Existence and efficiency of stationary states in a renewable resource based OLG model: A comparison across types of harvest costs

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Abstract

Harvesting costs can reduce the incentive to overexploit a renewable resource stock, and particularly so when costs are stock dependent. In this paper, we compare different types of harvest costs in a renewable resource based OLG model and analyze under which conditions a nontrivial stationary state solution exists and whether this solution is efficient or not. We find that stock dependent harvest costs favor the existence of a nontrivial stationary state and that for intergenerational efficiency a positive own rate of return on the resource stock is no longer necessary. Whether constant or inversely stock dependent, harvest cost in general equilibrium necessitate the inquiry of a positive resource stock price ton ensure the existence of a stationary state market equilibrium.

Keywords: natural resources, harvest costs, overlapping generations, existence, efficiency

JEL Codes: Q20; D90; C62;

1 Introduction

There is no such thing as free lunch—or in the context of natural resources, harvesting involves effort which translates into costs in terms of time and/or money. For any renewable resource, harvest costs can be constant, depend on the harvest volume or on the resource stock (Bjørndal et al., 1993; Grafton et al., 2007). Constant harvest cost characterize a resource which is difficult to access but once access is achieved harvesting leads to no additional costs. In real world situations, this is hardly found, yet it is a common (implicit) modelling assumption because it is merely a generalization of the case without harvest cost (Smith, 1968). Harvest cost which depend on the harvest volume are typical for many renewable resources which are available in abundance, such as aquaculture or wood. Finally, harvesting effort and hence costs can also depend on the resource stock following the general wisdom 'the larger the stock, the easier to catch.'

The importance of harvest costs is fully acknowledged in partial equilibrium models of resource dynamics (Clark and Munro, 1975; Levhari et al., 1981; Olson and Roy, 1996). The main finding in partial equilibrium models is that stock dependence of harvest cost reduces the likelihood of overexploitation of the resource stock, an effect which does not occur for harvest costs which depend only on the harvest volume. Despite this significant impact of harvest costs, they are hardly found in intertemporal general equilibrium models (exemptions being Krutilla and Reuveny, 2004; Elíasson and Turnovsky, 2004; Bednar-Friedl and Farmer, 2013). In this paper, we therefore investigate how different types of harvest costs affect the main results in a renewable resource based overlapping generations (OLG) economy. We focus on the conditions for existence and efficiency of stationary state solutions. In comparison to the standard model without harvest costs, we find that results are changed both when harvest costs depend on the harvest volume and the resource stock.

While the question of the existence of equilibria is never posed in infinitely lived agent models as a nontrivial solution always exists, non–trivial stationary state equilibria may not exist in OLG models without harvest costs (Koskela et al., 2002;

Farmer, 2000). In contrast to a model without harvest cost, the existence of a stationary state is complicated in case of stock dependent harvest cost by the fact that the stock price of the renewable resource in general equilibrium might become negative for some feasible resource stock values (Bednar-Friedl and Farmer, 2013). The reason for negative resource price is that the harvest unit cost, which is also dependent on the wage rate, may become larger than the sales price of the resource harvest.

For a renewable resource based economy, it is also important whether a stationary state equilibrium is efficient or not. In an OLG model with a renewable resource but without harvest costs, the nontrivial stationary state solutions may or may not be efficient (Koskela et al., 2002). When a solution is inefficient, then the opportunity costs of holding the resource stock are negative—a situation which would imply a negative rate of return on assets. We therefore investigate whether such a situation can emerge, or is more likely to emerge (in terms of stringency of conditions), with some types of harvest cost functions than with others.

The remainder of the paper is structured as follows. Section 2 provides the description of the model, including a characterization of the different types of harvest costs. Section 3 derives the conditions for existence of nontrivial stationary states by comparing different types of harvest costs. The conditions for the efficiency of these solutions are analyzed and compared in section 4. Section 5 summarizes our results and concludes.

2 Model description

In this chapter, we start with the description of the general modelling framework. This framework consists of an OLG model with a renewable resource stock with concave regeneration and a general harvest cost function. The renewable resource is used as input in commodity production. Moreover, the resource stock serves as asset to transfer income from working to the retirement period. Households split their working time between employment in the production sector and harvesting.

After having outlined the general modelling framework, we proceed to the analysis of intertemporal equilibrium dynamics by distinguishing three cases for the harvest cost function: no harvest cost, constant unit harvest cost, and unit harvest costs that depend inversely on the resource stock.

To be able to analytically elaborate the consequences of different types of harvest cost, we assume log–linear utility and Cobb–Douglas technology and a logistic resource growth. As Lloyd-Braga et al. (2007) point out, a log–linear intertemporal utility function and a Cobb–Douglas production function ensure a unique steady state solution in spite of the endogeneity of labor supply. More general utility functions generate multiple steady state solutions which we want to avoid in order to be able to focus on the existence and efficiency implications of different types of time–consuming harvest cost. Our model without harvest costs is closely related to the parametric example of log–linear utility and Cobb-Douglas technology in Koskela et al. (2002). Regarding harvest cost, we build on the renewable resource based OLG model with stock dependent harvest cost in Bednar-Friedl and Farmer (2013) but for the purpose of comparison we also consider the cases of no harvest cost and linear harvest cost.

2.1 Household and firm optimization

The representative consumer's intertemporal utility depends on consumption during the working period, C_t^1, and consumption during the retirement period, C_{t+1}^2, $0 < \beta < 1$ denoting the time discount factor. The representative young household's preferences are represented by a log–linear intertemporal utility function:

$$u = u(C_t^1, C_{t+1}^2) = \; ln\, C_t^1 + \beta\, ln\, C_{t+1}^2. \tag{1}$$

The young household faces a budget constraint in each period of life. In the first period, the household splits total working time (normalized to one) on employment in the commodity production sector and on resource harvesting effort h. As a consequence of exclusive private property rights (as for e.g. a fish pond or a fishing ground), the younger household acquires the resource stock from the older household in the competitive resource stock market at the beginning of the period and can also appropriate the revenues from resource harvest in the current market period.[1] Moreover, because of the concavity of the regeneration function, rents on the resource stock occur which are appropriated by the resource–owning young generation.

Thus, in addition to wage income, the young household gains income from selling of the resource harvest X_t. This income is spent on consumption C_t^1. For transferring income to their retirement period, young households save in terms of the natural resource stock R_t^d. The budget constraint in the working period is thus:

$$p_t\, R_t^d + C_t^1 = w_t(1 - \mathrm{h}(X_t, R_t^d)X_t) + q_t X_t, \tag{2}$$

where w_t denotes real wage, q_t the price of resource harvest, p_t the price of the resource stock demanded, and the price of the consumption good in period t serves as the numeraire.

The harvest cost function $\mathrm{h}(X_t, R_t^d)$ is assumed to have the following properties: $\partial \mathrm{h}(X_t, R_t^d)/\partial X_t \geq 0$, $\partial \mathrm{h}(X_t, R_t^d)/\partial R_t \gtreqless 0$. Thus, unit harvest effort is either constant or increasing in the harvest volume and decreases or increases in the resource stock.

From saving the resource stock, the household gains income in the retirement period, which is spent on consumption C_{t+1}^2:

$$C_{t+1}^2 = p_{t+1} R_{t+1}. \tag{3}$$

The resource stock is also an asset to households and therefore the dynamics of the resource stock form the third constraint:

$$R_{t+1} = R_t^d + \mathrm{g}(R_t^d) - X_t. \tag{4}$$

The representative household thus chooses C_t^1, C_{t+1}^2, R_t^d, and X_t to maximize (1) with respect to (2) (taking account of (9)), (3), and (4).

[1] In contrast to this beginning–of–period market equilibrium notion, Koskela et al. (2002) use the end–of–period asset market equilibrium concept.

This yields the following first order condition for intertemporal optimality in consumption:

$$\frac{C_{t+1}^2}{\beta\, C_t^1} = \frac{\left[1 + \mathrm{g}'(R_t^d)\right] p_{t+1}}{p_t + w_t \mathrm{h}_R(X_t, R_t^d) X_t}, \tag{5}$$

which requires that the intertemporal marginal rate of substitution between consumption when young and consumption when old equals the net return on the resource stock.

The second condition equates the price of the resource stock to the net return on resource harvest (i.e. the revenue on selling an incremental additional unit, net of the marginal harvest cost):

$$p_t = \left\{q_t - w_t \left[\mathrm{h}_X(X_t, R_t^d) X_t + \mathrm{h}(X_t, R_t^d)\right]\right\} \left[1 + \mathrm{g}'(R_t^d)\right] - \\ - w_t \mathrm{h}_R(X_t, R_t^d) X_t. \tag{6}$$

In (6), assume first that $\mathrm{h}(X_t, R_t^d) = 0$. In that case, the price of the resource stock has to be equal to the price of the harvest taking account the growth of the stock occurring in the respective period. In the more general case when harvest cost depend on harvest volume X (see second term in the curly brackets of 6), the revenue from selling the resource harvest is reduced by the time and hence costs involved in harvesting. If moreover harvest cost depend on the resource stock (see third term on right hand side of 6), then the revenues on harvesting are increased by keeping an additional unit unharvested because the larger the resource stock, the lower are is harvesting effort.

The firm is assumed to behave competitively and to maximize profits given output and input prices. Output Y_t is produced according to a constant–returns–to–scale Cobb–Douglas production function $Y_t = (X_t^d)^\alpha (N_t^d)^{(1-\alpha)}$, with labor N_t^d and resource harvest X_t^d as inputs. The firm's first order conditions read as follows:

$$q_t X_t^d = \alpha Y_t, \qquad\qquad w_t N_t^d = (1-\alpha) Y_t. \tag{7}$$

All markets are assumed to clear every period, i.e. the markets for the resource stock ($R_t^d = R_t, \forall t$), for resource harvest ($X_t^d = X_t, \forall t$), and for labor ($N_t^d = 1 - \mathrm{h}(X_t, R_t^d) X_t, \forall t$). Finally, commodity market clearing coincides with Walras'

Law and is therefore redundant:

$$(X_t)^{\alpha}(1 - \mathrm{h}(X_t, R_t^d)X_t)^{(1-\alpha)} = C_t^1 + C_t^2. \tag{8}$$

2.2 Specification of resource regeneration and harvest cost functions

For concave resource regeneration, the logistic function will be used: $\mathrm{g}(R_t^d) = r\left[R_t^d - (R_t^d)^2/R_{\max}\right]$, where $r > 0$ denotes the regeneration rate and $R_{\max}$ the carrying capacity.

In the following analysis, different versions of unit harvest cost functions will be used representing the idea that resource harvest requires labor (or effort) as input (see e.g. Krutilla and Reuveny, 2004; Elíasson and Turnovsky, 2004). We will compare two version of the general harvest cost function $\mathrm{h}(X_t, R_t^d)$. First, we will consider the case of constant unit harvest cost: $\mathrm{h}(X_t, R_t^d) = \lambda$ with $\lambda > 0$.[2] Thus, harvesting effort ($\mathrm{h}(X_t, R_t^d)X_t$) is linear in the harvest volume.

Second, we will assume that effort is inversely related to the resource stock:[3]

$$h(X_t, R_t^d) = \lambda \frac{X_t}{R_t^d}. \tag{9}$$

As a benchmark to which we compare the two types of harvest costs, we will also analyze the case of no harvest costs: $\mathrm{h}(X_t, R_t^d) = 0$.

2.3 The intertemporal equilibrium dynamics

As in Koskela et al. (2002), the intertemporal equilibrium dynamics can be reduced to a two–dimensional system in R_t and X_t by using the goods market clearing

[2]More in line with basic economic reasoning, we could also assume that harvest costs are quadratic. While the analysis turns out much more complicated, the qualitative results are similar as in the case of linear harvest cost.

[3]Solving (9) for X_t yields the well–known Schaefer (1954) harvest function, a functional specification popular in mostly partial equilibrium fishery models (see, e.g. Brown, 2000; Conrad, 1999; Clark, 1990).

condition and the household's and firm's first order conditions.

Focusing first on the case of no harvest costs, substituting for R_{t+1} in (3) yields for X_t:

$$X_t = R_t^d + \mathrm{g}(R_t^d) - \frac{C_{t+1}^2}{p_{t+1}}. \tag{10}$$

Using (5),(6), and (10) in (2) gives the first intertemporal equilibrium condition:

$$\frac{X_t\left[1-\gamma(1-\alpha)\right]}{\alpha} = \Psi(R_t), \quad \Psi(R_t) \equiv \left\{\gamma\Phi(R_t) + \left[1+\mathrm{g}'(R_t)\right]\right\} R_t, \tag{11}$$

where $\gamma \equiv 1/(1+\beta)$ and $\Phi \equiv \mathrm{g}(R_t)/R_t - \mathrm{g}'(R_t)$.

For the case of constant unit harvest costs, (11) changes to:

$$\frac{X_t\left[1-\gamma(1-\alpha)-\lambda X_t\right]}{\left[\alpha-\lambda X_t\right]} = \Psi(R_t). \tag{11'}$$

For the case of harvest costs which depend inversely on the resource stock, (11) changes to:

$$\frac{X_t R_t\left[1-\gamma(1-\alpha)\right] - \lambda {X_t}^2\left[1+(1-\gamma)(1-\alpha)\right]}{R_t\left[\alpha R_t - \lambda X_t\right]} = \Psi(R_t). \tag{11''}$$

Regardless of the type of harvest cost function, the second intertemporal equilibrium condition is given by resource dynamics (4).

3 Existence of nontrivial stationary state

3.1 Reference model without harvest cost

Setting $X_{t+1} = X_t = X, \forall t$ and $R_{t+1} = R_t = R, \forall t$ in (4) and (11) gives the following stationary state conditions:

$$X = \mathrm{g}(R) \tag{12}$$

$$\Psi(R) = \frac{X\left[1-\gamma(1-\alpha)\right]}{\alpha}. \tag{13}$$

Inserting (12) into (13), we can define the left hand side by $\mathrm{LHS0} \equiv \Psi(R)$ and the right hand side by $\mathrm{RHS0} \equiv \mathrm{g}(R)\left[1-\gamma(1-\alpha)\right]/\alpha$. In order to ensure that the resource stock price is strictly positive at a stationary state, we define

$\mathrm{B} \equiv (p/[1+\mathrm{g}'(R)])\,(X/Y)$ which, if larger than zero, implies that $p > 0$. Acknowledging (6), we obtain $\mathrm{B0} = \alpha > 0$. Thus, in the model without harvest cost we know for sure that the resource stock price is positive for all $R \in (0, R_{max})$ and hence the stationary state solution is economically feasible.

For a nontrivial stationary state to exist, it is sufficient that the $\mathrm{LHS0}$ cuts $\mathrm{RHS0}$ from below at the intersection point.[4] As summarized in Prop. 1, this is the case if the slope of the left hand side at the origin is flatter than the slope of the right hand side.

Proposition 1 (Existence of stationary state without harvest cost) *Let* $\mathrm{g}(R) = r\,[R - R^2/R_{\max}]$ *and* $\mathrm{h}(R,X) = 0$. *Then a unique nontrivial stationary state solution* $R \in (0, R_{max})$ *with* $p > 0$ *exists if* $\lim_{R\to 0^+} \mathrm{LHS0}'(R) \equiv 1 + r < \lim_{R\to 0^+} \mathrm{RHS0}'(R) \equiv [1 - \gamma(1-\alpha)]\,r/\alpha$.

Proof 1 *At the origin,* $\mathrm{LHS0}(0) = \mathrm{RHS0}(0) = 0$ *but by assumption* $\lim_{R\to 0^+} \mathrm{LHS0}'(R) < \lim_{R\to 0^+} \mathrm{RHS0}'(R)$. *On the other hand,* $\mathrm{LHS0}(R_{max}) = [1 - (1-\gamma) \cdot r]\,/R_{max}$ *and* $\mathrm{RHS0}(R_{max}) = 0$ *and hence* $\mathrm{LHS0}(R_{max}) > \mathrm{RHS}(R_{max})$. *Since both* $\mathrm{LHS0}(R)$ *and* $\mathrm{RHS0}(R)$ *are continuous functions on* $[0, R_{max}]$, *the intermediate value theorem ensures the existence of a* $0 < R < R_{max}$ *such that* $\mathrm{LHS0}(R) = \mathrm{RHS0}(R)$.

For the uniqueness of the stationary state solution, we need to distinguish the range of R *on which* $\mathrm{RHS0}(R)$ *and/or* $\mathrm{LHS0}(R)$ *are monotonically increasing or decreasing. Assume first that the model parameters are such that the stationary state solution lies in* $[0, R_{max}/2]$. *Knowing that both* $\mathrm{LHS0}(R)$ *and* $\mathrm{RHS0}(R)$ *are monotonically increasing in* $[0, R_{max}/2]$ *and moreover that* $\lim_{R\to R_{max}/2} \mathrm{LHS0}'(R) =$

[4] Dynamic stability of stationary state solutions we do not inquire in this paper. For the no–harvest cost case we refer to the proof of asymptotic stability in the log–linera, Cobb-Douglas model of Koskela et al. (2002) while the dynamic stability of stationary states with constant and inverse stock dependent harvest cost we checked numerically for a broad set of parameter values.

$1 - r(1-\gamma) > \lim_{R \to R_{max}/2} \mathrm{RHS0}'(R) = 0$, *functions* $\mathrm{RHS0}(R)$ *and* $\mathrm{LHS0}(R)$ *intersect once on the interval* $[0, R_{max}/2]$.

If, on the other hand, the stationary state lies in $[R_{max}/2, R_{max}]$, $\mathrm{RHS0}(R)$ *is monotonically decreasing. If* $\mathrm{LHS0}(R)$ *is increasing, the intersection point with* $\mathrm{RHS0}(R)$ *is unique. In the opposite case of decreasing* $\mathrm{RHS0}(R)$, *the slope of* $\mathrm{RHS0}(R)$ *is larger than that of* $\mathrm{LHS0}(R)$ *since* $\lim_{R \to R_{max}^+} |\mathrm{LHS0}'(R)| < \lim_{R \to R_{max}^+} |\mathrm{RHS0}'(R)|$. ***qed***

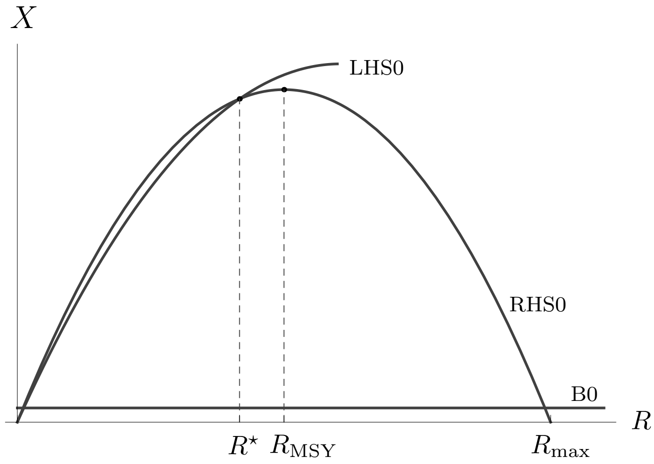

Figure 1: A unique stationary state $R^\star < R_{\mathrm{MSY}}$ in model without harvest cost

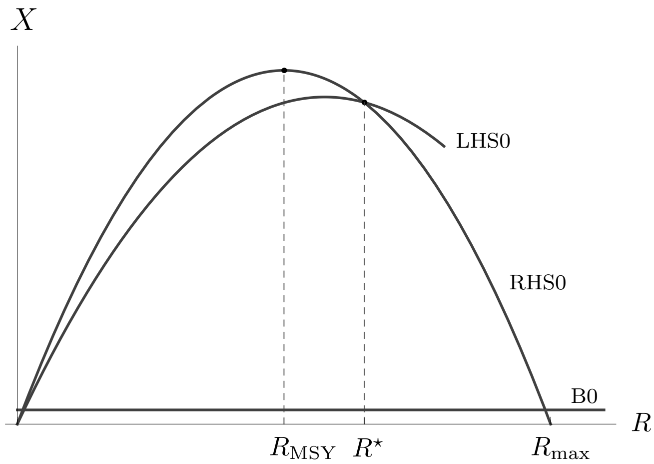

Figure 2: A unique stationary state $R^\star > R_{\mathrm{MSY}}$ in model without harvest cost

The economic intuition behind Prop. 1 is that two cases have to be distinguished.

In the first case, the model parameters are such that the stationary state lies between the origin and the maximum sustainable yield resource stock, $R_{MSY} = R_{max/2}$. This case is illustrated in Fig. 1.[5] Here, for a resource stock close to the origin, the growth potential of the resource stock is sufficiently large as compared to the productivity to the resource use in commodity production. Continuity of functions on the left and right hand side ensures that this also holds for larger levels. Moreover, as the regeneration function $\mathrm{g}(R)$ is concave and hence also $\mathrm{RHS0}$ is, there has to be an intersection with function $\mathrm{LHS0}$ as this function is monotonically increasing. As a consequence, there is a unique intersection point which is the nontrivial stationary state. The second case is depicted in Fig. 2 where the stationary state is larger than the maximum sustainable yield level but smaller than the carrying capacity. In that case, the slope of $\mathrm{RHS0}$ is negative but the slope of $\mathrm{LHS0}$ is certainly less steep leading also to a unique intersection point.

3.2 Model with constant unit harvest cost

For constant unit harvest cost, the stationary state condition changes to:

$$\Psi(R) = \frac{\mathrm{g}(R)\,[1-\gamma(1-\alpha)-\lambda\mathrm{g}(R)]}{[\alpha-\lambda\mathrm{g}(R)]}. \tag{14}$$

Proposition 2 (Existence with constant unit harvest cost) *Let* $\mathrm{g}(R) = r\,[R - R^2/R_{\max}]$ *and* $\mathrm{h}(R,X) = \lambda$. *If* $\lambda < 4\alpha/(rR_{max})$ *and moreover* $1+r < [1-\gamma(1-\alpha)]\,r/\alpha$, *or if* $\lambda \geq 4\alpha/(rR_{max})$, *then a unique nontrivial stationary state solution with* $p > 0$ *exists.*

Proof 2 *Denote the left hand side of* (14) *by* $\mathrm{LHSL}(R)$ *and the right hand side by* $\mathrm{RHSL}(R)$. $\mathrm{RHSL(R)}$ *exhibits two poles when* $\mathrm{BL}(R) \equiv \alpha - \lambda\mathrm{g}(R) = 0$. *For logistic regeneration, the poles can be calculated as* $\hat{R}_{1,2} = R_{max}/2 \pm \sqrt{\lambda r R_{max}\,(\lambda r R_{max} - 4\alpha)}$. *Both solutions are real if* $\lambda r R_{max} - 4\alpha \geq 0 \Longleftrightarrow$

[5] The figures are drawn for illustrative purposes based on the following parameter set: $\alpha = 0.3, r = 1.4, R_{max} = 10, \lambda = 0$. In Fig. 1, $\beta = 0.6$, while $\beta = 0.9$ in Fig. 2.

$\lambda \geq 4\alpha/(rR_{max})$. *If on the other hand* $\lambda rR_{max} - 4\alpha < 0$*, no pole emerges for* $\mathrm{RHSL}(R)$*.*

(i) Focusing first on the latter case, $\lambda rR_{max} - 4\alpha < 0$*, we have* $\mathrm{BL}(R) > 0$ *for* $R \in (0, R_{max}]$*. Analogously to the proof to Prop. 1 it is easy to verify that* $\mathrm{LHSL}(0) = \mathrm{RHSL}(0)$ *(see Fig. 3). On the other hand,* $\mathrm{LHSL}(R_{max}) = [1 - (1-\gamma)r]R_{max} > \mathrm{RHSL}(R_{max}) = 0$*. Since both functions are continuous on* $(0, R_{max}]$ *and by assumption* $\lim_{R\to 0^+} \mathrm{LHSL}'(R) < \lim_{R\to 0^+} \mathrm{RHSL}'(R)$*, at least one stationary state solution exists. Due to monotonicity of both functions for* $R \in (0, R_{max}/2)$*, the solution is unique.*

(ii) For the case of $\lambda rR_{max} - 4\alpha = 0$*, there is one pole* $\hat{R} = R_{max}/2$*. Since* $\mathrm{g}(R)$ *is maximal for* $R = R_{max}/2$ *and* $\mathrm{BL}(R_{max}/2) = 0$*, it follows that* $\mathrm{BL}(R) > 0$ *for all other admissible* R*. However, as* $\lim_{R\to 0^+} \mathrm{LHSL}'(R) < \lim_{R\to 0^+} \mathrm{RHSL}'(R)$ *and moreover* $\lim_{R\to R_{max}/2} \mathrm{RHSL}(R) = +\infty$*,* $\mathrm{LHSL}(R) \neq \mathrm{RHSL}(R)$ *for all* R *in* $[0, R_{max}/2)$*. To the right of the pole, i.e.* $R \in (R_{max}/2, R_{max}]$*,* $\mathrm{RHSL}(R)$ *decreases monotonically with larger* R *with* $\lim_{R\to R_{max}/2^+} \mathrm{RHSL}(R) = +\infty$ *and* $\lim_{R\to R_{max}} \mathrm{RHSL}(R) = 0$*. On the other hand,* $\mathrm{LHSL}(R_{max}/2) = 0$ *and* $\mathrm{LHSL}(R_{max}) > 0$*. Since both* $\mathrm{LHSL}(R)$ *and* $\mathrm{RHSL}(R)$ *are continuous functions of* $R \in (R_{max}/2, R_{max}]$ *an intermediate value theorem ensures a solution* $\mathrm{LHSL}(R) = \mathrm{RHSL}(R)$*. The solution is unique since* $\mathrm{RHSL}(R)$ *is monotonically decreasing and* $\mathrm{LHSL}(R)$ *is either monotonically increasing or when decreasing it decreases with an absolutely taken larger slope.*

(iii) If $\lambda rR_{max} - 4\alpha > 0$*, the two poles* $\hat{R}_1$ *and* $\hat{R}_2$ *occur (see Fig. 4). It is not difficult to see that* $\mathrm{BL}(R) > 0$ *for* $R \in [0, \hat{R}_1) \cup (\hat{R}_2, R_{max}]$ *and* $\mathrm{BL}(R) < 0$ *for* $R \in [\hat{R}_1), R_{max}]$*. By an analogous argument as in case (ii), it can be*

shown that $\mathrm{RHSL}(R) \neq \mathrm{RHSL}(R)$ *for* $R \in [0, \hat{R}_1)$ *while there is a unique solution in* $(\hat{R}_2, R_{max}]$. ***qed***

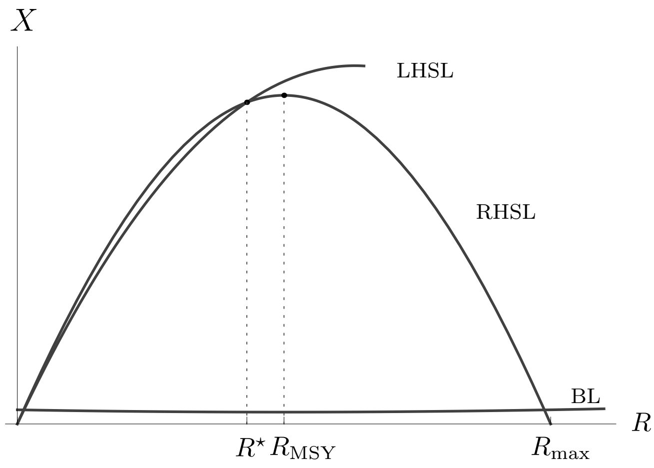

Figure 3: A unique stationary state $R^\star < R_{\mathrm{MSY}}$ in model with constant unit harvest cost

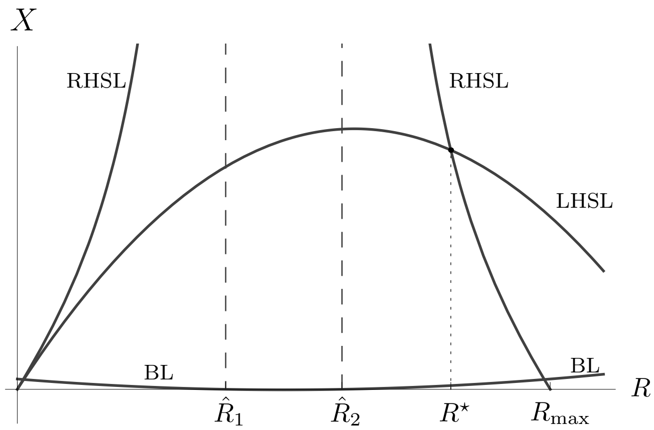

Figure 4: A unique stationary state $R^\star > R_{\mathrm{MSY}}$ in model with constant unit harvest cost

By comparing Prop. 2 to Prop. 1 it can be seen that the slope condition in the model without harvest cost translates to the similar condition in the model with constant unit harvest costs (see Fig. 3).[6]. In contrast to the model without harvest

[6]Fig. 3 is drawn for $\lambda = 0.014$, and Fig. 4 for $\lambda = 0.09$. For both figures, $\beta = 0.55$. All other model parameters are set as for Fig. 1.

cost, the positivity of the resource stock price p, which is equivalent to $\mathrm{BL}(R) > 0$, is no longer fulfilled for all feasible resource stocks as Fig. 4 illustrates. Between $\hat{R}_1$ and $\hat{R}_2$ function $\mathrm{BL}(R) < 0$ and hence the resource stock price p is negative precluding an economically feasible stationary state solution. While both to the left of $\hat{R}_1$ and to the right of $\hat{R}_2$ $\mathrm{BL}(R) > 0$, the relatively high harvest cost parameter $\lambda > 4\alpha/(rR_{max})$ enables a stationary state market equilibrium characterized by a high resource stock and a low harvest level.

The economic intuition of this result is that when harvest costs are large, decision makers have a higher incentive to keep the resource stock large. In contrast, when harvest costs are small, there is the danger that the resource stock is overexploited. It is therefore necessary that the natural growth rate r is large compared to the productivity of resource inputs in commodity production. If this latter condition is violated, a nontrivial steady state solution does not exist.

Since Krutilla and Reuveny (2004) conclude that harvest costs can lead to multiple solutions in the ILA context, it remains to be investigated whether harvest costs can lead to multiple solutions in an OLG framework, too. Likewise, Lloyd-Braga et al. (2007) find that endogenous labor supply can lead to multiple solutions. However, it follows from Prop. 2 that the nontrivial stationary state is unique also in the model with constant unit harvest cost, due to log–linear utility and Cobb-Douglas technology. Geometrically considered, uniqueness requires that both the slopes of the left and right hand side of (14) in the neighborhood of the stationary state are monotonic and that hence the left and right hand side intersect locally only once.

3.3 Model with inversely stock dependent harvest cost

For inverse stock dependent harvest cost, a stationary state is characterized by the following equation:

$$\Psi(R) = \frac{\mathrm{g}(R)\left\{[1-\gamma(1-\alpha)]\,R - [1+(1-\gamma)(1-\alpha)]\,\mathrm{g}(R)\lambda\right\}}{[\alpha R - \lambda \mathrm{g}(R)]}. \tag{15}$$

Proposition 3 (Existence with inverse stock dependent harvest cost) *Let* $\mathrm{g}(R) = r\,[R - R^2/R_{\max}]$ *and* $\mathrm{h}(R,X) = \lambda/R$. *If* $\lambda < \alpha/r$ *and moreover* $1 + r <$

$\{(1-\gamma)[1-(1-\alpha)\lambda r]-\gamma(1-\alpha)-\lambda r\}\, r/(\alpha-\lambda r)$*, or if* $\lambda \geq \alpha/r$*, then a unique nontrivial stationary state solution with* $p>0$ *exists.*

Proof 3 *Denoting again the left hand side of* (15) *by* $\mathrm{LHSR}(R) \equiv \Psi(R)$ *and the right hand side of* (15) *by* $\mathrm{RHSR}(R) \equiv \mathrm{g}(R)\{[1-\gamma(1-\alpha)]\,R-[1+(1-\gamma)(1-\alpha)]\,\mathrm{g}(R)\lambda\}\,/\mathrm{BR}(R), \mathrm{BR}(R) \equiv [\alpha R-\lambda \mathrm{g}(R)]$ *we find two poles of* $\mathrm{RHSR}(R)$ *which we get by setting* $\mathrm{BR}(R)=0$*:* $\hat{R}_1=0$ *and* $\hat{R}_2=[(\lambda r-\alpha)R_{max}]\,/(\lambda r)$*. For* $\lambda r=\alpha$*,* $\hat{R}_1=\hat{R}_2=0$ *while for* $\lambda>\alpha/r$ *only the second pole* $\hat{R}_2$ *exists.*

Commencing with the second case, $\lambda<\alpha/r$*, it is easy to show that* $\mathrm{BR}(R)>0$ *for all* $R \in (0,R_{max})]$ *(see Fig. 5). As in the proof to Prop. 2, we can show for this case that* $\mathrm{LHSR}(R)=\mathrm{LHSR}(R)$ *for* $0<R<R_{max}$ *if* $\lim_{R\to 0^+}\mathrm{LHSR}'(R)<\lim_{R\to 0^+}\mathrm{RHSR}'(R)$*.*

On the other hand, when $\lambda \geq \alpha/r$ *either* $\hat{R}_1=\hat{R}_2=0$ *or only the pole* $\hat{R}_2>0$ *exists. For* $R\in(0,\hat{R}_2)$*,* $\mathrm{BR}(R)<0$ *since* $\mathrm{BR}'(R)>0$*. Thus,* $\mathrm{LHSR}(R)$ *needs to intersect* $\mathrm{RHSR}(R)$ *to the right of the pole, i.e.* $R\in(\hat{R}_2,R_{max})$ *where* $\mathrm{BR}(R)>0$ *(see Fig. 6). Since* $\mathrm{LHSR}(R)>0$ *for all* $R\in(\hat{R}_2,R_{max}]$*, the denominator of* $\mathrm{LHSR}(R)$ *must be larger than zero. Hence,* $\lim_{R\to\hat{R}_2^+}\mathrm{RHSR}(R)=+\infty$ *while* $\mathrm{LHSR}(\hat{R}_2)<\infty$*. On the other hand,* $\mathrm{RHSR}(R_{max})=0$ *while* $\mathrm{LHSR}(R_{max})>0$*. As a consequence of the continuity of both* $\mathrm{LHSR}(R)$ *and* $\mathrm{RHSR}(R)$ *for* $R\in(\hat{R}_2,R_{max}]$*,* $\mathrm{LHSR}(R)=\mathrm{RHSR}(R)$ *for* $\hat{R}_2<R<R_{max}$*. The proof of the uniqueness is analogous to the proof of Prop. 2.* ***qed***

Again, the first case ($\lambda<\alpha/r$) of Prop. 3 is a generalization of the slope condition in Prop. 1 (model without harvest cost). As a consequence of stock dependent harvest cost, the first case (small harvest cost parameter) is valid for a larger range of R values as compared to the case with constant unit harvest cost (Fig. 5). This is the case as inverse stock dependent harvest cost imply for a small resource stock that unit harvest costs are high while they decrease with a larger resource stock.

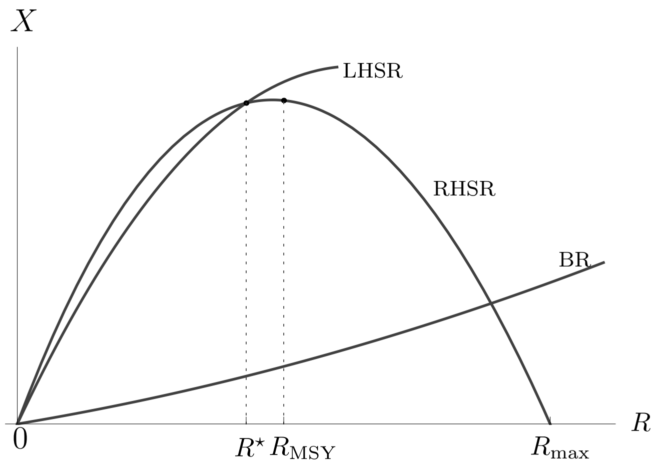

Figure 5: A unique stationary state $R^\star < R_{\mathrm{MSY}}$ in model with stock dependent harvest cost

For the second case (large harvest cost parameter), illustrated in Fig. 6, again no slope restriction is necessary. Yet, relative to the case with constant unit harvest cost, the harvest cost parameter needs to be larger when harvest costs depend inversely on the resource stock. The reason is again that for a large resource stock, unit harvest costs are driven downwards by the resource stock—an effect which cannot emerge when harvest costs only depend on harvest volume but not the resource stock.

4 Efficiency of nontrivial stationary state

Knowing that a nontrivial stationary state exists, we investigate whether (or under which conditions) the stationary state market solution is efficient. This is particularly relevant given the fact that the nontrivial stationary state solutions may or may not be efficient in an OLG model with a renewable resource but without harvest costs (Koskela et al., 2002).

To derive the conditions for (in-)efficiency of the stationary state solution, we set up the problem of a social planner who maximizes utility of each individual living in the stationary state and require that the utility of the oldest generation alive

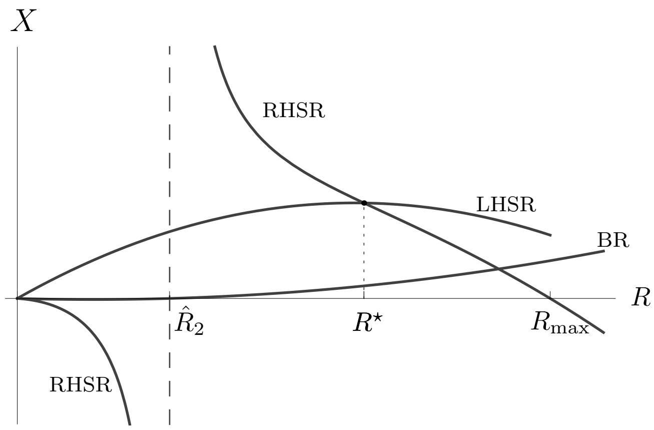

Figure 6: A unique stationary state $R^\star > R_{\text{MSY}}$ in model with stock dependent harvest cost

in the initial period (denoted by subscript $\circ$) achieves a predefined level:

$$\max W(C^1, C^2) = \ln\ C^1 + \beta\ \ln\ C^2$$

subject to

$$\begin{aligned}
(i) &\quad \ln\ C_o^2 = \ln\ \left(C_\circ^2\right)^\circ, \\
(ii) &\quad C^1 + C_\circ^2 = X^\alpha(1 - \mathrm{h}(R_\circ, X)X)^{1-\alpha}, \\
(iii) &\quad C^1 + C^2 = X^\alpha(1 - \mathrm{h}(R, X)X)^{1-\alpha}, \\
(iv) &\quad X = \mathrm{g}(R), \\
(v) &\quad R + X = R_\circ + \mathrm{g}(R_\circ), \\
(vi) &\quad (C_\circ^2, C^1, C^2) \geq 0,\ (R, R_\circ) \geq 0,\ X \geq 0,
\end{aligned}$$

where $R_\circ$ is the resource stock owned by the initially old generation.

Setting up the Lagrangian

$$\begin{aligned}
\mathcal{L} = \ln\ C^1 + \beta\ \ln\ C^2 + \mu_{-1}^C \left[\ln\ C_o^2 - \ln\ \left(C_\circ^2\right)^\circ\right] + \\
\phi_\circ^Y \left[X^\alpha(1 - \mathrm{h}(R_\circ, X)X)^{1-\alpha} - C^1 - C_\circ^2\right] + \\
+ \phi^Y \left[X^\alpha(1 - \mathrm{h}(R, X)X)^{1-\alpha} - C^1 - C^2\right] + \\
+ \phi^R \left[\mathrm{g}(R) - X\right] + \phi_\circ^R \left[R_\circ + \mathrm{g}(R_\circ) - R - X\right],
\end{aligned}$$

yields the following first order conditions:

$$\frac{C^2}{\beta C^1} = 1 + \frac{\phi^Y_\circ}{\phi^Y}, \tag{16a}$$

$$\frac{\mu^C_{-1}}{C^2_\circ} = \phi^Y_\circ, \tag{16b}$$

$$\begin{aligned}&\phi^Y_\circ \left\{ \alpha \frac{Y_\circ}{X} - (1-\alpha) \frac{Y_\circ \left[\mathrm{h}_X(R_\circ, X)X + \mathrm{h}(R_\circ, X)\right]}{(1 - \mathrm{h}(R_\circ, X)X)} \right\} + \\ &+ \phi^Y \left\{ \alpha \frac{Y}{X} - (1-\alpha) \frac{Y \left[\mathrm{h}_X(R, X)X + \mathrm{h}(R, X)\right]}{(1 - \mathrm{h}(R, X)X)} \right\} = \phi^R + \phi^R_\circ,\end{aligned} \tag{16c}$$

$$\phi^R \mathrm{g}'(R) = \phi^R_\circ + \phi^Y (1-\alpha) X^\alpha \left[1 - \mathrm{h}(R, X)X\right]^{(1-\alpha)} \mathrm{h}_R(R, X)X, \tag{16d}$$

$$\ln\ C^2_o = \ln\ \left(C^2_\circ\right)^\circ, \tag{16e}$$

$$Y_\circ = C^1 + C^2_\circ, \tag{16f}$$

$$Y = C^1 + C^2, \tag{16g}$$

$$R_\circ + \mathrm{g}(R_\circ) = R + X, \tag{16h}$$

$$\mathrm{g}(R) = X. \tag{16i}$$

To see whether individual utility and profit maximization leads to intergenerational efficiency, we compare (16) to household and firm first order conditions (5)–(7). We start again with the reference case without harvest cost before proceeding to linear and inverse stock dependent harvest cost.

4.1 Reference case without harvest cost

For the reference case without harvest cost, efficiency conditions (16c)–(16d) simplify to:

$$\alpha X^{\alpha-1} = \frac{\phi^R + \phi^R_\circ}{\phi^Y + \phi^Y_\circ},$$
$$\phi^R \mathrm{g}'(R) = \phi^R_\circ.$$

When moreover the utility of the initially old generation is disregarded as constraint for utility maximization of the young generation in the stationary state (Golden rule), $\mu^C_{-1} = 0$ and hence $\phi^Y_\circ = \phi^R_\circ = 0$. Then, the remaining efficiency conditions collapse to:

$$\frac{C^2}{\beta C^1} = 1, \tag{16a'}$$

$$\alpha X^{\alpha-1} = \frac{\phi^R}{\phi^Y}, \tag{16c'}$$

$$\mathrm{g}'(R) = 0, \tag{16d'}$$

$$X^\alpha = C^1 + C^2, \tag{16g'}$$

$$\mathrm{g}(R) = X. \tag{16i'}$$

Proposition 4 characterizes the properties of intergenerationally efficient stationary state solutions in the model without harvest cost, with the Golden Rule as a special case.

Proposition 4 (Efficiency of nontrivial stationary state (no harvest cost)) *Let* $\mathrm{g}(R) = r\left[R - R^2/R_{\max}\right]$ *and* $\mathrm{h}(R,X) = 0$*. The stationary state market equilibrium* $(X^\star, R^\star)$ *from* (12)–(13) *is intergenerationally efficient (Golden Rule) if* $\mathrm{g}'(R^\star) > 0$ *(*$\mathrm{g}'(R^\star) = 0$*). Otherwise, the stationary state market equilibrium is intergenerationally inefficient.*

Proof 4 *Assume first that the parameters of the OLG market economy are such that* $\mathrm{g}'(R^\star) > 0$*. Set provisionally* $q^\star = (\phi^R_\circ + \phi^R)/(\phi^Y_\circ + \phi^Y)$ *and* $\mathrm{g}'(R^\star) = \phi^Y_\circ/\phi^Y = \phi^R_\circ/\phi^R$*. Then, the market equilibrium conditions evaluated at the stationary state, i.e.*

$$\frac{(C^2)^\star}{\beta\,(C^1)^\star} = 1 + \mathrm{g}'(R^\star), \qquad p^\star = q^\star\left[1 + \mathrm{g}'(R^\star)\right], \tag{5'),(6'}$$

$$q^\star = \alpha(X^\star)^{1-\alpha}, \qquad w^\star = (1-\alpha)(X^\star)^\alpha, \tag{7'}$$

$$(X^\star)^\alpha = (C^1)^\star + (C^2)^\star, \qquad (X^\star)^\alpha = (C^1)^\star + (C^2_\circ)^\star, \tag{8'}$$

$$X^\star = \mathrm{g}(R^\star), \qquad R^\star + X^\star = R_\circ + \mathrm{g}(R_\circ), \tag{4'}$$

imply (16a')–(16i').

Second, assume that $\mathrm{g}'(R^\star) = 0$*. To show that this case is Golden Rule efficient, we start again from stationary state market equilibrium conditions. In the Golden Rule case,* $\mathrm{g}'(R^\star) = 0$ *holds which yields the modified stationary state market equilibrium conditions*

$$\frac{(C^2)^\star}{\beta\ (C^1)^\star} = 1, \qquad\qquad p^\star = q^\star. \qquad\qquad \text{(5''),(6'')}$$

But in the Golden Rule, also $\mu^C_{-1} = 0$ *which implies* $\phi^Y_\circ = \phi^R_\circ = 0$*. Hence we set* $q^\star = \phi^R/\phi^Y$ *in the intergenerational efficiency conditions* (16a'), (16c')–(16g'), (16i')*. These are identical to (5''), (6''), (7'), (8'), (4').* ***qed***

The two possible cases are illustrated by Figures 1– 2. In Figure 1, the stationary state market equilibrium is intergenerationally efficient—the stationary resource stock and thus, because of the no–arbitrage condition, the resource stock exhibits a positive own rate of return (underaccumulation of the resource stock occurs).[7] This resource stock is below the Golden Rule resource stock which coincides with the maximum sustainable yield level.

The opposite case is illustrated by Figure 2 in which $R^\star > R_{MSY}$ and hence the stationary state market equilibrium is intergenerationally inefficient. Thus, a central planner could increase welfare of the present and all future generations by a reduction in resource accumulation.

4.2 Constant unit harvest cost

If unit harvest costs are constant ($\mathrm{h}(R, X) = \lambda$), efficiency conditions (16c) changes to:

[7] Intergenerational (dynamic) efficiency is in line with stylized facts of advanced economies in which the real interest rate is strictly positive and hence also the own rate of return of the resource stock is positive.

$$\alpha X^{\alpha-1}\left[1-\lambda X\right]^{1-\alpha}-\lambda(1-\alpha)X^{\alpha}\left[1-\lambda X\right]^{-\alpha}=\frac{\phi^R+\phi^R_\circ}{\phi^Y+\phi^Y_\circ},$$

$$\phi^R \mathrm{g}'(R)=\phi^R_\circ. \tag{16c”}$$

while the other conditions are similar to the model without harvest cost.

In case of constant unit harvest cost, the stationary state market equilibrium as well the intergenerational efficiency conditions are complicated by additional terms. As a consequence, intergenerational efficiency or inefficiency depends on the level of the harvest cost parameter λ as summarized in Prop. (5).

Proposition 5 (Efficiency of stationary state (constant unit harvest cost)) *Let* $\mathrm{g}(R)=r\left[R-R^2/R_{\max}\right]$ *and* $\mathrm{h}(R,X)=\lambda$. *If unit harvest cost satisfies* $0<\lambda<\lambda^E$ *where* $\lambda^E\equiv\left[8\alpha-4(1-\gamma)r\right]/\left\{r\left[2-(1-\gamma)r\right]R_{max}\right\}$, *then the stationary state market equilibrium is intergenerationally efficient. The Golden Rule applies when* $\lambda=\lambda^E$. *When* $\lambda>\lambda^E$, *the stationary state market equilibrium is intergenerationally inefficient.*

Proof 5 *Evaluating again the market equilibrium conditions at the stationary state gives (5') and (4'),* $p^\star=(q^\star-w^\star\lambda)\left[1+\mathrm{g}'(R^\star)\right]$, $q^\star=\alpha(X^\star)^{\alpha-1}[1-\lambda X^\star]^{1-\alpha}$, $w^\star=(1-\alpha)(X^\star)^{\alpha}[1-\lambda X^\star]^{-\alpha}$, $(X^\star)^{\alpha}[1-\lambda X^\star]^{1-\alpha}=(C^1)^\star+(C^2)^\star$, $(X^\star)^{\alpha}[1-\lambda X^\star]^{1-\alpha}=(C^1)^\star+(C^2_\circ)^\star$. *Setting provisionally* $q^\star=(\phi^R_\circ+\phi^R)/(\phi^Y_\circ+\phi^Y)+\lambda w^\star$ *and again* $\mathrm{g}'(R^\star)=\phi^Y_\circ/\phi^Y=\phi^R_\circ/\phi^R$, *yields equivalence of the stationary state market equilibrium conditions with the intergenerational efficiency conditions* (16a)–(16i).

As in the case of no–harvest cost, the stationary state market equilibrium is stationary state efficient only if $\mathrm{g}'(R^\star)=\phi^Y_\circ/\phi^Y>0$, *i.e. for* $R^\star\in(0,R_{max}/2)$. *Acknowledging Prop. 2,* $\lambda\geq 4\alpha/(rR_{max})$ *implies inefficiency of the stationary state market equilibrium. However,* $\lambda<4\alpha/(rR_{max})$ *does not imply efficiency of* *the stationary state. The upper bound on λ ensuring intergenerational efficiency can be obtained by solving* $\mathrm{LHSL}(R_{max}/2) = \mathrm{RHSL}(R_{max}/2)$. *The solution is* $\lambda^E \equiv [8\alpha - 4(1-\gamma)r] \,/\, \{r\,[2-(1-\gamma)r]\,R_{max}\}$ *which is definitely smaller than* $4\alpha/(rR_{max})$. *For all* $\lambda \leq \lambda^E$, *the stationary state market equilibrium is intergenerationally efficient (Golden Rule included).* ***qed***

The two cases are illustrated in Figures 3–4. In Figure 3, $\lambda < \lambda^E$ and $R < R_{max}/2 = R_{MSY}$, and hence the stationary state market equilibrium is intergenerationally efficient. In contrast, in Figure 4, $\lambda > 4\alpha/(rR_{max}) > \lambda^E$, and hence the stationary state market equilibrium is inefficient.

To understand why a a stationary state market solution with large harvest cost parameter λ is inefficient, it is useful to evaluate the consequences which a higher harvest cost parameter has for the equilibrium resource stock. The higher harvesting costs, the more costly it is to harvest, the lower is resource harvest and the use of resource harvest in commodity production. As a consequence, most of labor will be devoted to commodity production instead of harvesting. But due to decreasing productivity of labor in commodity production, output and hence welfare could be increased when more labor would be devoted to harvesting.

4.3 Inverse stock dependent harvest cost

To evaluate the intergenerational efficiency of stationary state market equilibria with inversely stock dependent harvest cost, we rewrite the stationary state efficiency conditions (16a), (16c), and (16d) by assuming that $(C^2_\circ)^\circ = (C^2)^\star$ and $R_\circ = R^\star$.[8] As a consequence of (16g)–(16i), $\mathrm{g}(R^\star) = X^\star$ and $R = R^\star$. Moreover, $C^2 = (C^2)^\star$. Then, (16a) can be written as follows:

$$\frac{C^2}{\beta\left\{(\mathrm{g}(R^\star))^\alpha\left[1-\lambda\mathrm{g}(R^\star)/R^\star\right]^{1-\alpha} - (C^2)^\star\right\}} = 1 + \frac{\phi^Y_\circ}{\phi^Y}. \tag{18}$$

[8] This equality settings we used, although implicitly, already in the former case of constant unit cost.

The efficiency condition for efficient harvest can be rewritten as:

$$\alpha \mathrm{g}(R^\star))^{\alpha-1}\left[1-\lambda \mathrm{g}(R^\star))/R^\star\right]^{1-\alpha}-(1-\alpha)(\lambda/R^\star)\mathrm{g}(R^\star))^{\alpha}\left[1-\lambda \mathrm{g}(R^\star)\right]^{-\alpha}$$
$$=\frac{1+\phi^R_\circ/\phi^R}{1+\phi^Y_\circ/\phi^Y}\frac{\phi^R}{\phi^Y}, \quad (19)$$

with, from (16i),

$$\frac{\phi^R}{\phi^Y}=\frac{(1-\alpha)\mathrm{g}(R^\star)^{\alpha}\left[1-\lambda \mathrm{g}(R^\star)/R^\star\right]^{-\alpha}\lambda \mathrm{g}(R^\star)/(R^\star)^2}{-\mathrm{g}'(R^\star)+(\phi^R_\circ/\phi^R)}. \quad (20)$$

Proposition 6 (Efficiency of stationary state (stock dependent harvest cost)) *Let* $\mathrm{g}(R)=r\left[R-R^2/R_{\max}\right]$ *and* $\mathrm{h}(R,X)=\lambda$*. For* $\lambda\geq\alpha/R$*, the stationary state market equilibrium* $(X^\star,R^\star)$ *is inefficient. For* $\lambda<\alpha/R$*, those stationary state market equilibrium solutions are intergenerationally efficient for which there exists* $(\phi^Y_\circ/\phi^Y,\phi^R_\circ/\phi^R)\gg 0$ *such that* (18)–(19) *hold.*

Proof 6 *We start with the stationary state market equilibrium conditions* $(C^2)^\star/(\beta(C^1)^\star)=1+\mathrm{g}'(R^\star)+w^\star\lambda \mathrm{g}(R^\star)/\left[(R^\star)^2(q^\star-w^\star\lambda/R^\star\right]$, $p^\star-w^\star\lambda \mathrm{g}(R^\star)/(R^\star)^2=[1+\mathrm{g}'(R^\star)](q^\star-w^\star\lambda/R^\star)$, $q^\star=\alpha(\mathrm{g}(R^\star))^{\alpha-1}[1-\lambda \mathrm{g}(R^\star)/R^\star]^{1-\alpha}$, $w^\star=(1-\alpha)(\mathrm{g}(R^\star))^{\alpha}[1-\lambda \mathrm{g}(R^\star)/R^\star]^{-\alpha}$*. Setting provisionally* $\phi^Y_\circ/\phi^Y=\mathrm{g}'(R^\star)+\lambda w^\star \mathrm{g}(R^\star)/(R^\star)^2(\phi^Y_\circ+\phi^Y)/(\phi^R_\circ+\phi^R)$, $q^\star-\lambda w^\star/R^\star=(\phi^R_\circ+\phi^R)/(\phi^Y_\circ+\phi^Y)$, *and* $\phi^R_\circ/\phi^R=\mathrm{g}'(R^\star)+\lambda w^\star \mathrm{g}(R^\star)/(R^\star)^2(\phi^Y/\phi^R)$*. Using the latter two equations together with the first order market equilibrium conditions for* $w^\star$ *and* $q^\star$ *implies the efficiency conditions* (16a) *and* (16c)*.*

Note that for $\phi^R/\phi^Y>0$*, it is not necessary that* $\mathrm{g}'(R^\star)>0$*. However, the analytical complexity of* (18) *and* (19)*, together with* (20)*, precludes an analytical determination of upper and lower bounds for the harvest cost parameter which ensures positivity of* $\phi^Y_\circ/\phi^Y$ *and* $\phi^R_\circ/\phi^R$*, and thus intergenerational efficiency of the stationary state market equilibrium.* ***qed***

The two cases of Prop. 6 are again illustrated in Figures 5–6. In Fig. 6, the harvest cost parameter is large ($\lambda\geq\alpha/R$) and therefore the stationary state market equilibrium is again intergenerationally inefficient. For a smaller harvest cost parameter λ, it depends on the positivity of the relative shadow prices of the resource stock ($\phi^R_\circ/\phi^R$) and of the production output ($\phi^Y_\circ/\phi^Y$) whether the stationary state market equilibrium is intergenerationally efficient or not. In Figure 5, positivity is satisfied and thus efficiency is given.

Comparing these results to the model without harvest cost, it can be concluded that under inversely stock dependent harvest cost a positive own rate of return on the resource stock, i.e. $\mathrm{g}'(R) > 0$, is no longer necessary for intergenerational efficiency. Similarly to the case with linear harvest cost, it is also not sufficient to investigate whether the slope of resource regeneration is positive or negative because harvest costs have additional effects on both the size of the stationary state resource stock and the harvest level.

5 Conclusions

In this paper different specifications of harvest cost were introduced in an OLG model with a renewable natural resource. Harvesting requires effort (labor input) which has to compete with commodity production for labor input. Due to the general equilibrium setup, the real wage rate and hence unit harvest cost may be higher than the selling price of the resource harvest, leading to a negative resource stock price. Thus, we first investigated the existence of stationary state market solutions with positive resource stock price. For the model without harvest cost, the resource stock price is always positive but this does not apply to the cases of constant or inversely stock dependent unit harvest cost. Here we had to entangle the ranges of feasible resource stocks for which the resource stock price is positive. We find that the magnitude of the harvest cost parameter relative to the growth rate of the resource stock and elasticity of resource input in commodity production decide in which range of feasible resource stock range the stationary state market equilibrium occurs. We also investigate the uniqueness of stationary states and verify the general insight that in spite of endogenous labor supply the log–linear utility and

Cobb-Douglas technology preclude multiple stationary state solutions.

In addition to investigating the difference to the model without harvest cost, our objective was to study the potentially different impacts of different specifications of harvest costs. Here we find that inversely stock dependent harvest cost favor the existence of a nontrivial stationary state because harvests increase with a smaller resource stock. Thus, while unit harvest costs are small for a large resource stock, they become large for a small resource stock which provides a disincentive for overexploitation of the resource stock. This effect is not present when harvest costs depend only on the harvest volume but not the stock.

Of equal importance is that the condition that the own rate of return on the resource stock has to be positive, which ensures efficiency of the stationary state market equilibrium in the model without harvest cost, is not required in the model with inversely stock dependent harvest cost. Consequently, a stationary state market equilibrium in the model with stock dependent harvest cost may also be intergenerationally efficient when the own rate of return is negative.

Yet, on the other hand, the higher the harvest cost parameter the more likely a stationary state market equilibrium may eventually be intergenerationally inefficient. This is due to the fact that the higher harvesting costs, the more costly it is to harvest, the lower is resource harvest and the use of resource harvest in commodity production. As a consequence, most of labor will be devoted to commodity production instead of harvesting. But due to decreasing productivity of labor in commodity production, output and hence welfare could be increased when more labor would be devoted to harvesting.

Some directions for future research are easily identified. First, instead of harvest cost linearly depending the harvest cost level, a convex specification could be used. Second, the inverse impact of the resource stock could be reversed such that harvest costs increase with the resource stock, a specification suitable e.g. for species–rich ecosystems like tropical forests. Finally, also fixed costs could be considered, which may give rise to non–convexities in the net revenue function.

References

Bednar-Friedl, B., Farmer, K., 2013. Time consuming resource extraction in an overlapping generations economy with capital. Journal of Economics 110 (3), 203–224.

Bjørndal, T., Conrad, J. M., Salvanes, K. G., 1993. Stock size, harvesting costs, and the potential for extinction: the case of sealing. Land Economics 69 (2), 156–167.

Brown, G. M., 2000. Renewable natural resource management and use without markets. Journal of Economic Literature 38 (4), 875–914.

Clark, C. W., 1990. Mathematical Bioeconomics: The Optimal Management of Renewable Resources, 2nd Edition. Wiley, NY.

Clark, C. W., Munro, G. R., 1975. The economics of fishing and modern capital theory: a simplified approach. Journal of Environmental Economics and Management 2, 92–106.

Conrad, J. M., 1999. Resource Economics. Cambridge University Press, Cambridge, New York.

Elíasson, L., Turnovsky, S. J., 2004. Renewable resources in an endogenously growing economy: balancee growth and transitional dynamics. Journal of Environmental Economics and Management 48, 1018–1049.

Farmer, K., 2000. Intergenerational natural–capital equality in an overlapping–generations model with logistic regeneration. Journal of Economics 72 (2), 129–152.

Grafton, R., Kompas, T., Hilborn, R., 2007. Economics of overexploitation revisited. Science 318 (5856), 1601.

Koskela, E., Ollikainen, M., Puhakka, M., 2002. Renewable resources in an overlapping generations economy without capital. Journal of Environmental Economics and Management 43 (3), 497–517.

Krutilla, K., Reuveny, R., 2004. A renewable resource–based Ramsey model with costly resource extraction. Environmental and Resource Economics 27, 165–185.

Levhari, D., Michener, R., Mirman, L. J., 1981. Dynamic programming models in fishing: competition. American Economi Review 71 (4), 649–661.

Lloyd-Braga, T., Nourry, C., Venditti, A., 2007. Indeterminacy in dynamic models: When Diamond meets Ramsey. Journal of Economic Theory 134, 513–536.

Olson, L. J., Roy, S., 1996. On conservation of renewable resources with stock–dependent return and nonconcave production. Journal of Economic Theory 70, 133–157.

Schaefer, M. B., 1954. Some aspects of the dynamics of populations important to the management of commercial marine fisheries. Bulletin of Inter–American Tropical Tuna Commission 1, 25–56.

Smith, V. L., 1968. Economics of production from natural resources. American Economic Review 58, 409–432.