9. Star Formation in Galaxies

Galaxies are mostly observed through stars - any theory of galaxy formation has to address the question of how stars form.

- Baryonic gas in halos can cool within a time that is shorter than the age of the halo.
- Loses pressure support and flows to the center of the potential well - density increases.
- When density of cooling gas > dark matter density (in the central part of the halo) - gas becomes self-gravitating and collapses under its own gravity.

If cooling is efficient - self-gravitating gas is unstable and collapses catastrophically - formation of dense, cold gas clouds within which star formation can occur.

- What are the main processes that drive star formation?
- What is the rate at which stars form in a gas cloud?
- What mass fraction of a gas cloud can be converted into stars?
- What is the initial mass function (IMF), which describes the mass distribution of stars at birth?
9. Star Formation in Galaxies

Star formation history of a galaxy (mass in stars formed per unit time) +

IMF (types of stars formed) +

Stellar evolution models (describe how stars of different masses evolve with time)
- predict the luminosity and color of the galaxy as a function of time

Understanding SF is challenging:
typical mass of gas: $\sim 10^{11} M_\odot$, typical density of gas $\sim 10^{-24} \text{g cm}^{-3}$
typical mass of a star $\sim 1 M_\odot$, typical density of a star $\sim 1 \text{g cm}^{-3}$.

Formation of an individual stars spans 11 orders of magnitude in mass and 24 orders of magnitude in density.

However, for our purpose (galaxy formation and evolution) - properties of SF in a volume comparable to a galaxy

Question: how global properties of star formation depend on global properties of the gas: mass, density, temperature and chemical composition.

time and spatial scales relevant for SF much smaller than galaxy evolution - star formation can be described with some statistical relations between the star formation rate and the global properties of the star-forming gas.

Can be calibrated empirically, and may give some insight into the physical processes underlying star formation.
9.1 Giant Molecular Clouds: the Sites of Star Formation

Star formation is slow in present-day spiral galaxies, such as the Milky Way.

Time scale for star formation, the gas consumption time scale, is defined as the ratio between the gas mass available and the rate at which gas turns into stars, \( \tau \equiv \frac{M_{\text{gas}}}{M_*} \).

**Normal Spirals:**

- Spiral galaxies \( \tau \approx (1-5) \times 10^9 \text{ yr} \), \(~10\times\text{longer than the relevant dynamical time scales of the gas.}\)
- For the Milky Way (we have great detail), SF is always associated with giant molecular clouds (GMCs) embedded in the interstellar medium (ISM).
- The GMCs spatial correlate with young star clusters (ages \(< 10^7 \text{ yr}\)) but not with older star clusters: **typical lifetime of a GMC is \(~10^7 \text{ yr, much shorter than the age of a galaxy.}\**
- Rare to find GMCs not forming stars - star formation in a GMC starts when cloud forms.

**Rate of large-scale star formation is essentially proportional to the formation rate of GMCs.**
Star Bursts:

Star formation time scales $\sim 10^7 - 10^8$ yr, comparable to the dynamical times of the cold gas in these galaxies much shorter than those in normal disk galaxies

Observations of CO emission associated with large amounts of molecular gas ($10^8 - 10^{10} M_\odot$) confined to small volumes with sizes $<2$ kpc.

Molecular gas densities are 10-100 times higher than in the inner 1 kpc region of the Milky Way.

Although starbursts have very different star-formation properties, SF also driven by the availability of dense molecular clouds.

It is now generally believed that GMCs are the principal sites for SF, and SFR is determined by its ability to form GMCs. Star formation in galaxies can be divided into: ‘microphysics’ and ‘macrophysics’.

- the formation of individual stars in the dense regions of GMCs
- the formation and structure of GMCs in galaxies
9.1.1 Structure and Dynamical State of GMCs

Information on the structure of the interstellar medium (ISM) is available only for our own Galaxy. Since spatial and time scales for SF are typically much smaller than those of a galaxy - likely the relationship between SF and the fine structure of the ISM does not vary much from galaxy to galaxy.

ISM of Milky Way is contained in a flattened disk-like structure of predominantly cold gas. Dominated by neutral hydrogen, but \( \sim 20\% \) molecular, mainly \( \text{H}_2 \) + 1\% dust grains. Molecular fraction increases with gas density: 0 in the diffuse ISM, almost 100\% in dense regions, near Galactic center.

The state of a GMC can be described qualitatively with the aid of the virial theorem:

\[
\frac{1}{2} \dot{I} = 2K + W - \Sigma + B.
\]

Note that \( I \) is the moment of inertia, \( K \) is the kinetic energy of the system, and \( \Sigma \) is the work done by the external pressure. The potential energy, \( W \), is equal to the gravitational energy of the system and \( B \) is the magnetic energy. A system whose kinetic energy is much larger (smaller) than the negative value of its potential energy tends to expand (contract). For a static system, \( \frac{d^2 I}{dt^2} = 0 \), and

\[
2K + W + \Sigma = 0,
\]

(5.134)

giving a constraint on the global properties of any system in a static state. Finally, for \( \Sigma = 0 \) we have

\[
E = -K = \frac{W}{2},
\]

(5.135)

where \( E = K + W \) is the total energy.


9. Star Formation in Galaxies

\[
\frac{1}{2} \ddot{I} = 2K + W - \Sigma + B.
\] (9.1)

A cloud is said to be virialized if \( \ddot{I} = 0 \), expanding if \( \ddot{I} > 0 \), and contracting if \( \ddot{I} < 0 \).

Quantities on the right-hand side can be used to set relevant scales that describe the dynamic state of a cloud.

1. gravitational mass scale:

\[
M_G = \frac{\sigma^2}{(G^3 \rho)^{1/2}},
\] (9.2)

\( \sigma^2 \) is the 1-dimensional mean square velocity [including both the thermal component, \( \sigma^2 = k T / \mu m \), and the non-thermal (turbulent) component, \( \sigma^2 \)] and \( \rho \) is the mean density of the cloud.

For clouds with masses \( M > M_G \), thermal and turbulent motions are not capable of preventing gravitational collapse.
A cloud is said to be virialized if $\dddot{I} = 0$, expanding if $\dddot{I} > 0$, and contracting if $\dddot{I} < 0$. Quantities on the right-hand side can be used to set relevant scales that describe the dynamic state of a cloud.

2. maximum mass for an isothermal sphere embedded in a pressurized medium in hydrostatic equilibrium (in the absence of magnetic fields and turbulence), the Bonnor-Ebert mass:

\[
M_{\text{BE}} = 1.182 \frac{\sigma_{\text{th}}^3}{(G^3 \rho_0)^{1/2}} = 1.182 \frac{\sigma_{\text{th}}^4}{(G^3 P_{\text{th},0})^{1/2}},
\]

(9.3)

where $P_{\text{th},0} = \rho_0 \sigma^2$ is the surface pressure (Bonnor, 1956; Ebert, 1957). Clouds with $M > M_{\text{BE}}$ cannot be prevented from collapsing by thermal pressure alone.
A cloud is said to be virialized if $\ddot{r} = 0$, expanding if $\ddot{r} > 0$, and contracting if $\ddot{r} < 0$.

Quantities on the right-hand side can be used to set relevant scales that describe the dynamic state of a cloud.

3. $B = |W|$ defines a mass:

\[ M_\Phi = c_\Phi \frac{\Phi}{G^{1/2}}, \quad (9.4) \]

where $\Phi$ is the magnetic flux in the cloud, and $c_\Phi$ is a constant which depends on the configuration of the magnetic field. Magnetic fields alone cannot prevent gravitational collapse in clouds with $M > M_\Phi$. Clouds with $M > M_\Phi$ ($M < M_\Phi$) are said to be magnetically supercritical (subcritical).
9. Star Formation in Galaxies

GMC properties:

GMCs usually refer to molecular gas clouds with masses $10^4 - 10^6 \, M_\odot$.

Gravitationally bound (i.e. $M > M_\odot$), but not in global gravitational collapse (i.e. $\ddot{I}$ is not very negative).

Show high levels of internal turbulence. Typically embedded in more massive HI ‘supercclouds’, which themselves are gravitationally bound and have high levels of internal turbulence.

GMCs are highly inhomogeneous, consisting of smaller clumps with masses $\sim 10^3 - 10^4 \, M_\odot$, densities $\sim 10^2 - 10^4$ protons cm$^{-3}$, and temperatures $\sim 10 - 100$K.

The thermal Jeans masses corresponding to such temperatures and densities are only a few $M_\odot$, much smaller than the typical clump mass. This suggests that the clumps must be supported by non-thermal pressure.

Observations show substantial turbulent motion and dynamically significant magnetic fields.

Many clumps appear gravitationally unbound, but most of the mass is concentrated in the most massive clumps that are gravitationally bound.
Properties of Clumps:

Molecular clumps themselves are also highly inhomogeneous, containing dense cores with densities $> 10^{-20} \text{ g cm}^{-3}$ and temperatures $\sim 10 \text{ K}$.

**Low mass cores:** appear to be supported by thermal motion, rather than by turbulent motion or magnetic pressure. Mass comparable to Bonnor-Ebert indicating that these cores will contract further in the presence of effective cooling.

**Most massive cores:** have significant levels of internal turbulence and appear to be magnetically critical or supercritical. However, may also collapse once the turbulence energy has been dissipated.

**Cores in molecular clumps are the potential sites within which individual stars form. Indeed, young stars in the Milky Way are typically found to be associated with dense cores embedded in large molecular clouds.**
9. Star Formation in Galaxies

GMCs obey a number of relatively well-defined scaling relations. Masses and (turbulent) velocity widths of individual clouds and clumps are observed to be tightly correlated with their radii:

\[ M \propto R^2, \quad \Delta v \propto R^{1/2} \propto \rho^{-1/2} \quad (9.5) \]

reflection of the underlying power spectrum scaling expected for supersonic turbulence. Break down for clouds with low masses, in which turbulence becomes unimportant.

The observed mass distribution of GMCs is found to be a power law over a wide range in masses, with a relatively sharp cutoff:

\[ \frac{dN}{d\ln M} = N_0 \left( \frac{M}{M_0} \right)^{-\zeta} \quad (M \lesssim M_u), \quad (9.6) \]

where \( \zeta \sim 0.5 \). Upper limit, \( M_u \sim 5 \times 10^6 M_\odot \), is believed to be set by self-gravitating instabilities in the ISM.

Clump mass function is similar - also formed by turbulent processes within gravitationally-bound systems.

Mass distribution of cores is significantly steeper, with a power law index \( \zeta \sim 1.4 \) at \( M > \sim 1 M_\odot \). In addition, there are indications that the core mass function starts to decline below a mass \( M < \sim 1 M_\odot \). This core-mass distribution has the same characteristics as the IMF of young stars, suggesting again that dense molecular cores are directly linked to the formation of individual stars.
9.1.2 GMC Formation in a Supersonic Turbulent Medium

Turbulence is ubiquitous in the ISM over a large range of scales:
from that comparable to the host galaxy all the way down to that of massive cores of the molecular clouds.

Based on these observations, and modelling, large-scale, supersonic turbulence is assumed to govern the macrophysics of SF in galaxies.

How do molecular clouds form in a supersonic, turbulent medium, and what governs their structure?

What processes drive and maintain the turbulence?
Destruction of Molecules

The most abundant molecule in interstellar space, by far, is molecular hydrogen (H$_2$). Its abundance is set by its formation and destruction processes.

The main destruction process is photodissociation:
Absorption into a repulsive excited state leads to dissociation (direct photodissociation). However, for H$_2$ there are no allowed transitions to repulsive states from the ground state at energies less than 13.6eV, so that direct photodissociation is not possible.

Photodissociation of H$_2$ can occur via an indirect, two-step process, called spontaneous radiative dissociation.

The photodissociation rate of H$_2$ is $k_{pd} \approx 5 \times 10^{-11}s^{-1}$ (corresponding to a typical lifetime of a H$_2$ molecule of ~ 600 yr) in the unattenuated interstellar radiation field.

Inside an interstellar cloud - less radiation field due to:
- continuum attenuation by dust grains
- line attenuation of H$_2$ itself (self-shielding): H$_2$ molecules at the edge of the cloud absorb all photons at certain wavelengths, so that molecules deeper in the cloud ‘see’ virtually no photons at all, and are not dissociated.
Formation of Molecules

Main formation mechanism of $\text{H}_2$ in the ISM is:

recombination of pairs of adsorbed hydrogen atoms on the surfaces of dust grains.

This process is more efficient than $\text{H}_2$-formation via gas phase reactions by many orders of magnitude.

Gas in thermal equilibrium has three thermally stable phases, at temperatures of $\sim 10^6 \text{K}$, $\sim 10^4 \text{K}$ and $\sim 100 \text{K}$.

Only in the cold phase ($T \sim 100 \text{K}$) - density high enough to form molecular hydrogen in significant abundances.

Without a cool phase of interstellar gas, normal compressions do not assemble clouds dense enough and at high enough column density to shield themselves from UV starlight (compression by filament waves, turbulence, large scale instabilities, supernovae, stellar winds)

No shielding - no molecules - no further cooling to a state where self-gravity is strong.

Prerequisite for the formation of molecular clouds – and thus for the formation of stars – is the presence of gas in thermal equilibrium in the cold phase.
Formation of Molecules

In the presence of dust grains, the time scale for the formation of molecular hydrogen (H$_2$) is about $t_{\text{form}} \sim 1.5 \times 10^9 n^{-1}$ yr, where $n$ is the number density of gas particles.

The rate of molecule formation is enhanced in dense regions - the formation of molecules requires the compression of the ISM to a sufficiently high level.

The H$_2$/HI mass ratio increases roughly linearly with external pressure in the ISM.

Numerical simulations show that shock compression in a self-gravitating, turbulent gas can produce gas densities sufficiently high to form H$_2$ on a time scale of about $10^6$ years, suggesting that a supersonic, turbulent ISM is the ideal environment for the formation of molecules.
Formation of Molecular Clouds

In a self-gravitating turbulent medium, the collapse of gas clouds can be affected by turbulence in two different ways.

1. Turbulent motion increases the velocity dispersion of the gas - delay or suppress gravitational collapse.

2. Gas can be compressed by shocks produced by the supersonic flow - promote collapse by increasing the gas density.

For an isothermal gas with density $\rho$, the post-shock density is $\rho' = M^2 \rho$ (assuming strong shocks, the Mach number of the shock, $M$, $\gg 1$. In this case, the gravitational mass scale decreases by a factor of $M$.

Turbulent motion and shock compression combine to change the gravitational mass scale to:

$$M \propto \left(\sigma_{\text{th}}^2 + \sigma_{\text{nt}}^2\right)^{3/2} / \left(M \rho^{1/2}\right),$$

where $\sigma_{\text{th}}$ and $\sigma_{\text{nt}}$ are the thermal and turbulent velocity dispersions.

On scales comparable to the driving scale, the typical shock velocity is of the same order as $\sigma_{\text{nt}}$ and is much larger than $\sigma_{\text{th}}$, and so $M \propto \sigma_{\text{nt}}^2$. In this case, the net effect of turbulence is to increase $M$, i.e. to suppress gravitational collapse.

Since $\sigma_{\text{nt}}$ is expected to decrease with scale, small, dense clouds may have $\sigma_{\text{nt}}$ much smaller than the shock velocity. In this case, the net effect of turbulence is to promote gravitational collapse.

Thus, turbulence may suppress gravitational collapse globally, but can promote local gravitational collapse.

Question is whether shock velocity is comparable to turbulent velocity.
Formation of Molecular Clouds

Numerical models of highly compressible, self-gravitating turbulence show that clouds can form over large ranges in both mass and density.

For an isothermal gas, the volume-weighted probability distribution function (i.e. the fraction of volume as a function of gas density) is roughly lognormal:

$$ p(lnx)dlnx = \frac{1}{\sqrt{2\pi}\sigma_{lnx}^2} \exp \left[ -\frac{(lnx - \langle lnx \rangle)^2}{2\sigma_{lnx}^2} \right] dlnx, $$  \hspace{1cm} (9.9)

Massive, dense regions of this distribution become gravitationally bound and form GMCs.

Those that are not gravitationally bound may be redispersed by turbulence.

The densest regions may become self-gravitating cores and collapse further to form stars.

Efficient SF only occurs in a fraction of the gas clouds that gets compressed sufficiently to collapse, while the overall star formation efficiency may remain low because of the support by turbulence.

This may explain why the global star formation efficiency in a galaxy is in general quite low.
Impact of Magnetic Fields

The ISM is strongly magnetized.

Magnetic pressure can support molecular clouds and prevent gravitationally unstable regions from collapsing.

- Numerical simulations show that compression by supersonic turbulence is not capable of making a magnetically subcritical cloud collapse.

- On the other hand, magnetic fields are not able to prevent a supercritical clouds from collapsing although they can delay the collapse.

If the magnetic field is tightly coupled to the gas cloud, the magnetic flux $\Phi$, as well as the magnetic mass scale, $M_\Phi$, remains constant as the cloud collapses. $M_\Phi \approx 10^4 M_\odot$, much larger than the typical mass of molecular cloud cores.

Thus, in order for magnetized gas clouds to fragment and collapse further to form dense cores, their magnetic fluxes have to be reduced. A gas cloud may lose magnetic flux through a process called ambipolar diffusion.

For a cloud containing both ionized and neutral particles, the neutral population is only coupled to the $B$-field through collisions with ions. If ionized fraction goes down, collisions between neutral and ions are insufficient to couple the neutral gas with the magnetic field - neutral particles diffuse through the magnetic field, reducing the magnetic flux.

Still, ambipolar diffusion timescale is much longer than the dynamical timescale of the cloud. Thus, even with ambipolar diffusion, a magnetically subcritical cloud may still not be able to collapse before it is destroyed. On the other hand, magnetically critical or supercritical clouds may lose magnetic flux through ambipolar diffusion as its density and neutral fraction increase during the collapse. This is consistent with the observational fact that GMCs and their sub-structures are supercritical with respect to their magnetic fields.
Disruption of Molecular Clouds

Lifetime of a GMC is \( \sim 10^7 \) yr, much shorter than the typical age of a galaxy.

GMCs are believed to be destroyed by the energy feedback from massive (OB) stars that form in them.

Possible mechanisms include:
- the formation and expansion of HII regions (due to the ionization and heating by UV photons from massive stars)
- stellar winds
- supernova explosions.

Energy outputs are comparable over the lifetime of a young star cluster, but detailed evaluations of the overall efficiency suggest that the dominant destruction mechanism is probably photo-evaporation by HII regions.

May also play an important role in keeping a GMC turbulent before destroying it,
The cold gas in a star-forming galaxy is assembled through the cooling and accretion of gas in a dark matter halo. Accreted gas contains angular momentum - settles into a disk-like structure supported by angular momentum.

The accreted gas is expected to be clumpy – especially in mergers, where large chunks of gas are being added into the galaxy – and to generate high levels of turbulence in the gas.

Cosmological simulations of galaxy disk formation show that gravitational instabilities + shear flows + tidal interactions - generate high levels of turbulence in the ISM on large scales.

The large-scale turbulence may then cascade down in scales before it is dissipated, producing a supersonic turbulent medium similar to the ISM.

However, under molecular cloud conditions turbulence decays in a free-fall time of a cloud - processes to maintain the turbulence.

On global scales: collapses induced by gravitational and magnetodynamical instabilities, supernova explosions
small scales: energy from protostellar outflows, stellar winds, ionizing radiations
Disk Gravitational Instability

The cold gas in a spiral galaxy is distributed in a disk-like structure supported by rotation. Disk can become locally unstable when its surface density is too high - collapse of patches - turbulence driving

For local perturbations with axisymmetry, the dispersion relation (between the circular frequency $\omega$ and wave number $k$) is

$$\omega^2 = \kappa^2 - 2\pi G \Sigma_{\text{gas}} |k| + k^2 c_s^2, \quad (9.10)$$

where $\Sigma_{\text{gas}}$ is the local surface density of the disk, $c_s$ is the sound speed of the gas,

$$\kappa^2 \equiv 2 \frac{V_c}{R} \left( \frac{V_c}{R} + \frac{dV_c}{dR} \right) \quad (9.11)$$

is the epicyclic frequency of the disk, and $V_c(R)$ is the circular velocity at radius $R$. 
Disk Gravitational Instability

According to Eq. (9.10), the parameter that governs the instability of a rotation-supported gaseous disk is Toomre’s $Q$ parameter:

$$Q \equiv \frac{\kappa c_s}{\pi G \Sigma_{\text{gas}}} ,$$  \hspace{1cm} (9.12)

and a disk is locally unstable if $Q < 1$. This defines a critical surface density,

$$\Sigma_{\text{crit}} = \frac{\kappa c_s}{\pi G} ,$$  \hspace{1cm} (9.13)

above which a disk is locally unstable.

For an unstable disk with $Q < 1$, the time scale for perturbation growth can be obtained from the dispersion relation. To do this, let us define a Jeans wavelength, $\lambda_j = c^2_s / G \Sigma_{\text{gas}}$, which is the critical wavelength for collapse in the absence of differential rotation.

Since $Q$ is small for such disks, we may neglect the $\kappa^2$-term in the dispersion relation (9.10).

Thus, at a typical wavelength $\lambda \sim 2\lambda_j$, the time scale for instability growth is

$$\tau \sim \frac{1}{i\omega} \sim \frac{c_s}{\pi G \Sigma_{\text{gas}}} \sim 3 \times 10^7 \text{ yr} \left( \frac{c_s}{10 \text{ km s}^{-1}} \right) \left( \frac{\Sigma_{\text{gas}}}{30 M_\odot \text{ pc}^{-2}} \right)^{-1} .$$  \hspace{1cm} (9.14)
Since the gas velocity dispersion in galaxy disks is roughly constant ($c_s \sim 10\text{km\,s}^{-1}$), the formation rate of gas clouds due to disk instability depends mainly on the gas surface density.

For $\Sigma_{\text{gas}} = 30\, M_\odot\, \text{pc}^{-2}$, the above time scale is about $3 \times 10^7\, \text{yr}$, which is comparable to the lifetime of GMCs. The Jeans length corresponds to a mass

$$M_J \equiv \frac{c_s^4}{G^2 \Sigma_{\text{gas}}} \sim 2 \times 10^7\, M_\odot \left(\frac{c_s}{10\, \text{km\,s}^{-1}}\right) 4 \left(\frac{\Sigma_{\text{gas}}}{30\, M_\odot\, \text{pc}^{-2}}\right)^{-1},$$

which is comparable to the mass of a supercloud.

Thus, gravitational instability may be responsible for triggering the formation of superclouds, the gravitational collapse of which may produce a turbulent medium within which GMCs form.
Parker Instability

Another process that can trigger the growth and formation of massive clouds from the diffuse ISM is the Parker instability (Parker, 1966).

Also known as the magnetic buoyancy instability:

Consider a uniform disk of gas which is coupled to a magnetic field that is parallel to the disk.

Suppose that the disk is gravitationally stratified in the vertical direction, and is in dynamical equilibrium under the balance of gravity and pressure (thermal and magnetic). Now consider a small perturbation which causes the field lines to rise in certain parts of the disk and to sink in others. Because of gravity, the gas loaded onto the field lines tends to slide off the peaks and sink into the valleys.

The increase of mass load in the valleys makes them sink further, while the magnetic pressure causes the peaks to rise as their mass load decreases.

Consequently, the initial perturbation is amplified, causing the production of density fluctuations in an initially uniform disk. The characteristic scale for the Parker instability is about 1 - 2 kpc for Milky-way. Not enough for to drive collapse on large scales but may trigger gravitational instability in a marginally unstable disk.
Thermal Instability

The thermal instability is one of the basic elements in some of the most influential models of the ISM.

Naturally leads to the standard picture of a three-phase medium, in which a hot phase with temperature $T \sim 10^6 \text{K}$, a warm phase with $T \sim 10^4 \text{K}$, and a cold phase with $T \sim 10^2 \text{K}$, co-exist in rough pressure equilibrium.

Because of the large range in temperatures, the gas density in such a medium can cover a large range, consistent with observations of the ISM.

However, the highest density expected from the pressure of the ISM is still lower than the typical density of a GMC, suggesting that GMCs cannot be created directly from thermal instability.

However, when thermal instability occurs, it may trigger gravitational instability as the cooling gas is compressed.
Spiral Arms

As mentioned above, density enhancements (and hence molecular clouds) can form in a galaxy disk as a result of local gravitational instability. A galactic disk may also form grand-design spiral arms - spiral density waves.

Interstellar gas swept by a spiral density wave is compressed, which promotes gravitational instability. This gives a natural explanation for the fact that prominent spiral arms are observed to be associated with enhanced star formation.

Many disk galaxies reveal small, flocculant arm fragments that have a limited radial and angular extent, unlike the grand-design spirals.

Arm fragments are simply local density enhancements (probably created due to gravitational instability of the disk) that have been sheared into spiral structures due to the differential rotation of the disk.

Thus, whereas the star formation in a grand-design spiral is triggered by the spiral density wave, in the case of arm fragments the star formation is triggered by a local density enhancement subsequently sheared into a spiral pattern.
Gas in galaxies can also be disturbed by interactions with nearby galaxies or by merging with another galaxy.

Gas can be compressed by shocks as two galaxies merge, and be energized as the energy in the merger is converted into turbulent motion.

The most dramatic effect is to cause the gas in a galaxy to rapidly lose angular momentum and to flow towards the center of the galaxy.

In simulations of mergers of gaseous disk galaxies of comparable masses, an exceedingly high gas surface density, $\sim 10^4$ M$_\odot$ pc$^{-2}$, can be produced in the central region with a size of only about $\sim 100$ pc, making the gas capable of forming large quantities of molecular clouds to support a high star formation rate.

Indeed, starburst galaxies with star formation rates as high as $10 - 100$ M$_\odot$ yr$^{-1}$ are commonly observed to be associated with gas concentrations in the central regions of interacting or merging systems.
Induced Star Formation

The large-scale star formation driven by gravitational instability, spiral density waves, and galaxy interactions is usually referred to as ‘spontaneous’ star formation, because the resulting star formation activities in different parts of the galaxy are independent of each other.

Possible that initial conditions for large-scale star formation are generated by star-formation activity itself.

Star formation might spread as a wave of gravitational collapse — produced by the compression of gas associated with supernova shocks, stellar winds or ionization fronts — propagating through the ISM.

Such scenarios are usually called ‘induced’ star formation, because here star formation in one place is induced by the feedback of star formation activity in other places.

Environments for SF, density and velocity structure of the ISM, produced by stars.

Even if the large-scale star formation environments are produced by the other processes described above, compression by energy feedback from nearby stars may play an important role in triggering star formation in pre-existing clouds.
9.3 Star Formation Laws

SFR is characterised as: mass in stars formed per unit time per unit area; $\Sigma^\prime_* = M_*/\text{area}$.

Ideally this would from first principles as a function of the relevant physical conditions of the ISM (e.g., density, temperature, and metallicity of the gas, radiation field, magnetic field strength, etc.).

Still impossible. Still not clear which ISM properties is most important for governing the local star formation rate.

To make progress use observations to determine empirical star formation laws, i.e. empirical scaling relations between $\Sigma^\prime_*$ and ISM properties.

All empirical star formation laws suffer from systematic uncertainties.

dust extinction, uncertainties regarding the initial mass function and molecular gas densities.

The surface density of molecular hydrogen, $\Sigma_\text{H}_2$, is typically determined from the integrated CO intensity, $I_{\text{CO}}$, using a constant CO-to-H$_2$ conversion factor, $X_{\text{CO}} \equiv \frac{N(\text{H}_2)}{I_{\text{CO}}}$. Although it is common practice to assume that $X_{\text{CO}}$ is constant $\sim 2 \times 10^{20} \text{cm}^{-2}(\text{K}\text{km s}^{-1})^{-1}$, there are strong indications that it may actually be a function of metallicity and radiation field.

Another potential concern is that CO may be a more direct tracer of star formation rather than of H$_2$. This could come about if the energy input from young stars is responsible for exciting the CO emission.
9.3.1 The Kennicutt-Schmidt law

The most obvious requirement for star formation is the presence of gas. A powerlaw relation of the form:

$$\dot{\Sigma}_* \propto \Sigma_{\text{gas}}^N,$$

is known as the Schmidt law of star formation, after Schmidt (1959) who found that such a relation with $N \sim 2$ could adequately account for the observed distributions of HI and stars perpendicular to the Galactic plane.

Numerous observational studies have confirmed with $N$ typically in the range from 1 to 2.

Kennicutt (1998) extended such an analysis by also including starburst galaxies, and found that these systems follow the same power-law relation between $\Sigma^*$ and the total surface densities of gas (atomic plus molecular, again averaged over the entire star forming disk) as normal spiral galaxies.

Schmidt law gives a good parameterization over a large range of surface densities, from the most gas-poor spiral disks to the cores of the most luminous starburst galaxies. The best fit to the observational data gives:

$$\dot{\Sigma}_* = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_{\text{gas}}}{M_\odot \text{pc}^{-2}} \right)^{1.4 \pm 0.15} \text{M}_\odot \text{yr}^{-1} \text{pc}^{-2},$$

(9.19)
9.3.1 The Kennicutt-Schmidt law

\[ \dot{\Sigma}_* = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_{\text{gas}}}{M_\odot \text{pc}^{-2}} \right)^{1.4 \pm 0.15} \text{M}_\odot \text{yr}^{-1} \text{kpc}^{-2}, \]  

(9.19)

Kennicutt-Schmidt law - interpreted as star formation rate controlled by the self-gravity of the gas.

SFR will be proportional to: (gas mass / time-scale for gravitational collapse). For a gas cloud with mean density \( \rho \), the free-fall time is:

\[ t_{\text{ff}} = \left( \frac{3\pi}{32G\rho} \right)^{1/2}, \]  

(9.20)

which is the time for a pressure-free, spherical cloud to collapse to a point under its self-gravity. This is the shortest time within which a self-gravitating cloud can be completely converted into stars. Hence,

\[ \dot{\rho}_* = \epsilon_{\text{SF}} \frac{\rho_{\text{gas}}}{t_{\text{ff}}} \propto \rho_{\text{gas}}^{1.5}, \]  

(9.21)

where \( \epsilon_{\text{sf}} \) is the ratio of the free-fall time of the gas divided by the gas consumption time, which is a measure for the efficiency of star formation.

If all galaxies have \( \sim \) same scale-height, \( \Sigma' \propto \Sigma^{1.5} \), in good agreements with the empirical Kennicutt-Schmidt law.
9.3.1 The Kennicutt-Schmidt law

\[ \dot{\rho}_* = \varepsilon_{\text{SF}} \frac{\rho_{\text{gas}}}{t_{\text{ff}}} \propto \rho_{\text{gas}}^{1.5}, \quad (9.21) \]

However, the efficiency parameter \( \varepsilon_{\text{SF}} \ll 1 \): self-gravity of the gas cannot be the entire story.

Either the star formation timescale really is \( \tau_{\text{ff}} \), but only a small fraction \( \varepsilon_{\text{SF}} \) of the gas participates in star formation, or the actual star formation time scale is \( t_{\text{ff}}/\varepsilon_{\text{SF}} \). In either case, some additional physics is required.

Kennicutt (1998) also showed that the data reveals an equally tight correlation between \( \Sigma^* \) and \( \Sigma_{\text{gas}}/\tau_{\text{dyn}} \) (shown in the right-hand panel of Fig.9.3). Here \( \tau_{\text{dyn}} \) is defined somewhat arbitrarily as \( 2\pi R/V_{\text{rot}}(R) = 2\pi R/\Omega(R) \), the orbital time at the outer radius \( R \) of the star forming disk. The best fit obtained by Kennicutt (1998) is

\[ \dot{\Sigma}_* \approx 0.017\Sigma_{\text{gas}}\Omega, \quad (9.22) \]

which implies that about 10 percent of the available gas is consumed by star formation per orbital time.

A star formation law of the form \( \tau_{\text{SF}} = \Sigma_{\text{gas}}/\Sigma^* t_{\text{dyn}} \) follows naturally from models in which star formation is self-regulated such that disks maintain \( Q \approx 1 \). In that case the time-scale for instability growth is \( \tau \propto \kappa^{-1} \). Since disk galaxies have flat rotation curves \( \kappa \propto \Omega \), so that a star formation law of the form of Eq. (9.22) emerges as long as the star formation time scale is proportional to the instability time scale.

However, also true for star formation rate governed by the rate of collisions between gravitationally bound clouds, or triggered by spiral arms.
9.3.1 The Kennicutt-Schmidt law

\[ \log (\Sigma_{\text{SFR}} [M_{\odot} \text{yr}^{-1} \text{kpc}^{-2}]) = \alpha \log (\Sigma_{\text{gas}} [M_{\odot} \text{pc}^{-2}]) + \beta \]

\[ \log (\Sigma_{\text{gas}} [M_{\odot} \text{pc}^{-2}]) = \gamma \log (\Sigma_{\text{gas}} / \tau_{\text{dyn}}) + \delta \]
9.3.2 Local Star Formation Laws

Two empirical star formation laws that fit the data + multiple models that reproduce these laws - no insight

Problem - SF laws based on quantities averaged over the entire star forming disk.

We must look into local star formation laws, which correlate star formation rates, gas densities and orbital time scales that have been measured on much smaller scales.

High resolution data for $\Sigma^\star$, $\Sigma_{\text{gas}}$ and $\Omega$ as function of galactocentric radius - although there is a roughly linear trend between SFE and orbital frequency among galaxies no strong relation exists within galaxies. Orbital time not important.

Fitting individual Schmidt laws to individual galaxies on a pixel-by-pixel basis, yields $N = d\log \Sigma^\star/d\log \Sigma_{\text{gas}}$ that cover the entire range from $1 < N < 3$. A single power-law is actually a poor fit to the local relation between $\Sigma^\star$ and $\Sigma_{\text{gas}}$.

The $\Sigma^\star$-$\Sigma_{\text{gas}}$ relation reveals a pronounced break at low surface densities of the gas.
9.3.2 Local Star Formation Laws

First focus on what regulates \( \Sigma^\star \) in regions where star formation is prevalent (above drop).

Look at atomic and molecular gas separately.

left-hand - HI does not correlate with SF

Interestingly - surface density of atomic hydrogen seems to saturate at \( \Sigma_{\text{HI}} \sim 10 M_\odot \text{pc}^{-2} \)

-maybe HI to convert to \( H_2 \) at higher column densities due to self-shielding.
Well defined Schmidt law for the molecular gas:

\[
  \dot{\Sigma}_* = (8 + 4) \times 10^{-3} \left( \frac{\Sigma_{\text{H}_2}}{M_\odot \text{pc}^{-2}} \right)^{1.0 \pm 0.2} M_\odot \text{yr}^{-1} \text{kpc}^{-2},
\]

power-law index close to unity - implies a roughly constant SFE of \((5.3 \pm 2.5) \times 10^{-10} \text{yr}^{-1}\), equivalent to a molecular gas consumption time of \(\sim 2 \times 10^9 \text{yr}\).

Schmidt law between \(\Sigma^*\) and the total gas surface density is a mixture of two ‘laws’; the first governing the transformation of atomic to molecular gas, the second describing the formation of stars from molecular gas.
In the central regions of disk galaxies, where the gas is largely molecular, $R_{\text{mol}}$ is large resulting in $N \sim 1$.

In the outer regions, the molecular fraction decreases (due to a decline in $P_{\text{ext}}$), causing $N$ to increase towards $\sim 1.85$. 

9.3.2 Local Star Formation Laws
9.3.2 Local Star Formation Laws

Since SF only in GMCs and SFE of H$_2$ is constant: SF in GMCs is constant (no dependence on their formation mechanism or environment).

But, since the mass function of GMCs depend on environment, the SFE of GMCs cannot be a function of cloud mass.

The typical mean densities within GMCs are $n_H \sim 100 \text{cm}^{-3}$, implying a typical free-fall time of $\tau_{\text{ff,GMC}} \sim 4 \times 10^6 \text{yr}$. Compared to the empirical molecular gas consumption time, we infer that

$$\epsilon_{\text{SF,GMC}} \equiv \frac{\tau_{\text{ff,GMC}}}{\tau_{\text{SF,GMC}}} \sim 0.002$$

(9.26)

Key question: why these efficiencies are so low.

Both turbulence and magnetic fields can prevent or delay the collapse of a GMC.

However, on GMC scales, magnetic field strength seems insufficient to prevent rapid collapse, at least for GMCs in the Milky Way.

Hence, it is generally believed that the low value of $\epsilon_{\text{SF,GMC}}$ is an outcome of supersonic turbulence.

Mass in a cloud supported against collapse by supersonic turbulence should be in structures that are insufficiently dense to collapse. SF only occurs in sufficiently dense substructures that collapse before being redispersed by turbulence.
The properties of stellar populations depend on what kind of stars are being formed. Since the properties and evolution of individual stars are primarily determined by their masses, the initial mass function (IMF), is very important!

The IMF, $\phi(m)$, is defined so that $\phi(m)dm$ is the relative number of stars born with masses in the range $m \pm dm/2$. The IMF is assumed to be a continuous function, and we adopt the normalization:

$$\int_{m_l}^{m_u} m \phi(m) \, dm = 1 M_\odot,$$  \hspace{2cm} (9.28)

where $m_l$ and $m_u$ are the lower and upper mass limits for stars. $\phi(m)dm$ is the number of stars born with masses in the range $m \pm dm/2$ for every $M_\odot$ of newly-formed stars.

Thus, for a total mass, $M_*$, of newly-formed stars, the total number and total mass of stars with masses in the range $m \pm dm/2$ are given by:

$$dN(m) = \frac{M_*}{M_\odot} \phi(m) \, dm \quad \text{and} \quad dM(m) = \frac{M_*}{M_\odot} m \phi(m) \, dm,$$ \hspace{2cm} (9.29)

Central temperatures of stellar objects with masses $0.08M_\odot$ are too low for hydrogen fusion to take place, and stars with masses greater than about $100 M_\odot$ are unstable against radiation pressure: limits $m_l = 0.08M_\odot$ and $m_u = 100M_\odot$. 

9.4 Observationally Deriving the IMF

Mass is generally not observable - estimated from luminosity.

IMF derived in 4 steps:

1) measure the apparent magnitudes and distances of all stars in a particular volume down to some magnitude limit (only possible in the Milky way).

2) Observed apparent magnitudes are converted into absolute magnitudes, $M$, using their distances, and LF is derived, $\Phi(M)$, defined as the number density of stars as a function of their absolute magnitude.

3) LF of stars is transformed into a MF, using $M/L$ from stellar evolution models.

4) The IMF is obtained from the MF, correcting for stars that died (depends on the star formation history)
9.4 The Initial Mass Function

\[ \phi(m) \propto m^{-2.35} \]
9.4 The Initial Mass Function

\[ \phi(m) \propto m^{-2.35} \]  
for stars in the mass range \(0.4 M_\odot < m < 10 M_\odot\). More recent determinations suggest that the IMF deviates from a pure power law, becoming flatter at the lowest mass end and steeper at the

\[ \phi(m) \propto m^{-b} \text{ with } b = 2.35 \quad [\text{Salpeter IMF}] \]  

\[ \phi(m) \propto \begin{cases} m^{-2.45} & (m > 10 M_\odot) \\ m^{-3.25} & (1 M_\odot < m < 10 M_\odot) \\ m^{-1.80} & (0.2 M_\odot < m < 1 M_\odot) \end{cases} \quad [\text{Scalo IMF}] \]  

\[ \phi(m) \propto \begin{cases} m^{-2.7} & (1.0 M_\odot < m < 100 M_\odot) \\ m^{-2.3} & (0.5 M_\odot < m < 1.0 M_\odot) \\ m^{-1.3} & (0.08 M_\odot < m < 0.5 M_\odot) \\ m^{-0.3} & (0.01 M_\odot < m < 0.08 M_\odot) \end{cases} \quad [\text{Kroupa IMF}] \]  

\[ \xi(m) \propto \begin{cases} m^{-1.35} \exp\{-[\log(m/0.2 M_\odot)]^2/0.6) & (m > 1.0 M_\odot) \\ (m < 1.0 M_\odot) \end{cases} \quad [\text{Chabrier IMF}] \]
Self Regulated Accretion

Masses of stars limited by how much matter they can accrete during their formation.
Each gas clump produced only a single star.
Since mass and density of a clump are correlated with the velocity dispersion within the clump, the mass of the star is assumed to be determined primarily by the velocity dispersion, $\Delta v$.

$$m \propto (\Delta v)^\eta.$$  \hspace{1cm} (9.41)

Need to work out the relation between $m$ and $\Delta v$ to obtain $\eta$.

Assuming that the final mass of a star is determined by the condition that the wind from the star is strong enough to reverse the infall onto the star, $\eta = 11/6$, yielding an IMF slope $b \approx 2.1$, not too different from what is observed.

Note that this result is based on the assumption that the wind is spherically symmetric, while observations show that stellar winds are typically collimated. Hence, it is unclear to which extent the result of spherical model are applicable.
Hierarchical Accretion

Observations indicate that SF clouds consist of networks of self-similar filaments. If stars form by accretion in such a self-similar structure, and if their masses are related to the masses of the region where they form, a power-law IMF can be produced. In a self-similar fractal structure, the number density of subclusters with a typical size $l$ can be written as

$$
\frac{dN}{d\ln l} \propto l^{-D},
$$

(9.42)

where $D$ is the fractal dimension of the cloud boundary. If the mass of a subcluster is related to its size as $M \propto l^\gamma$, the mass function of the subclusters is

$$
\frac{dN}{d\ln M} \propto M^{-D/\gamma}.
$$

(9.43)

Assuming that the mass of a star is proportional to that of the subcluster in which it forms, the IMF has a logarithmic slope $b = 1 + D/\gamma$.

Observations show that the boundary of molecular clouds can be described as fractals with a dimension of about 1.3, which suggests that the surfaces of molecular clouds have a fractal dimension $\sim 2.3$ - IMF slope $b = 1 + 2.3/\gamma$. Thus, $b = 3.3$ for $\gamma \sim 1$ (i.e. for filament-like subclusters), and $b = 1.8$ for $\gamma \sim 3$ (i.e. for three-dimensional subclusters).

These slopes bracket that of the Salpeter IMF, suggesting that the origin of the IMF may be related to the geometry of the molecular ISM.
IMF Based on the Core Mass Function

Based on the mass distribution of molecular cloud cores expected in a turbulent medium.

Since the strength of compression in a shock must be related to the pre-shock spatial scale, the mass function of dense cores in the GMCs generated by supersonic turbulence with a power-law spectrum, $|\Delta v(l)| \propto l^q$ (which is the $rms$ velocity on scale $l$), should have a mass function

$$\frac{dN}{d\ln m} \propto m^{-3/(3-2q)}.$$  \hfill (9.44)

For the expected turbulence $q = 0.5$ and $N = -1.5$.

Since only high-density cores collapse, the IMF should be related to the gravitationally-bound fraction of the mass function. For a given gas density $n$, the thermal Jeans mass is:

$$m_J = m_{J,0} \left( \frac{n}{n_0} \right)^{-1/2}, \text{ where } m_{J,0} \approx 1.2 \times \left( \frac{T}{10^4 \text{ K}} \right)^{3/2} \left( \frac{n_0}{10^4 \text{ cm}^{-3}} \right)^{1/2} \text{M}_\odot \quad (9.45)$$

The distribution function of local Jeans mass is $P(y)dy = 2y^{-2}p[x(y)]dy$, where $y \equiv m/m_{J,0} = x^{-1/2}$. For cores with a given mass $m$, only the fraction with local Jeans mass $m_J < m$, $f(m) = m/m_{J,0} P(y)dy$, can collapse. Hence, the mass function of the cores that can collapse is:

$$\frac{dN}{d\ln m} \propto f(m)m^{-3/(3-2q)}.$$  \hfill (9.46)

If $p(x)$ is log-normal, then $f(m) \sim 1$ for $m \gg m_{J,0}$, and $f(m) \sim 0$ for $m \ll m_{J,0}$. Therefore, the resulting core mass function is a power law. Such a core mass function has the same characteristics as the observed IMF of stars!
(Non)-Universality of the IMF

Current observations based on local star-forming galaxies are consistent with universal IMF.

But uncertainties are large!

There is evidence for local variations of the IMF in different star-forming regions.

- star-forming regions such as Taurus contain no stars more massive than $\sim 2M_\odot$.
- the more massive Orion cloud contains both high- and low-mass stars.
- when molecular clouds are divided into cold ($< 10$ K) and warm ($> 20$ K) populations, small and cold clouds in general do not contain massive stars, while warm clouds do.

Important implications:

- starburst galaxies, with large amounts of concentrated gas, will likely have more large molecular clouds relative to that in normal spirals: larger fraction of massive stars might form in starburst galaxies than in normal galaxies.

maybe TOP HEAVY
(Non)-Universality of the IMF

Ability of a gas cloud to collapse and fragment is determined by the local thermal Jeans mass: fragmentation only when cloud mass > Jeans mass.

Since Jeans mass depends on density and temperature of gas, fragmentation depends on how efficient the gas can cool.

If cooling time < tdyn of the cloud - cloud can cool and fragment as it collapses

If cooling time > tdyn, collapse stops as the gas is heated up by adiabatic compression accompanying the collapse

The radiative cooling rate of a cloud depends on metals and the balance between radiative cooling and heating by adiabatic compression defines a critical metallicity, $Z_{\text{crit}}$.

Clouds with metallicity $Z > Z_{\text{crit}}$ can fragment to form low-mass systems. Suggests that for $Z > Z_{\text{crit}}$, normal IMF, while $Z < Z_{\text{crit}}$, may have a more top-heavy IMF.

Theoretical modeling shows that $10^{-4} < Z_{\text{crit}} < 10^{-3}$.

Since most stars have much higher metallicities - likely negligible at $z=0$.
But high-redshift galaxies have lower $Z$, so maybe IMF varies with.

The ambient radiation field may also play an important role.
Far-IR and CMB, whose temperature increase with redshift as $2.73(1 + z)$ K, may raise the minimum temperature and increase the Jeans mass in star-forming clouds. This may then shift the mass scale of the IMF upwards at high $z$ and in starburst regions, thereby producing a top-heavy IMF.