

Uncertainty and Coordination in Common-Property Fisheries*

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Abstract

In order to analyse the dynamics of a common-property fishery and the non-cooperative behaviour of harvesting agents, a dynamic coordination game in a discrete-time framework is studied. The model distinguishes between the effects of different sources of uncertainty on the behaviour of fishing agents. It is shown that if there is only uncertainty about future biological changes of the stock of resource, the possibility of sustainable harvest in the current period might be deterred by ‘the fear of future depletion’. While if the harvesting firms in addition to the biological uncertainty, face strategic uncertainty, by obtaining only private noisy assessments about the current stock, their coordination is influenced by ‘fear of miscoordination’ as well. In such a coordination problem with the multiplicity of equilibria as a generic characteristic, the equilibrium-refinement of Carlsson and van Damme (1993) is applied to derive a unique prediction about the outcome of the model. The effect of reduction of risk of miscoordination on the self-fulfilling equilibrium is discussed.

Key words: renewable resources, non-cooperative dynamic games, biological and strategic uncertainty, multiplicity and equilibrium refinement

JEL Classification: Q22; Q5; C72; C73; D8

1 Introduction

In the literature of resource extraction, the role of uncertainties and incomplete information has been extensively studied. The pioneer in this area is Reed (1978, 1979), who introduced the stochastic fluctuations in the stock recruitment of a renewable resource¹. In the literature of fisheries², the common reasons for the stochastic shocks in the ecosystem are listed as the variability of the fish stock, predation by other animals, changes of temperature etc. Here, similar to Levhari and Mirman (1980) there are

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¹For stochastic games of resource extraction with recruitment uncertainty, see Spulber (1982, 1985), Mirman and Spulber (1985), Clark and Kirkwood (1985), Clemhout and Wan (1985), Clarke and Reed (1994), Roughgarden and Smith (1996), Costello et al(2001), Weitzman (2002), Laukkanen (2003, 2005), Sethi et al(2005), Amir (2005), Koulvatianous (2010), Antoniadou et al(2013), also Fesselmeyer and Santugini (2013).

²Which according to van Long (2011) are known as the metaphor of all renewable resources.

two fishing agents, who share a fishing stock and interact non-cooperatively³ in a dynamic setting. After recruitment of the resource, the stock is publicly observed and then the appropriators face the biological shocks affecting the stock of fish⁴.

There is another source of uncertainty, yet to be studied in the theoretical literature of common-property resources, which is the strategic uncertainty. Here, strategic uncertainty refers to the uncertainty about the other players' behaviour (even in the equilibrium), and it is the result of noisy private observations⁵. In the present research, after facing the biological shocks, the harvesting agents are modelled to obtain private noisy information about the current stock. In a coordination game, it is shown that players form higher-order beliefs about information of the other fishing firm, and second guess its harvesting decision.

The uncertainties are introduced on two levels. In the first step, the model is analysed under the assumption of complete information about the current state of the fishery. However, there are random biological shocks that may affect the stock in the future periods⁶. Because of the complication of modelling the continuation values in this setup, the analysis starts by using the smallest dynamic game, which is a two-period model. Hence, in the last period, the dominant strategy of overfishing removes the uncertainty about the outcome of the game in that period. However, when another period is added to this model, it is shown that the existence of uncertainty about future cooperation may hinder today's cooperation on sustaining the resource, which we call 'the fear of future depletion'⁷.

In the next step, it is assumed that fishing firms do not observe the state of the environment precisely in any period, but they receive noisy private signals about it. A two-period model is sufficient to capture the idea of risk of miscoordination, as a result of the strategic uncertainty. Additionally, the three-period model with imprecise private information, depicts a situation where both the fear of future depletion and the fear of miscoordination exist, and it is shown how coordination may be more difficult under such a circumstance.

The present research contributes also to the strand of equilibrium refinement. The multiplicity of equilibria has been intensively investigated in the literature of common property resources. Among all, Dutta (1995) generalized the Folk Theorem of Aumann and Shapley (1994) to the stochastic games, also Benhabib and Radner (1992), Dutta and Sundaram (1993), Dockner and Sorger (1996) and Sorger (1998) show multiplicity of perfect equilibria in deterministic dynamic games. Although equilibrium selection has been one of the main challenges of game theory since the 1980s, equilibrium refinement has not been studied in the applications of natural resources.

³For analysis of non-cooperative behavior in stochastic games of resource extraction, in particular see Mirman and Spulber (1984,1985), Szaajowski (2006), Amir (2010), Koulovatianos (2010), Antoniadou et al (2013) and Fesselmeyer and Santugini (2013).

⁴Therefore, the term of biological uncertainty is chosen instead of the recruitment uncertainty. This is similar to Roughgarden and Smith (1996) and Sethi et al (2005), who distinguish between the uncertainty due to the variability of the stock or the recruitment uncertainty and the measurement error in estimate of the stock.

⁵For further discussion, see Morris and Shin (2003).

⁶This timing is similar to Clark and Kirkwood (1985), but here the non-regulated agents rather than the manager face the uncertainty.

⁷In the literature of renewable resources, for the effect of risk of an adverse event due to environmental uncertainty on the economic behaviour, among many others see , Tsur and Zemel (1995,1996,1998), Lafforgue (2005), Polasky et al (2011),Koulovatianos (2010), Antoniadou et al (2013), Fesselmeyer and Santugini (2013).

Indeed, while coordination and trust play a crucial role in the environmental economics, models of coordination under the assumption of precise information have multiplicity of equilibria. In this paper, the interaction of harvesting firms is modelled in a coordination setup⁸, such that if any fishing firm is going to overexploit, then there is a first-mover advantage to do so. It is shown that when the fishing firms have precise information about the current state of the fishery and there is only biological uncertainty in the future, then there is multiplicity of equilibrium outcomes, which reduces the predictive power of the model.

Gaining from the realistic underlying uncertainties in the model, the setup is suitable to apply the equilibrium-refinement of Carlsson and van Damme (1993), known as the global game. For the general imprecise-signal case of the model, where both biological and strategic uncertainties are present, this equilibrium-selection technique is used to obtain equilibrium uniqueness when the signals in the limit are extremely precise, although the level of the stock will never be common knowledge. It is shown that the results of Carlsson and van Damme (1993) generalise to the case of renewable resources that the stock in each period depends on its previous levels⁹. The model will give a unique prediction for any level of private information about the stock, and it is discussed under which conditions the harvesters coordinate on preserving the stock, and when in a self-fulfilling way they decide to deplete the resource. It is discussed how reducing the source of strategic uncertainty, helps the researcher or the potential manager of the resource in deriving a conclusion about the interaction of the harvesting agents. Furthermore, the global game equilibria are compared with the risk-dominant equilibria of Harsanyi and Selten (1988) in the precise-signal model¹⁰.

Most of the literature on global games is about static settings. The growing literature of dynamic global games can be classified into two main strands. In the first strand, which was originally studied by Morris and Shin (1999, 2000) and Chamley (1999), fundamentals are the driving force of the dynamics. In such frameworks, in each period the state variable, about which players obtain private information, is evolving according to a one-step random-walk process. They show that each period of the dynamic model can be considered as an independent static game, with a unique threshold equilibrium in the limit, which is a random variable itself. Morris and Shin (2003) call this type of dynamic settings ‘Recurring Incomplete Information’ global games. As Steiner (2008b) and Chassang (2010) have pointed out, the dynamics of outcomes in the models of this generation are produced exogenously by the fundamentals.

The other strand in dynamic global game literature, includes models that endogenise the underlying dynamics. This idea was first proposed by Angeletos, Hellwig and Pavan (2007) who studied a multi-period regime-change game where, the fundamentals were i.i.d. Survival of the status quo in at least one

⁸Dawes (1973) and Dasgupta and Heal (1979) are among the early studies that formalise the interaction of harvesting agents in a prisoner’s dilemma setup, but in a world of complete information. In stochastic games, Amir (2010) imposes the stronger assumption of supermodularity of the payoffs, but the information is not private, and he focuses on the existence of Markov Perfect Equilibria.

⁹Common applications of global games are currency crises (Morris and Shin, 1998), bank runs (Morris and Shin, 2001, Rochet and Vives, 2004, and Goldstein and Pauzner, 2005), foreign direct investment (Dasgupta, 2007), political regime change (Edmond, 2011) and debt-pricing (Morris and Shin, 2004, and Corsetti, Guimaraes and Roubini, 2006).

¹⁰The risk-dominance equivalence of the equilibrium thresholds of the global game and precise-signal case, was first proved by Carlsson and van Damme (1993) for the class of static 2×2 games and in general it is a generic characteristic of static global games. However, here it is shown that this result does not hold necessarily.

period weakens the posterior beliefs about the possibility of abandoning the regime. So players' beliefs about the fundamentals are the endogenous source of the dynamics. Similar to Morris and Shin (1999), they also assume the same fundamental-independent payoff structure. Therefore, this type of endogeneity is completely related to the evolution of beliefs. Giannitsarou and Toxvaerd (2007) and Steiner (2008a) captured the idea of endogeneity of dynamics by introducing another state variable in the payoff functions. Chassang (2010) and Dasgupta, Steiner and Stewart (2012) study respectively, infinite and finite-horizon global games, in which all payoffs are functions of an i.i.d state variable and players' incentives through the payoff structure are the source of endogeneity of dynamics.¹¹

The present work is a combination of these two strands. First, as a renewable resource, the state of a fishery depends on its previous value through the growth function¹². Beside the exogenous evolution of the fishery, all of the payoffs depend on the state of the resource. Hence the players' incentives are the other driving force of the dynamics of outcomes.

The structure of the paper will be as follows: The model is introduced in section 2. First, it is analysed under biological uncertainty in section 3. Then, in section 4 strategic uncertainty is added to the setup, and the equilibrium-refinement technique is introduced. Section 5 concludes and finally extended proofs and plots are provided in the appendix.

2 The General Framework

Following Chassang (2010), in each period, a 2×2 game with stay-exit structure is played. Two fishing firms share a stock of fish $w_t \in [0, 1]$ and they choose between two actions: sustainable-fishing, S, and over-fishing, O. If the fishing agents decide to maintain the resource then the game will continue to the next period, but they may choose to exhaust the stock in any period and end the game. If there is no uncertainty in the model, then the stock of fish grows according to the specific, but commonly-used growth function of Levhari and Mirman (1980), as an increasing, strictly concave and bounded growth function, $w_t = (\bar{w}_{t-1})^\alpha$, where $0 < \alpha < 1$, and \bar{w}_{t-1} is the escapement level of the stock at the end of the period $t - 1$. Also \bar{w}_0 is the initial stock which is chosen by nature at the beginning of the game. Furthermore, α is common knowledge among the harvesting firms.

However, in this model there are uncertainties about both the stock of fish and the harvesting action of the other player. The timing of the game is as follows:

At the beginning of the game, nature chooses the state of fishery, \bar{w}_0 . Then at the beginning of each period t , for $t \geq 1$, the escapement level, \bar{w}_{t-1} , which is the parent fish stock left from the previous period, reproduces and the players receive a public signal, $y_t \equiv (\bar{w}_{t-1})^\alpha$, about the state of fishery, w_t . Then the fishery will be subject to additive shocks, ξ_t . Therefore, $w_t = y_t + \xi_t$ ¹³. Indeed the public signal is $y_t = w_t - \xi_t$, and the state variable w_t shows the state of fishery at the stage of decision-making, when the

¹¹There is another strand in the dynamic global games, where two-period models are considered. For example, see Heidhues and Melissas (2006), Dasgupta (2007), and Kovac and Steiner (2012).

¹²Not completely similar to Morris and Shin(1999) and Chamley (1999), since the state variable does not follow a random-walk process.

¹³Henceforth, capital letters denote random variables, and lower case letters refer to the realization values that the random variables map into.

biological shocks have happened. Also, $\Xi \sim U[-c, c]$, and $0 < c < 1$. These shocks capture the biological uncertainty studied in the literature, and the parameter c can be interpreted as the precision of the public signal. The biological shocks are serially uncorrelated and independent of w_t . It is assumed that if the stock is fully depleted to zero, then there will not be any immigration to the stock, and the fishery can never recover.

After the biological shocks, the firms obtain a noisy private signal, x_t^i , about the stock, where $x_t^i = w_t + \varepsilon_t^i$, also $E \sim U[-a, a]$. The superscript $i \in \{1, 2\}$ refers to players' index, $-i$ refers to the other player, and subscript t is the time-period index. Without loss of generality, it is assumed that X_t maps into $[0, 1]$, and W_t maps into $(0, 1)$. Indeed, restricting the maximum level of noise of each period, $a + c$, to be small enough relative to the initial stock, ensures finite support of X_t and W_t in a T-period model¹⁴. Since ε_t^i can be interpreted as the measurement error in the assessment of the fish stock, parameter a may be the precision of the private information. These noises are i.i.d, and independent of w_t . Although the signals of the two fishing firms are highly correlated, this information is private. In addition, the structure of payoffs, the distributional assumptions of the private signals and the noise technology of the state of fishery are common knowledge among fishing firms.

The uncertainty is introduced in two steps. In the first step, the analysis is focused on precise private signals, where $a = 0$. Therefore firms can assess the current state of the fishery perfectly, but the biological shocks in future periods may still affect the fishery. In this framework, even when the private information is precise the model has incomplete information due to the future uncertainty in the continuation values. In the second step, where the general case with noisy signals is studied, there will be both strategic and biological uncertainties in the first period.

Finally, players choose between two harvesting levels simultaneously, and at the end of each period, actions are observed and payoffs are determined. Indeed, w_t in period t , refers to the level of stock at the stage that players are making decisions about the harvest of that period. The payoffs of the harvesting firms is equal to the amount of fish that they catch. As mentioned earlier, if the players coordinate on sustainable-fishing, then the game continues to the next period, and if one or both players overfish in any period, the game ends, both players receive the termination payoffs of that period, the whole stock will be extinguished, and no further positive or negative shocks in subsequent periods will affect the stock. Table 2.1 depicts the payoffs of each period. The fishing firms have linear symmetric strategies. Sustainable-fishing is harvesting according to the pre-committed rule, rw_t , where the sharing rule of r is exogenously fixed in the model, also $r \in (0, \frac{1}{2})$. While, payoff of (O,O) is sharing equally whatever is in the stock. If one of the players is going to exit the game, there is a first-mover advantage to do so. In other words, if player 1 fishes sustainably while player 2 overfishes, then player 1 will receive a smaller share of the stock, and vice versa.

This setup fits in the literature of stochastic repeated games, where the stage game changes from period to period and the transition function, showing the evolution of stock depends on both the actions and the random biological shocks. Here, the focus will be on Markovian strategies, which depend on the current signals about the state of the environment¹⁵. Also, in the present research the analysis is restricted

¹⁴For example in a two-period model, it is sufficient to assume $b(b(w_0)^\alpha - c - a)^\alpha - c - a \geq 0$ and $b(b(w_0)^\alpha + c + a)^\alpha + c + a \leq 1$.

¹⁵The restrictiveness of this assumption is discussed after deriving equilibrium uniqueness of Markovian strategies.

to a few number of periods. The explained stochastic resource game can be denoted by $G(\bar{w}_0, a, c, r, \alpha, \beta)$, and hence forth we suppress notation and express it as G .

		Firm 2	
		S	O
Firm 1	S	rw_t, rw_t	$rw_t, (1-r)w_t$
	O	$(1-r)w_t, rw_t$	$\frac{w_t}{2}, \frac{w_t}{2}$

Figure 2.1: Normal-form representation of the game for period t

In the last period, there is no continuation value. The strictly dominant strategy for both players is overfishing in the last period. Appealing to sequential rationality, by backward induction, one can summarise the augmented payoff of the row player in the first period of a two-period model as shown in Table 2.2, where $\beta \in (0, 1)$ is the discount factor, which is identical for both harvesting agents.

		Firm 2	
		S	O
Firm 1	S	$rw_1 + \frac{\beta}{2}\mathbb{E}[w_2 y_1, x_1^1]$	rw_1
	O	$(1-r)w_1$	$\frac{w_1}{2}$

Figure 2.2: Payoffs of firm 1 in the first period of a two period model, augmented with the continuation payoffs

Note that the strategy of firm i in period t is shown by σ_t^i , which maps from the set of signals to the action space. Before starting the analysis, a few standard definitions are listed below. The game can have different equilibria. For example, history-dependent equilibria or history-independent equilibria, e.g playing the Nash equilibrium of overfishing in the first period. Here, we are interested in the threshold-strategy Markov Perfect equilibria.

Definition 1. Threshold strategy: a strategy $\sigma_t^i : X_t \times Y_t \rightarrow \{S, O\}$ is a threshold strategy for player i , if there exists a cutoff $k_{(\cdot)}^i \in X_t \times Y_t$, below which the firms sustainably harvest, and (strictly) above that they overfish¹⁶.

Indeed, the reason of concentrating on threshold strategies with overfishing for high levels of the stock is concavity of the growth function.

Definition 2. Markovian Strategies: a strategy σ_t^i is Markovian if it depends only on player i 's current signal, for $i \in \{1, 2\}$.

Definition 3. The strategy profile σ_t^i is a threshold strategy Markov perfect equilibrium (MPE), if σ is a threshold Markov strategy and a perfect Bayesian Nash equilibrium.

Since the game is symmetric, the unique MPE strategy refers to a unique pair of MPE strategies.

¹⁶The number in parenthesis of the subscripts denotes the number of periods in the underlying model.

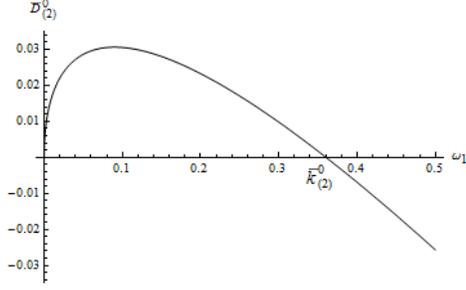


Figure 3.1: Differences between expected payoffs, when the other firm fishes sustainably in a two-period model, and $\alpha = 0.5$, $\beta = 0.7$, $r = 0.33$, therefore $\bar{k}_{(2)}^0 = 0.37$

3 Biological Uncertainty

In this section we will study the decision of fishing agents in the first period, under the assumption of $a = 0$. In other words, there is complete information about the current state of the fishery, but there is biological uncertainty about the future stock.

3.1 Two-Period Model

The set of MPE thresholds is not a singleton, and it admits an upper bound and a lower bound. To characterise this set for the game depicted in Table 2.1, note that at the upper bound equilibrium the payoff of sustainable fishing is more than deviating, when the other player is fishing sustainably, i.e. the sustainable fishing is an equilibrium if

$$\bar{D}_{(2)}^0(w_1) \equiv rw_1 + \frac{\beta}{2}\mathbb{E}[w_2|w_1, \sigma_1^i = \sigma_1^{-i} = S] - (1-r)w_1 > 0 \quad (3.1)$$

Let $\bar{k}_{(2)}^0$ (where 0 refers to the precise-signal case) be such a threshold that forms the upper bound of the MPE thresholds. Firm i harvests sustainably iff $w_1 < \bar{k}_{(2)}^0$, where $\bar{k}_{(2)}^0$ is defined by $\bar{D}_{(2)}^0(\bar{k}_{(2)}^0) = 0$. Let $(1-2r)^\alpha \equiv b$. Since $\mathbb{E}[w_2|w_1, \sigma_t^i = \sigma_t^{-i} = S] = b(w_1)^\alpha$, the unique non-trivial root of (3.1) will be

$$\bar{k}_{(2)}^0 = \left(\frac{\beta}{2}\right)^{\frac{1}{1-\alpha}} (1-2r)^{-1} \quad (3.2)$$

$\bar{D}_{(2)}^0(w_1)$ is a concave function and it has a maximum at $(\frac{\beta\alpha}{2})^{\frac{1}{1-\alpha}} (1-2r)^{-1}$, which proves the threshold behaviour of the upper bound equilibrium.

As an example Figure 3.1 plots $\bar{D}_{(2)}^0(w_1)$ for some specific parameter values. This behaviour of the difference of expected payoffs is a result of the biological reproduction function. In fact, at low levels of stock, the high recruitment rates provides a greater incentive for the firms to coordinate on sustainable fishing. However, as the stock grows, the temptation of over-exploitation also increases, and finally at the threshold level, the temptation dominates the incentive to coordinate in preserving the stock. In this equilibrium, although there are only two periods in the model, because the fish stock is reproducing, depending on the current level of the stock, firms may choose to wait in the first period.

Similarly, the lower bound of the set of MPE thresholds, $\underline{k}_{(2)}^0$, is characterised by setting the payoffs of overfishing more than sustainable fishing, when the other firm overfishes, i.e. overfishing is equilibrium

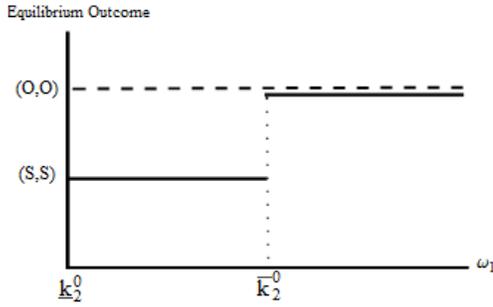


Figure 3.2: Multiplicity of equilibrium outcomes under the precise signals, where Solid lines: equilibrium outcome of the upper-bound threshold-strategy MPE, and Dashed line: equilibrium outcome of the lower-bound threshold-strategy MPE

if

$$\underline{D}_{(2)}^0(w_1) \equiv \frac{w_1}{2} - rw_1 > 0 \quad (3.3)$$

The lower bound $\underline{k}_{(2)}^0$, as expected, is zero, since overfishing in every period is always a MPE. Indeed, (3.3) is satisfied for any values of the precise signal or the state variable, as long as $r < \frac{1}{2}$.

Figure 3.2 depicts the bounds of the set of threshold strategy MPE. In the first period, for any state of the fishery above \bar{k}_2^0 , there is a unique equilibrium outcome, in which both players overfish. However, for any state (or precise signal) below \bar{k}_2^0 , there is a multiplicity of equilibrium outcomes that players either play (O,O) or (S,S). If (3.1) and (3.3) hold, there is a continuum of such threshold equilibria between $[0, \bar{k}_2^0]$. Therefore, if there is complete information about the current stock of fish, then the theory cannot predict the action of harvesting agents. This multiplicity of equilibrium outcome, which is the natural characteristic of coordination games under complete information, in the absence of any equilibrium refinement reduces the predictive power of the model to zero for any value of the stock below the upper-bound threshold.¹⁷

In subsequent sections, for the two-period model we will focus on a subset of games $G_{(2)} \subset G$, in which in the first period both (S,S) and (O,O) are pure Nash equilibria, i.e (3.1) and (3.3) hold. So, it is a coordination game and always admits multiplicity of equilibrium outcomes under the assumption of precise signals.

3.2 A Model of Fear of Future Depletion: The Three-Period Model

In this section, by adding another period to the two-period model it is possible to capture the biological uncertainty about the level of stock in a non-final period.

If in the first period the harvesting firms decide to coordinate on sustainable fishing and proceed to the second period, then they will choose their harvest level by comparing the state variable, w_2 , (or their precise signals, x_2^i) with the threshold equilibrium of the middle period, which is inherently the threshold equilibrium of a two-period model, that is already worked out in section 3.1.

To characterise the set of threshold strategy MPE of the first period, and the range of multiplicity

¹⁷This result is in line with Chassang (2010), in which depending on the strategy of the other player, the set of SPE is bounded by ‘the most and the least cooperative equilibria’, both of which have threshold-type forms. He studies an infinite-horizon model with an i.i.d. state variable.

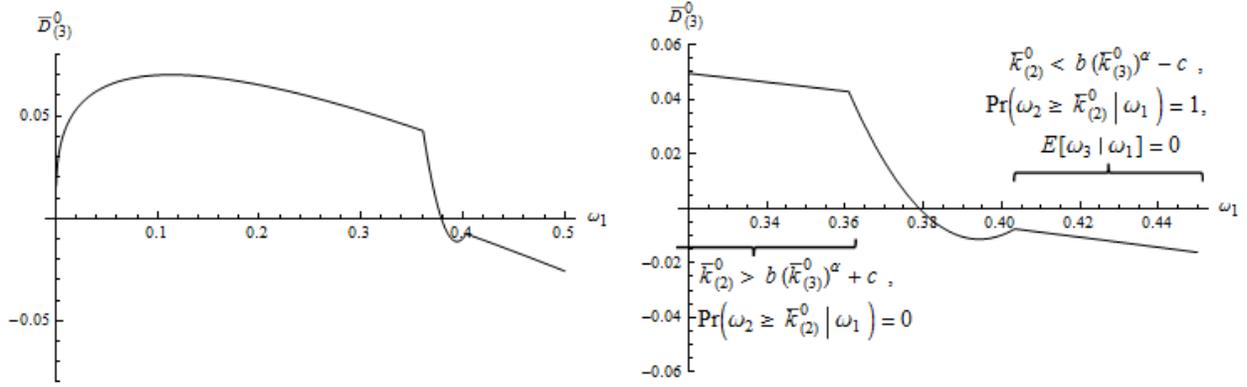


Figure 3.3: Differences of expected payoffs, when the other firm fishes sustainably in a three-period model, and in both panels $\beta = 0.7$, $c = 0.01$, $r = 0.33$, $\alpha = 0.5$

of equilibrium outcomes in the underlying game, again the upper bound of the set of MPE thresholds is worked out by the same intuition. The incentive to sustainable fishing, if the other firm coordinates on sustainable fishing, will be

$$\begin{aligned}
\bar{D}_{(3)}^0(w_1) &\equiv \\
&rw_1 + \beta \mathbb{E} \left[\left[rw_2 + \frac{\beta}{2} \mathbb{E}[w_3|w_2] \right] \mathbf{1}_{w_2 < \bar{k}_{(2)}^0} + \left[\frac{w_2}{2} \right] \mathbf{1}_{w_2 \geq \bar{k}_{(2)}^0} \mid w_1, \sigma_1^i = \sigma_1^{-i} = S \right] - (1-r)w_1 \\
&= \beta \left[\mathbb{E} \left[rw_2 + \frac{\beta}{2} w_3 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S \right] \Pr(w_2 < \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S) \right. \\
&\quad \left. + \mathbb{E} \left[\frac{w_2}{2} \mid w_1, \sigma_1^i = \sigma_1^{-i} = S \right] \Pr(w_2 \geq \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S) \right] - (1-2r)w_1
\end{aligned} \tag{3.4}$$

Let $\bar{k}_{(3)}^0$ be the upper bound of the set of MPE in the first period of a three-period model, so it is the root of $\bar{D}_{(3)}^0(\bar{k}_{(3)}^0)$. Note that $\mathbb{E}[\mathbb{E}[w_3|w_2] \mid w_1] = \mathbb{E}[w_3 \mid w_1]$. In addition, since $W_2|w_1 \sim U[b(w_1)^\alpha - c, b(w_1)^\alpha + c]$, equation (3.4) and indeed the conditional probabilities of $\Pr(w_2 < \bar{k}_{(2)}^0)$ and $\Pr(w_2 \geq \bar{k}_{(2)}^0)$, and $E[w_3]$ are needed to be conditioned on the fact that $\bar{k}_{(2)}^0$ must have the same support as $w_2|w_1$. See Appendix A.1 for these conditions.

It is not possible to solve equation (3.4) explicitly, however for specific parameter values it can be solved numerically. In the rest of this section, first the behaviour of $\bar{D}_{(3)}^0(w_1)$ is discussed. Then, the multiplicity of MPE, the comparison of the two and three-period models, and some comparative statistic results are provided.

First of all, as depicted in Figure 3.3, $\bar{D}_{(3)}^0(w_1)$ has two kinked points and a non-monotone region in between the two kinked points, which depending on the parameter values, might appear in either positive, or negative ranges of $\bar{D}_{(3)}^0(w_1)$, or even on the horizontal line affecting the threshold of the upper bound equilibrium (See Appendix B.1).

In order to shed light on the economic intuition of the behaviour of $\bar{D}_{(3)}^0(w_1)$ function, it is helpful to check the behavior of each of its expressions (e.g. see Appendix B.2). Indeed for any w_1 below the left kinked point, $\bar{k}_{(2)}^0 > b(w_1)^\alpha + c$. It can be deduced from Appendix A.1 that in this region the fishing

agents in the second period will certainly fish sustainably. On the other hand, for any w_1 beyond the right kinked point, $\bar{k}_{(2)}^0 \leq b(w_1)^\alpha - c$, which implies, the probability of overfishing in the second period is one and $\mathbb{E}[w_3|w_1] = 0$. This is summarised in panel (b) of Figure 3.3. In these two regions, $\bar{D}_{(3)}^0(w_1)$ follows the same pattern as in the two-period model, because there is no uncertainty about coordination in the middle period. Furthermore, the slope of $\bar{D}_{(3)}^0(w_1)$ in these two regions may be either positive or negative, depending on the parameter values and the size of the stock of fish.

However, between the two kinked points, the probabilities of coordination are interior. As the probability of overfishing in the second period increases, continuation payoff of coordination on sustainable fishing in the second period, i.e. $\mathbb{E}[[rw_2 + \frac{\beta}{2}w_3]\mathbf{1}_{w_2 < \bar{k}_{(2)}^0} | w_1, \sigma_1^i = \sigma_1^{-i} = S]$ decreases. We call this ‘the fear of future depletion’. In other words, because of the possibility of depletion of the stock in the future, players are more likely to overfish today, and it might even affect $\bar{k}_{(3)}^0$ in the first period, as plotted in Figure 3.3. Although there is no strategic uncertainty in the model so far, the biological uncertainty next to the dependency of the state of the fishery on its previous values, forces the fishing firms to take into account the risk of future exhaustion.

If the state variable was i.i.d.¹⁸, then thresholds of different periods would be constant, and continuation values would not be a function of threshold of subsequent periods. Moreover, by the i.i.d. state variable, $\Pr(w_2 < \bar{k}_{(2)}^0 | w_1)$ would not depend on w_1 . However, in the context of renewable resources, a serially correlated state variable is a plausible assumption.

On the other hand, the increase of $\Pr(w_2 \geq \bar{k}_{(2)}^0 | w_1)$ has another effect, which is increasing the expected payoff of overfishing in the second period as part of the continuation payoff of the first period. Hence, the $\bar{D}_{(3)}^0(w_1)$ function in between the two kinked points is non-monotone. Especially, close to the second kinked point, where the probability of future exhaustion is high enough, the temptation of overfishing in the second period dominates and $\bar{D}_{(3)}^0(w_1)$ will be upward-sloping.

Furthermore, as a result of the biological uncertainty in the middle period, in the three-period model, there may not be a unique equilibrium threshold, and as shown in Appendix B.1, the $\bar{D}_{(3)}^0(w_1)$ function might have no, one, two or three distinct real roots. To characterise the bounds of MPE, note that the condition for the lower bound of the MPE (where both fishing firms overfish) is the same as the two-period model, i.e. players will overfish for all values of $w_1 \in W_1$. But, the upper bound of the MPE in the three-period model, admits three possible cases¹⁹.

In the first case, similar to Figure 3.2 in the two-period model, there is a unique upper-bound threshold equilibrium, below which there is a multiplicity of equilibrium outcomes, and above that there is a unique prediction of overfishing. However, there is another possibility, depicted in Figure 3.4, where $\bar{D}_{(3)}^0(w_1)$ has three roots. Actually, in this case the fear of future depletion is affecting the root of $\bar{D}_{(3)}^0(w_1)$, and exacerbating the coordination on sustainable fishing. So, there is not a threshold-type equilibrium in this case and the range of multiplicity of equilibrium outcomes increases by the segment between the second and third roots of the $\bar{D}_{(3)}^0(w_1)$ function. In addition, for some parameter values, at the upper bound MPE, for any level of the stock firms harvest sustainably. So, in this case the range of multiplicity of

¹⁸as commonly assumed in the literature of global games

¹⁹If $\bar{D}_{(3)}^0(w_1)$ has two roots, where one of them is generated by tangency of $\bar{D}_{(3)}^0(w_1)$ to the horizontal axis, then again it will lead to a unique threshold strategy MPE.

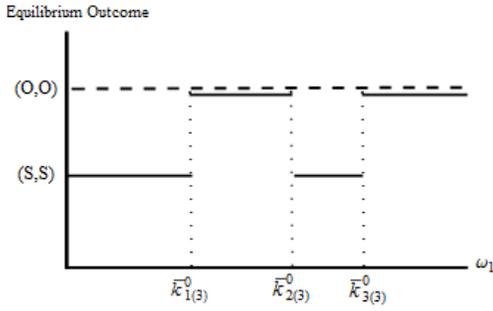


Figure 3.4: Multiplicity of MPE outcomes in the three-period model if the indifference condition has three roots. Solid line: Equilibrium Outcome of the upper bound of MPE (or range of multiplicity of equilibrium outcomes). Dashed line: Equilibrium Outcome of the lower bound of MPE.

equilibrium outcomes expands to all $w_1 \in [0, 1]$. From now onwards the analysis of the three-period model is focused on a subset of games $G_{(3)} \subset G$ that $\bar{D}_{(3)}^0(w_1)$ is greater than or equal to zero. Therefore, there is multiplicity of equilibrium outcomes for some ranges of the precise signal.

About the comparison of the upper bound of the MPE thresholds of the two and three-period models, beside the difference in the behaviour of expected payoffs at the upper bound of set of MPE, some other comparisons are summarised in the following remarks.

Remark 1. For any w_1 such that in the three-period model $\Pr(w_2 \geq \bar{k}_{(2)}^0 | w_1) = 1$, then $\bar{D}_{(3)}^0 = \bar{D}_{(2)}^0$.

See also the plots in Appendix B.4 for some parameter values. In other words, if $\bar{k}_{(2)}^0$ does not belong to the region of w_1 such that in a three-period model, $\Pr(w_2 \geq \bar{k}_{(2)}^0 | w_1) = 1$, then the (first) root of $\bar{D}_{(3)}^0$ is above $\bar{k}_{(2)}^0$. More precisely,

Remark 2. If $\beta > \frac{(w_1^\alpha)(1-2r)}{\mathbb{E}[w_2^\alpha | w_1, \sigma_1^i = \sigma_1^{-i} = S]}$, and $\sigma_1^i = \sigma_1^{-i} = S$, then $\bar{D}_{(3)}^0 \geq \bar{D}_{(2)}^0$ for all values of $w_1 \in W_1$.

Proof. See Appendix A.2.

In fact, in the three-period model, if the discount factor is sufficiently large, then possible gain of the middle period makes the future more valuable for the fishing firm. Therefore, it will be more profitable to coordinate on sustainable fishing, which translates to the shift of the indifference condition of the first period of the three-period model to the right. This in turn implies increase of the range of multiplicity of equilibrium outcomes.²⁰

Remark 3. If β increases, for a larger range of w_1 , the probability of coordination in the second period, $\Pr(w_2 < \bar{k}_{(2)}^0 | w_1)$, will be degenerated at 1.

Indeed, if the harvesting agents are more patient in all periods, then in the first period, at a higher level of the state variable, the fear of future depletion comes to exist, so it increases the possibility of today's coordination for any given w_1 . This is through the effect of the discount factor on the expected payoff of the third period, which increases the weight of payoff of sustainable harvest over excessive fishing in the second period.

Remark 4. If the public signal becomes more precise, i.e. c decreases, the two kinked points of $\bar{D}_{(3)}^0$ get closer to each other.

²⁰This result is in line with different theoretical studies in the application, which also are extensively reinforced by several field studies in renewable resources. For example, according to Ostrom (1992), Schlager and Ostrom (1992) and Baland and Platteau (1996), there is a consensus in field experiments that one of the attributes of harvesting agents which lead to successful self-governing of common-pool resources is the patience of appropriators.

As an example see Appendix B.3. Unlike the discount factor, the precision of the public signal is a parameter that policy makers can directly control through the media or press, by providing the firms with more accurate information about the resource. Indeed, in a three-period model and under precise-signal assumption, the role of public signal is decreasing the range of existence of the fear of future depletion²¹. It sheds light on the fact that in a two-period model, where there was no uncertainty about the future outcomes, the public signal was superfluous.

4 Biological and Strategic Uncertainty

Under precise information, the fishing firms could perfectly anticipate the other firm's harvest in equilibrium. However, in this section, the model is analysed under the more plausible assumption of having both biological and strategic uncertainties. Indeed, the fishing firms do not assess the state of environment precisely in any period, but they obtain public signals before the biological shocks and noisy private signals after them.

When there is private noisy information, fear of unilateral overfishing by the other firm, may affect players' incentives. In such a complicated environment, in which players potentially should take into account higher-order beliefs about the other player's beliefs about the fundamental (or the state variable), Carlsson and van Damme (1993) provide a tractable method, which ends up with a simple monotone strategy equilibrium that does not depend on higher-order beliefs. Indeed their analysis is based on vanishing noise of the private signals, although the state variable will never become common knowledge at the stage of decision making. Therefore, in such a setting, our fishing firms do not need to include the complete reasoning process of each others' beliefs in their decision making²², and the equilibrium takes a simple form that just depends on the firms' signals.

In the following sections, after introducing and analysing the general model under the assumption of imprecise signals, in order to improve the predictive power of the model, the equilibrium-selection technique of Carlsson and van Damme (1993) is applied. Then, its result is compared with the risk-dominance criterion of Harsanyi and Selten (1988), which is an equilibrium-selection technique in the unperturbed games.

For the infinite-horizon global games with i.i.d. state variables, Chassang (2010) adopts the dynamic programming approach of Abreu, Pearce and Stacchetti (1990), and introduces a tractable approach to find the Perfect Bayes Nash Equilibrium (PBNE) of the game. Indeed, the stationarity assumption in his model, which results from the i.i.d. noise, causes constant threshold equilibria in different periods. It simplifies the problem of finding the MPE of the infinite-horizon game to the analysis of a one-shot game augmented with the continuation payoffs²³. However, here the state variable, which is the state of

²¹This finding is also in line with the field studies of Elinor Ostrom(1992). She lists the reliability and predictability of resources (here decrease of c) as one of the main attributes of successful self-governed common pools.

²²See Morris and Shin (2003) for more discussion.

²³In fact, he derives a fixed-point equation between the equilibrium value of the infinite horizon game and the equilibrium value of playing a one-shot global game augmented with the continuation values. In addition, as noise vanishes, by convergence of the value of the one-shot game to the risk-dominance continuation value, finding the asymptotic PBNE of the infinite-horizon game will be even more tractable.

the fishery, depends on its previous values. So, the threshold equilibria are evolving over time, similar to Morris and Shin (1999, 2000) and Chamley (1999) where the state variable is a random-walk. But in contrast to them, all of the payoffs are function of the state variable and the evolution of players' incentives reinforces the dynamics²⁴. Hence, to handle the problem of a growing state variable in a context where payoffs are state-dependent, parallel to the precise-signal case, the analysis uses the backward induction in a two and three-period model. Also, it is shown that a three-period model is enough to study both the fear of future depletion and the fear of miscoordination.

4.1 A Model of Fear of Miscoordination: The Two-Period Model

To find the equilibrium threshold of the game in the first period for player i , $k_{(2)}^i$, she must postulate the probability of cooperation by the other player in that period, which is a function of his equilibrium threshold, $k_{(2)}^{-i}$. Assuming player i to be the player who exactly receives the threshold signal, she must be indifferent between the expected payoffs of sustainable fishing and overfishing in equilibrium. In other words, $\Delta_{(2)}^i(x_1^i, k_{(2)}^{-i})$ must be equal to zero, where

$$\Delta_{(2)}^i(x_1^i, k_{(2)}^{-i}) \equiv \mathbb{E} \left[(rw_1 + \frac{\beta}{2}w_2)\mathbf{1}_{\substack{\sigma_1^i=S \\ \sigma_1^{-i}=S}} + (rw_1)\mathbf{1}_{\substack{\sigma_1^i=S \\ \sigma_1^{-i}=O}} - ((1-r)w_1)\mathbf{1}_{\substack{\sigma_1^i=O \\ \sigma_1^{-i}=S}} - (\frac{w_1}{2})\mathbf{1}_{\substack{\sigma_1^i=O \\ \sigma_1^{-i}=O}} \middle| y_1, x_1^i \right] \quad (4.1)$$

The extended analysis of the model in this general case is discussed in Appendix A.3, however its overview is as follows. The first step is finding the posterior belief about the state of the fishery, and then firm i should find out the probability distribution of signal of the other player, x_1^{-i} . This distribution can be obtained by calculating a convolution of two independent distributions. It is shown that posterior distribution of the stock of fish, also the conditional distribution of signal of the other firm admits three different possible cases, where each may lead to a different MPE threshold of the game. Obtaining the distributions of w_1 and x_1^{-i} is sufficient to calculate the roots of $\Delta_{(2)}^i(x_1^i, k_{(2)}^{-i})$ for any of the three possible cases. Here in contrast to section 3, in equilibrium miscoordination may happen, and the private noise in the firms' assessment may deter the sustainability of the resource in this stay-exit setting.

Now let the private signals be extremely precise in the limit, which is not a totally abstract assumption²⁵. Using the global game analysis of Carlsson and van Damme (1993), it is possible to derive the unique MPE threshold, which is summarised in the next proposition.

Proposition 1. *In the imprecise-signal case of any game in $G_{(2)} \subset G$, and in the first period, as β converges to zero, there exists a unique symmetric MPE threshold, $k_{(2)}^*$, below which player i fishes sustainably and above which he will overfish. Furthermore, by assuming partial order on the strategy set, this will be the only PBNE of the game. In addition, it coincides with the risk-dominant threshold of the precise-signal game.*

²⁴In Morris and Shin (1999, 2000) and Chamley (1999), payoffs of choosing the 'safe' action (here overfishing) are constant.

²⁵For example, see Costello, Adams and Polasky(1998), and Costello, Polasky and Solow(2001) for improvement in the accuracy of forecast of the fish stocks.

For extended proof see Appendix A.4, however a sketch of the proof will be as follows. As the private signals become extremely precise, i.e. in the limit that a converges to zero, then $f_{W_t}(w_t | y_t, x_t^i)$ converges to $\frac{1}{2a}$, therefore $f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i)$ converges to the following symmetric triangular distribution²⁶,

$$f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) = \frac{1}{4a^2} \begin{cases} x_t^{-i} - x_t^i + 2a & \text{if } x_t^i - 2a \leq x_t^{-i} < x_t^i \\ -x_t^{-i} - x_t^i + 2a & \text{if } x_t^i \leq x_t^{-i} < x_t^i + 2a \end{cases} \quad (4.2)$$

and its cumulative distribution at player i 's signal will be $F_{X_t^{-i}}(x_t^i | y_t, x_t^i) = 1 - F_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) = \frac{1}{2}$.

Furthermore, given the symmetry of payoffs for both firms, the threshold equilibrium should be symmetric for both of them, i.e. $k_{(2)}^i = k_{(2)}^{-i} = k_{(2)}^*$. Hence, assuming that player i is the threshold-type player, then the equilibrium threshold, $k_{(2)}^*$, such that player i fishes sustainably iff $x_1^i < k_{(2)}^*$, solves $\Delta_{(2)}^i(k_{(2)}^*, k_{(2)}^*) = 0$. The unique root of $\Delta_{(2)}^i(k_{(2)}^*, k_{(2)}^*)$ is,

$$k_{(2)}^* = \left(\frac{\beta}{3}\right)^{\frac{1}{1-\alpha}} (1 - 2r)^{-1} \quad (4.3)$$

This threshold coincides with the risk-dominant threshold of the precise-signal case, $k_{(2)}^{RD}$, derived in appendix A.4. Also, note that in the limit the continuation value of the global game analysis defined in (4.1), converges to the continuation value of the risk-dominant case. In other words, the predictions of both models are similar, although here it is the private signal (rather than simply the state) which maps into a unique action, and always players in the equilibrium will coordinate on the less-risky action.

The two-period model can be interpreted as a glorified static game. So, the risk-dominance equivalence, which is a generic characteristic of the static global game, is inherited by the two-period model. However, as will be shown, in the three-period model, due to the fear of future depletion in the middle period, the risk-dominance equivalence does not necessarily hold.

Now for each level of the stock, it is possible to predict where the fishing firms harvest sustainably and where the exhaustion happens. Such a model with a unique prediction can serve as a benchmark for the institutional analysis or policy implications. For example, if the environment is not fragile, i.e. if the state of the fishery is at a level which is below the equilibrium threshold, then the theory of the global game suggests that by reduction in the noise of firms' assessment about the stock, without any ambiguity about the equilibrium outcome it can be deduced that the firms will trust each other and coordinate on preserving the stock. Intuitively, reduction of noise can be achieved through communication among the firms. Conversely, this result provides a possible explanation about the catastrophic situation of many fisheries around the world, where increasing the common knowledge about the adverse state of environment does not prevent its collapse, possibly because all of the harvesting agents have economically chosen to exhaust the resource.

²⁶Where $f(\cdot)$ refers to the probability density function and $F(\cdot)$ is the cumulative distribution function of the random variable.

4.2 A Model of Fear of Future Depletion and Fear of Miscoordination: The Three-Period Model

Now, again under the assumption that firms make noisy private assessment about the state of the fishery in each period, another period is added to the two-period model. Hence, the fear of future depletion, which is induced by the biological uncertainty in the middle period, through the channel of inertia in the growth function affects the continuation values. Also, here the risk of miscoordination, caused by the noise in the private information, makes cooperation even more difficult. Especially, as highlighted in the literature of global games²⁷ the risk of miscoordination is higher close to the roots of the indifference conditions, where players flip their behaviour.

Similar to equation (4.1), the MPE of the first period of the three-period model can be obtained by the following equation, showing the incentive to sustainable fishing,

$$\begin{aligned} \Delta_{(3)}^i(x_1^i, k_{(3)}^{-i}) \equiv & \\ \mathbb{E} \left[\left(rw_1 + \beta \left\{ (rw_2 + \frac{\beta}{2}w_3) \mathbf{1}_{\substack{x_2^i < k_{(2)}^* \\ x_2^{-i} < k_{(2)}^*}} + (rw_2) \mathbf{1}_{\substack{x_2^i < k_{(2)}^* \\ x_2^{-i} \geq k_{(2)}^*}} + ((1-r)w_2) \mathbf{1}_{\substack{x_2^i \geq k_{(2)}^* \\ x_2^{-i} < k_{(2)}^*}} + (\frac{w_2}{2}) \mathbf{1}_{\substack{x_2^i \geq k_{(2)}^* \\ x_2^{-i} \geq k_{(2)}^*}} \right\} \right) \mathbf{1}_{\substack{\sigma_1^i = S \\ \sigma_1^{-i} = S}} \right. & \\ \left. + (rw_1) \mathbf{1}_{\substack{\sigma_1^i = S \\ \sigma_1^{-i} = O}} - ((1-r)w_1) \mathbf{1}_{\substack{\sigma_1^i = O \\ \sigma_1^{-i} = S}} - (\frac{w_1}{2}) \mathbf{1}_{\substack{\sigma_1^i = O \\ \sigma_1^{-i} = O}} \mid y_1, x_1^i \right] & \end{aligned} \quad (4.4)$$

where again assuming player i as the threshold-type player, $\Delta_{(3)}^i(k_{(3)}^*, k_{(3)}^*) = 0$ implicitly gives the MPE of the game²⁸. Similar to the precise-signal case, in order to calculate the continuation value of the game and the probability of sustainable harvesting in the middle period, the MPE thresholds of the two-period model are used. In addition, since the continuation value depends on the private signals of the firms in the second period, the distribution functions of signal of the second period conditional on signals of the first period, i.e. $f_{X_2^i}(x_2^i \mid y_1, x_1^i)$ or $f_{X_2^{-i}}(x_2^{-i} \mid y_1, x_1^i)$ must be derived. In Appendix A.5, first $f_{X_2^i}(x_2^i \mid y_1, x_1^i)$ is worked out, then the conditional probability distribution of w_2 , i.e. $f_{W_2}(w_2 \mid y_1, x_1^i)$, along similar lines is calculated. Also, the probability of choosing sustainable fishing in the first period will be defined by the probability of the other firm choosing sustainable fishing. In other words, $f_{X_1^{-i}}(x_1^{-i} \mid y_1, x_1^i)$ is needed. But this probability is already calculated in the two-period model in (A.5), (A.6) and (A.7). Therefore, all the necessary elements to solve for the MPE of the three-period model are provided.

Additionally, as our special interest is in the equilibrium refinement, again a global game analysis is carried out. The following proposition, as the main result of the paper, is deduced from this analysis in the limit.

Proposition 2. *In the imprecise-signal case of any game in $G_{(3)} \subset G$, and in the first period, as the noise of private signal vanishes, then for any level of the private signal, there is a unique prediction about the outcome of the game. In addition, if $\Delta_{(3)}^i(k_{(3)}^*, k_{(3)}^*)$ admits a unique root, then there exists a unique*

²⁷Specifically discussed by Steiner(2008), and Chassang(2010).

²⁸Since the roots of $\Delta_{(3)}^i(k_{(3)}^*, k_{(3)}^*)$ may not be unique, here $k_{(3)}^*$ is not necessarily a threshold-type strategy

symmetric MPE threshold, $k_{(3)}^*$, below which player i fishes sustainably and above which he will overfish. Also this threshold coincides with the risk-dominant threshold of the precise-signal game.

Proof. Appendix A.6.

The first part of the proposition shows that the equilibrium-refinement results of Carlsson and van Damme (1993) generalises to a case where the state variable grows over time. It should be noted that in dynamic global games, there might be multiplicity of roots of the indifference conditions²⁹. Indeed this multiplicity of roots is different from having a continuum of equilibria in the precise-signal case. In our case, under precise signals there was a multiplicity of equilibrium outcomes for any level of the stock below the upper bound equilibrium, while by the global game analysis the predictive power of the model in the limit has increased from zero to one, although there might not be a threshold-type equilibrium.

However, in case of the multiplicity of roots of $\Delta_{(3)}^i(x_1^i, k_{(3)}^{-i})$ the risk-dominance equivalence result of Carlsson and van Damme (1993) does not hold in this model. In the limit, if the root of the indifference condition of player i is unique, then the conditional probability of sustainable fishing by the other firm in the first period at signal of player i , will be equal to half, i.e. as derived in (4.2), $\Pr(\sigma_1^{-i} = S \mid y_1, x_1^i) = F_{X_1^{-i}}(x_1^i \mid y_1, x_1^i) = \frac{1}{2}$. But if as the result of fear of future depletion, $\Delta_{(3)}^i(x_1^i, k_{(3)}^{-i})$ admits multiple roots (discussed in section 3.2), then the MPE of the perturbed game in the limit do not coincide with the risk-dominant equilibria. In the current application, its reason goes back to the renewability of the resource. In fact, if the state variable was i.i.d., the threshold MPE were fixed over time, and $F_{X_1^{-i}}(x_1^i \mid y_1, x_1^i)$ was always equal to half. But, here the biological uncertainty in the middle period which translates to the fear of future depletion, may cause non-uniqueness of the roots of $\Delta_{(3)}^i(x_1^i, k_{(3)}^{-i})$, in turn the risk-dominance equivalence does not generalise to the context of renewable resources³⁰.

Remark 5. For any x_1^i that $\Pr(x_2^i \geq k_{(2)}^* \mid y_1, x_1^i)$ converges to one, then as the noise of private signals vanishes, $\Delta_{(3)}^i(x_1^i, k_{(3)}^{-i}) \rightarrow \Delta_{(2)}^i(x_1^i, k_{(2)}^{-i})$

The remark states that in the first period of a three-period game if a harvesting firm, based on its current signal, postulates that tomorrow the private information that will be obtained is likely to lead to excessive exploitation, then in the limit the underlying problem will be reduced to a two-period model, because it is presumed that there is not any day after tomorrow. In addition, similar to section 3, if the threshold $k_{(3)}^*$ is determined in a region where the firm's incentive implies that they will definitely exhaust the resource in the second period, then the thresholds of two and three-period models will be the same. In other words, in the limit that information is extremely precise, adding any other period to the model will never affect the decision of harvesters today if the conditional belief of any of the firms about tomorrow's information lead it to excessive harvest.

Of course in a region where the probability of overfishing in the second period is interior, i.e. $0 < \Pr(x_2^i \geq k_{(2)}^* \mid y_1, x_1^i) < 1$, their cooperation might be threatened by the fear of future depletion (if the root appears in this region). In fact, with general private noise in the information structure, the second and third terms in the continuation payoff in (4.4) suggest that in a three-period model under strategic uncertainty, the fear of future depletion is exacerbated by the fear of miscoordination in the second period.

²⁹See Chassang (2010)

³⁰See appendix A.6.

Therefore, reducing the noise in the private information of harvesting firms (or reducing the fear of miscoordination) has two effects. The first effect is mitigating the fear of future depletion, which as discussed in the section 3.2 may affect the roots of the indifference condition. So, improving common knowledge among firms may help their cooperation in preserving the stock. The other effect of reduction of the risk of miscoordination, is through the channel of equilibrium refinement that serves the potential researcher or manager of the resource. As mentioned in the two-period model, any conclusion is just conditional on the information about the state of the stock and its reproducibility. For instance, if the stock is at a level where the reproduction rate is high, i.e. below the (first) root of the indifference condition, then as the information of different firms converge to the actual level of the stock, without any multiplicity in prediction, it can be inferred that as a self-fulfilling prophecy the harvesting firms coordinate on sustainable harvest.

5 Conclusions

This paper studies a dynamic non-cooperative fishery model, with stay-exit structure. Two types of biological and strategic uncertainties are embedded in the setup, and it is possible to distinguish the sources of different uncertainties and risk, while they may be interconnected in the continuation payoffs. Although the generalisation of the model to a finite number of periods is possible³¹, a three-period model is sufficient to deliver the message of the paper. It is shown how the biological uncertainty and the resulting fear of environmental collapse in the future may affect the behaviour of the harvesting agents, and how the fear of unilateral over-harvest by the other party, due to the strategic risk, adds to the adverse result of the biological uncertainty.

In addition, the paper presents a framework for refinement of equilibria and improving the predictive power of the underlying model. Answering questions of trust and coordination has a significant importance in resource economics, while the models of coordination with complete information have a multiplicity of equilibrium outcomes. Here, the incomplete information, introduced by the realistic uncertainties in the model, provides a suitable setup to apply the global game equilibrium-selection technique. The analysis suggests that as the private information of the harvesters become extremely precise, for any level of their private assessment, the theory is able to uniquely predict the equilibrium outcome. Furthermore, it is shown that departure from the i.i.d-state to a state-dependent global game may result in a difference of the global game equilibrium from its risk-dominance equivalence.

³¹The monotonicity of the growth function and Hormander's theorem assure the existence of the necessary probability distributions to analyse the model in a finite horizon.

A Appendix: The Extended Proofs

A.1 Conditioning $\bar{k}_{(2)}^0$ in $\bar{D}_{(3)}^0$

In order to derive the conditional probabilities of $\Pr(w_2 < \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S)$ and the $\mathbb{E}[w_3 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S]$, it is useful to define a function

$$J(w_1) \equiv \min \left\{ \bar{k}_{(2)}^0, bw_1^\alpha + c \right\}$$

Hence,

$$\begin{aligned} \Pr(w_2 < \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S) &= \int_{-\infty}^{J(w_1)} f_{W_2}(q \mid w_1) \mathbf{d}q \\ &= \begin{cases} \frac{J(w_1) - bw_1^\alpha + c}{2c} & \text{if } \bar{k}_{(2)}^0 > bw_1^\alpha - c \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where $f_{W_2} = \frac{1}{2c}$ defined on $[b(w_1)^\alpha - c, b(w_1)^\alpha + c]$. Similarly,

$$\begin{aligned} \mathbb{E}[w_3 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S] &= b \int_{-\infty}^{J(w_1)} q^\alpha f_{W_2}(q \mid w_1) \mathbf{d}q \\ &= \begin{cases} \frac{b}{\alpha+1} \left(\frac{[J(w_1)]^{\alpha+1} - [bw_1^\alpha - c]^{\alpha+1}}{2c} \right) & \text{if } \bar{k}_{(2)}^0 > bw_1^\alpha - c \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

A.2 Proof of Remark 2

If $\Pr(w_2 \geq \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S) = 0$, and $\sigma_1^i = \sigma_1^{-i} = S$ then $D_{(3)}^0(w_1) > D_{(2)}^0(w_1)$ if and only if $\beta > \frac{w_1^\alpha(1-2r)}{\mathbb{E}[w_2^\alpha \mid w_1, \sigma_1^i = \sigma_1^{-i} = S]}$. If $\Pr(w_2 \geq \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S) = 1$, and $\sigma_1^i = \sigma_1^{-i} = S$ then $D_{(3)}^0(w_1) = D_{(2)}^0(w_1)$ for all values of $w_1 \in W$. If $0 < \Pr(w_2 \geq \bar{k}_{(2)}^0 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S)$, and $\sigma_1^i = \sigma_1^{-i} = S$ then $D_{(3)}^0(w_1) > D_{(2)}^0(w_1)$ if and only if $\beta > \frac{w_1^\alpha(1-2r)}{\mathbb{E}[w_2^\alpha \mid w_1, \sigma_1^i = \sigma_1^{-i} = S]}$.

Therefore, if $\beta > \frac{w_1^\alpha(1-2r)}{\mathbb{E}[w_2^\alpha \mid w_1, \sigma_1^i = \sigma_1^{-i} = S]}$ and $\sigma_1^i = \sigma_1^{-i} = S$, then $D_{(3)}^0(w_1) \geq D_{(2)}^0(w_1)$ for all values of $w_1 \in W$. Note that the condition $\sigma_1^i = \sigma_1^{-i} = S$ is sufficient for $\frac{w_1^\alpha(1-2r)}{\mathbb{E}[w_2^\alpha \mid w_1, \sigma_1^i = \sigma_1^{-i} = S]}$ to be less than one, so the suggested set for β is not empty.

A.3 The General Analysis of the Two-Period Model with Imprecise Signals

First, since the fishing agents are Bayesian learners, their posterior belief about state of the fishery can be obtained by a simple Bayes rule or the following formula,

$$f_{W_t}(w_t | y_t, x_t^i) = \frac{f_{\Xi}(w_t - y_t) f_E(x_t^i - w_t)}{\int f_{\Xi}(q - y_t) f_E(x_t^i - q) \mathbf{d}q} \quad (\text{A.1})$$

$$= \frac{1}{\int_{\max\{y_t - c, x_t^i - a\}}^{\min\{y_t + c, x_t^i + a\}} \mathbf{d}q} \quad (\text{A.2})$$

Assuming the private signal to be more precise than the public signal, i.e. if $a < c$,³²,

$$f_{W_t}(w_t | y_t, x_t^i) = \begin{cases} \frac{1}{x_t^i + a - y_t + c} & \text{if } w_t \in [y_t - c, x_t^i + a] \text{ and } x_t^i - a < y_t - c < x_t^i + a < y_t + c \\ \frac{1}{2a} & \text{if } w_t \in [x_t^i - a, x_t^i + a] \text{ and } y_t - c < x_t^i - a < x_t^i + a < y_t + c \\ \frac{1}{y_t + c - x_t^i + a} & \text{if } w_t \in [x_t^i - a, y_t + c] \text{ and } y_t - c < x_t^i - a < y_t + c < x_t^i + a \end{cases} \quad (\text{A.3})$$

In addition, from the point of view of player i , the probability distribution of signal of the other player can be derived by calculating the convolution of two independent distributions, i.e.

$$f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) = \int f_{W_t}(x_t^{-i} - p | y_t, x_t^i) f_E(p) \mathbf{d}p \quad (\text{A.4})$$

which depending on the support of W_t , admits three possible cases:

Case I: If $w_t \in [y_t - c, x_t^i + a]$ and $x_t^i - a < y_t - c < x_t^i + a < y_t + c$,

$$f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) = \frac{1}{2a(x_t^i + a - y_t + c)} \int_{\max\{-a, x_t^{-i} - x_t^i - a\}}^{\min\{a, x_t^{-i} - y_t + c\}} \mathbf{d}p \quad (\text{A.5})$$

$$= \frac{1}{2a(x_t^i + a - y_t + c)} \begin{cases} x_t^{-i} - y_t + c + a & \text{if } x_t^{-i} \in [y_t - c - a, x_t^i] \\ 2a & \text{if } x_t^{-i} \in [x_t^i, y_t - c + a] \\ 2a - x_t^{-i} + x_t^i & \text{if } x_t^{-i} \in [y_t - c + a, x_t^i + 2a] \end{cases}$$

Case II: If $w_t \in [x_t^i - a, x_t^i + a]$ and $y_t - c < x_t^i - a < x_t^i + a < y_t + c$,

$$f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) = \frac{1}{4a^2} \int_{\max\{-a, x_t^{-i} - x_t^i - a\}}^{\min\{a, x_t^{-i} - x_t^i + a\}} \mathbf{d}p \quad (\text{A.6})$$

$$= \frac{1}{4a^2} \begin{cases} x_t^{-i} - x_t^i + 2a & \text{if } x_t^{-i} \in [x_t^i - 2a, x_t^i] \\ x_t^i - x_t^{-i} + 2a & \text{if } x_t^{-i} \in [x_t^i, x_t^i + 2a] \end{cases}$$

³²As later it will be assumed that the noise of private signal converges to zero, this assumption does not affect the results. For further reference see Morris and Shin(2003).

Case III: If $w_t \in [x_t^i - a, y_t + c]$ and $y_t - c < x_t^i - a < y_t + c < x_t^i + a$,

$$\begin{aligned} f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i) &= \frac{1}{2a(y_t + c - x_t^i + a)} \int_{\max\{-a, x_t^{-i} - y_t - c\}}^{\min\{a, x_t^{-i} - x_t^i + a\}} \mathbf{d}p \\ &= \frac{1}{2a(y_t + c - x_t^i + a)} \begin{cases} x_t^{-i} - x_t^i + 2a & \text{if } x_t^{-i} \in [x_t^i - 2a, y_t + c - a] \\ 2a & \text{if } x_t^{-i} \in [y_t + c - a, x_t^i] \\ a - x_t^{-i} + y_t + c & \text{if } x_t^{-i} \in [x_t^i, a + y_t + c] \end{cases} \end{aligned} \quad (\text{A.7})$$

Clearly, case *I* and *III* of $f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i)$ are trapezoidal distributions and case *II* is a symmetric triangular distribution. All of these three segments of $f_{W_t}(w_t | y_t, x_t^i)$ and therefore $f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i)$ are possible, and using the indifference condition of player i , each of these three cases lead to a different pair of threshold strategy MPE.

For all of the three cases, equation (4.1) is equivalent to:

$$\begin{aligned} \Delta_{(2)}^i(x_1^i, k_{(2)}^{-i}) &\equiv \\ \Pr(x_1^{-i} < k_{(2)}^{-i} | y_1, x_1^i) &\mathbb{E} \left[(rw_1 + \frac{\beta}{2}w_2) - (1-r)w_1 \mid y_1, x_1^i \right] \\ \Pr(x_1^{-i} \geq k_{(2)}^{-i} | y_1, x_1^i) &\mathbb{E} \left[(rw_1 - \frac{w_1}{2} \mid y_1, x_1^i \right] \end{aligned} \quad (\text{A.8})$$

where

$$\mathbb{E}(w_t | y_t, x_t^i) = \int_{-\infty}^{\infty} f_{W_t}(w_t | y_t, x_t^i) q \mathbf{d}q \quad (\text{A.9})$$

and

$$\mathbb{E}(w_{t+1} | y_t, x_t^i) = b \int_{-\infty}^{\infty} f_{W_t}(w_t | y_t, x_t^i) q^\alpha \mathbf{d}q \quad (\text{A.10})$$

should be calculated according to the appropriate support of $f_{W_t}(w_t | y_t, x_t^i)$. Given symmetry of payoffs for both harvesting firms, and assuming firm i as the threshold-type player, then the MPE can be found by solving $\Delta_{(2)}^i(k_{(2)}^*, k_{(2)}^*) = 0$.

A.4 The Global Game Analysis of the Two-Period Model with Imprecise Signals

Assume the private signals are extremely precise, i.e. in the limit, a converges to zero, but the state of the fishery will never be common knowledge at the stage of decision making. Then the support of w_t converges to $[x_t^i - a, x_t^i + a]$, where $y_t - c < x_t^i - a < x_t^i + a < y_t + c$. In other words, the other two possible supports of w_t either convert to this case or to the empty set. Therefore, $f_{W_t}(w_t | y_t, x_t^i)$ converges to $\frac{1}{2a}$, for a vanishing but non-zero a . Accordingly, in the limit, $f_{X_t^{-i}}(x_t^{-i} | y_t, x_t^i)$ converges to the symmetric triangular distribution (A.6), where its cumulative distribution at player i 's signal will be $F_{X_t^{-i}}(x_t^i | y_t, x_t^i) = 1 - F_{X_t^{-i}}(x_t^i | y_t, x_t^i) = \frac{1}{2}$.

In addition, using the linearity of (A.8),

$$\lim_{a \rightarrow 0} \mathbb{E}(w_t \mid y_t, x_t^i) = x_t^i \quad (\text{A.11})$$

and

$$\lim_{a \rightarrow 0} \mathbb{E}(w_{t+1} \mid y_t, x_t^i) = b(x_t^i)^\alpha \quad (\text{A.12})$$

hence, in the limit $\Delta_{(2)}^i(k_{(2)}^*, k_{(2)}^*)$ converges to

$$\frac{1}{2} \left[\left(3r - \frac{3}{2}\right) k_{(2)}^* + \frac{\beta}{2} b(k_{(2)}^*)^\alpha \right]$$

where its non-trivial root is

$$k_{(2)}^* = \left(\frac{\beta}{3}\right)^{\frac{1}{1-\alpha}} (1 - 2r)^{-1} \quad (\text{A.13})$$

According to Chassang (2010), by assuming a partial order on the set of pure strategies, the best response is monotone in the strategy of the other player. This is sufficient to use the results of Milgrom and Roberts(1990) and Vives(1990) that in this coordination game, the set of rationalisable equilibria is bounded by the Markovian equilibria. Hence, if there is a unique MPE in the game, this is the only equilibrium strategy.

The final step is deriving the risk-dominant threshold of the precise-signal game. In the first period of any game in $G_{(2)} \subset G$, based on Harsanyi and Selten (1988), sustainable-fishing strictly risk-dominates overfishing if the product of deviation losses of sustainable-fishing is larger than those of overfishing, i.e.

$$\left[rw_1 + \frac{\beta}{2} \mathbb{E}[w_2 \mid w_1, \sigma_1^i = \sigma_1^{-i} = S] - (1 - r)w_1 \right]^2 - \left[\frac{w_1}{2} - rw_1 \right]^2 > 0 \quad (\text{A.14})$$

Since in any game in $G_{(2)}$, (S, S) and (O, O) are pure Nash Equilibria of the game, therefore using (3.1) and (3.3), the equation (A.14) can be written as $D_{(2)}^{RD}(w_1) > 0$, where

$$D_{(2)}^{RD}(w_1) \equiv \left[\left(3r - \frac{3}{2}\right) w_1 + \frac{\beta}{2} b(w_1)^\alpha \right] \quad (\text{A.15})$$

Hence, the risk-dominant equilibrium admits a threshold form, below which firm i fishes sustainably and above which he will overfish. Let $k_{(2)}^{RD}$ be such a threshold which is the non-trivial root of (A.15), indeed

$$k_{(2)}^{RD} = \left(\frac{\beta}{3}\right)^{\frac{1}{1-\alpha}} (1 - 2r)^{-1}$$

Note that $k_{(2)}^{RD}$ is strictly smaller than $\bar{k}_{(2)}^0$ for any $0 < \alpha < 1$.

A.5 The General Analysis of the Three-Period Model with Imprecise Signals

The distribution of private noises is identical among the firms. Therefore, from the point of view of firm i in the first period, X_2^i and X_2^{-i} are identically distributed. In addition, due to the linear structure

of the indifference conditions, the order of taking limit of noise of different periods, (by differentiation between private noise of different periods) does not affect our results³³.

Since,

$$\begin{aligned} x_2^i &= b(w_1)^\alpha + \xi_2 + \varepsilon_2^i \\ &= y_2 + \xi_2 + \varepsilon_2^i \end{aligned}$$

and given our distributional assumptions, it is possible to gain from Hormander's theorem (1983), that if two distribution functions are compactly supported and continuous, then their convolution exists and it is also compactly supported and continuous. Hence, to derive the distribution of X_2^i , first the distribution of Y_2 is worked out, then the distribution of $\Xi_2 + E_2^i$ is calculated, and finally their joint convolution, as the distribution of X_2^i or X_2^{-i} is derived.³⁴

In order to find the probability distribution of a function of a random variable³⁵, because y_2 is a monotone transformation of w_1 , it is possible to define its inverse function,

$$\begin{aligned} w_1 &= \left(\frac{y_2}{b}\right)^{\frac{1}{\alpha}} \\ &\equiv g^{-1}(y_2) \end{aligned}$$

Therefore,

$$\begin{aligned} f_{Y_2}(y_2 | y_1, x_1^i) &= f_{W_1}(g^{-1}(y_2) | y_1, x_1^i) \left| \frac{d}{dy_2} g^{-1}(y_2) \right| \\ &= f_{W_1}(g^{-1}(y_2) | y_1, x_1^i) \frac{y_2^{\frac{1-\alpha}{\alpha}}}{\alpha b^{\frac{1}{\alpha}}} \end{aligned}$$

also $f_{W_1}(w_1 | y_1, x_1^i)$ is derived in (A.3), thus

$$\begin{aligned} &f_{Y_2}(y_2 | y_1, x_1^i) \\ &= \frac{y_2^{\frac{1-\alpha}{\alpha}}}{\alpha b^{\frac{1}{\alpha}}} \begin{cases} \frac{1}{x_1^i + a - y_1 + c} & \text{if } y_2 \in [b(y_1 - c)^\alpha, b(x_1^i + a)^\alpha] \text{ and } x_1^i - a < y_1 - c < x_1^i + a < y_1 + c \\ \frac{1}{2a} & \text{if } y_2 \in [b(x_1^i - a)^\alpha, b(x_1^i + a)^\alpha] \text{ and } y_1 - c < x_1^i - a < x_1^i + a < y_1 + c \\ \frac{1}{y_1 + c - x_1^i + a} & \text{if } y_2 \in [b(x_1^i - a)^\alpha, b(y_1 + c)^\alpha] \text{ and } y_1 - c < x_1^i - a < y_1 + c < x_1^i + a \end{cases} \quad (\text{A.16}) \end{aligned}$$

It is clear that depending on its support, $f_{Y_2}(y_2 | y_1, x_1^i)$ may or may not depend on the precision of public signal.

The next step is finding the distribution of $\Xi_2 + E_2^i$. Let $m_2 \equiv \xi_2 + \varepsilon_2^i$ and again $a < C$. The convolution of these two independent uniform random variables gives the following trapezoidal distribution:

³³The results are available on request.

³⁴Note that the biological shocks are serially independent, so Y_2 and Ξ_2 are independently distributed.

³⁵For example see Greene(2003)

$$\begin{aligned}
f_{M_2}(m_2 | y_1, x_1^i) &= \int f_{\Xi}(m_2 - q | y_1, x_1^i) f_E(q) \mathbf{d}q \\
&= \frac{1}{4ac} \int_{\max\{-a, m_2 - c\}}^{\min\{a, m_2 + c\}} \mathbf{d}q \\
&= \frac{1}{4ac} \begin{cases} m_2 + c + a & \text{if } -a - c < m_2 < a - c \\ 2a & \text{if } a - c < m_2 < -a + c \\ a - m_2 + c & \text{if } -a + c < m_2 < a + c \end{cases}
\end{aligned} \tag{A.17}$$

Now the probability distribution of X_2^i will be

$$f_{X_2^i}(x_2^i | y_1, x_1^i) = \int f_M(x_2^i - p | y_1, x_1^i) f_{Y_2}(p | y_1, x_1^i) \mathbf{d}p \tag{A.18}$$

where depending on the supports of the distribution of W_1 , and in turn Y_2 , has three general possible cases. However, here only the result for the case where $w_t \in [x_t^i - a, x_t^i + a]$ and $y_t - c < x_t^i - a < x_t^i + a < y_t + c$ is provided³⁶, which itself admits three possibilities:

$$\begin{aligned}
&f_{X_2^i}(x_2^i | y_1, x_1^i) \\
&= \frac{1}{8\alpha c a^2 b^{\frac{1}{\alpha}}} \begin{cases} \int_{\max\{b(x_1^i - a)^\alpha, x_2^i - a + c\}}^{\min\{b(x_1^i + a)^\alpha, x_2^i + a + c\}} (x_2^i - p + c + a) p^{\frac{1-\alpha}{\alpha}} \mathbf{d}p & \text{if } y_2 - a - c < x_2^i < y_2 + a - c \\ \int_{\max\{b(x_1^i - a)^\alpha, x_2^i + a - c\}}^{\min\{b(x_1^i + a)^\alpha, x_2^i - a + c\}} (2a) p^{\frac{1-\alpha}{\alpha}} \mathbf{d}p & \text{if } y_2 + a - c < x_2^i < y_2 - a + c \\ \int_{\max\{b(x_1^i - a)^\alpha, x_2^i - a - c\}}^{\min\{b(x_1^i + a)^\alpha, x_2^i + a - c\}} (a - x_2^i + p + c) p^{\frac{1-\alpha}{\alpha}} \mathbf{d}p & \text{if } y_2 - a + c < x_2^i < y_2 + a + c \end{cases}
\end{aligned} \tag{A.19}$$

Since, $w_2 = y_2 + \xi_2$, by the same line of argument, the distribution function of W_2 is

$$\begin{aligned}
&f_{W_2}(w_2 | y_1, x_1^i) = \int f_{\Xi}(w_2 - q | y_1, x_1^i) f_{Y_2}(q | y_1, x_1^i) \mathbf{d}q \\
&= \frac{1}{2\alpha c b^{\frac{1}{\alpha}}} \begin{cases} \frac{1}{x_1^i + a - y_1 + c} \int_{\max\{b(y_1 - c)^\alpha, w_2 - c\}}^{\min\{b(x_1^i + a)^\alpha, w_2 + c\}} q^{\frac{1-\alpha}{\alpha}} \mathbf{d}q & \text{if } w_2 \in [b(y_1 - c)^\alpha - c, b(x_1^i + a)^\alpha + c] \\ \frac{1}{2a} \int_{\max\{b(x_1^i - a)^\alpha, w_2 - c\}}^{\min\{b(x_1^i + a)^\alpha, w_2 + c\}} q^{\frac{1-\alpha}{\alpha}} \mathbf{d}q & \text{if } w_2 \in [b(x_1^i - a)^\alpha - c, b(x_1^i + a)^\alpha + c] \\ \frac{1}{y_1 + c - x_1^i + a} \int_{\max\{b(x_1^i - a)^\alpha, w_2 - c\}}^{\min\{b(y_1 + c)^\alpha, w_2 + c\}} q^{\frac{1-\alpha}{\alpha}} \mathbf{d}q & \text{if } w_2 \in [b(x_1^i - a)^\alpha - c, b(y_1 + c)^\alpha + c] \end{cases}
\end{aligned} \tag{A.20}$$

Now all the necessary elements to analyse (4.4) are derived. Since the roots of (4.4) may not be unique, then the problem of finding the MPE is more laborious. In fact, for each possible support of X_1^{-i} , using a guess and verify approach for the location of roots, an algorithm can numerically check whether

³⁶The complete analysis is available on request

$$\Delta_{(3)}^i(k_{(3)}^*, k_{(3)}^*) = 0.$$

A.6 The Global Game Analysis of the Three-Period Model with Imprecise Signals

Let in all periods $a \rightarrow 0$. Based on the argument in A.4, $f_{W_1}(w_1 | y_1, x_1^i)$ converges to $\frac{1}{2a}$. Consequently,

$$\lim_{a \rightarrow 0} f_{Y_2}(y_2 | y_1, x_1^i) = \frac{y_2^{\frac{1-\alpha}{\alpha}}}{2a\alpha b^{\frac{1}{\alpha}}}$$

where $y_2 \in [b(x_1^i - a)^\alpha, b(x_1^i + a)^\alpha]$ and $y_1 - c < x_1^i - a < x_1^i + a < y_1 + c$.

Therefore, in the limit only the middle case of distribution of W_2 is relevant, where $w_2 \in [b(x_1^i - a)^\alpha - c, b(x_1^i + a)^\alpha + c]$ and $w_2 - c < b(x_1^i - a)^\alpha < b(x_1^i + a)^\alpha < w_2 + c$, also $\lim_{a \rightarrow 0} f_{W_2}(w_2 | y_1, x_1^i) = \frac{1}{2c}$. Similarly for $f_{X_2^i}(x_2^i | y_1, x_1^i)$, the support of x_2^i converges to $[y_2 + a - c, y_2 - a + c]$ where $x_2^i + a - c < x_1^i - a < x_1^i + a < x_2^i - a + c$, for a vanishing, none-zero a . So $\lim_{a \rightarrow 0} f_{X_2^i}(x_2^i | y_1, x_1^i) = \frac{1}{2c}$.

Similar to the conditions in A.1, in a three-period model

$$\lim_{a \rightarrow 0} \mathbb{E}(w_3 | y_1, x_1^i, x_2^i < k_{(2)}^*, x_2^{-i} < k_{(2)}^*) = \frac{b \left((\min[k_{(2)}^*, b(x_1^i)^\alpha + c]^{\alpha+1} - (b(x_1^i)^\alpha - c)^{\alpha+1} \right)}{2c(\alpha + 1)} \quad (\text{A.21})$$

and

$$\lim_{a \rightarrow 0} \Pr(x_2^i < k_{(2)}^* | y_1, x_1^i) = \frac{\left(\min[k_{(2)}^*, b(x_1^i)^\alpha + c] - b(x_1^i)^\alpha + c \right)}{2c} \quad (\text{A.22})$$

In addition, in the limit the probability of $x_2^i < k_{(2)}^*$ and $x_2^i \geq k_{(2)}^*$ and vice versa, converges to zero. Therefore, (4.4) can be written as

$$\begin{aligned} & \lim_{a \rightarrow 0} \Delta_{(3)}^i(x_1^i, k_{(3)}^{-i}) = \\ & \lim_{a \rightarrow 0} \Pr(\sigma_1^{-i} = S | y_1, x_1^i) \left[r x_1^i + \beta \left\{ \lim_{a \rightarrow 0} \Pr(x_2^i < k_{(2)}^* | y_1, x_1^i) \right. \right. \\ & \left. \left. \left(r \lim_{a \rightarrow 0} \mathbb{E}(w_2 | y_1, x_1^i) + \frac{\beta}{2} \lim_{a \rightarrow 0} \mathbb{E}(w_3 | y_1, x_1^i, x_2^i < k_{(2)}^*, x_2^{-i} < k_{(2)}^*) \right) + \right. \right. \\ & \left. \left. \left(1 - \lim_{a \rightarrow 0} \Pr(x_2^i < k_{(2)}^* | y_1, x_1^i) \right) \left(\frac{1}{2} \lim_{a \rightarrow 0} \mathbb{E}(w_2 | y_1, x_1^i) \right) \right\} - (1 - r)x_1^i \right] + \\ & \lim_{a \rightarrow 0} \Pr(\sigma_1^{-i} = O | y_1, x_1^i) \left[r x_1^i - \frac{x_1^i}{2} \right] \end{aligned} \quad (\text{A.23})$$

substituting form (A.12), (A.21) and (A.22), then $\lim_{a \rightarrow 0} \Delta_{(3)}^i(k_{(3)}^*, k_{(3)}^*) = 0$ gives the MPE of the game in the limit. If this equation has a unique root, then $\lim_{a \rightarrow 0} \Pr(\sigma_1^{-i} = S | y_1, x_1^i) = \lim_{a \rightarrow 0} \Pr(x_1^{-i} < k_{(3)}^* | y_1, x_1^i) = \frac{1}{2}$, so the limiting roots of (A.23) will be the same as roots of $D_{(3)}^{RD}(w_1)$, defined below

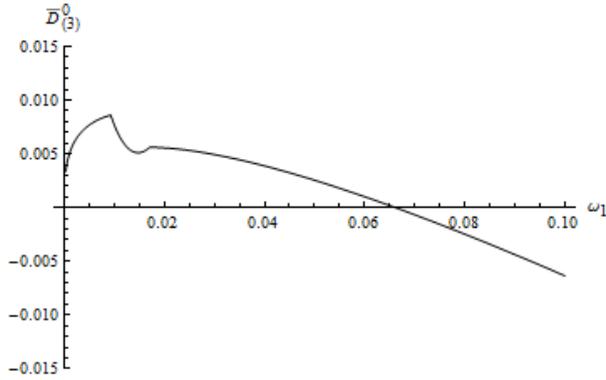
$$\begin{aligned}
D_{(3)}^{RD}(w_1) \equiv & \\
& \left[rw_1 + \beta \left\{ \Pr(w_2 < k_{(2)}^{RD} \mid w_1) \right. \right. \\
& \left. \left. \left(r\mathbb{E}(w_2 \mid w_1) + \frac{\beta}{2}\mathbb{E}(w_3 \mid w_1, w_2 < k_{(2)}^{RD}) \right) + \right. \right. \\
& \left. \left. \left(1 - \Pr(w_2 < k_{(2)}^{RD} \mid w_1) \right) \left(\frac{1}{2}\mathbb{E}(w_2 \mid w_1) \right) \right\} - (1-r)w_1 \right] + \\
& \left[rw_1 - \frac{w_1}{2} \right]
\end{aligned} \tag{A.24}$$

Also, $D_{(3)}^{RD}(w_1)$ has a similar behaviour to $\bar{D}_{(3)}^0(w_1)$, which sheds light on the limiting behaviour of $\Delta_{(3)}^i(x_1^i, k_{(3)}^{-i})$. As mentioned, the order of taking limit of the noise of different periods does not affect the results. However, if as the result of multiplicity of roots, $\lim_{a \rightarrow 0} \Pr(\sigma_1^{-i} = S \mid y_1, x_1^i) \neq \frac{1}{2}$, then although in the limit the continuation values converge to the risk-dominant continuation value of the precise-signal case³⁷, the risk-dominance equivalence of the roots does not hold.

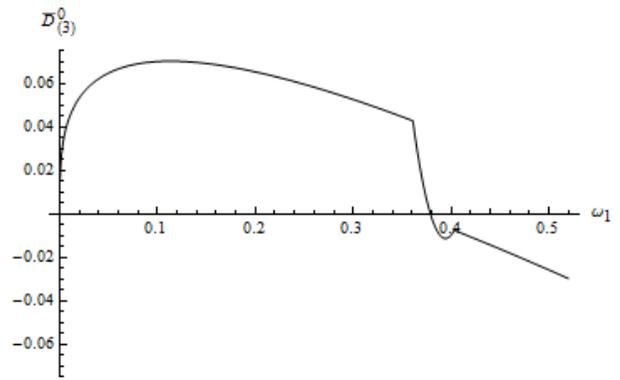
³⁷Similar to Chassang (2010)

B Appendix: The Plots

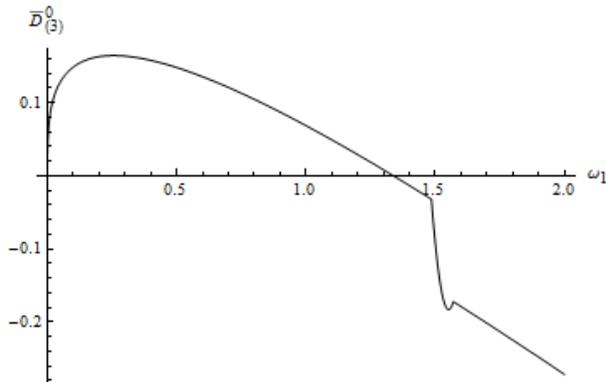
B.1 Numeric Examples of $D_{(3)}^0$ Function



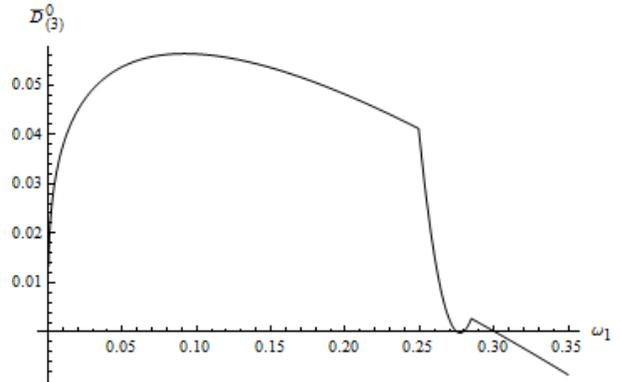
(a) $\beta = 0.3, c = 0.01, r = 0.33, \alpha = 0.5$



(b) $\beta = 0.7, c = 0.01, r = 0.33, \alpha = 0.5$



(c) $\beta = 0.99, c = 0.01, r = 0.33, \alpha = 0.5$



(d) $\beta = 0.64, c = 0.01, r = 0.33, \alpha = 0.5$

B.2 Different Expressions of $D_{(3)}^0$ Function

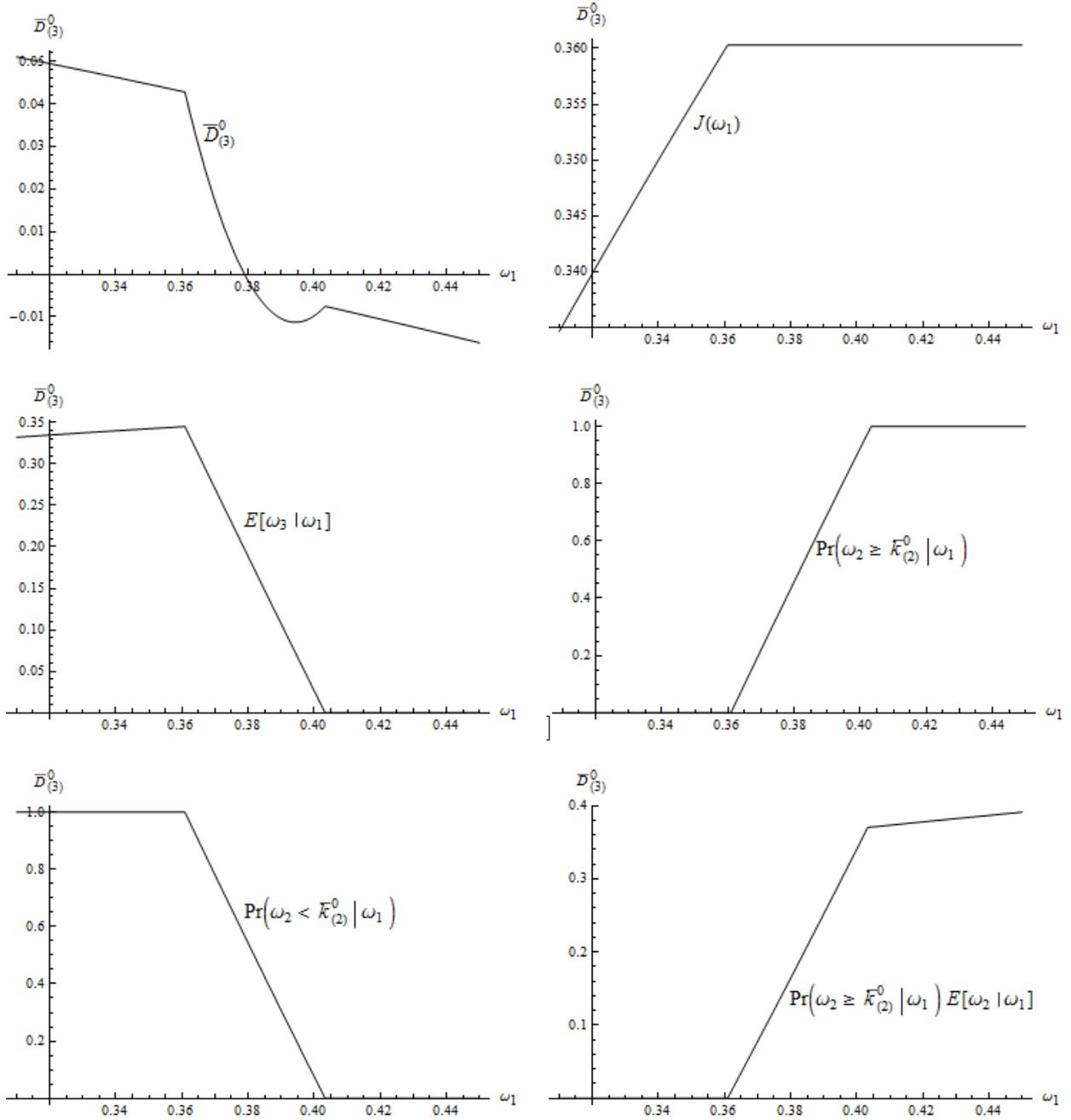
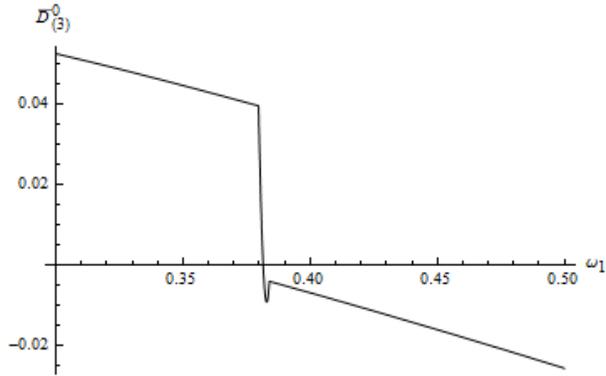
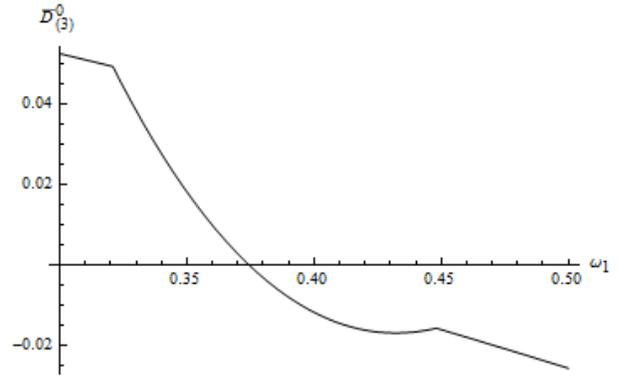


Figure B.1: $\beta = 0.7$, $c = 0.01$, $r = 0.33$, $\alpha = 0.5$

B.3 Comparative Statics of Parameter c

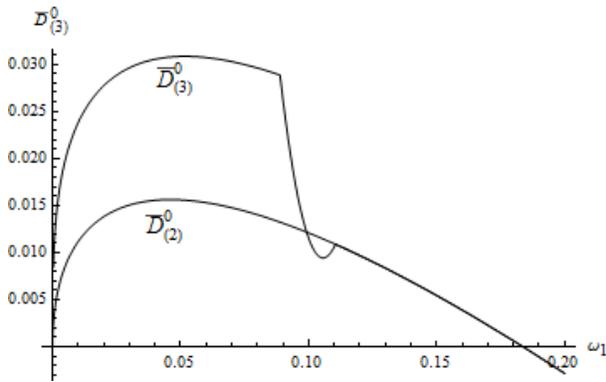


(a) $\beta = 0.7, c = 0.001, r = 0.33, \alpha = 0.5$

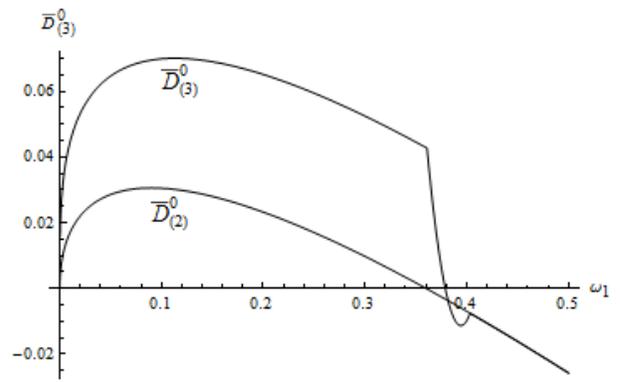


(b) $\beta = 0.7, c = 0.03, r = 0.33, \alpha = 0.5$

B.4 Comparison of $\bar{D}_{(3)}^0$ and $\bar{D}_{(2)}^0$



(c) $\beta = 0.5, c = 0.01, r = 0.33, \alpha = 0.5$



(d) $\beta = 0.7, c = 0.01, r = 0.33, \alpha = 0.5$

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