# Public and private health, pollution and growth

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**Abstract**

This paper presents an overlapping generations model where longevity is endogenously influenced by three factors: public health provision, pollution and the private health efforts undertaken by individuals. All public expenditures, that provide not only public health but also public abatement, are financed thanks to labour taxation. When pollution strongly affects the longevity, the economy may be dynamically unstable and experience endogenous cycles. An appropriate fiscal policy may rule out such fluctuations. However, depending on fundamentals, such a fiscal policy may deteriorate capital accumulation and welfare at the steady state, which may mitigate previous conclusions.

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## 1 Introduction

According to the World Health Organization, among the leading global risks for mortality is pollution and environmental deterioration (outdoor air pollution, indoor air pollution, unsafe water etc). Some disease and injury outcomes caused by risk exposures have been quantified in terms of deaths: For instance on a global scale, 8% of lung cancer deaths are attributable to deteriorated environmental conditions. Similarly, unsafe water might be responsible for 88% of diarrhoeal deaths. Indeed, worldwide health risks are in transition: while population is globally ageing, owing to successes against infectious diseases, people face nowadays new burdens of chronic and acute diseases and environmental factors are a big part of these "modern" risks. As suggested by the WHO, understanding the role and the origins of these health risks is vital to prevent from economic contraction or to fight against under-development.

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The economic literature has been very prolific with regards to the contribution of health and, in particular, longevity in the development process (Mirowsky and Ross (1998), Blackburn and Cipriani (2002), Chakraborty (2004), Cervelatti and Sunde (2005), Chakraborty and Das (2005)). In richer economies, individuals may afford medical care and medicines, they are often more educated and thus adopt healthier lifestyles meanwhile public authorities are capable of funding more public health services and to develop access to public health systems. To sum up, healthier agents are more productive, growth is promoted and in turn, the enhanced development process improves health status.

Yet, it is necessary to figure out the origins of health risks and their induced injuries. In fact, the causal chain of these risks and outcomes provides many entries points for intervention. We are particularly interested in these opportunities of intervention when dealing with environmental risks on health. Two types of actions are evident. On the one hand, the reduction of harmful pollution flows seems relevant; On the other hand, it could be the provision of health protection services or the improvement of preventive health programmes. Nevertheless, global costs of abating pollution or preserving environmental conditions are often disproportionate with regards to individuals capacity to afford them. Then, it seems that those actions are mainly undertaken by governments. With respect to health, the argument is more disputable. Even if governments remain main contributors to world health spending, individuals can invest in their own health capital. For instance, such efforts could include annual health screening, adoption of healthy life styles, but also out-of-pocket expenses for essential medication and private health services. All components are complement inputs in the production of health status as private efforts may induce larger improvements in life expectancy if they come along with public efforts to maintain a good environmental quality and to provide efficient public health care systems.

The aim of this paper is to show that these complementarities may lead to aggregate instability and endogenous cycles. In this case, we will investigate if the fiscal policy, designed here as the share of public spending devoted to public health, may restore stability. This possible stabilizing role of the policy will be compared to its effect on welfare and capital accumulation.

The paper presents a standard overlapping generations model with production augmented with endogenous longevity. Each young agent works for a competitive wage and survives to the start of the old age. Nevertheless, any agent is alive only for a fraction of the second period of life. In our set-up, agents only care about their second period consumption while alive. Then, in order to increase her time length, any agent may incur investment in her own health. In addition, her life expectancy is positively influenced by public expenditure in health while it reduces with the deterioration of the environment. Those public expenses are financed thanks to labour income taxation.

Under usual assumptions on preferences, technology and the law of motion of environmental quality overtime, there is a unique steady state. As soon as the effect of pollution on longevity is not too low, this steady state is unstable and endogenous cycles may occur. These fluctuations are explained by the complementary effects of health and pollution on longevity. An appropriate policy, through the choice of the share of public spending devoted to public health, may rule out endogenous cycles. Analyzing also the effects of such a policy on welfare, capital accumulation and pollution, evaluated at the steady state, we highlight that such a policy may be damaging, depending on fundamentals. Hence, a policy recommendation should strongly depends on how longevity is affected by pollution and public health, on the production technology and on the efficiency of abatement.

Our paper is close to Chakraborty (2004) who introduces endogenous mortality in a dynamic OLG set-up with production. In particular, the probability that an individual survives on to the old age depends on public health expenditures. Complementary with his approach, we introduce two additional components to the endogenous life expectancy. Following Bhattacharya and Qiao (2007), we consider simultaneously both private and public expenditures. We differ from these two papers as we focus on the environmental health risks. Consequently and more closely related with the papers by Varvarigos (2010) and Palivos and Varvarigos (2012), we also assume that environmental conditions are key determinants to longevity.

The paper is organized as follows. In Section 2, we present the model and discuss main assumptions. Then, in Section 3, we analyze the equilibrium in the long run. In Section 4, we explore the dynamic properties of our framework and finally, Section 5 concludes. Many technical details are relegated to an Appendix.

# 2 The model

## 2.1 Households

We consider an overlapping generations model, where time is discrete, $t = 0, 1, ..., \infty$ and the population size of a young generation is constant and normalized to one. When young, agents live during the whole period. However, they may survive to old age with a probability[1] $\pi_t \in [0, 1)$. When old, agents consume an amount $c$ at each moment of time. Since we focus on the links between pollution and longevity, we abstract from consumption choices in the first period of time. Consequently, preferences of an individual born at date $t$ are defined over the uncertain second period consumption and are represented by the following utility function:

$$U(c_{t+1}^t) = \pi_t u(c_{t+1}^t) \tag{1}$$

In our framework, longevity naturally is endogenous and crucially depends on a health indicator denoted by $\theta_t$. For tractability reasons, let us consider the following explicit function for life expectancy that satisfies usual properties (See for instance Blackburn and Cipriani (2002), Cervelatti and Sunde (2003) or

[1] $\pi_t$ can also be interpreted as longevity or life expectancy.

Castelló-Climent and Doménech (2008)):

$$\pi_t \equiv \pi(\theta_t) = \frac{b\theta_t}{1+\theta_t}, \tag{2}$$

with $0 < b < 1$. Therefore, $\pi(\theta_t)$ is increasing from $\pi(0) = 0$ to $\pi(+\infty) = b$ and is strictly concave.

In addition, the indicator of health status is partly shaped by private health expenditure among others variables that will be described below. These expenditure are realised when young so that, during their first period of life, agents use their labour income for two alternative purposes: private health expenditure ($x_t$) and savings ($s_t$). Moreover, a tax ($\tau \in [0, 1)$) is levied on labour income ($w_t$) in order to finance public expenditure. Thus, the first period budget constraint is written as:

$$x_t + s_t = (1-\tau)w_t \tag{3}$$

During their second period of life, they only consume the remunerated savings:

$$R_{t+1}s_t = c_{t+1}\pi_t, \tag{4}$$

where $R_{t+1}$ denotes the interest rate. Notice that following Chakraborty (2004), we assume a perfect annuity market, meaning that the return of total savings of young agents is distributed among survival old households.

Beyond the sole private health expenditure, we assume that longevity is also influenced by public health services $\eta_t$, which obviously improve health status, while the former may be cut-down by the current stock of pollution, denoted by $P_t$. Accordingly, let us express the index of health status as:

$$\theta_t = \frac{x_t^{\alpha}\eta_t^{1-\alpha}}{P_t^{\beta}} \tag{5}$$

with $0 \leqslant \alpha \leqslant 1$ and $\beta \geqslant 0$. As in Bhattacharya and Qiao (2007), this equation captures the possible interactions between public and private components in the health sector, *i.e.* their complementarity. In contrast and differently from their paper, we introduce an additional element which is the pollution stock that negatively affects the efficiency of both types of expenditures, as soon as $\beta > 0$.

Finally, in order to perform a deep and relevant analysis in this set-up, we consider the following explicit instantaneous constant inter-temporal elasticity of substitution utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{6}$$

with $0 < \sigma < 1$ the inter-temporal elasticity of substitution, which ensures that utility is increasing with life expectancy.

Pollution and public health care being given, individuals maximize the utility function (1) substituting (2), (6) and (5), facing the two budget constraints (3) and (4), and $x_t \geqslant 0$. The First Order Condition yields:

$$\frac{1}{1+\theta_t}\frac{\alpha}{x_t} \leq \frac{1-\sigma}{\sigma s_t} \tag{7}$$

with equality if $x_t > 0$.[2] This equation reflects the trade-off agents face to when choosing the amount of private health expenditure. On the one hand, agents invest in $x_t$ in order to live longer and thus to increase their utility; On the other hand, these expenditure reduce the revenue to be saved for future consumption and thus lower future utility. A deeper analysis of equation (7) allows us to state that there is no corner solution and no equilibrium without private health expenditure in our set-up:

**Lemma 1** *Agents always invest a strictly positive amount of their income in private health expenditure, so that* $x_t > 0$ $\forall t \geqslant 0$.

**Proof.** See Appendix A. ■

Beyond the complementarity between private and public health services, the reasoning is quite intuitive.[3] Starting from a very low value of private health investment (for instance $x_t$ close to 0), the marginal benefit of investing in private health is arbitrarily large while the marginal utility loss associated to a reduced consumption is low. Therefore, agents always have incentives to engage in private health.

## 2.2 Public sector

As mentioned previously, public authorities may intervene through the provision of two types of public services: first, the government provides public health expenditure; second, she may also engage environmental protection actions ($G_t$) in order to reduce the negative effect of pollution on health. We consider that due to the extensively large costs associated with pollution abatement, only the government may afford them (implying that agents do invest in private maintenance). Both expenses are financed by labour income taxation and can be expressed as:

$$\eta_t = \mu\tau w_t \tag{8}$$

$$G_t = (1-\mu)\tau w_t \tag{9}$$

[2] Let us note that the second order condition is satisfied. Indeed, the FOC can be rewritten:

$$\frac{\partial \pi_t}{\partial x_t}\sigma u(c_{t+1}) - R_{t+1}u'(c_{t+1}) \leqslant 0$$

We deduce that the Second Order Condition is given by:

$$\frac{\partial^2 \pi_t}{\partial x_t^2}\sigma u(c_{t+1}) - \left(\frac{R_{t+1}}{\pi_t} + \frac{c_{t+1}}{\pi_t}\frac{\partial \pi_t}{\partial x_t}\right)\left[\frac{\partial \pi_t}{\partial x_t}\sigma u'(c_{t+1}) - R_{t+1}u''(c_{t+1})\right] < 0$$

[3] Indeed, Lemma 1 holds even if we consider for instance a CES function to build the health indicator, $\theta_t$. Then, even tough there is no *a priori* complementarity between public and private health expenses, we can show that agents always invest in private health expenditure.

where $\mu \in (0,1]$ is the share of the public revenue devoted to public health spending.[4] In this framework, and different from Raffin and Seegmuller (2013), any increase in $\mu$ reduces just as much the effort of public abatement.

## 2.3 Pollution

In addition, we introduce an additional dynamic variable to account for the deterioration of the environmental quality: the stock of pollution. Then, we describe the evolution of the pollution stock according to the following law of motion:

$$P_{t+1} = (1-m)P_t + \epsilon_1 Y_t - \epsilon_2 G_t \tag{10}$$

with $m \in (0,1)$, $\epsilon_1 \geqslant 0$, $\epsilon_2 \geqslant 0$, and $P_0 \geqslant 0$ given. In other words, environmental quality is deteriorated by the stock of pollution induced by the production process but may be enhanced thanks to public spending. As for the parameter $m$, it captures natural inertia phenomena, since nature can not entirely absorb pollution flows at each period of time. Finally, parameters $\epsilon_1$ and $\epsilon_2$ reflect the cleanness degree of production and the efficiency of public abatement, respectively.

## 2.4 Production

Finally, there is a unique final good that is produced by a continuum of unit size competitive firms using the following neo-classical technology $Y_t = F(K_t, L_t)$, where $Y_t$ is aggregate production, $K_t$ aggregate capital and $L_t$ aggregate labour. The production function displays usual properties and is homogeneous of degree 1. Let $k_t \equiv K_t/L_t$ denote the capital per capita and $f(k_t) \equiv F(k_t, 1)$ the intensive form of the production function. In order to lead a proper analysis of the equilibrium and the dynamics of the economy, we make the following assumption:

**Assumption 1** *$f(k)$ is a continuous function defined on $[0,+\infty)$ and $C^2$ on $(0,+\infty)$, strictly increasing ($f'(k) > 0$) and strictly concave ($f''(k) < 0$). There exists $\widetilde{k} > 0$ such that $f(\widetilde{k})/\widetilde{k} - f'(\widetilde{k}) > \left(1 + \alpha\frac{\sigma}{1-\sigma}\right)/(1-\tau)$. Defining $s(k) \equiv f'(k)k/f(k) \in (0,1)$ as the capital share in total income and $\rho(k) \equiv [f'(k)k/f(k) - 1]\, f'(k) / [kf''(k)] > 0$ as the elasticity of capital-labour substitution, we further assume $\rho(k) > \max\{2s(k); 1 - s(k)\}$ and $s(k) < 1/2$.*

Let us remark that $R_t$ accounts for the real interest rate and that capital fully depreciates after one period of use. Then, profit maximization yields:

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \tag{11}$$

$$R_t = f'(k_t) \equiv R(k_t) \tag{12}$$

We can easily deduce that $\frac{w'(k)k}{w(k)} = \frac{s(k)}{\rho(k)}$ and $\frac{R'(k)k}{R(k)} = -\frac{1-s(k)}{\rho(k)}$.

[4] Note that we assume $\mu \neq 0$ to exclude a zero utility coming from no longevity.

# 3 Equilibrium

Being given that the population size of a young working generation equals one, the equilibrium in the labour market is ensured by $L_t = 1$. Equilibrium on the capital market is satisfied when $k_{t+1} = s_t$. From the budget constraint (3), this directly implies that $x_t = (1-\tau)w(k_t) - k_{t+1}$. Moreover, using (11), we can express the levels of public services as:

$$\begin{aligned} \eta_t &= \mu\tau w(k_t) && (13) \\ G_t &= (1-\mu)\tau w(k_t) && (14) \end{aligned}$$

Substituting (11) and (14) into equation (10), we obtain:

$$P_{t+1} = (1-m)P_t + \epsilon_1 f(k_t) - \epsilon_2(1-\mu)\tau w(k_t) \tag{15}$$

In addition, we can rewrite equation (7) with equality as:

$$k_{t+1} = \frac{1+\theta_t}{1+\alpha\frac{\sigma}{1-\sigma}+\theta_t}(1-\tau)w(k_t) \tag{16}$$

with

$$\theta_t = \frac{[(1-\tau)w(k_t) - k_{t+1}]^{\alpha}[\mu\tau w(k_t)]^{1-\alpha}}{P_t^{\beta}}$$

For a given level of wealth, the health status might differ according to the design of the public intervention. We can observe that, for a given amount of capital stock, if the whole public revenue is devoted to pollution abatement, the health indicator is reduced to 0. But, the latter is still positive when the public receipts are entirely devoted to public health. Nevertheless, it does not imply that the relationship between $\theta$ and $\mu$ is monotonous. In particular, it does not necessarily imply that $\mu = 1$ is the best solution to attain the highest possible level of health status. In particular, under proper conditions, an inverted U-shape could feature the relationship between the health indicator and the policy design. Formally, it could be the case that the health indicator reaches its highest level for a value $\hat{\mu}$, so that for any $\mu < \hat{\mu}$, it is relevant to increase the share of public receipts dedicated to health while beyond this threshold, it is more appropriate to reduce it. This is the result of the two opposite effects induced by a change in $\mu$. On the one hand, it enhances the health indicator through an increase in public health provision; but, on the other hand, it reduces the same indicator by increasing the stock of current pollution.

Given $k_0 \geqslant 0$ and $P_0 \geqslant 0$, equations (15) and (16) describe the dynamics of the economy through the evolution of the couple $(k_t, P_t)$, for all $t \geqslant 0$.

In order to stick with reality, we assume that the stock of pollution is an increasing function of physical capital accumulation. Since $w(k)$ is increasing and smaller than $f(k)$, this is satisfied for:

**Assumption 2** $\epsilon_1 > \epsilon_2$

Assumption 2 means that globally pollution flows exceed abatement flows as it can be observed in real world. In fact any economy is able to clean-up all emitted pollution flows. Nevertheless, as the economy gets richer, the difference between production-induced pollution and abatement may narrow. Formally, this parametric assumption also implies that the available technology of abatement is not so much efficient with respect to the emission rate of pollution per unit of production. Again, this seems to be consistent with empirical evidences.

Once we have identified the non-linear two-dimensional system that describes the evolution of the economy over time, we perform an analysis of the existence, uniqueness and further the stability of the stationary equilibrium, when it exists.

## 3.1 Stationary equilibrium

A steady state is defined by $k_t = k$ and $P_t = P$ for all $t$. Using equations (15) and (16), we obtain:

$$P = \frac{[(1-\tau)w(k)-k]^{\frac{\alpha+1}{\beta}}[\mu\tau w(k)]^{\frac{1-\alpha}{\beta}}}{\left[\left(\alpha\frac{\sigma}{1-\sigma}+1\right)k-(1-\tau)w(k)\right]^{\frac{1}{\beta}}} \equiv \varphi(k) \tag{17}$$

$$P = \frac{1}{m}[\epsilon_1 f(k) - \epsilon_2(1-\mu)\tau w(k)] \equiv \psi(k) \tag{18}$$

Using these two equations, we show the existence and the uniqueness of the stationary solution $(k^*, P^*)$:

**Proposition 1** *Let $\overline{k}$ and $\underline{k}$ be defined by:*

$$\overline{k} = (1-\tau)w(\overline{k}) \tag{19}$$

$$\underline{k} = \frac{(1-\tau)w(\underline{k})}{1+\alpha\frac{\sigma}{1-\sigma}} \tag{20}$$

*Under Assumptions 1-2, there exists a unique steady state $k^* > 0$ that belongs to $(\underline{k}, \overline{k}]$. This implies that there exists a unique value $P^* > 0$.*

**Proof.** See Appendix B. ■

The stationary solution eventually reached by one economy crucially depends on the public policy implemented by the government. We perform such an analysis in the following.

## 3.2 Comparative statics

In this section, we investigate the effects of an increase in the share of public revenue devoted to public health $\mu$ on the stationary level of capital $k^*$, pollution $P^*$ and welfare $W^*$. But, let us first define the stationary welfare $W_t = W$ such as:

$$W \equiv \pi(\theta)\frac{c^{1-\sigma}}{1-\sigma} \tag{21}$$

As a preliminary result, we can show that the welfare at the steady state can be defined as an increasing function of physical capital only:

**Lemma 2** *Under Assumptions 1-2, the welfare evaluated at the steady state $W^*$ is an increasing function of capital, i.e. $W^* \equiv W(k^*)$ with $W'(k^*) > 0$.*

**Proof.** See Appendix C. ■

Indeed, using (4) we can observe that consumption at the steady state is an increasing function of $k^*$ and $\theta$, $c = c(k^*, \theta)$. Similarly, exploring equation (5) and using (16), we deduce that the indicator of health merely depends positively on the stock of physical capital at the steady state, $\theta = \theta(k^*)$. This implies that the richer the economy, the better the health status, no matter the level of pollution. In other words, the increasing expenditure in private and public health always offset the negative effect of a growing stock of pollution.

We now study the consequences of a change in the public policy design, that is a raise of $\mu$, on the stationary levels of capital, welfare and pollution:

**Proposition 2** *Under Assumptions 1-2, an increase of $\mu$ has the following effects:*

1. *$k^*$ and $W^*$ are increasing if and only if $(1-\alpha)[\epsilon_1 f(k^*) - \epsilon_2(1-\mu)\tau w(k^*)] > \beta\epsilon_2\mu\tau w(k^*)$, which is satisfied for $1-\alpha > \beta\frac{\epsilon_2\mu\tau}{\epsilon_1 - \epsilon_2(1-\mu)\tau}$;*
2. *$k^*$ and $W^*$ are decreasing if and only if $(1-\alpha)[\epsilon_1 f(k^*) - \epsilon_2(1-\mu)\tau w(k^*)] < \beta\epsilon_2\mu\tau w(k^*)$, which is satisfied for $1-\alpha < \beta\frac{\epsilon_2\mu\tau}{2\epsilon_1}$;*
3. *$P^*$ is always increasing.*

**Proof.** See Appendix D. ■

The global effects on the welfare are deduced from Lemma 2. Dealing with pollution, the raising share of public revenue devoted to public health (detrimental to the effort of public abatement) displays a clear cut outcome: Pollution always raises. As for the welfare *per se* or the stock of capital, conclusions are more ambiguous. By direct inspection of (16), we know on the one hand, that capital accumulation raises with respect to longevity and the index of health status $\theta$. On the other hand, as already evoked above, a larger value of $\mu$ exhibits two opposite effects on this index of health: A positive one through a larger level of public health expenditure and a negative one because this simultaneously lowers public environmental protection, which raises pollution. Proposition 2 shows that the first (second) effect dominates when the weight of public health $1-\alpha$ on the index is sufficiently larger (lower) than the weight of pollution $\beta$. Proposition 2 implies that when the weight of public health is relatively large, the negative effect of a growing pollution stock is overweighted by an improvement in health status. Thus, overall, the welfare is enhanced at the steady state. On the contrary, when this weight is too small, a public policy in favour of health displays a negative effect on the quality of the environment, which seems to offset the positive effect on health and capital accumulation. Beyond this result, we may also investigate the distribution of public revenue consequences, depending on the initial value of $\mu$ it-self. In particular, we can show that if the share devoted to public health is already relatively high, then, it is more likely that configuration *ii)* occurs. Conversely, for a relatively low initial value of $\mu$, configuration *i)* is more probable. Thus, if the relationship between the capital stock and the share of public receipt devoted to public health is non-monotonous, it seems that there is a room for an optimal policy design, in order to maximise welfare at the steady state.

Note that Proposition 2 gives necessary and sufficient conditions as well as sufficient conditions to evaluate the effect of $\mu$ on capital and stationary welfare. While the last ones do not depend on the steady state $k^*$, the necessary and sufficient conditions do. Therefore, it is interesting to focus on the case of a Cobb-Douglas technology where more clear-cut conclusions can be obtained.

**Corollary 1** *Let*

$$\widehat{\mu} \equiv \frac{(1-\alpha)(\epsilon_1 - \epsilon_2\tau(1-s))}{\epsilon_2\tau(1-s)(\beta-(1-\alpha))} \tag{22}$$

*Under Assumptions 1-2, assume further that* $f(k) \equiv Ak^s$, *with* $A > 0$ *and* $0 < s < 1/2$.

1. *If* $\beta \leqslant (1-\alpha)\epsilon_1/[\epsilon_2\tau(1-s)]$, $k^*$ *and* $W^*$ *are increasing in* $\mu$ *for all* $\mu \in (0,1]$.

2. *If* $\beta > (1-\alpha)\epsilon_1/[\epsilon_2\tau(1-s)]$, $k^*$ *and* $W^*$ *are increasing in* $\mu$ *for all* $\mu \in (0,\widehat{\mu}]$ *and decreasing in* $\mu$ *for all* $\mu \in [\widehat{\mu},1]$.

**Proof.** See Appendix E. ■

The first result of this corollary confirms Proposition 2. When the weight of pollution in the health index is lower than the weight of public health, a larger share of public spending devoted to public health always induces a larger level of capital and welfare, through its positive effect on longevity that fosters saving.

The second result is even more interesting. It highlights an inverse U-shaped relationship between $\mu$ and $k^*$ or $W^*$. It also means that there is a level of the share of public spending devoted to public health that maximizes welfare and the stationary level of capital. We argue that this may still be explained through the effects of $\mu$ on the health index and therefore, on longevity. Capital and welfare both increase under a larger longevity. Recall that an increase of the share of public spending devoted to public health raises public health, but also pollution. When $\mu$ is low, because of the concavity of the longevity with respect to the health index, the first effect dominates, whereas the effect on pollution dominates when $\mu$ is sufficiently large and close to 1.

# 4 Dynamics

For ease of presentation, we establish that choosing an appropriate value of $m \in (0,1)$ allows us to normalize the steady state $k^*$, such that $k^* = 1$. Then, we lead the analysis of the dynamics and, in particular, we study the stability properties of the long run equilibrium.

**Proposition 3** *Let*

$$\bar{\epsilon}_1 \equiv \frac{[(1-\tau)w(1)-1]^{\frac{\alpha+1}{\beta}}[\mu\tau w(1)]^{\frac{1-\alpha}{\beta}}}{f(1)\left(\frac{\alpha\sigma}{1-\sigma}+1\right)^{1/\beta}} \tag{23}$$

*Assuming that* $\epsilon_1 < \bar{\epsilon}_1$,[5] $1 < (1-\tau)w(1) < 1+\alpha\frac{\sigma}{1-\sigma}$ *and Assumptions 1-2 hold, there exists a unique* $m^* \in (0,1)$ *that ensures that* $k^* = 1 \in (\underline{k}, \overline{k})$.

**Proof.** See Appendix F. ■

Consequently, we can deduce the level of pollution associated to the normalized steady state $k^* = 1$:

$$P^* = \frac{[(1-\tau)w-1]^{\frac{\alpha+1}{\beta}}[\mu\tau w]^{\frac{1-\alpha}{\beta}}}{\left[\alpha\frac{\sigma}{1-\sigma}+1-(1-\tau)w\right]^{\frac{1}{\beta}}}, \tag{24}$$

where we denote $w \equiv w(1)$, meanwhile we define $s \equiv s(1)$ and $\rho \equiv \rho(1)$. To analyse the stability properties of the steady state, we differentiate the dynamic system (15)-(16) around the normalized steady state. Then, we can claim the following:

**Proposition 4** *Assuming that* $\epsilon_1 < \bar{\epsilon}_1$, $1 < (1-\tau)w < 1+\alpha\frac{\sigma}{1-\sigma}$ *and Assumptions 1-2 hold, the characteristic polynomial evaluated at the steady state* $k^* = 1$, *is given by* $P(\lambda) = \lambda^2 - T(\rho)\lambda + D(\rho) = 0$, *where:*

$$T(\rho) = 1 - m^* + \frac{s}{\rho}\frac{1+\frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1+\frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \tag{25}$$

$$\begin{aligned} D(\rho) &= (1-m^*)\frac{s}{\rho}\frac{1+\frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1+\frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \\ &+ m^*\frac{s}{\rho}\frac{\rho\epsilon_1-\epsilon_2(1-\mu)\tau(1-s)}{\epsilon_1-\epsilon_2(1-\mu)\tau(1-s)}\frac{\frac{\beta\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1+\frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \end{aligned} \tag{26}$$

*and*

$$\theta = \frac{\alpha\sigma/(1-\sigma)+1-(1-\tau)w}{(1-\tau)w-1} \tag{27}$$

[5] Of course, this last inequality may be compatible with Assumption 2.

**Proof.** See Appendix G. ■

Since the two dynamic variables $k_t$ and $P_t$ are predetermined, the steady state can never be locally indeterminate. Nevertheless, we can determinate the stability properties of the steady state, *i.e.* whether it is a saddle, a source or a sink. In this last case only, the dynamic path locally converges to the steady state. We can also focus on the existence of endogenous cycles studying the occurrence of bifurcations. To simplify our proofs, we assume:

**Assumption 3** $s \geqslant 1/3$.

This parametric assumption directly implies that $2s \geqslant 1 - s$ and according to Assumption 1, $\rho \in (2s, +\infty)$. Then, we can establish a first result on the stability properties of the steady state:

**Proposition 5** *Assuming that* $\epsilon_1 < \bar{\epsilon}_1$, $1 < (1-\tau)w < 1 + \alpha\frac{\sigma}{1-\sigma}$ *and Assumptions 1-3 hold, the steady state can never be a saddle. It is either a sink or a source.*

**Proof.** See Appendix H. ■

This proposition shows that starting in a neighbourhood of the steady state, the economy either converges to this equilibrium or there is instability. Choosing the elasticity of capital-labour substitution $\rho$ as a bifurcation parameter, we clarify whenever one of these two stability configurations occurs. We discuss the results according to the level of $\beta$, *i.e.* the effect of pollution on longevity.

**Proposition 6** *Let*

$$A_\beta \equiv \frac{m^* s\epsilon_1}{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)} \frac{\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \tag{28}$$

$$B_\beta \equiv (1-m^*)\Lambda(2s) < 1 \tag{29}$$

$$C_\beta \equiv \frac{m^*}{2}\frac{2s\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)} \frac{\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \tag{30}$$

$$\Delta_\beta \equiv \left[1 - m^* - \frac{s}{\rho}\frac{1 + \frac{1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}\right]^2 \frac{\rho}{4m^* s} \frac{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}{\rho\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}\left[\frac{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma)}{\theta\alpha\sigma/(1-\sigma)} + \frac{\alpha}{(1-\tau)w-1}\right] \tag{31}$$

*Assuming that* $\epsilon_1 < \bar{\epsilon}_1$, $1 < (1-\tau)w < 1 + \alpha\frac{\sigma}{1-\sigma}$ *and Assumptions 1-3 hold, we have:*

1. *If* $\beta$ *is sufficiently low, satisfying* $\beta < \min\{A_\beta^{-1}; (1-B_\beta)C_\beta^{-1}\}$*, the steady state is a sink for all* $\rho \in (2s, +\infty)$*;*

2. *If $\mu < 1 - \epsilon_1 s/[\epsilon_2\tau(1-s)]$, $\alpha$ sufficiently low and $\beta$ larger but close to $1/A_\beta$, there exists $\rho_H \in (2s, +\infty)$ such that the steady state is a sink for $\rho \in (2s, \rho_H)$, a Hopf bifurcation generically occurs for $\rho = \rho_H$, and the steady state is a source for $\rho \in (\rho_H, +\infty)$;*

3. *If $\mu$ is close to 1 and $\beta$ smaller but close to $1/A_\beta$, there exists $\rho_H \in (2s, +\infty)$ such that the steady state is a source for $\rho \in (2s, \rho_H)$, a Hopf bifurcation generically occurs for $\rho = \rho_H$, and the steady state is a sink for $\rho \in (\rho_H, +\infty)$;*

4. *If $\beta$ is sufficiently large, satisfying $\beta > \max\{A_\beta^{-1}; (1-B_\beta)C_\beta^{-1}\}$, the steady state is a source for all $\rho \in (2s, +\infty)$.*

*In addition, when $\beta < \Delta_\beta$ the eigenvalues are real, whereas when $\beta > \Delta_\beta$, the eigenvalues are complex conjugates.*

**Proof.** See Appendix I. ■

When pollution weakly affects longevity ($\beta$ sufficiently low), the economy converges to the steady state. Of course, it may be the case that such a stable dynamic path experiences oscillations. On the contrary, when pollution strongly affects longevity, the steady state is unstable.

Between these two configurations, there is an interesting case. The economy converges if the elasticity of capital-labour substitution is not too large. For larger values of this elasticity, the steady state looses its stability through the occurrence of a Hopf bifurcation, meaning that an endogenous cycles appears around the steady state. This happens if $\beta$ is not too weak but $\alpha$ is sufficiently weak, i.e. an endogenous cycle occurs if the effects of public health and pollution on longevity are high enough.

Note that since $s < 1/2$, the required condition $\epsilon_1 < \epsilon_2(1-\mu)\tau(1-s)/s$ to have a Hopf bifurcation may be in accordance with Assumption 2. Of course, if the share of government revenue to public health $\mu$ is too close to 1, our sufficient conditions to have endogenous cycles are no more fulfilled. By direct inspection of the proof of Proposition 6, we may have instability for all $\rho \in (2s, +\infty)$.

We now investigate more deeply the effect of $\mu$ on the stability properties of the steady state focusing on the most interesting configuration, i.e. case 2 of Proposition 6.

**Corollary 2** *Assuming that $\epsilon_1 < \bar{\epsilon}_1$, $1 < (1-\tau)w < 1 + \alpha\frac{\sigma}{1-\sigma}$, Assumptions 1-3 hold, $\epsilon_1 < \epsilon_2(1-\mu)\tau(1-s)/s$, $\alpha$ sufficiently low and $\beta$ larger but close to $1/A_\beta$, the critical value $\rho_H$ is decreasing in $\mu$ and becomes smaller than $2s$ for $\mu$ sufficiently close to 1.*

**Proof.** See Appendix J. ■

This corollary establishes that when the share of government revenue devoted to public health $\mu$ increases, the range of elasticities of capital-labour substitution for instability $(\rho_H, +\infty)$ raises. When $\mu$ is sufficiently large, we can even get instability for all $\rho \in (2s, +\infty)$. Therefore, a larger share of government revenue to public health promotes instability. This means that if a government would like to rule out instability, it has to increase the share of its revenue to finance environmental maintenance.

Another implication of this corollary is that a Hopf bifurcation may occur for reasonable values of the elasticity of capital-labour substitution, i.e. close to 1. Indeed, for an appropriate choice of $\beta$ with respect to $1/A_\beta$ and of $\mu$, $\rho_H$ may take values close to 1. This means that endogenous cycles can occur for values of the elasticity of capital-labour substitution in accordance with empirical estimates.

# 5 Conclusion

This paper presents an overlapping generations model where longevity is endogenously influenced by three factors: public health provision, pollution and the private health efforts undertaken by individuals. All public expenditures, that provide not only public health but also public abatement, are financed thanks to labour taxation. Under usual assumptions on preferences, technology and the law of motion of environmental quality overtime, there is a unique steady state. As soon as the effect of pollution on longevity is not too low, this steady state is unstable and endogenous cycles may occur. These cycles stem from the complementarity between private and public health expense. An appropriate fiscal policy may rule out such fluctuations. However, depending on fundamentals, such a fiscal policy may deteriorate capital accumulation and welfare at the steady state, which may mitigate previous conclusions.

# 6 Appendices

## A Proof of Lemma 1

First of all, to prove Lemma 1, we can easily show that $\lim_{x_t \to 0} \theta_t = 0$. Let us note $\Gamma_t > 0$ the left-hand side of inequality (7), *i.e.* $\Gamma_t \equiv \frac{1}{1+\theta_t}\frac{\alpha}{x_t}$. Then, we can demonstrate that $\lim_{x_t \to 0} \Gamma_t = +\infty$. Therefore, $\Gamma_t < \frac{1-\sigma}{\sigma s_t}$ cannot hold for $x_t = 0$, which implies that $x_t > 0$ and $\Gamma_t = \frac{1-\sigma}{\sigma s_t}$.

## B Proof of Proposition 1

Any steady state should satisfy $P \geqslant 0$. Under Assumption 2, $P = \psi(k) \geqslant 0$ for all $k \geqslant 0$. In contrast, $P = \varphi(k) \geqslant 0$ requires:

$$w(k)/k \geqslant 1/(1-\tau) \tag{A.1}$$

$$\left(\alpha \frac{\sigma}{1-\sigma} + 1\right)/(1-\tau) > w(k)/k \tag{A.2}$$

Since $w(k)/k = f(k)/k - f'(k)$, we can easily show that $\lim_{k\to+\infty} w(k)/k = 0$. From Assumption 1, there exist values of $k > 0$ such that $w(k)/k > \left(\alpha\frac{\sigma}{1-\sigma} + 1\right)/(1-\tau)$. We also note that $w(k)/k$ is decreasing, because we have $d\ln[w(k)/k]/d\ln k = 1/k(s(k)/\rho(k) - 1) < 0$. This shows the existence of $\overline{k}$ and $\underline{k}$, defined by (19) and (20) respectively, such that $\underline{k} < \overline{k}$. Any $k \in (\underline{k}, \overline{k}]$ satisfies inequalities (A.1) and (A.2).

We show now the existence of a steady state that belongs to $(\underline{k}, \overline{k}]$. By direct inspection of (17), we deduce that $\varphi(\underline{k}) = +\infty$ and $\varphi(\overline{k}) = 0$. Moreover,

$$\begin{aligned}\psi(\underline{k}) &< \frac{1}{m}\epsilon_1 f(\underline{k}) < +\infty \\ \psi(\overline{k}) &> \frac{1}{m} f(\overline{k})[\epsilon_1 - \epsilon_2(1-\mu)\tau] > 0\end{aligned}$$

By continuity, there is one solution $k^* \in (\underline{k}, \overline{k})$ solving $\varphi(k) = \psi(k)$.

To show uniqueness, let us note $\epsilon_\varphi(k) \equiv \varphi'(k)k/\varphi(k)$ and $\epsilon_\psi(k) \equiv \psi'(k)k/\psi(k)$. Using (17) and (18), we get:

$$\epsilon_\psi(k) = \frac{s(k)}{\rho(k)}\frac{\epsilon_1\rho(k) - \epsilon_2(1-\mu)\tau(1-s(k))}{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s(k))} \tag{A.3}$$

$$\begin{aligned}\epsilon_\varphi(k) = {}& \frac{1}{\beta}\frac{2(1-\tau)w(k)s(k)/\rho(k) - [1+\alpha+(1-\alpha)s(k)/\rho(k)]k}{(1-\tau)w(k) - k} \\ & -\frac{1}{\beta}\frac{[1+\alpha\sigma/(1-\sigma)]k - (1-\tau)w(k)s(k)/\rho(k)}{[1+\alpha\sigma/(1-\sigma)]k - (1-\tau)w(k)}\end{aligned} \tag{A.4}$$

Under Assumptions 1 and 2, $\epsilon_\psi(k) > 0$. Moreover, the first term in (A.4) is lower than $1/\beta$, whereas the second one as larger than $1/\beta$. We conclude that $\epsilon_\varphi(k) < 0$, which shows the uniqueness of the steady state.

# C Proof of Lemma 2

In the long run, the welfare $W$ is defined by $W \equiv \pi(\theta)c^{1-\sigma}/(1-\sigma)$. Since consumption is given by $c = R(k)k/\pi(\theta)$, the welfare may be rewritten:

$$W = \pi(\theta)^\sigma (R(k)k)^{1-\sigma}/(1-\sigma) \tag{A.5}$$

Using (17), we have:

$$\theta = \frac{[(1-\tau)w(k) - k]^\alpha[\mu\tau w(k)]^{1-\alpha}}{P^\beta} = \frac{\left(\alpha\frac{\sigma}{1-\sigma} + 1\right)k - (1-\tau)w(k)}{(1-\tau)w(k) - k} \equiv \theta(k) \tag{A.6}$$

Substituting $\theta(k)$ into (21), the welfare becomes a function of $k$, namely $W \equiv W(k)$. Therefore, at the steady state $k^*$, the welfare is given by $W(k^*)$. Using Assumption 1, $R(k)k$ is increasing in $k$. We also know that $\pi(\theta)$ is increasing in $\theta$. Therefore, $W(k^*)$ is an increasing function of physical capital if $\theta'(k) > 0$. Using (A.6), we get:

$$\frac{\theta'(k)k}{\theta(k)} = \frac{\left(\alpha\frac{\sigma}{1-\sigma}+1\right)k-(1-\tau)w(k)\frac{s(k)}{\rho(k)}}{\left(\alpha\frac{\sigma}{1-\sigma}+1\right)k-(1-\tau)w(k)} - \frac{(1-\tau)w(k)\frac{s(k)}{\rho(k)}-k}{(1-\tau)w(k)-k} > 0$$

since according to Assumption 1 $\rho(k) > s(k)$.

## D Proof of Proposition 2

Let us note $\epsilon_\varphi(\mu) \equiv \frac{\partial\varphi(k)}{\partial\mu}\frac{\mu}{k}$ and $\epsilon_\psi(\mu) \equiv \frac{\partial\psi(k)}{\partial\mu}\frac{\mu}{k}$. The steady state $k^*$ is given by $\varphi(k^*) = \psi(k^*)$. Differentiating this equation with respect to $k$ and $\mu$, we get:

$$\frac{dk}{d\mu}\frac{\mu}{k^*} = \frac{\epsilon_\varphi(\mu)-\epsilon_\psi(\mu)}{\epsilon_\psi(k^*)-\epsilon_\varphi(k^*)} \tag{A.7}$$

We have $\epsilon_\psi(k^*) > \epsilon_\varphi(k^*)$. Therefore, the sign of $\frac{dk}{d\mu}\frac{\mu}{k^*}$ is given by $\epsilon_\varphi(\mu)-\epsilon_\psi(\mu)$. Using (17) and (18), we have:

$$\epsilon_\varphi(\mu) = \frac{1-\alpha}{\beta} \tag{A.8}$$

$$\epsilon_\psi(\mu) = \frac{\epsilon_2\mu\tau w(k^*)}{\epsilon_1 f(k^*)-\epsilon_2(1-\mu)\tau w(k^*)} \tag{A.9}$$

We easily deduce the necessary and sufficient conditions to have either $dk/d\mu > 0$ or $dk/d\mu < 0$. We now determine sufficient conditions that do not depend on the steady state level $k^*$.

One one hand, we note that $\epsilon_\psi(\mu) < \frac{\epsilon_2\mu\tau f(k^*)}{[\epsilon_1-\epsilon_2(1-\mu)\tau]f(k^*)} = \frac{\epsilon_2\mu\tau}{\epsilon_1-\epsilon_2(1-\mu)\tau}$. Using (A.8) and (A.9), we deduce that $\epsilon_\varphi(\mu)-\epsilon_\psi(\mu) > 0$ if $1-\alpha > \beta\frac{\epsilon_2\mu\tau}{\epsilon_1-\epsilon_2(1-\mu)\tau}$.

On the other hand, $\epsilon_\psi(\mu) > \frac{\epsilon_2\mu\tau w(k^*)}{\epsilon_1 f(k^*)} = \frac{\epsilon_2\mu\tau(1-s(k^*))}{\epsilon_1} > \frac{\epsilon_2\mu\tau}{2\epsilon_1}$ under Assumption 1. We deduce that $\epsilon_\varphi(\mu)-\epsilon_\psi(\mu) < 0$ if $1-\alpha < \beta\frac{\epsilon_2\mu\tau}{2\epsilon_1}$.

Of course, using Lemma 2, we deduce the effect of $\mu$ on the welfare $W^*$. We focus now on the effect of $\mu$ on pollution $P^*$. Using (18) and (A.7), we have:

$$\begin{aligned}\frac{dP}{d\mu}\frac{\mu}{P^*} &= \epsilon_\psi(k^*)\frac{dk}{d\mu}\frac{\mu}{k^*}+\epsilon_\psi(\mu)\\ &= \frac{\epsilon_\psi(k^*)\epsilon_\varphi(\mu)-\epsilon_\varphi(k^*)\epsilon_\psi(\mu)}{\epsilon_\psi(k^*)-\epsilon_\varphi(k^*)}\end{aligned}$$

Since $\epsilon_\psi(k^*) > 0$, $\epsilon_\varphi(k^*) < 0$, $\epsilon_\varphi(\mu) > 0$ and $\epsilon_\psi(\mu) > 0$, we easily deduce that $\frac{dP}{d\mu}\frac{\mu}{P^*} > 0$.

## E Proof of Corollary 1

First note that $f(k) = Ak^s$, with $A > 0$ and $s \in (0, 1/2)$, satisfies Assumption 1. Using this production function, we have $w(k^*)/f(k^*) = 1-s$. Therefore, using the proof of Proposition 2, $\epsilon_\varphi(\mu) > \epsilon_\psi(\mu)$ if and only if:

$$(1-\alpha)[\epsilon_1 - \epsilon_2\tau(1-s)] > \mu\epsilon_2\tau(1-s)[\beta - (1-\alpha)]$$

Since $\mu \in (0,1]$, this inequality is satisfied if $\beta \leqslant (1-\alpha)\epsilon_1/[\epsilon_2\tau(1-s)]$ or $\beta > (1-\alpha)\epsilon_1/[\epsilon_2\tau(1-s)]$ and $\mu < \widehat{\mu}$. On the contrary, when $\beta > (1-\alpha)\epsilon_1/[\epsilon_2\tau(1-s)]$ and $\mu > \widehat{\mu}$, we obtain $\epsilon_\varphi(\mu) < \epsilon_\psi(\mu)$. Then, the proof of Proposition 2 allows us to deduce the corollary.

## F Proof of Proposition 3

Using (17) and (18), there exists a unique value $m = m^*$ solving $\varphi(1) = \psi(1)$, with:

$$m^* = [\epsilon_1 f(1) - \epsilon_2(1-\mu)\tau w(1)] \frac{\left[\alpha\frac{\sigma}{1-\sigma} + 1 - (1-\tau)w(1)\right]^{\frac{1}{\beta}}}{[(1-\tau)w(1)-1]^{\frac{\alpha+1}{\beta}}\,[\mu\tau w(1)]^{\frac{1-\alpha}{\beta}}} \tag{A.10}$$

Of course, we have $m^* > 0$, whereas

$$m^* < \frac{\epsilon_1 f(1)\left(\alpha\frac{\sigma}{1-\sigma} + 1\right)^{\frac{1}{\beta}}}{[(1-\tau)w(1)-1]^{\frac{\alpha+1}{\beta}}\,[\mu\tau w(1)]^{\frac{1-\alpha}{\beta}}} < 1$$

for $\epsilon_1 < \bar{\epsilon}_1$. Finally, we clarify that $k^* = 1$ belongs to $(\underline{k}, \overline{k})$ for $1 < (1-\tau)w(1) < 1 + \alpha\frac{\sigma}{1-\sigma}$.

## G Proof of Proposition 4

We differentiate the dynamic system (15)-(16) around the normalized steady state $k^* = 1$. We get:

$$\begin{aligned}
\frac{dP_{t+1}}{P^*} &= (1-m^*)\frac{dP_t}{P^*} + m^*\frac{s}{\rho}\frac{\rho\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}\frac{dk_t}{k^*} \\
\frac{dk_{t+1}}{k^*} &= -\frac{\frac{\beta\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}\frac{dP_t}{P^*} \\
&+ \frac{s}{\rho}\frac{1 + \frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}\frac{dk_t}{k^*}
\end{aligned}$$

where $\theta$ is given by (27). We deduce the trace $T(\rho)$ and the determinant $D(\rho)$ of the associated Jacobian matrix, given by (25) and (26) respectively.

## H Proof of Proposition 5

The steady state s a saddle if either $P(-1) < 0 < P(1)$ or $P(1) < 0 < P(-1)$. Since $T(\rho) > 0$ and $D(\rho) > 0$, we have $P(-1) = 1 + T(\rho) + D(\rho) > 0$. To determine the sign of $P(1) = 1 - T(\rho) + D(\rho)$, let us note:

$$\Lambda(\rho) \equiv \frac{s}{\rho} \frac{1 + \frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1} \frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1} \frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} \tag{A.11}$$

Because $\Lambda(\rho)$ is decreasing in $\rho$ and $\rho \in (2s, +\infty)$, we have $\Lambda(\rho) < \Lambda(2s)$. One can easily prove that $\Lambda(2s) < 1$, that shows that $\Lambda(\rho) < 1$ for all $\rho \in (2s, +\infty)$.

Using (25), (26) and (A.11), we deduce that:

$$\begin{aligned} P(1) &= m^*(1 - \Lambda(\rho)) \\ &+ m^* \frac{s}{\rho} \frac{\rho\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}{\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)} \frac{\frac{\beta\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}}{1 + \frac{\alpha}{(1-\tau)w-1} \frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}} > 0 \end{aligned}$$

Since $P(1) > 0$ and $P(-1) > 0$, the steady state cannot be a saddle, but is either a source or a sink.

# I Proof of Proposition 6

Taking into account the results of Proposition 5, the steady state is a sink if $D(\rho) < 1$ and is a source if $D(\rho) > 1$. Moreover, choosing $\rho \in (2s, +\infty)$ as a bifurcation parameter, a Hopf bifurcation generically occurs if $D(\rho)$ crosses 1 for a value of $\rho$ that belongs to $(2s, +\infty)$.

By direct inspection of (26), we immediately see that $D(\rho)$ either monotonically increases or decreases with respect to $\rho$. We have:

$$D(+\infty) = A_\beta \beta \tag{A.12}$$

$$D(2s) = C_\beta \beta + B_\beta \tag{A.13}$$

where $A_\beta$ is given by (28) and:

Using (27), we note that $\theta$ does not depend on $\beta$. On the contrary, $m^*$ depends on $\beta$, but always belongs to $(0, 1)$ (see Appendix F).

Using (28) and (A.12)-(30), both $D(+\infty)$ and $D(2s)$ belongs to $(0, 1)$ when $\beta$ is low enough. Since $D(\rho)$ monotonically varies with respect to $\rho$, the steady state is a sink for all $\rho \in (2s, +\infty)$. When $\beta$ is sufficiently large, both $D(+\infty)$ and $D(2s)$ are larger than 1. In this case, the steady state is a source for all $\rho \in (2s, +\infty)$.

We now investigate whether there may be a change of stability through the occurrence of Hopf bifurcation when $\rho$ describes $(2s, +\infty)$. Using (A.12) and (A.13), $D(+\infty) > 1$ and $D(2s) < 1$ if and only if:

$$A_\beta \beta > 1 \text{ and } C_\beta \beta + B_\beta < 1 \tag{A.14}$$

Taking $\beta = 1/A_\beta$, the second inequality rewrites $C_\beta/A_\beta + B_\beta < 1$. Using (28) and (30),

$$\frac{C_\beta}{A_\beta} = \frac{2s\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)}{2s\epsilon_1} < \frac{1}{2}$$

if and only if $\epsilon_1 < \epsilon_2(1-\mu)\tau(1-s)/s$. Moreover, using (A.11) and (29), $\Lambda(2s)$ tends to 1/2 when $\alpha$ tends to 0, which ensures $B_\beta < 1/2$. By continuity, for $\alpha$ low enough and $\beta$ larger but sufficiently close to $1/A_\beta$, the two inequalities in (A.14) are satisfied. Therefore, we can conclude that there exists $\rho_H \in (2s, +\infty)$ satisfying $D(\rho_H) = 1$. Moreover, $D(\rho) < 1$ for $\rho \in (2s, \rho_H)$, whereas $D(\rho) > 1$ for $\rho \in (\rho_H, +\infty)$.

## J Proof of Corollary 2

We note $f \equiv f(1)$ and:

$$\Sigma \equiv \frac{\left[\alpha\frac{\sigma}{1-\sigma} + 1 - (1-\tau)w\right]^{\frac{1}{\beta}}}{[(1-\tau)w-1]^{\frac{\alpha+1}{\beta}}[\mu\tau w]^{\frac{1-\alpha}{\beta}}}$$

Using (A.10), we deduce that $m^* = \Sigma f[\epsilon_1 - \epsilon_2(1-\mu)\tau(1-s)]$. Using this expression, $D(\rho_H) = 1$ may be rewritten:

$$\begin{aligned}
&\left[\Sigma f s\epsilon_1 \frac{\beta\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))} - \frac{\alpha}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}\right.\\
&\left.-1\right]\rho_H = \Sigma f s\epsilon_2(1-\mu)\tau(1-s)\left[\frac{\beta\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))} - 1\right.\\
&\left.-\frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}\right]\\
&-s\left(1-\Sigma f\epsilon_1\right)\left[1+\frac{(1-\tau)w-(1-\alpha)}{(1-\tau)w-1}\frac{\theta\alpha\sigma/(1-\sigma)}{(1+\theta)(1+\theta+\alpha\sigma/(1-\sigma))}\right]
\end{aligned}$$

Note that on the left-hand side of this equality, the terms into brackets has the same sign than $\beta A_\beta - 1$, i.e. is positive. On the right-hand side, we have $\Sigma f\epsilon_1 < 1$ (see Appendix F) and the last term into brackets is positive. Hence, the first term into brackets on the right-hand side of the equality is also positive.

Since $\theta$ does not depend on $\mu$, we deduce that $\rho_H$ decreases with respect to $\mu$. Moreover, when $\mu$ tends to 1, the right-hand side of the equality becomes negative, meaning that $\rho_H$ reaches a negative value. Therefore, if $\mu$ is sufficiently close to 1, $\rho_H$ becomes smaller than $2s$.

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