Irreversible investment and transition to clean capital

Julie Rozenberg [1,*], Adrien Vogt-Schilb [1], Stephane Hallegatte [2]

[1] *CIRED, 45bis avenue de la Belle Gabrielle, 94736 Nogent-sur-Marne, France.*
[2] *The World Bank, Sustainable Development Network, Washington D.C., USA*

Abstract

This paper analyses the optimal transition from dirty to clean capital when investment is irreversible. The cost of the transition has two components: the technical cost of investing in clean capital instead of polluting capital, and a temporary irreversibility cost. With a carbon price, the irreversibility cost can be reduced by under-utilizing polluting capital. By preventing under-utilization of existing capital, instruments that focus on redirecting investments — such as feebate programs or environmental standards on new capital — reduce short-term losses but increase the intertemporal cost of the transition. We discuss implications regarding inter- and intra-generational distributional impacts of climate change mitigation.

Keywords: distributional impacts, intergenerational equity, energy efficiency standards, mothballing
JEL classification: L50, O44, Q52, Q54, Q58
To be considered for the best student paper award

For the past centuries, economic growth has been based on the accumulation of fossil-fueled capital that releases greenhouse gases (GHG) in the atmosphere. From a global welfare perspective, this accumulation of polluting capital is suboptimal because it does not internalize the future economic damages caused by climate change. Fossil-fueled capital has been overvalued for decades and the world has made suboptimal technological choices based on distorted prices. Mitigating climate change therefore requires stopping the accumulation of fossil-fueled capital, and future economic growth has to rely on clean capital.

In the absence of any other market failure, the optimal solution to trigger such a large-scale transition from polluting to clean capital is to tax carbon emissions. A carbon tax makes the utilization of polluting capital more expensive and clean investment more profitable. Emissions are thus reduced through two qualitatively different channels (Vogt-Schilb et al., 2014): immediate abatements if firms and households stop using part of their existing fossil-fueled capital (e.g. they drive less) and long-term abatements thanks to clean capital accumulation (e.g. they buy electric cars). Under-utilization of existing capital however has significant consequences on short-term output, employment and consumption

*Corresponding author
Email addresses: `rozenberg@centre-cired.fr` (Julie Rozenberg), `vogt@centre-cired.fr` (Adrien Vogt-Schilb), `shallegatte@worldbank.org` (Stephane Hallegatte)

January 30, 2014

and may be politically unacceptable to people who invested under pre-existing rules (Kaplow, 1993).

Instead of implementing a carbon price, governments seem to prefer redirecting investment, using instruments such as energy efficiency standards on new capital (e.g. CAFE standards, efficiency standards for new buildings and home appliances) or fiscal incentives, such as "feebates" in the automobile sector (Anderson et al., 2011). Such instruments – we call them *investment-based instruments* – are less efficient than a carbon price: for instance, they create rebound effects if greater energy efficiency leads to more extensive use (Goulder and Parry, 2008).

However, the lower efficiency of investment-based instruments comes with an increased social and political acceptability. Indeed, these instruments give firms and households the opportunity to make investments consistent with the turnover of their capital stock, that is to keep using existing capital until it depreciates, while investing in cleaner new capital. By spreading the costs over time and economic agents, they may ease the political economy of the transition to clean capital and make the implementation of the carbon tax easier in the long-run.

In this paper, we compare analytically a carbon price with instruments that focus on investment instead of emissions (e.g. standards or subsidies on investments). Some studies compare the efficiency of such instruments (e.g, Fischer and Newell, 2008; Goulder and Parry, 2008), however, they do not explicitly consider the intertemporal distribution of abatement efforts nor model capital. In this paper, we investigate how the timing of the transition to clean capital is modified when using alternative mitigation instruments.

We use a simple Ramsey model with two types of capital, as first proposed by Ploeg and Withagen (1991): "polluting" capital, which creates a negative externality (greenhouse gases emissions), and "clean" capital, which does not. In the model, reducing emissions can be done through two qualitatively different channels (Vogt-Schilb et al., 2014). First, through a substitution between polluting and clean capital, i.e. structural change (as in Acemoglu et al., 2012). This option is slow because it requires clean capital accumulation and depreciation of polluting capital. Second, it is possible to instantly reduce emissions through under-utilization of polluting capital (or equivalently early-scrapping), i.e. through a contraction of the output volume. This allows unlimited short-term abatement as in Nordhaus (1993).

We start from a *laissez-faire* economy with no externality, in which optimal investment are such that the marginal productivities of polluting and clean capital are equal. From this steady state, a social planner decides to implement a climate mitigation policy. maintain the concentration of greenhouse gases in the atmosphere below a certain threshold. This threshold can be interpreted as the *carbon budget* corresponding to an exogenous policy objective such as the UNFCCC "2C target",[1] or as a tipping point beyond which the environment (and welfare) can be highly damaged.

Two strategies are compared to comply with the ceiling. The first strategy uses a price on carbon emissions, that regulates all GHG emissions. The

[1]Climate research shows that global temperature change may be approximated by cumulative emissions (Zickfeld et al., 2009).

second strategy redirects investments towards clean capital, using a differentiation of investment costs between polluting and clean capital. This instrument is equivalent to a carbon tax completed by a temporary subsidy on polluting capital.

In the long run the carbon price and the investment-based instruments lead to the same steady state, in which the level of installed polluting capital is fixed. The strategies however induce different trajectories over the short run.

A carbon price yields the first-best optimum pathway, that includes an adjustment of polluting capital utilization in the short-run. The partial utilization of polluting capital has significant short-term impact on production and possibly consumption. Practically, this impact would primarily affect the owners of polluting capital and the workers who depend on them. With investment-based instruments, total discounted welfare is lower than in the optimum, but output is higher over the short run because polluting capital is used at full capacity even during the transition. These investment-based instruments thus smooth out the transition toward a low-carbon economy, and can make more acceptable a carbon price in the longer run.

Our results highlight a trade-off between the optimality of a climate mitigation policy and its short-term impacts, which influence implementation ease. If we compare the instruments in terms of welfare maximization, the carbon tax alone is always the best policy. However, when looking at other criteria such as short-term impacts, second-best strategies might appear preferable to many decision-makers. Indeed, strategies that focus on new capital allow reaching the same long-run objective as the optimal policy but delay efforts, with lower short-term impacts on output and higher efforts over the medium-run.

An important downside of investment-based policies is that they cannot curb emissions as fast as the carbon price can. If a carbon tax remains impossible to implement in the near future and the transition has to be triggered by investment-based instruments, their slowness makes their implementation (and enforcement) all the more urgent.

The remainder of the paper is structured as follows. Section 1 presents the model and section 2 solves for the *laissez-faire* equilibrium. In section 3 we analyze the optimal growth path, that can be obtained with a carbon price, and we compare it with investment-based second-best instruments in section 4. Section 5 concludes.

1. Model

We consider a Ramsey framework with a representative infinitely-lived household, who receives the economy's production from firms y_t, saves by accumulating assets[2], receives income on assets at interest r_t and purchases goods for consumption c_t. At time t, consuming c_t provides consumers with a utility $u\left(c_t\right)$. The utility function is increasing with consumption, and strictly concave ($u'(c) > 0$ and $u''(c) < 0$).
The household maximizes their intertemporal discounted utility W, given by:

$$W = \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{1}$$

[2]Assets are capital and loans to other households.

where ρ is the rate of time preference.
Firms produce one final good, using two types of available capital: polluting capital k_b and clean capital k_g. Clean capital encompasses existing clean technologies as well as patents, research and development expenses and human capital necessary to develop new clean technologies.
Firms may use only a portion q_t of installed capital k_t to produce the flow of output y_t given by:

$$y_t = F(A_t, q_{p,t}, q_{c,t}) \tag{2}$$

$$q_{p,t} \leq k_{p,t} \tag{3}$$

$$q_{c,t} \leq k_{c,t} \tag{4}$$

F is a classical production function, with decreasing marginal productivities,[3] to which we add the assumption that capital can be under-utilized. A_t is exogenous technical progress, and increases at an exponential rate over time.

In the remaining of this paper, q_t will be called utilized capital and k_t installed capital. Although it is never optimal in the *laissez-faire* equilibrium, the under-utilization of installed capital can be optimal when a carbon price is implemented[4]. For instance, coal plants can be operated part-time and low-efficiency cars can be driven less if their utilization is conflicting with the climate objective.

Production is used for consumption (c_t) and investments ($i_{p,t}$ and $i_{c,t}$).

$$y_t = c_t + i_{p,t} + i_{c,t} \tag{5}$$

Investment $i_{p,t}$ and $i_{c,t}$ increase the stock of installed capital, which depreciates exponentially at rate δ:

$$\dot{k}_{p,t} = i_{p,t} - \delta\, k_{p,t} \tag{6}$$

$$\dot{k}_{c,t} = i_{c,t} - \delta\, k_{c,t} \tag{7}$$

The doted variables represent temporal derivatives.

Investment is irreversible (Arrow and Kurz, 1970):

$$i_{p,t} \geq 0 \tag{8}$$

$$i_{c,t} \geq 0 \tag{9}$$

This means that for instance, a coal plant cannot be turned into a wind turbine, and only disappears through depreciation.
Polluting capital used a time t emits greenhouse gases ($G \times q_{p,t}$) which accumulate in the atmosphere in a stock m_t. GHG atmospheric concentration increases with emissions, and decreases at a dissipation rate[5] ε:

$$\dot{m}_t = G \cdot q_{p,t} - \varepsilon m_t \tag{10}$$

[3]We assume decreasing returns to scale even in the clean sector but a further extension of this work will be to assume increasing returns to scale in the short-run.

[4]In this paper, under-utilization of clean capital is never optimal so $q_{c,t} = k_{c,t}$.

[5]The dissipation rate allows maintaining a small stock of polluting capital in the steady state.

In the following section, we solve for the *laissez-faire* equilibrium. In the last two sections, we adopt a cost-effectiveness approach (Ambrosi et al., 2003) and analyze policies that allow maintaining atmospheric concentration m_t below a given ceiling $\bar{m}$, a proxy for the increase in global temperature (Meinshausen et al., 2009):

$$m_t \leq \bar{m} \tag{11}$$

This threshold can be interpreted as a tipping point beyond which the environment (and output) can be highly damaged. It can also be interpreted as an exogenous policy objective such as the UNFCCC "2C target".

2. *Laissez-faire* equilibrium

The *laissez-faire* equilibrium leads to classical results of a Ramsey model with two types of capital.

Proposition 1. *In the laissez-faire equilibrium, the marginal productivities of clean and polluting capital are equal. Consumption grows as long as the marginal productivity of capital — net of depreciation — is higher than the rate of time preference.*

PROOF. Firms rent the services of capital from households, who own it. We denote $R_{p,t}$ and $R_{c,t}$ the rental prices of a unit of polluting and clean capital respectively. A firm's total cost for capital is $R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot k_{p,t}$. The firm's flow of profit at time t is given by:

$$\Pi_t = F(A_t, q_{p,t}, q_{c,t}) - R_{c,t} \cdot k_{c,t} - R_{p,t} \cdot k_{p,t} \tag{12}$$

A competitive firm, which takes $R_{c,t}$ and $R_{p,t}$ as given, maximizes its profit by using all installed capital and by equalizing at each time t the marginal productivity of polluting and clean capital to their respective rental prices:

$$\partial_{q_b} F(q_{p,t}, q_{c,t}) = R_{p,t}$$
$$\partial_{q_g} F(q_{p,t}, q_{c,t}) = R_{c,t}$$

Since capital depreciates at the constant rate δ, the net rate of return to the owner of a unit of polluting or clean capital is respectively $R_{p,t} - \delta$ and $R_{c,t} - \delta$.[6] We model a closed economy, thus the assets owned by the households are installed capital, or loans to other households at rate r_t. At equilibrium, households should be indifferent between investing in polluting or clean capital, or lending to other households, so that

$$R_{p,t} = R_{c,t} = r_t + \delta \tag{13}$$

When solving for households' utility maximization, we find the Euler equation, that gives the basic condition for choosing consumption over time (see Appendix A):

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{c \cdot u''(c)} \cdot (r_t - \rho) \tag{14}$$

[6] We implicitly assumed that the price of capital in units of consumables is 1, but this will not always be the case when the GHG ceiling is introduced.

The intertemporal elasticity of substitution is positive ($-\frac{u'(c)}{cu''(c)} > 0$) so consumption grows if the rate of return to saving r_t (i.e. the marginal productivity of capital, net of depreciation) is higher than the rate of time preference. The interest rate r_t is the rate that converts future consumption into a current consumption that is equivalent in terms of social welfare. If the interest rate equals the rate of time preference, consumption is constant over time. □

As a consequence of Proposition 1, if the marginal productivity of polluting capital is higher than that of clean capital, the ratio of polluting capital over clean capital is higher than one. In other words, if using polluting capital is more productive than using clean capital, firms will invest more in polluting capital.

In this *laissez-faire* equilibrium, the shadow prices of polluting and clean capital are always equal to the marginal utility of consumption (J. Barro and Sala-i Martin, 2004).[7] In the next section, however, we show that when the climate externality is internalized the price of polluting capital decreases, and so does the real rate of returns for the owners of polluting capital.

3. Discounted welfare maximization: Carbon price

In this section, we solve for the welfare maximization program, in which institutions impose the social cost of emissions on producers and consumers (e.g. an optimal carbon tax, a universal cap-and-trade system) in order to internalize the GHG ceiling constraint. A social planner maximizes intertemporal utility given the economy budget constraint (eq. 5), the capital motion law (eq. 6 and eq. 7), the irreversibility constraint (eq. 8) and the GHG ceiling constraint (eq. 10 and eq. 11). The social planner program is:

$$
\begin{aligned}
&\max_{c,i,k} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \\
\text{subject to } & F(q_p, k_c) - c_t - i_{p,t} - i_{c,t} = 0 && (\lambda_t) \\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
& \dot{m}_t = G\, q_{p,t} - \varepsilon m_t && (\mu_t) \\
& m_t \leq \bar{m} && (\phi_t) \\
& i_{p,t} \geq 0 && (\psi_t) \\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}
$$

We indicated in parentheses the co-state variables and Lagrangian multipliers associated to each constraint. λ_t is the current value of income. ν_t and χ_t are the current values of polluting and clean capital. μ_t is the current cost of pollution in the atmosphere, expressed in terms of undiscounted utility at time t. The present value Hamiltonian associated to the maximization of social welfare can

[7] In other words, the price of polluting and clean capital, expressed in units of consumables is always 1.

be found in Appendix B.
We define τ_t as the current price of GHG, expressed in units of consumables.

$$\tau_t = \frac{\mu_t}{\lambda_t} \tag{15}$$

The steady state is reached when $m_t = \bar{m}$ and $\dot{m}_t = 0$. We call t_{ss} the date when the steady state is reached ($\forall t \geq t_{ss},\ m_t = \bar{m}$). In the steady state, polluting installed capital is constant at $k_{p,t} = \bar{m}\,\varepsilon/G$.

We show in Appendix B.2 that the carbon price exponentially grows at the endogenous interest rate plus the dissipation rate of GHG until the ceiling is reached.

$$\dot{\tau_t} = \tau_t[\varepsilon + r_t] \tag{16}$$

The main first-order conditions of our problem are (Appendix B.1):

$$u'(c_t) = \lambda_t = \nu_t + \psi_t = \chi_t \tag{17}$$

$$\partial_{k_c} F = \frac{1}{\lambda}\left((\delta+\rho)\chi_t - \dot{\chi}_t\right) \tag{18}$$

$$\beta_t = \frac{1}{\lambda}\left((\delta+\rho)\nu_t - \dot{\nu}_t\right) \tag{19}$$

$$\partial_{q_p} F = \beta_t + \tau_t \cdot G \tag{20}$$

Equations 18 and 19 display the rental prices of clean and polluting capital $R_{c,t}$ and $R_{p,t}$, as shown by Jorgenson (1967):

$$R_{c,t} = \frac{1}{\lambda}[(\delta+\rho)\chi_t - \dot{\chi}_t] \tag{21}$$

$$R_{p,t} = \frac{1}{\lambda}[(\delta+\rho)\nu_t - \dot{\nu}_t] \tag{22}$$

where χ_t and ν_t are respectively the marginal clean and polluting capital prices. The Lagrangian multiplier associated to the constraint $q_{p,t} \leq k_{p,t}$ (β_t) is therefore equal to the rental price of polluting capital $R_{p,t}$ (eq. 19). As a consequence, as long as the rental price of polluting capital is positive, polluting capital is fully-utilized (complementary slackness condition, see Appendix B.1). However, if the rental price of polluting capital goes down to zero, polluting capital may be under-utilized.
We can deduce the following proposition from the first-order conditions.

Proposition 2. *Along the optimal path, the marginal productivity of clean capital is equal to the rental price of clean capital, which is equal to the interest rate plus the depreciation rate.*

$$\partial_{k_c} F = R_{c,t} = r_t + \delta \tag{23}$$

Along the optimal path, the marginal productivity of polluting capital must be equal to the rental price of polluting capital plus the carbon price τ_t multiplied by the marginal emissions of production G.

$$\partial_{q_p} F = R_{p,t} + \tau_t\, G \tag{24}$$

Proof. See Appendix B.3. □

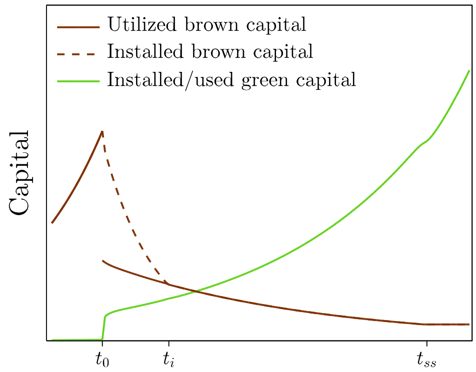

Figure 1: Polluting and clean installed capital, and utilized polluting capital in the first-best optimum. Before t_0, the economy is on the *laissez-faire* equilibrium. At t_0 the carbon price is implemented and polluting capital depreciates until t_i ($i_b = 0$). During this period, polluting capital may be under-utilized ($q_{p,t} < k_{p,t}$). Polluting investments then start again, and the steady state is reached at t_{ss}.

In the *laissez-faire* equilibrium, investments were made such that the net marginal productivity of polluting capital was equal to the interest rate. This is no longer true when the pollution externality is internalized, as firms have to pay the carbon tax when they use polluting capital. Their marginal cost is higher than the interest rate and they must therefore reduce the amount of polluting capital used for production, to adjust their marginal benefit.

We suppose that at t_0, when the climate mitigation policies are introduced, the level of installed polluting capital is higher than in the new steady state. Arrow and Kurz (1970) show that in this case, the irreversibility constraint can be binding ($\psi_t > 0$) and the shadow price of polluting capital is lower than the marginal utility of consumption ($\chi_t < \lambda_t$, see eq. 17).[8]

Proposition 3. *Two phases can be distinguished during the optimal transition to the new steady state with externality:*

- *A phase when polluting investments are strictly positive, and the price of polluting capital is equal to the price of clean capital. During this phase (and on the steady state),* $R_{p,t} = R_{c,t}$ *and:*

$$\partial_{q_p} F = \partial_{k_c} F + \tau_t\, G \tag{25}$$

The marginal productivity of polluting capital is higher than that of clean capital, to adjust to its higher social cost.

- *A phase when the value of polluting capital is lower than the marginal utility of consumption. During this phase, polluting investment is nil and the rental price of polluting capital is lower than that of clean capital.*

$$\begin{aligned} &R_{p,t} = R_{c,t} - p \\ &\textit{with } 0 < p \leq R_{c,t} \end{aligned} \tag{26}$$

[8]In other words, the price of polluting capital can decrease below 1.

We show in the following proposition that the phase when the value of polluting capital is lower than that of clean capital (and polluting investment is nil) necessarily happens first when a carbon price is implemented in the *laissez-faire* equilibrium.

Proposition 4. *At t_0, when a GHG ceiling is enforced in the laissez-faire equilibrium, the value of polluting capital decreases and polluting investment are nil.*

Proof. Appendix C. □

During this first phase, the irreversibility constraint prevents the economy from transforming polluting capital into either clean capital or consumption even though polluting capital is overabundant. The price of polluting capital therefore decreases, as well as its rental price. The decrease of the rental price allows adjusting the marginal cost of producing with polluting capital to compensate for the carbon tax in the short run. The phase when $i_{p,t} = 0$ may be separated into two different sub-phases depending on the carbon price, compared to the marginal productivity of the last unit of polluting capital.

Proposition 5. *During the first phase (when $i_{p,t} = 0$) if the carbon price is higher than the marginal productivity of the last unit of installed polluting capital (expressed in output per emissions), polluting capital is under-utilized until its marginal cost meets its marginal productivity.*

$$\partial_{q_p} F = \tau_t \, G \text{ with } q_{p,t} < k_{p,t} \tag{27}$$

During this phase of under-utilization, both the price and rental price of polluting capital are nil ($\psi_t = \lambda_t$ and $\nu_t = 0$).[9]

Proof. See Appendix D.

During the transition to clean capital, the shadow cost of emissions (or social cost of carbon) τ_t is equal to a "technical" marginal abatement cost (the cost of replacing polluting capital by clean capital) plus an irreversibility cost during the first phase:

$$\tau_t = \frac{\partial_{q_p} F - \partial_{k_c} F}{G} + \frac{p}{G} \tag{28}$$
$$\text{with } 0 \leq p \leq \partial_{k_c} F \text{ (eq. 26)}$$

When polluting capital is under-utilized, $p = \partial_{k_c} F$ and the shadow cost of emissions is equal to the cost of not using a unit of installed polluting capital. On the steady state, the irreversibility cost is nil and the shadow cost of emissions is equal to the technical marginal abatement cost.

Because investment is irreversible, the society that we model has to live with past mistakes for a while, once it realizes it has been on a non-optimal growth path. A way to bypass this obstacle is to give up part of installed

[9]Note that when the rental price of polluting capital is nil, the marginal productivity of polluting capital is transfered to households through the tax revenue $\tau_t \, G$ only.

polluting capital in order to reduce emissions faster (Fig. 1 and Prop. 4). Such a strategy reduces short-term output, but for stringent climate objectives (with regard to past accumulation of polluting capital), it is optimal.

Under-utilization of existing capital may be politically difficult. First, it appears as a waste of resources and creates unemployment (even though labor is not modeled here). Second, it affects primarily the owners of polluting capital and the workers whose jobs depend on this capital, transforming them into strong opponents to climate policies.

4. Investment-based instruments

It is possible to reach the GHG ceiling by changing investment decisions without creating an incentive to reduce the utilization rate of polluting capital, i.e. with no effect on production decisions. In practice, it can be done with investment-based instruments such as energy efficiency standards, fiscal incentives (as feebates) or differentiated interest rates depending on the carbon content of capital (Rozenberg et al., 2013).

In this section, we first solve the social planner program with an additional constraint on the full utilization of polluting capital. We show that it amounts to subsidizing the use of obsolete polluting capital in the short-run, in addition to the carbon tax. We then show that the same outcome can be reached with a differentiation of investment costs between polluting and clean capital.

These instruments allow reaching the same steady state as in the first-best optimum in the long-run, and they induce a full utilization of polluting capital in the short-run.

4.1. Social optimum with a carbon tax and a temporary subsidy

We use the same social planner program as in 3 and we add a full-utilization constraint:

$$q_{p,t} = k_{p,t} \ (\alpha_t)$$

The condition on the marginal productivity of polluting capital becomes (Appendix E):

$$\partial_{q_p} F = \beta_t - \alpha_t + \tau_t \cdot G \tag{29}$$

Note that due to complementary slackness conditions (Appendix E), if $\beta_t > 0$ then $\alpha_t = 0$ and if $\alpha_t > 0$ then $\beta_t = 0$. In the first phase when polluting investment is nil, if the carbon tax is higher than the marginal productivity of the last unit of brown capital, the value of brown capital is nil, $\beta_t = 0$ and the equation becomes:

$$\partial_{q_p} F = -\alpha_t + \tau_t \cdot G \tag{30}$$

α_t is a subsidy to the utilization of polluting capital.

Since the complementary subsidy is a second-best strategy, the optimal value of the carbon tax is higher in the short-run than the one found in the first-best solution (section 3). However, during the first phase firms see a lower resulting carbon tax (thanks to the subsidy) and thus have no incentive to under-utilize polluting capital. Consistently with Kaplow (1993), government relief reduces the efficiency of the policy, but may make it more acceptable.

Here, the temporary subsidy amounts to imposing a lower carbon tax to firms in the beginning because we model only one kind of polluting capital. If there was a continuum of polluting capital with different carbon intensities, the subsidy would only go to carbon-intensive capital that would otherwise be discarded.

In practice, a unique carbon price can be implemented to act as a signal for investments, and it can be completed by temporary subsidies to the most vulnerable firms or households, so that they can keep using their polluting capital.

Such a subsidy to obsolete polluting capital is comparable to Japanese industrial policies, that supported declining traditional sectors during the transition towards higher productivity sectors in the middle of the 20th century (Beason and Weinstein, 1996).

This compensation has similar effects as the grandfathering phase of the European Carbon Trading Scheme during which quotas were allocated for free, or the announced-in-advance carbon tax proposed by Williams (2011).

4.2. Differentiation of capital costs

In practice, it may not be feasible to monitor and compensate each individual loser of climate mitigation policies (e.g., Kanbur, 2010). The transfers themselves may appear unacceptable to other actors, especially if they go to a relatively wealthy population.

Another solution to prevent polluting capital under-utilization is to differentiate capital costs, for instance with fiscal incentives such as subsidies on clean investment ($\theta_{c,t} < 0$) or taxes on polluting investment ($\theta_{p,t} > 0$). Here we model lump-sum taxes on installed capital, which correspond to feebates programs that would apply on both new investments and the secondary market for capital. In our model, since investment is irreversible, these differentiated capital costs have the same impact on investment decisions as taxes/subsidies on investment.

The firm's flow of profit at time t is given by:

$$\Pi_t = F(q_{p,t}, q_{c,t}) - (R_{c,t} + \theta_{c,t})\, k_{c,t} - (R_{p,t} + \theta_{p,t})\, k_{p,t} \tag{31}$$

The optimal values of $\theta_{c,t}$ and $\theta_{p,t}$ can be obtained with a maximization of social welfare given the ceiling constraint. We solve the firm's maximization problem in Appendix F and find that for all t it is optimal to have:

$$\begin{aligned} q_{p,t} &= k_{p,t} \\ \partial_{q_p} F &= R_{p,t} + \theta_{p,t} \\ \partial_{q_c} F &= R_{c,t} + \theta_{c,t} \end{aligned}$$

Under-utilizing polluting capital is never optimal because firms do not pay carbon emissions directly (in other words, the marginal cost of polluting capital does not depend on capital utilization). Instead, firms see that the marginal cost of polluting capital is higher than that of clean capital, such that investment in polluting capital is not profitable.

Over the short-run, as in the social optimum the economy does not invest in new polluting capital. As explained in Appendix F, when $\theta_{c,t}$ is implemented, the rental price of clean capital increases (as well as the interest rate). Similarly,

$\theta_{p,t}$ induces a temporary decrease in the rental price of polluting capital. Once polluting capital has depreciated to a level compatible with the GHG ceiling, polluting investments become profitable and start again.

In the steady state, the amount of installed polluting capital is fixed (so that emissions are constant and $\dot{m} = 0$), polluting and clean investments are positive and $R_{p,t} = R_{c,t}$. In this case the marginal productivity of polluting capital is equal to that of clean capital plus the sum of the tax and the subsidy ($(-\theta_{c,t})$ is positive):

$$\partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{q_c} F(q_{p,t}, q_{c,t}) + (\theta_{p,t} - \theta_{c,t})$$

The same steady state as in the social optimum is reached if the optimal value of the tax plus the subsidy is equal to the carbon tax multiplied by the marginal emissions of polluting capital:

$$\forall t \geq t_{ss},\ \theta_{p,t} - \theta_{c,t} = \tau_t \cdot G$$

with t_{ss} the date at which the steady state is reached.

This capital cost differentiation is similar to existing fiscal incentives, feebates programs or concessional loans for high efficiency homes or appliances. With a continuum of polluting capital, the differentiation should be proportionate to the carbon content of each new investment. In practice, this tax on polluting investment or subsidy to clean investment can be done at the firm level but can also be subject to regulatory capture. Capital costs can also be differentiated using financial markets, as in Rozenberg et al. (2013). In this case, the differentiation would be calibrated on the carbon content of investments.

If the cost differentiation is made through a subsidy on clean capital only (as in Rozenberg et al. (2013)), the price of polluting capital does not decrease in absolute terms when the policy is implemented. Instead, the interest rate increases and becomes higher than the rental price of polluting capital. This transition may appear more acceptable to the owners of polluting capital.

4.3. Comparison with the social optimum

As already noted, if the concentration ceiling is not stringent, these second-best instruments are equivalent to the carbon tax alone, because it is optimal to always use all polluting capital in the short-run. On the other hand, if the ceiling is too stringent, such that waiting for polluting capital depreciation is not sufficient to remain below the ceiling, these instruments cannot be used to reach the target. This is illustrated in Figure 5 and discussed in section 4.4.

In the middle zone, the one that is interesting for this section, mitigation efforts are reduced in the short run with the second-best alternatives, compared to the first-best optimum.

Proposition 6. *With instruments that focus on redirecting investment, output is higher in the short-run than in the first-best solution with a carbon price.*

PROOF. We showed with Proposition 4 that in the first-best optimum, when a carbon price is introduced, the utilization rate of polluting capital may be discontinuous. Therefore, in the short-run — that is, when the irreversibility constraint is binding — output is lower in the first-best optimum than in the second-best solution, in which the utilization rate of polluting capital is continuous. □

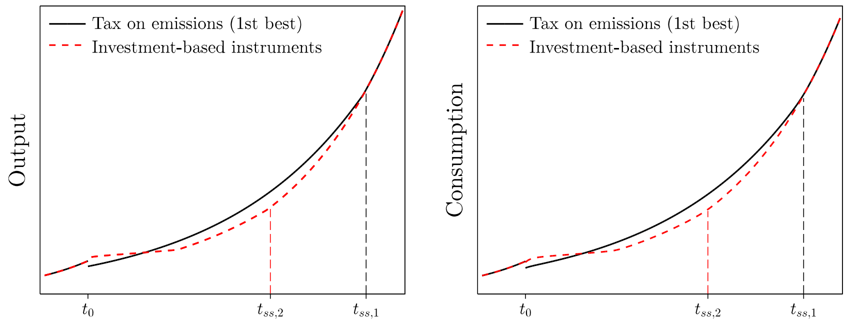

Figure 2: On the left, output y in the two cases. In the short-run output is lower in the first-best case because of the adjustment of polluting capital utilization. On the right, consumption c is higher in the second-best case because of a higher output y. t_{ss} is the date at which the steady state is reached, it is reached sooner in the second-best case ($t_{ss,2} < t_{ss,1}$).

If production is higher over the short-run in the second-best mitigation strategy, consumption can also be higher (we find so in the illustrative simulation of this paper). Analytically, however, the effect on consumption is ambiguous because it involves the offsetting impacts from a substitution effect and an income effect: short-term output is higher, but investments in clean capital may also increase since the saving rate is endogenous.

Eventually, all instruments lead to the same steady state, but instruments focused on investment result in lower discounted welfare than in the social optimum, while they may increase the utility of current generations (Fig. 2). These policies generates higher short-term emissions (Fig. 4) than a carbon price, and because they are sub-optimal, they also generate a higher emissions shadow price (Fig. 3).[10]

The dynamics of capital accumulation mean that the social cost of abatement cannot be translated into consumption losses in a trivial way (see also Vogt-Schilb et al., 2014). For instance, investment-based instruments are more expensive at each time t in terms of emissions shadow cost (Fig. 3) while output (and possibly consumption) is higher over the short-run (Fig. 2). The carbon price is not a good indicator to estimate the policy effect on *instantaneous* output and consumption (instead, it gives the impact on intertemporal welfare). On the other hand, the policy design influences the intertemporal distribution of mitigation efforts.

Choosing the best instrument in terms of welfare results in choosing the lowest social cost of abatement but not the highest consumption at each time t. There is however a trade-off between efficiency (intertemporal welfare), intergenerational equity (distribution of efforts over time) and implementation obstacles (political economy). Other criteria than social welfare maximization can be used to decide on the best policy to implement. For instance, Llavador et al. (2011) use the Intergenerational Maximin criterion, which maximizes the

[10]In a cost-benefit analysis, abatement would therefore be lower with policies that target investment than with the optimal policy with a carbon price.

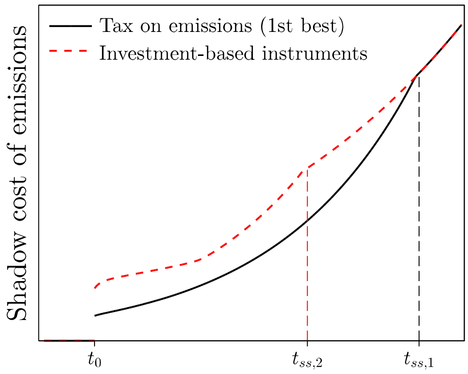

Figure 3: The shadow price of emissions (or carbon price) is higher with investment-based instruments than with a carbon price.

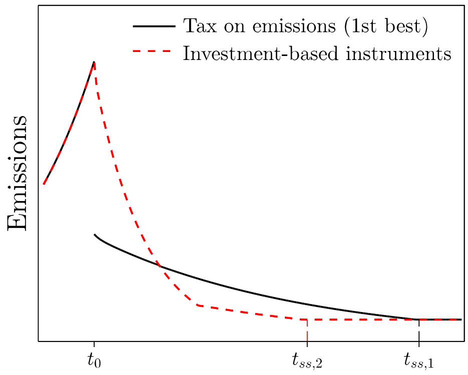

Figure 4: GHG emissions in the two cases. The carbon price induces spare polluting capital and thus reduces carbon emissions faster in the short-run.

minimum utility over the whole trajectory. Using this criterion, investment-based policies would be preferred to the carbon tax alone.

4.4. Lock-in

The carbon price does not always lead to under-utilize polluting capital. In fact, this depends on the stringency of the climate target and the level of emissions embedded in installed capital (Fig. 5). As long as climate policies are absent (or very lax), the global economy accumulates polluting capital, making GHG emissions grow and reducing the carbon budget for a given climate target (the arrow "conventional growth" in Fig. 5).

At a low development level (left hand side, Fig. 5), a carbon tax does not lead to under-utilization of polluting capital and reaching the climate target is possible and optimal without a downward step in income. In this case, the carbon price consistent with the climate target leads to the exact same growth path as investment-based policies. This is a situation of "flexibility" in which a country can chose a polluting or a clean development path at low cost, using either a carbon price or investment-based instruments.

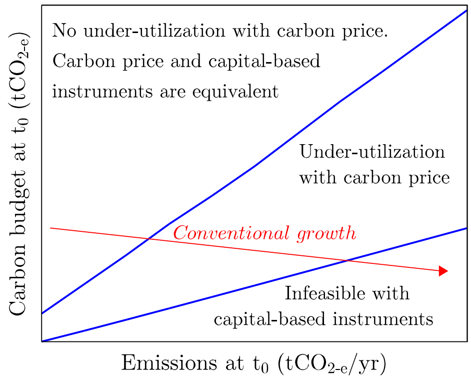

Figure 5: Depending on initial emissions (i.e. initial polluting capital $k_{b,0}$) and on the carbon budget ($\bar{m} - m_0$), the carbon tax and investment-based instruments can lead to different or similar outcomes (for a given set of parameters, and in particular ρ and δ). If the carbon budget is too stringent, such that waiting for polluting capital depreciation is not sufficient, the investment-based instruments cannot be used. If the carbon budget is not stringent, there is no under-utilization of polluting capital in the first-best optimum with the carbon tax and investment-based instruments are equivalent. While the economy is on the laissez-faire growth path (red arrow), polluting capital accumulates and the carbon budget is reduced for a given climate objective.

At one point, polluting capital reaches the level when a carbon price induces capital under-utilization and its negative political economy consequences. From there, a carbon price becomes more difficult to implement. But the alternative option of using investment-based instruments is available, leading to higher inter-temporal costs but no immediate drop in income. There is a window of opportunity, during which alternative investment-based instruments may induce a smooth and acceptable transition to a low-carbon economy.

If this occasion is missed (right hand side, Fig. 5), it becomes impossible to reach the climate target without under-utilization of polluting capital and investment-based options are not available any more (if the climate objective is not revised). This "investment-based infeasibility zone", i.e. the zone in which polluting capital must be under-utilized to remain below the ceiling, depends on the capital depreciation rate δ, the GHG dissipation rate ε, initial GHG concentration m_0 and initial polluting capital k_0. It is expressed analytically in Appendix G and can be approximated by:

$$\bar{m} < m_0 + \frac{G\, k_0}{\delta}$$

According to Davis et al. (2010), the level of existing polluting infrastructure in 2010 is still low enough to achieve the 2°C target without under-utilizing polluting capital, suggesting that the global economy is not in this last region yet. They show that if existing energy infrastructure was used for its normal life span and no new polluting devices were built, future warming would be less than 0.7 degrees Celsius. Yet, reaching the 2 degrees target might imply to stop investing in polluting capital tomorrow, which depends on our ability to overcome infrastructural inertia and develop clean energy and transport services (Davis et al., 2010; Guivarch and Hallegatte, 2011). Note that Davis et al. (2010)

do not discuss whether an optimal climate policy (i.e. a policy that minimizes the discounted cost) would lead to under-utilization, that is whether we are in the top or the middle triangle in Fig. 5.

When we get in the last area (right hand side, Fig. 5), not only the economic cost of reaching the climate target is higher, but the political economy also creates a carbon lock-in: the only option to reach the climate target has to have a significant short-term cost, making it more difficult to implement successfully a climate policy consistent with the target.

5. Conclusion

There is a trade-off between the inter-temporal optimality of a climate policy and the acceptability of its short-term impacts. The carbon tax is the best tool to maximize discounted welfare. But public policy is especially difficult in contexts where costs are immediate, concentrated and visible; and benefits are diffuse over time and over citizens (Olson, 1971). Triggering a transition towards clean capital with a carbon tax is likely to impose the early retirement of polluting capital, which could be operated for several more years (a process sometimes referred to as early-scrapping or mothballing). Such a strategy can be unacceptable to people who made economic choices under pre-existing rules.

We find that when all polluting capital is used in the short run, the outcome in terms of discounted intertemporal welfare is lower but current generations have a higher income than when a carbon price is implemented alone. Such a transition towards a low-carbon economy can be triggered by clean incentives that shift investments towards clean capital without penalizing existing polluting capital (e.g., feebate programs). It is equivalent to subsidizing polluting capital to ensure it is fully used until the end of its lifetime, despite the carbon tax.

With these compensations, current generations keep using their inefficient buildings and combustion engines, while redirecting their investment towards clean capital. After some time, the only remaining "polluting capital" is the one that does not need to be substituted by clean capital, as in the optimal case. A carbon price can then be implemented more easily, since all instruments are then equivalent. The second-best strategy focused on redirecting investments therefore only differ temporarily from the first-best pathway, in a way that smooths the transition costs: it decreases efforts in the short-run, increases them in the medium-run, and leaves them unchanged in the long-run.

Standard economic theory establishes that lump-sum cash transfers to compensate the losers are the best instrument to tackle the equity issues faced when implementing a carbon tax. In practice, however, it may not be feasible to monitor and compensate each individual loser of climate mitigation policies (e.g., Kanbur, 2010). The transfers themselves may appear unacceptable to other actors, especially if they go to a relatively wealthy population. Instead, policy instruments that focus on investment only, such as energy-efficiency standards or feebates programs, have the potential to tackle both the efficiency (they trigger a transition to clean capital) and the equity (they compensate losers) functions of a climate mitigation policy.

Acknowledgements

The authors thank Patrice Dumas, Louis-Gaetan Giraudet, Marianne Fay and the participants of the 2014 AEA conference for their useful comments on a previous version of this article. All remaining errors are the authors'. The views expressed in this paper are the sole responsibility of the authors. They do not necessarily reflect the views of the World Bank, its executive directors, or the countries they represent.

References

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., Feb. 2012. The environment and directed technical change. American Economic Review 102 (1), 131–166.

Ambrosi, P., Hourcade, J., Hallegatte, S., Lecocq, F., Dumas, P., Ha Duong, M., 2003. Optimal control models and elicitation of attitudes towards climate damages. Environmental Modeling and Assessment 8 (3), 133–147.

Anderson, S. T., Parry, I. W. H., Sallee, J. M., Fischer, C., 2011. Automobile fuel economy standards: Impacts, efficiency, and alternatives. Review of Environmental Economics and Policy 5 (1), 89–108.

Arrow, K. J., Kurz, M., 1970. Optimal growth with irreversible investment in a ramsey model. Econometrica 38 (2), 331–344.

Beason, R., Weinstein, D. E., 1996. Growth, economies of scale, and targeting in japan (1955-1990). The Review of Economics and Statistics, 286–295.

Davis, S. J., Caldeira, K., Matthews, H. D., 2010. Future CO2 emissions and climate change from existing energy infrastructure. Science 329 (5997), 1330 –1333.

Fischer, C., Newell, R. G., 2008. Environmental and technology policies for climate mitigation. Journal of Environmental Economics and Management 55 (2), 142–162.

Goulder, L. H., Mathai, K., 2000. Optimal CO2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39 (1), 1–38.

Goulder, L. H., Parry, I. W. H., 2008. Instrument choice in environmental policy. Review of Environmental Economics and Policy 2 (2), 152–174.

Guivarch, C., Hallegatte, S., 2011. Existing infrastructure and the 2C target. Climatic Change 109 (3-4), 801–805.

J. Barro, R., Sala-i Martin, X., 2004. Economic Growth.
URL http://localhost:8080/xmlui/handle/123456789/1237

Jorgenson, D., 1967. The theory of investment behavior. In: Determinants of investment behavior. NBER.

Kanbur, R., 2010. Macro crisis and targeting transfers to the poor. In: Globalization and Growth: Implications for a Post-Crisis World. Michael Spence, Danny Leipziger, Commission on Growth and Development, The World Bank, Washington DC, p. 342.

Kaplow, L., Jan. 1993. Government relief for risk associated with government action. Working Paper 3006, National Bureau of Economic Research.
URL http://www.nber.org/papers/w3006

Llavador, H., Roemer, J. E., Silvestre, J., 2011. A dynamic analysis of human welfare in a warming planet. Journal of Public Economics 95 (11), 1607–1620.

Meinshausen, M., Meinshausen, N., Hare, W., Raper, S. C. B., Frieler, K., Knutti, R., Frame, D. J., Allen, M. R., 2009. Greenhouse-gas emission targets for limiting global warming to 2C. Nature 458 (7242), 1158–1162.

Nordhaus, W. D., 1993. Optimal greenhouse-gas reductions and tax policy in the" DICE" model. The American Economic Review 83 (2), 313–317.
URL http://www.jstor.org/stable/10.2307/2117683

Olson, M., 1971. The Logic of collective action public goods and the theory of groups. Harvard University Press, Cambridge, Mass.

Ploeg, F. V. D., Withagen, C., 1991. Pollution control and the ramsey problem. Environmental and Resource Economics 1 (2), 215–236.

Rozenberg, J., Hallegatte, S., Perrissin-Fabert, B., Hourcade, J.-C., 2013. Funding low-carbon investments in the absence of a carbon tax. Climate Policy 13 (1), 134–141.

Vogt-Schilb, A., Meunier, G., Hallegatte, S., 2014. Optimal timing, cost and sectoral dispatch of emission reductions: abatement cost curves vs. abatement capital accumulation. ASSA, Philadelphia.

Williams, R. C., Nov. 2011. Setting the initial time-profile of climate policy: The economics of environmental policy phase-ins. NBER, 245–254.
URL http://www.nber.org/chapters/c12144

Zickfeld, K., Eby, M., Matthews, H. D., Weaver, A. J., Aug. 2009. Setting cumulative emissions targets to reduce the risk of dangerous climate change. Proceedings of the National Academy of Sciences 106 (38), 16129–16134.
URL http://www.pnas.org/cgi/doi/10.1073/pnas.0805800106

Appendix A. Maximization of the household's utility

We consider a Ramsey framework with a representative infinitely-lived household, who receives the economy's production from firms y_t, saves by accumulating assets a_t, receives income on assets at interest r_t and purchases goods for consumption c_t. The assets dynamics are given by:

$$\dot{a}_t = r_t \cdot a_t + y_t - c_t \tag{A.1}$$

At time t, consuming c_t provides consumers with a utility $u(c_t)$. The utility function is increasing with consumption, and strictly concave ($u'(c) > 0$ and $u''(c) < 0$).

The household maximizes their intertemporal utility, given by

$$W = \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{A.2}$$

where ρ is the rate of time preference.
The present value Hamiltonian is:

$$H_h = e^{-\rho t} \cdot \{u(c_t) + \lambda_t [r_t \cdot a_t + y_t - c_t]\} \tag{A.3}$$

where λ_t is the shadow price of income at time t.
The first order conditions for a maximum of W are:

$$\forall t, \partial_c H_h = 0 \Rightarrow \lambda_t = u'(c_t) \tag{A.4}$$

$$\forall t, \partial_a H_h = -\frac{\partial(e^{-\rho t}\lambda_t)}{\partial t} \Rightarrow \dot{\lambda}_t = (\rho - r_t)\lambda_t \tag{A.5}$$

The doted variables represent temporal derivatives.
If we differentiate eq. A.4 with respect to time and substitute for λ from this equation and $\dot{\lambda}$ from eq. A.5, we get the Euler equation, which gives the basic condition for choosing consumption over time:

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{c \cdot u''(c)} \cdot (r_t - \rho) \tag{A.6}$$

$-\frac{u'(c)}{cu''(c)} > 0$ so consumption grows if the rate of return to saving is higher than the rate of time preference. If the interest rate equals the rate of time preference, consumption is constant over time.

Appendix B. Social optimum (section 3)

The social planner program is:

$$\begin{aligned}
&\max_{c,i,k} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \\
&\text{subject to } F(q_p, k_c) - c_t - i_{p,t} - i_{c,t} = 0 && (\lambda_t) \\
&\dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
&\dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
&\dot{m}_t = G\, q_{p,t} - \varepsilon m_t && (\mu_t) \\
&m_t \leq \bar{m} && (\phi_t) \\
&i_{p,t} \geq 0 && (\psi_t) \\
&q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

The variables in parentheses are the co-state variables and Lagrangian multipliers associated to each constraint. λ_t is the current value of income. ν_t and χ_t are the current values of brown and green capital. μ_t is the current cost of pollution in the atmosphere, expressed in terms of undiscounted utility at time t.

The present value Hamiltonian associated to the maximization of social welfare is:

$$\begin{aligned} H_t = e^{-\rho t} \cdot \{ & u(c_t) + \lambda_t [F(q_p, k_c) - c_t - i_{p,t} - i_{c,t}] + \nu_t [i_{p,t} - \delta k_{p,t}] \\ & + \chi_t [i_{c,t} - \delta k_{c,t}] - \mu_t \cdot [G\, q_{p,t} - \varepsilon m_t] + \phi_t \cdot [\bar{m} - m_t] \\ & + \psi_t \cdot i_{p,t} + \beta_t [k_{p,t} - q_{p,t}] \} \end{aligned} \tag{B.1}$$

All multipliers are positive.
The complementary slackness conditions associated to the irreversibility constraint are:

$$\forall t,\ \psi_t \geq 0 \text{ and } \psi_t \cdot i_{p,t} = 0 \tag{B.2}$$

The complementary slackness condition associated to the "under-utilization" are:

$$\forall t,\ \beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{B.3}$$

Appendix B.1. First order conditions

First order conditions give:

$$\begin{aligned}
\frac{\partial H_t}{\partial c_t} &= 0 \Rightarrow & u'(c_t) &= \lambda_t \\
\frac{\partial H_t}{\partial i_{p,t}} &= 0 \Rightarrow & \lambda_t &= \nu_t + \psi_t \\
\frac{\partial H_t}{\partial i_{c,t}} &= 0 \Rightarrow & \lambda_t &= \chi_t \\
\frac{\partial H_t}{\partial k_{p,t}} &= -\frac{\partial (e^{-\rho t} \nu_t)}{\partial t} \Rightarrow & -\nu_t \delta + \beta_t &= -\dot{\nu}_t + \rho \nu_t \\
\frac{\partial H_t}{\partial k_{c,t}} &= -\frac{\partial (e^{-\rho t} \chi_t)}{\partial t} \Rightarrow & \lambda_t \partial_{k_c} F(k_{p,t}, k_{c,t}) - \chi_t \delta &= -\dot{\chi}_t + \rho \chi_t \\
\frac{\partial H_t}{\partial q_{p,t}} &= 0 \Rightarrow & \lambda_t \partial_{q_p} F(q_{p,t}, k_{c,t}) - \mu_t \cdot G &= \beta_t \\
\frac{\partial H_t}{\partial m_t} &= \frac{\partial (e^{-\rho t} \mu_t)}{\partial t} \Rightarrow & -\phi_t + \varepsilon \mu_t &= \dot{\mu}_t - \rho \mu_t
\end{aligned}$$

They can be reduced to the following equations:

$$u'(c_t) = \lambda_t = \nu_t + \psi_t = \chi_t \tag{B.4}$$

$$\partial_{k_c} F = \frac{1}{\lambda}((\delta + \rho)\chi_t - \dot{\chi}_t) \tag{B.5}$$

$$\beta_t = \frac{1}{\lambda}((\delta + \rho)\nu_t - \dot{\nu}_t) \tag{B.6}$$

$$\partial_{q_p} F = \beta_t + \tau_t \cdot G \tag{B.7}$$

As in Jorgenson (1967), the rental prices of green and brown capital $R_{c,t}$ and $R_{p,t}$ follow:

$$R_{c,t} = \frac{1}{\lambda}[(\delta + \rho)\chi_t - \dot{\chi}_t] \tag{B.8}$$

$$R_{p,t} = \frac{1}{\lambda}[(\delta + \rho)\nu_t - \dot{\nu}_t] \tag{B.9}$$

where χ_t and ν_t are respectively the values of green and brown capital. The Lagrangian multiplier associated to the constraint $q_{p,t} \leq k_{p,t}$ (β_t) is therefore equal to the rental price of brown capital $R_{p,t}$ (eq. B.6). As a consequence, as long as the rental price of brown capital is positive, brown capital is fully-utilized (complementary slackness condition B.2).

Appendix B.2. Carbon price

We define τ_t as the current price of GHG, expressed in units of consumables.

$$\tau_t = \frac{\mu_t}{\lambda_t} \tag{B.10}$$

We call t_{ss} the date at which GHG concentration reaches the ceiling[11]: $\forall t \geq t_{ss}$, $m_t = \bar{m}$. Eq. B.7 gives the evolution of μ_t. Using $\dot{\mu_t} = (\dot{\lambda_t}\tau_t + \lambda_t\dot{\tau_t})$ and $\frac{\dot{\lambda_t}}{\lambda_t} = (\rho - r_t)$, it can be written as the evolution of τ_t (the carbon price):

$$\dot{\tau_t} = \tau_t[\varepsilon + r_t] - \frac{\phi_t}{\lambda_t}$$

Before m reaches the ceiling, it is not binding and $\phi_t = 0$. In that case the carbon price follows the following motion rule:

$$\dot{\tau_t} = \tau_t[\varepsilon + r_t]$$

The carbon price thus exponentially grows at the endogenous interest rate plus the dissipation rate of GHG until the ceiling is reached. Once the constraint is reached, $\forall t \; m_t = \bar{m}$, and $\phi_t > 0$.

These dynamics may be interpreted as a generalized Hotelling rule applied to clean air: along the optimal pathway, and before the ceiling is reached, the discounted abatement costs are constant over time. The appropriate discount rate is $r + \varepsilon$, to take into account the natural decay of GHG in the atmosphere (see for instance Goulder and Mathai, 2000, footnote 11, p6).

When the ceiling is reached, $m_t = \bar{m}$, brown installed capital is constant at $k_{p,t} = \bar{m}\,\varepsilon/G$ and the economy is on the steady state.

Appendix B.3. Capital: proof of proposition 2

If we differentiate eq. B.4 with respect to time and substitute λ_t and $\dot{\lambda_t}$, we can write:

$$\frac{c_t \cdot u''(c_t)}{u'(c_t)} \cdot \frac{\dot{c_t}}{c_t} = (\rho + \delta - R_{c,t}) \tag{B.11}$$

From eq. A.6 we can thus write:

$$R_{c,t} = r_t + \delta \tag{B.12}$$

Combining eq. B.6 and eq. B.7 we find:

$$\partial_{q_p} F = R_{p,t} + \tau_t \cdot G'(q_{p,t})$$

□

[11] We assume that $m_t = \bar{m}$ on the interval $[t_{ss}, +\infty[$, which is compatible with usual functional forms, like the ones we use here for numerical illustrations.

Appendix C. Irreversibility constraint: proof of proposition 4

The GHG ceiling is imposed at $t = t_0$. Before that, the economy is in the competitive equilibrium so green and brown capacities have the same marginal productivity and capacities are fully used (Proposition 1). At t_0^-, i.e. just before the ceiling is internalized ($t < t_0$), we thus have the following limits for $q_{p,t}$ and $\partial_{q_p} F$:

$$\lim_{t \to t_0^-} q_{p,t} = k_{p,t} \tag{C.1}$$

$$\lim_{t \to t_0^-} \partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{k_c} F(q_{p,t}, q_{c,t}) \tag{C.2}$$

We use a proof by contradiction to show that at t_0^+ (when the constraint is internalized) the irreversibility condition is necessarily binding. Suppose that when $t > t_0$, the irreversibility condition is not binding, i.e. $\psi_t = 0$ (eq. B.2). According to proposition 3, it leads to:

$$\lim_{t \to t_0^+} q_{p,t} = k_{p,t} \tag{C.3}$$

$$\lim_{t \to t_0^+} \partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{k_c} F(q_{p,t}, q_{c,t}) + \tau_t \cdot G \tag{C.4}$$

So from eq. C.2 and eq. C.4:

$$\lim_{t \to t_0^+} \partial_{q_p} F \neq \lim_{t \to t_0^+} \partial_{q_p} F \tag{C.5}$$

$\partial_{q_p} F$ is a continuous function of $q_{p,t}$ so eq. C.5 implies that $\lim_{t \to t_0^+} q_{p,t} \neq \lim_{t \to t_0^+} q_{p,t}$, and that is incompatible with eq. C.1 and eq. C.3.
Therefore, the irreversibility condition is necessarily binding at $t = t_0^+$, i.e. $\psi_t > 0$.
Two cases then need to be distinguished, whether brown capacities are fully used or not. If brown capacities are under-utilized ($q_{p,t} < k_{p,t}$ and $\beta_t = 0$), there is a discontinuity in output. □

Appendix D. Under-utilization of brown capital: proof of proposition 5

When the irreversibility constraint is binding ($\psi_t > 0$), the price of brown capital ($\frac{\nu_t}{\lambda_t} = 1 - \frac{\psi_t}{\lambda_t}$) can decrease below 1. If $0 < \psi_t < \lambda_t$ the price of brown capital is positive, so brown capital is fully-utilized even though no brown investment is made. If $\psi_t = \lambda_t$ the price of brown capital is nil and it can be under-utilized. When $\psi_t > 0$ we can differentiate eq. B.4 ($\nu_t = \chi_t - \psi_t$) and substitute ν_t and $\dot{\nu}_t$ in eq. B.6 to obtain

$$\frac{\beta_t}{\lambda_t} = \frac{1}{\lambda_t} \Big((\delta + \rho)\chi_t - \dot{\chi}_t + \dot{\psi}_t - (\rho + \delta)\psi_t \Big)$$

and using eq. B.5 we get

$$\frac{\beta_t}{\lambda_t} = \partial_{k_c} F - \frac{1}{\lambda_t} \Big(-\dot{\psi}_t + (\rho + \delta)\psi_t \Big)$$

We call $p = \frac{1}{\lambda_t}\left((\rho+\delta)\psi_t - \dot{\psi}_t\right)$ and get

$$\partial_{q_p} F = \partial_{k_c} F - p + \tau_t \cdot G'(q_{p,t})$$

$0 < \psi_t \leq \lambda_t \Rightarrow 0 \leq \frac{1}{\lambda_t}\left((\rho+\delta)\psi_t - \dot{\psi}_t\right) \leq \left(\rho + \delta - \frac{\dot{\lambda}_t}{\lambda_t}\right)$,
so that $0 \leq p \leq \partial_{k_c} F$.
When $p = \partial_{k_c} F$, the rental rate of brown capacities is nil and brown capacities may be under-utilized (slackness condition, eq. B.3) in order to adjust the marginal productivity of brown capital to the carbon price:

$$\partial_{q_p} F = \tau_t \cdot G'(q_{p,t})$$

Appendix E. Maximization of social welfare with full utilization constraint

We use the same social planner program as in Appendix B and we add a full-utilization constraint:

$$q_{p,t} = k_{p,t} \ (\alpha_t)$$

The present value Hamiltonian associated to the maximization of social welfare is:

$$\begin{aligned} H_t = e^{-\rho t} \cdot \{ & u(c_t) + \lambda_t[F(q_p, k_c) - c_t - i_{p,t} - i_{c,t}] + \nu_t[i_{p,t} - \delta k_{p,t}] \\ & + \chi_t[i_{c,t} - \delta k_{c,t}] - \mu_t \cdot [G\, q_{p,t} - \varepsilon m_t] + \phi_t \cdot [\bar{m} - m_t] \\ & + \psi_t \cdot i_{p,t} + \beta_t[k_{p,t} - q_{p,t}] + \alpha_t[q_{p,t} - k_{p,t}] \} \end{aligned}$$

All multipliers are positive.
Equations B.6 and B.7 become:

$$\begin{aligned} \beta_t - \alpha_t &= \frac{1}{\lambda}((\delta+\rho)\nu_t - \dot{\nu}_t) \\ \partial_{q_p} F &= \beta_t - \alpha_t + \tau_t \cdot G \end{aligned}$$

The rental price of brown capital is therefore equal to $\beta_t - \alpha_t$. Note that due to complementary slackness conditions, if $\beta_t > 0$ then $\alpha_t = 0$ and if $\alpha_t > 0$ then $\beta_t = 0$. In this latter case, the rental rate of brown capital is negative (as with differentiation of capital costs). We have:

$$\begin{aligned} \partial_{k_c} F &= R_{c,t} = r_t + \delta \\ \partial_{q_p} F &= R_{p,t} + \tau_t \cdot G \end{aligned}$$

In the long run when $i_b > 0$ the equilibrium is equivalent to the social optimum. In the short run when $i_b = 0$, $\psi_t > 0$ and $R_{p,t} < R_{c,t}$, except that in this case $R_{p,t}$ becomes negative if the carbon price is higher than the marginal productivity of the last unit of brown capital (expressed in output per emissions). Thus brown capital is always fully-utilized.

Appendix F. Firms' maximization problem with differentiation of capital costs

Capital costs can be differentiated with fiscal incentives, e.g. subsidies on new green capacities ($\theta_{c,t} < 0$) or taxes on new brown capacities ($\theta_{p,t} > 0$). Here we model lump-sum taxes on all capacities, but they only have an impact on new investment decisions and thus lead to the same investment as taxes on new capacities. The optimal values of $\theta_{c,t}$ and $\theta_{p,t}$ can be obtained with a maximization of social welfare given the ceiling constraint. The firm's flow of profit at time t is given by:

$$\Pi_t = F(q_{p,t}, q_{c,t}) - (R_{c,t} + \theta_{c,t})\, k_{c,t} - (R_{p,t} + \theta_{p,t})\, k_{p,t} \tag{F.1}$$

The Lagrangian corresponding to the firm's maximization program is:

$$L(t) = \Pi_t + \beta_t(k_{p,t} - q_{p,t}) + \gamma_t(k_{c,t} - q_{c,t}) \tag{F.2}$$

First order conditions are:

$$\partial_{q_g} L = 0 \Rightarrow \qquad \partial_{q_g} F(q_{p,t}, q_{c,t}) = \gamma_t \tag{F.3}$$
$$\partial_{q_b} L = 0 \Rightarrow \qquad \partial_{q_b} F(q_{p,t}, q_{c,t}) = \beta_t \tag{F.4}$$
$$\partial_{k_g} L = 0 \Rightarrow \qquad \gamma_t = R_{c,t} + \theta_{c,t} \tag{F.5}$$
$$\partial_{k_b} L = 0 \Rightarrow \qquad \beta_t = R_{p,t} + \theta_{p,t} \tag{F.6}$$

For all t, the complementary slackness conditions are

$$\gamma_t \geq 0 \text{ and } \gamma_t \cdot (k_{c,t} - q_{c,t}) = 0$$

$$\beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0$$

Note that with the carbon tax, β_t was equal to the rental price of brown capacities while here it is equal to rental price plus the tax on brown capital (eq. F.6).
With eq. F.3 we have $\gamma_t = \partial_{q_b} F(q_{p,t}, q_{c,t}) > 0$, so $q_{c,t} = k_{c,t}$ for all t.
Similarly, $\beta_t = \partial_{q_b} F(q_{p,t}, q_{c,t}) > 0$, so $q_{p,t} = k_{p,t}$ for all t.
The combination of eq. F.3 and eq. F.5, and eq. F.4 and eq. F.6

$$\partial_{q_b} F(q_{p,t}, q_{c,t}) = R_{p,t} + \theta_{p,t}$$
$$\partial_{q_g} F(q_{p,t}, q_{c,t}) = R_{c,t} + \theta_{c,t}$$

The irreversibility constraint is never binding for green investments, so as in the laissez-faire equilibrium, the value of green capital in units of consumables is always 1. Green capacities and loans are perfect substitutes as assets, and $R_{c,t} = r_t + \delta$, with r_t the interest rate. Note that when the policies are implemented, the continuity of capacities imposes that $R_c(t_0^+) = R_c(t_0^-) - \theta_c(t_0^+)$ (with $\theta_c(t_0^+) < 0$). In other words, the rental price of green capacities suddenly increases when the subsidy is implemented.

At the same time, the irreversibility constraint is binding for brown capacities and their value decreases below that of green capacities. For the same reason as for the green rental price, when the tax is implemented we have $R_p(t_0^+) = R_p(t_0^-) - \theta_p(t_0^+)$ so the rental price of brown capacities decreases and the economy does not invest in new brown capacities during the first phase. Note that, contrary to the carbon tax case, this policy may lead to a negative rental price for brown capacities when brown investments are nil, if $\theta_{p,t}$ is higher than the marginal productivity of brown capital.

In the steady state, the amount of installed brown capital is fixed (so that emissions are constant and $\dot{m} = 0$), brown and green investments are positive, the irreversibility constraint is not binding and $R_{p,t} = R_{c,t}$. In this case the marginal productivity of brown capital is equal to that of green capital plus the sum of the tax and the subsidy (note that $(-\theta_{c,t})$ is positive):

$$\partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{q_c} F(q_{p,t}, q_{c,t}) + (\theta_{p,t} - \theta_{c,t})$$

The same steady state as in the social optimum is reached if the optimal value of the tax plus the subsidy is equal to the carbon tax multiplied by the marginal emissions of brown capital:

$$\forall t \geq t_{ss},\ \theta_{p,t} - \theta_{c,t} = \tau_t \cdot G$$

with t_{ss} the date at which the steady state is reached.

Appendix G. Second-best infeasibility zone

This zone defines the cases when the ceiling is reached before brown capacities have depreciated to a sustainable level. If no investment is made in brown capacities, we have:

$$k_{p,t} = k_0\, e^{-\delta t}$$

Therefore, the stock of pollution follows this dynamic:

$$\dot{m} = k_0\, e^{-\delta t} - \varepsilon\, m$$

The solution to this differential equation is:

$$m_t = -\frac{G\, k_0}{\delta - \varepsilon}\, e^{-\delta t} + \left(m_0 + \frac{G\, k_0}{\delta - \varepsilon} \right)\, e^{-\varepsilon t}$$

This function first increases to a maximum $m_{max} = \frac{G\, k_0}{\delta} e^{-\delta t}$ and then decreases. The maximum date is

$$t_{max} = -\frac{1}{\delta}\, \ln(\frac{m_{max}\, \varepsilon}{G\, k_0})$$

The expression of m at the maximum date gives the limit of the infeasibility zone if $m_{max} = \bar{m}$:

$$\bar{m} = -\frac{G\, k_0}{\delta - \varepsilon}\, e^{\ln(\frac{\bar{m}\, \varepsilon}{G\, k_0})} + \left(m_0 + \frac{G\, k_0}{\delta - \varepsilon} \right)\, e^{\frac{\varepsilon}{\delta} \ln(\frac{\bar{m}\, \varepsilon}{G\, k_0})}$$

This can be rewritten:

$$\bar{m} = \left[\left(m_0 + \frac{G\, k_0}{\delta - \varepsilon} \right) \left(\frac{\varepsilon}{G\, k_0} \right)^{\frac{\varepsilon}{\delta}} \left(\frac{\delta - \varepsilon}{\delta} \right) \right]^{\frac{\delta}{\delta - \epsilon}}$$

The "green incentives infeasibility zone" depends on the capital depreciation rate, the GHG dissipation rate, initial GHG concentration and initial brown capacities.