Tradable Permits Schemes and New Technology Adoption

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Abstract

First, this paper models a permits market where the interaction of strategically acting firms determines prices. The permit price is the Cournot equilibrium of a game where firms in permits excess choose the amount of permits they submit to the exchange. Second, the paper characterises the inter–dependence between the incentives to adopt a new technology and the permit price in the presence of a technological shock. Finally, a mechanism that allows the regulator to respond to the effect, in terms of permits allocations, that the technological shock has on the market's workings is proposed and assessed. At each stage, we analyse the effect on social welfare of the different systems.

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1 Introduction

Our aim in this paper is to investigate a tradable–permits scheme designed by a regulator who has perfect foresight in terms of the participating firms' behavior, but who cannot anticipate the advent of new, less polluting production technologies. We present model for a permits market where the interaction of strategically acting firms determines prices. In a nutshell, firms are allocated a certain number of permits and a penalty for non–compliance is announced. Firms then decide how much to produce and, therefore, how much they pollute. These decisions determine how many permits firms have available for trading. The permit price is then the Cournot equilibrium of a game where firms in permits excess choose the amount of permits they submit to the exchange. We interpret this model as the long–term state of a system that has remained unchanged for a long time. On a second step, we assume a *technological shock* occurs, i.e. a cleaner production technology suddenly becomes available. We characterize the inter–dependence between the incentives to adopt a new technology and the permit price, one of the relevant driving factors of new technology adoption. Then, we investigate the technology adoption strategies and the permit–trading strategies of regulated firms. On a third step, we introduce a mechanism, which we dub *cash–for–permits* (C4P for short), that allows the regulator to respond to the effect, in terms of permits allocations, that the technological shock has on the market's workings. The introduction of the C4P may also foster the incentives to adopt new technologies. At each stage, we analyze the effect on social welfare of the different systems. To this end, a *damage function* is used to quantify the impact that emissions have on society.

It has been shown in the literature that tradable pollution permits may not be the most desirable policy instrument in a dynamic setting, even if there is complete information and the permit market is perfectly competitive – see Milliman and Prince (1989a), Malueg (1989), and Biglaiser et al. (1995). The reason being that the original policy levels may astray from optimality if new technologies are adopted. In principle, under a tradable–permits regime a regulatory agency chooses a fixed, total amount of permits so that the marginal private cost of pollution control is equal to the pollution's social damage. Suppose now a new, advanced technology becomes available. As investments in the new technology take place, the costs of pollution control fall, reducing the value of the permits (scarcity rents). The system deviates from what the regulatory agency initially deemed optimal. By adjusting the total number of permits, however, the marginal private cost of pollution control and the pollution's social damage may be equalized again. In reality, though, an amendment of a policy level is the exception rather than the rule. The arrival of a new technology is hardly predictable and, typically, once the regulatory agency implements its policy, levels remain constant for long periods of time. The European Union Emission Trading Scheme (EU ETS), EU's flagship market–based policy for capping carbon emissions, constitutes a blatant example of a firm (unresponsive) policy level. Currently, permit prices reflect an extremely low demand for permits in the market. There has been considerable attention on what needs to be done in order to support long–term investment incentives and facilitate the orderly functioning of the system. Several options have been listed.[1] To reduce the immediate surplus, the European Commission proposed to temporarily take allowances out of the trading system. This "backloading" is a temporary measure that may, politically, pave the way for structural measures that would tighten the demand–supply balance for emission allowances. However, there are strong reservations about changing the policy by tightening the cap, making the political discussion about amending the policy levels an extraordinary long and impervious legislative process.

In Malueg (1989), Milliman and Prince (1989a) and Jung et al. (1996), tradable–permits regimes analysis has been centered on the aggregate cost savings resulting from an industry–wide adoption of some new technology, where the authors assume that all the firms adopt the new technology, and then they compare aggregate costs before and after technology adoption. More recently, Requate and Unold (2003) and Requate (2005a) have shown that these aggregate cost savings differ substantially from a single firm's incentives to adopt a new technology. It is observed in these papers that decreasing permit prices (scarcity rents) provide incentives for some firms to free–ride on the other firms' technology investments by taking

[1] For an overview of the proposed market reform we refer to the report by the European Commission Commission (2012) and to the responses to the Commission consultation by Marcu (2013) and Taschini (2013).

advantage of decreasing compliance costs via permit purchase. Hence, in equilibrium both types of firms, using old and new technologies, are active in the market. Focusing on the firms' incentives to adopt a new technology, this recent literature attempts to determine how many firms will invest if new technologies becomes available and the policy levels remain constant for long periods of time. The number of firms investing in new technologies is determined endogenously in a framework where firms behave as price–takers on the permit market and are sufficiently small, relative to the whole market, that they cannot affect the market price for permits by adopting a new, less polluting technology. We modify such an assumption and investigate the incentives of strategically–interacting firms. Firms have two mutually non–exclusive compliance options: adoption of a new technology and exchange of permits. When firms cannot fully offset their emissions by trading permits, they face a fixed, per–unit penalty for excess emissions. We explicitly model the inter–dependence between technology adoption and the permit price. When the firms' choices can affect the market price, we argue that there is a natural trade–off between offering a higher number of cheap permits, or less of them, but at a higher value[2]. Consequently, we show that the aggregate supply is not necessarily equal to the number of unused permits. Instead, the aggregate permit supply is the solution of a non–cooperative trading (Nash) game.

While our primary motivation lies with the adoption of new technology and the effect of the latter on permit prices, our ideas carry over to any form of interplay between a pure quantity–based scheme and complementary policies that involves the regulation of the same underlying pollutant. Lately the European Commission has been grappling with the perverse consequences of the interaction between the EU ETS and EU energy policies, where the impact of energy efficiency and renewable energy incentives was to lower demand for permits with an obvious impact on permit prices. The interplay between ETS and (non–anticipated) complementary policies constitutes, therefore, another interesting application of the framework proposed in this paper. As mentioned above, when policy levels remain constant for long periods of time, an internal mechanism that responds to the adoption of new technologies could be necessary. One such promising mechanism (suggested by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995)) consists of offering guarantees of the future allowance price. This could be organized in the form of a regulator who stands ready to buy permits in the market at an announced price. We consider a similar mechanism: a contract that is written on the final holdings of permits and is contingent on the technology status. Hence, by adopting new technologies, the firms virtually create put–option like contracts, which we dub *Cash–for–permits* and denote by *C4P*. Once created, these contracts are assumed to be non–transferable. We show that this mechanism creates a floating price floor. Moreover, the Nash–games methodology that we use to model the generation of the allowance prices in the no–anticipation setting can be extended to this one. This allows us to show, theoretically and numerically, that the C4P generate a price floor that can be interpreted as a (floating) minimum price guarantee.

The remainder of the paper is organized as follows: Section 2 contains the benchmark model; the case of an unanticipated technological shock is presented in Section 3; the cash–for–permits are introduced in Section 4, which is followed by our conclusions and an appendix that contains all mathematical proofs.

2 The Benchmark Model

We work in a single–period setting, where we study the strategic interactions of a group of m firms[3]. In our emissions–constrained model economy, the firms operate under a tradable–emission permits scheme. The latter is designed and enforced by a central authority (the regulator), who is responsible for issuing permits and penalizing firms that emit more than the permitted amount of pollution, i.e. admissible emissions are capped. We denote by $\mathcal{I}$ the set $\{1, \dots, m\}$. A total of N permits is issued by the regulator, and they are allocated among the m firms via an auction or some other suitable mechanism. The authority may, for

[2]This bears some similarities to the problem of large portfolio liquidation

[3]We use a static stetting as a proxy for one where the policy has remained unchanged for a long time

instance, initially distribute permits for free (grandfathering) and then allow firms to trade them. N_i is the number of emission permits allocated to firm i.[4]

The i-th firm's profits from engaging in the pollutant–generating activity, excluding permit trading and payments of fines for noncompliance, are given by

$$F_i(y_i) := p_i f_i(y_i) - C_i(y_i).$$

Here y_i represents the input to production, $f_i(y_i)$ is the total output, p_i is the unitary price of firm i's output product and $C(y_i)$ corresponds to the total costs of production. We assume that $f_i(\cdot)$ is smooth, increasing and concave. Furthermore, we assume that $C_i(\cdot)$ is a smooth, increasing, strictly convex function, that $f_i(0) = C_i(0) = 0$ and that the mapping $y \mapsto p_i f_i(y) - C_i(y)$ is coercive. The firms are assumed to be exclusively price–takers in their output markets. In the absence of regulation, firms strive to maximize their profits:

$$\mathcal{P}_i := \max_{y \geq 0} F_i(y).$$

Under the assumptions made on $C_i(\cdot)$ and $f_i(\cdot)$, there is a unique $\overline{y}_i$ that solves $\mathcal{P}_i$. The interval $S_i := [0, \overline{y}_i]$ is the strategy set for firm i in the non–cooperative games we study below. The vector $(y_1, \ldots, y_m)$ will be referred to as the *inputs vector*. We assume, for simplicity, that emission levels are linear with respect to inputs. In other words, the regulated emissions corresponding to an input y_i are $\alpha_i y_i$. A lower α_i indicates a cleaner firm.

The regulatory agency uses an increasing, non–negative, convex *damage function* $D : \mathbb{R}_+ \to \mathbb{R}_+$ to quantify the damage that emissions bring upon society. The regulator's objective is to design a tradable permit system in such a way that the firms' equilibrium behavior yields a level of net benefits, production benefits minus by–product pollution damages, as close as possible to the first–best level

$$SW_{fb} := \max \Big\{ \sum_{i=1}^m F_i(y_i) - D\Big(\sum_{i=1}^m \alpha_i y_i \Big) \Big\}.$$

By construction, the map

$$(y_1, \ldots, y_m) \mapsto \sum_{i=1}^m F_i(y_i) - D\Big(\sum_{i=1}^m \alpha_i y_i \Big)$$

is strictly concave and coercive, which yields the following

Lemma 2.1 *There is a unique allocation of inputs* $\big(y_1^{fb}, \ldots, y_m^{fb}\big)$ *such that*

$$\sum_{i=1}^m F_i(y_i^{fb}) - D\Big(\sum_{i=1}^m \alpha_i y_i^{fb} \Big) = \max \Big\{ \sum_{i=1}^m F_i(y_i) - D\Big(\sum_{i=1}^m \alpha_i y_i \Big) \Big\}.$$

Let $S := \bigotimes_i S_i$; as a consequence or the fact that $D(\cdot)$ is increasing and strictly convex, we have that the allocation $\big(y_1^{fb}, \ldots, y_m^{fb}\big)$ belongs to the interior of S. In other words, the business–as–usual input levels are socially suboptimal, and there should be room for improvement via regulation.

In terms of noncompliance, whenever a firm's emissions exceed the number of permits it holds, it must pay a penalty. We consider a fixed, per–unit of (not offset) emissions penalty, which we denote by P.[5] While the regulator is assumed not to be constrained in the number of permits she issues, this is not the case for P. It would be, for instance, politically unviable to set $P = \infty$. This would provide the regulatory agency with full control via the allocation of permits, but it would also destroy the main feature of the

[4] Besides being transferable, permits are assumed to be homogeneous and infinitely divisible.

[5] Paying a fixed, per–unit of emissions penalty is an alternative to compliance and corresponds to the imposition of a price ceiling, as first discussed by Jacoby and Ellerman (2004).

system: introducing a mechanism via which the industry internalizes the cost of polluting. The income of firm i from a small input ϵ is $F_i(\epsilon)$, and the corresponding emissions are $\epsilon\alpha_i$. The latter, if uncovered, would trigger a penalty payment of $\epsilon P\alpha_i$. It should then be the case that $F_i(\epsilon) \geq \epsilon P\alpha_i$, which implies $F_i'(0) \geq P\alpha_i$. This condition must hold for all firms, therefore

$$P \leq \min_{i\in\mathcal{I}} \frac{F_i'(0)}{\alpha_i}.$$

Remark 2.2 *The analysis in Malik (1990) indicates that the principal effect of noncompliance in a tradable permits market is to alter the equilibrium permit price. Malik considers a large, competitive permit market where firms are price takers. However, he argues that the impact on the equilibrium permit price may have a significant effect on the other firms' emissions levels, particularly in non perfectly competitive markets –the matter under investigation in this paper. Endorsing this claim, in our model firms recognize their influence on permit price and behave accordingly.*

2.1 The firms' strategy sets

In a typical market for a generic good, sellers and buyers are naturally identified *ex ante*. In contrast, in a permits market the demand for and supply of permits originates from the same set of firms. The literature on market power in transferable permit systems typically describes the starting seller–buyer position with a simple dichotomy. When firms' permits endowments are above their efficient allocation, they become net sellers of permits in the market. On the other hand, when firms' permits endowments are below their efficient allocation, they become net buyers of permits. In tradable permit markets, however, whether regulated firms come forward as buyers or sellers should be endogenously determined. We present below a game that models such mechanism.

In the *outer loop* of the game, the firms' strategy sets are $\{[0,\overline{y}_i]\}_{i\in\mathcal{I}}$. Firms choose their input levels $\{y_i\}$, which determine the required amount of allowances. We denote the latter by $x_i := N_i - \alpha_i y_i$. The firms' decisions are based on a simple cost–opportunity analysis: foregone profits vs. compliance costs. Let

$$s := \Big\{ i \in \mathcal{I} \,\Big|\, x^i > 0 \Big\} \quad \text{and} \quad d := \Big\{ i \in \mathcal{I} \,\Big|\, x_i \leq 0 \Big\},$$

be the supply and demand sides of the market, respectively. Firms whose choice of y_i has positioned them in d prefer to produce more than what may be covered with the allowances that have been allocated to them. They anticipate that the additional profits from their production activities will outweigh the expenses made in permit purchases and penalty payments. Firms whose choice of y_i has positioned them in s have reached the polar conclusion. We shall show that the opportunity cost of being noncompliant depends on the penalty vs. output–market prices ratio.

The *inner loop* of the game is played exclusively among firms whose x_i's are positive. The corresponding strategy sets are $\{[0,x_i]\}_{i\in S}$. It is here where the price of the allowances is determined. We argue that firms in d are passive when it comes to price generation, the reason being the following: Firms in permit shortage face severe penalties if they fail to deliver an amount of allowances equal to their emissions. Hence, it is in the buyers' best interest to offset all their emissions for any price lower than the penalty level. The firms in permit shortage are, therefore, expected to submit all their demand to the exchange. On the contrary, sellers do not necessarily offer the entire bulk of their unused allowances. It may be in the self–interest of firms in allowance excess to limit their offers, keep the exchange value of permits high and, possibly, collect higher revenues.

2.2 The generation of permit prices (the game's inner loop)

In a typical market for a generic good, a market clearing price is reached when the quantity supplied equals the quantity demanded. Typically, it is assumed that the market clearing process occurs instantaneously.

When firms are not price takers, however, the market clearing process does not occur instantaneously: timing matters. The identification of a market clearing price, therefore, requires to specify a trading mechanism. Liski and Montero (2005, 2006) assume a Stackelberg timing, whereas we consider simultaneous trading.

2.2.1 Price generation

Permits are submitted to an exchange and traded simultaneously. The force driving the exchange value of an allowance is the supply and demand balance for permits. We postpone the presentation of how firm i strategically determines its net permits position x_i until Section 2.3; for the time being we take the x_i's as given and analyze the game's inner loop. The expressions

$$\mathcal{S} := \sum_{i \in s} x_i \quad \text{and} \quad \mathcal{D} := -\sum_{i \in d} x_i,$$

represent the number of unused permits and the number of non–offset emissions, respectively. The following ratio is used below to determine the allowances' value:

$$\mathcal{R} := \begin{cases} -\frac{\mathcal{S}}{\mathcal{D}}, & \text{if } \mathcal{D} > 0, \\ 0, & \text{otherwise.} \end{cases}$$

We denote by $\mathcal{S}^e$ the total number of unused permits submitted to the exchange and available for sale. As argued in Section 2.1, in general $\mathcal{S}^e < \mathcal{S}$. We study in Section 2.2.2 how $\mathcal{S}^e$ is determined. To account for a lower sensitivity of the allowance value in case of extreme permit demand, i.e. the supply–demand ratio is close to 0, or extreme permit supply, i.e. the supply–demand is close to 1, we define for $a > 0$ the parameterized family of (reaction) functions $\eta_a : [0, a] \to [0, 1]$ as

$$\eta_a(x) := \begin{cases} \exp\left\{\frac{x^2}{x^2 - a^2}\right\}, & \text{if } x \in [0, a), \\ 0, & \text{otherwise.} \end{cases}$$

The function η_a is the right half of a scaled mollifier (see Evans (1998)). It is infinitely smooth at 0 and a, with derivatives of all orders equal to zero at these points. The exchange value of an allowance is:

$$\Pi \quad := \quad P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}^e}{\mathcal{D}}\Big) = P \cdot \exp\Big\{\frac{(\mathcal{S}^e)^2}{(\mathcal{S}^e)^2 - \mathcal{D}^2}\Big\}.$$

Firms in permit excess must reach a compromise between offering a higher number of cheap permits, or less of them, but at a higher value.[6].

Remark 2.3 *Since we assume that the penalty is an alternative to compliance, this quantity represents an upper bound for the price of an allowance. In particular, one should observe that the indifference buy–price for an allowance that safeguards a firm that is in permits shortage from paying the penalty P is precisely P. Hence, by construction* $0 \le \Pi \le P$.

2.2.2 The sellers' payoffs and permit hoarding

Let $\{x_i\}_{i \in \mathcal{I}}$ be given. We denote by x_i^e firm i's submission to the exchange. Notice that $x_i^e = x_i$ if the latter is negative. In order to analyze how sellers choose their supply schedules, we consider the income generated from the permit exchange of firm $i \in s$ as a function of the *supply vector* $\big(x_i^e, x_{-i}^e\big)$:

$$\Psi^i\big(x_i^e, x_{-i}^e\big) := x_i^e P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}_{-i}^e + x_i^e}{\mathcal{D}}\Big), \quad \text{where} \quad \mathcal{S}_{-i}^e := \sum_{j \in s \setminus \{i\}} x_j^e. \tag{1}$$

[6]Notice that this trade–off follows partly from the non–linearity in prices introduced by the functions $\eta_a(x)$.

Equation (1) showcases the sellers' trade–off mentioned in Section 2.2.1. Below, we follow the well–worn trail of studying the best–response correspondences of each seller's response to the remaining sellers' submission of allowances to the exchange, and we show the existence of Nash equilibria of this strategic interaction. To this end we have the following

Lemma 2.4 *For any supply vector* $\left(x_i^e, x_{-i}^e\right)$*, the mapping* $\tilde{x}_i^e \mapsto \Psi^i\left(\tilde{x}_i^e, x_{-i}^e\right)$ *is maximized at a single point. In other words, the correspondence*

$$\Phi^i(x_i^e, x_{-i}^e) = argmax\Big\{\Psi^i\big(\tilde{x}_i^e, x_{-i}^e\big) \,\big|\, \tilde{x}_i^e \in [0, x_i]\Big\}$$

is single valued.

Notice that out of the three requirements to apply Kakutani's Fixed–point Theorem (see for example Meyerson (1991)), Lemma 2.4 takes care of the non–vacuity and the convexity. We then need an upper–semicontinuity result, which we present in the following.

Lemma 2.5 *Let* m_s *denote the number of firms in permits excess, and let the mapping* $\Phi : \mathbb{R}^{m_s} \to \mathbb{R}^{m_s}$ *be defined via*

$$\Phi(x_{i_1}, \ldots, x_{i_{m_s}}) := \bigotimes_{j=1}^{m_s} \Phi^{i_j}(x_{i_j}, x_{-i_j})$$

for $(x_{i_1}, \ldots, x_{i_{m_s}}) \in \bigotimes [0, x_{i_j}]$*, then* Φ *is continuous.*

Lemmas 2.4 and 2.5, together with Kakutani's Fixed–point Theorem imply that the mapping

$$(x_1, \ldots, x_{m_s}) \mapsto \Phi(x_1, \ldots, x_{m_s})$$

has a fixed point. In other words, we have proved the following

Theorem 2.6 *The (non–cooperative) game* $\mathcal{G} = \big\{[0, x_i], \Psi^i\big\}_{i \in s}$ *possesses a pure–strategy Nash Equilibrium.*

The Nash equilibrium mentioned in Theorem 2.6 is in fact unique. This follows from the fact that it coincides with the unique solution of the system of equations:

$$D_{z_{i_j}} \Psi(z_{i_j}, z_{-i_j}) + \lambda_{i_j} = 0, \quad \lambda_{i_j}(z_{i_j} - x_{i_j}) = 0, \quad \sum_{j=1}^{m_s} x_{i_j} \leq \mathcal{S}, \quad j = 1, \ldots, m_s,$$

where the λ_{i_j}'s are the Lagrange multipliers associated to the constraints $z_{i_j} - x_{i_j} \leq 0$. From now on $x_{i_j}^*$ represents the j–th entry of the Nash–equilibrium that results from the solution of the game among the firms in permit excess, and $\mathcal{S}^*$ is the corresponding sum. The *equilibrium exchange value* of an allowance is:

$$\Pi^* := P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}^*}{\mathcal{D}}\Big).$$

A further comment should be made regarding the generation of prices in the case where some of the constraints $(z_{i_j} - x_{i_j} \leq 0)$ prove to be binding. If $x_{i_j}^* = x_{i_j}$, then the following point is of course moot. Otherwise, if some firms are not able to increase their supply schedules up to the (unconstrained) equilibrium level, then the firms that still have availability of permits have extra room to increase their exchange offers. Interestingly, the additional potential number of permits does not restore the original aggregate supply of the unconstrained problem, i.e. the aggregate supply of permits (in equilibrium) in the presence of binding constraints is strictly below that of the unconstrained problem. When some firms' supply–constraints bind, therefore, the equilibrium price increases. Moreover, the firms with non–satiated constraints collect higher profits than in the unconstrained case by virtue of a lower (aggregate) supply. We formalize these claims in the following:

Lemma 2.7 *Let $\{x_i^* \leq x_i\}$ and Π^* be the equilibrium supply profile and the corresponding permit price of the game $\mathcal{G} = \big\{[0, x_i], \Psi^i\big\}_{i \in S}$, where the supply constraint $z_i - x_i \leq 0$ does not bind for any $i \in s$. Next assume at least one of the N_{i_j}'s is decreased as to make the corresponding supply constraint binding. Let $\tilde{\mathcal{G}} = \big\{[0, x_i], \Psi^i\big\}_{i \in s}$ denote the (now constrained) game, and let $\{\tilde{x}_i\}$ and $\tilde{\Pi}$ be the corresponding equilibrium supply profile and price. It then holds that:*

1. *If $x_i < x_i^*$, then $\tilde{x}_i = x_i$.*
2. *If $x_i > x_i^*$, then $\tilde{x}_i > x_i^*$.*
3. *$\tilde{\Pi} > \Pi^*$.*

2.2.3 The mechanics of order execution and the buyers' payoffs

In order to complete the description of the trading mechanism that clears the permit market, we must specify how buy– and sell–orders are matched: All orders are submitted to a centralized exchange market, in which they are randomly (and uniformly) matched one–by–one. All of the sellers' orders get executed. The probability that the order of firm $i \in d$ is matched is

$$-\frac{x_i}{\mathcal{D}}.$$

In other words, firm i's access to the sell–side of the market corresponds to its relative contribution to the aggregate demand schedule. In view of the latter, the executed orders of firm i are:

$$X_i := \frac{x_i}{\mathcal{D}} \cdot \mathcal{S}^e.$$

Let $\phi^i((y_i, y_{-i}))$ denote the payoff for firm i given the inputs vector y. If $i \in s$, then

$$\phi^i((y_i, y_{-i})) = F_i(y_i) + \Pi^* \cdot x_i^*.$$

On the other hand, if $i \in d$, then

$$X_i = \frac{x_i}{\mathcal{D}} \cdot \mathcal{S}^*,$$

and the firm's profits would be

$$\phi^i((y_i, y_{-i})) = F_i(y_i) - P \cdot x_i \frac{\mathcal{D} - \mathcal{S}^*}{\mathcal{D}} - \Pi^* \cdot X_i.$$

The quantity $x_i\big(\mathcal{D} - \mathcal{S}^*\big)/\mathcal{D}$ represents the number of emissions that are not offset by allowances, and for which the the prescribed penalty would be levied. We may rewrite

$$\phi^i((y_i, y_{-i})) = F_i(y_i) - x_i\Big[P - (P - \Pi^*)\frac{\mathcal{S}^*}{\mathcal{D}}\Big].$$

Remark 2.8 *When firms are price takers in the permit market and fully compliant, a tradable permit system is essentially equivalent to a tax (for buyers) and subsidy (for sellers) system. When firms are noncompliant, the quantity $\Big[P - (P - \Pi^*)(\mathcal{S}^*/\mathcal{D})\Big]$ could be thought of as a "rebate" price at which buyers, by virtue of their participation in the permit market, can achieve compliance.*

2.3 The firms' strategic interaction (the game's outer loop)

Below we study the non–cooperative game $\mathcal{G} = \left\{S_i, \phi^i\right\}_{i\in\mathcal{I}}$. We first analyze a two–player version, which can be extended with relative ease to the general case. For a given x_{-i}, we look at the continuity and quasiconcavity properties of the mapping $y \mapsto \phi^i((y, y_{-i}))$. We have two cases:

(i) **Case $\mathbf{x_{-i} > 0}$**. For $y \in [0, N_i/\alpha_i^k)$, we have $x_i > 0$. There is no trading, since both firms are in permits excess, hence

$$\phi^i((y, y_{-i})) = F_i(y).$$

Let us assume that $N_i/\alpha_i^k < \overline{y}_i$, then for $y \in (N_i/\alpha_i^k, \overline{y}_i]$, we have $x_i \leq 0$. For the purpose of price generation, once it has decided on x_i, firm i does not act strategically, i.e. $\mathcal{D} = x_i$. The FOCs for firm $-i$'s supply of permits are

$$\frac{d}{dx} x \cdot \exp\left(\frac{x^2}{x^2 - x_i^2}\right) = 0.$$

The above occurs if $x = \sqrt{2-\sqrt{3}}x_i$. As long as $x_{-i} > \sqrt{2-\sqrt{3}}x_i$, firm $-i$'s price–generating problem has an interior solution, with

$$\Pi^* = P \cdot \exp\left(\frac{\sqrt{2-\sqrt{3}}}{1-\sqrt{3}}\right) \quad \text{and} \quad \frac{\mathcal{S}^e}{\mathcal{D}} = \sqrt{2-\sqrt{3}}.$$

This leads to

$$\phi^i((y, y_{-i})) = F_i(y) + (\alpha_i^k - N_i)\Big[\big(P - P \cdot \exp\left(\frac{\sqrt{2-\sqrt{3}}}{1-\sqrt{3}}\right)\big)\sqrt{2-\sqrt{3}} - P\Big].$$

Since this map is the difference of a strictly concave one (F_i) and a linear one, it is itself strictly concave. In the case where firm $-i$ does not hold enough free permits to optimally respond to firm i's demand, i.e. when

$$y_i > \frac{(1-\sqrt{3})^{-1/2}x_{-i} + N_i}{\alpha_i^k} =: x_{-i}^s,$$

then $\mathcal{S}^* = x_{-i}$ (we have a corner solution to firm $-i$'s optimization problem). As a consequence

$$\phi^i((y, y_{-i})) = F_i(y) - Px_i + P\Big[\big(1 - \exp\left(\frac{x_{-i}^2}{x_{-i}^2 - x_i^2}\right)\big)\Big],$$

Ii is not hard to show that for $y > x_{-i}^s$ $\frac{d^2}{dy^2}\phi^i((y, y_{-i})) < 0$. In other words, this branch of ϕ^i is also strictly concave. Furthermore, we have that

$$\left.\frac{d}{dy}\phi^i((y, y_{-i}))\right|_{y \searrow x_{-i}^s} < \left.\frac{d}{dy}\phi^i((y, y_{-i}))\right|_{y \nearrow x_{-i}^s}.$$

From this expression we conclude that the mapping $y \mapsto \phi^i((y, y_{-i}))$ is strictly quasiconcave.

Remark 2.9 *There are two points of non–smoothness of the mapping $y \mapsto \phi^i((y, y_{-i}))$. One is the point $y = N_i/\alpha_i^k$, and the other one $y = x_{-i}^s$. At the former there is a downwards jump in marginal utility for firm i, where it looses strategic power. At the latter there is au upwards jump in marginal utility for firm i, because firm $-i$ looses its market power.*

(ii) **Case $\mathbf{x_{-i}} < \mathbf{0}$**. For $y \in (N_i/\alpha_i^k, \overline{y}_i)$, we have $x_i < 0$. There is no trading, since both firms are in permits shortage, hence

$$\phi^i((y, y_{-i})) = F_i(y).$$

For $y \in [0, N_i/\alpha_i^k)$, we have $x_i > 0$. Firm i then acts strategically in terms of price generation. Namely, for an input level y_i, the price is determined via

$$\max_{x \in [0, x_i]} P \cdot \exp\left(\frac{x^2}{x^2 - x_{-i}^2}\right).$$

Let us assume that for $y_i = 0$ the maximizer is interior. Then there exists $y_i^* < N_i/\alpha_i^k$ such that if $y_i < y_i^*$, the price is determined by the FOCs

$$\frac{d}{dx} \exp\left(\frac{x^2}{x^2 - x_{-i}^2}\right) = 0.$$

As before, the above results in $x = \sqrt{2 - \sqrt{3}}x_{-i}$. Hence, for $y \in [0, y_i^*]$ we have

$$\phi^i((y, y_{-i})) = F_i(y) + \sqrt{2 - \sqrt{3}}x_{-i}P \cdot \exp\left(\frac{\sqrt{2 - \sqrt{3}}}{\sqrt{1 - \sqrt{3}}}\right).$$

Notice that the second summand of the righthand side of the expression above is constant. As long as firm i is able to keep the optimal supply/demand ratio, its optimal strategy implies disposing of a certain number of unused permits. This is in contrast to the scenario where we introduce the price–support mechanism. When $y \in (y_i^*, N_i/\alpha_i^k]$, then the solution to firm i's price–generation problem is a corner one, and

$$\phi^i((y, y_{-i})) = F_i(y) + (N_i - \alpha_i^k y)P \cdot \exp\left(\frac{(N_i - \alpha_i^k y)^2}{(N_i - \alpha_i^k y)^2 - x_{-i}^2}\right).$$

We have that the mapping $y \mapsto \phi^i((y, y_{-i}))$ is smooth at y_i^*; however

$$\left.\frac{d}{dy}\phi^i((y, y_{-i}))\right|_{y \nearrow N_i/\alpha_i^k} > \left.\frac{d}{dy}\phi^i((y, y_{-i}))\right|_{y \searrow N_i/\alpha_i^k}.$$

In other words, there is a downwards jump in firm i's marginal utility when it switches from being a supplier to being a demander of permits.

We have obtained that all players' best–response functions are continuous and strictly quasiconcave. From arguments similar to those in Section 2.2.2, we conclude that the two–firms game has a (unique) Nash equilibrium. We analyze below the case $m = 3$, and we argue that the corresponding results are general. Let us analyze the reaction function of firm 1. If firm 2 and 3 are both in permits shortage (surplus), we are back to the two–players setting. Let us assume that $x_2 > 0$ and $x_3 < 0$.

(i) **Case $\mathbf{y_1} \in [\mathbf{0}, \mathbf{N_1}/\alpha_\mathbf{1}^\mathbf{k})$**. When firm 1's input decision leaves it in permits excess, the total demand for permits is $\mathcal{D} = x_3$. Let $\mathcal{S}^*$ be the corresponding optimal supply of permits (for the two firms). Further, let $\mathcal{S}_2^*$ be the number of permits that firm 2 would optimally supply to the exchange in the absence of firm 1. To present the most general scenario, let us assume that $N_1 + x_2 > \mathcal{S}^*$. In other words, it is possible for the two firms on the supply side of the market to match the optimal supply. There exists then $\tilde{y}_1 \in (0, N_1/\alpha_1^k)$ such that

$$\Pi^* = P \cdot \exp\left((\mathcal{S}^*)^2/((\mathcal{S}^*)^2 - \mathcal{D}^2)\right) \quad y_1 \in [0, \tilde{y}_1].$$

Therefore, on $[0, \tilde{y}_1]$

$$\phi^1((y_1, y_{-1})) = \phi^1(y_1) + x_1^e P \cdot \exp\Big(\frac{(\mathcal{S}^*)^2}{(\mathcal{S}^*)^2 - \mathcal{D}^2}\Big).$$

Notice that on this interval, ϕ^1 coincides with F_1 "bumped up" by the constant $x_1^e \Pi^*$. Next we look at ϕ^1 over $(\tilde{y}_1, N_1/\alpha_1^k)$. Over this interval firm 1 will fully submit x_1 to the exchange. We distinguish two subcases:

(a) **Subcase $\mathbf{x_2} > \mathcal{S}_\mathbf{2}^*$.** In this case, as soon as $y_1 > \tilde{y}_1$, the supply x_2^* of firm 2 decreases in such a way that $x_1 + x_2^* < \mathcal{S}^*$. This has the following interpretation: As the number of permits that firm 1 has available for submission dwindles, it looses market influence. Firm 2 benefits by gradually reducing its supply of permits. Interestingly, this results in an increase in the permit price. The submission level $x_2^*(x_1)$ can be characterized, for a given y_1 by the FOCs

$$\frac{d}{dx} x \cdot \exp\left(\frac{(x_1 + x)^2}{(x_1 + x)^2 - x_3^2}\right) = 0.$$

This leads to the fourth order polynomial equation

$$\Big((x_1 + x)^2 - x_3^2\Big) - 2x x_3^2 (x_1 + x) = 0.$$

The closed–form expression for $x_2^*(x_1)$, which is rather unpalatable, leads to

$$\phi^1((y_1, y_{-1})) = \phi^1(y_1) + x_1 P \cdot \exp\Big(\frac{(x_1 + x_2^*(x_1^e))^2}{(x_1 + x_2^*(x_1^e))^2 - \mathcal{D}^2}\Big).$$

We provide a plot of ϕ^1 in Figure **??**. The mapping $y_1 \mapsto \phi^1(y_1, y_{-1})$ is smooth at $\tilde{y}_1$.

(b) **Subcase $\mathbf{x_2} < \mathcal{S}_\mathbf{2}^*$.** In this case there exists $\breve{y}_1 > \tilde{y}_1$. such that for $y_1 > \breve{y}_1$, firm 2 submits all of x_2 to the exchange. This results in

$$\phi^1((y_1, y_{-1})) = \phi^1(y_1) + x_1 P \cdot \exp\Big(\frac{(x_1 + x_2)^2}{(x_1 + x_2)^2 - \mathcal{D}^2}\Big).$$

(ii) **Case $\mathbf{y_1} \in (\mathbf{N_1}/\alpha_\mathbf{1}^\mathbf{k}, \bar{\mathbf{y}}_\mathbf{1})$.** When firm 1's input decision leaves it in permits shortage, the total demand for permits is $\mathcal{D} = x_1 + x_3$. Since firms 1 and 3 do not (directly) participate in the strategic generation of prices, this case mimics the one–firm–in–excess, one–firm–in–shortage case in the two–firms scenarios:

(a) If $x_2 \geq \sqrt{2 - \sqrt{3}}(x_1 + x_3)$, then x_2^* is precisely this quantity, and

$$\phi^i((y_1, y_{-1})) = F_1(y) + (\alpha_1^k - N_1)\Big[\big(P - P \cdot \exp\left(\frac{\sqrt{2 - \sqrt{3}}}{\sqrt{1 - \sqrt{3}}}\right)\big)\sqrt{2 - \sqrt{3}} - P\Big].$$

(b) If $x_2 < \sqrt{2 - \sqrt{3}}(x_1 + x_3)$, then $x_2 = x_2$, and

$$\phi^i((y_1, y_{-1})) = F_1(y) + x_1\Big[\big(P - \Pi^*\big)\frac{x_2}{x_1 + x_3} - P\Big],$$

where

$$\Pi^* = P \cdot \exp\left(\frac{x_2^2}{x_2^2 - (x_1 + x_3)^2}\right).$$

We may conclude that the best–response functions in the three–players game are continuous and strictly quasiconcave. The n–player case follows analogously, with the set of firms in permits excess playing the role of firm 2, and those in permits shortage playing the role of firm 3. Clearly, there could be a longer sequence of "exits" points, where firms that are barely in permits excess simply submit all of their available permits to the exchange. At these points the curvature of ϕ^i changes, as it did in $\breve{y}_1$ and $\tilde{y}_1$. The properties of ϕ^i, however, remain unaltered, and we have the following

Theorem 2.10 *The (non–cooperative) game $\mathcal{G}^* = \left\{[0, x_i], \phi^i\right\}_{i \in \mathcal{I}}$ possesses a pure–strategy Nash Equilibrium.*

We now have a precise description of how the strategic interaction among firms occurs.

2.4 The impact on social welfare

3 No–anticipative technology adoption

In a tradable permit scheme where firms can adopt a new technology, the price of permits depends on the number of firms that adopt new technologies. At the same time, the number of firms that decide to adopt new technologies depends on the (rational) expectations of the permit price. Most models assume that firms do not view their decisions on technology adoption as affecting the price of permits (see Requate and Unold (2003) and references therein). In contrast, we analyze below a model where the inter–dependence between technology adoption and permit value is explicitly modeled. We describe in this section the impact of an unanticipated emergence of a cleaner production technology in the workings of the permits market, as well as on social welfare.

3.1 The technological shock

We model the technological shock to the firms' productive technologies via an enlargement of their strategy sets and a modification to their payoff functions. We assume that the newly available technology can be gradually and continuously adopted, with the variable $a_i \in [0, 1]$ describing the range from no adoption to total adoption, which we call firm i's *curbing input*. We refer to $a = (a_1, \ldots, a_m)$ as the curbing vector. This fits technologies such as scrubbers or division of energy generation into different fuels better than, for example, a large investment on new machinery. We have made this modeling choice for mathematical simplicity.

Let $r_i : [0, 1] \to (0, 1]$ be a smooth, increasing function such that $r_i(0) > 0$ and $r_i(1) = 1$. A choice of a_i reduces the firm's emissions profile from α_i to $\alpha_i r(a_i)$. The strictly convex, increasing and non–negative function $K_i : [0, 1] \to \hat{R}_+$ determines the cost in which firm i incurs by choosing a_i, namely $K_i(a_i)$. As before, we assume that $K_i(0) = 0$. Firms must now choose not only their production inputs, but also their curbing ones. We note by $\tilde{F}_i : [0, \overline{y}_1] \times [0, 1] \to \hat{R}$ the corresponding profit functions net of permits–trading profits or penalty payments:

$$\tilde{F}_i(y_i, a_i) := p_i f_i(y_i) - C_i(y_i) - K_i(a_i).$$

Notice that we have assumed the firms' productivity is unaffected by the choice of curbing input. In the absence of a permits market, no firm would choose $a_i > 0$. Since a positive choice of a_i reduces the firm's emissions, however, the potential increase in permits–market profits or reduction in penalty payments may imply that in a regulated environment choosing $a_i = 0$ may be suboptimal.

3.2 The impact of the firms' enlarged strategy space on trading

The firms' choices of curbing inputs has no impact on the mechanics of the generation of permit prices, i.e. on the game's inner loop described in Section 2.2. This follows from the fact that in say section we took the firms' positions in permits as given, and then we proceeded to compute the equilibrium price. On the other hand, the firms' positions in permits $\tilde{x}_i := N_i - \alpha_i r(a_i) y_i$, are impacted by the curbing choices. Therefore, the way in which firms play the game's inner loop changes with the enlarged strategy space. Indeed, let $\tilde{\phi}^i\big((y_i, a_i); (y_{-i}, a_{-i})\big)$ denote the payoff for firm i given the inputs and curbing vectors y and a, respectively. If $i \in s$, then

$$\tilde{\phi}^i\big((y_i, a_i); (y_{-i}, a_{-i})\big) = \tilde{F}_i(y_i, a_i) + \Pi^* \cdot x_i^*.$$

The choice of a_i enters $\tilde{\phi}^i$ not only through $\tilde{F}_i$, but also through x_i^*, since firm i is less constrained in its permit position, hence in how many permits it can submit for trade, the larger a_i is. When it comes to the buy–side of the market, if we keep the order–matching mechanism of Section 2.2.3 and if $i \in d$, then

$$\tilde{X}_i = \frac{\tilde{x}_i}{\mathcal{D}} \cdot \mathcal{S}^*,$$

and the firm's profits would be

$$\tilde{\phi}^i\big((y_i, a_i); (y_{-i}, a_{-i})\big) = \tilde{F}_i(y_i, a_i) - P \cdot \tilde{x}_i \frac{\mathcal{D} - \mathcal{S}^*}{\mathcal{D}} - \Pi^* \cdot \tilde{X}_i.$$

The quantity $\tilde{x}_i\big(\mathcal{D} - \mathcal{S}^*\big)/\mathcal{D}$ represents the number of emissions that are not offset by allowances, and for which the the prescribed penalty would be levied. We may rewrite

$$\phi^i((y_i, y_{-i})) = F_i(y_i) - \tilde{x}_i\Big[P - (P - \Pi^*)\frac{\mathcal{S}^*}{\mathcal{D}}\Big].$$

Again, we highlight the fact that the curbing vector impacts $\mathcal{S}^*$ and $\mathcal{D}$, and therefore Π^*

3.3 The impact on social welfare

4 No–anticipative technology adoption with a price support contract

As investments in new technologies take place, the costs of pollution control fall. Under no–anticipation of new technology adoption, the allocation schedule remains unchanged, depreciating the permits value. The system deviates from what the regulatory agency initially deemed optimal. It can be shown that, by adjusting the total number of permits, optimality can be reached *ex-post*. However, an amendment of a policy level is a rather exceptional event. An alternative would be for the regulator to stand ready to buy back permits as needed, at an announced price. Arbitrage would ensure that the exchange value of allowances would stay above the announced price. Among others, this contract has been suggested by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995). We consider a similar mechanism and introduce a free–of–charge contract that is written on the final holdings of permits and is contingent on the technology status. We call this contract *cash–for–permits* (C4P for shorthand). We assume that adoption of new technologies is perfectly verifiable by the regulator, thus ruling out moral hazard. All firms that have adopted a new technology have access to C4P. In fact, their investment entitles these firms to as many C4P (put options) as allowances they hold. These options, however, are non–transferable. When firms decide to exercise their options, the regulator buys back the permits at the pre–announced price, P_g.[7] One may think of the C4P (strike) price P_g as a minimum price guarantee contingent on the technology status. Below we show that the C4Ps create a (floating) price floor for the exchanged permits.

[7]It should be noted that under this contract, the outstanding number of permits is modified via actions of the firms, and not due to direct intervention of the regulator.

4.1 The impact of the C4P on the aggregate supply and the permit price

The introduction of the C4P has an impact on the number of the permits that firms in permit excess submit to the exchange and, therefore, affects the allowance exchange value. We maintain the notation $x_i(h)$, $\mathcal{D}(h)$ and $\mathcal{S}(h)$ used in Section 3. If firm i has adopted a low pollution–emitting technology and it is in permit excess, then the quantity $x_i(h)$ can be divided into $x_i^e(h)$ and $x_i^c(h)$. The former indicates the number of permits submitted to the exchange, and the latter those that are "cashed–for–permits".

In parallel to Section 3.2, we must now study the generation of allowance prices considering how firms in permit excess balance their positions in C4P and market–exchanged permits. Assume that firm i is in permit excess and that it operates under new technology. Given that the other firms in excess have submitted $\mathcal{S}_{-i}^e = \sum_{j\neq i} x_j^e$ permits into the exchange market, firm i's choice of x_i^e yields a profit equal to:

$$\Psi_i^4\big(x_i^e, x_{-i}^e\big) := P_g(x_i - x_i^e) + x_i^e P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}_{-i}^e + x_i^e}{\mathcal{D}}\Big). \tag{2}$$

Notice that the mapping $x \mapsto P_g(x_i - x)$ has constant slope $-P_g$. A relatively high P_g could then result in $x_i^e \equiv 0$ being the optimal exchange strategy for all firms that are in permit excess, and which operate under the new technology. Obviously, the condition $P_g < P$ should hold. This condition is sufficient to guarantee that markets will not shut down: it will not be optimal for all the firms to submit zero–supply schedules and exercise their C4P. The latter claim follows from the fact that

$$\frac{d}{dx}\,\Psi_i^4\big(x,0\big)\Big|_{\{x=0\}} = P - P_g.$$

If all other firms were to exercise their C4P, the marginal utility of firm i at zero would be increasing in its submissions to the exchange, hence it would find it suboptimal to abstain from trading permits.

For the ease of exposure, let us now split the sets $s(t+1,h)$ into $s^o(t+1,h)$ and $s^n(t+1,h)$. These sets represent, respectively, the firms that are in permit excess at the end of the period $[t,t+1]$ and that operate under the old technology throughout the period, and those that are in permit excess, but which have already adopted a new technology. Firms that belong to $s^o(t+1,h)$ simply operate as before; however, those in $s^n(t+1,h)$ will not submit permits into the exchange unless they can make at least P_g per unit of allowances traded. Again, we assume without loss of generality that total demand equals one, so that the payoff of a firm in $s^n(t+1,h)$, which submits x_i^e to the exchange, given that the other firms in permit excess have submitted K_1 is:

$$\Psi_i^4\big(x_i^e, x_{-i}^e\big) = P_g(x_i - x_i^e) + x_i^e P \cdot \exp\left\{\frac{(K_1 + x_i^e)^2}{(K_1 + x_i^e)^2 - 1}\right\}.$$

Similarly to Lemma 2.4, we have the following

Lemma 4.1 *For $i \in s^n(h)$, and any supply vector x_e^{-i}, the mapping $x \mapsto \Psi_i^4\big(x, x_{-i}^e\big)$ is maximized at a single point of $(0, x_i]$.*

With Lemma 4.1 at hand, the analysis of the existence of equilibria of the game $\mathcal{G} = \big\{[0, x_i(h)],\ \Psi_i^4\big\}_{i\in s}$ is analogous to that of Section 3.2, and the corresponding equilibrium will be denoted by $\{x_i^4\}_{i\in s}$. We may then conclude the existence of a (unique) equilibrium price $(\Pi^4)^*(t+1,h)$, which in turn keeps our description of the mechanics of trading mostly unchanged. The notable difference being that a firm in permit excess, operating under the new technology, has a profit:

$$\phi_i^4(h) = (\Pi^4)^*(h) \cdot |x_i^4(h)| + P_g|x_i(h) - x_i^4(h)| + R_i.$$

Furthermore, we have the following

Proposition 4.2 *Under identical primitives and identical pairs* $\big(\{N_i\}_{i\in\mathcal{I}}, P\big)$, *it holds that*

$$(\Pi^4)^*(h) \geq \Pi^*(h).$$

Remark 4.3 *By construction, if* $i \in s(h)$, *for any* h *we have that*

$$\phi_i^4(h) \geq \phi_i(h).$$

The equality corresponds to the cases of boundary solutions or firms in permit excess that operate under the old technologies. From Proposition 4.2, we also get that if $i \in d(h)$ *then*

$$\phi_i^4(h) \leq \phi^i(h).$$

We have seen that the Nash–games that determine the allowance prices and govern the evolution of the technological vector are still well defined in the C4P–setting. It should be kept in mind that the payoff functions Ψ_4^i depend on the technology vector h. Furthermore, Proposition 4.2 indicates that it is in the interest of those firms operating under the new technology to reduce permits availability and increase their scarcity rents. Such a strategy's impact on prices increases the compliance and non–compliance costs of the firms in permit shortage. This in turn affects the evolution of the technological vector.

4.2 Social welfare under C4P

5 Conclusions

Under a transferable permit scheme, the value of the allowances (scarcity rents) determines the incentives of regulated companies to invest in new, low pollution–emitting technologies. Permit schemes have traditionally been analyzed by focusing on the aggregate cost savings resulting from an industry–wide adoption of some new technology. In such a setting, most authors make the assumption that all firms adopt the new technology and then compare aggregate costs before and after adoption (Malueg (1989), Milliman and Prince (1989b), and Jung et al. (1996)). It has recently been shown that these aggregate cost savings differ substantially from a single firm's incentives to adopt a new technology (Unold and Requate (2001) and Requate (2005b)). These recent contributions highlight the fact that the possibility that a firm may free–ride on a decreasing permit price caused by other firms' investment in new technologies had previously been ignored. This firm may find it optimal not to adopt a new technology. Hence, both type of firms (those using old and new technology) coexist. As such, the number of firms adopting new technologies should be determined endogenously. By assuming perfect competition in the permit market, Kennedy and Laplante (1999) and Requate and Unold (2003) endogenize the number of firms that, in equilibrium, invest in new technologies. Here the price of permits depends on the number of firms that have adopted the new technology. The firms, however, fail to internalize the externalities they impose on others via their investment decisions an their corresponding impact on the market price of permits. We have relaxed such an assumption and investigated technology adoption behaviour under imperfect competition in the permit market. The inter–dependence between technology adoption and the permit price has been explicitly modelled in our framework: the number of firms that decide to adopt a new technology depends on their (rational) expectations of the future permit price; the future price of permits hinges on the number of firms that adopt a new technology. In particular, technology adoption is contingent on the future permits' scarcity and is ultimately reflected in the allowances price. Both aggregate demand and the aggregate supply are the driving forces of the permit price. The latter being a function of uncovered pollution emissions (permits demand), the level of unused emission permits (permits supply), and the current technological status. We have argued that under imperfect permit market competition, the aggregate permits supply does not necessarily equal to the number of unused permits, and based on the latter we have modelled the aggregate permit supply as the (unique, pure–strategies) equilibrium of a non–cooperative Nash game.

Under no–anticipation of new technology adoption, the system might deviate, as investments in new technology take place, from what the regulatory agency initially deemed optimal. By adjusting the total number of permits, optimality could be recovered ex–post. Policy levels, however, tend to remain constant for long periods of time. This is a typical scenario in real politics. Following the suggestions by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995), we have introduced a price–support contract: The Cash–for–permits (C4P). This instrument provides an alternative to permits adjustment and may be used to foster the firms' long–term incentives to adopt new technologies. We have modified the Nash–game that served to set the expected allowance prices and governed the evolution of the technological vector in the no–anticipative setting, which yielded a well–defined price–generation methodology. This has allowed us to show, theoretically and numerically, that the price support contact creates a price floor that can be interpreted as a (floating) minimum price guarantee. We have numerically examined the impact of the minimum price guarantee on prices, as well as on the number of firms that adopt low pollution–emitting technologies. The timing of such adoptions was also part of our analysis. We have found that the higher the minimum price guarantee, the higher the aggregate level of firms adoption and, quite interestingly, the earlier the adoption of the new technology.

A Appendix

Proof of Lemma 2.4 We assume that $\mathcal{D} > 0$, otherwise there is no demand for allowances, hence no market, and the maximizer is trivial. We must show that the mapping

$$x \mapsto x \exp\left\{ \Big(\frac{K_1 + x}{K_2}\Big)^2 \Big/ \Big(\Big(\frac{K_1 + x}{K_2}\Big)^2 - b \Big) \right\},$$

where $b = \left(\frac{K_1 + x^i}{K_2}\right)^2$, $K_1 = \mathcal{S}_e^{-i}$ and $K_2 = \mathcal{D}$ is maximized at a single point of $[0, x^i]$. By rescaling if necessary, we may assume without loss of generality that $K_2 = 1$. Moreover, we may assume that $K_1 + x_i \leq 1$, given that under the previous assumption $\eta_{\mathcal{R}} \equiv 0$ for any value larger than 1. Initially we assume that $K_1 + x_i = 1$, i.e. firm i has the ability, given K_1, to fully satisfy the demand for allowances. Since in such a case the values of the mapping under investigation are strictly positive on $(0, x^i)$, we need only to seek interior maximizers. The first order conditions yield the equation

$$L(x) := (K_1 + x)^4 - 4(K_1 + x)^2 + 2K_1(K_1 + x) + 1 = 0.$$

We have that $L(0) = (K_1 - 1)^2 > 0$, and $L(x^i) = -2 + 2K_1 < 0$. It follows from the Intermediate Value Theorem that L has a root x_{i_0} in $(0, x^i)$. To show uniqueness, we note that L'' changes sign only once on $[0, \infty)$, which given the general shape of the graph of a fourth–degree polynomial implies there are only two roots in this interval. Since $L(x^i) < 0$ and $\lim_{t \to \infty} L(t) = \infty$, we conclude that the remaining root lies beyond $x = x_i$. If it were the case that $K_1 + x_i < 1$, then either $x_{i_0} \leq K_1 + x_i$, in which case the previous result holds, or the maximizer is precisely x_i. $\square$

Proof of Lemma 2.5 For $x \in [0, x^i]$, the mapping

$$K_1 \mapsto x \exp\left\{ \Big(\frac{K_1 + x}{K_2}\Big)^2 \Big/ \Big(\Big(\frac{K_1 + x}{K_2}\Big)^2 - b \Big) \right\}$$

is continuous. Notice that x_{-i_j} is a relevant statistic for $\Phi_i(x_{i_j}, x_{-i_j})$ only through $\sum_{k \neq j} x_{i_k}$, and clearly the mapping

$$(x_{i_1}, \ldots, x_{i_{m_s}}) \mapsto \bigotimes \sum_{k \neq j} x_{i_k}$$

is continuous. It follows immediately that the mapping $(x_{i_j}, x_{-i_j}) \mapsto \Phi_i(x_{i_j}, x_{-i_j})$ is continuous over $\bigotimes [0, x_{i_j}]$, which finalizes the proof. $\square$

Proof of Lemma 2.7 The first point follows from the fact that the best–response path of a firm whose supply–constraint is binding reaches and is absorbed by the corresponding $x^i(h)$. Next we assume $\#s = 2$ for notational simplicity. Since for a fixed x_1, the expression

$$1 - \frac{2x_1(x_1 + x_2)}{((x_1 + x_2)^2 - 1)^2},$$

which corresponds to the first order conditions of firm 1, is decreasing in x_1, if $x_{1_h} < x_1^*(t+1,h)$, then $\tilde{x}_2(t+1,h) > x_2^*(t+1,h)$. In what follows we drop the arguments $(t+1,h)$ for clarity. The question remains whether or not the increased supply by firm 2 over the unconstrained–equilibrium level fully compensates the decreased supply of firm 1, as to leave aggregate supply unchanged. The answer is no. If firm 2 were to offer $x_1^* - \tilde{x}_1 + x_2^*$, we would have

$$1 - \frac{2\big(x_1^* - \tilde{x}_1 + x_2^*\big)\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} = 1 - \frac{2\,x_2^*\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} - \frac{\big(x_1^* - \tilde{x}_1\big)\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} < 0.$$

The inequality follows from the fact that the first two terms on its left hand side add up to zero (the first order condition for the unconstrained equilibrium) and $x_1^* - \tilde{x}_1 > 0$. We conclude that $\tilde{x}_1 + \tilde{x}_2 < x_1^* + x_2^*$, which in turn implies $\tilde{\Pi}(t+1,h) > \Pi^*(t+1,h)$. □

Proof of Lemma 4.1 As in the proof of Lemma 2.4, we first assume that $K_1 + x_i = 1$. Let

$$f(x) := P_g(x_i - x) + xP \cdot \exp\left\{\frac{(K_1 + x)^2}{(K_1 + x)^2 - 1}\right\}$$

and

$$g(x) := f'(0)x + P_g x_i.$$

The graph of the function g in simply the tangent to the graph of f at $x = 0$. We observe that for $x \in (0, x_i)$,

$$\begin{aligned} g(x) - f(x) &= (f'(0) + P_g)x - xP \cdot \exp\left\{\frac{(K_1 + x)^2}{(K_1 + x)^2 - 1}\right\} \\ &= xP \cdot \exp\left\{\frac{(K_1)^2}{(K_1)^2 - 1}\right\} - xP \cdot \exp\left\{\frac{(K_1 + x)^2}{(K_1 + x)^2 - 1}\right\} \\ &> 0. \end{aligned}$$

In other words, the graph of f is strictly under the graph of its tangent at $x = 0$. We also have that $f'(0) = -P_g + P \cdot \exp\left\{\frac{(K_1)^2}{(K_1)^2 - 1}\right\}$. If this quantity were to be non–positive, then f would be maximized at $x = 0$. On the other hand, if $f'(0) > 0$ and $f'(x_i) < 0$, then there is $x_{i_0} \in (0, x_i)$ such that $f'(x_{i_0}) = 0$. Moreover, $x \mapsto xP \cdot \exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}$ is a quasiconcave (thus single–cusped) mapping, hence so is $x \mapsto f(x)$. We may then conclude that x_{i_0} is unique. The case where $K_1 + x_i < 1$ follows in a similar fashion, but $f(x_i) > 0$. Nevertheless, f remains quasiconcave, hence maximized at a single point.

□

Proof of Proposition 4.2 It suffices to show that under any circumstance, the best response of a firm that is in permit excess is lower or equal (in terms of units or permits submitted to the exchange) with C4Ps than it would be without C4Ps. Trivially firms that find themselves in permit excess, but which have not change technology, will have the same best responses as before to a given submission level K_1. We assume that the best responses of firm i to K_1 are interior (i.e. they belong to $(0, x_i)$), since otherwise we find boundary solutions where x_i is submitted. Below we write the first order conditions for the interior solutions. The case of no C4P corresponds to the solution of the equation

$$\exp\left\{\frac{(K_1 + x)^2}{(K_1 + x)^2 - 1}\right\}\left(1 - \frac{2x(K_1 + x)}{((K_1 + x)^2 - 1)^2}\right) = 0, \tag{3}$$

whereas in the presence of EC4P we must find the root of

$$\exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}\left(1-\frac{2x(K_1+x)}{((K_1+x)^2-1)^2}\right)=\frac{P_g}{P}. \tag{4}$$

Since the mapping $x \mapsto -\frac{2x(K_1+x)}{(K_1+x)^2-1)^2}$ is decreasing, the root of Equation (4) on $(0, x^i)$ is smaller than that of Equation (3). By virtue of Lemma 2.7, we know that any additional permits submitted by firms who are not eligible to cash–4–permits will not restore the total supply to its pre–C4P levels, which concludes the proof.

□

References

Biglaiser, G., Horowitz, J. K., and Quiggin, J. (1995). Dynamic Pollution Regulation. *Journal of Regulatory Economics*, 8:33–44.

Commission, E. (2012). The state of the European carbon market in 2012. Technical report, European Commission, Brussels, Belgium.

Evans, L. C. (1998). *Partial Differential Equations.* American Mathematical Society, AMS - United States of America.

Jacoby, H. D. and Ellerman, A. D. (2004). The safety valve and climate policy. *Energy Policy*, 32(4):481–491.

Jung, C., Krutilla, K., and Boyd, R. (1996). Incentive for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives. *Journal of Environmental Economics and Management*, 30(1):95–111.

Kennedy, P. and Laplante, B. (1999). Environmental Policy and Time Consistency: Emissions Taxes and Emissions Trading. In E.Petrakis, Sartzetakis, E., and Xepapadeas, A., editors, *Environmental Regulation and Market Power*. Edward Elgar Publisher, Cheltenham, U.K.

Laffont, J. J. and Tirole, J. (1996). A Note on Environmental Innovation. *Journal of Public Economics*, 62:127–140.

Liski, M. and Montero, J. P. (2005). A note on market power in an emissions permit market with banking. *Environmental and Resource Economics*, 31:159–173.

Liski, M. and Montero, J. P. (2006). On pollution permit banking and market power. *Journal of Regulatory Economics*, 29:282–302.

Malik, A. S. (1990). Markets for Pollution Control when Firms are Noncompliant. *Journal of Environmental Economics and Management*, 18:97–106.

Malueg, D. (1989). "emission credit trading and the incentive to adopt new pollution abatement technology". *Journal of Environmental Economics and Management*, 18:52–57.

Marcu, A. (2013). Consultation on structural options to strengthen the EU Emissions Trading System . Technical report, Centre for European Policy Studies, Brussels, Belgium.

Meyerson (1991). *Game Theory: Analysis of Conflict.* Harvard University Press, United States of America.

Milliman, S. and Prince, R. (1989a). Firm incentives to promote technological change in pollution control. *Journal of Environmental Economics and Management*, 17(3):247–265.

Milliman, S. R. and Prince, R. (1989b). Firm incentives to promote technological change in pollution control. *Journal of Environmental Economics and Management*, 17:247–265.

Requate, T. (2005a). Dynamic incentives by environmental policy instruments - a survey. *Ecological Economics*, 54:175–195.

Requate, T. (2005b). Timing and Commitment of Environmental Policy, Adoption of New Technology, and Repercussions on R&D. *Environmental and Resource Economics*, 31(2).

Requate, T. and Unold, W. (2003). Environmental Policy Incentives to Adopt Advanced Abatement Technology - will the True Ranking Please stand up? *European Economics Review*, 47(1):125–146.

Roberts, M. and Spence, M. (1976). Effluent charges and licenses under uncertainty. *Journal of Public Economic*, 5(3):193–208.

Taschini, L. (2013). Options for structural measures to improve the European Union Emissions Trading System: response to a European Commission consultatio. Technical report, Grantham Research Institute on Climate Change and the Environment, London, UK.

Unold, W. and Requate, T. (2001). Pollution control by options trading. *Economics Letters*, 73:353–358.