Lobbying over Exhaustible-Resource Extraction*

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Abstract

Consider a lobby group of exhaustible-resource suppliers, which bargains with the government over the extraction of an exhaustible resource and over contribution payments. We characterize the path of contributions and the resulting extraction path, taking into account how the environmental damage of resource usage and the demand elasticity change optimal extraction. A high marginal environmental damage reduces the government's preferred extraction, a high price elasticity of resource demand reduces that of the lobby. We show that if the former effect dominates, the equilibrium contributions in a setting of repeated bargaining exceed those under full commitment.

Keywords: Environmental Policy; Exhaustible Resources; Political Economy; Lobbying; Nash Bargaining; Dynamic Programming

JEL Codes: D72; Q31; Q38; Q58

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1 Introduction

It is commonplace among environmental economists to assume that the influence of resource suppliers' interest groups distorts policy away from a "social planner's" ideal. For example, many suspect fossil-fuel owners to lobby against appropriate Pigouvian taxation. Once we accept the assumption that lobbyists can have an influence on policy, and if we additionally assume that there are no sufficient counterforces from other interest groups, this distortion is true almost by definition.

The literature in the tradition of the Grossman and Helpman (1994) common-agency interest-group model assumes that interest groups offer conditional bribes to the government to induce political distortions in their preferred direction. While such a depiction may be an acceptable caricature of lobbies' political influence for static problems, the political economy of exhaustible resources raises some specific questions due to its inherently dynamic nature. Firstly, if an interest group offers contribution payments for a favorable policy path, will the government have an incentive to deviate from this distorted path after following it for a while? This question arises because assuming that the government can be bribed into concrete policy changes in the short run seems much more plausible than buying a whole policy path. Secondly, how do contributions develop over time? And combining the questions, will policy and contributions develop differently if commitment to a policy path is possible than if it is not?

The aim of the current paper is to characterize the influence of resource suppliers on their extraction in a dynamic model. We assume that the government can choose the extraction path (which is a shortcut for indirect means like extraction quotas or competition policy), while the suppliers get the revenues. As usual in the literature, the government's utility is a weighted sum of a utilitarian welfare function and utility from lobby contributions. To focus on the relation between the government and the resource owners' lobby, we do not consider a common-agency setting with many competing lobbies, but assume that only one lobby group exists. Usually, interest-group models assume that lobby groups are first movers and offer contribution schedules that maximize their surplus given that the government is indifferent about accepting them. In our model, we instead assume that the government has some influence on the outcome as well, that is, there is a (Nash) bargaining which determines policy and contributions.

If the government has this more active role in shaping policy, the question arises how the threat of ending cooperation in the future shapes the bargaining outcome. Thus, we compare different commitment assumptions. We first derive a solution that assumes the government and the lobby to sign a binding contract at the beginning of time, specifying complete paths till infinity (open-loop solution). Afterwards, we alternatively assume that commitment for the future is impossible. Instead, the negotiating parties bargain in each period about the current values of their variables. To derive a time-consistent path, we apply the recursive Nash bargaining solution (RNBS) concept of Sorger (2006). In the RNBS, the negotiating parties take the effects of present agreements on future bargaining positions into account.

In our model, the government considers the economy's surplus net of environmental externalities while the lobby group's sole objective is the firms' intertemporal profit, i.e., it takes the price elasticity of resource demand into account. It turns out that whether extraction in the lobbying equilibrium occurs faster or slower than in the social optimum crucially depends on the magnitudes of these damage and monopoly effects. We show how these considerations cause a difference between the contribution payments in the commitment case and those where commitment is not possible.

In particular, we demonstrate that, firstly, contributions in all equilibria converge to zero while extraction costs increase towards a prohibitively high level. Secondly, with utility being linear in payments, the commitment assumption does not affect the extraction path. Lastly, in situations where the socially optimal initial extraction is higher than the initial extraction preferred by the monopoly, lobby payments are lower if there is no commitment possibility (and vice versa).

The resource market in our model is fairly general. Given that we assume that there is no intertemporal behavior on the demand side, the resource might best be thought of as a fuel like coal or oil. To make the model tractable, there is only flow pollution, so that we would suppose it to be, for example, sulfur dioxide, soot or dust. The political model is a pure lobbying model without elections. Thus, the best application may be smog and similar phenomena in developing or newly industrialized countries with weak institutions.

Our paper proceeds as follows. In the next section, we discuss the paper's relation to the literature. Section 3 introduces the basic resource-economic model, and derives the welfare-maximizing and the monopolistic profit-maximizing extraction path. Section 4 models lobbying over the extraction path. In Section 5, we discuss the results and comparative dynamics. Section 6 concludes.

2 Relation to the Literature

Different strands of literature are related to different aspects of our paper, namely resource extraction, lobbying, and the dynamic bargaining game.

Firstly, concerning resource extraction, we assume that extraction costs increase with cumulative past extraction (cf. Levhari and Liviatan, 1977). Extraction ceases (asymptotically) because marginal extraction costs become prohibitively high. The stock of the resource, by contrast, is assumed not to be a binding constraint. This implies that there is no intertemporal physical resource scarcity, and there are no Hotelling rents, but Ricardian rents due to increasing costs – for this, see Hartwick (1982).

An additional kind of rents in our model are monopoly rents. There is a large literature analyzing how the governments of resource-importing countries try to reap the rents of foreign resource suppliers, either Hotelling rents (see, e.g., Bergstrom, 1982 and Keutiben, 2014) or monopoly rents or both (see, e.g., Rubio, 2011 or, with stock pollution, Wirl, 1994, 1995, Rubio and Escriche, 2001, Liski and Tahvonen, 2004, and Daubanes, 2008). In these papers, the governments maximize their own country's utilitarian welfare. In our model, resource suppliers are part of the same country as consumers, so that a welfare-maximizing government has no particular interest in distributing rents away from them. However, monopoly rents distort the market outcome exactly because they are linked to monopolistic supply behavior, and this is what the government would like to avoid.

The monopolistic supply path of resources (which is the lobby group's preferred path) generally differs from both the competitive and the welfare-maximizing path. The assumed functions of our model lead to the typical behavior (cf. Krautkraemer, 1998) that a resource monopolist prefers a decelerated extraction, compared to the extraction of competitive, unregulated suppliers.[1] What is more important for our model is the comparison of the monopolistic path with the welfare-maximizing one. It has long been known that both a monopoly's tendency to restrict supply and an unregulated industry's ignorance of environmental externalities have to be taken into account for welfare judgments, so that a monopoly may be a second-best solution (see, in the context of static Pigouvian taxation, Buchanan, 1969 and Barnett, 1980).[2] Such a trade-off in the judgment of market power also exists in the dynamic context; if a monopolist prefers to restrict supply, this benefits the environment. Though we are not concerned with second-best solutions, it is important to keep in mind that governmental welfare-maximization does not always mean reducing extraction, even if there are environmental externalities. We will see that both the absolute difference between the optimal initial extraction quantities of the lobby and of the government and the sign of this difference shape our model's outcome.

The kind of interest-group model featured in this paper follows the tradition of the Grossman and Helpman (1994) common-agency lobbying model in so far as the government in the model has a mixed motivation of welfare maximization and contributions, and firms pay for a favorable policy. Grossman and Helpman (2001) provide an excellent overview on this kind of model. Instead of the lobby group offering a contribution schedule, our paper assumes a bargaining process to share the surplus of the

[1] Situations where a monopolist chooses a faster extraction are possible, but less common. For an overview of the literature on monopolistic resource supply, see the list of Fischer and Laxminarayan (2005).

[2] There even exist calculations of the optimal level of market power for an industry – see Gopinath and Wu (1999).

favorable policy. This is suggested in Grossman and Helpman (2001, Section 7.5), as a generalization of the surplus sharing (and, as a shortcut to the results of the usual lobbying game structure, in Goldberg and Maggi, 1999). In our setup, the asymmetric Nash bargaining solution serves to analyze the influence of the lobby's bargaining power on the outcome.

The common-agency lobbying model has been applied to a (static) environmental policy setting by Aidt (1998) and Fredriksson (1997, 1998). More relevant to our model are dynamic versions of the framework. Damania and Fredriksson (2000) model a repeated game of collusion between lobbying firms. The first similar model of resource dynamics we know of is Barbier et al. (2005). Boyce (2010) models lobbying in the context of renewable resources and common-pool resources. In contrast to both papers, we explicitly take different price elasticities of demand for the extracted resource into account. Barbier et al. (2005) assume a small open economy with an exogenous price, and Boyce (2010) assumes that harvesters have a logarithmic utility function of their resource extraction. Moreover, we explicitly model the development of contributions under different commitment assumptions, which is not an issue in the mentioned papers.

To solve for a time-consistent equilibrium, Boyce (2010) applies the truthful Markov perfect equilibrium of Bergemann and Välimäki (2003). Our solution concept for the time-consistent equilibrium is the recursive Nash bargaining solution of Sorger (2006). This concept assumes that bargaining parties meet in each period, and commit to the negotiation outcome for that period only. Because cooperation is always beneficial, they rationally expect to cooperate in the future. Their threat point is to leave the bargaining table for the current period and choose their non-cooperative strategies, that in turn take their impact on future negotiations into account. Before discussing this solution concept in detail, we introduce the economy with benchmarks for the lobbying model in the following section.

3 The Economy

3.1 Basics

We consider a partial-equilibrium model.[3] The supply side is a sector of resource owners, who optimize intertemporally. The demand side is represented by a stationary demand function. Policy is determined by a government and the resource owners' lobby group. In this section, we introduce the model setting by deriving and characterizing two benchmarks for the lobbying model: welfare maximization and monopolistic profit

[3] Our model economy is a standard partial-equilibrium resource-economic setting with Ricardian (instead of Hotelling) rents, as for example in Hartwick (1982).

maximization. These are the polar cases that span the bargaining range of our later political model. We also shortly discuss the case of competitive, unregulated resource owners who do not consider environmental externalities and take the price path of the resource as given.

Let $q(t)$ denote resource extraction in period t and $z(t)$ cumulative extraction of all previous periods. Then, the equation of motion of z is

$$z(t+1) \equiv z_+(t) \equiv z(t) + q(t). \tag{3.1}$$

In the following, we drop t where no ambiguities arise. Gross consumer surplus in each period is $u \equiv u(q)$, and net consumer surplus is $u - pq$, where p is the market price of the resource. Consumers take the price as given, which implies

$$p \equiv p(q) = \tfrac{\partial u(q)}{\partial q} \tag{3.2}$$

in equilibrium. Because there are no intertemporal effects on the demand side, q may best be thought of as an energy resource like coal or oil. There are resource owners whose flow profit π is

$$\pi(p, z, q) = p \cdot q - c(q, z), \tag{3.3}$$

where $c \equiv c(q, z)$ are extraction costs that positively depend on both current and cumulative extraction. We assume that z is not limited to a fixed resource stock. Instead, intertemporal optimization in our model means balancing benefits of extraction today and increased extraction costs in the future.[4] The agents in our model have a discount rate r and a discount factor $\beta \equiv 1/(1+r)$, and an infinite planning horizon. The present value of profits, discounted to period t then is

$$\Pi(t) = \sum_{s=0}^{\infty} \beta^s \cdot \pi(t+s) = \sum_{s=0}^{\infty} \beta^s \cdot [p(t+s) \cdot q(t+s) - c(q(t+s), z(t+s))] \tag{3.4}$$

where $s \in \mathbb{N}$ is the summation index. The economy's instantaneous utilitarian welfare w is the sum of net consumer surplus and profit minus the environmental flow damage $k \equiv k(q)$, which is caused by resource consumption. In this sum, payments cancel out, so we have

$$w \equiv w(z, q) = u(q) - c(q, z) - k(q). \tag{3.5}$$

[4]Thus, a resource owner in our model does not really own a given stock of resources, but a mine which can be dug deeper and deeper, which makes it more and more costly to extract additional resources.

Intertemporal welfare is the discounted sum of instantaneous welfare:

$$\begin{aligned} W(t) &= \sum_{s=0}^{\infty} \beta^s \cdot w(t+s) \\ &= \sum_{s=0}^{\infty} \beta^s \cdot \left[u(q(t+s)) - c(q(t+s), z(t+s)) - k(q(t+s))\right]. \end{aligned} \tag{3.6}$$

3.2 Benchmarks

Consider a social planner choosing a welfare-maximizing extraction path. She maximizes intertemporal welfare (3.6) subject to the equation of motion (3.1). The planner's Bellman equation is

$$W(z) = \max_q \left[u(q) - c(q,z) - k(q) + \beta \cdot W(z_+)\right]. \tag{3.7}$$

Letting a double-asterisk denote the planner's optimal solution, the first-order condition is

$$\frac{\partial u(q^{**})}{\partial q} - \frac{\partial c(q^{**},z)}{\partial q} - \frac{\partial k(q^{**})}{\partial q} + \beta \cdot \frac{\partial W(z_+^{**})}{\partial z_+} = 0. \tag{3.8}$$

Differentiating the Bellman equation yields the Envelope Condition:

$$\frac{\partial W(z)}{\partial z} = -\frac{\partial c(q^{**},z)}{\partial z} + \beta \cdot \frac{\partial W(z_+^{**})}{\partial z_+} = -\frac{\partial c(q^{**},z)}{\partial z} - \frac{\partial u(q^{**})}{\partial q} + \frac{\partial c(q^{**},z)}{\partial q} + \frac{\partial k(q^{**})}{\partial q}. \tag{3.9}$$

Shifting this in time and substituting in the first-order condition gives the planner's Euler equation, which is the Hotelling rule modified for stock-dependent cost and flow-dependent damage effects:

$$\frac{\partial u(q^{**})}{\partial q} - \frac{\partial c(q^{**},z)}{\partial q} - \frac{\partial k(q^{**})}{\partial q} = \beta \cdot \left[\frac{\partial c(q_+^{**},z_+)}{\partial z_+} + \frac{\partial u(q_+^{**})}{\partial q_+} - \frac{\partial c(q_+^{**},z_+)}{\partial q_+} - \frac{\partial k(q_+^{**})}{\partial q_+}\right]. \tag{3.10}$$

Thus, the flow utility of today's marginal resource net of extraction cost and flow externalities has to equal the discounted net utility that could be gained from the resource if it was extracted a period later, plus the additional stock cost effect.

Now suppose that a monopolist supplies the resource. The monopolist maximizes the present value of profits (3.4) subject to the equation of motion (3.1), and internalizes the price reaction (3.2). The resource owner's Bellman equation is

$$\Pi(z) = \max_q \left[p(q) \cdot q - c(q,z) + \beta \cdot \Pi(z_+)\right]. \tag{3.11}$$

Letting a (single-)asterisk denote the monopolist's optimal solution, following the same

Functions		Explicit forms
$u(q)$	$=$	$\gamma_1 q - \frac{\gamma_2}{2} q^2$
$p(q) = \frac{\partial u(q)}{\partial q}$	$=$	$\gamma_1 - \gamma_2 q$
$c(q, z)$	$=$	$\left(\omega_z z + \omega_1 + \frac{\omega_2}{2} q\right) \cdot q$
$\frac{\partial c(q,z)}{\partial q}$	$=$	$\omega_z z + \omega_1 + \omega_2 q$
$\frac{\partial c(q,z)}{\partial z}$	$=$	$\omega_z q$
$k(q)$	$=$	$\frac{\kappa}{2} q^2$
$\frac{\partial k(q)}{\partial q}$	$=$	κq

Table 1: Explicit functions. *The parameter index indicates the power of the variable that the parameter relates to.*

steps as in the welfare-maximizing case yields the following Euler equation:

$$p(q^*) + \frac{\partial p(q^*)}{\partial q} q^* - \frac{\partial c(q^*, z)}{\partial q} = \beta \cdot \left[\frac{\partial c(q_+^*, z_+)}{\partial z_+} + p(q_+^*) + \frac{\partial p(q_+^*)}{\partial q} q_+^* - \frac{\partial c(q_+^*, z_+)}{\partial q_+} \right]. \tag{3.12}$$

The interpretation is similar to that of the planner's Euler equation, except that the monopolist does not care for environmental damage, but for his influence on the price.

Comparing (3.10) and (3.12), it is apparent that the planner's and the monopolist's extraction paths coincide whenever the marginal damage, $-\frac{\partial k(q)}{\partial q}$, is equal to the marginal loss of monopoly rent, $\frac{\partial p(q)}{\partial q} q$, in each period. Otherwise, it depends on the magnitudes of these effects whether extraction occurs faster or slower in the planner's than in the monopolist's case. We discuss the interaction of these effects in detail in the context of our political model in Section 5. Competitive, unregulated resource owners would neither internalize the environmental damage nor their influence on the price. Thus, their Euler equation can be derived from (3.12) by substituting the price derivatives by zero, or from (3.10) by dropping the environmental-damage derivatives. In what follows, we neglect the competitive, unregulated case and derive explicit solutions for the planner's and the monopolist's optimization problems only.

3.3 Benchmarks: Explicit Solutions

To allow for an explicit solution, we assume that the demand price is a linear function of the quantity, that marginal extraction costs in a given period are a linear function of cumulative extraction and of extraction within that period, and that marginal environmental damage in a given period is proportional to the extraction quantity within that period. The assumed functions are summarized in Table 1. Collecting terms, we have

$$w = (b - \omega_z z) \cdot q - \frac{a_g}{2} q^2, \tag{3.13a}$$

$$\pi = (b - \omega_z z) \cdot q - \frac{a_h}{2} q^2, \tag{3.13b}$$

where

$$a_g \equiv \omega_2 + \gamma_2 + \kappa, \tag{3.14a}$$

$$a_h \equiv \omega_2 + 2\gamma_2, \tag{3.14b}$$

$$b \equiv \gamma_1 - \omega_1. \tag{3.14c}$$

$b - \omega_z z$ are the (instantaneous) net gains of the first extracted unit. Due to the assumed functions, these are the same for welfare and profit. a_g and a_h are the slopes of the marginal welfare function and the marginal profit function, respectively.

One way to derive a solution for the optimal (welfare-maximizing or profit-maximizing) extraction would be to use (3.13) in the Euler equations of the previous section. Together with the equation of motion of cumulative extraction, (3.1), this constitutes a solvable system of difference equations. An alternative approach is to guess the form of the policy functions and verify it afterwards. In Section 4.5, we need this guess-and-verify method to derive the equilibrium of the time-consistent policy model. For a consistent presentation, we therefore also use it in this section.

We guess that there exist constants $Y_{w,0}$, $Y_{w,1}$, $Y_{\pi,0}$, $Y_{\pi,1}$, such that the following (Markovian) extraction functions exist:

$$q^{**}(z) = Y_{w,0} + Y_{w,1} z, \tag{3.15a}$$

$$q^{*}(z) = Y_{\pi,0} + Y_{\pi,1} z, \tag{3.15b}$$

that is, we expect the quadratic utility functions to lead to extraction functions that are linear in the state. To solve for these coefficients, we first use them to formulate the maximized value functions (3.7) and (3.11) in an explicit form:

Lemma 1 (Benchmark Value Functions). *Assume that*[5]

$$0 < 1 + Y_{w,1} \leq 1, \tag{3.16}$$

$$0 < 1 + Y_{\pi,1} \leq 1. \tag{3.17}$$

[5] (3.16) and (3.17) are stability conditions; cf. Gandolfo (1996, Chapter 3).

Then,

$$W^{**}(z) = \frac{-\frac{a_g}{2} - \frac{1}{Y_{w,1}}\omega_z}{1-\beta\left(1+Y_{w,1}\right)^2} q^{**}(z)^2 + \frac{b+\frac{Y_{w,0}}{Y_{w,1}}\omega_z}{1-\beta\left(1+Y_{w,1}\right)} q^{**}(z), \tag{3.18}$$

$$\Pi^{*}(z) = \frac{-\frac{a_h}{2} - \frac{1}{Y_{\pi,1}}\omega_z}{1-\beta\left(1+Y_{\pi,1}\right)^2} q^{*}(z)^2 + \frac{b+\frac{Y_{\pi,0}}{Y_{\pi,1}}\omega_z}{1-\beta\left(1+Y_{\pi,1}\right)} q^{*}(z). \tag{3.19}$$

Proof. See Appendix A.1. □

Using these value functions, we can explicitly derive the respective coefficients:

Lemma 2 (Benchmark Extractions). *In the social planner's benchmark extraction function* (3.15a)*, we have the following coefficients:*

$$Y_{w,0} = \psi_g \cdot b, \tag{3.20a}$$

$$Y_{w,1} = -\psi_g \cdot \omega_z, \tag{3.20b}$$

implying $q^{**}(z) = \psi_g \cdot (b - \omega_z z)$ *where*

$$\psi_g \equiv \frac{r \cdot \left[a_g - \sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}\right]}{2\omega_z\left(\omega_z - a_g\right)}. \tag{3.21}$$

For the monopolist's benchmark extraction function (3.15b)*,* g *has to be replaced by* h *in* (3.20) *and* (3.21)*, so that we get* $q^{*}(z) = \psi_h \cdot (b - \omega_z z)$*.*

Proof. See Appendix A.2. □

With these results, we can explicate some parameter restrictions implied by (3.16):

Lemma 3 (Parameter Restrictions). (3.16) *implies*

$$a_g > \frac{2\omega_z}{r}\left(\sqrt{1+r} - 1\right), \tag{3.22a}$$

$$1 > \frac{r \cdot \left[a_g - \sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}\right]}{2\left(\omega_z - a_g\right)}, \tag{3.22b}$$

which can only be fulfilled if

$$a_g > \omega_z. \tag{3.22c}$$

The same must be true for a_h*.*

Proof. (3.22a) is required for real roots in (3.21). Then, if $\omega_z - a_g < 0$ (> 0), both the numerator and the denominator are negative (positive), so that ψ_g is positive. This fulfills one of the two inequalities implied by (3.16) and (3.20b). The other inequality implied by these equations is $1 - \psi_g \omega_z > 0$. Substituting (3.21) yields (3.22b). (3.22c) is proven by contradiction. Suppose that $a_g < \omega_z$. Then (3.22b) implies

$$r a_g - 2\left(\omega_z - a_g\right) < r\sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}. \tag{3.23}$$

Suppose that the left-hand side is positive. Then the inequality must still hold for the square of both sides, but squaring and cancelling terms leads to $a_g > \omega_z$, which is a contradiction. Now suppose that the left-hand side is negative, so that 3.23 would always be fulfilled. For this, it must hold that $a_g < \frac{2}{2+r}\omega_z$. At the same time, (3.22a) must hold. For this we would need $\frac{\sqrt{1+r}-1}{r} < \frac{1}{2+r}$, which cannot be true. □

These results allow to characterize the quantity relation between the planner's and the monopolistically optimal extraction:

Lemma 4 (Benchmark Extractions: Relation). *For any given level of cumulative extraction z, we have $q^{**}(z) > q^*(z)$ if $a_g < a_h \Leftrightarrow \kappa < \gamma_2$ (and vice versa).*

Proof. Differentiating (3.21), rearranging and simplifying, we see that $\partial\psi_g/\partial a_g < 0$ if

$$\omega_z < \frac{\left(\omega_z - a_g\right)\left(a_g + \frac{2}{r}\omega_z\right)}{\sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}} + \sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}. \tag{3.24}$$

Multiplying by the root term and rearranging again, we obtain

$$2\left(\omega_z - a_g\right) < r \cdot \left[a_g - \sqrt{a_g^2 - \frac{4}{r}\omega_z\left(\omega_z - a_g\right)}\right]. \tag{3.25}$$

By (3.22c), this is equivalent to (3.22b). □

Lastly, using the derived coefficients in Lemma 1, we can simplify the value functions as follows:

Lemma 1' (Simplified Benchmark Value Functions). *Assume that* (3.16) *and* (3.17) *hold. Then,*

$$W^{**}(z) = \frac{-\frac{a_g}{2} + \frac{1}{\psi_g}}{1 - \beta\left(1 - \psi_g\omega_z\right)^2} q^{**}(z)^2, \tag{3.18'}$$

$$\Pi^{*}(z) = \frac{-\frac{a_h}{2} + \frac{1}{\psi_h}}{1 - \beta\left(1 - \psi_h\omega_z\right)^2} q^{*}(z)^2. \tag{3.19'}$$

Note that (3.16) *and* (3.17) *imply that* $0 < 1 - \psi_g \omega_z \leq 1$ *and* $0 < 1 - \psi_h \omega_z \leq 1$*, respectively.*

From Lemma 2, it is easy to see that the social planner's and the monopolist's optimal extraction for a given level of cumulative extraction z are proportional. Furthermore, the greater ψ_g (or ψ_h), the higher the initial extraction and the faster the extraction declines. Additionally, extraction increases with net extraction gains b, and the influence of cumulative extraction grows with its cost effect ω_z. Both paths imply convergence towards $z = b/\omega_z$. This is the cumulative extraction level that makes extraction costs so high that there is no net consumer surplus and no flow profit left. Thus, the linear demand function and the ever-growing marginal extraction costs ensure that total extraction is finite.

As we will see in the illustration of extraction paths in Section 5, this level is reached only asymptotically along any optimal path. Moreover, both the welfare-maximizing and the monopolistic path converge to the same level. This is due to the fact that the specific environmental damage cost function $k(q)$ has zero marginal damage for $q = 0$.[6] Thus, there is never any disagreement about the end point of extraction.[7] The properties of the benchmark extraction functions are compared to the properties of the lobbying equilibrium and threat extraction functions in Section 5. There we also discuss which extraction is higher, depending on parameter relations.

4 Lobbying and Policy Determination

4.1 Political Agents

We assume that policy is set by a government, which interacts with a resource owner lobby group. More precisely, we assume that in each period the government chooses the firms' extraction quantity q. It may be more realistic to assume that the government sets a maximum quantity $q \leq q^{\max}$ for the firms' extraction, for a example by implementing an allowance trading system or by allocating extraction quotas. This does not make a difference if the limit is binding, so that $q = q^{\max}$ and firms would like to produce more. In the model, there are also cases where the government would enforce welfare maximization by choosing a higher quantity than the (monopolistically) profit-maximizing one. A minimum production $q \geq q^{\min}$ may not be realistic, but this case may be understood as enforcing competition policy which raises the quantity. The outcome of lobbying, which leads to a quantity below the welfare-maximizing one, would

[6] If there were positive marginal externalities of zero extraction, the government's b parameter would have to be corrected downwards – see (3.14c). This would lead to different convergence levels.

[7] The fact that the extraction quantities converge to zero asymptotically can be seen by evaluating the extraction path expressions (A.4) and (A.7) in Appendix A.1.

then imply that the government turns a blind eye to cartelization.

Let m denote the lobby's contributions to the government. The discounted sum of contributions then is

$$M(t) = \sum_{s=0}^{\infty} \beta^s \cdot m(t+s). \tag{4.1}$$

In each period, the government has the following utility function:

$$g(z, q, m) = w(z, q) + \zeta m = u(q) - c(q, z) - k(q) + \zeta m. \tag{4.2}$$

Thus, the government cares for welfare w, but it also derives utility ζm from the lobby's payments. The intertemporal utility of the government is the discounted sum of utility:

$$\begin{aligned}\Gamma(t) &= \sum_{s=0}^{\infty} \beta^s \cdot g(z(t+s), q(t+s), m(t+s)) = W(t) + \zeta M(t)\\ &= u(q(t)) - c(q(t), z(t)) - k(q(t)) + \zeta m(t) + \beta \cdot \Gamma(t+1).\end{aligned} \tag{4.3}$$

The (collective) utility function of resource owners is

$$h(z, q, m) = \pi(p(q), z, q) - \sigma m = p(q) \cdot q - c(q, z) - \sigma m, \tag{4.4}$$

consisting of the sector's profits, π, and the lobby's cost of paying contributions, σm, which may, for example, reflect the coordination problems within the group. The lobby group's intertemporal utility is the discounted sum of this utility:

$$\begin{aligned}H(t) &= \sum_{s=0}^{\infty} \beta^s \cdot h(p(t+s), z(t+s), q(t+s), m(t+s)) = \Pi(t) - \sigma M(t)\\ &= p(q(t)) \cdot q(t) - c(q(t), z(t)) - \sigma m(t) + \beta \cdot H(t+1).\end{aligned} \tag{4.5}$$

The assumptions that resource owners have overcome the collective action problem to form a lobby group (instead of explicitly modeling this process) and that the government's utility function (4.2) is additively separable between contributions and utilitarian welfare are usual in the interest-group literature (cf. Grossman and Helpman, 1994 or Grossman and Helpman, 2001). A more important feature, which is also typical in the literature, is the assumption of constant marginal contribution utility, which simplifies the derivation of the time paths in the following.[8]

We assume that the government and the lobby bargain about a vector consisting of lobby contributions and an extraction quantity. If no agreement is reached in the bargain, both parties leave the bargaining table, and then the government unilaterally

[8] Cf. Klein et al. (2008) for the complications that can arise when current choices affect future marginal utility from the control variables in settings without commitment.

decides the quantity and the lobby pays no contributions (non-cooperative solution). The utility in this “threat-point” scenario determines how much of the gains from cooperation each bargaining party can appropriate.

This raises the question of commitment and time consistency. We first derive an open-loop solution, in which the threat point in each period is determined by the non-cooperative outcome that would have been chosen at the beginning of the model’s planning horizon. We derive a time-consistent solution afterwards, in which the threat point is determined by considering in each period the utility the parties would gain by deviating from cooperation for that period only. The open-loop solution would require a commitment device, which typically does not exist in lobbying relations. Moreover, we assume that the government has an active role in the bargain, and its strength is represented by the respective parameter in the asymmetric Nash bargaining solution. Such an active role is in contrast to the common-agency literature mentioned above, which assumes that the lobbies make a take-it-or-leave-it offer. It is this active role of both bargaining parties that makes the question of commitment relevant, as active bargaining parties must also be able to leave the bargaining table during any negotiation.

4.2 Open-Loop Nash Bargaining Solution

Let us first discuss the open-loop solution, where parties fix a path of extractions and contributions in the beginning of the planning horizon. To distinguish these results from the later time-consistent solution, we use the index o. The bargaining outcome is marked by $\star$, and the outside option by $\#$. We define the open-loop solution as follows:[9]

Definition 1 (Open-Loop Nash Bargaining Solution). *The open-loop Nash bargaining solution of our lobbying game consists of two pairs of time-indexed strategies,* $(q^{o\star}(t), m^{o\star}(t))$ *and* $(q^{o\#}(t), m^{o\#}(t))$*, and two pairs of value functions,* $(\Gamma^{o\star}(t), H^{o\star}(t))$ *and* $(\Gamma^{o\#}(t), H^{o\#}(t))$*, such that the following is true. For all* z *it holds that*

$$z_+^{o\star}(t) = z(t) + q^{o\star}(t), \tag{4.6a}$$

$$z_+^{o\#}(t) = z(t) + q^{o\#}(t), \tag{4.6b}$$

[9]This definition is formulated in a way to allow an easy comparison with the later recursive Nash bargaining solution.

$$\Gamma^{o\star}(t) = g(z(t), q^{o\star}(t), m^{o\star}(t)) + \beta \cdot \Gamma^{o\star}(z_+^{o\star}(t)), \tag{4.7a}$$

$$H^{o\star}(t) = h(z(t), q^{o\star}(t), m^{o\star}(t)) + \beta \cdot H^{o\star}(z_+^{o\star}(t)), \tag{4.7b}$$

$$\Gamma^{o\#}(t) = g(z(t), q^{o\#}(t), m^{o\#}(t)) + \beta \cdot \Gamma^{o\#}(z_+^{o\#}(t)), \tag{4.7c}$$

$$H^{o\#}(t) = h(z(t), q^{o\#}(t), m^{o\#}(t)) + \beta \cdot H^{o\#}(z_+^{o\#}(t)), \tag{4.7d}$$

$$q^{o\#}(t) \in \arg\max_q \left[g(z(t), q(t), m^{o\#}(t)) + \beta \cdot \Gamma^{o\#}(z_+(t)) | q(t) \geq 0 \right], \tag{4.8a}$$

$$m^{o\#}(t) \in \arg\max_m \left[h(z(t), q^{o\#}(t), m(t)) + \beta \cdot H^{o\#}(z_+^{o\#}(t)) | m(t) \geq 0 \right], \tag{4.8b}$$

$$\begin{aligned} & N(z(t), q(t), m(t)) \\ & \equiv \chi \cdot \ln \left[g(z(t), q(t), m(t)) + \beta \cdot \Gamma^{o\star}(z_+(t)) - \Gamma^{o\#}(t) \right] \\ & \quad + (1-\chi) \cdot \ln \left[h(z(t), q(t), m(t)) + \beta \cdot H^{o\star}(z_+(t)) - H^{o\#}(t) \right], \end{aligned} \tag{4.9}$$

$$(q^{o\star}(t), m^{o\star}(t)) \in \arg\max_{q,m} \left[N(z(t), q(t), m(t)) | q(t) \geq 0, m(t) \geq 0 \right]. \tag{4.10}$$

(4.6a) and (4.6b) are just the definitions of the equation of motion, (3.1), along the cooperative and non-cooperative path, respectively. Accordingly, the present-value equations (4.3) and (4.5) have a cooperative form, (4.7a) and (4.7b), and a non-cooperative form, (4.7c) and (4.7d). For the non-cooperative form, (4.8a) and (4.8b) define the choice variables. The objective function of the cooperative game is the (logarithm of the) Nash product (4.9). The maximands are defined by (4.10).

We start deriving the solution by discussing the threat points. The government can enforce any desired quantity path if it wants to. If there were no lobby offering contributions, the government would be best off by choosing the welfare-maximizing path described by (3.10). At the same time, the lobby would be best off paying no contributions, as their contributions are costly in the present and have no intertemporal effect. The government's utility would be the maximized discounted welfare, and the lobby's utility would equal the discounted profits associated with the welfare-maximizing quantities:

$$\begin{aligned} \Gamma^{o\#}(t) &= W^{**}(t) \\ &= \sum_{s=0}^{\infty} \beta^s \cdot \left[u(q^{**}(t+s)) - c(q^{**}(t+s), z^{**}(t+s)) - k(q^{**}(t+s)) \right], \end{aligned} \tag{4.11a}$$

$$H^{o\#}(t) = \Pi^{**}(t) = \sum_{s=0}^{\infty} \beta^s \cdot \left[p^{**}(t+s) \cdot q^{**}(t+s) - c(q^{**}(t+s), z^{**}(t+s)) \right]. \tag{4.11b}$$

Now consider the bargaining outcome. From (4.9) and (4.10), we have for $q^{o\star}(t)$:

$$\begin{aligned}&\frac{\chi}{\Delta^o_\Gamma(t)}\cdot\left[\tfrac{\partial g(z(t),q^{o\star}(t),m^{o\star}(t))}{\partial q(t)}+\beta\cdot\tfrac{\partial\Gamma^{o\star}(z^{o\star}_+(t))}{\partial z_+(t)}\right]\\&+\frac{1-\chi}{\Delta^o_H(t)}\cdot\left[\tfrac{\partial h(z(t),q^{o\star}(t),m^{o\star}(t))}{\partial q(t)}+\beta\cdot\tfrac{\partial H^{o\star}(z^{o\star}_+(t))}{\partial z_+(t)}\right]=0,\end{aligned}\tag{4.12}$$

where

$$\Delta^o_\Gamma(t)\equiv\Gamma^{o\star}(t)-W^{**}(t),\tag{4.13a}$$

$$\Delta^o_H(t)\equiv H^{o\star}(t)-\Pi^{**}(t)\tag{4.13b}$$

are the gains from cooperating for the government and the lobby, respectively. Rearranging and noting that q does not directly affect current contribution utility yields:

$$\tfrac{\partial w(z(t),q^{o\star}(t))}{\partial q(t)}+\beta\cdot\tfrac{\partial\Gamma^{o\star}(z^{o\star}_+(t))}{\partial z_+(t)}+\frac{1-\chi}{\chi}\frac{\Delta^o_\Gamma(t)}{\Delta^o_H(t)}\cdot\left[\tfrac{\partial\pi(z(t),q^{o\star}(t))}{\partial q(t)}+\beta\cdot\tfrac{\partial H^{o\star}(z^{o\star}_+(t))}{\partial z_+(t)}\right]=0.\tag{4.14}$$

Thus, q is chosen so as to maximize a weighted sum of, on the one hand, current welfare and discounted government utility, and, on the other hand, current profit and discounted lobby utility. The weight depends on the bargaining power and the respective gains from cooperating. Likewise, the first-order condition for the contribution m is equivalent to

$$\tfrac{\partial g(z(t),q^{o\star}(t),m^{o\star}(t))}{\partial m(t)}+\frac{1-\chi}{\chi}\frac{\Delta^o_\Gamma(t)}{\Delta^o_H(t)}\tfrac{\partial h(z(t),q^{o\star}(t),m^{o\star}(t))}{\partial m(t)}=0,\tag{4.15}$$

which is simpler than the condition for the extraction quantity because m has no stock effect. By (4.2) and (4.4), utilities are linear in m, so that the respective marginal utilities are constant. Substituting them in (4.15) and rearranging yields:

$$\frac{1-\chi}{\chi}\frac{\Delta^o_\Gamma(t)}{\Delta^o_H(t)}=\lambda\equiv\frac{\zeta}{\sigma},\tag{4.16}$$

so that the lobby's weight, which is denoted by λ in the following, is constant and defined by the relative valuation of the monetary contributions. Note that the bargaining maximizes the weighted sum

$$R(t)=\Gamma(t)+\lambda\cdot H(t).\tag{4.17}$$

Substituting from (4.3) and (4.5) and simplifying yields:

$$\begin{aligned} R(t) = &\sum_{s=0}^{\infty} \beta^s \cdot [w(z(t+s), q(t+s)) + \lambda \cdot \pi(p(q(t+s)), z(t+s), q(t+s))] \\ &+ \sum_{s=0}^{\infty} \beta^s \cdot \underbrace{(\zeta - \lambda \cdot \sigma)}_{=0} \cdot m(t+s) \\ = &W(t) + \lambda \cdot \Pi(t). \end{aligned} \tag{4.18}$$

To find the q path, we can thus formulate the following Bellman equation:

$$R(z) = \max_q \left[w(z,q) + \lambda \cdot \pi(p(q), z, q) + \beta \cdot R(z_+)\right] \tag{4.19}$$

and again maximize along the lines of Section 3.2. The Euler equation is

$$\begin{aligned} &p(q^{o\star}) - \tfrac{\partial c(q^{o\star},z)}{\partial q} - \tfrac{\partial k(q^{o\star})}{\partial q} + \lambda \cdot \left[p(q^{o\star}) + \tfrac{\partial p(q^{o\star})}{\partial q} q^{o\star} - \tfrac{\partial c(q^{o\star},z)}{\partial q}\right] \\ =&\beta \cdot \left\{ \tfrac{\partial c(q_+^{o\star},z_+)}{\partial z_+} + p(q_+^{o\star}) - \tfrac{\partial c(q_+^{o\star},z_+)}{\partial q_+} - \tfrac{\partial k(q_+^{o\star})}{\partial q_+} \right. \\ &\left. + \lambda \cdot \left[\tfrac{\partial c(q_+^{o\star},z_+)}{\partial z_+} + p(q_+^{o\star}) + \tfrac{\partial p(q_+^{o\star})}{\partial q_+} q_+^{o\star} - \tfrac{\partial c(q_+^{o\star},z_+)}{\partial q_+}\right]\right\}. \end{aligned} \tag{4.20}$$

Note that the resource price p in the square brackets stems from the suppliers' marginal returns, while the price outside the square brackets just reflects the consumers' marginal utility. Thus, (4.20) is a weighted sum of the planner's (3.10) and the monopolist's (3.12) Euler equation. The higher the government values the contributions, the more the lobby determines the path (and vice versa).

How do we obtain the contribution path? Substituting (4.16) in the maximized version of the weighted sum (4.17) and rearranging yields:

$$\Gamma^{o\star}(t) = \chi \cdot [R^{o\star}(t) - \lambda \cdot \Pi^{**}(t)] + (1-\chi) \cdot W^{**}(t), \tag{4.21}$$

$$H^{o\star}(t) = \frac{1}{\lambda} \cdot \left[(1-\chi) \cdot [R^{o\star}(t) - W^{**}(t)] + \chi \cdot \lambda \cdot \Pi^{**}(t)\right]. \tag{4.22}$$

$R^{o\star}(t)$ is known from (4.19), $\Pi^{**}(t)$ and $W^{**}(t)$ are known from the welfare-maximizing extraction path. Using $q^{o\star}(t)$ in (4.3), we derive the equilibrium contribution payments:

$$m^{o\star}(t) = \frac{1}{\zeta}\left[\Gamma^{o\star}(t) - [w(z(t), q^{o\star}(t)) + \beta \cdot \Gamma^{o\star}(t+1)]\right], \tag{4.23}$$

which we can calculate using the derived $\Gamma^{o\star}(t)$. To shed some light on the outcome of the open-loop bargaining setting, we proceed with an explicit solution in the following section.

4.3 Open-Loop Nash Bargaining Solution: Explicit Solution

We continue the model from Section 3.3 and thus use the values and functions from Table 1. Thereby, the planner's explicit instantaneous welfare (3.13a) and the monopolist's explicit flow profit (3.13b) are complemented by utility from and cost of contributions, respectively. Thus we have

$$g = (b - \omega_z z) \cdot q - \frac{a_g}{2} q^2 + \zeta m, \tag{4.24a}$$

$$h = (b - \omega_z z) \cdot q - \frac{a_h}{2} q^2 - \sigma m. \tag{4.24b}$$

As in Section 3.3, we guess that there exist constants $X_{g,0}$, $X_{g,1}$, $Y_{g,0}$, $Y_{g,1}$, $X_{h,0}$, $X_{h,1}$, $X_{h,2}$, such that the following policy functions exist:

$$q^{o\star}(t) = X_{g,0} + X_{g,1} z(t), \tag{4.25a}$$

$$q^{o\#}(t) = Y_{g,0} + Y_{g,1} z(t), \tag{4.25b}$$

$$m^{o\star}(t) = X_{h,0} + X_{h,1} z(t) + X_{h,2} z(t)^2, \tag{4.25c}$$

$$m^{o\#}(t) = 0, \tag{4.25d}$$

that is, we expect the quadratic utility functions to lead to extraction functions that are linear in the state, because we already have confirmed this for the benchmark cases, and a quadratic payment function because the payments are compensations for choosing a different extraction, and the benchmark value functions from Lemma 1' are quadratic in extraction. (4.25d) stems from the discussion above. From this discussion, we also know that the threat extraction path must be the same as in the welfare-maximizing case, so we can write

$$q^{o\#}(t) = q^{**}(t) = Y_{w,0} + Y_{w,1} z(t) \tag{4.25b'}$$

with the coefficients from Lemma 2. Furthermore, we know that the government's threat value function is equal to (3.18') and the lobby's threat value function has the same form as (3.19'), except that in the latter ψ_h has to be replaced by ψ_g stated in Lemma 2, because the government determines the (threat) extraction. Thus we have

$$\Gamma^{o\#}(t) = W^{**}(t), \tag{4.26}$$

$$H^{o\#}(t) = \frac{-\frac{a_h}{2} + \frac{1}{\psi_g}}{1 - \beta \left(1 - \psi_g \omega_z\right)^2} q^{**}(t)^2. \tag{4.27}$$

For the following discussion, keep in mind that in the open-loop solution the bargaining is determined using hypothetical threat points. This means that the payments at time $t + s$ are determined by considering how the cooperative situation differs from the hypothetical non-cooperative situation that would exist at $t+s$ if the bargaining parties had never cooperated.

We can now determine the explicit value functions in equilibrium.

Lemma 5 (Equilibrium Value Functions). *Assume that*

$$0 < 1 + X_{g,1} \leq 1. \tag{4.28}$$

Then,

$$\Gamma^{o\star}(t) = \frac{-\frac{a_g}{2} - \frac{1}{X_{g,1}}\omega_z}{1-\beta\left(1+X_{g,1}\right)^2} q^{o\star}(t)^2 + \frac{b + \frac{X_{g,0}}{X_{g,1}}\omega_z}{1-\beta\left(1+X_{g,1}\right)} q^{o\star}(t) + \zeta M^{o\star}(t), \tag{4.29}$$

$$H^{o\star}(t) = \frac{-\frac{a_h}{2} - \frac{1}{X_{g,1}}\omega_z}{1-\beta\left(1+X_{g,1}\right)^2} q^{o\star}(t)^2 + \frac{b + \frac{X_{g,0}}{X_{g,1}}\omega_z}{1-\beta\left(1+X_{g,1}\right)} q^{o\star}(t) - \sigma M^{o\star}(t), \tag{4.30}$$

where

$$\begin{aligned} M^{o\star}(t) \equiv & \frac{\frac{1}{X_{g,1}} X_{h,1}}{1-\beta\left(1+X_{g,1}\right)} q^{o\star}(t) + \frac{X_{h,0} - \frac{X_{g,0}}{X_{g,1}} X_{h,1}}{1-\beta} \\ & + \frac{X_{h,2}}{\left(X_{g,1}\right)^2} \left[\frac{q^{o\star}(t)^2}{1-\beta\left(1+X_{g,1}\right)^2} - \frac{2X_{g,0} q^{o\star}(t)}{1-\beta\left(1+X_{g,1}\right)} + \frac{\left(X_{g,0}\right)^2}{1-\beta} \right] \end{aligned} \tag{4.31}$$

is the present value of contributions. $q^{o\star}(t)$ *is defined in* (4.25a).

Proof. See Appendix A.3. □

The next step is to determine the equilibrium extraction path, because we know by the discussion above that this path maximizes (4.17) (and is therefore relatively easily characterizable):

Lemma 6 (Equilibrium Extraction). *In the government's equilibrium policy function* (4.25a)*, we have the following coefficients:*

$$X_{g,0} = \psi \cdot b, \tag{4.32a}$$

$$X_{g,1} = -\psi \cdot \omega_z, \tag{4.32b}$$

so that $q^{o\star}(t) = \psi \cdot (b - \omega_z z(t))$*, where*

$$\psi = \frac{r \cdot \left[a - \sqrt{a^2 - \frac{4}{r}\omega_z(\omega_z - a)}\right]}{2\omega_z\left(\omega_z - a\right)} > 0, \tag{4.33}$$

and where $a \equiv \frac{a_g + \lambda a_h}{1+\lambda} > \omega_z$.

Proof. See Appendix A.4. The inequalities follow from Lemma 3. □

Thus, expectedly, the form of the policy function is the same as that in the benchmark cases, and its concrete value for any given level of cumulative extraction $z(t)$ shows a distortive influence of the lobby. By Lemma 4, a larger value of a in (4.33) implies higher extraction given $z(t)$. Note that a increases if either a_g or a_h increases, or if λ increases (decreases) given that $a_g < a_h$ ($a_g > a_h$). Lastly, we can derive the contributions that will be paid along the equilibrium path:

Lemma 7 (Equilibrium Contribution Payments). *In the lobby's equilibrium contribution payment function* (4.25c)*, we have the following coefficients:*

$$X_{h,0} = \Theta \cdot b^2, \tag{4.34a}$$

$$X_{h,1} = -\Theta \cdot 2b\omega_z, \tag{4.34b}$$

$$X_{h,2} = \Theta \cdot \omega_z^2, \tag{4.34c}$$

so that $m^{o\star}(t) = \Theta \cdot (b - \omega_z z(t))^2$*, where*

$$\Theta \equiv \frac{\beta\left(\psi_g - \psi\right)}{1 - \beta\left(1 - \psi_g\omega_z\right)^2}\left[\rho_1\left(\psi_g\psi\omega_z^2 + r\right) - \rho_2\left[\psi_g\psi\omega_z + \frac{1}{2}\left(\psi_g + \psi\right)r\right]\right] \geq 0 \tag{4.35}$$

and where $\rho_1 \equiv \frac{1-\chi}{\zeta} - \frac{\chi}{\sigma}$ *and* $\rho_2 \equiv \frac{1-\chi}{\zeta}a_g - \frac{\chi}{\sigma}a_h$.

Proof. See Appendix A.5. □

Along the equilibrium extraction path, $z(t)$ converges to b/ω_z. Thus, contributions converge to zero. They are overproportionally high when $z(t)$ is small, because then the absolute quantity distortion is high, whose impact on welfare is quadratic. This relation reflects the fact that a deviation from the welfare-maximizing extraction entails quadratic welfare losses due to the quadratic environmental damage-cost function and the deadweight loss of supply reductions,[10] which have to be compensated by lobbying contributions.

Using the coefficients from Lemmas 6 and 7 in Lemma 5, we can simplify the equilibrium value functions as follows:

Lemma 3' (Simplified Equilibrium Value Functions). *Assume that* (4.28) *holds. Then,*

$$\Gamma^{o\star}(t) = \frac{-\frac{a_g}{2} + \frac{1}{\psi}}{1 - \beta\left(1 - \psi\omega_z\right)^2} q^{o\star}(t)^2 + \zeta M^{o\star}(t), \tag{4.29'}$$

[10]See, for example, Varian (1996, Chapter 23), for the deadweight loss of monopoly.

$$H^{o\star}(t) = \frac{-\frac{a_h}{2} + \frac{1}{\psi}}{1 - \beta\left(1 - \psi\omega_z\right)^2} q^{o\star}(t)^2 - \sigma M^{o\star}(t), \tag{4.30'}$$

where

$$M^{o\star}(t) \equiv \frac{\Theta/\psi^2}{1 - \beta\left(1 - \psi\omega_z\right)^2} q^{o\star}(t)^2. \tag{4.31'}$$

Note that (4.28) *implies that* $0 < 1 - \psi\omega_z \leq 1$.

4.4 Recursive Nash Bargaining Solution

The open-loop bargaining solution above presupposes that the government and the lobby can commit to extraction and contribution paths at the beginning of the planning horizon. The contribution payments reflect the lobby's or the government's gain from cooperating in the respective period, compared to their situation in the same time period if they had never let themselves in for the cooperation. However, this way of determining the gains from cooperation may be misleading if binding contracts over all times cannot be signed. In this case, we have to take into account that the threat points may be different if cooperation is canceled in some later moment.

To cope with time consistency in such situations, Sorger (2006) proposes the recursive Nash bargaining solution (RNBS). It assumes that bargaining takes place every period. If cooperation failed, the government and the lobby would choose their non-cooperative Nash equilibrium strategies, but only for that period. These non-cooperative strategies are different from those discussed in the open-loop situation. In contrast to the latter, they consider how bargaining positions are changed a period later, when the bargaining parties will cooperate again. The bargaining parties rationally expect themselves to do so exactly because there will again be gains from cooperating. Because the parties never commit themselves to future behavior, the recursive Nash bargaining solution is time-consistent.

In the following, we borrow Sorger's definition, but adjust the notation so that it fits our model. Again, we let $\star$ denote the bargaining values, and $\#$ the threat values; c denotes the closed-loop solution. The building blocks of the recursive Nash bargaining solution are defined as follows:

Definition 2 (Recursive Nash Bargaining Solution). *The recursive Nash bargaining solution of our lobbying game consists of two pairs of stationary Markovian strategies,* $(q^{c\star}, m^{c\star})$ *and* $(q^{c\#}, m^{c\#})$*, and two pairs of value functions,* $(\Gamma^{c\star}, H^{c\star})$ *and* $(\Gamma^{c\#}, H^{c\#})$*,* *such that the following is true. For all z it holds that*

$$z_+^{c\star} = z + q^{c\star}(z), \tag{4.36a}$$
$$z_+^{c\#} = z + q^{c\#}(z), \tag{4.36b}$$

$$\Gamma^{c\star}(z) = g(z, q^{c\star}(z), m^{c\star}(z)) + \beta \cdot \Gamma^{c\star}(z_+^{c\star}), \tag{4.37a}$$
$$H^{c\star}(z) = h(z, q^{c\star}(z), m^{c\star}(z)) + \beta \cdot H^{c\star}(z_+^{c\star}), \tag{4.37b}$$
$$\Gamma^{c\#}(z) = g(z, q^{c\#}(z), m^{c\#}(z)) + \beta \cdot \Gamma^{c\star}(z_+^{c\#}), \tag{4.37c}$$
$$H^{c\#}(z) = h(z, q^{c\#}(z), m^{c\#}(z)) + \beta \cdot H^{c\star}(z_+^{c\#}), \tag{4.37d}$$

$$q^{c\#}(z) \in \arg\max_q \left[g(z, q, m^{c\#}(z)) + \beta \cdot \Gamma^{c\star}(z_+) | q \geq 0 \right], \tag{4.38a}$$
$$m^{c\#}(z) \in \arg\max_m \left[h(z, q^{c\#}(z), m) + \beta \cdot H^{c\star}(z_+^{c\#}) | m \geq 0 \right], \tag{4.38b}$$

$$\begin{aligned} N(z, q, m) \equiv{}& \chi \cdot \ln \left[g(z, q, m) + \beta \cdot \Gamma^{c\star}(z_+) - \Gamma^{c\#}(z) \right] \\ &+ (1 - \chi) \cdot \ln \left[h(z, q, m) + \beta \cdot H^{c\star}(z_+) - H^{c\#}(z) \right], \end{aligned} \tag{4.39}$$
$$(q^{c\star}(z), m^{c\star}(z)) \in \arg\max_{q,m} \left[N(z, q, m) | q \geq 0, m \geq 0 \right]. \tag{4.40}$$

The interpretation of the equations follows along the lines of Section 4.2. Equations (4.36) show the equations of motion for the stock in case of bargaining and non-cooperative behavior, respectively. Equations (4.37a) and (4.37b) recursively define the value functions of successful bargaining. Likewise, equations (4.37c) and (4.37d) recursively define the threat points, that is, the value functions of unsuccessful bargaining, which is the result of the behavior defined by (4.38). Finally, the last two lines define the Nash product and the outcome of the bargaining. Note that this definition of a bargaining solution presupposes that even the threat points today assume a successful bargain in the following period; the threat is to behave uncooperatively for the current period. Next-period bargaining then starts with the state that the game produces in the current period. Maximizing the Nash product implies the following optimality condition for $q^{c\star}(z)$:

$$\begin{aligned} &\frac{\chi}{\Delta_\Gamma^c(z)} \cdot \left[\tfrac{\partial g(z, q^{c\star}(z), m^{c\star}(z))}{\partial q} + \beta \cdot \tfrac{\partial \Gamma^{c\star}(z_+^{c\star})}{\partial z_+} \right] \\ &+ \frac{1 - \chi}{\Delta_H^c(z)} \cdot \left[\tfrac{\partial h(z, q^{c\star}(z), m^{c\star}(z))}{\partial q} + \beta \cdot \tfrac{\partial H^{c\star}(z_+^{c\star})}{\partial z_+} \right] = 0, \end{aligned} \tag{4.41}$$

where the gains from cooperation are defined by the same token as (4.13a) and (4.13b):

$$\Delta_\Gamma^c(z) \equiv \Gamma^{c\star}(z) - \Gamma^{c\#}(z), \tag{4.42a}$$

$$\Delta_H^c(z) \equiv H^{c\star}(z) - H^{c\#}(z). \tag{4.42b}$$

As in Section 4.2, we note that (4.41) is equivalent to

$$\frac{\partial w(z,q^{c\star}(z))}{\partial q} + \beta \cdot \frac{\partial \Gamma^c(z_+^{c\star})}{\partial q} + \frac{1-\chi}{\chi}\frac{\Delta_\Gamma^c(z)}{\Delta_H^c(z)} \cdot \left[\frac{\partial \pi(z,q^{c\star}(z))}{\partial q} + \beta \cdot \frac{\partial H^c(z_+^{c\star})}{\partial q}\right] = 0. \tag{4.43}$$

Thus, the extraction again maximizes a weighted sum of the government's and the lobby's intertemporal utility. The respective condition for the contribution path is

$$\frac{\partial g(z,q^{c\star}(z),m^{c\star}(z))}{\partial m} + \frac{1-\chi}{\chi}\frac{\Delta_\Gamma^c(z)}{\Delta_H^c(z)}\frac{\partial h(z,q^{c\star}(z),m^{c\star}(z))}{\partial m} = 0. \tag{4.44}$$

Substituting the functions and rearranging (4.44) yields the same constant weight λ as in Section 4.2, due to the constant marginal utility of contributions:

$$\frac{1-\chi}{\chi}\frac{\Delta_\Gamma^c(z)}{\Delta_H^c(z)} = \lambda \equiv \frac{\zeta}{\sigma}. \tag{4.45}$$

Thus, the q path is determined as in in Section 4.2, namely as the maximand of the Bellman equation (4.19). Thus, it must be the same as in the open-loop solution. The fact that the contributions do not have a stock effect implies, from (4.38b), that the threat level is $m^{c\#} = 0$. What about the equilibrium contribution path? To derive it, we need the equilibrium and threat value functions. Deriving the equilibrium value functions along the lines of Section 4.2, we have:

$$\Gamma^{c\star}(z) = \chi \cdot \left[R^{c\star}(z) - \lambda \cdot H^{c\#}(z)\right] + (1-\chi)\cdot \Gamma^{c\#}(z), \tag{4.46}$$

$$H^{c\star}(z) = \frac{1}{\lambda}\cdot\left[(1-\chi)\cdot\left[R^{c\star}(z) - \Gamma^{c\#}(z)\right] + \chi\cdot\lambda\cdot H^{c\#}(z)\right]. \tag{4.47}$$

$R^{c\star}$ is known from (4.19). The threat functions, however, are not known, and will in general differ from the open-loop case. The threat extraction quantity is not as easy to derive, however, so we will spare it for the explicit solution in the following section.

4.5 Recursive Nash Bargaining Solution: Explicit Solution

To shed some light on the outcome of the time-consistent bargaining setting, we proceed along the lines of Section 4.3. Thus, the instantaneous objective functions are given by (4.24). Again, we guess that there exist constants $X_{g,0}$, $X_{g,1}$, $y_{g,0}$, $y_{g,1}$, $x_{h,0}$,

$x_{h,1}$, $x_{h,2}$, such that the following policy functions exist:

$$q^{c\star}(z) = q^{o\star}(z) = X_{g,0} + X_{g,1}z, \tag{4.48a}$$

$$q^{c\#}(z) = y_{g,0} + y_{g,1}z, \tag{4.48b}$$

$$m^{c\star}(z) = x_{h,0} + x_{h,1}z + x_{h,2}z^2, \tag{4.48c}$$

$$m^{c\#}(z) = 0. \tag{4.48d}$$

(4.48d) follows from the discussion in the previous section; the optimal non-cooperative strategy of the lobby in a one-period deviation is not to pay contributions. From this discussion, we also know that the equilibrium extraction path must be the same as in the open-loop solution, which explains the equality in (4.48a), for which we can use the coefficients from Lemma 6. The definition of the equilibrium value functions follows along the lines of Section 4.3:

Lemma 8 (Equilibrium Value Functions). *For the coefficients in* (4.48a)*, we have already assumed that* (4.28) *holds. Then, the lobby's equilibrium value functions have exactly the same form as those in the open-loop case stated in Lemma 5, except that all X_h coefficients have to be replaced by their x_h counterparts.*

Proof. The proof follows along the lines of Lemma 5.

Likewise, we can state the following threat value functions:

Lemma 9 (Threat Value Functions). *The threat value functions are given by*

$$\Gamma^{c\#}(z) = -\frac{a_g}{2}q^{c\#}(z)^2 + (b - \omega_z z)\cdot q^{c\#}(z) + \beta\left[\frac{-\frac{a_g}{2}+\frac{1}{\psi}}{1-\beta(1-\psi\omega_z)^2}q^{c\star}(z_+^{c\#})^2 + \zeta M^{c\star}(z_+^{c\#})\right], \tag{4.49}$$

$$H^{c\#}(z) = -\frac{a_h}{2}q^{c\#}(z)^2 + (b - \omega_z z)\cdot q^{c\#}(z) + \beta\left[\frac{-\frac{a_h}{2}+\frac{1}{\psi}}{1-\beta(1-\psi\omega_z)^2}q^{c\star}(z_+^{c\#})^2 - \sigma M^{c\star}(z_+^{c\#})\right], \tag{4.50}$$

where

$$M^{c\star}(z_+^{c\#}) \equiv -\frac{\frac{1}{\psi\omega_z}x_{h,1}}{1-\beta(1-\psi\omega_z)}q^{c\star}(z_+^{c\#}) + \frac{x_{h,0}+\frac{b}{\omega_z}x_{h,1}}{1-\beta} + \frac{x_{h,2}}{(\psi\omega_z)^2}\left[\frac{q^{c\star}(z_+^{c\#})^2}{1-\beta(1-\psi\omega_z)^2} - \frac{2\psi b q^{c\star}(z_+^{c\#})}{1-\beta(1-\psi\omega_z)} + \frac{(\psi b)^2}{1-\beta}\right] \tag{4.51}$$

is the present value of contributions in the future, discounted to the next period. $q^{c\#}(z)$ *is defined in* (4.48b)*, and* $q^{c\star}(z_+^{c\#})$ *then is defined by* (4.48a) *and* (4.36b).

Proof. Using the respective definitions of the threat value functions from (4.37), we can substitute the instantaneous utility functions from (4.24) and the discounted equilibrium value functions from Lemma 8. The threat policy is then substituted from (4.48b) and (4.48d). □

Note that the equilibrium values of the next period are used because then the parties return to cooperation. Because $m^{c\#}(z) = 0$ is already known, we can now determine the threat extraction of the government, $q^{c\#}(z)$, along with the equilibrium contribution payments $m^{c\star}(z)$.

Lemma 10 (Threat Extraction). *In the government's threat policy function* (4.48b)*, we have the following coefficients:*

$$y_{g,0} = \tilde{\psi} \cdot b, \tag{4.52a}$$

$$y_{g,1} = -\tilde{\psi} \cdot \omega_z, \tag{4.52b}$$

so that $q^{c\#}(z) = \tilde{\psi} \cdot (b - \omega_z z)$*, where*

$$\tilde{\psi} = \frac{1}{\omega_z}\left[1 - \frac{1-\beta\left(1-\psi\omega_z\right)^2}{\frac{2}{1+r}\left[\psi\omega_z\left(\omega_z - a_g\right) + \tilde{\Theta}\omega_z^2\zeta\right] - \frac{r}{1+r}a_g}\left(\omega_z - a_g\right)\right]. \tag{4.53}$$

Proof. See Appendix A.6. □

Lemma 11 (Equilibrium Contribution Payments). *In the lobby's equilibrium contribution payment function* (4.48c)*, we have the following coefficients:*

$$x_{h,0} = \tilde{\Theta} \cdot b^2, \tag{4.54a}$$

$$x_{h,1} = -\tilde{\Theta} \cdot 2b\omega_z, \tag{4.54b}$$

$$x_{h,2} = \tilde{\Theta} \cdot \omega_z^2, \tag{4.54c}$$

so that $m^{c\star}(z) = \tilde{\Theta} \cdot (b - \omega_z z)^2$*, where*

$$\tilde{\Theta} \equiv \frac{\beta\left(\tilde{\psi} - \psi\right)}{1-\beta\left(1-\tilde{\psi}\omega_z\right)^2}\left[\rho_1\left(\tilde{\psi}\psi\omega_z^2 + r\right) - \rho_2\left[\tilde{\psi}\psi\omega_z + \frac{1}{2}\left(\tilde{\psi}+\psi\right)r\right]\right] \geq 0 \tag{4.55}$$

and where $\rho_1 \equiv \frac{1-\chi}{\zeta} - \frac{\chi}{\sigma}$ *and* $\rho_2 \equiv \frac{1-\chi}{\zeta}a_g - \frac{\chi}{\sigma}a_h$.

Proof. See Appendix A.6. □

Using the coefficients from Lemmas 10 and 11 in (4.51), we can simplify the present value of contributions in the future, discounted to the next period, as follows:

$$M^{c\star}(z_+^{c\#}) \equiv \frac{\tilde{\Theta}/\psi^2}{1-\beta\,(1-\psi\omega_z)^2} q^{c\star}(z_+^{c\#})^2. \tag{4.51'}$$

Like the equilibrium payments in the open-loop case (Lemma 7), the payments in the recursive Nash bargaining case (Lemma 11) are a quadratic function of the difference between the marginal net gain of the first extracted unit and current extraction costs. The reason is the same, namely that welfare losses grow quadratically in the absolute quantity deviation from welfare-maximizing extraction. As z grows, contributions of both solution approaches vanish.

However, the time-consistent solution is much more complex than the open-loop solution. $\tilde{\psi}$ and $\tilde{\Theta}$ are still interdependent, and we cannot solve them explicitly. Thus we can analytically describe the extraction path and the qualitative properties of lobbying contributions in general (and have done so), but cannot derive the difference between open-loop and time-consistent contribution payments in the same manner. Therefore, we present and discuss graphical illustrations of numerical examples in the following section.

5 The Lobbying Equilibrium

We now illustrate the equilibrium in our policy model with specific parameter values for the functions in Table 1 (see page 8). The parameters used in the figures, if nothing else is noted, are $\beta = 15/16$, $\omega_z = 1/10$, $\gamma_1 = 100$, $\gamma_2 = 1/2$, $\omega_1 = 0$, $\omega_2 = 2$. In the beginning of time ($t = 0$), cumulative extraction is assumed to be zero, $z = 0$. We assume $\sigma = \zeta = 1$, so that the lobby group's policy weight is $\lambda = 1$, and the bargaining weight is symmetric, $\chi = 1/2$. The remaining economic parameter, κ, will be indicated in each of the following cases.

We start by illustrating a specific case, in which $\kappa = \gamma_2 (= 1/2)$. Figure 1 shows the paths of extraction (left-hand side figure) and cumulative extraction (right-hand side figure). The gray curve is the path that an unregulated competitive market would produce, hence, the *laissez-faire* path. The black curve is the path for a monopoly extractor, the welfare-maximizing path, and also the lobbying-equilibrium path. Why would a monopolist and a welfare-maximizing government want the same extraction path in this case? By the linear functions, the consumers' marginal utility of extraction is $\gamma_1 - \gamma_2 q$. To get instantaneous marginal welfare, subtract the marginal flow damage, κq. By (3.2), the consumers' marginal utility also is the demand price. Thus, the

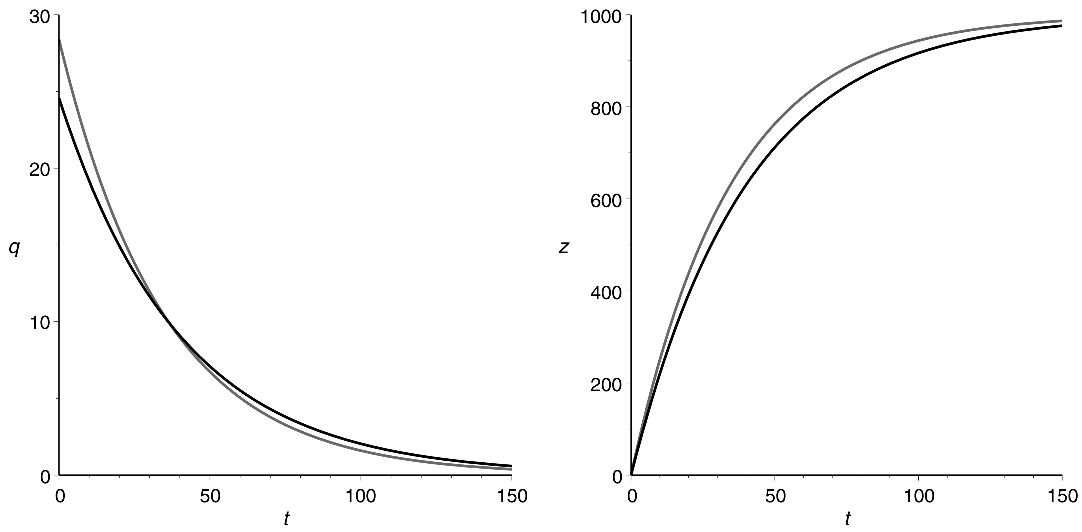

Figure 1: Extraction paths and cumulative extraction paths if there is no conflict of interest.

monopolist's marginal revenue is $\gamma_1 - 2\gamma_2 q$. Given that both deciders face the same level of cumulative extraction, their (marginal) extraction costs are identical. Thus, in the first period, instantaneous marginal extraction utility of the government equals that of the monopoly if

$$\gamma_1 - \gamma_2 q - \kappa q = \gamma_1 - 2\gamma_2 q, \tag{5.1}$$

which is exactly fulfilled because we have assumed $\kappa = \gamma_2$. In words, the curve reflecting marginal consumer utility net of marginal environmental damage has the same slope as the suppliers' marginal revenue curve, so that a monopoly would always want to extract the socially efficient quantity. Because this will also be true in the future, a coincidence of marginal flow utilities implies a coincidence of extraction paths.

By comparison, the laissez-faire extraction starts from a higher level because resource owners neither internalize their effect on the price nor the environmental damage. Because they extract more in the beginning of time, their costs rise faster than along the welfare and revenue maximizing path. Therefore, extraction along the laissez-faire path is lower than that along the latter path in later periods.

All paths converge to zero production, which is reached when so much has been extracted that marginal extraction costs are as high as the consumers' willingness to pay. However, along all of the extraction paths, this cumulative-extraction level is only reached asymptotically. Along the way, cumulative extraction of competitive suppliers is always higher than the welfare-maximizing or monopolistic cumulative extraction. To summarize, we cannot say that laissez-faire extraction is always *higher*, but it is *faster* than on the other paths. Because this characterization of the laissez-faire case always holds, we leave it out of the following figures for clarity.

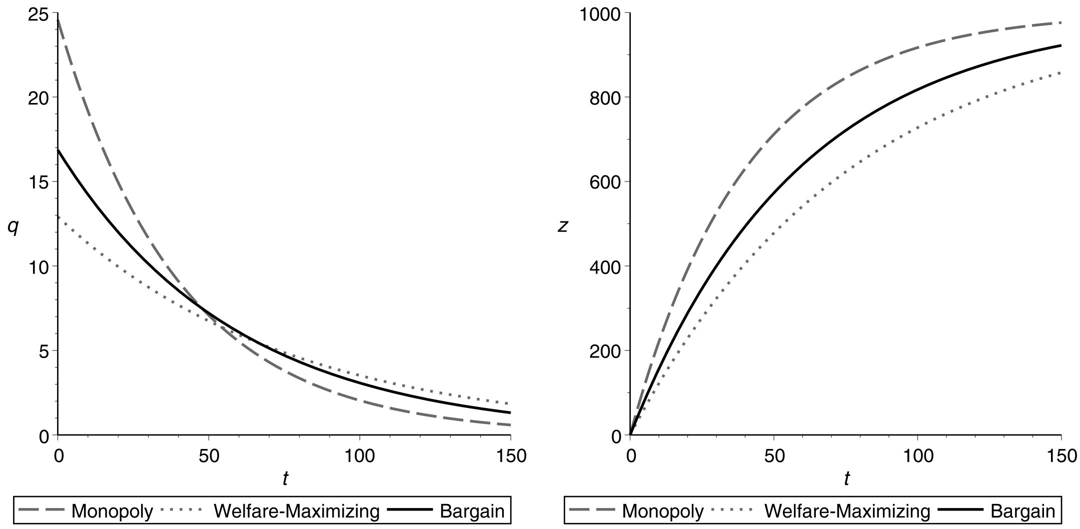

Figure 2: Extraction paths and cumulative extraction paths if lobbying accelerates extraction.

What about the contribution payments? In the described case, there is no conflict of interest between the government and the lobby. The monopoly path would be the best the lobby could get, and it coincides with the welfare-maximizing path (and thus, the government's threat). Thus, there are also no lobby contributions; the lobby does not pay anything because the government will act in the lobby's interest anyway.[11]

The next case is the one most people would expect for environmental policy: The welfare-maximizing extraction is smaller than the monopolistic one in the beginning of time, so that environmental policy restricts supply. According to Lemma 4, this is true for $\kappa > \gamma_2 = 1/2$. To illustrate in Figure 2, we use $\kappa = 4$. The solid black curve is the bargaining outcome, the dashed gray curve is monopolistic extraction, and the dotted gray curve is welfare-maximizing.

By the logic of the equation (5.1) discussion, now the (instantaneous) marginal net welfare from extracting is smaller than the marginal net revenue in the beginning of time (when marginal extraction costs are identical). Thus, a monopolist would prefer a faster extraction than the government, leading to a higher cumulative extraction. The bargaining outcome maximizes a weighted sum of welfare and the resource owners' profit, leading to a compromise path in the lobbying equilibrium, where extraction is faster than along the welfare-maximizing path, but slower than the monopolistic extraction.

Remember that this path is valid for both the open-loop bargaining and the recursive Nash bargaining solution (RNBS). What is different between the two commitment assumptions is the development of lobbying contributions. The left-hand side of Fig-

[11]Formally, this can be seen by noting that $a_g = a_h$ implies that ψ in (4.33) equals ψ_g in (3.21). For, e.g., the open-loop case, we then have $m^{o\star} = 0$ – see Lemma 7.

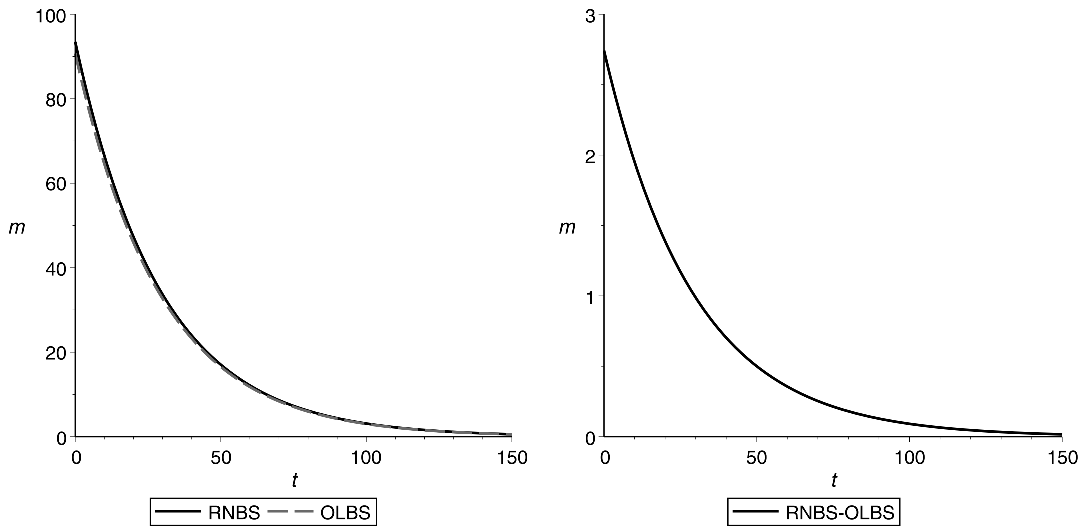

Figure 3: Contribution paths if lobbying accelerates extraction.

ure 3 shows the paths for the two different commitment assumptions. The solid black curve is the result of repeated bargaining in the RNBS, the gray dashed curve is the contribution path agreed upon in the open-loop solution. The right-hand side of the figure shows the difference between the two.

The first fact about these paths is that in both cases, contributions converge to zero. The reason for this is very intuitive in the open-loop solution. While both bargaining parties' preferred extraction paths are very different at the beginning, they both converge to zero. The conflict of interest vanishes, and so does the necessity to compensate the government for not being on the welfare-maximizing path and the resource owners' gain from this. The reason in the RNBS case is similar, but not exactly the same, because the government can never "jump" back to the welfare-maximizing path. However, the government's threat in the RNBS is to behave uncooperatively for one period, thereby maximizing a sum of current welfare and the future bargaining position. In any given period, this threat also has to take the cumulative extraction into account. If up to then a lot has been extracted, so that marginal extraction costs have become very high and the paths converge to zero, a one-period deviation cannot change much about this anymore.

Where do the differences between the contribution paths come from? Figure 4 helps to explore this question. The black curve shows the bargaining q path from Figure 2, and the dotted gray curve is the welfare-maximizing q path, which here is the open-loop threat path. The solid gray curve is the RNBS threat extraction. At any moment of time, it shows what the government would do given that too much (compared to the welfare-maximizing extraction) has already been accumulated, and too much will be accumulated in the future. The high cumulative extraction in the past implies high extraction costs today, which reduces the one-period deviation quantity. The anticipated

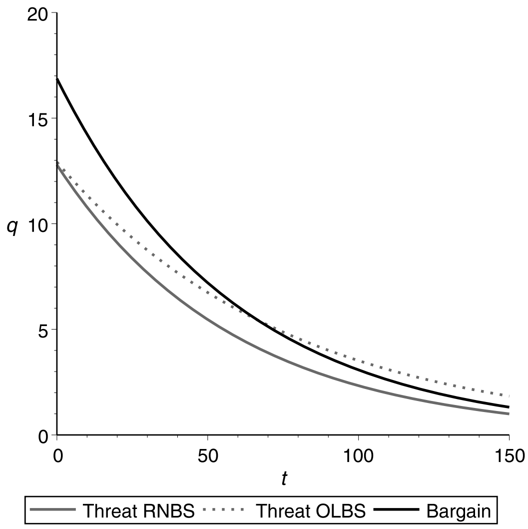

Figure 4: Threat paths if lobbying accelerates extraction.

high future extraction also reduces the deviation quantity, because this reduces future extraction costs at least a bit.

However, having followed the lobbying equilibrium for some time, the government can never "jump" back to the welfare-maximizing path. Even if it chose that path from some period on, the past extraction could not be undone, and so costs would be permanently higher. Anticipating this, the government would need higher compensation to agree to the lobbying-equilibrium policy in the RNBS case.

Now consider the opposite case where the marginal revenue curve is steeper than the marginal welfare curve, $\kappa < \gamma_2$. Lemma 4 then implies that the lobby would like to produce *less* than the social optimum at the beginning of time; it would prefer a slower extraction. Put differently, the government, due to the presence of environmental externalities, would always want extraction to be slower than the laissez-faire extraction path, but now the parameter constellation $\kappa < \gamma_2$ implies that the lobby would want extraction to be even further decelerated.[12] The most extreme version of the monopoly effect exists when there is no externality, and lobbying only implies quantity reductions. Then the model may be a shortcut for "resource-suppliers lobbying against competition policy".

The lobby thus pays the government to allow lower production. After some periods, the actual extraction has been the compromise path for some time, so that extraction costs are lower than they would have been along the welfare-maximizing path. Thus,

[12]Note that this results from the assumption that for any quantity, the resource owners receive the whole consumer price. This allows the interpretation that we have, in each period, a quantity-allowance trading system in which all certificates are allocated to the industry for free – so called "grandfathering". The situation would be different if, for example, such allowances were auctioned or extraction was taxed (and the tax receipts were not redistributed to the firms). See Hepburn et al. (2013) for a detailed discussion of the impact of emissions allocation on profits in a static setting. For a comprehensive discussion of the distributional effects of environmental policy see Fullerton (2001).

if the government deviated from cooperation for one period, it would choose a higher quantity than q^{**}. The relation between contributions from Figure 3 is turned around: The payments in the open-loop case are permanently higher than they are with repeated bargaining. In the latter case, the government could always choose a short-run deviation to the original welfare-maximizing path and have even lower costs, because *less* extraction has happened in the past. Therefore, the compensation does not have to be as high as it is along the open-loop path.

To summarize, if the marginal revenue curve is steeper (flatter), the lobbying costs will be higher (lower) in the open-loop than in the RNBS case. However, the open-loop and the RNBS case produce the same extraction path. Thus, in our setting, the (in-)ability of the bargaining parties to commit to a long-run path does not affect welfare.

However, the fact that lobbying affects extraction is unambiguously bad for welfare. The only exception to this rule is the case in which the marginal revenue curve has the same slope as the marginal welfare curve, so that monopolistic extraction and the welfare-maximization path coincide.[13]

To further illustrate, consider an interesting special case, namely, a small open economy facing a fixed (and static) world price for the extracted resource. Thus, it could both import and export the resource for a price of γ_1, and we have $\gamma_2 = 0$. In this setting, there are no consumer rents, so that there cannot be a monopoly rent. Then any supply reduction (compared to the laissez-faire path) always reduces firms' profit. Moreover, either there are no externalities (and there is also no lobbying), or we have $\kappa > \gamma_2 = 0$. Thus, in such cases we should see higher payments of resource extractors if official commitment is impossible. This setting may be interpreted as a model where resource suppliers export to a competitive world market, but pollution has local effects. This may, for example, roughly describe coal production in many countries.

Finally, let us consider some comparative dynamics. The first point results from the discussions above: An increased environmental marginal-damage parameter κ implies higher contribution payments if $\kappa > \gamma_2$. Thus, while higher marginal damage costs would in general be bad, they only imply higher lobby payments if the conflict of interest between the government and the resource owners concerning the extraction quantity is intensified. This would be valid in a static model as well, and we mainly stretched the point in the previous paragraphs as it determines which kind of commitment assumption leads to higher contributions.

We now concentrate on the two "political" parameter sets, namely the marginal-utility parameters of the contributions and the bargaining weights. Concerning the marginal-utility parameters, the lobby's policy weight λ is given by ζ/σ – see (4.16). If the government's preference ζ for payments is higher, or the lobby's cost of collecting

[13]Note that lobbying is good for the environment when the marginal revenue curve is steeper than the marginal welfare curve. However, then the monopoly path would be even better for the environment.

contributions σ is lower, then the lobby receives a higher weight in the equilibrium extraction choice. By contrast, we do not need the bargaining weight χ to determine the extraction path – it only influences the compensation payments.

6 Conclusions

In this article, we have derived resource extraction determined by the bargaining of a government and a lobby group. Equilibrium extraction is a compromise path between welfare maximization and profit maximization. If contributions and extraction were independent, the government would prefer the former, and the lobby would prefer the latter. The weight of the lobby influence on the equilibrium path increases in the government's preference for contribution payments and decreases in the lobby's cost of collecting and paying them. Depending on demand and damage functions, this implies that extraction is either too fast or too slow, compared to welfare maximization. Along all equilibrium paths, extraction converges to zero as marginal costs increase with cumulative extraction. Thus, the conflict of interest between welfare maximization and profit maximization vanishes in the long run, and so do the contribution payments.

We have also analyzed the impact of different degrees of commitment on the equilibrium. One extreme is that full commitment, till the end of time, is possible (open-loop Nash bargaining solution). The other variant is that the parties bargain each period, which requires a time-consistent solution strategy (recursive Nash bargaining solution). The commitment assumptions do not change the equilibrium extraction path. However, they lead to different levels of contribution payments. If the lobbying-equilibrium extraction is faster (slower) than the welfare-maximizing extraction, payments are higher (lower) in the time-consistent equilibrium than in the open-loop equilibrium, to compensate for the government's worse (better) outside option.

We believe the political-economy analysis of resource extraction to be a promising field of research, given that in this policy area many people seem to be convinced of the resource owners' distortive influence. In particular, an interesting research topic would be the political determination of backstop technologies' development, which would broaden the perspective on political distortions from resource consumption to investment.

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A Appendix

A.1 Proof of Lemma 1

Substituting (3.15a) in (3.1) yields:

$$z(t+1) = z(t) + Y_{w,0} + Y_{w,1} z(t). \tag{A.1}$$

Hence,

$$\begin{aligned} z(t+s) &= z(t) + \sum_{\nu=0}^{s-1} q(t+\nu) \\ &= z(t) + sY_{w,0} + Y_{w,1} \sum_{\nu=0}^{s-1} z(t+\nu). \end{aligned} \tag{A.2}$$

After some substitutions and rearrangements, we get

$$\begin{aligned} z(t+s) &= \left(1+Y_{w,1}\right)^s z(t) + Y_{w,0} \sum_{\nu=0}^{s-1} \left(1+Y_{w,1}\right)^\nu \\ &= \frac{\left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right] - Y_{w,0}}{Y_{w,1}}, \end{aligned} \tag{A.3}$$

where we have to assume (3.16) for stability. Substituting in (3.15a) yields:

$$q^{**}(t+s) = \left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right]. \tag{A.4}$$

From (3.13a), we then have (after some rearrangements):

$$w(t+s) = \left(\omega_z Y_{w,0} + bY_{w,1}\right) \frac{\left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right]}{Y_{w,1}} \\ - Y_{w,1}\left(\frac{a_g}{2} Y_{w,1} + \omega_z\right) \left[\frac{\left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right]}{Y_{w,1}}\right]^2 . \tag{A.5}$$

Then from (3.6):

$$W^{**}(t) = \left(\omega_z Y_{w,0} + bY_{w,1}\right) \sum_{s=0}^{\infty} \beta^s \frac{\left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right]}{Y_{w,1}} \\ - Y_{w,1}\left(\frac{a_g}{2} Y_{w,1} + \omega_z\right) \sum_{s=0}^{\infty} \beta^s \left[\frac{\left(1+Y_{w,1}\right)^s \left[Y_{w,0} + Y_{w,1} z(t)\right]}{Y_{w,1}}\right]^2 . \tag{A.6}$$

Evaluating the infinite sums yields (3.18). Likewise, by (3.1) and (3.15b) and assuming (3.16), we get

$$q^*(t+s) = \left(1+Y_{\pi,1}\right)^s \left[Y_{\pi,0} + Y_{\pi,1} z(t)\right]. \tag{A.7}$$

From (3.13b), we then have (after some rearrangements):

$$\pi(t+s) = \left(\omega_z Y_{\pi,0} + bY_{\pi,1}\right) \frac{\left(1+Y_{\pi,1}\right)^s \left[Y_{\pi,0} + Y_{\pi,1} z(t)\right]}{Y_{\pi,1}} \\ - Y_{\pi,1}\left(\frac{a_h}{2} Y_{\pi,1} + \omega_z\right) \left[\frac{\left(1+Y_{\pi,1}\right)^s \left[Y_{\pi,0} + Y_{\pi,1} z(t)\right]}{Y_{\pi,1}}\right]^2 . \tag{A.8}$$

Then from (3.4):

$$\Pi^*(t) = \left(\omega_z Y_{\pi,0} + bY_{\pi,1}\right) \sum_{s=0}^{\infty} \beta^s \frac{\left(1+Y_{\pi,1}\right)^s \left[Y_{\pi,0} + Y_{\pi,1} z(t)\right]}{Y_{\pi,1}} \\ - Y_{\pi,1}\left(\frac{a_h}{2} Y_{\pi,1} + \omega_z\right) \sum_{s=0}^{\infty} \beta^s \left[\frac{\left(1+Y_{\pi,1}\right)^s \left[Y_{\pi,0} + Y_{\pi,1} z(t)\right]}{Y_{\pi,1}}\right]^2 . \tag{A.9}$$

Evaluating the infinite sums yields (3.19).

A.2 Proof of Lemma 2

q^{**} must maximize (3.18), which we can split in instantaneous welfare of period t and discounted welfare of all periods afterwards,

$$\begin{aligned} W^{**}(t) = &-\frac{a_g}{2}q(t)^2 - \omega_z z(t) \cdot q(t) + bq(t) \\ &+ \beta \Bigg\{ \frac{\omega_z Y_{w,0} + bY_{w,1}}{1-\beta\left(1+Y_{w,1}\right)} \frac{Y_{w,0} + Y_{w,1}z(t+1)}{Y_{w,1}} \\ &\quad - \frac{\frac{a_g}{2}Y_{w,1} + \omega_z}{1-\beta\left(1+Y_{w,1}\right)^2} \frac{\left[Y_{w,0} + Y_{w,1}z(t+1)\right]^2}{Y_{w,1}} \Bigg\}. \end{aligned} \tag{A.10}$$

Substituting the equation of motion (3.1), differentiating with respect to $q(t)$, and substituting (3.15a), we get the following first-order condition:

$$\begin{aligned} 0 = &-a_g\left[Y_{w,0} + Y_{w,1}z(t)\right] - \omega_z z(t) + b \\ &+ \beta \Bigg\{ \frac{\omega_z Y_{w,0} + bY_{w,1}}{1-\beta\left(1+Y_{w,1}\right)} \\ &\quad - \frac{a_g Y_{w,1} + 2\omega_z}{1-\beta\left(1+Y_{w,1}\right)^2} \left[Y_{w,0} + Y_{w,1}\left[Y_{w,0} + \left(1+Y_{w,1}\right)z(t)\right]\right] \Bigg\}. \end{aligned} \tag{A.11}$$

We can shift this to period $t+1$, substitute $z(t+1) = z(t) + q(t)$, and again substitute (3.15a). This generates two equations in two unknowns. These contain quadratic terms, but taking (3.16) into account, we can select the solution (3.20). Likewise, q^* must maximize (3.19). Taking (3.17) into account, we can select the monopoly-solution coefficients as stated in Lemma 2.

A.3 Proof of Lemma 5

Along the same lines as in Appendix A.1, by (3.1) and (4.25a) and assuming (4.28), we get

$$q^{o\star}(t+s) = \left(1+X_{g,1}\right)^s \left[X_{g,0} + X_{g,1}z(t)\right], \tag{A.12a}$$

$$\begin{aligned} m^{o\star}(t+s) = &X_{h,0} + X_{h,1}\frac{\left(1+X_{g,1}\right)^s\left[X_{g,0} + X_{g,1}z(t)\right] - X_{g,0}}{X_{g,1}} \\ &+ X_{h,2}\left[\frac{\left(1+X_{g,1}\right)^s\left[X_{g,0} + X_{g,1}z(t)\right] - X_{g,0}}{X_{g,1}}\right]^2. \end{aligned} \tag{A.12b}$$

From (4.24a), we then have (after some rearrangements):

$$\begin{aligned} g(t+s) = {} & \Big(\omega_z X_{g,0} + b X_{g,1}\Big) \frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}} \\ & - X_{g,1}\left(\frac{a_g}{2}X_{g,1}+\omega_z\right)\left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}}\right]^2 \\ & + \zeta \left\{ X_{h,0} + X_{h,1}\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}} \right. \\ & \left. + X_{h,2}\left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}}\right]^2 \right\}. \end{aligned} \tag{A.13}$$

Then from (4.3) and (4.7a):

$$\begin{aligned} \Gamma^{o\star}(t) = {} & \sum_{s=0}^{\infty} \beta^s \cdot g(t+s) \\ = {} & \Big(\omega_z X_{g,0} + b X_{g,1}\Big) \sum_{s=0}^{\infty} \beta^s \frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}} \\ & - X_{g,1}\left(\frac{a_g}{2}X_{g,1}+\omega_z\right)\sum_{s=0}^{\infty} \beta^s \left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}}\right]^2 \\ & + \zeta \sum_{s=0}^{\infty} \beta^s \left\{ X_{h,0} + X_{h,1}\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}} \right. \\ & \left. + X_{h,2}\left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}}\right]^2 \right\}. \end{aligned} \tag{A.14}$$

Evaluating the infinite sums yields (4.29). Likewise, (4.24b) yields:

$$\begin{aligned} h(t+s) = {} & \Big(\omega_z X_{g,0} + b X_{g,1}\Big) \frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}} \\ & - X_{g,1}\left(\frac{a_h}{2}X_{g,1}+\omega_z\right)\left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big]}{X_{g,1}}\right]^2 \\ & - \sigma \left\{ X_{h,0} + X_{h,1}\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}} \right. \\ & \left. + X_{h,2}\left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0}+X_{g,1}z(t)\Big] - X_{g,0}}{X_{g,1}}\right]^2 \right\} \end{aligned} \tag{A.15}$$

and from that we get

$$
\begin{aligned}
H^{o\star}(t) &= \sum_{s=0}^{\infty} \beta^s \cdot h(t+s) \\
&= \Big(\omega_z X_{g,0} + b X_{g,1}\Big) \sum_{s=0}^{\infty} \beta^s \frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0} + X_{g,1} z(t)\Big]}{X_{g,1}} \\
&\quad - X_{g,1}\left(\frac{a_h}{2} X_{g,1} + \omega_z\right) \sum_{s=0}^{\infty} \beta^s \left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0} + X_{g,1} z(t)\Big]}{X_{g,1}}\right]^2 \\
&\quad - \sigma \sum_{s=0}^{\infty} \beta^s \Bigg\{ X_{h,0} + X_{h,1} \frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0} + X_{g,1} z(t)\Big] - X_{g,0}}{X_{g,1}} \\
&\qquad + X_{h,2} \left[\frac{\Big(1+X_{g,1}\Big)^s \Big[X_{g,0} + X_{g,1} z(t)\Big] - X_{g,0}}{X_{g,1}}\right]^2 \Bigg\}. \qquad \text{(A.16)}
\end{aligned}
$$

Evaluating the infinite sums yields (4.30).

A.4 Proof of Lemma 6

From (4.19) and (4.24), $q^{o\star}$ must maximize

$$
\begin{aligned}
R(t) = &-\frac{a_g}{2} q(t)^2 - \omega_z z(t)\cdot q(t) + b q(t) + \lambda \left[-\frac{a_h}{2} q(t)^2 - \omega_z z(t)\cdot q(t) + b q(t)\right] \\
&+ \beta\left[\Gamma^{o\star}(t+1) + \lambda H^{o\star}(t+1)\right], \qquad \text{(A.17)}
\end{aligned}
$$

where λ is defined by (4.16). By Lemma 5:

$$
\begin{aligned}
\Gamma^{o\star}(t+1) = &\frac{\omega_z X_{g,0} + b X_{g,1}}{1-\beta\Big(1+X_{g,1}\Big)} \frac{X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]}{X_{g,1}} \\
&- \frac{\frac{a_g}{2} X_{g,1} + \omega_z}{1-\beta\Big(1+X_{g,1}\Big)^2} \frac{\Big[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\Big]^2}{X_{g,1}} \\
&+ \zeta \Bigg\{ \frac{X_{h,0} - \frac{X_{g,0}}{X_{g,1}} X_{h,1}}{1-\beta} + \frac{X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]}{1-\beta\Big(1+X_{g,1}\Big)} \frac{X_{h,1}}{X_{g,1}} \\
&\quad + \frac{X_{h,2}}{\Big(X_{g,1}\Big)^2} \Bigg[\frac{\Big[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\Big]^2}{1-\beta\Big(1+X_{g,1}\Big)^2} \\
&\qquad - \frac{2X_{g,0}\Big[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\Big]}{1-\beta\Big(1+X_{g,1}\Big)} + \frac{\Big(X_{g,0}\Big)^2}{1-\beta} \Bigg] \Bigg\} \qquad \text{(A.18)}
\end{aligned}
$$

and

$$
\begin{aligned}
H^{o\star}(t+1) = {} & \frac{\omega_z X_{g,0} + b X_{g,1}}{1-\beta\left(1+X_{g,1}\right)} \frac{X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]}{X_{g,1}} \\
& - \frac{\frac{a_h}{2} X_{g,1} + \omega_z}{1-\beta\left(1+X_{g,1}\right)^2} \frac{\left[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\right]^2}{X_{g,1}} \\
& - \sigma \Bigg\{ \frac{X_{h,0} - \frac{X_{g,0}}{X_{g,1}} X_{h,1}}{1-\beta} + \frac{X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]}{1-\beta\left(1+X_{g,1}\right)} \frac{X_{h,1}}{X_{g,1}} \\
& \qquad + \frac{X_{h,2}}{\left(X_{g,1}\right)^2} \Bigg[\frac{\left[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\right]^2}{1-\beta\left(1+X_{g,1}\right)^2} \\
& \qquad\qquad - \frac{2 X_{g,0}\left[X_{g,0} + X_{g,1}\left[z(t)+q(t)\right]\right]}{1-\beta\left(1+X_{g,1}\right)} + \frac{\left(X_{g,0}\right)^2}{1-\beta} \Bigg] \Bigg\}.
\end{aligned} \tag{A.19}
$$

Substituting (A.18) and (A.19) into (A.17), differentiating with respect to $q(t)$, and substituting (4.25a), we get the following first-order condition:

$$
\begin{aligned}
0 = {} & -\left(a_g + \lambda a_h\right)\left[X_{g,0} + X_{g,1} z(t)\right] + (1+\lambda)\left[b - \omega_z z(t)\right] \\
& + \beta \Bigg\{ \frac{(1+\lambda)\left[b X_{g,1} + \omega_z X_{g,0}\right]}{1-\beta\left(1+X_{g,1}\right)} \\
& \quad - 2 \frac{X_{g,0} + X_{g,1}\left[z(t) + X_{g,0} + X_{g,1} z(t)\right]}{1-\beta\left(1+X_{g,1}\right)^2} \left[\frac{a_g + \lambda a_h}{2} X_{g,1} + (1+\lambda)\,\omega_z\right] \\
& \quad + \underbrace{(\zeta - \lambda\sigma)}_{=0} \Bigg[\frac{X_{h,1} - 2\frac{X_{g,0}}{X_{g,1}} X_{h,2}}{1-\beta\left(1+X_{g,1}\right)} \\
& \qquad\qquad + 2 \frac{X_{g,0} + X_{g,1}\left[z(t) + X_{g,0} + X_{g,1} z(t)\right]}{1-\beta\left(1+X_{g,1}\right)^2} \frac{X_{h,2}}{X_{g,1}} \Bigg] \Bigg\}.
\end{aligned} \tag{A.20}
$$

We can shift this to period $t+1$, substitute $z(t+1) = z(t) + q(t)$, and again substitute (4.25a). This generates two equations in two unknowns. These contain quadratic terms, but taking (4.28) into account, we can select the solution (4.32).

A.5 Proof of Lemma 7

We can use the equilibrium value functions (from Lemma 5) and the threat value functions (from 4.26 and 4.27) in (4.13) and substitute in (4.16). This relation must hold in every time period, so we can formulate it for t, $t+1$, and $t+2$. Using the equilibrium equation of motion (4.6a) together with the assumed function (4.25a) and the coefficients derived in Lemma 6, we can reduce the three equations so that the only unknowns are the coefficients that we derive in Lemma 7.

A.6 Proof of Lemma 10 and 11

Following the steps in Appendix A.4 for the value functions in Lemma 9, we have the following coefficients in the government's threat policy function (4.48b):

$$y_{g,0} = \left[\tilde{\psi} - \frac{\frac{1-\beta(1-\psi\omega_z)^2}{1-\beta(1-\psi\omega_z)} \cdot \left[\frac{2x_{h,2}}{\omega_z} + \frac{x_{h,1}}{b}\right]\zeta}{2\psi\omega_z\left(\omega_z - a_g\right) + 2x_{h,2}\zeta - ra_g} \right] \cdot b, \tag{A.21a}$$

$$y_{g,1} = -\tilde{\psi} \cdot \omega_z, \tag{A.21b}$$

where

$$\tilde{\psi} = \frac{\psi\omega_z\left(\omega_z - a_g\right) + \frac{2x_{h,2}}{\psi\omega_z}\zeta - \frac{r}{\psi}}{2\psi\omega_z\left(\omega_z - a_g\right) + 2\tilde{\Theta}\cdot\omega_z^2\zeta - ra_g} \cdot \psi. \tag{A.22}$$

Along the same lines as in Appendix A.5, we can use the equilibrium value functions (from Lemma 5) and the threat value functions (from Lemma 9) to derive the following coefficients in the lobby's equilibrium contribution payment function (4.48c):

$$\begin{aligned}
x_{h,0} &= \tilde{\Theta}\cdot b^2 \\
&\quad - \frac{\left(\tilde{\psi} - \frac{y_{g,0}}{b}\right)\beta b^2}{1-\beta\left(1-\tilde{\psi}\omega_z\right)^2}\left[\rho_1\frac{\left(\psi - \frac{y_{g,0}}{b}\right)\omega_z(1-\beta)\left(1-\tilde{\psi}\omega_z\right)}{1-\beta\left(1-\tilde{\psi}\omega_z\right)}\right] \\
&\quad - \frac{\left(\tilde{\psi} - \frac{y_{g,0}}{b}\right)\beta b^2}{1-\beta\left(1-\tilde{\psi}\omega_z\right)^2}\left[\rho_1\left(\psi\omega_z\tilde{\psi}\omega_z + r\right) + \rho_2\left[\psi\omega_z\tilde{\psi} + \frac{1}{2}\left(\tilde{\psi} + \frac{y_{g,0}}{b}\right)r\right]\right], \qquad \text{(A.23a)} \\
x_{h,1} &= -\tilde{\Theta}\cdot 2b\omega_z \\
&\quad + \frac{\left(\tilde{\psi} - \frac{y_{g,0}}{b}\right)\left[1-\beta\left(1-\psi\omega_z\right)\right]b\omega_z}{1-\beta\left(1-\tilde{\psi}\omega_z\right)^2}\left[\rho_1\frac{1-\beta\left(1-\left(\tilde{\psi}\omega_z\right)^2\right)}{1-\beta\left(1-\tilde{\psi}\omega_z\right)} + \rho_2\tilde{\psi}\right], \qquad \text{(A.23b)} \\
x_{h,2} &= \tilde{\Theta}\cdot\omega_z^2, \qquad \text{(A.23c)}
\end{aligned}$$

where

$$\tilde{\Theta} \equiv \frac{\beta\left(\tilde{\psi} - \psi\right)}{1-\beta\left(1-\tilde{\psi}\omega_z\right)^2}\left[\rho_1\left(\tilde{\psi}\psi\omega_z^2 + r\right) - \rho_2\left[\tilde{\psi}\psi\omega_z + \frac{1}{2}\left(\tilde{\psi} + \psi\right)r\right]\right] \tag{A.24}$$

and where $\rho_1 \equiv \frac{1-\chi}{\zeta} - \frac{\chi}{\sigma}$ and $\rho_2 \equiv \frac{1-\chi}{\zeta}a_g - \frac{\chi}{\sigma}a_h$. These coefficients, (A.21) and (A.23), form a system of equations which is solved in Lemma 10 and 11.