Population, Technology, and Malthusian constraints: A quantitative growth theoretic perspective*

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Preliminary version – comments welcome

Abstract

We study the interactions between population dynamics, technological progress, income growth and food production at the global level. We formulate a two-sector Schumpeterian growth model with endogenous population and structurally estimate the key parameters of the model using simulation methods. Our quantitative model closely fit the last fifty years of data on world GDP, population, TFP growth, and crop land area. Projections from the fit suggest a population of around 10 billion by 2050, and despite a strong decline in population growth rates, 14 billion by 2100. As population and per capita income grow, agricultural production increases, but we find that agricultural land increases by only 10 percent relative to 2010.

Keywords: Economic growth; Population projections; Endogenous innovations; Demand for food; Land conversion

JEL Classification numbers: O31, O44, Q15, Q16.

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1 Introduction

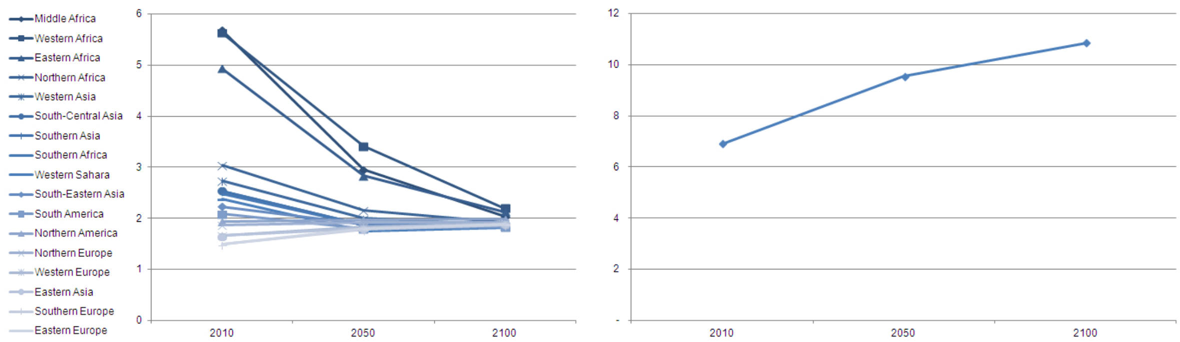
The world population has increased four-fold over the past century (United Nations, 1999). During this period and in most parts of the world, productivity gains in agriculture have confounded the famous Malthusian prediction that population growth would outstrip food supply. Population and income have determined the demand for food and thus agricultural production, rather than food availability determining population. However, despite an apparent potential for population to grow further in the future, the dominant view is that population will reach a steady state in the next century (United Nations, 2008). Our aim in this paper is to understand how this view can be reconciled with technological progress and income growth that occurred in recent history.

This question is located at the intersection of several important topics in economic research, including Malthusian theory, the role of agriculture in development, and the determinants of population growth. In spite of this, few economists have contributed to the debate about forecasting the future population. Instead, the *de facto* global standard source of demographic projections (Lutz and Samir, 2010) is the United Nations' series of *World Population Prospects*, updated every two years. The latest version projects a global population, on a medium scenario, of 9.6 billion in 2050 and 10.9 billion in 2100, by which time the population growth rate is close to zero (United Nations, 2012). The UN uses a so-called 'cohort-component projection method', i.e. it works from the basic demographic identity that the number of people in a country at a particular moment in time is equal to the number of people at the last moment in time, plus the number of births, minus the number of deaths, plus net migration, all of this done for different age groups. Assumptions about future fertility, mortality and international migration rates are exogenous and crucial. The medium scenario, displayed in Figure 1, assumes that all the countries of the globe converge to a replacement fertility rate of 2.1 over the next 100 years, irrespective of their starting point.

The sensitivity of the projections to assumptions about fertility, in particular, is evidenced by the spread in the forecast population size in 2100 between the UN's two other scenarios, the low and high variants: 6.8 to 16.6 billion. The difference between these two scenarios is entirely driven by varying the convergence fertility rate by increments of half a child above and below the medium assumption (giving target rates of 1.6 and 2.6 children per woman for the low and

Figure 1: United Nations projections 2010 – 2100

(a) Regional fertility (b) World population



high variants respectively). From an economist's point of view, while extremely detailed, the shortcoming of projections like these is that they lack an explicit economic rationale.

The purpose of this paper is to contribute a new, macro-economic approach to population forecasting and in doing so to integrate the possible role of Malthusian (food-supply) constraints with the analysis. We develop a modeling framework to jointly assess economic growth, technological progress, population dynamics, and food production at the global level. The framework comprises a two-sector Schumpeterian growth model – the two sectors are manufacturing and agriculture – combined with endogenous population dynamics. As population and income grow, the demand for food increases, raising the share of land in agricultural use. Ultimately there is of course a constraint on the area of land that can be converted into agriculture, but whether this constraint binds and constitutes a serious brake on population growth depends on technological progress in agriculture, which is the result of labor-intensive Research and Development. We estimate the key parameters of the model to match the last 50 years of data on GDP, population, technological progress and crop land area at the global level. We then employ the fitted model to suggest future trajectories for a number of key variables that are typically studied in isolation, including of course population.

Our quantitative macroeconomic growth model links population and technological progress in two important dimensions. The first key component of the model is technological progress and how it relates to human capital. We model the process of knowledge accumulation in a standard Schumpeterian framework, but 'neutralize' the scale effect that characterized early versions of such models (see Jones, 1995a).[1] With technological progress, effective labor units require more human capital, capturing the complementarity between human capital and technology (Goldin and Katz, 1998). In the long run this implies an increasing cost of fertility, which is a key driver in causing population growth to slow down (Galor and Weil, 2000).

Aside from the rising opportunity cost of fertility, the second constraint on population growth is food availability. Agriculture is represented as a separate sector providing food to sustain population, with food demand increasing in both the size of the population and per capita income, the latter capturing the possibly important role of changes in diet as affluence rises. By isolating the role of agriculture in meeting food demand and since land is a key input to agricultural production, limited land reserves function as a Malthusian constraint in our model.[2]

Our model is able to closely replicate observed trajectories between 1960 and 2010 of GDP, population, technological progress and crop land area, all at the global level. Our quantitative results suggest a population of around 10 billion by 2050, which is in line with other major forecasts, but it suggests a population of at least 14 billion by 2100, at the high end of what is currently forecast. While the model requires agriculture to use land as an input to food production, we show that in fact the key factor behind the agricultural sector being able to supply enough food to meet this very large population is technological progress. Land in agricultural use only increases by around 10 percent relative to today. Importantly, our projections are rather conservative in terms of technological progress, with TFP growth in both sectors is below one percent per year and declining from 2010 onwards. We carry out sensitivity analysis on a number of structural assumptions and show that our forecast of significant continued population growth throughout this century is robust to variations in assumptions.

The remainder of the paper is structured as follows. The structure of the model is laid out in Section 2. Section 3 describes ensuing optimization problem and estimation strategy. Section 4 reports projections with the model. Sensitivity analysis is provided in Section 5. Some

[1] As shown by Chu et al. (2013), the qualitative behavior of the model is in line with more recent representations of technological progress put forward by Dinopoulos and Thompson (1998), Peretto (1998), and Young (1998) among others.

[2] Despite the potential importance of the land constraint, only a few papers on long-run economic growth consider it explicitly (Hansen and Prescott, 2002; Strulik and Weisdorf, 2008). Other papers considering more general natural resource constraints that can potentially include land are Peretto and Valente (2011); Bretschger (2013); Brander and Taylor (1998).

concluding comments are provided in Section 6.

2 The economy

We study an economy extending over infinite horizon and treat time as discrete. The economy consists of two production sectors. The first sector, "agriculture", produces food with the sole purpose of sustaining current population. In other words, food does not directly contribute to individual welfare, but it is a determinant of population dynamics by constraining its evolution. The second sector, "manufacturing", produces a homogeneous aggregate good which can be consumed or invested to build up a stock of capital. Aside from capital, there are two other primary input factors: labor and land.

We consider the problem of allocating labor and capital from the perspective of a dynastic household, and abstract from externalities arising in a decentralized setting by solving the social planner problem.[3] We consider two core dynamic processes. First, R&D determines how production technology evolves over time, as described by TFP (Romer, 1994). We use a Schumpeterian model for innovations, where sectoral TFP growth results from allocating labor to produce innovations. Second, endogenous fertility determines increments to the effective labor force. Child rearing is time intensive and hence fertility competes with other labor-market activities in the sense of Barro and Becker (1989). We posit a positive relationship between fertility costs and technological process, so that additions to the stock of effective labor units require more time as technological progress raises human capital requirements. As in Galor and Weil (2000), this is the main driver of a transition to a low fertility equilibrium.

2.1 Production

In both sectors aggregate output is represented by a constant-returns-to-scale production function with endogenous Hicks-neutral technological change.[4] In manufacturing, aggregate output

[3] This is mainly a technical simplification to exploit efficient non-linear programming solvers. Since we take the model to fit historical data, the presence of imperfections in the economy will be reflected in the estimated parameters.

[4] Assuming technological change is Hicks-neutral, so that improvements to production efficiency does not affect the relative marginal productivity of input factors, considerably simplifies the analysis at the cost of abstracting from a number of interesting issues related with the direction or bias of technical change (see Acemoglu, 2002).

in period t is given by a standard Cobb-Douglas production function:

$$Y_{t,mn} = A_{t,mn} K_{t,mn}^{\vartheta} N_{t,mn}^{1-\vartheta} , \tag{1}$$

where $Y_{t,mn}$ is real manufacturing output in time t, $A_{t,mn}$ is an index of productivity in manufacturing, $K_{t,mn}$ is capital allocated to manufacturing, $N_{t,mn}$ is the workforce allocated to manufacturing, and $\vartheta \in (0,1)$ is a share parameter. Since technical change is Hicks-neutral, the assumption that output is Cobb-Douglas is consistent with long-term empirical evidence (Antràs, 2004).

In agriculture, land services from the stock of converted land, X_t, is included as a factor input, and we posit a two-stage constant elasticity of substitution (CES) functional form (e.g. Kawagoe et al., 1986; Ashraf et al., 2008):

$$Y_{t,ag} = A_{t,ag} \left[(1-\theta_X) \left(K_{t,ag}^{\theta_K} N_{t,ag}^{1-\theta_K} \right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} , \tag{2}$$

where $\theta_{X,K} \in (0,1)$, and σ is the elasticity of substitution between a capital-labor composite factor and agricultural land. This specification provides flexibility in representing how capital and labor can be substituted for land, and it nests the Cobb-Douglas specification as a special case ($\sigma = 1$). While a Cobb-Douglas function is often used to characterize aggregate agricultural output (e.g. Mundlak, 2000; Hansen and Prescott, 2002), $\sigma \geq 1$ implies that land is not an essential input in agriculture,[5] and long-run empirical evidence reported in Wilde (2013) indeed suggests that $\sigma < 1$.

We make the simplifying assumption that, in each period, agricultural production is consumed entirely to sustain the current population. In manufacturing, output can either be consumed by households or invested into an aggregate capital stock:

$$Y_{t,mn} = C_t + I_t , \tag{3}$$

where C_t and I_t are aggregate consumption and investment respectively. The accumulation of

[5] This issue has been thoroughly discussed in the context of oil scarcity (see Dasgupta and Heal, 1979, for a seminal contribution).

capital is then given by:

$$K_{t+1} \;=\; K_t(1-\delta_K) + I_t\,, \quad K_0 \text{ given}\,, \tag{4}$$

where δ_K is the per-period depreciation rate. Because the model is formulated as a social planner problem, savings cum investments decisions mirror those of a one-sector economy (see Ngai and Pissarides, 2007, for a similar treatment of savings in a multi-sector growth model).

2.2 Innovations and technological progress

The evolution of sectoral TFP is based on a discrete time version of the Schumpeterian model by Aghion and Howitt (1992). In this framework innovations are drastic, so that a firm holding the patent for the most productive technology temporarily dominates the industry until the arrival of the next innovation. The step size of productivity improvements associated with an innovation is denoted $s > 0$, and we conventionally assume that is it the same in both sectors.[6] Without loss of generality, we assume that there can be at most $I > 0$ innovations over the length of a time period, so that the maximum growth rate of TFP each period is $S = (1+s)^I$. For each sector $f \in \{mn, ag\}$, we further denote the number of innovations arriving over the course of period t by $i_{t,f}$, and define the arrival *rate* of innovations each period as $\rho_{t,f} = i_{t,f}/I$. For s small, the evolution of TFP can thus be closely approximated by:

$$A_{t+1,f} \;=\; A_{t,f}\cdot \rho_{t,f} S\,, \quad f \in \{mn, ag\}\,. \tag{5}$$

Effectively then, sectoral per-period growth rate of TFP is represented as the share of the maximum feasible TFP growth, which in turn depends on the number of innovations arriving within each time period.[7]

R&D in manufacturing and in agriculture hire labor as the main determinant of innovations

[6] In general the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure a right over the proposed innovation. For purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not marginal change) which is usually represented as a minimum one-time shift.

[7] The arrival of innovations is a stochastic process, and we implicitly made use of the law of large number to smooth out the random nature of growth over the discrete time intervals. Our representation is qualitatively equivalent, but somewhat simpler, to the continuous time version of the model where the arrival rate of innovations is described by a Poisson process.

in each period:

$$\rho_{t,f} = \overline{\lambda}_{t,f} \cdot N_{t,A_f}\,, \quad f \in \{mn, ag\}\,, \tag{6}$$

where N_{t,A_f} is labor employed in R&D for sector f and $\overline{\lambda}_{t,f}$ measures labor productivity. An important issue with this representation of technological progress has to do with the "scale effect", or an implied positive relationship between population size and long-run growth, which contradicts empirical evidence (e.g. Jones, 1995b; Laincz and Peretto, 2006). In the present paper we work with the scale-invariant version of Aghion and Howitt (1992) proposed by Chu et al. (2013), where $\overline{\lambda}_{t,f}$ is formulated as a decreasing function of the scale of the economy. In particular, we define $\overline{\lambda}_{t,f} = \lambda_f N_{t,A_f}^{\mu_f - 1} / N_t^{\mu_f}$, where $\lambda_f > 0$ is a productivity parameter and $\mu_f \in (0,1)$ is an elasticity. Including population N_t as the scale of the economy neutralizes the scale effect and is in line with more recent representations of technological change in which R&D simultaneously develops new products and improve existing ones (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for example).[8] Furthermore, our representation of R&D implies decreasing returns to labor in R&D through the parameter μ_f, which captures the duplication of ideas among researchers (Jones and Williams, 2000).

2.3 Labor and population dynamics

In each period, the change in population derives from fertility n_t and morality d_t:

$$N_{t+1} = N_t + n_t - d_t\,, \quad N_0 \text{ given}\,. \tag{7}$$

Because population equals total labor force, n_t and d_t respectively represent the increment and decrements to the stock of effective labor units. The mortality rate is assumed to be constant, so that $d_t = \delta_N$. The parameter δ_N essentially captures the expected lifetime and is the usual representation of mortality in related studies considering growth and endogenous population

[8] In a 'product line' representation of technological progress, the number of product grows with population, thereby diluting R&D inputs, so that long-run growth doesn't necessarily rely on population growth rate, but rather on the share of labor in the R&D sector. An other strategy to address the scale effect involves postulating a negative relationship between labor productivity in R&D and the existing level of technology, giving rise to "semi-endogenous" growth models (Jones, 1995a, 2001). In this setup however, long-run growth is only driven by population growth, which is also at odds with the data (Ha and Howitt, 2007).

(e.g. Strulik and Weisdorf, 2008; Peretto and Valente, 2011; Bretschger, 2013).[9]

In the model, fertility derives from the allocation of labor to child rearing activities:

$$n_t = \overline{\chi}_t \cdot N_{t,N}\,, \tag{8}$$

where $N_{t,N}$ is labor allocated to child rearing activities and $\overline{\chi}_t$ is an inverse measure of the time cost of producing effective labor units. This formulation for fertility mirrors the standard model of households' fertility choices (Becker, 1960; Barro and Becker, 1989), where fertility requires time as an input, so that child rearing competes with other labor-market activities.

We characterize the well-documented complementarity between human capital and the level of technology (Goldin and Katz, 1998) by postulating an increasing relationship between the time cost of child rearing and the level of technology: $\overline{\chi}_t = \chi N_{t,N}^{\zeta-1}/A_t^{\omega}$, where $\chi > 0$ is a productivity parameter, $\zeta \in (0,1)$ is an elasticity representing scarce factors required in child rearing, A_t is an index of technology,[10] and $\omega > 0$ measures how the cost of children increase with the level of technology. This formulation implies that as the stock of knowledge in the economy grows, additions to the stock of effective labor units become increasingly costly in terms of child rearing time. Because we consider population in terms of effective labor unit without explicitly modeling the accumulation of human capital, we capture the positive relationship between technology and human capital requirements of Galor and Weil (2000), and over time a demographic transition will occur as the 'quality' of children required to keep up with technology will be favored over the quantity. Furthermore, ζ captures the fact that the costs of child rearing over a period of time increases more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p.412 and Bretschger, 2013).

Population dynamics are constrained by food availability. Specifically, denoting per capita food demand by $\overline{c_t}(\cdot)$, we require that $N_t\overline{c_t}(\cdot) = Y_t^{ag}$.[11] Per capita food consumption $\overline{c}(\cdot)$, i.e. the quantity of food required for maintaining an individual in a given society, is a concave function

[9] The implied expected lifetime of the agent is given by $1/\delta_N$.

[10] Specifically, A_t is an average of the technological efficiency in each sector weighted by output.

[11] Food consumption does not contribute directly to social welfare. As discussed below however, the level of population enters the social welfare criteria (together with the utility of per capita consumption of the manufacturing good). Thus through the impact of the subsistence requirements on population dynamics, food availability will affect social welfare.

of per capita income: $\overline{c} = \xi \cdot \left(\frac{Y_{t,mn}}{N_t}\right)^{\kappa}$, where ξ is a scale parameter and $\kappa > 0$ is the income elasticity of food consumption. Food demand thus captures both physiological requirements (e.g. minium per capita caloric intake) and the positive relationship between the demand for food and per capita income.

2.4 Land

As a primary factor, land input to agriculture has to be converted from a total stock of available land $\overline{X}$ by applying labor. Thus the allocation of labor to convert land determines the amount of land available for agriculture each period, and over time the stock of land used in agriculture develops as:

$$X_{t+1} \;=\; X_t(1-\delta_X) + \psi \cdot N_{t,X}^{\varepsilon}\,, \quad X_0 \text{ given}\,, \quad X_t \leq \overline{X}\,, \tag{9}$$

where $\psi > 0$ measures labor productivity in land clearing activities, $\varepsilon \in (0,1)$ is an elasticity, $N_{t,X}$ is labor allocated to land clearing activities, and the depreciation rate δ_X measures how fast converted land reverts back to natural land.

2.5 Preferences and social objective

Households are assumed to have preferences over own consumption c_t, the number of children n_t, and the utility that his children will experience in the future $U_{i,t+1}$. We use the class of preference by Barro and Becker (1989) defined recursively as

$$U_t = u(c_t) + b(n_t)\sum_{i=0}^{n_t} U_{i,t+1}\,,$$

where $u(\cdot)$ is the per-period utility function, and the function $b(\cdot)$ specifies preferences for fertility.

We further make a number of standard assumption to specialize households' utility function. First, to simplify aggregation of preferences, we assume that all individuals alive in a given period experience the same utility. Second, per period utility takes the standard form $u(c_t) = \frac{c^{1-\gamma}-1}{1-\gamma}$,, where γ is the inverse of the intertemporal elasticity of substitution. Third, we follow the original Barro and Becker (1989) formulation and set $b(n_t) = \beta\, n_t^{-\eta}$, where $\beta < 1$ is the discount factor and η is an elasticity determining how the utility of parents changes with n_t. Fourth, as individuals in the model face an age-independent death probability δ_N, we follow the certainty equivalent formulation of Jones and Schoonbroodt (2010) to introduce the idea that parents can survive more than one period. In particular, we assume that parents treat similarly their own future utility (conditional on survival probability $\pi = 1 - \delta_N$) and that of their children, so that total expected family size enters the utility function as an argument. Under these additional assumptions, the utility of adults alive in period t is given by:

$$U_t = \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta \left(\pi + n_t\right)^{1-\eta} U_{t+1} \, .$$

Following Alvarez (1999), we now define a dynasty as the set of household with common ancestor, and use the equivalent dynastic representation for household's preferences. [12] In each period, the size of the dynasty is defined as the number of household alive, which coincides with total population $N_t = N_0 \cdot \Pi_{\tau=0}^{t-1}(\pi + n_t)$. By sequential substitution, the utility of the dynasty head is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[N_0 \, \Pi_{\tau=0}^{t-1}(\pi + n_t)\right]^{1-\eta} \frac{c_t^{1-\gamma} - 1}{1-\gamma} \, ,$$

which implies that the problem can be reformulated in terms of the aggregate variables as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \, N_t^{\eta} \, \frac{(C_t/N_t)^{1-\gamma} - 1}{1-\gamma} \, , \tag{10}$$

where $C_t = c_t N_t$ is aggregate consumption in t.

For $\gamma > 1$, which is consistent with empirical evidence on the intertemporal elasticity of substitution, it can be shown that concavity of Equation (10) in (C_t, N_t) requires that $\eta \in (0, 1)$. This implies that, depending on η, preferences of the the dynastic head have both Classical and Average Utilitarian objective as extreme cases.[13] We will come back to this issue when we discuss the selection of parameters.

[12] As Alvarez (1999) notes, this formulation is useful because it makes the problem technically simpler, ensures overall concavity of the objective, and makes the horizon of the households and a social planner coincide. Moreover, because household preferences are defined recursively, the sequence of decisions by household in period 0 will be the same as those that would be decided by household in period t.

[13] See Baudin (2010) for a discussion on the relationship between dynastic preferences and different classes of social welfare functions.

3 Solution method and estimation strategy

This section discusses how we take a numerical version of the model to the data. The aim is to solve a numerical version of the model with key parameters selected to fit observed trajectories for GDP, TFP growth, world population, and crop land over the period 1960 to 2010, the period for which reliable data exist for these quantities.[14] The full set of parameters determining the quantitative trajectories are listed in Table 1. Our empirical strategy proceeds in three steps. First, a number of parameters are determined exogenously. Second, we calibrate some of the parameters to match observed quantities, mainly to initialize the model based on 1960 data. Third, we estimate the remaining parameters with a simulation methods. This section discusses the numerical solution concept and the estimation steps in turn.

3.1 Numerical solution concept

We consider a social planner choosing paths for C_t, $K_{t,f}$ and $N_{t,f}$ by maximizing social welfare (10), subject to technological constraints (2), (1), (3), (4), (5), (7), (9), and market clearing conditions for capital and labor:

$$K_t \;=\; K_{t,mn} + K_{t,ag}\,, \quad N_t \;=\; N_{t,mn} + N_{t,ag} + N_{t,A_{mn}} + N_{t,A_{ag}} + N_{t,N} + N_{t,X}\,. \tag{11}$$

This formulation implies the simplifying assumption that capital and labor are perfectly mobile across sectors. But in a deterministic setting where there are no unexpected shocks to factor prices, this assumption has no qualitative implications for our results.[15] Perhaps more importantly, relying on a social planner representation implies that we assume away externalities associated with the R&D sector.

The optimization problem builds upon the standard core components of economic growth but the number of variables – nine control and five states – render numerical methods necessary for studying its solution. The problem is an infinite horizon dynamic optimal control problem,

[14] We have worked with data covering the period from 1900 to 2010. While the fit of the model is not as good as with a shorter time period, our quantitative conclusions are not affected by restraining the period of observation.

[15] In the presence of uncertainty, perfect factor mobility will imply that the transitional dynamics following the unexpected events will appear less costly than in a situation where capital and/or labor are at least partially sector specific.

Table 1: List of parameters of the model and associated numerical values

Imposed parameters		
ϑ	Share of capital in manufacturing	0.3
θ_K	Share of capital in capital-labor composite for agriculture	0.3
θ_X	Share of land in agriculture	0.25
σ	Elasticity of substitution between land and the capital-labor composite	0.6
δ_K	Yearly rate of capital depreciation	0.1
S	Maximum increase in TFP each year	0.05
$\lambda_{mn,ag}$	Labor productivity parameter in R&D	1
δ_N	Exogenous mortality rate	0.02
κ	Income elasticity of food demand	0.25
β	Discount factor	0.99
γ	Inverse of the intertemporal elasticity of substitution	2
η	Elaticity of altruism towards future members of the dynasty	0.01
Calibrated parameters		
$A_{0,mn}$	Initial value for TFP in manufacturing	5.7
$A_{0,ag}$	Initial value for TFP in agriculture	1.55
K_0	Initial value for capital stock	27.6
N_0	Initial value for population	3.03
X_0	Initial the stock of converted land	1.348
ξ	Food consumption for unitary income	0.4
δ_X	Rate of natural land reconversion	0.02
Estimated parameters		
μ_{mn}	Elasticity of labor in manufacturing R&D	0.53
μ_{ag}	Elasticity of labor in agricultural R&D	0.51
χ	Labor productivity parameter in childrearing	0.103
ζ	Elasticity of labor in child rearing	0.516
ω	Elasticity of labor productivity in child rearing w.r.t. technology	0.091
ψ	Labor productivity in land conversion	0.077
ε	Elasticity of labor in land-conversion	0.186

and there are two main classes of solution methods that can be employed (see Judd, 1998). The first is dynamic programming, exploiting Bellman's recursive formulation of the problem and its contraction mapping property. This solution method is however subject to the curse of dimensionality with respect to the number of continuous states variable. As an alternative, we use mathematical programming techniques, and in particular constrained non-linear optimization, which directly search for a local optimum of the objective function (the discounted sum of utility), starting from a candidate solution to the problem, and subject to the requirement of maintaining feasibility as defined by the constraints of the problem. The solution method thus mimics the social planner welfare maximization program.[16]

Direct optimization methods can be applied to problems with potentially large number of state variables, but an important drawback as compared to dynamic programming is that they cannot accommodate an infinite horizon. Indeed solving the model over an infinite horizon involves maximizing a sum with an infinite number of terms subject to an infinite number of constraints. As long as $\beta < 1$ however only a finite number of terms matter for numerical optimization, and we determine a truncation period T such that increasing it does not change the optimal solution to the problem over the first $T' = 200$ periods, which is a relevant horizon for our analysis. Specifically, we find that truncating the horizon of the problem to $T = 400$ periods is enough to approximate the solution to the infinite horizon problem with a precision of 1e-5 for all the variables in the model. Similarly, we find that re-initializing the model in T' indeed provides an optimal solution to the problem. This suggests that differences between the infinite horizon problem and its finite horizon counterpart is only driven by 'terminal effects', as the shadow values of stock variables are optimally zero in the terminal time period.

3.2 Choice of imposed parameters

Starting with production technology, we need to select values for share parameters ϑ, θ_K and θ_X, and for the elasticity of substitution σ. In manufacturing the Cobb-Douglas assumption implies that the output factor shares, or cost components of GDP, are constant over time, and we use a standard value of 0.3 for the share of capital (see for example Gollin, 2002). In agriculture the CES assumption implies that the factor shares are not constant, and we chose θ_X to approximate a value for the share of land in global agricultural output of 0.25 in 1960. In general the land factor share is negatively correlated with income (Caselli and Feyrer, 2007), and our estimate is in line with data for developing countries reported in Fuglie (2008).[17] For the capital-labor composite, we follow Ashraf et al. (2008) and also use a standard value of 0.3 for the share of capital. Taken together, these estimates of the output value shares in agriculture are also in

[16] The program is implemented in the GAMS software and solved with the KNITRO (Byrd et al., 1999, 2006) which alternates between interior-point and active-set methods.

[17] Because evidence about the value share varies across sources, we do not attempt to directly calibrate the parameter of the CES function for agriculture.

agreement with factor shares reported in Hertel et al. (2012).[18]

As mentioned previously, the long-run elasticity of substitution between land and the capital-labor composite input is expected to be less than one. If this is not the case, land may become irrelevant to agricultural production. Wilde (2013) provides long-run evidence on the elasticity of substitution between land and other inputs from pre-industrial England, finding robust evidence that σ is around 0.6. While external validity of this estimates may be an issue, in particular to developing countries with rapidly growing population, it is based on data for a closed agrarian economy and is consistent with our functional form assumption in (2). In the sensitivity analysis, we derive implications from assuming $\sigma = 0.2$ and $\sigma = 1$.

Among the remaining 'imposed' parameters, the yearly rate of capital depreciation δ_K is set to 0.1 (see Schündeln, 2013, for a survey and evidence for developing countries), and maximum TFP growth per year S is set to 5 percent. The latter number is consistent with evidence about yearly country-level TFP growth rate reported in Fuglie (2012), as it does not report growth rate above 3.5 percent. Labor productivity parameter in R&D, $\lambda_{mn,ag}$, is not separately identified from S, and we set it to 1 without affecting our results. The income elasticity of food demand is 0.25, which is consistent with evidence across countries and over time reported in Subramanian and Deaton (1996), Beatty and LaFrance (2005), and Logan (2009).

We make the following assumptions regarding the preference parameters. The elasticity of intertemporal substitution is set to 0.5 in line with estimates by Guvenen (2006). In the model, this corresponds to selecting $\gamma = 2$. Given the constraint on η highlighted above, we initially set it to 0.01 so that our objective is effectively in line with Classical Utilitarism. Intuitively, this implies that altruism towards future members of the dynasty is roughly constant with the number of members in future dynasty. As an alternative, we report sensitivity analysis for the case where altruism declines with n_t, in particular $\eta = 0.5$.[19] We also assume a high degree of altruism by setting the discount factor to 0.99.

[18] For 2007, the factor shares for the global agricultural sector reported in Hertel et al. (2012) are 0.15 for land, 0.47 for labor, and 0.37 for capital. However, shares for developing countries are probably a better estimates of the value shares prevailing in the mid-20th century.

[19] Note that in our setting, an average utilitarian objective is not globally concave.

3.3 External calibration

Starting values for the states variables are all calibrated to observed quantities in 1960. For initial population N_0, we use an estimate of the world population in 1960 of 3.03 billions (United Nations, 1999). Initial crop land area X_0 is from Goldewijk (2001), 1.348 billion hectares. For the remaining state variables, sectoral TFP $A_{0,ag}, A_{0,mn}$ and the stock of capital K_0 there is no available estimates, and we adapt the calibration strategy of Nordhaus (2008) to a two-sector setting. Recognizing that the initial capital stock is allocated across agriculture and manufacturing, we use four calibration targets. First, we use an estimate of world GDP in 1960 of 9.8 trillions 1990 international dollars (Maddison, 1995; Bolt and van Zanden, 2013). Second, we assume that the share of agriculture in total GDP in 1960 is 15%, which corresponds to GDP share of agriculture reported in Echevarria (1997). Third, we assume that the marginal product of capital ($MPK_{t,f}$) in 1960 is 15 percent. While this may appear as relatively high, it is not implausible for developing economies (see Caselli and Feyrer, 2007).

These calibration targets provide us with the following four expressions:

$$
\begin{aligned}
Y_{0,mn} &= A_{0,mn} K_{0,mn}^{\vartheta} N_{0,mn}^{1-\vartheta}\,, && (12)\\
= Y_{0,ag} &= A_{t,ag}\left[(1-\theta_X)\left(K_{t,ag}^{\theta_K} N_{t,ag}^{1-\theta_K}\right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\,, && (13)\\
MPK_{0,mn} &= \vartheta A_{0,mn} K_{0,mn}^{\vartheta-1} N_{0,mn}^{1-\vartheta}\,, && (14)\\
MPK_{0,ag} &= \theta_K(1-\theta_X) A_{0,ag}^{\sigma}\left(\frac{Y_{0,ag}}{K_{0,ag}}\right)^{\frac{1}{\sigma}}\left(\frac{L_{0,ag}}{K_{0,ag}}\right)^{\frac{(\sigma-1)(1-\alpha)}{\sigma}}\,, && (15)
\end{aligned}
$$

These expressions are imposed as constraints when we solve the problem numerically, which gives an initial value of 1.55 and 5.7 for TFP in agriculture and manufacturing respectively, and a stock of capital of 27.6. Note that these value are determined based on the solution of the model and hence depend on vectors of parameters estimated from the data.

We further calibrate exogenous mortality rate by assuming an average adult working life (after infant mortality) of 45 years, which implies $\delta_N = 0.022$, and set $\delta_X = 0.02$ which corresponds to period of regeneration of natural land of 50 years. The parameter measuring food consumption for unitary income (ξ), is calibrated such that the demand for food in 1960 represents about 15% of world GDP, which is consistent with the calibration targets for initial TFP and capital. This implies $\xi = 0.4$.

3.4 Estimation of the remaining parameters

The seven remaining parameters $\{\mu^{mn,ag}, \chi, \zeta, \omega, \psi, \varepsilon\}$ are conceptually more difficult to tie down because of the highly aggregated structure of the model. We thus directly estimate them using simulation-based structural estimation.

The aim of the estimation is for the model to fit observed trajectories over the period 1960 to 2010 for world GDP (Maddison, 1995; Bolt and van Zanden, 2013), world population (United Nations, 1999, 2008), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and sectoral TFP (Martin and Mitra, 2001; Fuglie, 2012).[20] In the model these correspond respectively to $Y_{t,mn} + Y_{t,ag}$, N_t, X_t, $A_{t,mn}$ and $A_{t,ag}$. In addition, because of the central role of population in the model, we also target population growth rate.

For each parameter to be estimated from the data, we start by specifying bounds of a uniform distribution and draw 10,000 parameters to form vectors of possible combinations of parameters. Solving the model for each vector of parameters, we then compute squared deviations between the solution of the model over 5-year time steps and observed outcomes for GDP, population, crop lands and TFP.[21] In order to compare the size of the error across different variables, we scale the sum of squared error by the sum of the observed quantities, i.e. we select the vector of parameters that minimizes:

$$\sum_j \left[\sum_\tau (Z^*_{j,\tau} - Z_{j,\tau})^2 / \sum_\tau Z_{j,\tau} \right] , \tag{16}$$

where $Z_{j,\tau}$ denotes the targeted quantities j at time τ and $Z^*_{j,\tau}$ is the corresponding value simulated from the model.

Having identified a narrower range of parameters for which the model approximates observed data relatively well, we reduce the range of values considered for each parameter and

[20] Data on TFP is derived from TFP growth estimates and are thus surrounded with uncertainty given they rely on the choice of a particular econometric methodology. Nevertheless, a robust finding of the literature is that the growth rate of TFP in both the agriculture and economy-wide is on average around 1.5-2% per year. To remain conservative on future technological progress, we set the growth rate of TFP to 1.5 percent per year in 1960, declines to 1.2 percent in 1970 and further to 1 percent from 1990 onwards.

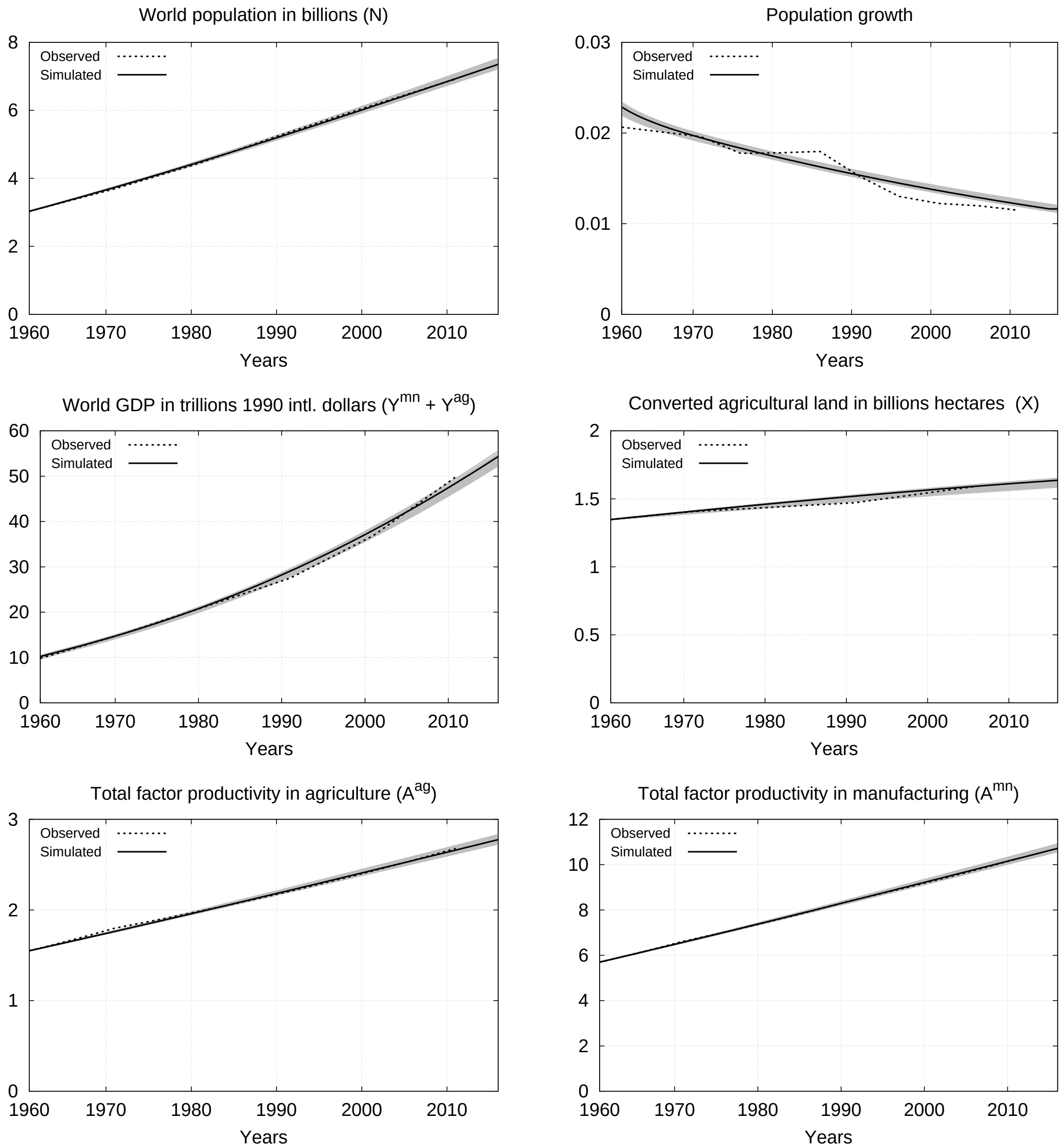
[21] Hence for each 5 quantities in the model, as well as the growth rate of population, we compute the simulation error for 11 data points.

draw an other 10,000 vectors to solve the model. This procedure gradually converges to the estimated values reported in Table 1. Note that as for many simulation-based estimation procedure involving highly non-linear models, the uniqueness of the solution cannot be formally proved (see Gourieroux and Monfort, 1996). Our experience with the model suggest however that the solution is stable, with the sum of squared errors increasing significantly for vectors of parameters that differ from our estimates. This is due to the fact that we target a large number of data points for several variables, and that changing each one parameter will impact trajectories across all variables in the model, which makes the selection criteria for parameters very demanding.

Observed trajectories and simulated trajectories based on the vector that minimizes expression (16) are displayed in Figure 2. The sum of relative squared error is around 10.1 percent. The size of the error is mainly driven by the error for GDP (7.3 percent), land (1.6 percent) and population (0.6 percent). Figure 2 also report a shaded area representing runs for which the objective is relaxed by 20% relative to the best fit achieved (i.e. the set of runs with an error of 12% maximum). Overall simulations with the model closely matches the data, and simulated paths suggest that the model can approximate observed dynamics very well.

Given the aggregated nature of the model and the highly stylized representation of the production function we consider, it is difficult to make statements about the plausibility of the parameters estimates. The only exception is the elasticity of labor in R&D (μ_f), but even there there is disagreement on what this parameter should be. In particular, Jones and Williams (2000) argues that it is around 0.75, while Chu et al. (2013) use a value of 0.2. These two papers however rely on thought experiments to justify their choices rather than some estimation procedure. According to our results, a doubling of the share of labor allocated to R&D would increase TFP growth by around 50%.

Figure 2: Estimation of the model 1960 – 2010



4 Quantitative analysis: Global projections from the model

Figure 3 displays projections for growth rates of several important variables from 2010 to 2100. The key feature of these paths is that they all decline towards a balanced growth trajectory. Population growth rapidly drops below 1 percent, and TFP growth for both sectors remain below 1 percent. This is a conservative assumption, and it implies that GDP growth is mostly driven by capital accumulation.

Figure 3: Estimated model: Growth rate of selected variables

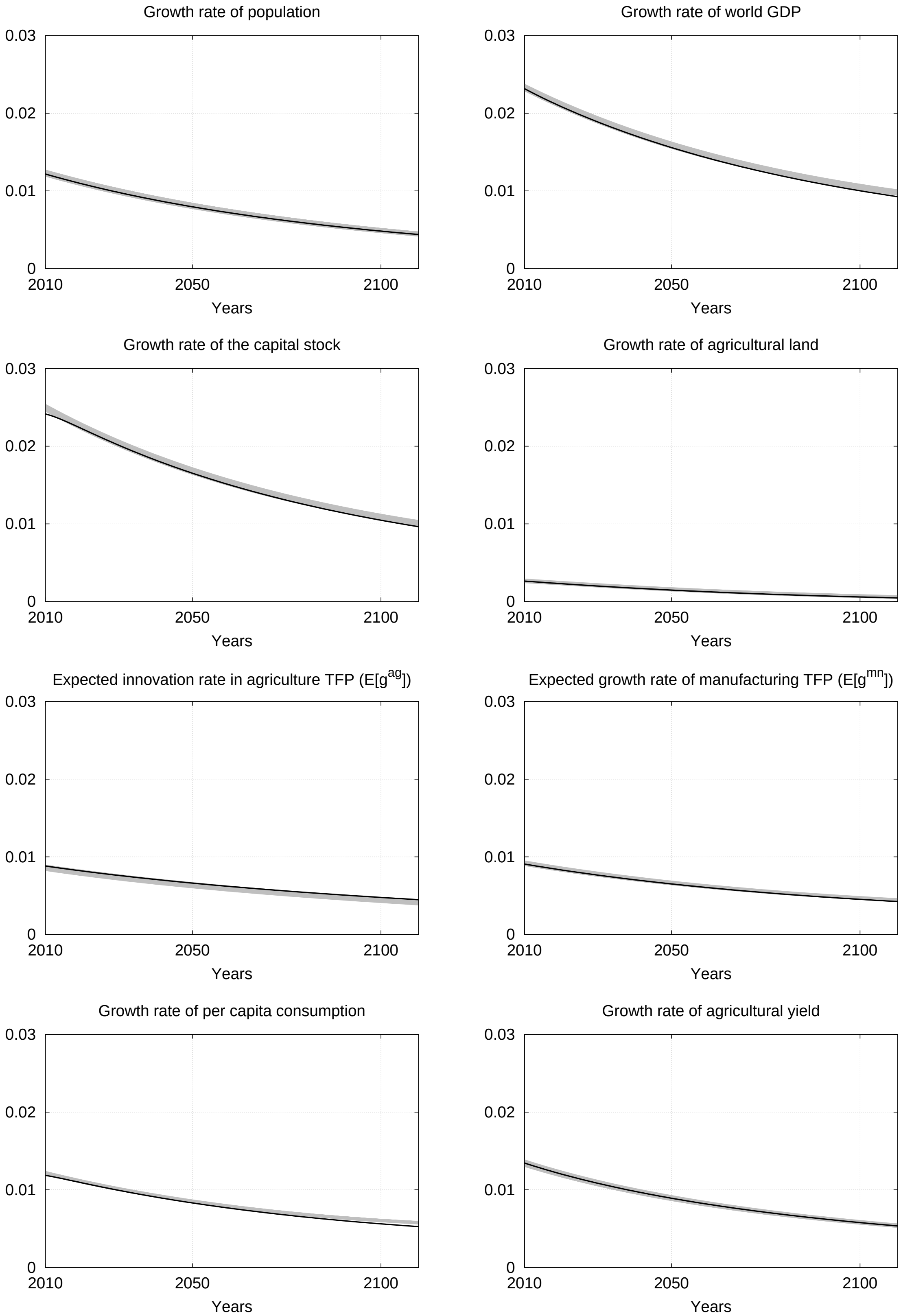
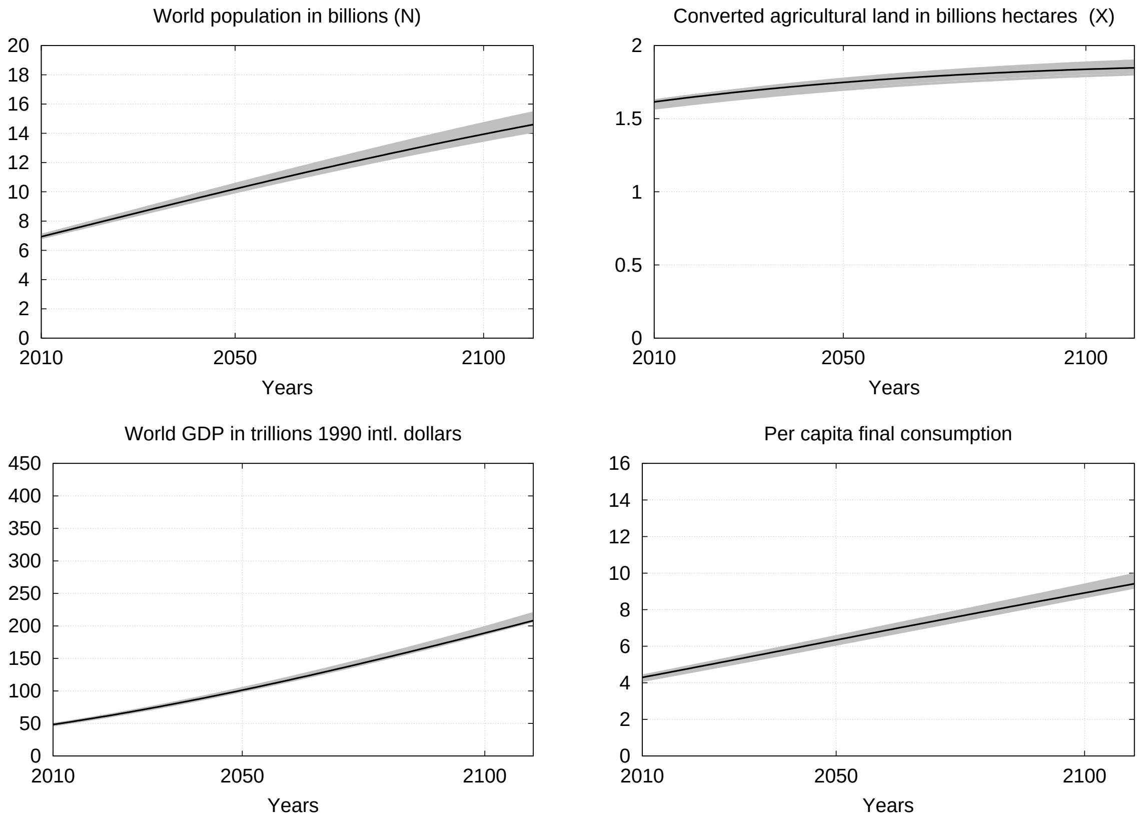


Figure 4: Estimated model: Projections for selected variables



One notable exception is the path for agricultural land area, whose growth rate converges to zero. This suggests that the elasticity of substitution (σ) is high enough to allow agricultural output to grow from other inputs (including technology), with the growth rate if agricultural yield converging to some positive number. We will return to the importance of σ in the section on sensitivity analysis.

In Figure 4 we report *levels* for key variables. World population is around 10 billions by 2050 and reaches 14 billions by 2100. For this variable, the shaded area that lies close to the projected path and represents alternative pathways for a slightly lower fit of the model over 1960 to 2010 suggests a range of 13 to 15 billions by 2100. These projections are consistent with figures from the UN although for the scenarios assuming that fertility remains slightly higher than 2.1 as discussed in Section 1. Converted agricultural land remains below 2 billon hectares over the long run. This is also consistent with alternative projections on land conversion (see Alexandratos and Bruinsma, 2012).

5 Sensitivity analysis

We now report the results of sensitivity analysis with respect to two key component of the model: the representation of agricultural production and estimated productivity parameters in the R&D sectors. Starting with agricultural production, we consider the impact of our assumption about the elasticity of substitution between land and the capital-labor composite input in the agricultural production function. Our baseline case is derived under the assumption that $\sigma = 0.6$, which follows from the work by Wilde (2013). However, evidence with regard to this parameter remains scarce, and it is an important driver of the demand for land and in turn on the ability to produce food and thus sustain the population.

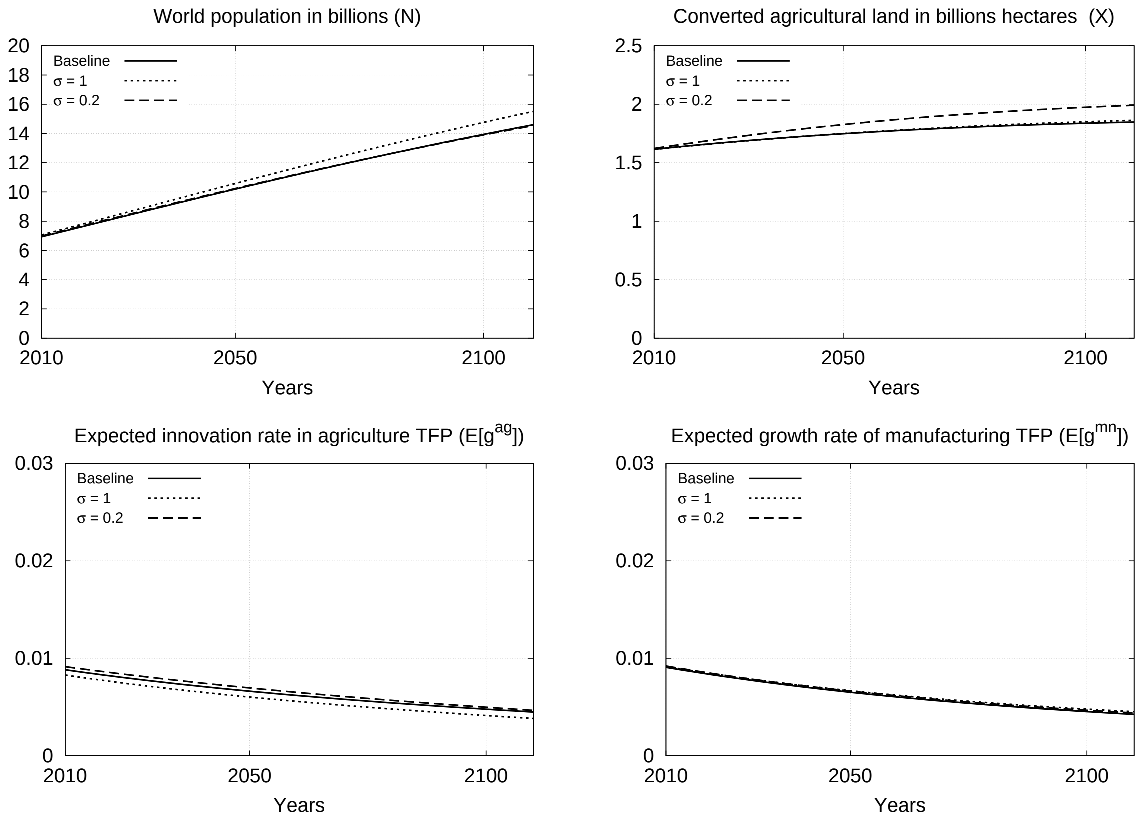
In order to establish the importance of this assumption for our results, we re-estimate the parameters of the model assuming that $\sigma = 1$, so that agricultural production is Cobb-Douglas, and $\sigma = 0.2$, which provides a lower bound on substitution possibilities in agriculture.[22] The results reported in Figure 5 suggest a number of interesting results. First, while the difference between $\sigma = 0.6$ and $\sigma = 0.2$ in terms of land use is extremely small, it significantly affects the trajectory for population. The main reason is that it renders food production cheaper, so relaxing the constraint on population growth. Second, the fact that more labor and capital can be used in production implies that technological progress is slightly slower under a Cobb-Douglas assumption. Third, while increasing σ reduces the demand for cropland, we observe a slight increase in the area of land under cultivation which is driven by the higher population and the ensuing food demand.

Decreasing σ has the expected effect of making land more important in agriculture, so that the overall amount of converted land is higher than under the baseline path. Strikingly, assuming that inputs are less substitutable implies a positive growth rate for land also in the long run. In turn, innovation in agriculture is also slightly higher to compensate the larger cost of using productive inputs. The higher cost in agriculture, however, do not translate in differences in population growth.

The second sensitivity test we conduct relates to technological progress. Here we do not re-estimate the parameters of the model but rather seek to illustrate the effect of alternative as-

[22] The fit of the model is comparable, with an error of around 10 percent.

Figure 5: Sensitivity analysis: Agricultural production function



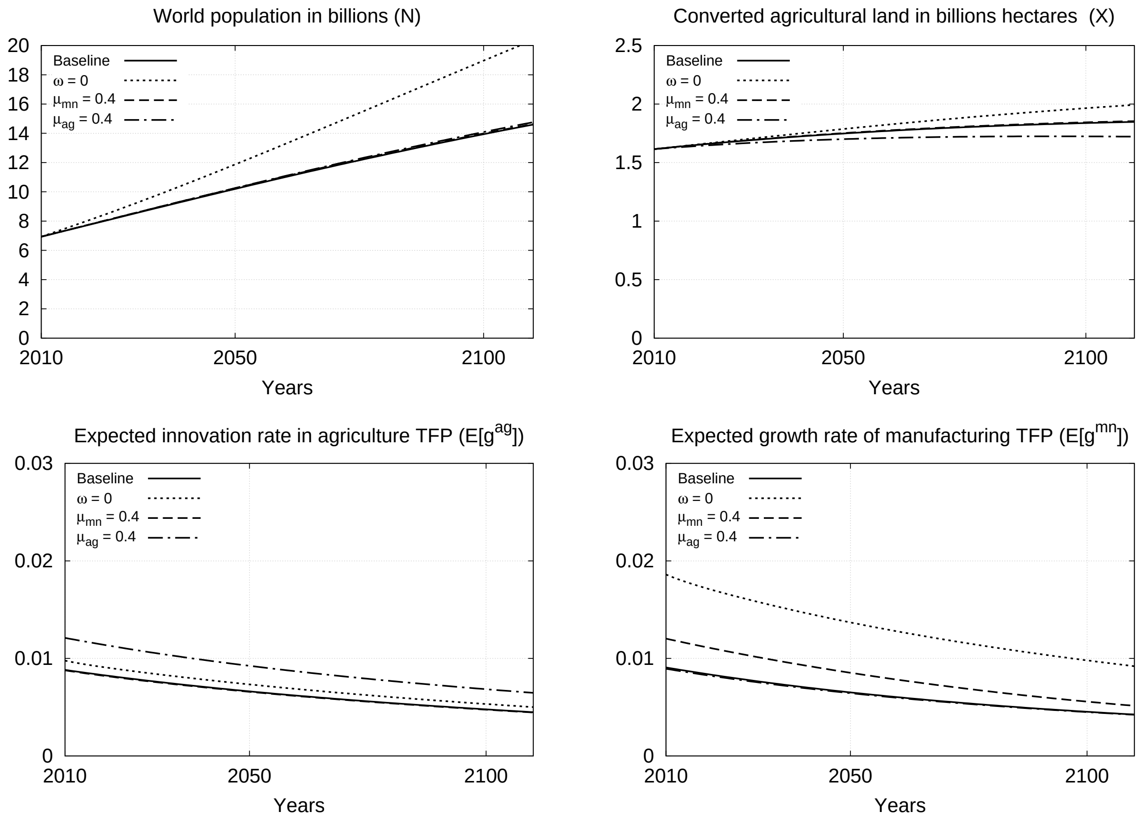
sumptions about technological progress.[23] First, we assess the effect of the relationship between technological progress and fertility cost by setting $\omega = 0$. This implies that improved technology does not require effective labor units to embody more human capital. Second, we reduce the elasticity of innovation with respect to the share of labor in R&D from about one half to 0.4. For a given share of labor, this increases productivity, but reduces marginal productivity. Results for these runs are reported in Figure 6

The impact of ω is substantial as both population and the level of technology grow significantly more rapidly. This quantifies the role of this parameter in the slowdown of population growth over time, but also as a constraint to technological progress as it increases the cost of population growth. As a consequence of the higher population and higher demand for food, the amount of agricultural land is also higher in every period.

Reducing μ_f increases the pace of technological progress without affecting population. There

[23] Because the parameters we vary are estimated, so that the fit of the model over 1960 to 2010 is lower under the assumption made in this section.

Figure 6: Sensitivity analysis: Technology and human capital

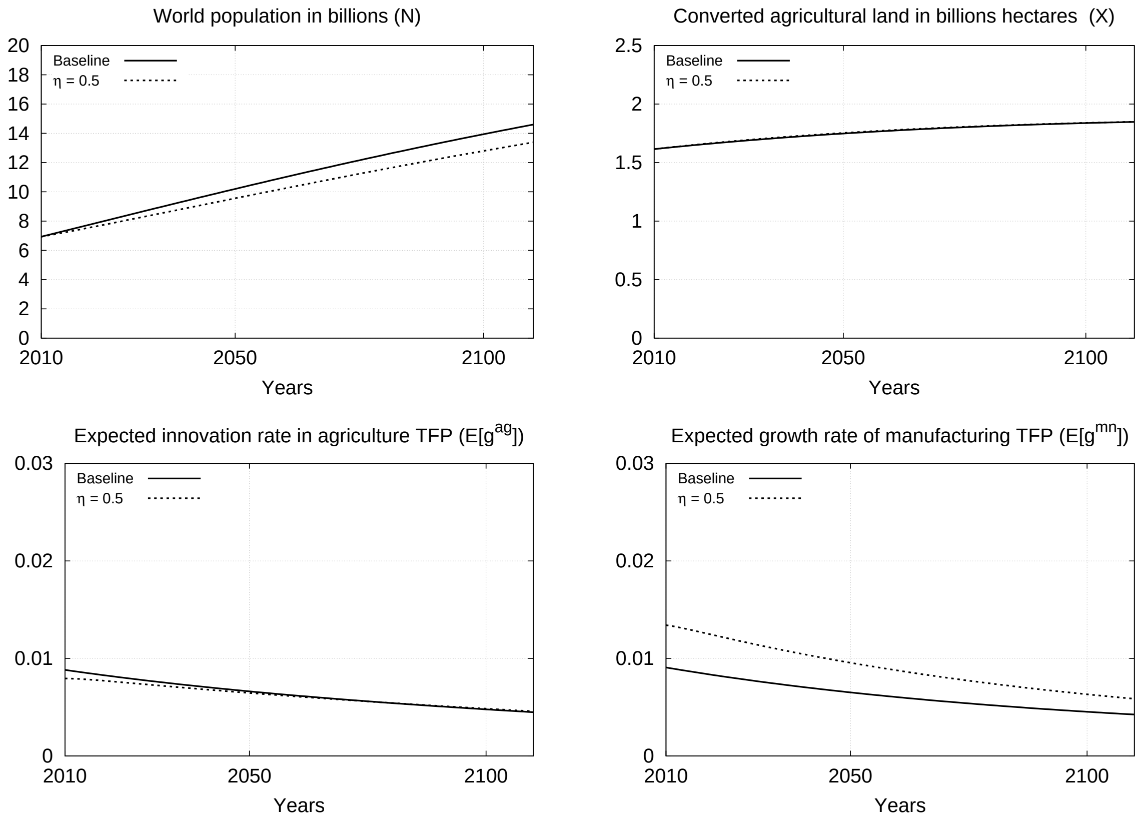


is thus a shift of labor from R&D sectors to education. Also there is little interaction between sectors, as changing μ_{ag} does not affect technological progress in manufacturing and vice-versa. However, improved technological prospect in agriculture reduces the amount of land converted land through a substitution effect.

The final parameter we vary is the elasticity of preferences for the welfare of future members of the dynasty, η. In particular, we consider the case of $\eta = 0.5$, so that the marginal utility of fertility (and population) declines much more rapidly than under our baseline assumption of $\eta = 0.01$. We again estimate the parameters of the model as described in Section 3.4, so that the model fits observed trajectories under different preferences for fertility. The resulting paths are reported in Figure 7.

As expected, we find that reducing η reduces long run population growth by reallocating resources to the production of the manufacturing good, and hence increase the level of per capita consumption. The two channels for this effect is a direct allocation of labor to the manufacturing sector, but also a higher share of labor allocated to manufacturing R&D.

Figure 7: Sensitivity analysis: Preferences over future utility (η)



6 Concluding comments

We have studied the implications of a macroeconomic growth model to make projections for population and land use over the long-run. This provides a very rich framework to study interactions among key drivers of per capita income growth. Our model confirms the widespread expectation that the long-standing processes of growth in population, land conversion are in a process of decline. This is resulting from a systematic shift in the production of social welfare away from quantity-based economies (large levels of population growth, and associated land conversion and food production) and toward quality-based economies (large investments in technology and capital and education for lower levels of population).

The model provides a framework to evaluate policies that jointly impact food production and population, such as those affecting agricultural productivity. Climate change, for example, will impact agricultural productivity and is not reflected in the past 50 years of data that we have used to inform the estimation of the model. An other source of uncertainty is the impact of declining biodiversity, and in particular of the growing share of monoculture agriculture. This could lead to the spread of pathogens that could put at risk a significant share of agricultural output. As the population relying on the agricultural system grows over time, the potential impact of agricultural production shocks increases, which calls for a careful management of agricultural technology.

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