Business Cycles and Environmental Policy; Give the Dirty Firms a Break

Inge van den Bijgaart*, Sjak Smulders†

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Abstract

This paper determines optimal environmental policy responses to real and financial shocks in a two-period partial equilibrium model with heterogeneous firms, an environmental externality and credit constraints. We show that, to alleviate credit constraints and encourage investment, the optimal emission tax falls short of marginal emission damages. The optimal response to shocks depends on how the shock affects the size of the environmental and credit market failures and the effectiveness of the tax in alleviating these market failures. For a conventional functional form, the optimal response to a (persistent) negative productivity shock or a tightening of credit is to reduce the emission tax. Our results are informative for how climate change policy should optimally change with the business cycle.

JEL classifications: E32, H23, Q54, Q58,

Keywords: business cycles; credit constraints; optimal emission tax

[presenter (Ph.D. student) would like to be considered for the young economist sessions]

1 Introduction

On the 10th of December, 2007, the IPCC and Al Gore received the Nobel Peace Prize for "their efforts to build up and disseminate greater knowledge about man-made climate change, and to lay the foundations for the measures that are needed to counteract such change" Nobelprize.org (2007). That same year, the IPCC's Fourth Assessment Report was released, expressing grave concern regarding climate change and the speed at which it is occurring. With the Fifth Assessment Report underway in 2014, and GHG emissions from especially the developing world rising rapidly, the call for policy action has become even stronger since. Nine months after the 2007 Nobel Award Ceremony, Lehman Brothers filed for bankruptcy, triggering the largest global financial crisis since the great Depression. US GDP fell by more than 3% in 2009, and by 2010, unemployment had reached almost 10%. Early 2014, while many European economies are still struggling, the US economy is recovering, albeit slowly.

*Corresponding author. Tilburg University Department of Economics, CentER, and TSC. PO Box 90153, 5000 LE Tilburg, The Netherlands; phone: +31 13 466 8785; email: I.M.vdnBijgaart@uvt.nl

†Tilburg University Department of Economics, CentER, and TSC.

The economic downturn and call for climate action inspired a large policy debate regarding the desirability of implementing or strengthening environmental policies in a recession. Some see the economic downturn as an opportunity for climate actions. Inspired by Roosevelt's New Deal, proposals were put forward for a so-called 'Green New Deal'. These proposals consisted of fiscal policy actions benefiting both the economy and the environment. Examples of such actions are the weatherization of homes, investments in green R&D and tax credits for hybrid vehicles. Such a deal would take advantage of the low opportunity cost and high benefits of expansionary fiscal policy in the downturn, simultaneously benefiting the environment in the long run (UNEP, 2009; Houser et al., 2009; Bowen and Stern, 2010). The global recession also proved a threat to the implementation of environmental policy measures. In 2009, the introduction of an Australian carbon trading scheme was delayed due to concerns that it would undermine the economy's recovery (Guardian, 2009). Similar concerns were raised in California, where in 2010 a proposal was put forward to suspend its climate bill until unemployment fell below 5.5% (Wall Street Journal, 2010). The proposal was not approved, but such examples are not stand-alone: Jacobsen (2013) found a significant negative correlation between unemployment rates and US Senate support for environmentally favorable policies.

This paper contributes to this debate by formally evaluating the optimal response of environmental policy to a recession. For this purpose, we construct a model in which emissions are a harmful byproduct of output and firms can reduce their emissions by investing in emission-saving technologies, but may be constrained in their investment choice because of the lack of credit. We focus on a specific type of policy, an emission tax, and find that, to alleviate constraints, it is always optimal for the social planner to set this tax below marginal environmental damages. Next, we consider two types of economic shocks, a productivity shock that affects all firms, and a credit shock that affects constrained firms only. For a conventional functional form, the optimal emission tax is procyclical: it rises in response to a favorable productivity or credit shock.

This paper provides further insight in the different mechanisms that determine the optimal response of the tax to the economic shock. Abstracting from consumption smoothing effects and assuming marginal environmental damages are independent of the shock, we can identify four different mechanisms through which an economic shock affects the optimal tax. The first mechanism is the investment value effect. Improved productivity or access to credit allows constrained firms to invest more, which reduces the underinvestment problem; as a result the tax mainly serves to internalize environmental externalities rather than alleviate the financing problem. If however, we are dealing with a persistent favorable productivity shock, the benefits of investment in emission savings also rise, calling for a lower tax. This second, negative effect is the persistence effect. The optimal tax does not only depend on the marginal benefit of increasing investment or reducing emissions, but also on the degree to which investment and emissions are sensitive to tax changes. For example, if emissions are highly sensitive to the tax rate whereas constrained firms' investments hardly respond to tax changes, a relatively high tax is optimal. Hence, shocks affect the optimal tax through two additional mechanisms: the investment and emission sensitivity effects. The former evaluates the shocks' effect on the marginal effect of a tax change on emission-saving investment. It determines the degree to which the shock enhances, or worsens, the effectiveness of tax reductions in alleviating the credit constraint. In a similar manner, the emission sensitivity effect assesses the shock's effect on degree to which the tax is effective in reducing emissions. For a general functional form, the investment and emissions sensitivity effects are both ambiguous. In general, the combined effect from the four mechanisms cannot be signed. However, for a plausible specification we find that the optimal emission tax is procyclical: in a recession, tax reductions have a smaller effect on profits and investments, but since credit constraints are more harmful and also emission reductions are harder to achieve, a tax reduction is the best response to a recession.

This paper builds on several literatures. First, in recent years, a small literature on business cycles and environmental policy has emerged.[1] Most work focuses on the optimality of emission taxes versus quota or

[1] See Heutel and Fischer (2013).

intensity targets in the presence of economic shocks (Angelopoulos et al., 2010, Fischer and Springborn, 2011 and Dissou and Karnizova, 2012). For our paper, the work by Heutel (2012) is most relevant. Heutel (2012) uses a real business cycle model with a pollution externality to determine the optimal environmental policy response to economic fluctuations. It features two counteracting forces: in booms, the marginal utility of consumption is low, yet emissions, and thereby damages as a share of output are high. Taken together, he finds that procyclical carbon taxes and emissions are optimal. Our research question is similar, yet our assumptions differ in two core ways. First, we abstract from consumption smoothing and introduce credit constraints. Financial sector shocks and lack of access to credit have been a major factor in the recent US recession. Hence, they feature prominently in the rapidly growing DSGE literature studying the effects of economic shocks.[2] Also in the literature on growth, Aghion et al. (2010) and subsequent empirical results (Aghion et al. (2009) and Aghion et al. (2012)), point at the importance of credit constraints when evaluating the effect of recessions on investment and economic growth. Due to credit constraints which become especially pressing in recessions, firms may be unable to invest, offering a rationale for cyclical (fiscal) policy aimed at alleviating credit constraints. Second, when dealing with climate policies, due to slow global temperature adjustment, there is a significant lag between the timing of emissions and damages.[3] This implies that, unless an individual, temporary boom or recession has substantial effect on growth and the future emission path, the present value cost of climate change is unlikely to be greatly affected by the cycle.[4] We approximate this by assuming marginal emission damages as independent of the current level of emissions and output. This setup is complementary to Heutel (2012), who assumes emissions are immediately damaging. Hereby we focus on credit constraints as the main rationale for implementing cyclical policy. Allowing for procyclical damages in our setup would add effects and slightly complicate the analysis, yet not alter core results

This paper proceeds as follows. Section 2 introduces the basic model. Section 3 solves for the equilibrium given taxes and Section 4 the optimal emission tax. Section 5 discusses economic shocks, and how tax policy should respond. The model is solved for a specific functional form in Section 6. Section 7 discusses further extensions and concludes. Appendix A discusses optimal emission tax policy under an alternative tax recycling scheme.

2 The model

2.1 Production and investment

We use a two-period partial equilibrium model, with a unit mass of profit-maximizing heterogeneous firms $i \in [0,1]$. Throughout this paper, we will use lowercase letters to denote first-period variables, and uppercase letters for second-period variables, subscripts denote partial derivatives. Each firm has access to a common technology y, with a common productivity parameter a, which transforms energy $e(i)$ into output:

$$y(a, e(i)), \tag{1}$$

where $y_a > 0$, $y_e > 0$, $y_{ee} < 0$ and $y_{ea} > 0$. We normalize the price of a unit of output to unity. The heterogeneity across firms stems from their emission efficiency, $z(i) \in (0, \infty)$, which expresses energy use per unit of emissions, i.e. $e(i) = z(i)m(i)$ where $m(i)$ equals firm i's first period emissions. Emissions impose an externality on current

[2] See for instance Gertler and Kiyotaki (2010), and Bassetto et al. (2013) and Khan and Thomas (Forthcoming 2013) for an evaluation of the effect of economic shocks in a model with credit market imperfections and heterogeneous firms. Earlier references include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999).

[3] Nordhaus (2007) for example uses a temperature adjustment speed of approximately 2 percent a year (see Gerlagh and Liski, 2012). Caldeira and Myhrvold (2013) estimate slightly faster temperature adjustment, with 50% of the adjustment taking place in the first decade.

[4] This has also been pointed out by Lintunen and Vilmi (2013).

or future generations, and to correct for this externality the social planner may impose an emission tax. Such a tax implies that the heterogeneity in $z(i)$ translates into a heterogeneity in the marginal cost of energy use: $q+t/z(i)$, where t is the emission tax rate. This emission tax rate is equal across firms, due to the social planners inability implement differential tax rates or observe firm-specific characteristics. We assume energy is supplied by fully competitive extraction firms, with marginal extraction costs q, such that q is the cost of a unit of energy to the firm. Then, firms i's profits equal

$$\pi(i) = y(a, e(i)) - (q + t/z(i))\, e(i) - f, \tag{2}$$

where $f \geq 0$ is some fixed production cost. We abstract from bankruptcies, and assume $z(i)$ and f are such that $\pi > 0$ for all firms. Initial emission efficiency, $z(i)$, is given for each firm, but $Z(i)$ is determined by the investment undertaken by the firm at the start of the second period. Investment costs read

$$C\left(z(i), Z(i)\right), \tag{3}$$

with $C_z < 0$, $C_Z > 0$, $C_{ZZ} > 0$ and $C_{Zz} < 0$. Improving emission efficiency is costly, and more so the greater efficiency is obtained. An initially high efficiency level provides an investment advantage however, as this reduces investment cost. This can be interpreted as convex adjustment. Investments are financed out of retained earnings and bank loans. Bank lending is not unlimited however. Firms are subject to a credit constraint and can only borrow a multiple $[\xi - 1]$ of their first-period profits π. This gives the following constraint:

$$C \leq \xi \pi, \tag{4}$$

where we require $\xi \geq 1$. With $\xi = \infty$, constraints are absent and firms can borrow funds without limit. Any retained earnings not used for investment purposes can be deposited at the bank. We abstract from other financial market failures and assume the return received on deposits equals the loan rate. Taxes are recycled lump-sum, but do not affect firms' ability to invest. This represents the case in which the tax is imposed in only a single sector, or only a small subset of firms pollute, making the lump-sum negligible for the investing firms. Appendix A discusses an alternative case in which the lump sum is recycled to firms, and firms can use this lump sum for investment and loan collateral. We find that such a recycling scheme significantly complicates the analysis, while all propositions continue to hold.

2.2 Environmental externality

So far we have not been explicit about the type of emission externality we consider. The literature typically distinguishes two types of emission externalities: flow externalities and stock externalities. Examples of flow externalities are air and water pollution. These externalities are short-lived: the period in which the pollutant is emitted roughly coincides with the period during which damages are incurred and the pollutant is again depreciated. Stock externalities behave differently. Here the timing of emission and damages is distinct: they may be years, or even decades apart. The most common example here is the emission of CO2 and resulting damages through climate change. Because temperature changes are slow following the emission of CO2 and the corresponding increase in atmospheric CO2 concentrations, the damages caused by a given emission take years, of not decades, to materialize. Also, CO2 is a persistent greenhouse gas (the atmospheric half-life of CO2 is approximately 40 years, see Joos et al., 2012), and damages will be incurred for many decades. These properties imply that marginal damages from emissions are rather insensitive to temporary shocks, unless these shocks have sizable long-run effects. To the contrary, one should expect the marginal damages from flow pollutant emissions to be very responsive to current economic conditions. The existing literature on business cycles and environmental policy has mostly abstracted from this issue by focusing on the cost effectiveness of policies reducing emissions to a certain level (e.g. Angelopoulos et al. (2010); Fischer and Springborn (2011); Dissou and Karnizova (2012)). An exception is Heutel (2012), who models CO2 as a persistent greenhouse gas, but fails to account for the lag in atmospheric temperature, and thereby, damage adjustment. Since Heutel (2012) models damages as proportional to output, and quadratic in atmospheric concentrations, this failure to account for the temperature lag increases the procyclicality of marginal damages and thereby the procyclicality optimal emission tax.

In this paper, we analyze the effect of a temporary shock on the optimal taxation of a stock externality such as CO2 emissions. Our setup is partial equilibrium, and the shock may thus be reinterpreted as occurring in one sector of the economy only. For this reason, taking the present value of marginal damage, Δ, as independent from the current flow of emissions and output of the economy is most appropriate and we will hence assume this is that case.

3 Equilibrium

This section demonstrates the equilibrium of the model defined above. To save on notation, we express all variables in present value terms and suppress the firm identifier, i, when no confusion arises. Each firm maximizes the present value of profits, subject to the credit constraint, by choosing energy use, e and E and emission efficiency Z:

$$\max_{e,E,Z} \pi + \Pi - C$$
$$\text{subject to } C \leq \xi \pi,$$

The first-order conditions with respect to energy use read

$$\pi_e = 0 \text{ and } \Pi_E = 0, \tag{5}$$

and Z follows from the first-order condition

$$C_Z = \Pi_Z, \tag{6}$$

if $C < \xi \pi$ and (4) and $C_Z \leq \Pi_Z$ otherwise. By (5), we find $e_t < 0$ and $\pi_t < 0$: energy use and profits are falling in the tax rate. In a similar manner, by $e_q < 0$ and $\pi_q < 0$, higher energy cost q will reduce firms' energy use and profits. The heterogeneity in z causes heterogeneity in energy use across firms. A high z firm has a relatively low marginal cost of energy use, and chooses high e, causing profits to be high as well. This follows from (2) and (5), which give $e_z > 0$ and $\pi_z > 0$. The effect of z on emissions is not directly obvious. On the one hand, given energy use, a high emission efficiency implies emissions are low. On the other hand, energy use is increasing in emission efficiency. Recently, a literature has pointed out under what conditions improvements in green technology reduces harmful emissions, and in what cases stricter environmental policy leads to green technology adoption to begin with.[5] We restrict attention to the most intuitive case by assuming that improvements in green technology reduce harmful emissions:

Assumption 1 $m_z < 0$

By Assumption 1, low z firms do not only have low energy use and profit, but also high emissions. This assumption also immediately implies that $Z^U_T > 0$: stricter environmental policy leads to more green technology

[5] See for example Gil-Molto and Dijkstra (2011), Brechet and Meunier (2012), Gans (2012), Perino and Requate (2012) and Smulders and Di Maria (2012).

adoption in the optimum. Finally, one can determine the effect of z on the degree to which firms are credit constrained. Here, we first define the following

Definition 1 Let $Z^R(i)$ be the maximal (restricted) choice of Z for firm i, which is implicitly defined by (4). And let $Z^U(i)$ be the optimal (unrestricted) choice of Z for firm i, implicitly determined by (6). Then if $Z^R(i) < Z^U(i)$, firm i is constrained. If $Z^R(i) \geq Z^U(i)$, we refer to firm i as unconstrained.

By (4) and (6), both Z^R and Z^U are increasing in z. This is due to the following. First, with high z, (marginal) investment costs are relatively low. This increases both the maximal Z, Z^R, and optimal Z, Z^U. Second, for a positive emission tax, profits π are increasing in z. Higher z thus alleviates the credit constraint, increasingZ^R for all firms. This increases constrained firm's investment in efficiency improvements, yet will leave unconstrained firms' choice of Z unaffected. Because a higher z implies higher maximal and optimal Z, it is not directly obvious whether the low z or the high z firms are particularly constrained. Motivated by a literature that points out that firms need time to "outgrow" their constraints (see for instance Bassetto et al., 2013, Buera and Shin, 2013 and Khan and Thomas, Forthcoming 2013), we assume that small, low z firms are more likely unable to choose the optimal emission efficiency Z^U:

Assumption 2
Let firms be ranked in ascending order according to $z(i)$. Then we can define some firm n such that firms $i < n$ are constrained, and firms $i \geq n$ are unconstrained.[6]

For a slightly more specific investment cost function, Assumption 2 is not required, as we can show the following:

Remark 1
If we can express investment costs, $C(z,Z)$, as $C(1,\tilde{z}(i))$ with $\tilde{z}(i) \equiv Z(i)/z(i)$, and rank firms in ascending order according to $z(i)$, we can define some firm n such that firms $i < n$ are credit constrained, and firms $i \geq n$ are unconstrained.

Proof
By definition, we must have that for firm $i = n$, $Z^R = Z^U$. Then, if for $i = n$, $Z^R_z > Z^U_z$, we must have that all firms $i < n$ are credit constrained, and firms $i > n$ are unconstrained. First, we use (4) and (6) to establish $Z^R_z = C_Z^{-1}[\xi\pi_z - C_z]$ and $Z^U_z = [\Pi_{ZZ} - C_{ZZ}]^{-1} C_{Zz}$. By $C(1,\tilde{z})$, we have $C_z = -\tilde{z}z^{-1}C_{\tilde{z}}$, $C_Z = z^{-1}C_{\tilde{z}}$, $C_{Zz} = -z^{-2}[\tilde{z}C_{\tilde{z}\tilde{z}} + C_{\tilde{z}}]$ and $C_{ZZ} = z^{-2}C_{\tilde{z}\tilde{z}}$. By (2) and (5) we know $\Pi_Z = Z^{-2}E$ which gives $\Pi_{ZZ} = Z^{-1}\left[Z^{-2}[ZE_Z - E] - \Pi_Z\right]$. Using that $M_Z = Z^{-2}[ZE_Z - E]$ and for firm $i = n$, $\Pi_Z = C_Z$ we have $\Pi_{ZZ} = Z^{-1}\left[M_Z - z^{-1}C_{\tilde{z}}\right]$. We can then establish $Z^R_z = C_{\tilde{z}}^{-1} z\xi\pi_z + \tilde{z} > \tilde{z}$ and $Z^U_z = \tilde{z}\frac{\tilde{z}C_{\tilde{z}\tilde{z}} + C_{\tilde{z}}}{\tilde{z}C_{\tilde{z}\tilde{z}} + C_{\tilde{z}} - zM_Z} < \tilde{z}$, from which it follows that $Z^R_z > Z^U_z$. ■

The implications of Assumption 2 are represented in Figures 1a and 1b below. The green curve depicts the optimal second-period emission efficiency, Z^U, for different levels of z, and the red curve depicts the relationship between z and the corresponding maximal level of Z, Z^R. Here, $z(n)$ is the level of first-period emission efficiency at which the two curves cross. The black curve then shows the Z selected. For firms with am initial emission efficiency below $z(n)$, the optimal second-period emission efficiency exceeds the maximal choice of Z. This firm is constrained, and will thus have $Z = Z^R$. A higher z increases both maximal and optimal Z. For $z(i) \geq z(n)$, the red curve exceeds the green curve, and firms will be able to invest optimally.

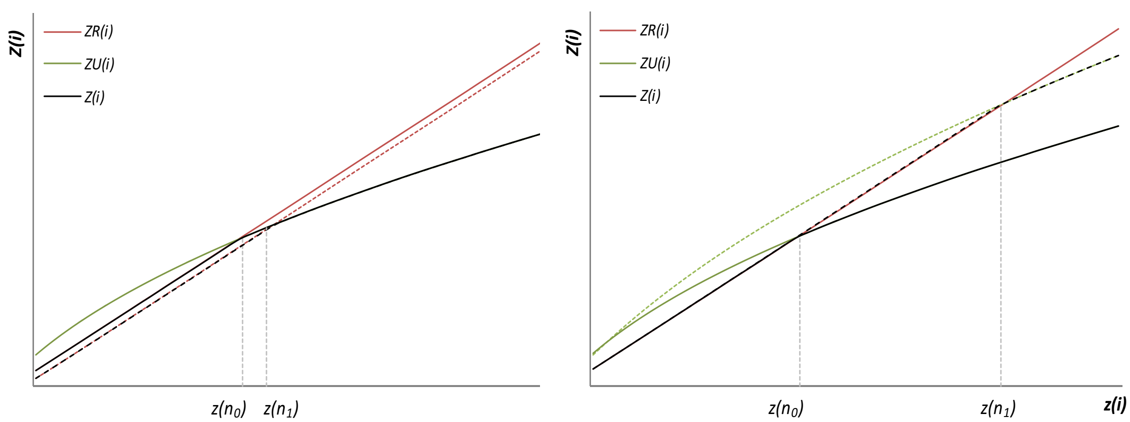
[6] Unless each firm i has its unique $z(i)$ such that $z(i) \neq z(j)$ for firm $i \neq j$, n may be a group of firms: $n = [\underline{n}, \overline{n}]$ where $z(i) = z(j)$ for all $i, j \in n$. Without affecting any of the main results, we will treat n as a single firm in the remainder of this paper.

The figures below allow us to evaluate the effects of tax increases on Z, Z^U, Z^R and $z(n)$. Figure 1a shows the effect of an increase in the first-period tax rate, t. In this figure, the dashed curves are the curves corresponding to the higher tax. As profits are falling in the tax rate, the tax increase reduces Z^R for all firms, but leaves Z^U is unaffected. With the red curve shifting down, and the green curve unaffected, we find that a tax increase causes $z(n)$ to rise from $z(n_0)$ to $z(n_1)$. Hence, the increase in first-period taxes reduces investment by constrained firms, and increases the number of firms that are constrained.

Figure 1b shows the effect of an increase in second-period taxes, T. Future tax increases, by increasing the cost of emissions, increase the return to emission efficiency improvements and thereby Z^U. With Z^R unchanged however, not all firms will be able to raise sufficient funds to finance the (full) additional investment. Even though the increase in T increases investment by all (previously) unconstrained firms, more firms will now be constrained: n rises from n_0 to n_1.

Figure 1: $Z(i)$ as a function of $z(i)$

(a) Effect of an increase in t (b) Effect of an increase in T



Curves are constructed using the functional forms presented in Section 6, with $\beta = 0.25$, $q = 10$, $\gamma = 1.1$, $a = 0$, $A = 0$, $t_0 = 1$, $T_0 = 1.25$, $f = 0.2$ and $\xi = 1.2$. The 0 subscript denotes the pre-shock value of the variable, the subscript 1 refers the the post-shock value. If the subscript is absent, values apply to both cases. For Figure 1a, we use $t_1 = 2$ and $T_1 = T_0$. For Figure 1b we have $t_1 = t_0$ and $T_1 = 2.5$. Solid lines represent the relationship prior to the tax increase, dashed lines after the tax increase.

4 Optimal environmental policy

The social planner maximizes the discounted sum of firm output, net of environmental damages, subject to the individual firm's credit constraint. We assume the social planner cannot identify the firm type, and is therefore unable to levy firm-specific emission taxes or investment subsidies. Its instruments are thus the first- and second-period emission taxes, t and T. For now, we assume the lump-sum tax revenue, $s = t\int_0^1 m(i)di$, is either a direct gain for the government, or redistributed but not used for investment purposes (i.e. it does not affect the firms' credit constraints). Appendix A discusses the case where the lump-sum tax is redistributed to the firm and can be used for investment purposes and as collateral. The social planner faces the following maximization problem

$$\max_{t,T} \int_0^1 \left[\pi + s - \Delta m + \Pi - C + S - \Delta M\right] di$$
$$\text{subject to } C \leq \xi \pi.$$

By equation (2), $\Pi_E = 0$, $S = T \int_0^1 Z^{-1} E di$ and $C_Z = \Pi_Z$ for unconstrained firms, the first order condition with respect to T reads

$$(T - \Delta) \int_0^1 \frac{E_T}{Z} di = 0, \tag{7}$$

which implies that the optimal second-period tax equals $T^* = \Delta$. This result is not surprising: in the second period, the only externality is the environmental externality, and therefore the tax should be set at the Pigouvian level. The second-best nature of the problem shows up in the first order condition for the first-period tax t, however:

$$(t^* - \Delta) \int_0^1 -\frac{e_t}{z} di = \int_0^n \left[\Pi_Z - C_Z\right] Z_t^R di, \tag{8}$$

where t^* is the first-period optimal emission tax. We can then establish the following

Proposition 1 As long as some firms are credit constrained ($n > 0$), the first-period tax falls short of the Pigouvian tax ($t^* < \Delta$).

Proof Since constrained firms invest suboptimally, we have $\Pi_Z - C_Z > 0$, and using (4) we can establish $Z_t^R < 0$. Then by (5) $e_t < 0$ and using (8), we must have $t^* < \Delta$. ■

This result can be explained as follows. In setting the tax, the social planner needs to take into account not only the harmful effect of emissions, but also the effect of the tax policy on firm's investment decisions. As firm profits are falling with the tax rate, a higher tax further constrains the constrained firms in their investment choice. To encourage investment, it will be optimal for the social planner to reduce taxes below the marginal environmental cost. More specifically, in the optimum, the marginal environmental loss due to insufficiently priced emissions multiplied by the sensitivity of emissions to the tax rate (LHS of (8)) must equal the gain from increasing investment of the constrained firms multiplied by the sensitivity of Z^R to the tax rate (RHS of (8)). If no firms are constrained ($n = 0$), the optimal tax equals marginal emission damages, Δ.

5 Environmental policy and economic shocks

The aim of this paper is to determine the optimal environmental policy response to economic shocks. Here, we consider two types of shock: a productivity shock and a credit shock. The productivity shock is a shock to a, with persistence $\mu \in [0, 1)$, such that $dA = \mu da$. The credit shock is a shock to ξ. As we take marginal environmental damages as exogenous, the optimal second-period tax, T^*, is independent of the productivity parameter, a, and credit constraints, ξ. The optimal tax in the first period, t^*, however, is, indirectly, a function of a, A, and ξ, and thus sensitive to changes in these parameters. In this section, we first assess the effect of the economic shocks on firm's energy use, profits, optimal and maximal investment. Next, we present the optimal response of $t^*$to the two types of shocks. This allows us to distinguish four mechanisms through which the shocks affect t^*. The mechanisms work in opposite directions, or may be ambiguously signed, and as a consequence, we are unable to establish whether taxes should rise or fall in response to the shocks. The model defined in Section 2 is very general however. In Section 6, we present an example with a conventional functional form, and conclude that for both shocks, t^* is procyclical.

5.1 Equilibrium outcomes

A shock to productivity, a, affects output y, through two channels. First, by (5), $e_a > 0$ and $y_e < 0$, an adverse productivity shock reduces y through e: by reducing the returns to energy use, the shock reduces energy use, which has a negative effect on output. Additionally, by $y_a > 0$, the adverse shock reduces output directly. Through its direct effect on output, we have $\pi_a < 0$ and which implies also profits are negatively affected by the adverse productivity shock. This fall in output and profits, caused by the productivity shock, may be persistent for two reasons. First, the shock itself may be persistent ($\mu > 0$), which reduces future productivity and thereby output and profits directly. Also, by reducing the marginal return to efficiency improvements, it reduces the incentive to invest.[7] Unconstrained firms will then select a lower Z, which has an additional negative effect on second-period output and profits for the those firms.

Also investment by constrained firms us affected. As a negative shock to a reduces profits, it further tightens the constraint, reducing Z^R for all firms. Constrained firms will then be forced to select a lower Z, which reduces their second-period energy use and profits. Hence, even with $\mu = 0$, credit constraints may cause a recession to persist. In the macro literature, this effect is known as the "financial accelerator" (Bernanke et al., 1999).

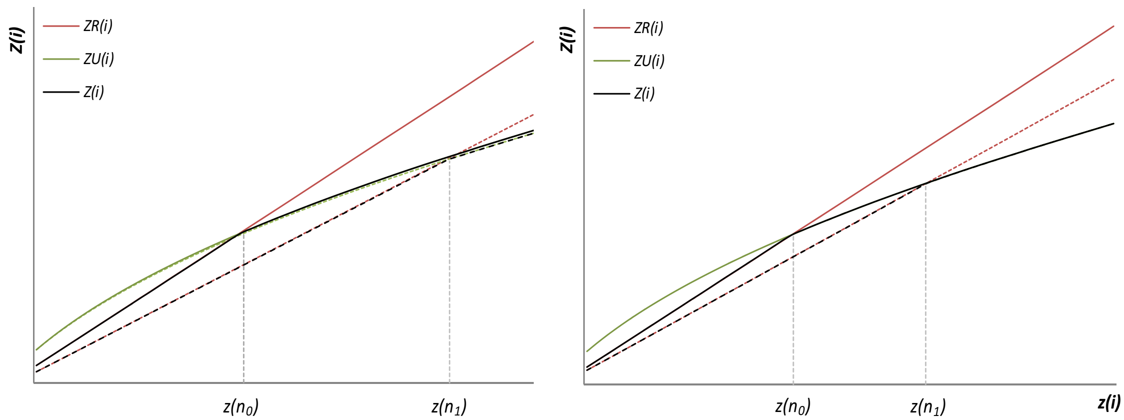
As a persistent negative productivity shock reduces both Z^R and Z^U, the effect of this shock on n, the number of constrained firms, is ambiguous. In Figure 2a below, we find that the effect of the shock on Z^U is small relative to its effect on Z^R, and thereby n will rise.

In addition to a shock to a, we evaluate the optimal policy response to a tightening of credit, i.e. a drop in ξ. Unlike the productivity shock, a credit shock does not affect all firms: it will only affect those firms who are credit constrained or become so due to the shock. These firms will be forced to reduce investment in emission efficiency. The lower Z implemented by those firms will then cause a drop in output Y and profit Π (relative to the no shock case). The effect of this shock on Z^U, Z^R and $z(n)$ is depicted in Figure 2b. It shows that a tightening of credit reduces Z^R for all firms, and as a consequence, investment is reduced for all with $z \leq z(n_1)$. By $z(n_0) < z(n_1)$, the credit shock increases the share of firms that are credit constrained.

[7] This can be seen by $\Pi_Z = Z^{-2}TE$ and $E_A > 0$.

Figure 2: $Z(i)$ as a function of $z(i)$

(a) Effect of an adverse productivity shock (b) Effect of a tightening of credit



Curves are constructed using the functional forms presented in Section 6, wit$\beta = 0.25$, $q = 10$, $\gamma = 1.1$, $a_0 = 0$, $A_0 = 0$, $t = 1$, $T = 1.25$, $f = 0.2$ and $\xi_0 = 1.2$. The 0 subscript denotes the pre-shock value of the variable, the subscript 1 refers the the post-shock value. If the subscript is absent, values apply to both cases. For Figure 2a, we use $a_1 = -0.02$, $\xi_1 = \xi_0$ and $\mu = 0.9$. For Figure 2b we have $a_1 = a_1$ and $\xi_1 = 1$. Solid lines represent the pre-shock relationship, dashed lines post-shock relationship.

5.2 Optimal environmental policy

Equation (8) implicitly solves for t^* as a function of marginal emission damages, Δ, the sensitivity of emissions to the tax, e_t/z, the marginal value of investment (which is positive for constrained firms), $\Pi_Z - C_Z$, and the sensitivity of a constrained firm's investment to tax changes, Z_t^R. The response of t^* to a productivity and credit shock will then depend on the degree to which the respective shock affects these four factors.

To solve for the optimal response of t^* to a productivity shock first, we take the total derivative of (8) and find

$$t_a^* = B^{-1}\left[\underbrace{(t^*-\Delta)\int_0^1 \frac{e_{ta}}{z}di}_{\substack{\text{emission sensitivity}\\ \text{effect } (+/-)}} + \int_0^n \left[\underbrace{[\Pi_Z - C_Z]\,Z_{ta}^R}_{\substack{\text{investment sensitivity}\\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}]\,Z_a^R Z_t^R}_{\substack{\text{investment value}\\ \text{effect } (+)}} + \underbrace{\mu \Pi_{ZA} Z_t^R}_{\substack{\text{persistence}\\ \text{effect } (-)}}\right] di\right], \tag{9}$$

where $B > 0$.[8] The optimal response of the emission tax depends on four effects, which may work in opposite directions.

The first effect is the emission sensitivity effect, which evaluates the shock's effect on the sensitivity of emissions to the tax. The sign of the effect is governed by e_{ta}, which is ambiguous absent further functional specification. As energy use is increasing in a, a reasonable assumption is that its sensitivity to taxes is too. This gives $e_{ta} < 0$, which implies that a positive technology shock makes energy use (and emissions), more

[8] More precisely, we have $B \equiv -\left[\int_0^1 z^{-1}\left[e_t + (t^* - \Delta)\, e_{tt}\right] di + \int_0^n \left[[\Pi_{ZZ} - C_{ZZ}]\, Z_t^R Z_t^R + [\Pi_Z - C_Z]\, Z_{tt}^R\right] di\right]$. As t^* maximizes $V \equiv \pi + s - \Delta m + \Pi - C + S - \Delta M$ subject to (4), we must have that for $t = t^*$, $V_t = 0$ and $V_{tt} < 0$. Here one can show $B = -V_{tt}$.

responsive to taxes. Since additional emissions have net social cost of $\Delta - t^* > 0$ (see Proposition 1), this calls for a higher tax when a is high.

The second effect is called the investment sensitivity effect. It evaluates Z^R_{ta}, the effect of a technology shock on Z^R_t, the sensitivity of maximal investment, Z^R, to the tax. Here Z^R_t can be interpreted as the effectiveness of a tax reduction in alleviating credit constraints, and $\Pi_Z - C_Z$ as the marginal value of alleviating these constraints. For our general functional form, Z^R_{ta} is ambiguous. This is the consequence of two counteracting effects. Energy use is increasing in a, which implies that a marginal increase in the tax rate causes greater cost increases, and thus profit losses, for a larger a. A positive shock to a thus increases the sensitivity of π, and, by (4), Z^R to, t.[9] Additionally, for a given increase in a, Z^R is now higher. By $C_{ZZ} > 0$, this implies marginal investment cost is higher as well. Any additional increase in Z^R now requires a greater increase in net profits, reducing the effectiveness of tax reductions in increasing Z^R. In case we find $Z^R_{ta} < 0$, the former effect dominates and Z^R is more sensitive to tax changes in booms: a reduction in t now provides greater benefits, as it allows constrained firms to more rapidly increase Z. This effect then calls for a lower tax in booms.

We named the third effect the investment value effect. As noted before, the productivity shock affects the credit constraint and thereby Z^R directly. As a consequence the shock affects the constrained firm's marginal value of investment: $\Pi_Z - C_Z$. One can easily show that this marginal value is falling in Z. Intuitively, higher productivity allows constrained firms to invest more, which reduces marginal value of additional investments and thereby the underinvestment problem. As a result, the tax can serve to internalize environmental externalities rather than alleviate the financing problem; this effect calls for a higher tax in booms. Finally, there exists a persistence effect: a positive technology shock increases (the expectation of) A, increasing the marginal benefits from investment in Z. This effect calls for a lower t^* if a is high.

All in all, additional assumptions are required regarding the functional forms of investment cost and output to determine the sign of t^*_a. Section 6 solves for t^*_a for a conventional functional form. For this functional form, the positive effects dominate: taxes should be raised in booms. This result is consistent with Heutel (2012), yet its rationale is distinct. Heutel found a procyclical tax due to the procyclicality of marginal damages, i.e. a procyclicality in Δ, whereas we assume Δ to be acyclical. A procyclical Δ would however further strengthen the procyclicality of t^*.

In a manner similar to the productivity shock, we can determine the optimal response of the tax to a credit shock. We find that two effects, the investment sensitivity effect and the investment value effect, are still present, whereas the externality and persistence effects are specific to the productivity shock. Using (8), we can write the total derivative and arrive at:

$$t^*_\xi = B^{-1} \int_0^n \left[\underbrace{[\Pi_Z - C_Z]\, Z^R_{t\xi}}_{\substack{\text{investment sensitivity} \\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}]\, Z^R_\xi Z^R_t}_{\substack{\text{investment value} \\ \text{effect } (+)}} \right] di. \tag{10}$$

Two effect determine the optimal response of the tax to the credit shock. Like with the productivity shock, the investment sensitivity effect evaluates the effect of the credit shock on the sensitivity of Z^R to the tax. It includes two counteracting effects, and as a consequence, the sign of $Z^R_{t\xi}$ is ambiguous. First, by (4), less stringent constraints (higher ξ) increase the change in investment credit due to a change in profits. Now, a marginal increase in taxes, causes a greater drop in Z^R. Second, given profits, a higher ξ allows for higher Z^R to begin with. With marginal investment costs increasing in Z, the increase in Z^R is now lower for a given increase in π. The first effect contributes towards a negative $Z^R_{t\xi}$, whereas the second effect increases $Z^R_{t\xi}$ (note $Z^R_t < 0$). If we find $Z^R_{t\xi} < 0$, a favorable credit shock increases the sensitivity of Z^R to the tax. A marginal reduction in t

[9] More formally, we have $\pi_t = -z^{-1}e$ and $e_a > 0$.

now causes greater benefits through credit alleviation, calling for a lower tax if constrained firms obtain credit relatively easily.

The second, investment value effect, summarizes the effect of the credit shock on the marginal value of investment. A favorable credit shock increases Z^R, which implies that $\Pi_Z - C_Z$ falls. By reducing the benefits of increasing Z, this effect calls allows for lower taxes in response to favorable credit shocks.

Comparing (10) to (9), we observe that the emission sensitivity effect and persistence effect are absent with the credit shock. For the productivity shock, the emission sensitivity effect captured the effect of the shock on the sensitivity of emissions to taxes. As credit shocks do not affect firms' decisions regarding e, it also leaves e_t unaffected and no emission sensitivity effect is present. The persistence effect expresses the increase in the marginal benefit from investment due to the increase in (expected) second-period productivity. Credit shocks do not affect firm's (future) productivity, and thus do not affect the marginal investment benefits through this channel.

Absent further functional specification, no conclusions can be drawn regarding the sign of t^*_ξ. The next section will use a conventional functional form to solve for (10) and establish that $t^*_\xi$is unambiguously positive: taxes should be reduced during credit crises.

6 Example: specific functional form

Section 5 established the implicit solution for the optimal emission tax, t, and its optimal response to a productivity and credit shock respectively. We found that the optimal response to a productivity shock is governed by a positive investment value effect, a negative persistence effect and ambiguous externality and investment sensitivity effects. As a consequence, the optimal response to a productivity shock is ambiguous without further assumptions regarding the functional forms of y and C. This ambiguity also arises in the optimal response to a credit shock. The sign of this response is governed by a positive investment value effect, and an ambiguous investment sensitivity effect. This section will provide an example of a specific functional form for the production function that allows us to arrive at unambiguous conclusions regarding the cyclicality of t^*.

Production now reads

$$y(a, e(i)) = \exp(a)^{1-\beta} e(i)^{\beta}, \tag{11}$$

where $\beta \in (0, 1)$. Investment cost are now defined as

$$C\left(z(i), Z(i)\right) = \tilde{z}(i)^{\gamma}, \tag{12}$$

with $\tilde{z}(i) \equiv Z(i)/z(i)$ and $\gamma > 1$.

With energy cost q and emission efficiency z, firm i's marginal cost of energy use still reads $q + t/z$. The optimizing firm sets $y_e = q + t/z$, which implies we find equilibrium energy use

$$e(i) = \exp(a) \left[\frac{\beta}{q + \frac{t}{z(i)}} \right]^{\frac{1}{1-\beta}}, \tag{13}$$

and profits can be reduced to

$$\pi(i) = \exp(a) \left[q + \frac{t}{z(i)} \right]^{-\frac{\beta}{1-\beta}} \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}} \right] - f. \tag{14}$$

With these functional forms, Figures 1 and 2 apply, and all results established in Section 5.1 continue to hold. In Section 5.2, no unambiguous conclusions could be drawn regarding the signs of the investment sensitivity effects, t_a^* and t_ξ^*. Before we solve for the sign of t_a^*, we first determine the sign and strength of the investment sensitivity effect in (9), and assess the strength of the negative persistence effect relative to the other (positive) effects. Using (13) and (12), one can show $Z_{ta}^R = Z_t^R/\gamma$. By $Z_t^R < 0$, the investment sensitivity effect is negative: in a boom a tax reduction allows for a greater increase in Z^R. Put differently, tax increase are more damaging for constrained firms choice of Z in booms, calling for lower taxes in booms. Then, we can establish the following

Proposition 2 As long as some firms are credit constrained, the first-period tax should rise in response to a positive productivity shock.

Proof By (11), (12), (13) and (14) we have $e_{ta} = e_t$, $Z_a^R = Z/\gamma$, $E_A = E$, $C_Z + ZC_{ZZ} = \gamma C_Z$, $\Pi_Z = Z^{-2}E$, and $Z\Pi_{ZZ} = [M_Z - \Pi_Z]$ by $M_Z = Z^{-2}[ZE_Z - E]$. Then (9) reduces to $t_a^* = B^{-1}\left[(t^* - \Delta)\int_0^1 z^{-1} e_t di + \gamma^{-1}\int_0^n [M_Z - \gamma C_Z + \mu \Pi_{ZA}] Z_t^R di\right]$. By $(t-\Delta)e_t > 0$, $Z_t^R < 0$, $C_Z > 0$ and $M_Z < 0$ this implies that if the shock is not persistent, i.e. if $\mu = 0$, we immediately have $t_a^* > 0$. A persistent shock ($\mu > 0$) reduces t_a^*, but will never turn t_a^* negative. We know $\mu < 1$, then using (8), we have $\lim_{\mu \to 1} t_a^* = [\gamma B]^{-1}\int_0^n TM_Z Z_t^R di > 0$, which implies we must have $t_a^* > 0$. ■

In a recession, tax reductions are less attractive because they allow for a lower increase in Z^R and less additional second-period profits due to lower productivity next period. Yet, in a recession, also emission reductions are harder to achieve, and firms face tougher constraints on credit. By Proposition 2, the former, negative, investment sensitivity and persistence effects are outweighed by latter, positive, externality and investment effects, and it is optimal to reduce taxes when the economy is hit by an adverse productivity shock. Likewise, emission taxes should be raised in booms.

We can establish a similar proposition regarding t_ξ^*. Using (13) and (12), we solve for the sign of the investment sensitivity effect in (10): $Z_{t\xi}^R = [\xi\gamma]^{-1} Z_t^R < 0$. Again, the investment sensitivity effect is negative: more stringent constraints imply a marginal tax reduction has a smaller impact on Z^R, which argues in favor of higher taxes following adverse financial conditions. This negative investment sensitivity effect does however not outweigh the positive investment value effect, as we arrive at the following result:

Proposition 3 As long as some firms are credit constrained, the first-period tax should rise in response to a positive credit shock.

Proof By (12) we have $C_Z + ZC_{ZZ} = \gamma C_Z$ and $Z_\xi^R = [\xi\gamma]^{-1} Z^R$. $Z_{t\xi}^R = [\xi\gamma]^{-1} Z_t^R$ then reduces (10) to $t_\xi^* = [B\xi\gamma]^{-1}\int_0^n [TM_Z - \gamma C_Z] Z_t^R di$. Then by $M_Z < 0$ and $C_Z > 0$, integral is negative, which gives $t_\xi^* > 0$. ■

One the one hand, a drop in ξ, by reducing the effect of a marginal profit increase on the credit available to the firms, reduce the sensitivity of Z^R to the tax rate. Hence, during financial crises, tax reductions are less effective in alleviating credit constraints. However, because a low ξ reduces investment to begin with, the marginal value of additional investment rises. By Proposition 3, the latter effect dominates, and emission taxes should be reduced in response to a financial contraction.

7 Discussion and conclusion

This paper evaluates optimal environmental policy in a two-period setting with heterogeneous firms. Firms use energy in production, with harmful emissions as a byproduct. To reduce these emissions, firms can invest in pollution intensity improvements, but investment may be suboptimal due to credit constraints. In this setup, an higher emission tax has two effects: it reduces energy use and thereby emissions, but further constrains financial retained profits, reducing investment of firms facing a binding credit constraints. The framework thus features a tradeoff between relieving firms' credit constraints, and reducing the degree of harmful emissions. We find that if constraints are binding for a subset of firms, the optimal emission tax falls short of marginal damages. The response of the optimal emission tax to a productivity or credit shock then depends on several factors. First, the shock will affect the degree to which firms are constrained, and thereby the value of alleviating the constraint. This effect, the investment value effect, is always positive: a positive productivity or credit shock reduces the degree to which firms are constrained, and thereby allows for higher taxes in booms. Next, the shock may affect the degree to which the tax instrument is effective in reducing emissions and increasing investment respectively. We call these the emission and investment sensitivity effects. As credit constraints do not affect emissions directly, the emission sensitivity effect is absent with a credit shock. For a very general functional specification, both effects are ambiguously signed, yet we find that for a conventional functional forms, the emission sensitivity effect is positive, while the investment sensitivity is negative. This implies that a positive productivity or credit shock increases the effectiveness of reducing emissions and alleviating the credit constraint, which in the former (latter) case calls for higher (lower) taxes during booms. Finally, if the shock is persistent, the future return to investment may be affected. This persistence effect is negative, calling for lower taxes during booms, and only present with a productivity shock. For a conventional functional form, the positive effects dominate, and taxes should rise in response to a positive credit or productivity shock. In the main section, we abstracted from tax recycling schemes to arrive at this conclusion. As shown in Appendix A, this results continues to hold if the tax is recycled lump sum, and this lump sum can be used for investment purposes.

In the policy debate regarding the desirability of environmental policy in an economic downturn, a major argument raised is that as (more stringent) environmental policies impose further hardship on business, they should be either postponed or canceled. Our result lends support to this argument by showing and shows that the optimal emission tax is procyclical, and thus lower following an adverse productivity or credit shock. This however, need not imply that further environmental legislation should be postponed until more virtuous times have arrived. Our analysis evaluates the optimal tax, and there is no reason to suppose emissions are currently optimally priced. To determine the desirability of strengthening, or weakening, environmental regulations, an assessment regarding the stringency of this regulation, relative to its optimal level, is required. In such an evaluation, as our analysis points out, the regulation's effect on firm's access to credit may be an important factor, and should hence be included in such assessment.

Several aspects have not been discussed in this current paper and are thus open for further work. First, we have taken the present value of the marginal emission damages as independent of the level of output and emissions. We believe that, especially when dealing with CO2 emissions, this is a valid approximation. Global average temperatures need time to adjust to changes in atmospheric concentrations, implying that damages are unlikely to be sensitive to year-to-year fluctuations. Our results are not sensitive to this assumption however. Procyclical damages can be easily included in our framework, and would be an additional reason to increase the emission tax in response to productivity or credit shocks. Second**,** we abstracted from consumption smoothing by focusing on production maximization instead of utility maximization. This is equivalent to assuming a utility function that is linear in consumption. Alternatively, one could reinterpret our model as a representation of one of many markets, where the shock is idiosyncratic to the market. In this case, with a representative consumer, consumption would be virtually unaffected by the shock. Defining a utility function with diminishing marginal utility would be an interesting extension to the model, yet one that is unlikely to overturn any of the main conclusions as it adds an additional force towards tax procyclicality. Third, according to the Schumpeterian view (see Aghion and Howitt, 1998), investment costs are procyclical, as opportunity costs, of for example labor, rises during booms. We here have abstracted from such procyclicality. The rationale behind this is that ample empirical evidence points out that investment in productivity improvements takes place during booms (see for example Aghion et al., 2012), indicating that channels, such as credit constraints, that favor investment in booms are stronger than those that turn investment countercyclical. Hence, introducing investment costs that are strongly procyclical such that they call for countercyclical tax policy would be unrealistic. Fourth, we have focused our analysis on the optimal tax policy. Assessing the cyclicality of the optimal emission quota would be a natural extension. Here, as absent of tax adjustments, emissions rise in booms, a procyclical emission tax need not imply countercyclical optimal emission quota. Finally**,** the model distinguishes only two periods. Firms face a single investment decision at the end of the first period, and no credit constraint is operational in the second period. Some initial efforts have been undertaken to arrive at an analytical solution for optimal taxes in a multi-period model. In such a model, because current investment reduces future investment cost, benefits accumulate and the marginal value of alleviating the credit constraint increases. However, if credit constraints continue to bind for some firms for a number of periods, optimal future taxes (T^* in our current setup) will also fall short of marginal damages. Compared to first-best, this causes underinvestment in efficiency improvements by the unconstrained firms, and excessive emissions by all firms. Further research must determine the effect of economic shocks on the optimal emission tax through these additional mechanisms, and how these effects balance with the effects identified in the two-period setup.

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Appendix

A Emission tax policy with lump-sum recycling

Throughout the analysis, we have assumed that the tax recycling scheme leaves the firm investment constraint, (4), unaffected. A rationale is that this lump-sum rebate is spread across many sectors in the economy not subject to carbon tax, implying the rebate is of a negligible size. Alternatively, one could argue the tax is rebated to consumers or comes at a direct benefit to the government. This setup greatly simplifies the analysis, as it allows us to abstract from the redistributional effects of the tax: as firms taxes are a function of their emission efficiency, not all firms are taxed equally, and the lump sum rebate would imply a redistribution of funds across firms. This appendix will determine the properties of the optimal tax, and its response to shocks, when the tax is redistributed lump sum, and this lump sum can, like profits, be used for investment purposes and as loan collateral. This appendix is structured similarly to the main text. First, we solve the firm optimization problem. Next, we determine t^* and T^* and their cyclical properties. As in the main text, we do not arrive at unambiguous solutions for t^*_a and t^*_ξ for the general functional form. Hence we solve t^*_a and t^*_ξ for the functional forms from (6). We find that all propositions established in the main text continue to hold.

A.1 Equilibrium

The firms' optimization problem now reads:

$$\begin{aligned}
&\max_{e,E,Z} \pi + s + \Pi - C + S \\
&\text{subject to } C \leq \xi \left[\pi + s\right],
\end{aligned} \tag{15}$$

where $s = t\int_0^1 m(i)di$ is exogenous to the firm (likewise for S) and output, profits and investment costs are defined as in (1)-(3). Firms take the lump sum as exogenous, which gives the first order conditions as in the main text: $\pi_e = 0$ and $\Pi_E = 0$. Also still $C_Z = \Pi_Z$ for unconstrained firms, yet now $C = \xi\left[\pi + s\right]$ for constrained firms. From the above, one can directly conclude that the lump-sum rebate alleviates the credit constraint. This is the consequence of increasing the gross profit, $\pi + s$, of firms, and a similar result would have been obtained for alternative redistribution schemes. As all firms receive the same lump sum, Assumption 2 and Remark 1 hold without any additional assumptions and we can define two groups: firms $i \in [0,n]$ who are credit constrained, and the unconstrained firms $i \in [n,1]$. The redistribution scheme however allows us to make an additional separation across firms: those for whom the tax increase is a net benefit, and those to whom it is a net cost. The most straightforward way to see this is if there are only 2 (types of) firms: one with $z = \infty$ (no emissions due to energy use) and one with $z < \infty$ (positive emissions). Now any positive emission tax will impose a cost on the latter group only. However, both groups participate in the redistribution scheme. It must thus follow that if a tax increase implies additional revenues are raised, this tax increase is a net gain to firms with $z = \infty$, and a net cost if $z < \infty$. By the following Lemma, the lump-sum, s, is increasing in the tax rate.

Lemma A1 $s_t > 0$

Proof We know $s = t\int_0^1 z^{-1}edi$, so $s_t = \int_0^1 z^{-1}\left[e + te_t\right]di$. By (2) and (5) we have $e_t = \left[zy_{ee}\right]^{-1}$ and $e_z = -z^{-2}y_{ee}^{-1}t$ which implies $te_t = -ze_z$. By Assumption 1, $m_z < 0$, where $m_z = z^{-2}\left[ze_z - e\right]$. From here it follows that $s_t = -\int_0^1 zm_z di$ and we have $s_t > 0$. ■

The above result can be explained as follows. The effect of an increase of z on emissions is twofold. On the one hand, a greater emission efficiency reduces emissions, given energy use. On the other hand, a higher z reduces the marginal cost of energy use, which increases e. For emissions to fall in z we thus need e to be relatively insensitive to changes in the tax component of energy costs, t/z. In a similar manner, the effect of an increase in t on tax revenue, s, can be separated in two effects. One the one hand, given emissions, an increase in t increases tax revenue. On the other hand, the increase in t reduces energy use, reducing emissions and tax revenues. By assuming $m_z < 0$, we implicitly assume e is relatively insensitive to changes in t/z, and as a consequence we also find $s_t > 0$. Note that this does not imply the downward-sloping part of the Laffer curve does not arise in our specification. For our analysis, we restrict our attention to the case where we have $m_z < 0$ and thereby $s_t > 0$.

Lemma A1 then allows us to establish the following:

Lemma A2 We can define some (hypothetical) firm $g > 0$ such that for firms $i > g$, $Z_t^R(i) > 0$ and for firms $i < g$, $Z_t^R(i) < 0$.[10]

Proof By (15), we have $Z_t^R(i) = C_Z^{-1}(i)\xi\left[\pi_t(i) + s_t\right]$ where $\pi_t(i) = -e(i)/z(i) < 0$ and $s_t > 0$ and equal across firms. Since firms are ranked according to z, where $z(0) < z(i)$ for all $i > 0$, this expression is unambiguously

[10] Here, a comment similar to footnote 5 applies. Unless each firm i has its unique $z(i)$, g may be a group of firms. Without affecting any of the main results, we will treat g as a single firm in the remainder of this appendix.

negative for firm $i = 0$ and larger (less negative) for higher i. This implies there is exists some unique emission efficiency $z' < \infty$, which satisfies $Z_t^R = 0$. Now define some firm g, such that $z(g) = z'$. Then we must have $Z_t^R(i) > 0$ for firms $i > g$ and $Z_t^R < 0$ for firms $i < g$. Note that g need not fall inside the support, i.e. we may have $g > 1$ and no firm may gain from tax increases. ■

Since emissions are falling in emission efficiency z, the least efficient firms pay the most in taxes. As profits are rising in z, this implies that the tax scheme is regressive: it relatively benefits high profit (high z) firms, and harms low profit (low z) firms. With a lump sum redistribution scheme, we must thus find that more efficient firms are more likely to gain from tax increases.

A.2 Optimal environmental policy

The social planner's problem now reads

$$\max_{t,T} \int_0^1 [\pi + s - \Delta m + \Pi - C + S - \Delta M]\, di$$
$$\text{subject to } C \leq \xi\, [\pi + s]$$

Using $\Pi_E = 0$, $S = T\int_0^1 Z^{-1} E di$, the first order condition with respect to T gives $(T - \Delta)\int_0^1 Z^{-1} E_T di = 0$ and we still find $T^* = \Delta$ and independent of a and ξ. Also for t, we still find (8), which we slightly rewrite:

$$-(t^* - \Delta)\int_0^1 \frac{e_t}{z} di = \int_g^n [\Pi_Z - C_Z]\, Z_t^R di + \int_0^g [\Pi_Z - C_Z]\, Z_t^R di. \tag{16}$$

As Z_t^R is positive for firms $i \in [g, n]$ and negative for firms $i \in [0, g]$, the sign of the RHS of (16) is not directly obvious. This raises the question whether the fact that some constrained firms gain from tax increases implies that the optimal tax exceeds marginal damages. By some tedious algebra, we can however show that Proposition 1 still applies:

Proposition A1 As long as some firms are credit constrained ($n > 0$), the first-period tax falls short of the Pigouvian tax ($t^* < \Delta$).

Proof Two cases can be distinguished. First, if $n \leq g$, Z^R is decreasing in t for all constrained firms. In this case, (16) collapses to (8) and the proof to Proposition 1 applies. If $g < n$, Z^R is increasing in t for some constrained firms and the proof runs as follows.

1. As $g < n$, we must have $\Pi_Z(g) - C_Z(g) > 0$. Next, we have $\int_0^1 Z_t^R(i) di = \xi \int_0^1 C_Z^{-1}(i)\left[-e(i)/z(i) + \int_0^1 s_t di\right] di$ where we know $\int_0^1 \left[-e(i)/z(i) + \int_0^1 s_t di\right] < 0$ and smaller (more negative) the lower $z(i)$. As $Z_z^R > 0$ and $C_{ZZ} > 0$, this must imply $\int_0^1 Z_t^R(i) di < 0$. By Lemma A2, we must have $\int_n^1 Z_t^R > 0$ and thus $\int_0^n Z_t^R(i) di < 0$. This gives $[\Pi_Z(g) - C_Z(g)]\left[\int_0^g Z_t^R(i) di + \int_g^n Z_t^R(i) di\right] < 0$

2. Using $\Pi_{ZZ} - C_{ZZ} < 0$, we have $\Pi_Z(g) - C_Z(g) < \Pi_Z(i) - C_Z(i)$ for $i < g$. By Lemma A2, we have $Z_t^R < 0$ for $i < g$ which gives $[\Pi_Z(g) - C_Z(g)]\int_0^g Z_t^R(i) di > \int_0^g [\Pi_Z(i) - C_Z(i)]\, Z_t^R(i) di$. In a similar manner, we have $\Pi_Z(g) - C_Z(g) > \Pi_Z(i) - C_Z(i)$ and $Z_t^R > 0$ for $i > g$ which implies $[\Pi_Z(g) - C_Z(g)]\int_g^n Z_t^R(i) di > \int_g^n [\Pi_Z(i) - C_Z(i)]\, Z_t^R(i) di$.

3. By $[\Pi_Z(g) - C_Z(g)]\left[\int_0^g Z_t^R(i) di + \int_g^n Z_t^R(i) di\right] < 0$, $[\Pi_Z(g) - C_Z(g)]\int_0^g Z_t^R(i) di > \int_0^g [\Pi_Z(i) - C_Z(i)]\, Z_t^R(i) di$ and $[\Pi_Z(g) - C_Z(g)]\int_g^n Z_t^R(i) di > \int_g^n [\Pi_Z(i) - C_Z(i)]\, Z_t^R(i) di$ we know

$\int_g^n [\Pi_Z(i) - C_Z(i)] Z_t^R(i)di + \int_0^g [\Pi_Z(i) - C_Z(i)] Z_t^R(i)di < 0$. Using (16) and $e_t < 0$ this implies $t^* < \Delta$. ■

Even if, under the lump-sum redistribution scheme, a tax increase allows some constrained firms to increase investment, this is no rationale for increasing the emission tax above the marginal emission damages. Next to the constrained firms that can increase investment, there are always constrained firms that are forced to reduce their investment due to a tax increase, and the social costs of this reduction in investment by the latter outweighs the benefits of increased investment opportunities to the former. This is due to the fact that the highest-emission firms are harmed disproportionally by tax increases. So, even with a lump-sum redistribution, low taxes continue to favor constrained firms' investment overall, which implies that we find an optimal tax that falls short of marginal emission damages, Δ.

A.3 Optimal policy response to shocks

By $T^* = \Delta$, $T^*$is acyclical. As in the main text, the sensitivity of t^* to a and ξ is governed by the externality, investment sensitivity, investment, and persistence effects. However, as, through the recycling scheme, the tax affects firms $i \in [g,n]$ differently than firms $i \in [0,g]$, we must now further separate the effects across groups.

We first solve for t_a^*. Using (16), we find

$$t_a^* = B^{-1} \left[\underbrace{(t^* - \Delta) \int_0^1 \frac{e_{ta}}{z} di}_{\substack{\text{emission sensitivity} \\ \text{effect } (+/-)}} + \left[\begin{array}{c} \int_{\min\{n,g\}}^n \left[\underbrace{[\Pi_Z - C_Z] Z_{ta}^R}_{\substack{\text{investment sensitivity} \\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}] Z_a^R Z_t^R}_{\substack{\text{investment value} \\ \text{effect } (-)}} + \underbrace{\mu \Pi_{ZA} Z_t^R}_{\substack{\text{persistence} \\ \text{effect } (+)}} \right] di \\ + \int_0^{\min\{n,g\}} \left[\underbrace{[\Pi_Z - C_Z] Z_{ta}^R}_{\substack{\text{investment sensitivity} \\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}] Z_a^R Z_t^R}_{\substack{\text{investment value} \\ \text{effect } (+)}} + \underbrace{\mu \Pi_{ZA} Z_t^R}_{\substack{\text{persistence} \\ \text{effect } (-)}} \right] di \end{array} \right] \right], \tag{17}$$

with $B > 0$ defined as in the main text. If $g \geq n$, the top line equals zero and the result collapses to (9). Note however that the partial derivatives of Z now include the effect of the shock though the lump sum rebate. If $g < n$, we find that for firms who benefit from tax increases and for firms to whom a marginal tax increase is a net cost, the investment sensitivity effect continues to be ambiguous (and may be of opposite signs for both groups). The market failure effect is still positive for the most constrained group (calling for a higher tax in booms), yet turns negative for firms $i \in [g,n]$. This can be explained as follows: a positive productivity shock reduces the size of the market failure, which allows the tax to be refocused on emissions. As for firms $i \in [g,n]$, a high tax alleviates their constraint more than a low tax, a boom creates room for lower taxes. Also the persistence effect differs across groups. A persistent boom increases the benefit of investment for all constrained firms, which calls for a tax favoring investment. For firms $i \in [g,n]$, this is a higher tax, firms $i \in [0,g]$ would call prefer a lower tax. As in the main text, t_a^* is ambiguous without further functional specification.

In a similar manner, we can evaluate the effect of a credit shock on t^*. Taking the total derivative of (16)

$$t_\xi^* = B^{-1} \left[\begin{array}{c} \int_{\min\{n,g\}}^n \left[\underbrace{[\Pi_Z - C_Z] Z_{t\xi}^R}_{\substack{\text{investment sensitivity} \\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}] Z_\xi^R Z_t^R}_{\substack{\text{investment value} \\ \text{effect } (-)}} \right] di \\ + \int_0^{\min\{n,g\}} \left[\underbrace{[\Pi_Z - C_Z] Z_{t\xi}^R}_{\substack{\text{investment sensitivity} \\ \text{effect } (+/-)}} + \underbrace{[\Pi_{ZZ} - C_{ZZ}] Z_\xi^R Z_t^R}_{\substack{\text{investment value} \\ \text{effect } (+)}} \right] di \end{array} \right], \tag{18}$$

Again, if $g \geq n$, the top line equals zero and the result collapses to (10). The investment sensitivity effect continues to be ambiguous (and may be of opposite signs for both groups). For the same reason as with the productivity shock, the investment value effect is negative for firms $i \in [g,n]$, and positive for firms $i \in [g,n]$, and absent further functional specifications, no conclusions can be drawn regarding the sign of t_{ξ}^{*}.

A.4 Example: specific functional form

Let's check if we can say more about the signs of t_a^* and t_{ξ}^* if we apply a reasonable (conventional) specification. Here we use the model from section 6. We show that both Proposition 2, establishing $t_a^* > 0$, and Proposition 3, which proves $t_{\xi}^* > 0$, continue to hold.

Using (12) and (14), (17) can be reduced to

$$t_a^* = B^{-1}\left[(t^* - \Delta)\int_0^1 \frac{e_t}{z}di + \gamma^{-1}\left[\begin{array}{c} \int_{\min\{n,g\}}^{n}\left[TM_Z - \gamma C_Z + \mu\Pi_Z\right]Z_t^R di \\ +\int_0^{\min\{n,g\}}\left[TM_Z - \gamma C_Z + \mu\Pi_Z\right]Z_t^R di \end{array}\right]\right], \tag{19}$$

and we can show Proposition 2 continues to hold:

Proposition A2 As long as some firms are credit constrained, the first-period tax should rise in response to a positive productivity shock.

Proof First, we know $M_Z = Z^{-2}\left[ZE_Z - E\right] < 0$ and $M_{ZZ} = Z^{-1}\left[E_{ZZ} - 2M_Z\right]$. We have $E_{ZZ} > 0$, which implies $M_{ZZ} > 0$. Hence, $T\int_0^n Z_t^R M_Z di > 0$. Next, $C_Z Z_t^R = -\xi z^{-1} e + s_t$, this gives $-\gamma\int_0^n C_Z Z_t^R > 0$. So if the shock is not persistent, i.e. $\mu = 0$, we must have $t_a^* > 0$. A persistent shock ($\mu > 0$) reduces t_a^*, but will never turn t_a^* negative. We know $\mu < 1$, then using (8), we have $\lim_{\mu\to 1} t_a^* = \gamma^{-1} T\int_0^n M_Z Z_t di > 0$, and we must have $t_a^* > 0$. ■

In a manner similar to the productivity shock, we can use (12), (14) and (18) to find

$$t_{\xi}^* = \left[\gamma\xi B\right]^{-1}\left[\begin{array}{c} \int_{\min\{n,g\}}^{n} Z_t^R\left[TM_Z - \gamma C_Z\right]di \\ +\int_0^{\min\{n,g\}} Z_t^R\left[TM_Z - \gamma C_Z\right]di \end{array}\right]. \tag{20}$$

This allows us to conclude that, with lump sum redistribution, taxes should fall in response to a financial contraction:

Proposition A3 As long as some firms are credit constrained, the first-period tax should rise in response to a positive credit shock.

Proof First, we know $M_Z = Z^{-2}\left[ZE_Z - E\right] < 0$ and $M_{ZZ} = Z^{-1}\left[E_{ZZ} - 2M_Z\right] > 0$ by $E_{ZZ} > 0$. This implies $T\int_0^n Z_t^R M_Z di > 0$. Now $C_Z Z_t^R = -\xi z^{-1} e + s_t$, this gives $\int_0^n C_Z Z_t^R di = \int_0^n z^{-1}\left[(1-\xi)e + te_t\right]di < 0$. Then by (20) we must have $t_{\xi}^* > 0$. ■