[ \frac{dN_1(t)}{dt} = -\gamma_L r N_1(t) - \gamma_c n_1(t) \]

\[ s = s_{1,2} = \frac{-r \gamma_L}{2} \pm \sqrt{\left(\frac{r \gamma_L}{2}\right)^2 - \gamma_L \gamma_c (r - 1)} \]

Solution: Ansatz
\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]
Over-critical damping (no relaxation oscillations)

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]

\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

Solution: Ansatz

\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]

\[ s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L\gamma_c(r-1)} \]

s is real and negative:

\[ \left(\frac{r\gamma_L}{2}\right)^2 > \gamma_L\gamma_c(r-1) \]

two solutions: two over-critically damped relaxation constants:

\[ r\gamma_L \gg \gamma_c \Rightarrow \begin{cases} s_1 = -r\gamma_L \\ s_2 = -\gamma_c \frac{r-1}{r} \end{cases} \]

Stimulated decay rate of excited atoms

Photon decay rate inside laser cavity

\[ s_2 = -\gamma_c \frac{r-1}{r} \Rightarrow s_2 \approx -\gamma_c, \text{ for } r \gg 1 \]
Over-critical damping (no relaxation oscillations)

Example HeNe Laser (p. 10)

\( l = 632.8 \text{ nm}, \ t_L \approx 100 \text{ ns}, \ 2l = 0.02 \) (2% output coupler), \( T_R = 2 \text{ ns} \)

\[
\Rightarrow \ t_c = \frac{T_R}{2l} \approx 100 \text{ ns} \quad \Rightarrow \ t_L \approx t_c, \text{ or } \gamma_L \approx \gamma_c
\]

For \( r >> 1 \) (when HeNe is pumped sufficiently far above threshold)
Condition for over-critical damping is fulfilled:

\[
r \gamma_L \gg \gamma_c
\]

HeNe laser falls back into steady state after about 100 ns
Under-critical damping (relaxation oscillations)

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]
\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

Solution: Ansatz
\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]

\[ s = s_{1,2} = -\frac{r \gamma_L}{2} \pm \sqrt{\left(\frac{r \gamma_L}{2}\right)^2 - \gamma_L \gamma_c (r - 1)} \]

\[ s \text{ is complex: } \left(\frac{\gamma_L r}{2}\right)^2 < \gamma_L \gamma_c (r - 1) \]

relaxation oscillations with an attenuation factor:

\[ n(t) = n_s + n_1 e^{-\gamma_{relax} t} \cos(\omega_{relax} t) \]

\[ \gamma_{relax} = \frac{r \gamma_L}{2} \]

\[ \gamma_c \gg r \gamma_L : \quad \omega_{relax} \approx \sqrt{\frac{r - 1}{\tau_L \tau_c}} = \sqrt{\frac{1}{\tau_{stim} \tau_c}} \]
Relaxation oscillations in the time domain and frequency domain (microwave spectrum analyzer - not optical frequencies!)

\[ n(t) = n_s + n_1 e^{-\gamma_{relax}t} \cos(\omega_{relax}t) \]

\[ \gamma_{relax} = \frac{r\gamma_L}{2} \]

\[ \gamma_c \gg r\gamma_L : \quad \omega_{relax} \approx \sqrt{\frac{r-1}{\tau_L} \frac{1}{\tau_c}} = \sqrt{\frac{1}{\tau_{stim}} \frac{1}{\tau_c}} \]
Relaxation oscillations

Example:
diode-pumped cw Nd:YLF laser

\[ f_{\text{relax}} = 116 \text{ kHz} \]
Measurement of small signal gain

\[ g = \frac{g_0}{1 + 2I / I_{sat}} \]

\[ f_{\text{relax}} \approx \frac{1}{2\pi} \sqrt{\gamma_L \gamma_c (r - 1)} = \frac{1}{2\pi} \sqrt{\frac{r - 1}{\tau_L \tau_c}} = \frac{1}{2\pi} \sqrt{\frac{g_0}{l} - 1} = \frac{1}{2\pi} \sqrt{\frac{2g_0 - 2l}{\tau_L T_R}} \]

\[ \tau_c = \frac{T_R}{2l} \quad 2g_0 \approx 4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l \]

\[ r = \frac{g_0}{l} \quad G_0 \approx e^{2g_0} = \exp \left( 4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l \right) \]
Measurement of small signal gain

Measured relaxation oscillation frequency for different output couplers, for $g_0 \gg l$

$$f_{\text{relax}} \approx \frac{1}{2\pi} \sqrt{\frac{2g_0 - 2l}{\tau_L T_R}} \quad (g_0 \gg l) \approx \frac{1}{2\pi} \sqrt{\frac{2g_0}{\tau_L T_R}}$$

relaxation oscillations independent of output coupler

$$2g_0 \approx 4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l$$

$$G_0 \approx e^{2g_0} = \exp\left(4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l\right)$$

Example: diode-pumped Nd:YLF laser