The Fair Carbon Tax

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January 30, 2014

Abstract

Environmental taxes are known to have a regressive impact. On top of that, those in the double-dividend literature suggest the revenues from such taxes can be used to reduce labor market distortions by lowering the income tax rate. This allows for yet even less redistribution, and thus more income inequality. This paper addresses these issues by incorporating an aversion from inequality in the individual utility function in a classical Mirrlees-type public-finance model. A new formula for the optimal income tax is derived, and in both optimal and suboptimal cases it is studied how the revenues from an environmental tax are optimally spent: lowering income tax rates, or redistributing more. The answer is dependent on the valuation of inequality and the level of preexisting tax rates.

Keywords: Double Dividend, Environmental taxation, Public Finance, regressive taxes.

JEL: E60 Q56 Q58 H23

Available for the Young Economist Award

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1 Introduction

This paper talks about the interaction of two of the most pressing issues of our time: inequality, and the environment. In order to reduce global warming, a substantial carbon tax is required (Sinn, 2008; Stern, 2007; van der Ploeg and Withagen, 2010). A big problem in raising carbon taxes is that those with a low income are often hit harder Combet, Ghersi, Hourcade, and Théry (2009), as they spend a larger share of their income on energy. At the same time, it is reasonable for the poor to care less about carbon taxes as their marginal utility from consumption is higher compared to their marginal utility from environmental benefits than for rich people. Both these things add to the fact that carbon taxes redistribute towards more inequality. On top of that, an approach suggested often by economists (Bovenberg and Goulder, 1994), is to use the proceeds of a carbon tax to diminish the distortions on the labor market, creating a "double dividend". By reducing distortions on the labor market through a lower income tax, there will however be even less redistribution and therefore even more income inequality. An alternative approach could be to use the proceeds of a carbon tax to hand out energy coupons equally per capita. This will create higher distortions on the labor market, but will also create lower income inequality due to income taxes. The central objective is thus to find the optimal way to decrease carbon emissions, taking into account the fundamental tradeoff between both labor market distortions, and income inequality.

Environmental taxation is looked at as an important special case in a framework with inequality aversion in public-finance models. On the topic of inequality there is a huge body of literature, for instance (Sen, 1973, 1992) to name one, who tried to include income inequality in the welfare function. With for example the occupy movement(Gamson, 2012) and Russel Brand's interview on BBC, questions about inequality are not only heavily present in the economic literatureKanbur and Lustig (1999), but also take an important place in the public atmosphere. This paper is not intended to settle any discussion about how inequality should be valued, but takes the standpoint that the individual utility of inequality is negative, and explores the effects thereof in a public finance framework, specifically in the case of an environmental taxation. Here I discuss some of what I regard to be important themes in the discussion.

In the classical public-finance model as in Mirrlees (1971), there is a balance between efficiency and redistribution, where redistribution is done because low incomes are assumed to have a higher marginal utility of consumption. But reducing income equality might well stimulate societies in ways that are hidden by consumption levels, reminiscent of Fréderic Bastiast's idea, *ce qu'on voit pas*: some virtues are hidden by measurement of pure growth, fixing a broken window or financing of a war both spur consumption growth. More concretely, income inequality is measured to have a negative effect on reported hapiness (Alesina, Di Tella, and MacCulloch, 2004; Wilkinson, Pickett, and Chafer, 2009), income inequality within rich societies can, and generally does, increase crime rates (Blau and Blau, 1982), mental health and drug use (Wilkinson, Pickett, and Chafer, 2009), health and social problems (Wilkinson and Pickett, 2006), and deteriorate educational performance and social mobility (Isaacs, Sawhill, and Haskins, 2008; OECD, 2012). Income inequality might increase the centralization of power, as described in (Stiglitz, 2012) and increases inequality of opportunity(Stiglitz, 2012). It can fuel populist, protectionist and anti-globalization sentiments (OECD, 2012), and is seen as one of the great challenges of our society by many. Those with high incomes are also hit by the negative effects of crime, and suffer the consequences of lower growth when the poor are not only poor in income but also poor in opportunities. I thus assume that income inequality is also bad for the high incomes.

As result of directly including an aversion from income inequality in the individual utility function, in this paper a familiar mechanism in political discussion about the level of redistribution is found: some level of inequality is wanted to incentivize labour participation and risk taking, but too much inequality has a negative effect on welfare, and even on growth. When this is the case, one policy decision is to increase income tax rates. How big the effect of inequality on labor participation is, is unclear. This depends on how exactly income inequality enters the utility function. Kanbur, Pirttilä, and Tuomala (2006) discuss some behavioral effects in public finance. Consuming or working more because of a greater income inequality might be facilitated through a "keeping up with the Joneses"- or "Rat Race"-mechanism (see for instance Ljungqvist and Uhlig (2000), Clark and Oswald (1996)).

Without any income inequality taken into account, the control of externalities in the presence of other tax distortions is a complex second-best problem in itself. Here Pigou (1920) gave us the simple first-best rule where marginal tax rates equal to marginal external damages. Sandmo (1975) was one of the first to study externality control in the presence of other distortions. An ensuing discussion focused on finding possible double dividends: to implement a tax that would not only internalize externalities but also reduce distortions by other taxes due to the raised revenues of a tax on an externality(Bovenberg and van der Ploeg, 1994). A review of this body of literature can be found in Bovenberg and Goulder (2001). A seminal analysis of commodity taxation in the presence of an optimal income tax is given by Atkinson and Stiglitz (1976), but their work also differs from this paper in that the social resource cost of the commodities does not include externalities. Although the second-best literature is strongly related to this paper, most of it addresses the question of how externalities influence the level at which externality taxes should be set. In this paper solely the question of how to spend its revenues is addressed, and therefore it is assumed the level of an environmental tax is set. Specifically, the question is adressed how to spend the revenue of a carbon tax in a model where other taxation, like the income tax, is already in place.

Likewise, the optimal income taxation as a second-best problem with assymetric information has been studied by many since Mirrlees (1971). The problem of how to spend tax revenues of a carbon tax is examined in the recent study by the IMF (de Mooij, Parry, and Keen, 2012), although income inequality effects of a carbon tax are not addressed in their report. Reports on how to use taxes to reduce inequality also exist (Joumard, Pisu, and Bloch, 2012), as do studies that do report on the distributional effects of carbon taxes (Combet, Ghersi, Hourcade, and Théry, 2009; Casler and Rafiqui, 1993; West and Williams, 2004). But none combine these effects. In the special case of carbon emissions, specific distributional effects have been studied. Here we think of effects like capital gains from cleaner air in certain neighborhoods(Fullerton, 2011), or the consequences for a coal-mining town that has lost its major source of income. However, none of these contain a broader public-finance study. A broader public-finance approach to the inquiry about how to tax externalities contained in Kaplow (2010) and Jacobs and de Mooij (2011). Jacobs and de Mooij (2011) find that the presence of preexisting distortions is largely irrelevant for the level of a externality-controlling Pigouvian tax. Their findings are based on a framework that has a heterogenous framework as that of Mirrlees. At the same time they are the first to study externality taxation in a Mirrleesian income tax model. This paper can be seen as a complement to the work of Jacobs and de Mooij (2011) and Kaplow (2010) by including pure income inequality aversion into their models. Looking at countries around the world we see that some, with a relatively small government (and thus relatively small labor distortions), should use the proceedings of a carbon tax for handing out energy coupons in order to redistribute more. Other countries, with a large government and more pressing labor distortions, should focus on efficiency and use the proceedings of a carbon tax for reduction of these labor distortions.

This writing is organized as follows: The model is presented in section 2, the optimal labor-tax is derived in section 3, and the environment is included in section 3.1. Section 4 contains a simulation, calibration, and results, and section 5 concludes.

2 Model

In this section a model is developed to derive a new expression for the optimal linear income tax. The model contains heterogeneous households, a consumption good and a government and can easily incorporate the environment and a measure for income inequality.

2.1 Households

There is a total mass of individuals equal to N, and they may differ by a skill parameter $n \in \mathcal{N} = [\underline{n}, \bar{n}]$. The density of individuals with type n is given by $f(n)$. Each individual n receives utility from commodity c_n. There is also a disutility from labor l_n and income inequality I. Households take the level of income inequality to be exogenous. In this paper, like in Mirrlees (1971), intertemporal problems are ignored, as are differences in taste, family size and composition and involuntary transfers. Individuals are assumed rational, there is no migration. Further, derivatives with respect to any variable x of person n's utility u_n will be denoted by u_x:

$$u_n \equiv u(c_n, l_n, I), \quad u_c, -u_l, -u_I > 0, \quad u_{cc}, u_{ll} < 0, \quad \forall n \tag{1}$$

The dependency of u_n on I with $u_I < 0$ is new in this model compared to other public finance models. Households are tied to their budget constraint:

$$c_n = (1-t)nl_n + g, \quad \forall n \tag{2}$$

Here nl_n is a person's ability level multiplied by the number of hours worked, and so $z_n \equiv nl_n$ is the income of an individual household n. g is a lump-sum transfer. The household maximization problem bounded to the budget constraint gives the FOC:

$$\frac{u_c}{u_l} = -\frac{1}{(1-t)n}, \tag{3}$$

the marginal rate of substition between the consumption good and leisure equals their relative prices. It can be used to find the form of Roy's Lemma. Further, I define $v(t, g, I)$ as the indirect utility at an optimal choice of l_n and c_n, l_n^*, c_n^*: $v_n(t, g, I) \equiv u_n(l_n^*, c_n^*, I)$. This together gives me the expressions for Roy's Lemma:

$$\frac{\partial v_n}{\partial t} = \frac{du_n}{dt}_{(dg=dI=0)} = -nl_n\lambda_n,$$

$$\frac{\partial v_n}{\partial g} = \frac{du_n}{dg}_{(dt=dI=0)} = \lambda_n,$$

$$\frac{\partial v_n}{\partial I} = \frac{du_n}{dg}_{(dt=dg=0)} = u_I.$$

These will be used to solve the model later on.

2.2 The Environment

In this section the model is expanded so as to include environmental taxation. The model remains largely the same, with a few notable exceptions. First, we separate out consumption c_n into two parts: dirty consumption x_n and clean consumption y_n, $c_n = x_n + y_n$. On the dirty consumption good tax τ will be levied. We assume no tax on the clean commodity, this is an abstraction from the real world. Further, we assume fixed producer prices and linear technology. Relaxing the assumption of fixed producer prices and linear technology does not affect the results as long as producer prices result from competitive behaviour and producers face constant returns to scale (c.f. Bovenberg and van der Ploeg (1994)). Utility is now of the form

$$u_n = u_n(c_n, x_n, l_n, I, E), \qquad u_E > 0, \quad u_{EE} < 0$$

.

Except for the fact that u_n depends on I with $u_I < 0$, this model is the same as that used in Jacobs and de Mooij (2011). The environment is modeled as

$$E = E_0 - \alpha N \int_{\mathcal{N}} x_n dF(n)$$

,

and the household budget becomes

$$y_n + x_n(1+\tau) = (1-t)nl_n + g, \quad \forall n.$$

This gives the first-order conditions:

$$\frac{u_y}{u_l} = -\frac{1}{(1-t)n}, \quad \frac{u_x}{u_l} = -\frac{1+\tau}{(1-t)n}, \quad \frac{u_x}{u_y} = 1+\tau.$$

The marginal rate of substitution between the consumption good and leisure thus equals their relative prices.

2.3 The Government

The government maximizes a Bergson-Samuelson social welfare function, which is a concave sum of individual utilities:

$$max_{t,g} W = N \int_{\mathcal{N}} \Psi(u_n) dF(n), \quad \Psi'(u_n) > 0, \quad \Psi''(u_n) \leq 0. \tag{4}$$

Where $dF(n) = f(n)dn$, and Ψ can be such that the government is Rawlsian ($\Psi'(u_n) = 0$, except for $n = \underline{n}$), or utilitarian ($\Psi'(u_n) = 1 \ \forall n$), or anywhere in between. There is also a government budget requirement G, which needs to be satisfied through:

$$N \int_{\mathcal{N}} [tnl_n + \tau x_n] dF(n) = Ng + G$$

The last important ingredient is that the government can only see the household income $z_n = nl_n$, not the hours worked or the individual ability levels, and has to base all its decision on this. The model is therefore a clear second-best. If no inequality is taken into account, the optimal tax result should be as in the Mirrleesian optimal tax model(Mirrlees, 1971). If we do take inequality into account, one might find a different labor-income tax and lump-sum return.

2.4 Inequality

Lastly, for inequality we can choose between different measures, like the Robin Hood coefficient, the income gap between the lowest and highest ten percent, or the Gini coefficient. Note that for any measure of inequality I assume it to be a function $I : z \to \mathbb{R}$, where z is the set of all incomes; $z = \{z_n\}, n \in \mathcal{N}$.

3 Optimal Mirrleesian Taxation Revisited

Here a revision of the Mirrlees optimal income tax is discussed. First, set $\tau = 0$. As there is no difference between using dirty or clean consumption goods in that case, write c_n instead of $x_n + y_n$. The government has instrument choice g and t, which leads to the Lagrangian [1]:

$$max_{t,g,I}\mathcal{L} = \int_{\mathcal{N}}[\Psi(v_n) + \eta(tnl_n - g - \frac{G}{N})]dF(n) + \mu(I - I(z)).$$

The first-order condition on I, works out to:

$$\mu = -\int_{\mathcal{N}}[\Psi'(v_n)u_I + \eta tn\frac{\partial l_n}{\partial I}]dF(n). \tag{5}$$

Here μ is the loss of social welfare is as a consequence of increased income inequality. The contributions of individual households fall apart into two terms: $\Psi'(v_n)u_I$ is the decreased welfare due to a smaller individual utility for person n due to increased income inequality. This term is always negative, because $u_I < 0$. The second term, $\eta tn\frac{\partial l_n}{\partial I}$, is the change in welfare due to the effects of this household on the labor market. Here $\frac{\partial l_n}{\partial I}$ is the amount that person n will work more, or work less. Through the factor ηtn this is converted into units of welfare. This term is not ex ante positive or negative. Therefore, μ is positive only if the utility effect dominates the labor effect.

Next, the first-order condition of the government optimization problem:

$$\int_{\mathcal{N}}[\frac{\Psi'(v_n)\lambda_n}{\eta} + tn\frac{\partial l_n}{\partial g}]dF(n) - \frac{\mu}{\eta}I_g(z) = 1, \tag{6}$$

or,

$$\int_{\mathcal{N}}\alpha_n^* dF(n) = 1. \tag{7}$$

$\alpha_n^* \equiv \frac{\Psi'(v_n)\lambda_n}{\eta} + tn\frac{\partial l_n}{\partial g} - \frac{\mu}{\eta}I_g$ is the added welfare generated through person n by increasing the transfer g(Jacobs, 2013). This includes any possible shifts in her tax payments through a change in his or her labor supply, and her different valuation of inequality. The left-hand side of equation 7 contains the term $\frac{\mu}{\eta}I_g(z_n)$. Here, I_g is the change in inequality due to a change in the lump-sum

[1] Note that in the optimization problem I is treated as an instrument, where it actually is a consequence of the distribution. This is equivalent as the identity for inequality is added as a constraint: $I = I(z)$.

transfer g, μ converts it to utility units and division by η converts it further to monetary units. The left hand side is therefore a measure of the marginal benefits to household n of raising the transfer g. Equation 7 thus equates the marginal benefits to household n of raising the transfer g, to its average costs per household: one euro.

This leads to the revised optimal Mirrleesian income tax rate:

Proposition 1. *The optimal income tax is given by*

$$\frac{t}{1-t}\bar{\epsilon^c_{lt}} + \frac{\mu}{\eta}\left(\frac{I_t(z)}{\int_{\mathcal{N}} z_n dF(n)} + I_g\right) = \xi^*, \tag{8}$$

where $\bar{\epsilon^c_{lt}} \equiv \frac{\int_{\mathcal{N}} z_n \epsilon^c_{lt} dF(n)}{\int_{\mathcal{N}} z_n dF(n)}$ *is the income-weighted compensated labor elasticity, and the redistributive characteristic is given by* $\xi^* = \frac{\int_{\mathcal{N}} z_n(1-\alpha^*_n)dF(n)}{\int_{\mathcal{N}} \alpha^*_n dF(n) \int_{\mathcal{N}} z_n dF(n)}$ *denotes the normalized covariance between welfare weights* α_n *and income* z_n *(Feldstein, 1972).*

Proof. Rewrite the first order condition with respect to the tax rate t. Using the Slutsky-equation $\frac{\partial l_n}{\partial t} = \frac{\partial l^c_n}{\partial t} - nl_n\frac{\partial l_n}{\partial g}$, this gives $\int_{\mathcal{N}}[nl_n(1-[\frac{\Psi'(v_n)\lambda_n}{\eta} + tn\frac{\partial l_n}{\partial g}]) + nl_n\frac{t}{(1-t)}\frac{\partial l^c_n}{\partial t}\frac{(1-t)}{l_n}]dF(n) = \frac{\mu}{\eta}I_t(z)$. Implementing all definitions and dividing by $\int_{\mathcal{N}} z_n dF(n)$ leads to the result.

□

Note that $\frac{\mu}{\eta}I_t(z)$ is the monetary loss of social value of the change in income inequality due to a changing income tax rate, and $\frac{\mu}{\eta}I_g$ is its counterpart for the lump-sum transfer. $\int_{\mathcal{N}} z_n dF(n)$ is the total income of households, and so $\frac{\frac{\mu}{\eta}I_t(z)}{\int_{\mathcal{N}} z_n dF(n)}$ is the monetary value of a change in income inequality due to a change in the income tax rate as a fraction of total income. The effect of the income tax rate is normalized by the total household income, as an infinitesimal change in the income tax rate costs every individual a fraction of their income, whereas the costs related to an infinitesimal change in the lump-sum transfer are independent of income, and thus normalized by $\int_{\mathcal{N}} dF(n) = 1$. The redistributional effect of a lump-sum transfer is observed to be dominated by the redistributional effect of the income tax(Wu, Perloff, and Golan, 2002).

We see this that this expression and its derivation are strongly similar to that of the derivation of the Mirrlees optimal linear income tax(Mirrlees, 1971): taxes are lower for a high elasticity $\bar{\epsilon^c_{lt}}$ and rise for a larger redistributive characteristic. On the one hand, efficiency is higher when $\bar{\epsilon^c_{lt}}$ is lower, indicating that for a higher tax people will hardly change their labor supply. On the other hand redistribution will be stronger for higher ξ^*, indicating a higher welfare can be achieved by donating to the low incomes with a high marginal utility of consumption: There is thus the familiar balance between efficiency and redistribution.

However, now there is also an effect of income inequality. The total effect is increasing with an increasing income tax unless the labor effect of a larger income inequality dominates the utility effect. In the end, it depends on the sign of $\frac{\mu}{\eta}\frac{I_t(z)}{\int_{\mathcal{N}} z_n dF(n)}$. In this the sign of μ is crucial. If we assume the social value of a euro the government is positive, $\eta > 0$, and that the inequality measure is such that both $I_t < 0$ and $I_g < 0$, this automatically leads to the follwing theorem:

Proposition 2. *The optimal income tax rate increases if and only if $\mu > 0$, or equivalently, labor effects dominate utility effects in the valuation of inequality:*

$$-\Psi'(v_n)u_I < \eta tn\frac{\partial l_n}{\partial I}. \tag{9}$$

Proof. By assumption $\eta > 0$. The increase (decrease) of the income tax rate follows directly from the optimal income tax formula 8. As $\Psi'(v_n)u_I < 0$ as $u_I < 0$. The right-hand term, $\eta tn\frac{\partial l_n}{\partial I}$, deals with the welfare effects of a possible increase or decrease of labor supply because of an higher income inequality. Therefore, the effect of the optimal linear income tax is ambiguous: There is a clear utility effect, but if the effects of inequality on the labor supply are strong enough, taxes should be set lower in order to redistribute less. □

Those who argue that inequality has a positive welfare effect, might base their argument on this: higher inequality would lead to more earnings competition, and thus to more labor supply if people cannot adjust their ability levels. One special case that is easy to analyse is that of seperable utility:

Proposition 3. *When utility is seperable in the sense that $u(c,l,I) = \tilde{u}(c,l)\phi(I)$ for some $\tilde{u}$ and ϕ, income tax rates increase.*

Proof. Note that if utility is seperable from equation 3, we see that the ratio between consumption and labor will be independent of income inequality. Therefore $\frac{\partial l_n}{\partial I} = 0$, and thus μ is negative as follows from equation 5. From the optimal income tax rate equation 8, we find that the optimal income tax rate will be set higher when $u_I < 0$.

□

The above proposition describes that the simplest intuitive case, where an aversion from inequality leads to higher taxation, is the case for a seperable utility function. But, although inequality might be perceived as creating unhappiness, it also incentivizes higher labor efforts. It is hidden in the structure of the utility function under which conditions a higher labor effort has enough positive effect to raise the optimal level of the income tax.

Proposition 4. *If $u_{c,I} = 0$, a necessary condition for income taxes to be set higher than in the Mirrleesian benchmark is that:*

$$u_{l,I} > 0 \tag{10}$$

Moreover, if $u_{l,I} \geq 0$ and $u_{c,I} > 0$, $\frac{\partial l_n}{\partial I} > 0$.

Proof. If $u_{l,I} < 0$, note that the marginal utility of leisure increases with rising income inequality, and people have still equal marginal utility of consumption. Thus $\frac{\partial l_n}{\partial I} < 0$, $\mu > 0$, and the income tax rate increases. If $u_{l,I} < 0$, we are in the case of seperable utility. If $u_{l,I} > 0$, and thus $\frac{\partial l_n}{\partial I} > 0$, there may be negative μ and thus lower tax rates. For the last part note that if the marginal utility of both consumption increases and the marginal utility of leisure decreases, so labor efforts $\frac{\partial l_n}{\partial I} > 0$. □

Proposition 5. *(conjecture) For any continuous skill distribution $f(n)$ taking values on $[0, \infty)$, there exists a function u such that in the optimal tax system $\frac{\partial l_n}{\partial I} > -\Psi(u_n)u_I$, $\forall n$ and constraints 1 hold.*

The meaning of this proposition would be that compared to the pure Mirrleesian optimal tax, in this model there can be a utility function chosen in such a way that the optimal tax is set higher or lower. This means that the conclusion of the model is driven by the choice of utility function. It would therefore perhaps be better to find other, empirical, ways to measure the size and sign of $\frac{\partial l_n}{\partial I}$. This is outside the scope of this paper.

Two extensions are of particular interest. First, if one also wishes to account for Keeping up with the Joneses and Rat Race effects, one might take a utility function of the form

$$u_n = u_n(c_n - \alpha\bar{c}, l_n - \beta\bar{l}) - f(IM) \tag{11}$$

Second, one might also want to discuss the optimal nonlinear tax. These issues are described in an Appendix.

3.1 Optimal Mirrleesian Taxation Revisited, Again

In this section the environment is included by generalizing towards $\tau \neq 0$ and $u_n(y_n, x_n, l_n, I, E)$. Taking the tax rates as given, we want to show when the tax revenues should be used for redistribution, and when for efficiency purposes. The Lagrangian is:

$$max_{t,g,\tau,I,E} \;\; \mathcal{L} = \int_{\mathcal{N}} [\Psi(v_n) + \eta(tnl_n + \tau x_n - g - \frac{G}{N})]dF(n)$$
$$+\mu(I - I(z)) - \rho(\frac{E - E_0}{N} - \alpha \int_{\mathcal{N}} x_n dF(n)$$

Giving the first-order condition on inequality:

$$\mathcal{L}_I = \int_{\mathcal{N}} [\Psi'(v_n)u_I n + \eta tn\frac{\partial l_n}{\partial I} + (\eta\tau - \alpha\rho)\frac{\partial x_n}{\partial I}]dF(n) + \mu = 0, \qquad (12)$$

The interpretation here is similar.$-\mu$ is the change in welfare is as a consequence of an increased income inequality. There is now an extra term, $(\eta\tau - \alpha\rho)\frac{\partial x_n}{\partial I}$, the effect on the government revenue and the change in green welfare due to a change in consumption of the dirty consumption good. The revenue effect is converted from monetary units to utility units by the factor η. Likewise, the first-order condition on g now gives:

$$\int_{\mathcal{N}} \alpha_n^{E*} dF(n) = 1. \qquad (13)$$

Now, $\alpha_n^{E*} \equiv \frac{\Psi'(v_n)\lambda_n}{\eta} + tn\frac{\partial l_n}{\partial g} - \frac{\mu}{\eta}I_g + (\tau - \frac{\alpha\rho}{eta})\frac{\partial x_n}{\partial g}$. The first part is still interpreted as the extra welfare generated by an increase in the utility of person n, inequality effects, and any possible shifts in his tax payments through a change in his or her labor supply. The new term is the extra welfare generated by a change in tax revenues from the dirty consumption good. The interpretation of the right-hand side remains the same, and thus equation 13 represents again the balance between the average marginal benefits of raising the transfer g by one euro and the costs of doing so.

Proposition 6. *The optimal income tax is given by*

$$\frac{t}{1-t}\overline{\epsilon_{lt}^c} + \frac{\tau - \frac{\alpha\rho}{\eta}}{\tau + 1}\overline{\gamma\epsilon_{xt}^c} + \frac{\mu}{\eta}(\frac{I_t(z)}{\int_{\mathcal{N}} z_n dF(n)} + I_g) = \xi^*, \qquad (14)$$

where $\gamma \equiv \frac{1+\tau}{1-t}\frac{x_n}{nl_n}$ *is the net expenditure share of dirty commodities in net labor* *income.*

Proof. Similar to before. See also Jacobs and de Mooij (2011) who do the same without income inequality. □

Note that we find the original Mirrlees' taxation back again when there is no environmental tax and inequality is not taken into consideration at all, $\tau = \mu = 0$, and the result from Jacobs and de Mooij (2011) when just $\mu = 0$: If income taxes decrease demand for the dirty consumption good, their positive effect on green welfare is more important than their negative effect on after-tax distortions in the consumption bracket as long as τ is below the Pigouvian level. If income taxes increase demand for the dirty good, the logic works the other way around. When $\mu \neq 0$, the price of one unit of environmental quality, $\frac{\rho}{\eta}$, changes to include inequality effects (see first-order condition on E below), and thus the Pigouvian level of taxation also changes.

$$\int_{\mathcal{N}} [\frac{u_E}{u_c}\frac{\Psi'(v_n)\lambda_n}{\eta} + tn\frac{\partial l_n}{\partial E} + (\tau - \alpha\frac{\rho}{\eta})\frac{\partial x_n}{\partial E}]dF(n) - \frac{\mu}{\eta}I_E(z) = \frac{\rho}{\eta} \tag{15}$$

This does not, however, change the intuition. In order to compute the optimal income tax, a balance is struck between the redistributive characteristic, efficiency in the labor market and the consumption market, and a term resulting from inequality aversion. Another special case of this result, is the government that imposes an environmental tax τ due to some external political reasons, but does not care about the environment at all. This government looks at $u_n = u(y_n, x_n, l_n, I)$, and thus ignores green welfare effects.

Proposition 7. *The optimal income tax for a government that implements an environmental tax, but takes individual utility to be independent of environmental quality, is:*

$$\frac{t}{1-t}\bar{\epsilon^c_{lt}} + \frac{\tau}{\tau+1}\overline{\gamma\epsilon^c_{xt}} + \frac{\mu}{\eta}(\frac{I_t(z)}{\int_{\mathcal{N}} z_n dF(n)} + I_g) = \xi^*, \tag{16}$$

where $\gamma \equiv \frac{1+\tau}{1-t}\frac{x_n}{nl_n}$ *is the net expenditure share of dirty commodities in net labor income.*

Proof. Similar to before. □

The interpretation is clear: an increase in the income tax rate still increases after-tax distortions in the consumption bracket as long as they decrease demand for dirty consumption. Now, however, the exacerbating effect of the income taxes on consumption distortions is no longer balanced by an increase in green welfare, as the price of a unit of environmental quality is set to zero.

4 Simulation

In order to answer the question what to do with the proceeds of an environmental tax, results were derived above in the case of optimal taxation. However, no country finds itself close to such an optimum. Therefore, we need to consider what to do when the preexisting taxation is suboptimal. To this end I calibrate the economy to a specific country, the Netherlands, and I calculate the welfare effects of different policy choices. The goal of this exercise is to find out which countries are to use their carbon tax revenues for redistribution, and which for efficiency purposes. Specifically, I try to answer this question: What is the optimal mix of the two policy instruments, raising the lump-sum transfer g and lowering income tax t for a country that currently finds itself in disequilibrium? To answer this question, I take the level of the pollution tax to be given and set exogenously. The question remains: what policy implications are there for the government. Here I make a distinction between different types of government: taking different amounts of inequality aversion and green welfare into account. First, a government is considered like in equation 16, that levies an environmental tax due to political reasons, but does not care about the environment. Then, a government is considered that does take environmental quality into account. Details about algorithms can be found in the Appendix.

4.0.1 Calibration

In this section I explain the numbers and functional forms put into my simulation. In this draft, the numbers are not very precise yet. This is a documentation of what I used. For utility I use a Cobb-Douglas type function

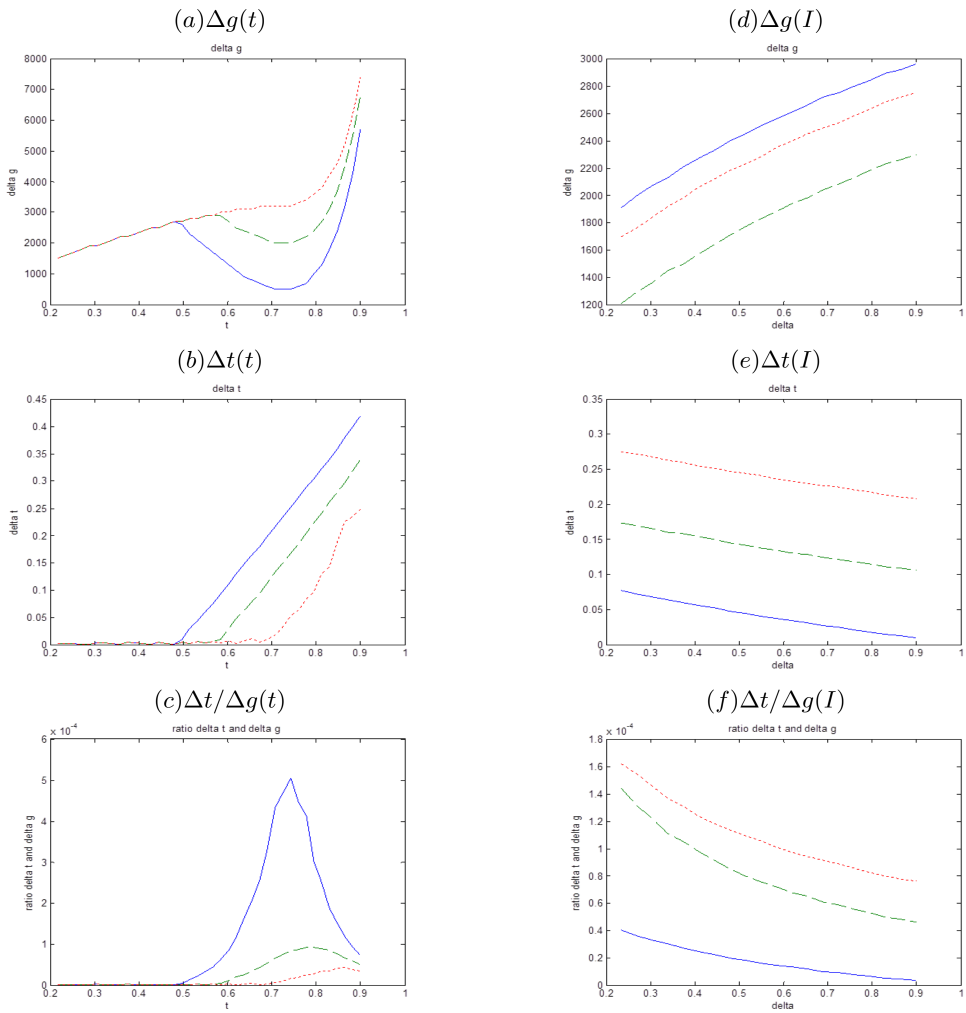
$$u_n = x_n^{\nu} y_n^{\beta} (1-l)^{\gamma} (1-I)^{\delta} - \zeta \int_{\mathcal{N}} x_n dF(n), \tag{17}$$

with $\nu = 0.05, \beta = 0.55, \gamma = 0.4$, and for I use various values for δ and ζ. The measure on Inequality I use is the Robin Hood-measure, measuring how big a share of the total income is required to make the income distribution completely equal. Note that for such a specification $\gamma \bar{\epsilon}_x^c t > 0$, and thus as income taxation inreases, demand for the dirty consumption good falls. This means that as long as $\tau < \frac{\alpha\rho}{\eta}$, distortions on the labor market exacerbate damages to the environment. The skill distribution is taken to be similar to the income distribution in the Netherlands (Zoutman, Jacobs, and Jongen, 2011); a lognormal distribution with a Pareto tail. For an approximation of the parameters of the Pareto tail, the World Top Income Database (Alvaredo, Atkinson, Piketty, and Saez, 2012) was used, and calibrated again to the Netherlands. For the Pareto tail function $(\frac{x_m}{x})^{\alpha}$, $\alpha = 3.23$ and $x_m = 336365$. The lognormal part of the income distribution is characterized by its mean, 43600, and its mode, 33000. The skill distribution is sliced into a thousand equidistant quantiles of 1000 Euros each. This means we effectively only study households with an income between zero and one million per year, and each person's yearly income is rounded up to the nearest thousand Euros.

4.0.2 Policy Simulations

First I describe a government that cares not about the environment, but does impose a carbon tax and wonders how to spend it. This is displayed in figure 1. If we do not value inequality at all, the optimal size of the lump-sum transfer generally increases as the size of the preexisting income taxes are larger, figure 1a. For relatively low levels of income tax, it is optimal to spend all revenues from a carbon tax on increasing the lump-sum transfer, as the distortions created by the income tax are not substantial enough to justify a lowering of the income tax. Redistribution is of greater importance in order to gain welfare, by giving to those who have a higher marginal utility of consumption. As we considers economies with a larger preexisting income tax, we see that if no inequality is taken into account (continuous line, $\delta = 0$), a portion of the proceedings of a carbon tax should be spent on lowering the income tax, which means only a smaller fraction can be spent on increasing the lump-sum transfer. For yet higher values of the preexisting income tax, the lump-sum transfer rises again. This regime is over the top of the Laffer curve(Wanniski, 1978), where lowering the income tax only gives more money to the government, and it is thus possible to increase g as well. We see that, if we do value inequality (long dashes, $\delta = 0.7$), that only for higher preexisting tax schemes the proceedings of a carbon tax are used partly to decrease the income tax, and even then it happens only later. If inequality is valued even stronger (dots, $\delta = 3$), we see that for higher preexisting income taxes, the lump-sum transfer hardly decreases at

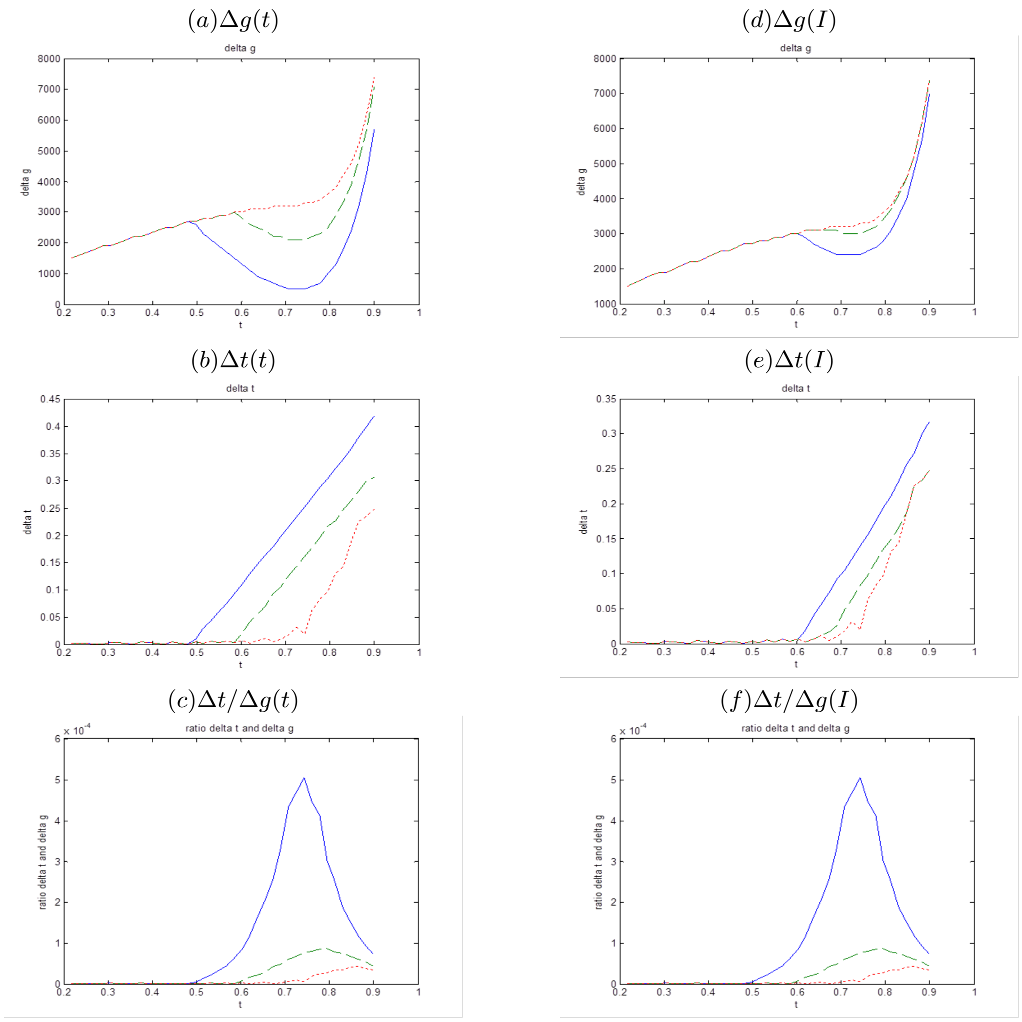
Figure 1: policy depending on preexisting tax rate t and inequality valuation δ



Notes: Left: (top) The optimal change in the lump-sum transfer as a function of the size of the preexisting tax rate t, (middle) the optimal change in the income tax rate as a function of the size of the preexisting tax rate t, (bottom) the ratio of the optimal change in the lump-sum transfer and the optimal income tax rate, as a function of the size of the preexisting tax rate t. On the right: The optimal change in the lump-sum transfer as a function of the valuation of inequality δ, (middle) the ratio of the optimal change in the lump-sum transfer and the optimal income tax rate, as a function of the valuation of inequality δ, (bottom) the ratio of the optimal change in the lump-sum transfer and the optimal income tax rate, as a function of the valuation of inequality δ. For each picture, the continuous line has no valuation of inequality, $\delta = 0$, the dashed line has $\delta = 0.7$, and the dotted line $\delta = 3$.

all. In figure 1b we see that before a threshold, it is optimal to never decrease the income tax. Above this threshold, it is optimal to decrease the taxes. The

Figure 2: policy depending on preexisting tax rate t and inequality valuation δ



Notes: On the left, there is no inequality aversion, i.e. $\delta = 0$, and on the right, there is inequality aversion, $\delta = 1$. Top: The optimal change in the lump-sum transfer as a function of the size of the preexisting tax rate t, (middle) the optimal change in the income tax rate as a function of the size of the preexisting tax rate t, (bottom) the ratio of the optimal change in the lump-sum transfer and the optimal income tax rate, as a function of the size of the preexisting tax rate t. For each picture, the continuous line has no valuation of the environment, $\alpha = 0$, the dashed line has $\alpha = 10^{-6}$, and the dotted line has $\alpha = 2 * 10^{-6}$.

point where the threshold lies depends strongly on the level of inequality valuation. In figure 1c we see that the ratio between the change in the income tax and the change in the lump-sum transfer, $\Delta t/\Delta g$, goes up at first. This is when it becomes pertinent to decrease the income taxes as tax distortions weigh up against arguments for redistribution. Later, over the top of the Laffer curve, the ratio goes down again.

The other pictures in figure 1 describe how the optimal policy depends on the valuation of equality, modeled in this case through δ. Figure 1d shows that the lump-sum transfer g should be increased more if inequality is weighed more. We see that in the continuous line (preexisting tax rate $t = 0.6$), the lump-sum transfer increase is highest. For the dashed line ($t = 0.7$), the lump-sum transfer should be lower, as a big part of the budget is now spent on decreasing the income tax. For a yet higher preexisting income tax rate (dotted line, $t = 0.8$), the lump-sum transfer should be higher again due to the reduction in distortions on the labor market. Figure 1e shows how the change in the income tax rate depends on different valuations of inequality. The effect is decreasing, as a reduction in the income tax rate will increase inequality. Further, the effect is seen to be increasing in the size of the preexisting tax rate, as this makes labor market distortions larger and a more pressing issue. Figure 1f shows how the ratio between the change in the income tax rate and the change in the lump-sum transfer depends on how to value income equality. The effect of a lowering change in the income tax rate is dominant on the ratio, as the ratio is decreasing in δ. Further, for a preexisting tax rate that is highest (dotted line, $t = 0.8$), ratio is highest because the higher tax rate makes for more incentives to reduce distortions trough redistribution than for lower preexisting tax rates (dashed line, $t = 0.7$ and continuous line $t = 0.6$).

Next, in figure 2 it is described how the optimal tax policy for a government that not only implements a carbon tax, but also cares about the environmental distortions created by in-place taxes. The pictures on the left; 2a, 2b, and 2c show the results when there is no inequality aversion at all, i.e. $\delta = 0$. In each picture, each line represents a different valuation of the environmental damages. Here the continuous line has $\alpha = 0$ from equation 17, the dashed line has $\alpha = 10^{-6}$, and the dotted line $\alpha = 2{*}10^{-6}$. We see that valuation of the environment is similar to valuation of inequality: If the environment is valued higher, then α has taken a higher value. Now, income taxes exacerbate damages as long as $\epsilon^{\bar{c}}_{xt} > 0$ and $\tau < \frac{\alpha\rho}{\eta}$, where now α is larger. This means that income taxes are only reduced later, because their distorting effect has to outweigh redistributive characteristic ξ and the exacebating effect on environmental damages. It is thus understood that the pictures on the left of figure 2 look similar to those on the left of figure 1. On the right, pictures are shown that include both a varying valuation of environmental damages α, and a valuation of inequality $\delta = 1$. This shows, as expected that the two effects sum in the same direction: as the two effects compound, the proceeds of a carbon tax are used to reduce income taxes only in case of large distortions, often on the right side of the Laffer curve.

5 Conclusion

The purpose of this paper is to pay attention to the redistributional effects of a carbon tax. As in the double dividend literature, the distortions of such a tax on the labor market are studied. However here, we study it in a public-finance framework, and ask what this implies as to how to best recycle the revenue of a carbon tax. The importance of studying different ways to recycle funds lies in the regressive nature of the carbon tax, and the notion that the suggested policy in the double dividend literature, a lower income tax, will lead to both less labor market distortions and more income inequality.

In order to look closer at the distributional preferences, we take several levels of inequality aversion, where we find in a new way the familiar tradeoff between work incentives and equity. Using this framework, the optimal taxes are studied as a benchmark, where revisions of the optimal linear Mirrleesian tax are found. A simulation is used to look at some more realistic, sub-optimal cases. The purpose thereof is to show how to recycle revenue of a carbon tax depends on both political preferences of inequality aversion and the level preexisting tax rates: Lower income taxes for excessively high preexisting tax rates, and, for seperable utility, redistribute more for a larger inequality aversion.

A classical remark by Margaret Thatcher accusing her opponent in a debate in the house of parliament is 'So long as the gap is smaller (between incomes of the rich and the poor) – so long as the gap is smaller – they'd rather have the poor poorer. You do not create wealth and opportunity that way'. She has gotten much praise for that, but the world has changed. Though Thatcher increased social mobility by cutting taxes, today in some parts of the world it seems it is income inequality that jams social mobility, more than excessively high income taxes. Especially in the case of the environment, one needs to pay special attention to distributional issues when implementing a tax reform.

In Kuznets (1955), it is argued inequality eventually goes down as income goes up. However, in recent times it can be seen that inequality in many rich countries goes up, whatever measure is used. Work by Schumacher (1973) describes that large parts of the world become richer and richer, and ask if there is a point where societies can have enough, and other things than efficiency, like income inequality, become important. The idea that the love of consumption and money is not always related to optimizing happines is one that is related to basic virtues and vices in most cultures. A similar question as in Schumacher (1973) was raised earlier by Keynes (1932): "The love of money as a possession — as distinguished from the love of money as a means to the enjoyments and realities of life — will be recognized for what it is (...) But beware! The time for all this is not yet. For at least another hundred years we must pretend to ourselves and to everyone that fair is foul and foul is fair; for foul is useful and fair is not". In this paper, more than in previous income tax models, the question is addressed when fair is useful.

The paper looks at inequality put into the utility function in a very general way: one needs to wonder about how it got there. For the case of "keeping up with the Joneses" or "rat race" effects, one needs to look deeper into specific forms of the utility function. Also, Stone-Geary preferences should be added to the simulation. This is work that the author plans to adress in a later version of this paper or a follow-up. Another interesting expansion might be to make this problem dynamic instead of static. In that case, temporal effects of household and government behavior need to be take into account.

6 Acknowledgements

Inge van den Bijgaart for initial discussions. I also am grateful to Bas Jacobs for teaching me about public finance, and to Rick van der Ploeg and Cees Withagen for comments. I gratefully acknowledge the ERC for financial support from FP7-IDEAS-ERC Grant No. 269788.

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7 Appendix

7.1 Keeping up with the Joneses

In order to account for "Keeping up with the Joneses" and "Rat race"-effects, we can alter the utility function as such:

$$u_n = u_n(c_n - \alpha\bar{c}, l_n - \beta\bar{l}) - f(IM)$$

here $\bar{c}$ and $\bar{l}$ are the average amount of consumption used and work supplied in the economy. The distribution of income, consumption, and labor is denoted by z_1, z_2 and z_3, respectively. The problem is still very similar to what it was before:

$$max_{t,g,,\bar{c},\bar{l},I}\mathcal{L} = \int_{\mathcal{N}} [\Psi(v_n) + \eta(tnl_n - g - \frac{G}{N})]dF(n) + \mu_1(I - I(z_1)) + \mu_2(\bar{c} - \bar{c}(z_2)) + \mu_3(\bar{l} - \bar{l}(z_3)).$$

This results in the first-order conditions:

$$\mu_1 = -\int_{\mathcal{N}} [\Psi'(v_n)u_I + \eta tn\frac{\partial l_n}{\partial I}]dF(n) + \mu_2\frac{\partial\bar{c}(z_2)}{\partial I} + \mu_3\frac{\partial\bar{l}(z_3)}{\partial I}, \quad (18)$$

$$\mu_2 = \int_{\mathcal{N}} [\Psi'(v_n)\lambda_n - \eta tn\frac{\partial l_n}{\partial\bar{c}}]dF(n) + \mu_3\frac{\partial\bar{l}(z_3)}{\partial\bar{c}} + \mu_1\frac{\partial I(z_1)}{\partial\bar{c}}, \quad (19)$$

$$\mu_3 = \int_{\mathcal{N}} [\Psi'(v_n)u_{l_n-\bar{l}} - \eta tn\frac{\partial l_n}{\partial\bar{l}}]dF(n) + \mu_2\frac{\partial\bar{c}(z_2)}{\partial\bar{l}} + \mu_1\frac{\partial I(z_1)}{\partial\bar{l}}, \quad (20)$$

This is similar to what we found for μ in the main text with just an aversion from inequality, for each externality we can give an interpretation of the social value of a change in the externality associated with μ_i: As before there are the direct utility effects, labor effects, but now also the mixed effects that each externality has on the other. This leads to the following modification of the optimal linear Mirrlees income tax

$$\xi = \frac{t}{1-t}\epsilon_{lt}^{\bar{c}} + \frac{\mu_1(\frac{I_t}{\int_{\mathcal{N}} z_n dF(n)} + I_g) + \mu_2(\frac{\bar{c}_t}{\int_{\mathcal{N}} z_n dF(n)} + \bar{c}_g) + \mu_3(\frac{\bar{l}_t}{\int_{\mathcal{N}} z_n dF(n)} + \bar{l}_g)}{\eta}$$

We see that for each externality included in the model, an extra term pops up in the formula for the income tax.

7.2 Optimal Nonlinear Tax

In this section I will describe how inequality and a nonlinear income tax interact. I will do this in two ways. First, by including an aversion from inequality in the utility function: $u_n = u_n(c_n, l_n, I)$. Second, by also including "rat-race" and "Keeping up with the Joneses"-effects in the utility function $u_n = u_n(c_n - \bar{c}, l_n - \bar{l}, I)$, where $\bar{c}$ and $\bar{l}$ are the average amomunts worked and consumed. Following Diamond (1998) and Jacobs (2013), we find the optimization problem.

The model is thus changed slightly. The household budget constraint now contains a nonlinear tax scheme instead of a linear tax rate and a lump-sum transfer:

$$c_n = nl_n - T(nl_n) \tag{21}$$

which leads directly to the first order condition

$$\frac{-u_l(c_n, l_n, I)}{u_c(c_n, l_n, I)} = n(1 - T'(z_n)), \tag{22}$$

On the other hand the government still optimizes the same Benthamite social welfare function

$$\int_{\mathcal{N}} \Psi(u_n) dF(n).$$

The government has to do with its budget constraint:

$$\int_{\mathcal{N}} T(z_n) dF(n) = G. \tag{23}$$

The goods-market clearing condition for the entire economy tell us that the total income should equal total private and public spending combined.

$$\int_{\mathcal{N}} nl_n dF(n) = \int_{\mathcal{N}} c_n dF(n) + G \tag{24}$$

As the productivity of every individual n is not observable to the government, we assume that the bundle c_n, z_n of individual n is such that she does not want any other bundle c_m, z_m than her own, see also Jacobs (2013) and Jacobs and de Mooij (2011). The Lagrangian for the resulting problem is given by

$$max_{l_n,c_n,u_n,I}\mathcal{L} = \int_{\mathcal{N}} [\Psi(u_n) + \eta(nl_n - c_n - \frac{G}{N}) + \mu_n(u_n(c_n, l_n, I) - u_n)]dF(n)$$

$$+ \int_{\mathcal{N}} (\theta_n \frac{l_n u_l(l_n)}{n} - u_n \frac{d\theta_n}{dn})dn + \theta_{\bar{n}} u_{\bar{n}} - \theta_{\underline{n}} u_{\underline{n}} + \zeta(I - I(z)),$$

where integration by parts has taken on the incentive-compatibility constraint. The first-order condition for inequality gives an expression for the valuation of inequality:

$$\int_{\mathcal{N}} \frac{u_I}{u_c} dF(n) + \int_{\mathcal{N}} \frac{\theta_n(u_l \frac{\partial l_n}{\partial I} + u_{ll} l_n \frac{\partial l_n}{\partial I})}{n\eta} dn = -\frac{\zeta}{\eta} \tag{25}$$

This leads to the following adjustment to the optimal nonlinear Mirrleesian income tax:

Proposition 8. *The optimal nonlinear income tax rate is given by*

$$\frac{T'}{1-T'} = (1 + \frac{1}{\epsilon^*} + \zeta I_l) \frac{u_c \int_n^{\bar{n}} (\frac{1}{u_c} - \frac{\Psi'}{\eta})}{nf(n)}$$

Proof. Writing out the first-order conditions:

$$-\eta + \mu_n u_c = 0$$

$$\eta n + \mu_n u_l + \theta_n \frac{u_l}{n} + \theta_n \frac{l_n u_{ll}}{n} - \zeta I_{l_n} = 0$$

$$(\Psi'(u_n) - \mu_n) f(n) - \frac{d\theta_n}{dn} = 0$$

$$\theta_{\bar{n}} = 0, \quad \theta_{\underline{n}} = 0$$

Integrating the first-order condition for u from n to $\bar{n}$ gives an expression for θ_n. Substitution in the condition for labor supply, whilst using the first-order conditions for consumption, the transversality conditions and the first-order condition for the household gives the result. □

This is just the old Mirrlees optimal nonlinear income tax formula with an extra correction for inequality aversion added to the distortion on the labor market. Note that for $n \to \underline{n}$ ($n \to \bar{n}$), $\zeta = -\int_{\mathcal{N}} \eta \frac{u_I}{u_c} dF(n) > 0$ and $I_{l_{\underline{n}}} < 0$ ($I_{l_{\bar{n}}} > 0$). For all practically used inequality measures, this means that the highest incomes have a higher marginal income tax rate, and the lowest incomes have lower marginal income tax rate. Adding the environment to the utility function, along with a tax on dirty consumption, does not change this result, see Jacobs and de Mooij (2011).

7.3 simulation

Here I explain the methodology behind my simulation. Consider first the case where there is no environmental taxation yet, i.e. $\tau = 0$. For a given tax system (t, g), it is straightforward to calculate the household choices $l_{n,new}(t, g), x_{n,new}(t, g)$, and $y_{n,new}(t, g)$ given a specific functional form of the utility function $u_n(x_n, y_n, l_n, I)$. One can also find the corresponding welfare $\int_{\mathcal{N}} \Psi(u_n) dF(n)$. We assume the government to have a balanced budget, so $G = \int_{\mathcal{N}} n l_n t dF(n) - Ng$.

Now, introduce a tax τ on the dirty consumption good. This changes the household consumption choices $l_{n,new}(t, g), x_{n,new}(t, g)$, and $y_{n,new}(t, g)$ that now need to be recalculated. The mathematical problem we face is to find the minimal t or maximal g such that

$$\int_{\mathcal{N}} \tau x_{n,new} + n l_{n,new} t dF(n) \geq Ng + G,$$

and the corresponding welfare. This problem can be solved numerically by a grid or binary search algorithm.

To find the optimal mix of taxes given a new carbon tax, the procedure is slightly different. Similar to before, I first find $G = \int_{\mathcal{N}} n l_n t dF(n) - Ng$. Then, $\forall g' \in [g, \bar{g}]$, I find the maximal variation in t by the procedure decribed above. Out of these, I choose the one that has maximal welfare and satisfies the budget constraint $\int_{\mathcal{N}} \tau x_{n,new} + n l_{n,new} t dF(n) \geq Ng + G,$.