Endogenous versus exogenous dynamics and scaling laws in Cyber-risks

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Our Group

• Collective dynamics and organization of social agents
  (Commercial sales, YouTube, Open sources, Cyber risks)

• Agent-based models of bubbles and crashes

• Predictability and prediction of complex systems / stock markets

• Asset pricing, hedge-funds, risk factors…

• Human cooperation for sustainability
CRISSES

- dramatic and rapid change of a system which is the culmination of a complex preparatory stage.
- fundamental societal impacts
- large natural catastrophes
  1. earthquakes,
  2. volcanic eruptions,
  3. hurricanes and tornadoes,
  4. landslides, avalanches,
  5. lightning strikes,
  6. meteorite/asteroid impacts,
  7. catastrophic events of environmental degradation,
• failure of engineering structures,
• crashes in the stock market,
• social unrest leading to large-scale strikes and upheaval,
• economic drawdowns on national and global scales,
• regional power blackouts,
• traffic gridlock,
• diseases and epidemics, etc.
• Self-organization? Extreme events are just part of the tail of power law distribution due to “self-organized criticality”? (endogenous)

• “Catastrophism”: extreme events require extreme causes that lie outside the system (exogenous)

• A mixture? How would it work?
• Self-organization?
  Extreme events are just part of the tail of power law distribution due to “self-organized criticality”? (endogenous)

• “Catastrophism”: extreme events require extreme causes that lie outside the system (exogenous)

• A mixture? How would it work?
• The “infinite memory” of the Internet
• Long memory in computer vulnerabilities
• Long memory in many social and natural phenomena
• The concept of RESPONSE FUNCTION and the fluctuation-susceptibility theorem
• Endogenous vs Exogenous organization of complex systems
• Globalization: smaller small risk - larger extreme risks
• Managing complex systems: illusion of control
The “infinite memory” of Internet

Unique source IPs sending scans from AS559 to the Internet aggregated in 15min bins (Unique source IPs generating TCP SYN scans to destination port 135 aggregated in 15min intervals. Only hosts placed inside AS559 are counted when they send more than 20 scans.)

\[ y(t) = \frac{A}{(t - t_c)^\alpha} \]

Dynamics of Blaster

Dynamics of Blaster (after desaisonalization)

(A= 8925,
alpha= 0.687
t = 2003.08.11 02:49).

Jonathan Gysel . ETH Zurich Master Thesis
Qualitative evolution of infection attempts from AS559 versus time, both axis logarithmic (The number of TCP SYNs sent to destination port 135 aggregated in 15min intervals. Only scans from AS559 to Internet are shown)
Activity memory:

\[ y(t) = \frac{A}{(t - t_c)^\alpha} \]

Interpretations:
1) Long-memory: correlation function of activity \( \gg 1/t \)

2) Long-term persistence, intermittency and clustering
   (total activity does not converge and grows as power law)

Mechanisms:
   a) Persistence phenomenon (random field with diffusion,
      Monte Carlo updates of spin models, etc)

   b) Activation processes (Omori-like as for aftershocks)

   c) \( \alpha<1 \) => network amplification effect of persistence by
      epidemic processes
      (Sornette, Critical Phenomena in Natural Sciences (2004))

Extrapolation to “Botnets”?
Scale Invariance and Fractals

Under a change of scale of the control parameter $x \rightarrow \lambda x$

$O(x) = \mu O(\lambda x)$

Functional equation whose solution is

$O(x) = C x^\alpha$, \hspace{1cm} \text{with} \hspace{1cm} \alpha = -\frac{\ln \mu}{\ln \lambda}$

Power laws are the hallmark of scale invariance:

$\frac{O(\lambda x)}{O(x)} = \lambda^\alpha$
illustration of scale invariance
Large-scale computer vulnerability

\[ F(x) = 1 - \left( \frac{k}{x} \right)^\alpha \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>Pareto</td>
<td>( \alpha = 0.260, k = 0.00161 )</td>
</tr>
<tr>
<td>2001</td>
<td>Pareto</td>
<td>( \alpha = 0.192, k = 0.00002 )</td>
</tr>
<tr>
<td>2002</td>
<td>Pareto</td>
<td>( \alpha = 0.199, k = 0.00032 )</td>
</tr>
<tr>
<td>2003</td>
<td>Pareto</td>
<td>( \alpha = 0.246, k = 0.00450 )</td>
</tr>
<tr>
<td>2004</td>
<td>Pareto</td>
<td>( \alpha = 0.395, k = 0.01401 )</td>
</tr>
<tr>
<td>2005</td>
<td>Pareto</td>
<td>( \alpha = 0.552, k = 0.05029 )</td>
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</tbody>
</table>

Table 2: Best fits to the exploit availability (after disclosure)

Figure 6: Gap between exploit- and patch-availability after vulnerability disclosure

(Frei et al. 2006)
Software vulnerability dynamics

S. Frei and T. Maillart

• vulnerability process is a good proxy of software resilience to bugs
• we identify 4 steps in vulnerability process:
  1. discovery (red)
  2. exploit (green)
  3. public disclosure (time reference)
  4. patch release (blue)
• exploits and patch can appear before disclosure (crosses) or after (circles)
• once again, response distribution in this process is heavily tailed
• very characteristic is the distribution of exploits (before disclosure, green crosses) which shows some patterns of power-law with phase transition, in lower tail
Software vulnerability dynamics

• Here we show comparison between types of softwares:
  - Microsoft (blue)
  - Linux (red)
  - Oracle (purple)
  - Mozilla (green)

• We can see that time to patch distribution is also heavily tailed.

• While it varies differently according to considered software the allure remains somehow the similar, especially when we consider Microsoft (blue) and Linux (red).
Understanding the Web browser threat: Examination of vulnerable online Web browser populations and the "insecurity iceberg"

S. Frei¹, T. Duebendorfer², G. Ollmann³, M. May¹
1 Communication Systems Group, ETH Zurich, Switzerland
2 Google Switzerland GmbH
3 IBM Internet Security Systems, USA

More than 600 million users surf at high risk

Zurich, July 1, 2008. Security researchers from ETH Zurich (The Swiss Federal Institute of Technology) and Google have shown in a first-of-kind study that more than 600 million Internet users have vulnerable Web browsers and are therefore easy targets of ‘drive-by download’ attacks.

-59.1% percent of Internet users worldwide use the latest major version of their preferred Web browser.

-Firefox users are the most attentive: 92.2% of them surfed with Firefox 2, the latest version before the recently released 3.0.

-Only 52.5% of Microsoft Internet Explorer users, however, employ the latest, most secure Internet Explorer 7 to surf the Net.

Slow reaction to latest browser version
Analogy: Temporal decay of aftershocks

The seismicity rate after a mainshock at time $t=0$ follows the modified Omori law:

Temporal decay of the rate $N(t)$ of aftershocks after a mainshock at $t=0$

$$N(t) = \frac{K}{(t+c)^p}$$

$p$ is in the range $[0.3, 2]$, often close to 1

[Omori, 1894; Utsu, 1960]
Temporal variation of activity

Analogy with earthquake rate:

- foreshocks?
- $N(t) \sim 1/t^{p'}$ ? $p' < p$ ?
- $N(t) \sim \exp(t)$ ?
- aftershocks
  - Omori law:
  - $N(t) \sim 1/(t+c)^p$

seismicity rate

background

foreshocks?

quiescence?

mainshock
time
The method of critical events in economics and social sciences

Fig.1: Aftershocks of two critical events. December 6, 1992 was marked by the destruction of the Ayodhya mosque in India which sparked a wave of anti-Hindu reactions; September 11, 2001 was marked by the destruction of the World Trade Center in New York which sparked a wave of anti-Islamic reactions. The origin of the horizontal scale corresponds to the day when the critical event occurred. The two solid lines show the number of articles writing on the destruction of Hindu temples or mosques respectively (scale on the left-hand side); the dotted line shows the number of mosques actually destroyed or damaged (scale on the right-hand side).

Roehner, Sornette and Andersen, cond-mat0402408 (2004)
Fig. 2: Relaxation curves after the shock of September 11. The solid line curve is the same as in Fig. 1 but over a larger time interval; the broken line (scale on the right-hand side) shows the number of anti-arab aggressions in California in the three months after September 11; the dotted line shows the changes in the level of the Dow Jones Index with respect to its pre-Sep.11 level as given by the difference DJI(pre-9/11)-DJI(current). The tails of all three curves are well-approximated by power laws $\sim t^{- \alpha}$, with exponents $\alpha$ comprised between -1.4 and -2.2: $\alpha_1 = -1.8\pm0.7$ (newspaper articles), $\alpha_2 = -1.4\pm0.5$ (anti-arab aggressions) and $\alpha_3 = -2.2\pm1.6$ (DJI). Roehner, Sornette and Andersen, cond-mat0402408 (2004)
The method of critical events in economics and social sciences

Roehner, Sornette and Andersen, cond-mat0402408 (2004)

Fig. 3a: How the price of precious metals react to high inflation. The price series for diamonds, gold or cobalt are very similar; they have been omitted for the sake of clarity; in contrast the price of non-precious metals such as copper or aluminum did not experience a peak. It can be observed that the strong correlation between prices ends after the burst of the speculative bubble; in normal times, the correlation between the prices of precious metals is very weak. Sources: The Economist (25 April 1987, Chalmin (1999).
Internet Download Shocks

Cumulative number of downloads $N$ as a function of time $t$ from the appearance of the interview on Wednesday 14th April 1999.

$$N(t) = \frac{\alpha}{1-p} t^{1-p} + ct \text{ with } p \approx 0.58 \pm 0.03.$$  


One-minute return

Omori law fit: $p=0.95$

OMORI LAW:
Example for the Landers aftershock sequence (1992, $M=7.3$, California)

Long-term relaxation of the implied volatility can also be accounted for by the LPPL. It describes the progressive fragmentation of the market which recaptures its usual heterogeneity.
FIG. 1: The probability distribution of the response time till a message is ‘answered’ (see text for definitions). Inset: same but measured in ‘ticks’, i.e. units of messages sent in the system. Solid lines follow $\sim \Delta t^{-1}$ and are meant as a guide to the eye.

Eckmann et al. (2004)
Subprime crisis: Google trend
YouTube viewer dynamics

**Exogenous**

**Endogenous**
Typical Relaxation after a burst of activity
Classification of Blockbusters

Source of Extreme Events: Endogenous vs Exogenous

Epidemic propensity: “branching ratio” $n$

0

1

Model: Epidemic Propagation of Interactions

Bare Response – Long memory process

$$\left(\frac{1}{t-t_c}\right)^{1+\theta}$$

Exogenous Response

$$\left(\frac{1}{t-t_c}\right)^{1-\theta}$$

Endogenous Response

$$\left(\frac{1}{t-t_c}\right)^{1-2\theta}$$

Strategy: -fostering larger branching ratios -targeting larger branching ratios
YouTube viewer dynamics

![Graph showing viewer dynamics over time, with two curves: Most Viewed Today and Front Page. The graph plots Aggregate Daily View Count against Time (days centered on peak).](image-url)
Fig. 2. Classifying movies according to time-evolution of the gross (income): (Left) Daily gross of a typical blockbuster movie (Spiderman) showing weekly periodic fluctuations (with gross peaking on weekends), while the overall trend is exponential decay. (Right) Comparing examples of blockbusters (Spiderman), bombs (Bulletproof Monk) and sleepers (My Big Fat Greek Wedding) in terms of the time-evolution of weekend gross. Time is measured in weekends to remove intra-week fluctuations.
Guidelines from Physics: perturb and study the response

Linear Response Theory

- External weak perturbation
- Response function $R(t)$

Equilibrium state → Non-equilibrium state

$V_{ext}(t)$ → Relaxation (dissipation)

Thermal fluctuations
- Autocorrelation function $C(t)$

Related through the Fluctuation Dissipation Theorem (FDT)
EXO: Drag resistance under an external force

ENDO: Random walk

\[ r^2 = 2d \, Dt \]

\[ D = \frac{k_B T}{\gamma} \]

(Einstein, 1905)
Basic technique: dynamic signatures

Exogenous

Endogenous

tsunami

harry potter movie

Google Trends
<table>
<thead>
<tr>
<th>Endogenous versus Exogenous</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Extinctions</strong></td>
</tr>
<tr>
<td>- meteorite at the Cretaceous/Tertiary KT boundary</td>
</tr>
<tr>
<td>- volcanic eruptions (Deccan traps)</td>
</tr>
<tr>
<td>- self-organized critical events</td>
</tr>
<tr>
<td><strong>Financial crashes</strong></td>
</tr>
<tr>
<td>- external shock</td>
</tr>
<tr>
<td>- self-organized instability</td>
</tr>
<tr>
<td><strong>Immune system</strong></td>
</tr>
<tr>
<td>- external viral or bacterial attack</td>
</tr>
<tr>
<td>- “internal” (dis-)organization</td>
</tr>
<tr>
<td><strong>Brain (learning)</strong></td>
</tr>
<tr>
<td>- external inputs</td>
</tr>
<tr>
<td>- internal self-organization and reinforcements (role of sleep)</td>
</tr>
<tr>
<td><strong>Aviation industry recession</strong></td>
</tr>
<tr>
<td>- September 11, 2001</td>
</tr>
<tr>
<td>- structural endogenous problems</td>
</tr>
<tr>
<td><strong>Recovery after wars?</strong></td>
</tr>
<tr>
<td>- internally generated (civil wars)</td>
</tr>
<tr>
<td>- externally generated</td>
</tr>
<tr>
<td><strong>Discoveries</strong></td>
</tr>
<tr>
<td>- serendipity</td>
</tr>
<tr>
<td>- maturation</td>
</tr>
<tr>
<td><strong>Volatility bursts in financial time series</strong></td>
</tr>
<tr>
<td>- external shock</td>
</tr>
<tr>
<td>- cumulative effect of “small” news</td>
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<tr>
<td><strong>Earthquakes</strong></td>
</tr>
<tr>
<td>- tectonic driving</td>
</tr>
<tr>
<td>- triggering</td>
</tr>
<tr>
<td><strong>Parturition</strong></td>
</tr>
<tr>
<td>- mother/foetus triggered?</td>
</tr>
<tr>
<td>- mother-foetus complex?</td>
</tr>
<tr>
<td><strong>Commercial success and sales</strong></td>
</tr>
<tr>
<td>- Ads</td>
</tr>
<tr>
<td>- epidemic network</td>
</tr>
<tr>
<td><strong>Social unrests</strong></td>
</tr>
<tr>
<td>- triggering factors</td>
</tr>
<tr>
<td>- rotting of social tissue</td>
</tr>
</tbody>
</table>
• Amazon.com posts a “live” ranking of all its products
• Book ranks in the top 10,000 are updated every hour according to a secret weighting of recent sales and entire history
The Original “Crisis”

• On Friday January 17, 2003, Sornette’s recent book jumped to rank # 5 on Amazon.com’s sales ranking (with Harry Potter as #1!!!)
• Two days before: release of an interview on MSNBC’s MoneyCentral website
June 4, 2002: New York Times article crediting the "groundbreaking research" of Dr. Nelson

June 5, 2002: Endogenous

"Heaven and Earth (Three Sisters Island Trilogy)" by N. Roberts

"Strong Women Stay Young" by Dr. M. Nelson
Predictions of the model

Criticality of social network

Type of disturbance

Endogenous

Exogenous

Sub - Critical

Critical

Exponent of Decay

$1-\phi$

$1+\phi$

$1-\phi$

$1+\phi$
Single Book Analysis

\[ \theta = 0.3 \pm 0.1 \]

Endogenous growth

Endogenous decay

Exogenous decay

\[ 1 - 2\theta \]

Sales/Day (arbitrary)

Time \(|t_c - t|\) (days)
The Aggregate

Real data averaged over +100 books

\[ \theta = 0.3 \pm 0.1 \]

Endogenous growth and decay

Exogenous decay

Exogenous growth

Average sales/Day

Time \(|t-t_0|\) (days)
What about precursory information?
Qualitative classifications

Sub - Critical

Endogenous

Views

Time

Critical

Viral

Exogenous

Fraction of Views in peak day

Views

Time

Exponent of Decay

$1 - 2\theta$

$1 + \theta$

$1 - \phi$

$1 + \phi$

Fraction of views in peak day

Views

Time

Exponent of Decay

$1 - \phi$

$1 + \phi$

Junk

Quality
Financial Volatility


Daily Returns

Time

1-1-80 31-12-81 1-1-84 31-12-85
Linear response to an external shock
(Multifractal Random Walk model)

\[ E_{\text{exo}}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}} \]

October 1987 crash: totally different mechanism

August 19, 1991: coup against President Gorbachev
September 11, 2001: Attack against the WTC

Endogenous shocks and Multifractal Random Walk model

\[ E_{\text{endo}}[\sigma^2(t) | \omega_0] \sim t^{-\alpha(s)} \]

\[ \ln(\tau) \text{ (ln-day)} \]

\[ \ln E[\sigma^2(\tau) | s] \]

\[ \ln(\tau) \text{ (ln-day)} \]

\[ \alpha(s) \]

\[ z_s \]
black: highly connected nodes
red: infected
green: un-infected
grey: infection not yet revealed

http://www.orgnet.com
Securitization of credit risks: is it the next “systemic collapse”?

- Securitization of credit risks leads to smaller risks
- But more inter-connected ⇒ global risk?

-Finance industry (investment, retail, wealth management)

-"size effect" of Cyber-Risk (cf Thomas Maillart’s talk)

CDS and CDO: form of insurance contracts linked to underlying debt that protects the buyer in case of default.

The market has almost doubled in size every year for the past five years, reaching $20 trillion in notional amounts outstanding last June 2007, according to the Bank for International Settlements.

Bundling of indexes of CDSs together and slicing them into tranches, based on riskiness and return. The most toxic tranche at the bottom exposes the holder to the first 3% of losses but also gives him a large portion of the returns. At the top, the risks and returns are much smaller-unless there is a systemic failure.
Securitization leads to larger inter-connectivity

Separation of financial and credit risks

Coupling strength increases
“We, at the Federal Reserve…recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence… Moreover, it was far from obvious that bubbles, even if identified early, could be preempted short of the Central Bank inducing a substantial contraction in economic activity, the very outcome we would be seeking to avoid.”

A. Greenspan  (Aug., 30, 2002)
FIRE CONTROL

The primary response from government has been to initiation aggressive fire suppression and management in an attempt to eliminate fire from native lands. In spite of these aggressive fire suppression efforts large wildfires continue to consume vast acreages of chaparral in Southern California.

Minnich (1983, 1997) comparing the chaparral fire regimes in southern California and Baja California found that in Baja California numerous small fire events fragment stands into a fine mixture of age classes, a process which appears to help preclude large fires. While the pattern of large fires in Southern California appears to be an artifact of suppression.
Fig. 1. Wildland fires in southern California and northern Baja California, 1972 to 1989, with vegetation noted. A broad gradient of increasing fire area northward in Baja California shifts to a pattern of infrequent small to very large fires north of the border. Divergences in fire size between the two countries are most evident in chaparral. Fire data was mapped from LandSat imagery; vegetation was mapped from aerial photography (43).

Fig. 2. Noncumulative frequency-area distributions for actual forest fires and wildfires in the United States and Australia (A) 4284 fires on U.S. Fish and Wildlife Service lands (1980–1995) (9). (B) 140 fires in the western United States (1150–1960) (10). (C) 164 fires in Alaskan boreal forests (1990–1991) (11), and (D) 298 fires in the ACT (1926–1991) (12). For each data set, the noncumulative number of fires per year (–dNf/dA) with area (A) is given as a function of A, with a reasonably good correlation over many decades of A is obtained by using the power-law relation (Eq. 1) with \( \alpha = 1.31 \) to 1.49. \( \alpha \) is the slope of the best-fit line in log-log space and is shown for each data set.


Minnich, R.A. (1983), Science 219
Information processing: normal people’s high level of general intelligence makes them too smart for their own good.

✓ Individuals appear hard-wired to over-attribute success to skill, and to underestimate the role of chance, when both are in fact present

✓ After a full cycle of rise and fall after which stocks were valued just where they were at the start, all his clients lost money (Don Guyon, 1909)

✓ Many academic works suggest that most managers underperform “buy-and-hold” strategy; persistence of winners is very rare, etc.

✓ Rats beat humans in simple games: People makes STORIES! Normal people have an “interpreter” in their left brain that takes all the random, contradictory details of whatever they are doing or remembering at the moment, and smooths everything in one coherent story. If there are details that do not fit, they are edited out or revised! (T. Grandin and C. Johnson, Animals in translation (Scribner, New York, 2005)}
Definition of “Illusion of control” in set-ups a priori defined to emphasize the importance of optimization:

- **Low entropy (more informative) strategies** under-perform
  **high-entropy (random) strategies**

**Demonstration in Minority games and Parrondo games**
(J. Satinover and D. Sornette)

Other examples where uncertainty and risks can be amplified by attempt to manage and control:
- control algorithm with optimal parameter optimization based on past observations generate power law PDF of fluctuations
- quality control
- management strategy during times of crises (distressed firms…)

**How can we falsify the value of control and management?**
• Dynamical approach to complex systems

• Ecology of risks: the immune system view (illnesses are not due to microbes!)
Critical Phenomena in Natural Sciences

Chaos, Fractals, Selforganization and Disorder: Concepts and Tools

First edition
2000

Second enlarged edition
2004

Why Stock Crash

Critical Events in Complex Financial Systems

DIDIER SORNETTE

Princeton University Press
Jan. 2003