ABANDONING FOSSIL FUEL: HOW FAST AND HOW MUCH?

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Abstract

Climate change must deal with two market failures, global warming and learning by doing in renewable use. The social optimum requires an aggressive renewables subsidy in the near term and a gradually rising carbon tax which falls in long run. As a result, more renewables are used relative to fossil fuel, there is an intermediate phase of simultaneous use, the carbon-free era is brought forward, more fossil fuel is locked up and global warming is lower. The optimal carbon tax is not a fixed proportion of world GDP. The climate externality is more severe than the learning by doing one.

Keywords: climate change, integrated assessment, Ramsey growth, carbon tax, renewables subsidy, learning by doing, directed technical change, multiplicative damages, additive damages

JEL codes: H21, Q51, Q54

Revised February 2014

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‡ This version of the paper has benefited from helpful comments at seminars at Oxford, Tilburg and the Tinbergen Institute and workshops at Annecy and Oxford. The first author is grateful for support from the OeNB Anniversary Fund grant on Green Growth (no. 14373). The second is grateful for support from the ERC Advanced Grant 'Political Economy of Green Paradoxes' (FP7-IDEAS-ERC Grant No. 269788).

1. Introduction

Climate change is the biggest externality our planet faces and the best way to deal with this is to correct for that is to price carbon appropriately, either by levying a carbon tax or by having a market for carbon emission permits. The key questions are what the level of the optimal price of carbon should be and what the time profile of this price should be.[1] The answer is that, in the absence of distortions in raising public revenue and other second-best issues, it must be set to the social cost of carbon (*SCC*): the present value of all future marginal global warming damages from burning one extra unit of fossil fuel. The answer is not straightforward in a world with exhaustible fossil fuel, increasing efficiency of carbon-free alternatives, gradual and abrupt transitions from fossil fuel to renewables, and endogenous growth and structural change. Our aim is thus to provide an answer to this question within the context of a fully calibrated integrated assessment model of climate change and Ramsey growth with exhaustible fossil fuel, gradual transition to carbon-free renewables and endogenous technical progress in the production of renewables.

We show that pricing carbon appropriately has the following consequences: it curbs fossil fuel use and promotes the substitution away from fossil fuel towards renewables, it leaves more untapped fossil fuel in the crust of the earth, and it brings forward the carbon-free era where fossil fuel has been completely abandoned. An initial phase where only fossil fuel is used is followed by an intermediate phase where fossil fuel and renewables are used alongside each other and a final carbon-free phase where the economy has fully transitioned to renewables. Global warming is curbed by optimally trading off reductions in global warming damages against the welfare losses from less economic growth and consumption. In contrast to earlier integrated assessment models following Nordhaus (2008) the timing of the phases of fossil fuel use, simultaneous use, and renewable use are endogenous in our model and driven by economic considerations and Hotelling rents on exhaustible resources play a prominent role.

So far international efforts have failed to establish a stringent system of carbon pricing and some national governments have instead moved forward to spur the transition away from fossil energy by using policy which increases the competitiveness of renewable energy production (e.g. feed-in tariffs). We show that such a policy mix of a zero carbon tax and an optimal subsidy helps bring forward the carbon-free era but is not sufficient to limit climate

[1] From now on we will refer to the optimal carbon tax on the understanding that it refers to the optimal price of carbon and could equally refer to the price fetched for carbon on an efficient emissions market or the shadow price of direct control legislation.

change to levels usually deemed to be 'safe'. Cooperation in combating climate change yields higher consumption and welfare levels than cooperation in the generation of renewable energy. The optimal policy mix must involve an aggressive subsidy for renewable energy sources to bring those sources into use and a gradually increasing carbon tax to price out fossil energy sources. Our analysis also highlights the following features.

First, the trade-off between global warming damages and welfare from consumption is analyzed within the context of a tractable fully optimizing Ramsey model of economic growth with a temporary population boom and ongoing technical progress. The seminal study of Nordhaus (2008) deals with the trade-off between growth and global warming but it optimizes welfare by choosing the *shares* of output allocated to saving and end-of-pipe mitigation. Our model is forward looking and maximizes global welfare by choosing the *levels* of consumption and carbon-intensive or carbon-free energy consumption in a Ramsey growth model as is also done in Rezai et al. (2012) and Golosov et al. (2014). However, we analyze not only the optimal carbon tax but also the optimal transition times for introducing the renewable alongside fossil fuel and abandoning fossil fuel altogether as well as the amount of untapped fossil fuel. We derive our results based on a calibrated and richer version of the analytical growth and climate models put forward in van der Ploeg and Withagen (2014).

Second, fossil fuel extraction costs rise as the remaining stock of reserves falls and less accessible fields need to be explored. This allows us to consider the important role of untapped fossil fuel in the fight against global warming whereas in most integrated assessment models such as the DICE model of Nordhaus (2008) no answer is given on how much fossil fuel should be left untapped. Our goal is thus to get an estimate of the maximum cumulative carbon emissions and the corresponding optimal carbon budget which is the maximum amount of fossil fuel that can be burnt before global warming reaches unacceptable levels. We also estimate the durations of the initial phase where only fossil fuel phase in used, the intermediate phase where fossil fuel is used alongside renewables, and the final carbon-free phase where only renewables are used. The intermediate phase arises because we allow for endogenous technical progress in renewables.

Third, in integrated assessment models fossil fuel is typically abundant so that fossil fuel demand at any point of time does not depend on expectations about the price of the future renewable backstops and consequently the transition times simply occur when the price of fossil fuel inclusive of the carbon tax reaches the price of the renewable. In our analysis fossil fuel is exhaustible and extraction costs are stock dependent so that the price of fossil fuel contains two forward-looking components: namely, the scarcity rent of fossil fuel (the present discounted value of all future increases in extraction costs resulting from an extracting an extra unit of fossil fuel) and the *SCC* (the present discounted value of all future marginal increases in global warming damages).This makes the calculation of the transition times much more complicated, since expectations about future developments such as learning by doing in using the renewable matter.

Fourth, production of renewable energy is subject to increasing returns to scale because we suppose its cost falls as cumulative use increases (Arrow, 1962). Technological progress, diffusion, and adoption are important issues in the economics of climate change and are widely studied in energy models but these studies usually do not allow for finite fossil fuel reserves, stock-dependent fossil fuel extraction costs and do not offer a fully calibrated integrated assessment model.[2] Most only discuss the first best outcome. If these cost reductions from learning-by-doing externalities are not fully internalized, a subsidy is required alongside the carbon tax. We first compare the no policy ("laissez faire") scenario with a global first-best scenario where both the global warming and learning-by-doing externalities are fully internalized and then with a scenario where only learning-by-doing externalities are internalized and the global warming externality is not (due to the lack of binding international agreements). This latter scenario leads to faster extraction of fossil fuel and acceleration of global warming, since fossil fuel owners fear that their resources will be worth less in the future. This phenomenon has been coined the Green Paradox by Sinn (2008).

Finally, our model generates a hump-shaped relationship between the optimal carbon tax and world GDP. In contrast, Golosov et al. (2014) offer a tractable Ramsey growth model which generates an optimal carbon tax which is proportional to GDP.[3] Their result depends on bold assumptions: logarithmic utility, Cobb-Douglas production, 100% depreciation of capital in each period, fossil fuel extraction which only requires labour and not capital, and

[2] Tsur and Zemel (2005) are an exception in explicitly considering finite resources and Popp (2004) is a prominent study of endogenous technical progress in a fully calibrated IAM. Popp et al. (2010) review the implications of technical innovation and diffusion for the environment. Goulder and Mathai (2000) study the theoretical implications in a stylized model of climate change. Manne and Richels (2004) argue that endogeneity of technical progress does not alter climate policy recommendations, specifically the transition timing. Jouvet and Schumacher (2012) find the opposite and so does Popp (2004) who studies optimal policies in an adapted version of DICE of Nordhaus (2008). Fisher and Newell (2008) study optimal interaction of policy instruments in a calibrated model of heterogenous energy producers limited to the US energy sector.

[3] This formula is already being used a lot (e.g., Hassler and Krusell, 2012; Gerlagh and Liski, 2013).

multiplicative production damages captured by a negative exponential function. We find that their result is not robust in a general integrated assessment model of climate change and Ramsey growth with exactly the same carbon cycle, especially if the coefficient of intergenerational inequality aversion differs very much from unity. The proportional carbon tax performs especially poorly if policy needs to address multiple market failures.

We focus on the effects of fossil fuel use on global warming in a detailed calibrated model of growth and climate change, but following Golosov et al. (2014) and based on Archer (2005) and Archer et al. (2009) we adopt a tractable model of the carbon cycle which is linear and allows for decay of only part of the stock of atmospheric carbon. This model of the carbon cycle abstracts from a delay between the carbon concentration and global warming (e.g., Gerlagh and Liski, 2013), which biases the estimate of the *SCC* and the carbon tax upwards. A more realistic model of the carbon cycle should also model the dynamics of the stocks of carbon in the upper and lower parts of the ocean and the time-varying coefficients originally put forward in the path-breaking paper of Bolin and Erikkson (1958). Although we capture catastrophic losses at high levels of atmospheric carbon, we abstract from positive feedback effects and the uncertain climate catastrophes that can occur in climate and growth models once temperature exceeds certain thresholds (e.g., Lemoine and Traeger, 2013; van der Ploeg and de Zeeuw, 2013).

Section 2 discusses a simple two-stock model of carbon accumulation in the atmosphere and global mean temperature based on Golosov et al. (2014) and discusses our specification of climate damages which are bigger at higher temperatures than Nordhaus (2008) following recent suggestions by Stern (2013). We do not adopt the approximation to damages used in Golosov et al. (2014), since this leads to unrealistic low damages at higher temperatures. Section 3 formulates our general equilibrium model of climate change and Ramsey growth with factor substitution between energy and the capital-labour aggregate, depreciation of manmade capital, endogenous technical progress in renewables, and stock-dependent fossil fuel extraction costs. Section 4 uses the functional forms and calibration discussed in an appendix to highlight the different outcomes for the optimal carbon tax, the renewables subsidy, untapped fossil fuel, the time it takes to phase in renewable energy and to reach the carbon-free era, and welfare under the social optimum and the market outcomes with no or only partial policy interventions using a carbon tax and a renewable subsidy. It also shows that the simple formula for the carbon tax as a fixed proportion of output from Golosov et al.

(2014) adapts incorrectly to multiple market failures, especially if intergenerational inequality aversion differs from unity. Section 5 discusses the sensitivity of climate policy to lower intergenerational inequality aversion, a lower discount rate, a higher climate sensitivity, the initial capital stock, the rate of endogenous technical change, changes in the growth rate and plateau of the population dynamics, higher substitutability between capital and energy in production, and more elaborate climate dynamics. Section 6 concludes.

2. The carbon cycle, temperature and global warming damages

We use the tractable decadal model of the carbon cycle put forward by Golosov et al. (2014):

(1) $E_{t+1}^P = E_t^P + \varphi_L F_t, \quad \varphi_L = 0.2, \quad E_0^P = 103 \text{ GtC},$

(2) $E_{t+1}^T = (1-\varphi)E_t^T + \varphi_0(1-\varphi_L)F_t, \quad \varphi = 0.0228, \quad \varphi_0 = 0.393, \quad E_0^T = 699 \text{ GtC},$

where E_t^P denotes the part of the stock of carbon (GtC) that stays thousands of years in the atmosphere, E_t^T the remaining part of the stock of atmospheric carbon (GtC) that decays at rate $\varphi = 0.0228$, and F_t the rate of fossil fuel use (GtC/decade). The unit of time t is a decade. About 20% of carbon emissions stay up 'forever' in the atmosphere and the remainder has a mean life of about 300 years, so $\varphi_L = 0.2$. The parameter $\varphi_0 = 0.393$ is calibrated so that about half of the carbon impulse is removed after 30 years. This carbon cycle has time-invariant coefficients and abstract from this and other subtleties discussed by Bolin and Erikkson (1958). It also abstracts from the three-reservoir system used by Nordhaus (2008) for describing the exchange of carbon with the deep oceans arising from the acidification of the oceans limiting the capacity to absorb carbon or the linear version of that put forward by Gerlagh and Liski (2013). In addition, it abstracts from diffusive rather than advective transfers of heat to the oceans as in Allen et al. (2009) which as has been pointed out by Bronselaer et al. (2013) and Baldwin (2014) leads to longer and greater warming, and abstracts from positive feedback effects in natural emissions which are likely to be more severe at higher temperatures.

The equilibrium climate sensitivity (*ECS*) is the rise in global mean temperature following a doubling of the total stock of carbon in the atmosphere, $E_t = E_t^P + E_t^T$. An often used estimate for the highly uncertain *ECS* is 3 (IPCC, 2007). To facilitate comparison with Golosov et al.

(2014), we postulate that the global mean temperature as a difference in degrees Celcius from the pre-industrial temperature (T) increases with the current total stock of atmospheric carbon:

(3) $$T_t = \omega \ln\left(E_t / 596.4\right) / \ln(2), \quad \omega = 3, \quad E_t \equiv E_t^P + E_t^T,$$

where 596.4 GtC (280 ppmv CO2) is the IPCC figure for the pre-industrial stock of atmospheric carbon.[4] We measure the stock of carbon in GtC. To obtain the stock of carbon in GtCO2, we multiply by the factor 44/12.

The evolution of the stock of fossil fuel reserves follows from the depletion equation:

(4) $$S_{t+1} = S_t - F_t, \quad S_0 = 4000 \text{ GtC},$$

where S_t denotes the stock of fossil fuel reserves at the start of period t and initially this is 4000 GtC. We allow for an exhaustible stock of fossil fuel reserves, but many other integrated assessment models (e.g., Nordhaus, 2008) suppose that fossil fuel reserves are abundant. This may be reasonable for coal, but seems unrealistic for conventional oil or gas.

Nordhaus (2008) has combined for purposes of his DICE-07 model detailed micro estimates of the costs of global warming to obtain aggregate macroeconomic costs of global warming. This has led to estimated climate damages of 1.7% of world GDP when global warming is 2.5° C. This figure can be used to calibrate the following function for the fraction of global production output that is left after damages from global warming:

(5) $$\tilde{Z}(T_t) = \frac{1}{1 + \zeta_1 T_t^{\zeta_2} + \zeta_3 T_t^{\zeta_4}}, \quad \text{so} \quad Z(E_t) \equiv \tilde{Z}\left(\omega \ln\left(E_t / 596.4\right) / \ln(2)\right),$$

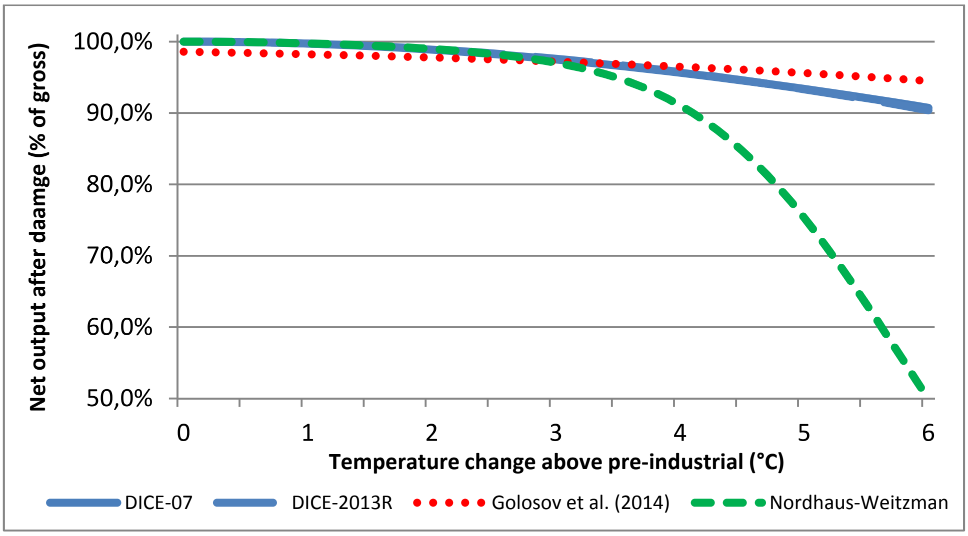
with $\zeta_1 = 0.00284$, $\zeta_2 = 2$, and $\zeta_3 = \zeta_4 = 0$ and plotted as the solid blue line in fig. 1.[5] Golosov et al. (2014) approximate this solid blue line with the exponential reduced-form net output function $Z(E_t) \cong \exp\left[\tilde{\zeta}(E_t - 581)\right]$, where $\tilde{\zeta} = -2.379 \times 10^{-5}$, which gives rise to the dotted red line in fig. 1. The fit to the Nordhaus (2008) damages is reasonably good for small degrees of

[4] We abstract from a lag between temperature and atmospheric carbon stock as discussed by Gerlagh and Liski (2013), but appendix C discusses how our analysis is modified with such a lag and the numerical results reported in section 5.3 illustrate that this lag lowers the social cost of carbon.

[5] The damage function resulting from the DICE-07 model is almost distinguishable (up to temperatures of 7 Celsius) from that of the DICE-2013R model (see http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm)

global warming from the present, but at higher degrees of global warming the fit is much worse and is much too optimistic about the size of global warming damages.[6]

Figure 1: What is left of output after damages from global warming



Stern (2013) criticizes the current generation of IAMs for focusing on too limited a set of functional and parametric relationships for climate damages. Weitzman (2010) argues that damages rise more rapidly at higher levels of temperature than suggested by (5), but empirical studies on the costs of global warming at higher temperatures are not available. But making the heroic assumptions that output damages equal 50% of world GDP at 6° C and 99% at 12.5° C, Ackerman and Stanton (2012) recalibrate equation (5) with $\zeta_1 = 0.00245$, $\zeta_2 = 2$, $\zeta_3 = 5.021 \times 10^{-6}$, and $\zeta_4 = 6.76$. The extra term in the denominator captures potentially catastrophic losses at high temperatures. The dashed green line in fig. 1 plots this recalibrated net output function. Although the parameter values used in Nordhaus (2008) can be justified on the basis of empirical studies of moderate degrees of global warming, the calibration based on Weitzman's arguments may be more appropriate for higher temperatures. In fact, this recalibration matches the function of Nordhaus (2008) very closely with deviations of less than half a percentage point up to 3°C of warming. In our analysis we use this calibration as the benchmark and will then compare it with damages favoured by Nordhaus (2008).

[6] Golosov et al. (2014) choose this approximation for mathematical convenience, but $Z(E_t) = 1.0435 - 0.0001\, E_t$ is a much better fit, also at higher temperature and carbon stocks ($R^2 = 0.9994$). In fact, their damage function deviate by less than one percentage point only between 2°C and 4°C warming.

3. An integrated assessment model of Ramsey growth and climate change

The social optimum

The social planner's objective is to maximize the utilitarian social welfare function:

$$(6) \qquad \sum_{t=0}^{\infty}(1+\rho)^{-t} L_t U_t(C_t / L_t) = \sum_{t=0}^{\infty}(1+\rho)^{-t} L_t \left[\frac{(C_t / L_t)^{1-1/\eta} - 1}{1-1/\eta} \right].$$

Here L_t denotes the size of the world population, which has an exogenous growth profile, C_t is aggregate consumption, U is the instantaneous CES utility function, $\rho > 0$ is the rate of pure time preference and $\eta > 1$ is the elasticity of intertemporal substitution. The ethics of climate policy depend on how much weight is given on the welfare of future generations (and thus on how small ρ is) and on how small intergenerational inequality aversion is or how easy it is to substitute current for future consumption per head (i.e., on how low $1/\eta$ is). The most ambitious climate policies result if society has a low rate of time preference and little intergenerational inequality aversion (low ρ, high η).

Optimal climate policy takes place under a number of constraints governing the global economy. First, output at time t is produced using three inputs: manmade capital K_t, labour, L_t, and energy. Two types of energy are used: renewables R_t, such as solar and wind energy, and fossil fuels like oil, natural gas and coal, F_t. Besides these inputs the mean global mean temperature or the concentration of atmospheric carbon plays a role, through the damages that are caused by climate change. The production function $H(.)$ has constant returns to scale, is concave and satisfies the usual Inada conditions. Renewables are subject to learning. Their marginal production cost $b(B_t)$ decreases with cumulated past production B_t, so $b' < 0$. Fossil fuel extraction costs at time t is $G(S_t)F_t$, with S_t the remaining stock of fossil fuel reserves. Extraction becomes harder as less accessible fields have to be explored, so that we assume that $G' < 0$. We also allow for technical progress and population growth.

What is left of production after covering the cost of resource use is allocated to consumption C_t, investments in manmade capital $K_{t+1} - K_t$, and depreciation δK_t with a constant rate of depreciation δ:

$$(7) \qquad K_{t+1} = (1-\delta)K_t + Z(E_t)H(K_t, L_t, F_t + R_t) - G(S_t)F_t - b(B_t)R_t - C_t,$$

where damages follow from (5) and temperature from (3). The initial stock of capital K_0 is given. The development of the permanent and transient parts of the atmospheric carbon stocks follows from (1) and (2). The development of the fossil fuel stock is given by (4) and the development of the knowledge stock for renewable use is given by:

(8) $$B_{t+1} = B_t + R_t, \quad B_0 = 0.$$

Current technological options favour fossil energy use; complete decarbonisation of the world economy requires substantial reductions in the cost of renewables versus that of fossil fuel. Apart from carbon taxes, technological progress is an important factor in determining the optimal combination of fossil and renewable energy sources as highlighted in recent contributions by Acemoglu et al. (2012) and Mattauch et al. (2012). We thus capture the importance of learning and lock-in effects by making the cost of renewables a decreasing function of past cumulated renewable energy production, $b' < 0$ with $B_t = \sum_{s=0}^{t} R_s$.

Proposition 1: *The social optimum maximizes (6) and satisfies equations (1)-(8), the Euler equation for consumption growth*

(9) $$\frac{C_{t+1} / L_{t+1}}{C_t / L_t} = \left(\frac{1 + r_{t+1}}{1 + \rho} \right)^{\eta}, \quad r_{t+1} \equiv Z_{t+1} H_{K_{t+1}} - \delta,$$

and the efficiency conditions for energy use

(10a) $$Z(E_t) H_{F_t + R_t}(K_t, L_t, F_t + R_t) \le G(S_t) + \theta_t^S + \theta_t^E, \quad F_t \ge 0, \quad \text{c.s.},$$

(10b) $$Z(E_t) H_{F_t + R_t}(K_t, L_t, F_t + R_t) \le b(B_t) - \theta_t^B, \quad R_t \ge 0, \quad \text{c.s.},$$

where the scarcity rent , the SCC and the social benefit of learning by doing in final goods units are, respectively, given by

(11) $$\theta_t^S = -\sum_{s=0}^{\infty} \left[G'(S_{t+1+s}) F_{t+1+s} \Delta_{t+s} \right],$$

(12) $$\theta_t^B = -\sum_{s=0}^{\infty} \left[b'(B_{t+1+s}) R_{t+1+s} \Delta_{t+s} \right] \text{ and}$$

(13) $$\theta_t^E = -\sum_{s=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0 (1 - \varphi_L)(1 - \varphi)^s \right\} \Delta_{t+s} Z'(E_{t+1+s}^P + E_{t+1+s}^P) H(K_{t+1+s}, L_{t+1+s}, F_{t+1+s} + R_{t+1+s}) \right],$$

with the compound discount factors given by $\Delta_{t+s} \equiv \prod_{s'=0}^{s} (1 + r_{t+1+s'})^{-1}, s \ge 0.$

Proof: see appendix B.

The Euler equation (9) is also known as the Keynes-Ramsey rule. It indicates that the growth rate in consumption per capita increases in the social return on capital (r_{t+1}) and decreases in the rate of time preference, especially if the elasticity of intertemporal substitution (η) is high or intergenerational inequality aversion ($1/\eta$) small.

Equation (10a) implies that, if fossil fuel is used, its marginal product should equal total marginal cost, which consists of the sum of marginal extraction cost $G(S_t)$ scarcity rent θ_t^S and the *SCC* θ_t^E. The scarcity rent and the *SCC* are defined in units of final goods (not utility units). If fossil fuel is not used, its marginal product is below its total marginal cost. Equation (10b) states that, if the renewable is used, its marginal product must equal its marginal cost $b(B_t)$ *minus* the social benefit of learning by doing θ_t^B (again defined in units of final goods).

Equation (11) states that the scarcity rent of keeping an extra unit of fossil fuel unexploited must equal the present discounted value of all future reductions in fossil fuel extraction costs. It follows from the Hotelling rule which requires that the return on extracting an extra unit of fossil, selling it and getting a return on it, i.e., the rate of interest $\left(r_{t+1}\theta_t^S\right)$ minus the increase in future extraction cost $(-G'(S_{t+1})F_{t+1})$, must equal the expected capital gain from keeping an extra unit of fossil fuel in the earth $\left(\theta_{t+1}^S-\theta_t^S\right)$. Equation (12) indicates that the social benefit of learning by doing equals the present discounted value of all future learning-by-doing reductions in the cost of renewable energy. It follows from the marginal reduction in the cost of renewable energy $(-b'(B_{t+1})R_{t+1})$ having to be equal to its user cost, i.e., the interest charge minus the capital gains $\left(r_{t+1}\theta_t^B-(\theta_{t+1}^B-\theta_t^B)\right)$.

Equation (13) states that the *SCC* is given by the present discounted value of all future marginal global warming damages from burning an additional unit of fossil fuel. This formulation for the *SCC* takes into account that one unit of carbon released from burning fossil fuel affects the economy in two ways: the first part remains in the atmosphere for ever and the second part gradually decays over time at a rate corresponding to roughly 1/300 per year.

Proposition 2: *If the utility function is logarithmic, the production function is Cobb-Douglas, global warming damages are* $Z(E_t)\cong\exp\left[-\tilde{\zeta}(E_t-581)\right]$, *depreciation of physical capital is*

100% every period and energy production does not require capital input, the social cost of carbon (13) per Giga ton of carbon equals

$$(13') \quad \theta_t^{E,\,Golosov\,c.s.} = \tilde{\zeta}\left[\left(\frac{1+\rho}{\rho}\right)\varphi_L + \left(\frac{1+\rho}{\rho+\varphi}\right)\varphi_0(1-\varphi_L)\right]Z(E_t)H(K_t,L_t,F_t+R_t).$$

Proof: see Golosov et al. (2014).

This result follows from aggregate consumption and investment being proportional to GDP if production is Cobb-Douglas, utility is logarithmic and depreciation is 100% each period. It also holds if the production function contains a constant-returns-to-scale sub-production function for the energy aggregate and each of the types of energy only needs labour to extract or produce it. Equation (13′) implies that the optimal *SCC* is proportional to world GDP. The factor of proportionality is independent of the elasticity of intertemporal substitution and the factor shares in production, but is proportional to the damage parameter $\tilde{\zeta}$ and increases if the social rate of discount ρ is smaller, the permanent fraction of the atmospheric stock of carbon φ_L is larger, and the lifetime of the transient component of the atmospheric stock of carbon $1/\varphi$ is larger. Expression (13′) serves as a useful rule of thumb in more general integrated assessment models of climate change and economic growth.

Decentralized market outcome

The world economy is best described by a decentralized market economy. Firms thus choose capital, labour and energy inputs to maximize profits, $\tilde{Z}(T_t)H(K_t,L_t,F_t+R_t)-w_tL_t-(r_{t+1}+\delta)K_t$ $-(p_t+\tau_t)F_t-[q_t-\upsilon_t]R_t,$ under perfect competition taking the wage rate w_t, the market interest rate r_{t+1}, the market price for fossil fuel p_t, the specific tax τ_t on carbon emissions, the market price for renewable energy q_t, the specific subsidy υ_t on the use of renewable energy, the economy-wide stock of accumulated knowledge about using renewable energy B_t and temperature T_t as given. Capital accumulation follows from $K_{t+1}=(1-\delta)K_t+I_t$, where I_t denote the investment rate at time t. Fossil fuel owners also operate under perfect competition and maximize the present value of their profits, $\sum_{t=0}^{\infty}\Delta_t\,[p_tF_t-G(S_t)F_t]$ with

$\Delta_t \equiv \prod_{s=0}^{t}(1+r_{1+s})^{-1},\, t\geq 0,$ subject to the depletion equation (4), taking the market price of fossil fuel p_t as given and internalizing the effect of depletion on future extraction costs. Producers of renewable energy again operate under perfect competition and maximize the present value of their profits, $\sum_{t=0}^{\infty}\Delta_t\left[\{q_t - b(B_t)\}R_t\right]$, taking the market price of renewable energy q_t and the economy-wide stock of accumulated knowledge about using renewable energy B_t as given. Households maximize the present discounted value of utility (6) subject to their budget constraint $A_{t+1}^H = (1+r_{t+1})A_t^H + w_t L_t + T_t - C_t$, where A_t^H denotes household assets and T_t lump-sum transfers from the government at time t. The government budget constraint is $A_{t+1}^G = (1+r_{t+1})A_t^G + \tau_t F_t - \upsilon_t B_t - T_t$, where denotes A_t^G government assets and we abstract from government consumption and investment. Without loss of generality, we can assume that the government balances its books, $A_t^G = 0$, in which case the government hands back revenue from carbon taxes minus cost of renewable energy subsidies as lump-sum transfers. Asset market equilibrium requires that $A_t^H = K_t$ and the final goods market equilibrium condition is $\tilde{Z}(T_t)H(K_t, L_t, F_t + R_t) = C_t + I_t + G(S_t)F_t + b(B_t)R_t$, so that the market must produce enough final goods to satisfy demand for consumption and investment goods and to cover extraction costs for fossil fuel and production costs of renewable energy.

Proposition 3: *The decentralized market outcome satisfies the equations (1)-(8), the Euler equation for consumption growth (9), the energy producers' optimality conditions*

(10a′) $p_t \le G(S_t) + \theta_t^S, \quad F_t \ge 0, \quad \text{c.s.},$

(10b′) $q_t \le b(B_t), \quad R_t \ge 0, \quad \text{c.s.},$

where the scarcity rent θ_t^S *is given by (11), and the final good firms' optimality conditions* $w_t = \tilde{Z}(T_t)H_{L_t}$, $r_{t+1} + \delta = \tilde{Z}(T_t)H_{K_t}$, $p_t - \tau_t = q_t - \nu_t = \tilde{Z}(T_t)H_{F_t+R_t}$.

Proof: From the optimality conditions for firms, fossil fuel producers and households. €

Proposition 4: *The social optimum is replicated in the decentralized market economy if* $\tau_t = \theta_t^E$ *and* $\nu_t = \theta_t^B$, $\forall t \ge 0$, *where these follow from (12) and (13).*

Proof: From comparing optimality conditions of proposition 2 with those of proposition 3. €

The social optimum is thus sustained in the market economy if the specific carbon tax is set to the *SCC*, the specific subsidy on renewable subsidy is set to the social benefit of learning by doing, and the net revenue is handed back in lump-sum fashion to households.

Policy scenarios

In our simulations we consider the following four policy scenarios:

Definition 1: Consider the following outcomes for a decentralized market economy.

I. *First best or social optimum which corresponds to* $\tau_t = \theta_t^E$ *and* $\upsilon_t = \theta_t^B, \forall t \geq 0.$

II. *Constrained first best with only learning by doing in use of renewable energy internalized, which corresponds to* $\tau_t = 0$ *and* $\upsilon_t = \theta_t^B, \forall t \geq 0.$

III. *Constrained first best with only the SCC internalized, which corresponds to* $\tau_t = \theta_t^E$ *and* $\upsilon_t = 0, \forall t \geq 0.$

IV. *"Laissez faire" or business as usual which corresponds to* $\tau_t = \upsilon_t = 0, \forall t \geq 0.$ €

There are two externalities in the decentralized economy stemming from missing markets for carbon permits and the benefits of learning in producing renewable energy. Policy scenario I corresponds to the social optimum and sets the specific carbon tax τ_t to the *SCC* (13) and sets the renewable subsidy υ_t to the social benefit of learning by doing in using renewable energy (12). Policy scenarios II and III correspond to two market scenarios with partial policy intervention. Scenario II internalizes the benefits of learning to use the renewable but not the *SCC*, so that the optimal renewable subsidy is given by $\upsilon_t = \theta_t^B$ and the carbon tax τ_t is set to zero (and thus equation (13) is irrelevant). In this case no international climate agreement can be reached but national governments move ahead using subsidies (e.g. feed-in tariffs) to stimulate cost reduction in the production of renewable energy. Scenario III internalizes the *SCC* but not learning to use the renewable, so that $\tau_t = \theta_t^E$ and the optimal renewable subsidy is zero (and thus equation (12) is irrelevant). Policy scenario IV corresponds to decentralized economy in the absence of any policy ("laissez faire") and corrects neither for the climate nor for the learning-by-doing externality, so that $\tau_t = \upsilon_t = 0$.

If initially renewable energy sources are not competitive, the decentralized economy starts with an initial phase of only fossil fuel use so that fossil fuel demand follows from setting its marginal product, $\tilde{Z}(T_t)H_{F_t}$, to the sum of extraction cost, scarcity rent and carbon tax, $G(S_t) + \theta_t^S + \tau_t$. After some time, the relative cost of renewable energy has fallen (due to the rising carbon tax and technical progress) and extraction costs have risen sufficiently (as less accessible fields have to be extracted) so that it has become attractive to phase in clean energy. During this intermediate phase with simultaneous use of fossil fuel and renewable energy demand follows from $\tilde{Z}(T_t)H_{F_t+R_t} = G(S_t) + \theta_t^S + \tau_t = b(B_t) - \upsilon_t$. After some more time, fossil fuel is phased out and the final carbon-free era starts. Since fossil fuel extraction costs become infinitely large as reserves are exhausted, fossil fuel reserves will not be fully exhausted and thus some fossil fuel will be left untapped in the crust of the earth at the end of the intermediate phase. Since fossil fuel and renewable energy are perfect substitutes, simultaneous use is not feasible if there is no learning by doing or if there is no renewable subsidy or carbon tax (except possibly for a single period of time).[7] But learning by doing introduces convexity in the production cost of the renewable so it is possible that an intermediate phase with simultaneous use emerges. From that moment on the *in-situ* stock of fossil fuel remains unchanged, but the carbon in the atmosphere gradually decays leaving ultimately only the permanent component of the carbon stock. During the final phase we have $\tilde{Z}(T_t)H_{R_t} = b(B_t) - \upsilon_t$, which shows that renewable use increases in capital, the stock of renewable knowledge and the renewable subsidy but decreases in global mean temperature.

One of our main objectives is to study the optimal timing of transitions from introducing the renewable alongside fossil fuel and from phasing out fossil fuel altogether because in most of the prevailing integrated assessment models these transitions are exogenous or ad hoc. We are interested in how the timing of these transitions differs across scenarios I-IV; for example, we wish to know by how much carbon taxes and renewable subsidies bring forward the carbon-free era. These transition times follow from the conditions that energy prices at these transition times should not jump. The stock of fossil fuel to leave untapped in the earth at the end of the intermediate phase follows from the condition that the economy is indifferent between using fossil fuel and renewable energy and that the scarcity rent has vanished:

$$(14) \qquad G(S_t) + \tau_t < b(B_t) - \upsilon_t \ \ for\ 0 \le t < t_{CF} \quad \text{and} \quad G(S_t) + \tau_t \ge b(B_t) - \upsilon_t, S_t = S_{t_{CF}}, \forall t \ge t_{CF}.$$

where t_{CF} is the time at which the economy for the first time relies on using only renewable energy. The stock of untapped fossil fuel increases in the renewable subsidy and carbon tax.

4. Policy simulation and optimization

In our numerical simulations time runs from 2010 till 2600 and is measured in decades, t=0,1,.., 59, so period 0 corresponds to 2010-2020, period 1 to 2020-2030, etc. The final time period is T = 59 or 2600-2610, but we highlight the transitional dynamics in the earlier parts

[7] Simultaneous use would imply that it is optimal to sell energy in the future rather than meeting full demand today which cannot be the case under perfect substitutability and positive discounting.

of the simulation. The functional forms and calibration of the carbon cycle, temperature module and global warming damages have been discussed in section 2. The functional forms and benchmark parameter values for the economic part of our integrated assessment model of growth and climate change are discussed in appendix D. On the whole our benchmark parameter values assume relatively low damages, low fossil fuel extraction cost and a high cost for renewable energy. This biases our model toward fossil fuel use. The reported simulations use the Leontief production function (with elasticity of substitution between energy and the capital-labour aggregate equal to $\vartheta = 0$).[8]

Table 1 summarizes the definition of scenarios I-IV as stated in definition 1. It also presents two further scenarios in which policy makers implement a carbon tax based on the proportionality rule (13′) of Golosov et al. (2014) to correct the climate externality in the market economy with either (V) choosing the renewable subsidy optimally (correcting for the learning externality and sub-optimal proportional carbon tax; blue, dotted) or (VI) ignoring the learning-by-doing externality in renewable use (orange, dotted). The results for these six policy scenarios are reported in tables 2 and 3 and fig. 2.

Table 1: Policy Scenarios

		Carbon tax, τ_t		
Renewable subsidy, ϑ_t		θ_t^E *from (13)*	0	*Proportional to GDP (13′)*
	θ_t^B *from (12)*	(I) first-best	(II) only subsidy	(V) proportional carbon tax and optimal renewable subsidy
	0	(III) only carbon tax	(IV) “laissez faire“	(VI) only proportional carbon tax

Section 4.1 compares the social optimum (I) with the “laissez faire” scenario (IV) and concludes that the carbon tax and renewable subsidy speed up the transition to the carbon-free era and leave more fossil fuel left untapped. We also show how well the scenarios with only one market failure (imposing either the optimal renewable subsidy (II) or the optimal carbon tax (III)) perform. Section 4.2 explains the different phases of the market price of fossil fuel and renewable energy in the various scenarios. Section 4.3 explores the sources of the large welfare losses under “laissez faire” in detail. Section 4.4 examines how well the simple

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[8] Appendix E shows how the benchmark policy simulations are affected by CES technology ($\vartheta = 0.5$).

formula for the carbon tax (13′) put forward by Golosov et al. (2014) performs (V and VI), especially in the face of multiple market failures.

4.1. How to transition more quickly to the carbon-free era and leave more fossil fuel untapped

The first three panels of fig. 2 show aggregate consumption, world GDP (net output net of global warming damages) and the aggregate capital stock. Under the first-best policy scenario I (blue, solid) consumption, GDP and the capital stock are monotonically increasing over the entire period of time under consideration. The transition to renewable energy takes place smoothly as soon as 2030 and fossil energy is phased out completely by 2050. Over this period 400 GtC are burnt, so most of the 4000 GtC of fossil fuel reserves are abandoned. This leads to a maximum increase in temperature of 2.3°C corresponding to an atmospheric carbon stock of 1015 GtC and slight overshooting of the 2°C warming limit corresponding to a carbon stock of 947 GtC (from (3)). This rapid and unambiguous first-best transformation towards a carbon-free economy is achieved through the implementation of a carbon tax and a renewable subsidy policy. Both follow an inverted U-shaped time profile. The global carbon tax is set to the *SCC* which starts at a level of 95 \$/tC or 26 \$/tCO2 and reaches a maximum of 560 \$/tC or 153 \$/tCO2 in the year 2170 long after the transition to renewable energy has occurred. The renewable subsidy starts at 160 \$/tC or 44\$/tCO2 and rises to 380 \$/tC or 104 \$/tCO2 in the year 2030 and rapidly falls to zero as all learning has taken place by the end of this century. The optimal policy mix, therefore, combines an aggressive subsidy to phase in renewable energy quite early and a carbon tax which gradually rises (and falls) to price fossil energy out of the market once renewable energy sources are competitive.

Table 2: Transition times and carbon budget

	Only fossil fuel	Simultaneous use	Renewable Only	Carbon used
I Social optimum	2010-2020	2030-2040	2050 –	400 GtC
II Carbon tax only	2010-2050	x	2060 –	730 GtC
III Renewable subsidy only	2010-2050	2060-2080	2090 –	1250 GtC
IV Laissez faire	2010-2110	x	2120 –	2510 GtC

In the no-policy or "laissez faire" scenario IV (orange, dashed) both externalities remain uncorrected: no international agreements are reached and no subsidy scheme implemented. As a result the economy uses much more fossil fuel, 2510 GtC in total, so much less fossil fuel is left in the crust of the earth. Global mean temperature increases by a maximum of 5.3°C matching recent IPCC and IEA estimates for business as usual. The transition to renewable energy occurs much later, in 2120, and abruptly. This is due to the fact that the benefits of renewable energy production in terms of climate change mitigation and learning-by-doing are not taken into account; hence the potential reductions in the cost of renewable production are not recognized and take longer to materialize. Table 3 indicates that these inefficiencies (see section 5.2 for more detail) cause a converted welfare loss of 73% of today's world GDP. [9]

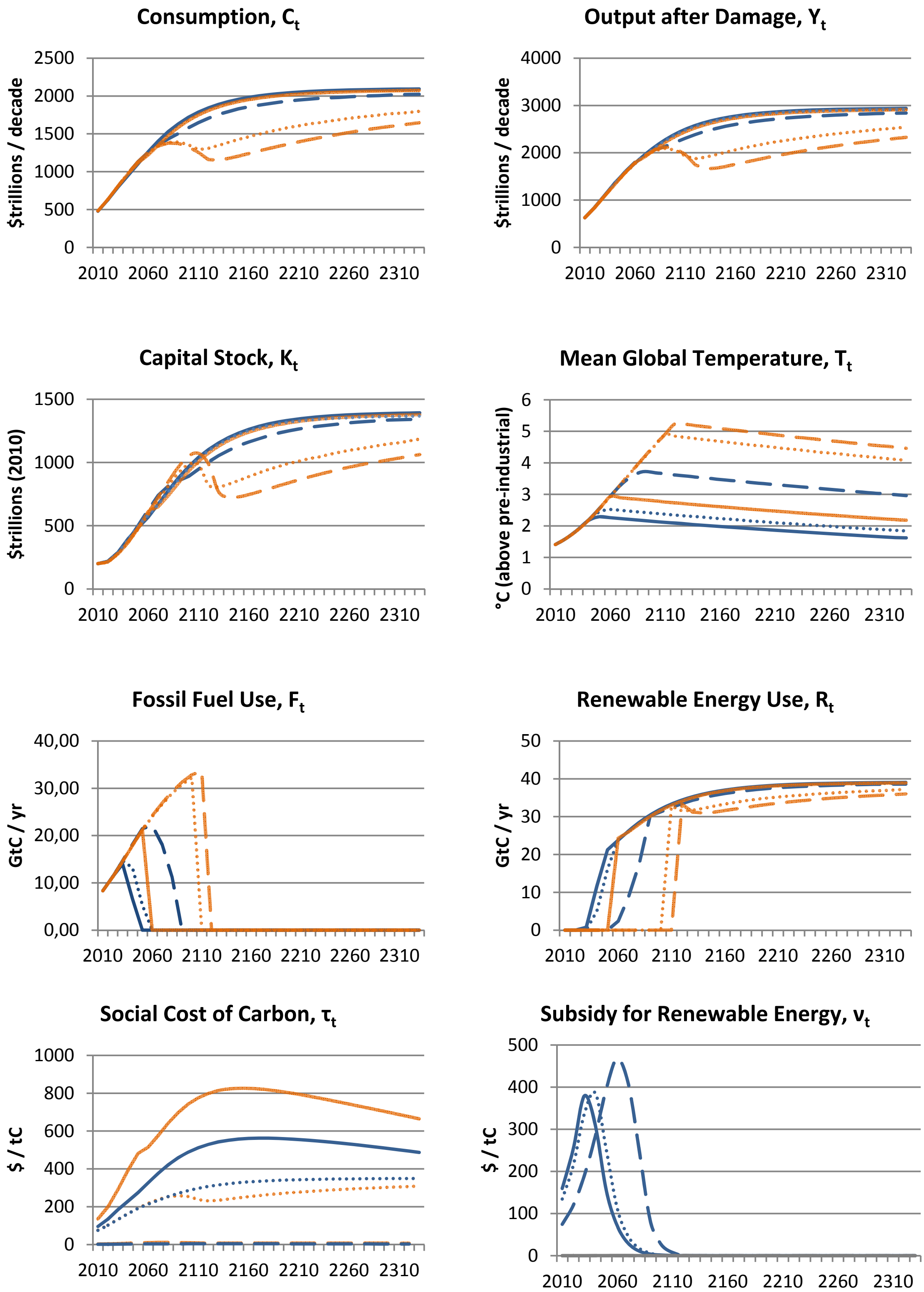
Table 3: Social costs of carbon, renewable subsidies, and welfare losses

	Welfare Loss (% of GDP)	Maximum carbon tax τ ($/tC)	Maximum renewable subsidy ($/tC)	max T (°C)
I Social optimum		560 $/tC	380 $/tC	2.3 °C
II Carbon tax only	-3%	830 $/tC		2.9 °C
III Renewable subsidy only	-10%		550 $/tC	3.7 °C
IV Laissez faire	-73%			5.3 °C

In scenarios II and III we consider the cases where policy makers internalize only one of the two externalities (reaching a global agreement on climate policy or introducing a scheme subsidizing renewable energy). Failing to reach an international climate agreement and implementing only a subsidy to encourage learning by doing up to a rather higher level of 550 $/GtC or 150 $/tCO2 is insufficient to bring forward the transition to renewable energy and to avoid severe climate change. This 'renewable subsidy only' case III performs better than "laissez faire" in terms of welfare and environmental outcomes but temperature rises by as much as 3.7°C above pre-industrial levels and 1250 GtC are used in total (half of the no-policy case but still three times the optimal amount). Subsidizing renewable energy induces simultaneous use of renewable along with fossil energy sources as early as 2060 with a complete transition by 2090. The presence of the climate externality, however, still lowers welfare by 10% compared to the social optimum. Interestingly, there are no Green Paradox effects as fossil fuel use is not increased in anticipation of fossil fuel being made less competitive relative to the renewable energy, since we have a Leontief production function with zero substitution between energy and the labour-capital composite. However, there are significant Green Paradox effects once these two inputs are substitutes (see blue and orange dashed lines in fossil fuel use panel in fig. E.1). Owners of fossil fuel deposits deplete their

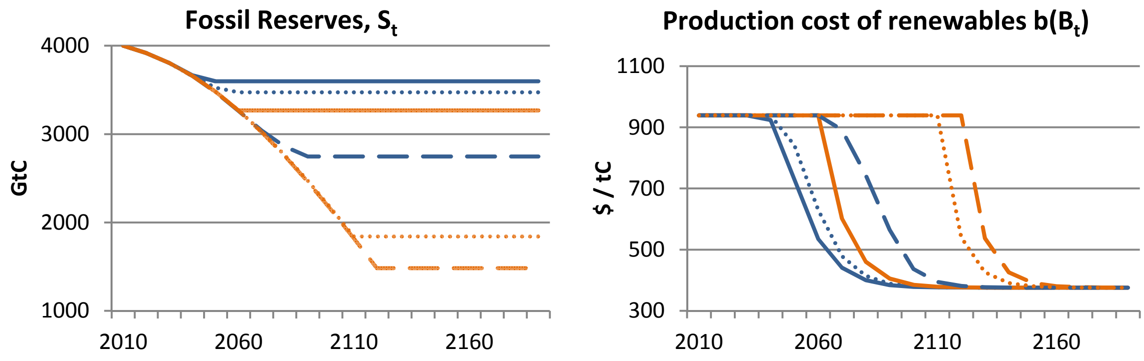
[9] This welfare metric expresses the difference in total discounted utility evaluated at today's (shadow) price of consumption to its first best equivalent as a share of initial GDP. Stern (2007) chooses to express the cost of in action in annuitiy terms. Nordhaus (2008) compares discounted utility using today's consumption as a numéraire.

Figure 2: Benchmark policy simulations



Key: laissez faire (— —), carbon tax only (——), proportional carbon tax only (·········), renewable subsidy only (— —), social optimum (——), prop. tax & optimal subsidy (·········)

Figure 2: Benchmark policy simulations (continued)



Key: laissez faire (— —), carbon tax only (——), proportional carbon tax only (·········), renewable subsidy only (— —), social optimum (——), prop. tax & optimal subsidy (·········)

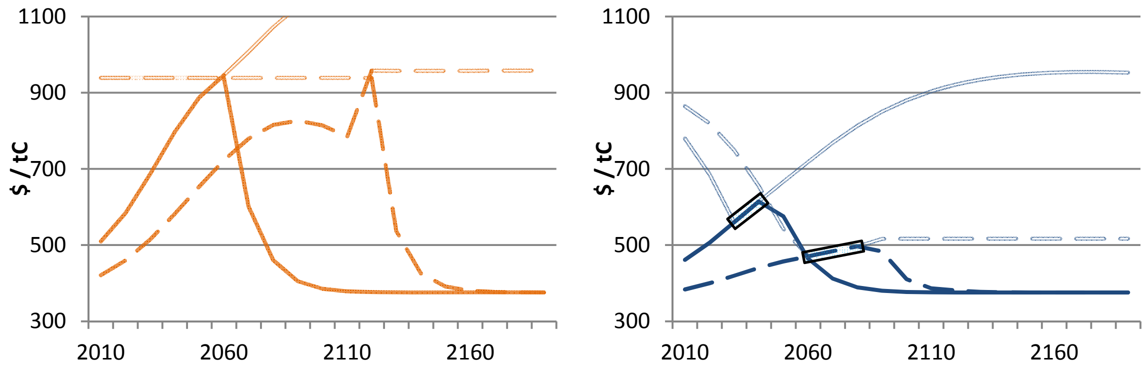
reserves more rapidly to avoid them becoming obsolete as they realize that renewables will become cheaper in the future.

The implementation of a climate policy by establishing carbon markets, taxing emissions, or direct control but ignoring the learning externality (see the orange, solid lines) is more effective in avoiding climate change and increasing welfare than the provision of a subsidy for renewable energy. In scenario III the carbon tax rises to a much higher level of 830 \$/tC or 226 \$/tCO2 than the first-best carbon tax. A carbon tax ends fossil fuel use by 2060 but does this abruptly as the learning externality is still not recognized. This policy effectively limits global warming to 2.9°C and carbon use to 730 GtC. The inefficiency of the learning externality is minor in that not internalizing it only lowers welfare by 3%. The comparison of welfare indicators across partial policy scenarios gives further credence to the assessment of Stern (2007) that climate change is the biggest externality our planet faces and leads us to conclude that policy makers should prioritize climate negotiations over renewable policy. If such negotiations fail, e.g., due to insurmountable free-riding problems associated with international treaties, (national) renewable subsidies are yet a relatively efficient instrument to avert the most severe aspects of global warming.

4.2. Time paths for the market price of fossil fuel and the renewable in the various scenarios

Market prices for both types of fossil and renewable energy are depicted in fig. 3. Regular lines depict prices of fuels in use, faint lines prices of fuels not in use. The price of fossil energy consists of the sum of marginal extraction cost and the Hotelling rent plus any carbon tax (see equation (10a)). The market price of renewable energy is set to its production cost minus any learning subsidy (see equation (10b)). Initially prices are rising in all scenarios and only on these rising sections are fossil fuels used. A subsidy for renewable energy enables a period of simultaneous use (indicated by boxed sections) and smoothes the transition to the carbon-free era. First, consider the laissez faire scenario IV (orange, dashed, left panel) where the carbon tax and the subsidy are set to zero.

Figure 3: The market price of fossil and renewable energy ($/tC)



Key left: laissez faire (– –), carbon tax only (——)

right: renewable subsidy only (——), social optimum (– –). Boxes indicate simultaneous use.

The market cost of renewable energy is above the market price of fossil energy and constant in the absence of any production and/or subsidy. The market price of fossil fuel is non-monotonic despite strictly rising extraction costs due to the non-linear path of the Hotelling rent: the Hotelling rent increases initially but soon starts to fall as the benefit of keeping carbon in situ drops to zero. The falling Hotelling rent compensates some of the increases in production costs, leading to a flattening of the market price of fossil fuel over several decades below the cost of renewables (see dashed, orange hump in the left panel of fig. 3). However, as production costs of fossil energy rise beyond those of renewable energy, prices spike and renewable sources take over all of energy production. After this switch point, the cost of fossil energy remains constant (coloured in faint orange as not in use). The cost of producing renewable energy decreases quickly and approaches its lower floor due to learning by doing. This scenario is clearly sub-optimal, since renewable energy is too expensive for much too long relative to fossil fuel.

Next, consider the case where only the climate externality is internalized using a carbon tax, scenario II (orange, solid, left panel). Adding the social cost of carbon to the price of fossil energy increases its initial price and its growth rate significantly. It surpasses the market price of renewable energy much earlier but the transition to the carbon-free era is still abrupt with a price spike in 2050 as the learning-by-doing externality is still not recognized.

If instead of the carbon tax, a subsidy for renewable energy is introduced, scenario III (dark blue, dashed, right panel), the market price of energy falls below its "laissez faire" level since fossil fuel owners fear that their resources will be worth less in the future. Lower market prices potentially stimulate higher fossil fuel use, faster extraction of fossil fuel, and acceleration of global warming; a phenomenon coined the Green Paradox by Sinn (2008). Our baseline simulations do not allow substitution between energy and the capital-labour composite and we do not find a Green Paradox here. Appendix E presents the same scenarios with a positive elasticity of substitution and lower market prices for fossil energy induce a significant increase in fossil energy use compared to the no policy scenario there (see fossil fuel panel in fig. E.1). The subsidy policy lowers the market price of renewable energy to allow for simultaneous use (indicated by boxed sections). Once renewable energy is brought into use, its price rises with that of fossil energy. The market price of renewable energy starts declining only as the transition to carbon-free energy is complete.

To implement the social optimum, scenario I (dark blue, solid, right panel), a carbon tax needs to be added to the renewable subsidy. The carbon tax on fossil energy increases the price if fossil energy beyond its "laissez faire" level. This lowers the required subsidy for renewable energy and brings forward the switch points to simultaneous use and to full renewable energy use.[10]

4.3. The decentralized equilibrium overinvests in dirty and underinvests in clean capital

How does the decentralized market economy, the no-policy scenario IV, differ from the social optimum? Initially, output, consumption and capital accumulation take place at very similar levels. However, the impacts of the climate and learning externalities are large enough to drastically change accumulation paths as global temperatures rise. This is also reflected in total welfare which is about 73% lower in the no policy case.

[10] If the social cost of carbon is added to the production cost of fossil fuel, the price of fossil energy continues to rise even as no fossil energy is produced (see the faint solid orange and dark blue lines). The social cost of carbon rises initially, because decay is limited and consumption is increasing. This yields a smaller marginal utility of consumption and thus a higher social cost of carbon, expressed in the numeraire (see equation (13)). However, after some point of time decay of atmospheric carbon dominates the decrease in marginal utility of consumption and the social cost of carbon – and with it the market price of fossil energy – start to fall. If the fall is sufficiently large, fossil energy can become competitive again. As fig. 3 indicates, the time horizon that we consider is too short and the learning-based reduction in renewable costs too large to make such re-switching optimal. A carbon tax proportional to GDP or consumption does not permit this economically sensible re-switching unless a major shock weakens the capital endowment or technological knowledge in the economy.

"Laissez faire" leads to inefficient allocation of resources, because economic decision makers do not recognize the deleterious effects of fossil fuel use and the positive effects of renewable energy on global warming. Private and social cost calculations diverge; agents overvalue the returns to conventional capital accumulation and undervalue investments in green energy sources. Failure to cooperate induces excessive fossil fuel extraction and capital accumulation leading to high global warming damages over the time horizon. This inefficient use of resources lowers welfare because it keeps consumption low in early periods of the program to allow for capital accumulation and consumption low in future periods due to high global warming damages. Damages in the no-policy scenario are large enough to lower factor returns sufficiently to induce decumulation of capital and a fall in consumption. From 2110-2140 the capital stock falls by 33% from a peak of $1070 trillion to a trough $725 trillion, consumption drops by 17% from a peak of $1375 trillion to a trough of $1150 trillion. This climate crisis is ended as extraction costs rise above the cost of renewable energy. At this point all of energy use is sourced from renewables and the climate and the economy recover from previous excessive use of fossil fuels. The inefficiency of this scenario is also reflected in the high share of GDP expended on energy which rises from around 6% in 2010 to 16% in 2110. Once the economy switches to the renewable and stocks of atmospheric carbon recede, the return to capital and the interest rate increase. This leads to a resumption of global growth. As only a fraction of carbon dissipates, welfare remains below the social optimum even in steady state.[11]

Introduction of a renewable subsidy to internalize positive effects of learning ameliorates the climate externality. Since the climate externality is still not recognized, the subsidy rises to compensate for the missing climate policy. By encouraging the usage of renewable energy, the subsidy reduces the carbon content of energy. This leads to more fossil fuel being locked up in situ and lower accumulation of carbon in the atmosphere, and allows higher consumption levels relative to the no-policy outcome. Since there is less underinvestment in clean energy, the welfare loss is curbed to 10% of current GDP.

Close inspection shows that consumption in the social optimum is the lowest for some initial periods. This implies that internalization of the climate externality would have to be phased in

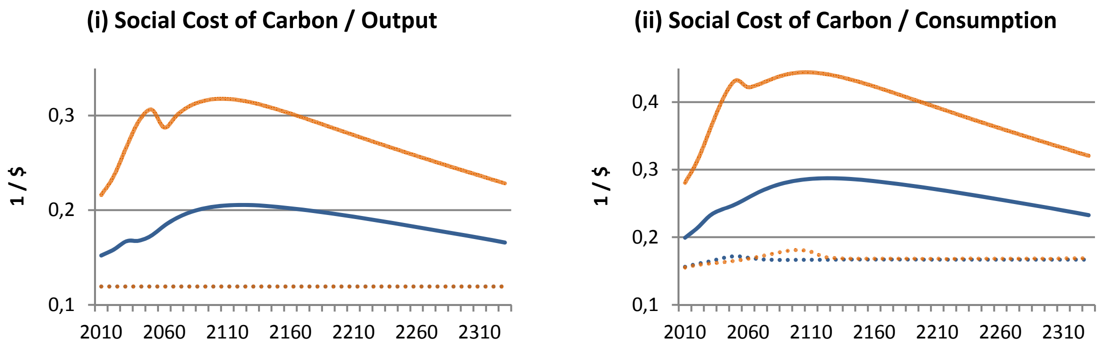
[11] Rezai et al. (2012) demonstrate the important implications of this inefficiency for the debate on the (opportunity) cost of climate change in a simple model of Leontief production technology and unlimited stocks of fossil energy.

more slowly which would be the case with a higher coefficient of intergenerational inequality aversion (lower η).[12]

4.4. The optimal carbon tax is not proportional to aggregate consumption or world GDP

To examine whether the linear formula for the optimal carbon tax (13′) put forward by Golosov et al. (2014) holds up in a more general integrated assessment model of Ramsey growth and climate change, fig. 4 plots the ratio of the optimal carbon tax to both world GDP and aggregate consumption. We immediately see that the optimal carbon tax (dark blue line) is not well described by a constant proportion of world GDP or aggregate consumption. The general pattern is that during the initial phases of fossil fuel use the social cost of carbon rises as a proportion of world GDP as more carbon emissions raise marginal damages of global warming and during the carbon-free phases the *SCC* falls as a proportion of world GDP as a significant part of the stock of carbon in the atmosphere is gradually returned to the surface of the oceans and the earth. The required rise and fall of the *SCC* is substantial.

Figure 4: The social cost of carbon as ratio of aggregate world GDP and consumption



Key: social optimum (——), carbon tax only (——),
proportional tax & optimal subsidy (········), proportional carbon tax only (········)

The dotted lines in fig. 2 provide further details on the time profiles of key variables under scenarios V and VI which set climate policy according to (13′). This proportional tax internalizes part of the *SCC* and improves welfare relative to no policy. A carbon tax proportional to GDP increases the price of fossil fuel and encourages a transition to renewable energy earlier on. However, the proportional tax starts at a too low level and rises too slowly

[12] Alternatively, the social optimum might have to be abandoned and instead one has to devise second-best intergenerational compensation schemes for policy to increase consumption in all periods. Karp and Rezai (2013) and Karp (2013) discuss the identification of generations in Ramsey models, intergenerational discounting, and the various effects of environmental policy in an OLG setting.

(and even falls in scenario VI) to mimic first best. The welfare loss of such an inefficient policy is equivalent to 48% of current GDP. Once the proportional carbon tax is supplemented with an optimal renewable subsidy, compensating the non-optimal climate policy, the social optimum is matched quiet closely and the welfare loss is less than 1%.

5. Robustness of the social cost of carbon

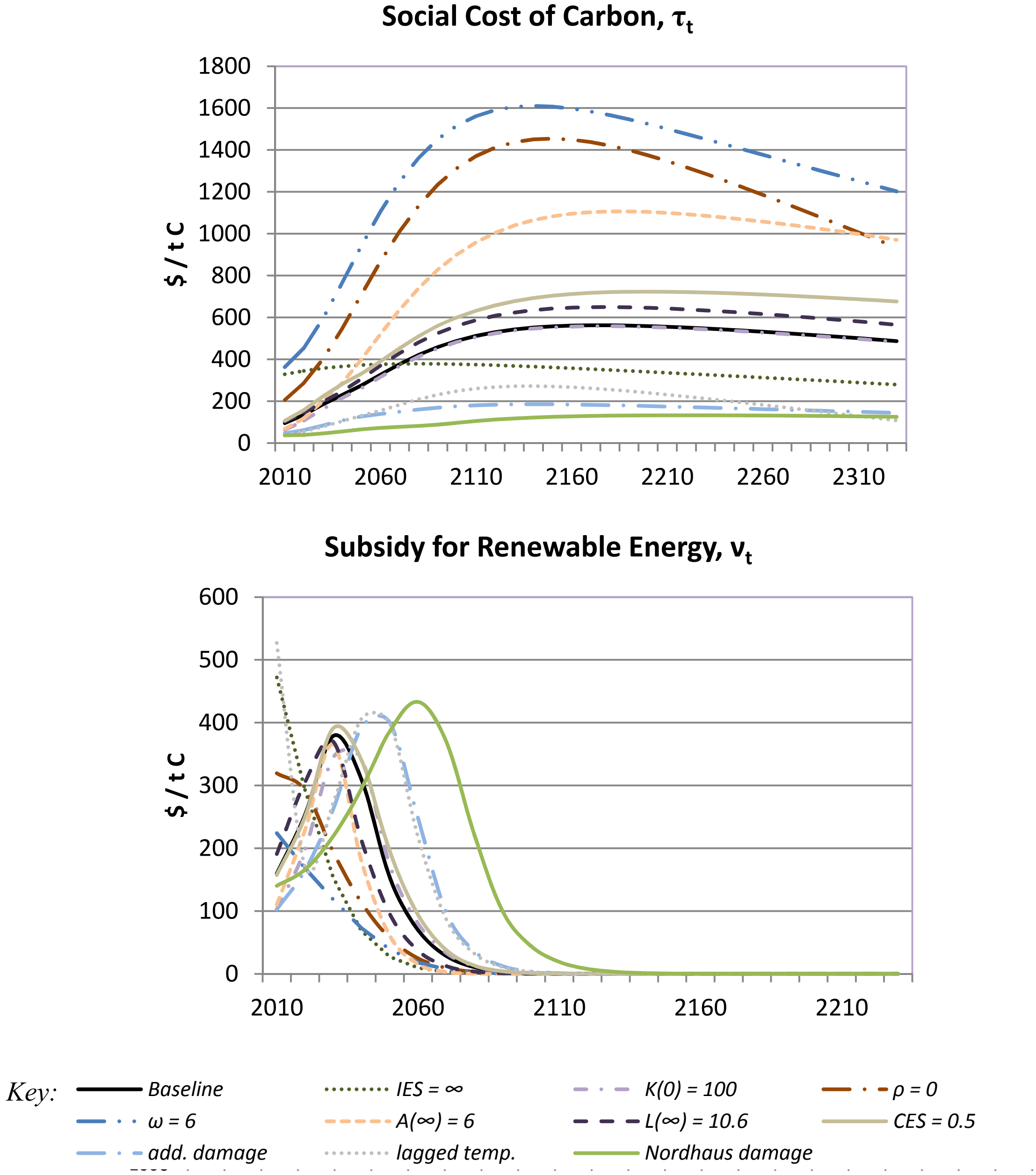
Fig. 4 shows the sensitivity of the first-best of the social optimum to some other key parameters. Our general finding of an inverted-U time profile for the social cost of carbon is robust. The exact timing and magnitude depends on specific parameters.

Table 4: Transition times and carbon budget (extended)

Scenario	*Fossil fuel*	*Simultaneous use*	*Renewable Only*	*Carbon used*
baseline	2010-2020	2030-2040	2050 –	400 GtC
$\rho = 0$	2010	2020	2030 –	120 GtC
$\eta \to \infty$	x	x	2010 –	0 GtC
$\omega = 6$	x	2010	2020 –	20 GtC
K_0=100	2010-2030	2040	2050 –	390 GtC
$A(\infty) = 6$	2010-2020	2030-2040	2050 –	400 GtC
$L(\infty) = 10.6$	2010-2020	2030-2040	2050 –	340 GtC
$\vartheta = 0.5$	2010-2030	2040	2050 –	390 GtC
proportional tax & optimal subsidy(V)	2010-2030	2040-2050	2060 –	530 GtC
prop. tax only (V)	2010-2100	x	2010 –	2,160 GtC
Nordhaus damage	2010-2050	2060-2080	2090 –	930 GtC
Temperature lag	2010-2040	2050-2060	2070 –	710 GtC

A much lower social rate of discount ($\rho = 0$) leads to a much more ambitious climate policy with a much higher social cost of carbon, earlier phasing in of renewables and more fossil fuel left in situ. A higher elasticity of intertemporal substitution implies less intergenerational inequality aversion. For example, with zero intergenerational inequality aversion ($\eta \to \infty$ instead of $\eta = ½$ as in figs. 2-4), this implies in a growing economy that the carbon tax hurts earlier generations much more than later generations. The social planner is relatively more concerned with fighting global warming than with avoiding big differences in consumption of different generations. Climate policy is also more aggressive with a higher equilibrium climate sensitivity ($\omega = 6$). Starting with half the initial capital stock ($K_0 = 100$) or increasing population and productivity growth ($A(\infty) = 6$ and $L(\infty) = 10.6$) hardly affect the *SCC*.

Figure 4: Sensitivity analysis for the time paths of the optimal social cost of carbon



Increasing the substitutability between the capital-labour aggregate and energy in the CES production function (ϑ = 0.5) leads to increased (fossil and renewable) energy use, which causes higher stocks of atmospheric carbon and a higher social cost of carbon.[13] Climate

[13] The possibility of substituting energy for capital increases the price sensitivity of energy demand. As explained in section 4.2, the case where there is only a renewable subsidy exhibits lower fossil energy prices than the no-policy scenario due the Green Paradox. Fossil resource owners are accepting lower prices today due to the anticipation of less demand for fossil energy in the future leading to higher fossil energy consumption today (see also appendix E).

policy is less ambitious with a proportional carbon tax set according to (13′) where the presence of an optimal subsidy reduces the amount of carbon burnt. Climate policy is also less ambitious if damages are lower as in Nordhaus (2008). Introducing a time lag in the temperature response to carbon increases, lowers the social cost of carbon and slows the transition to carbon-free sources of energy.

The subsidy for renewable energy use and the optimal carbon tax are complements. The increase in the *SCC* under lower discounting and intergenerational inequality aversion and a more sensitive climate allows for less aggressive renewable subsidies. The subsidy policy only increases markedly in cases in which the carbon tax (and with it the market price of fossil energy) falls: lower damages or less responsive temperature.

6. Conclusion

Our main findings based on an integrated assessment model of climate change and Ramsey growth are that it is important to not only internalize the climate externality but also learning by doing in using renewables. Pricing carbon and subsidizing renewable use curbs fossil fuel use and promotes substitution away from fossil fuel towards renewables, increases untapped fossil fuel, and brings forward the carbon-free era. Our benchmark results give rise to a global carbon tax which rises from about 100$/tC initially to 275 $/tC in 2050 and a renewable subsidy which rises from 160 $/tC initially to 380 $/tC in 2030 and then falls quickly to zero. The optimal policy mix, therefore, consists of an aggressive subsidy making renewable energy competitive early on and a gradually rising carbon tax pricing fossil energy out of the market. The total amount of carbon burnt is 400 GtC which is much less than the 2510 GtC under "laissez faire". Consequently, the social optimum manages to limit the maximum temperature to 2.3 °C instead of 5.3 °C under "laissez faire". The welfare loss without policy is 73% of today's world GDP. Climate policy becomes less ambitious in the sense that the social cost of carbon is higher, fossil fuel is abandoned less quickly and more carbon is used in total if the discount rate is higher, intergenerational inequality aversion is weaker, the equilibrium climate sensitivity is lower, and global warming damages are additive rather than multiplicative. More substitutability between energy and the capital-labour aggregate leads to more energy use especially in capital-scarce economies and more global warming. Hence, the required global carbon tax is higher.

If international agreements on cooperation in climate change mitigation cannot be reached, governments can move forward unilaterally by introducing subsidies to renewable energy. Such policy internalizes the learning externality and accelerates fossil fuel extraction and global warming as fossil fuel owners fear that their resources will be worth less in the future. The level and the duration of the subsidy policy increases compared to the social optimum to compensate for the missing carbon tax. This limits global warming to 3.7 °C and the loss in welfare to 10% relative to the social optimum. If a carbon tax is introduced instead of the subsidy, the welfare loss reduces to 3%. The comparison of welfare indicators across partial policy scenarios gives further credence to the assessment of Stern (2007) that climate change is the biggest externality our planet faces and leads us to conclude that policy makers should prioritize climate negotiations over renewable policy. If such negotiations fail, e.g., due to insurmountable free-riding problems associated with international treaties, (national) renewable subsidies are yet a relatively efficient instrument to avert the most severe aspects of global warming.

Renewable subsidies in isolation, however, are insufficient to combat climate change. Making total factor and energy productivities also endogenous (using the empirical estimates of the determinants of growth rates given in Hassler et al., 2011) allows further substitution possibilities between energy and the capital-labour aggregate in the longer run and justifies a more ambitious climate policy. R&D subsidies should be set such that the price of renewable energy follows the (rising) price of fossil energy and need to be complemented by a carbon tax in order to avoid excessive extraction associated with the Green Paradox.

A recent interagency working group (2010) suggests that US institutions use a social cost of carbon of initially \$80/tC rising to 165 \$/tC in 2050 in project appraisal based on a discount rate of 3% per annum. A discount rate of 2.5% per annum would imply an initial social cost of carbon of 129 \$/tC rising to 238 \$/tC in 2050, which in line with our estimates.[14] These figures are typically based on existing integrated assessment models which are often very elaborate and in which consumers rarely maximize utility as in the Ramsey model. Hence, it is not always clear what the underlying assumptions and the crucial parameters deriving the results are. Golosov et al. (2014) offer a tractable fully consistent general equilibrium model

[14] Such models of climate change yield estimates of the social cost of carbon starting from 5 to 35 \$ per ton of carbon in 2010 and rising to \$16 to \$50 per ton in 2050 (e.g., the DICE, PAGE and FUND models of Nordhaus (2008), Alberth and Hope (2007) and Tol (2002), respectively), but the Stern Review obtains much higher estimates of the social cost of carbon with a much lower discount rate (Stern, 2007).

of climate change and Ramsey growth, but employ unrealistically low damages at higher temperatures and need to make some very bold assumptions to ensure that both aggregate consumption and the carbon tax are a fixed proportion of world GDP. Much too much carbon is burnt and the welfare losses are substantial with this proportional carbon tax, especially if there is no policy in place for encouraging learning by doing in renewable production.

Our model of Ramsey growth and climate change has more realistic damages and finds that the optimal carbon tax is a hump-shaped function of world GDP. Our analysis also pays careful attention to how fast and how much fossil fuel should be abandoned and how quickly and how much renewables should be phased in. Our results suggest a 'third way' in climate policy which consists of a quick and aggressive path of upfront renewable subsidies to stimulate use of renewables and enjoy the fruits of learning by doing, confirming the logic of direct technical change and kick-starting green innovation developed in Acemoglu et al. (2012) and Mattauch (2012), and a gradually rising carbon tax as advocated in most integrated assessment studies including Nordhaus (2008) and Stern (2007).

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Appendix A: A simple formula for the social cost of carbon

Golosov et al. (2014) use a Cobb-Douglas production function with negative exponential damages:

(A1) $$Y_t = Z(E_t)AK_t^{\alpha}F_t^{\beta} = \exp\left[-\gamma(2.13E_t - 581)\right]AK_t^{\alpha}F_t^{\beta}, \quad \gamma = 2.379\times 10^{-5},$$

where K_t is the capital stock at the start of period t, A is the calibrated total factor productivity (including the contribution of fixed factors such as labour and land), α is the share of capital in value added, and β is the share of energy in value added. Apart from (1), (2) and (A1), Golosov et al. (2014) assume logarithmic utility, 100% depreciation of manmade capital each period, zero fossil fuel extraction costs and thus full exhaustion of initial fossil fuel reserves. With these bold assumptions one can show that the social cost of carbon or the optimal carbon tax is a constant fraction of world GDP:

(A2) $$\tau_t^{Golosov\ c.s.} = 1000\tilde{\zeta}\left[\frac{\varphi_L}{1-(1+\rho)^{-1}} + \frac{(1-\varphi_L)\varphi_0}{1-(1-\varphi)(1+\rho)^{-1}}\right]Y_t = \left[\frac{0.004758}{1-(1+\rho)^{-1}} + \frac{0.00748}{1-0.9772(1+\rho)^{-1}}\right]Y_t,$$

where $\rho > 0$ is the rate of time preference (see equation (13′)). Decadal world GDP is 630 US$ trillion in 2010, so that using a discount rate of 10% per decade (or 0.96% per year), $\rho = 0.1$, we get for 2010 a social cost of carbon of 75 US$/tC. This estimate is lower if society is less patient. The beauty of (A2) is that no detailed integrated assessment model of growth and climate change is needed. The carbon tax follows directly from the rate of time preference ρ, world GDP and some technical damage and carbon cycle parameters. Formula (A2) breaks down if intergenerational inequality aversion or factor substitution differs from unity, extraction costs are non-zero (especially if they are stock dependent) and the bad fit of the exponential approximation of (5) plays up. We will therefore investigate the robustness of this formula for the optimal carbon tax in a general integrated assessment model of Ramsey growth and climate change.

Appendix B: Proof of proposition 1

The adjoined Lagrangian for our model of Ramsey growth and climate change reads:

$$L \equiv \sum\nolimits_{t=0}^{\infty}(1+\rho)^{-t}\left[L_tU_t(C_t/L_t) - \mu_t^S(S_{t+1} - S_t + F_t) - \mu_t^B(B_{t+1} - B_t - R_t)\right]$$

$$+\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^{PE}(E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE}\left\{E_{t+1}^T - (1-\varphi)E_t^T - \varphi_0(1-\varphi_L)F_t\right\}\right]$$
$$-\sum_{t=0}^{\infty}(1+\rho)^{-t}\lambda_t\left[K_{t+1} - (1-\delta)K_t - Z(E_t^P + E_t^T)H(K_t, L_t, F_t + R_t) + G(S_t)F_t + b(B_t)R_t + C_t\right],$$

where μ_t^S denotes the shadow value of *in-situ* fossil fuel, μ_t^B the shadow value of learning by doing, μ_t^{PE} and μ_t^{TE} the shadow disvalue of the permanent and transient stocks of atmospheric carbon, and λ_t the shadow value of manmade capital. Necessary conditions for a social optimum are:

(A3a) $U'(C_t / L_t) = (C_t / L_t)^{-1/\eta} = \lambda_t,$

(A3b) $Z_t H_{F_t+R_t} \le G(S_t) + \left[\mu_t^S + \varphi_L \mu_t^{PE} + \varphi_0(1-\varphi_L)\mu_t^{TE}\right]/\lambda_t, \quad F_t \ge 0, \quad \text{c.s.},$

(A3c) $Z_t H_{F_t+R_t} \le b(B_t) - \mu_t^B / \lambda_t, \quad R_t \ge 0, \quad \text{c.s.},$

(A3d) $(1-\delta + Z_{t+1}H_{K_{t+1}})\lambda_{t+1} = (1+\rho)\lambda_t,$

(A3e) $\mu_{t+1}^S = (1+\rho)\mu_t^S + G'(S_{t+1})F_{t+1}\lambda_{t+1},$

(A3f) $\mu_{t+1}^B = (1+\rho)\mu_t^B - b'(B_{t+1})R_{t+1}\lambda_{t+1},$

(A3g) $\mu_{t+1}^{PE} = (1+\rho)\mu_t^{PE} + Z'(E_{t+1}^P + E_{t+1}^T)H_{t+1}\lambda_{t+1},$

(A3h) $(1-\varphi)\mu_{t+1}^{TE} = (1+\rho)\mu_t^{TE} + Z'(E_{t+1}^P + E_{t+1}^T)H_{t+1}\lambda_{t+1}.$

Equations (A3a) and (A3d) yield the Euler equation (9). The Kuhn-Tucker conditions (A3b) and (A3c) can be rewritten as (10a) and (10b) after defining $\theta_t^S \equiv \mu_t^S / \lambda_t$, $\theta_t^B \equiv \mu_t^B / \lambda_t$ and $\theta_t^E \equiv \left[\varphi_L \mu_t^{PE} + \varphi_0(1-\varphi_L)\mu_t^{TE}\right]/\lambda_t$ in final good units. Equations (A3e) and (A3d) yield the Hotelling rule

(A4) $\theta_{t+1}^S = (1+r_{t+1})\theta_t^S + G'(S_{t+1})F_{t+1}.$

Forward summation over time of (A4) gives (11). Equations (A3f) and (A3d) yield

(A5) $\theta_{t+1}^B = (1+r_{t+1})\theta_t^B + b'(B_{t+1})R_{t+1}.$

Forward summation over time of (A5) yields (12). Defining $\theta_t^{PE} \equiv \mu_t^{PE} / \lambda_t$ and $\theta_t^{TE} \equiv \mu_t^{TE} / \lambda_t$ in final good units we use (A3g), (A3h) and (A3a) to get:

(A6a) $\theta_{t+1}^{PE} = (1+r_{t+1})\theta_t^{PE} + Z'(E_{t+1}^P + E_{t+1}^T)H_{t+1},$

(A6b) $(1-\varphi)\theta_{t+1}^{TE} = (1+r_{t+1})\theta_t^{TE} + Z'(E_{t+1}^P + E_{t+1}^T)H_{t+1}.$

Solving (A6a) and (A6b) it can be shown that $\theta_t^E \equiv \varphi_L \theta_t^{PE} + \varphi_0(1-\varphi_L)\theta_t^{TE}$ is given by (13).

Appendix C: A lag between temperature and the atmospheric stock of carbon

We now suppose a lag between atmospheric stock of carbon and global mean temperature:

(3′) $T_{t+1} = (1-\varphi_T)T_t + \varphi_T \omega \ln\left((E_t^P + E_t^T)/581\right),$

where φ_T is the speed at which a higher stock of atmospheric carbon gets translated into a higher global mean temperature. Since it takes on average about 70 years, we set φ_T = 1.70. With this additional equation the Lagrangian function becomes:

$$
\begin{aligned}
L \equiv & \sum_{t=0}^{\infty}(1+\rho)^{-t}\left[L_t U_t(C_t/L_t) - \mu_t^S(S_{t+1}-S_t+F_t) - \mu_t^B(B_{t+1}-B_t-R_t)\right] \\
& -\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^T\left\{T_{t+1}-(1-\varphi_T)T_t - \varphi_T\omega\ln\left((E_t^P+E_t^T)/581\right)\right\}\right] \\
& +\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^{PE}(E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE}\left\{E_{t+1}^T - (1-\varphi)E_t^T - \varphi_0(1-\varphi_L)F_t\right\}\right] \\
& -\sum_{t=0}^{\infty}(1+\rho)^{-t}\lambda_t\left[K_{t+1}-(1-\delta)K_t - Z(T_t)H(K_t,L_t,F_t+R_t) + G(S_t)F_t + b(B_t)R_t + C_t\right].
\end{aligned}
$$

where μ_t^T is the marginal disvalue of global warming. Necessary conditions for a social optimum are (11a)-(11f) as before and:

(11g′) $\mu_{t+1}^{PE} = (1+\rho)\mu_t^{PE} + \varphi_T\omega(E_{t+1}^P + E_{t+1}^T)^{-1}\mu_{t+1}^T,$

(11h′) $(1-\varphi)\mu_{t+1}^{TE} = (1+\rho)\mu_t^{TE} + \varphi_T\omega(E_{t+1}^P + E_{t+1}^T)^{-1}\mu_{t+1}^T,$ and

(11i′) $(1-\varphi_T)\mu_{t+1}^T = (1+\rho)\mu_t^T + Z'(T_{t+1})H_{t+1}\lambda_{t+1}.$

We now get as before (12)-(15). The dynamics of the permanent and transient components of the social cost of carbon become:

$$
(16')\quad
\begin{aligned}
&\theta_{t+1}^{PE} = (1+r_{t+1})\theta_t^{PE} + \omega(E_{t+1}^P + E_{t+1}^T)^{-1}\theta_{t+1}^T, \\
&(1-\varphi)\theta_{t+1}^{TE} = (1+r_{t+1})\theta_t^{TE} + \omega(E_{t+1}^P + E_{t+1}^T)^{-1}\theta_{t+1}^T,
\end{aligned}
$$

where $\theta_t^T \equiv \mu_t^T/\lambda_t$. Solving (16′) yields the social cost of carbon as the present discounted value of all future marginal damages from global warming:

(A1) $\theta_t^C = \sum_{s=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s\right\}\Delta_{t+s}\omega(E_{t+s+1}^P + E_{t+s+1}^T)^{-1}\theta_{t+s+1}^T\right].$

Solving (11i) together with (11d) yields:

(A2) $(1-\varphi_T)\theta_{t+1}^T = (1+r_{t+1})\theta_t^T + Z'(T_{t+1})H_{t+1}.$

This equation yields the marginal cost of global warming:

(A3) $\theta_t^T = -\sum_{s=0}^{\infty}\left[(1-\varphi_T)^s \Delta_{t+s} Z'(T_{t+s+1})H_{t+s+1}\right].$

Upon substituting (A3) into (A1) we get the social cost of burning an extra unit of fossil fuel.

Relationship to Golosov et al. (2014)

Golosov et al. (2014) assume that there is no lag between an increase in the stock of atmospheric carbon and global mean temperature. Following Gerlagh and Liski (2012) we do allow for such a lag. Under this set of assumptions, (A3) shows that the marginal cost of global warming at the social optimum is proportional to global GDP (cf. equation (6)):

(A3′) $\theta_t^T = 2.379\times 10^{-5}\left(\dfrac{1+\rho}{\rho+\varphi_T}\right)Z(T_t)H(K_t, L_t, F_t + R_t).$

Upon substitution of (A3′) into (A1), we obtain:

(A4) $\theta_t^C = 2.379\times 10^{-5}\left(\dfrac{1+\rho}{\rho+\varphi_T}\right)\sum_{s=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s\right\}\Delta_{t+s}\omega(E_{t+s+1}^P + E_{t+s+1}^T)^{-1} Z(T_{t+s+1})H_{t+s+1}\right].$

This expression does not simplify to a simple expression depending only on current global GDP. However, if we use a dynamic reduced-form temperature module with a distributed lag between carbon stock and damages,

(A5) $Z_t = Z(E_t), \quad E_{t+1} = (1-\varphi_T)E_t + \varphi_T(E_{t+1}^P + E_{t+1}^T),$

we have the Lagrangian function

$$
\begin{aligned}
L \equiv & \sum_{t=0}^{\infty}(1+\rho)^{-t}\left[L_t U_t(C_t/L_t) - \mu_t^S(S_{t+1} - S_t + F_t) - \mu_t^B(B_{t+1} - B_t - R_t)\right] \\
& -\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^E\left\{E_{t+1} - (1-\varphi_T)E_t - \varphi_T(E_{t+1}^P + E_{t+1}^T)\right\}\right] \\
& +\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^{PE}(E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE}\left\{E_{t+1}^T - (1-\varphi)E_t^T - \varphi_0(1-\varphi_L)F_t\right\}\right] \\
& -\sum_{t=0}^{\infty}(1+\rho)^{-t}\lambda_t\left[K_{t+1} - (1-\delta)K_t - Z(E_t)H(K_t, L_t, F_t + R_t) + G(S_t)F_t + b(B_t)R_t + C_t\right].
\end{aligned}
$$

and thus we get:

(11g″) $\mu_{t+1}^{PE} = (1+\rho)(\mu_t^{PE} + \varphi_T \mu_t^E),$

(11h″) $(1-\varphi)\mu_{t+1}^{TE} = (1+\rho)(\mu_t^{TE} + \varphi_T \mu_t^E)$,

(11i′) $(1-\varphi_T)\mu_{t+1}^E = (1+\rho)\mu_t^E + Z'(E_{t+1})H_{t+1}\lambda_{t+1}$.

It thus follows that

(A6) $\theta_t^C = \sum_{s=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s\right\}\Delta_{t+s}\theta_{t+s+1}^E\right]$,

(A7) $\theta_t^E = -\sum_{s=0}^{\infty}\left[(1-\varphi_T)^s \Delta_{t+s} Z'(E_{t+s+1})H_{t+s+1}\right]$.

Under the Golosov et al. assumptions, we get

(A7′) $\theta_t^E = 2.379\times 10^{-5}\left(\dfrac{1+\rho}{\rho+\varphi_T}\right)Z(E_t)H_t$

and thus the following simple expression for the social cost of carbon:

(A6′) $\theta_t^C = 2.379\times 10^{-5}\left(\dfrac{1+\rho}{\rho+\varphi_T}\right)\left[\left(\dfrac{1+\rho}{\rho}\right)\varphi_L + \left(\dfrac{1+\rho}{\rho+\varphi}\right)\varphi_0(1-\varphi_L)\right]Z(E_t)H(K_t,L_t,F_t+R_t)$.

A lag between the atmospheric stock of carbon and damages ($\varphi_T > 0$) thus pushes down the social cost of carbon so our estimates of the optimal global carbon tax will be biased upwards.

Appendix D: Functional forms and calibration

Preferences

As was already clear from (8), we suppose a CES utility function. We set the elasticity of intertemporal substitution to $\eta = ½$ and thus intergenerational inequality aversion to 2. The rate of pure time preference ρ is set to 10% per decade which corresponds to 0.96% per year.

Cost of energy

We employ an extraction technology of the form $G(S) = \gamma_1(S_0/S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 5%-7% depending on the policy scenario. This translates to fossil production costs of $350/tC ($35/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon. This gives approximately $G(S_0) = \gamma_1 = 0.75$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume a more doubling but less than quadrupling of production costs if a total 3000 GtC is extracted. With $S_0 = 4000$ GtC,[1] this gives $\gamma_2 = 0.75$.[2] In general, we assume very low extraction costs and a high initial stock of reserves.

Initial capital stock and depreciation rate

The initial capital stock is set to 200 (US$ trillion), which is taken from Rezai et al. (2012). We set δ to be 0.5 per decade, which corresponds to a yearly depreciation rate of 6.7%.

Global production and global warming damages

Output before damages is $H_t = \left[(1-\beta)\left(AK_t^{\alpha} (A_t^L L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta (\frac{F_t + R_t}{\sigma})^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$, $\vartheta \geq 0, 0 < \alpha < 1$ and $0 < \beta < 1$. This is a constant-returns-to scale CES production function in energy and a capital-labour composite with θ the elasticity of substitution and β the share the parameter for energy. The capital-labour composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital, A total factor productivity and A_t^L the efficiency of labour. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same level of global warming damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as

$H_t = H_0 \left[(1-\beta) \left(\frac{AK_t^{\alpha}(A_t^L L_t)^{1-\alpha}}{H_0} \right)^{1-1/\vartheta} + \beta \left(\frac{F_t + R_t}{\sigma H_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$. We set the share of capital to $\alpha =$ 0.35 and the energy share parameter to $\beta = 0.06$. For the elasticity of factor substitution ϑ, we consider two alternatives: $\vartheta = 0$ (Leontief) for the benchmark run and $\vartheta = 0.5$ which we will refer to as the CES run. World GDP in 2010 is 63 $trillion. The energy intensity of output σ is calibrated to current energy use. In the Leontief case energy demand (only fossil fuel initially) is $F_0 = \sigma Z_0 H_0$. With carbon input of 8.36GtC in 2010, we get $\sigma = (8.36 / 2.13) / 63 = 0.062$. Finally, given $A_1^L = 1$ we can back out $A = 34.67$. Under CES we arrive at a different value.

[1] Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2-scenario that 7000 $GtCO_2$ (with 3.66 tCO2 per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

[2] Since $G(1000)/G(4000) = (4000/1000)^{\gamma_2} = 4^{\gamma_2}$ and $4^{0.75} = 2.8$.

The approach is to keep σ fixed and to use actual values for energy, labour and capital to get the initial global output level of 630 $ trillion per decade.

Population growth and labour-augmenting technical progress

Population in 2010 (L_1) is 6.5 billion people. Following Nordhaus (2008) and UN projections population growth is given by $L_t = 8.6 - 2.1e^{-0.35t}$. Population growth starts at 1% per year and falls below 1% percent per decade within six decades and flattens out at 8.6 billion people. In the sensitivity analysis in section 4.3 we assume faster growth and a higher plateau to reflect more recent forecasts. Without loss of generality the efficiency of labour $A_t^L = 3 - 2e^{-0.2t}$ starts out with $A_1^L = 1$ and an initial Harrod-neutral rate of technical progress of 2% per year. The efficiency of labour stabilizes at 3 times its current level.

Cost of the renewable and learning by doing

We model learning by doing with initial cost reductions and a lower limit for the cost of the renewable, i.e., $b(B_t) = \chi_1 + \chi_2 e^{-\chi_3 B_t}$, $\chi_1, \chi_2, \chi_3 \geq 0$. This formulation differs from the usual power law definition of learning curves (Manne and Richels, 2004) but allows us to better calibrate initial learning rates (which can reach infinity for power law) and formulate specific lower limit for unit cost. We calibrate unit cost of renewable energy to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\vartheta \to 0$, energy demand is $\sigma Z_t H_t$. The cost of generating all energy carbon free is $\sigma Z_t H_t b / Z_t H_t = \sigma b_t$. Nordhaus (2008) states that it costs 5.6% of GDP to decarbonise today's economy in a model of back-stop mitigation. In the model considered here this cost estimate needs to be added to the cost of producing conventional energy ranging between 5%-7%. This gives $\sigma b_1 = 0.12$ or with $\sigma = 0.062$ we get $b_1 = b(0) = \chi_1 + \chi_2 = 2$ or \$940/tC. Through learning by doing this cost can be reduced by 60% to a lower limit of 5% of GDP, so that $b(\infty) = \chi_1 = 0.8$ and thus $\chi_2 = 1.2$. We assume that cumulative renewable production lowers unit cost at a falling rate and the parameter χ_3 measures this speed of learning. We calibrate learning such that costs would decrease slowly. We suppose a 20% cost reduction if all of world energy use would be supplied by renewable sources for a whole decade or, equivalent, capacity would increase more than 5 times, so that χ_3 = 0.008.[3] This assumption is very

[3] This calibration is done for a Leontief technology and assumes that renewable energy is a viable but expensive alternative to fossil fuels. We assume that for a more general technology the same parameter values can be used.

conservative: Manne and Richels (2004) assume the same cost reduction if cumulative production doubles (but have to impose unrealistic constraints to on renewable use due to the strong curvature of the power law learning curve). Alberth and Hope (2007) assume a cost reduction of 5%-25% for a doubling of cumulative experience. Our calibration assumes a shallow learning curve and, together with the assumption with the calibration of abundant fossil fuel, biases the model toward fossil fuel use.

Computational implementation

In our simulations we solve the model for finite time and use the turnpike property to approximate the infinite-horizon problem. All equilibrium paths approach the steady state quickly such that the turnpike property renders terminal conditions essentially unimportant. We allow for continuation stocks to reduce the impact of the terminal condition on the transitions paths in the early periods of the program. We use the computer program GAMS and its optimization solver CONOPT3 to solve the model numerically. The social planner solution, OPT, in which the externality is taken into account fit the program structure readily. To solve the business-as-usual (BAU) equilibrium paths, we adopt the iterative approach discussed in detail in Rezai (2011). To approximate the externality scenario, the aggregate economy is fragmented into *N* dynasties. Each dynasty has *1/N*th of the initial endowments and chooses consumption, investment and energy use in order to maximize the discounted total utility of per capita consumption. The dynasties understand the contribution of their own emissions to the climate change and learning in renewable energy, but take carbon emissions and knowledge generation of others as given. The climate and knowledge dynamics are affected by the decisions of all dynasties. This constitutes the market failures.

It might seem easier to simply assume that there is one dynasty that ignores the externality but this would not be a rational expectations equilibrium. The problem of a planner in a fragmented economy is not an optimization problem. The CONOPT3 solver of GAMS is very powerful in solving maximization problems and it is more efficient to adopt an iterative routine than to attempt solving the equilibrium conditions directly. Given a technological specification, the computation of all four scenarios takes less than one minute. To introduce this approximate externality, we divide the initial stocks of capital, labour, and fossil fuel reserves by *N*. All production and cost functions are homogeneous of degree 1 and therefore invariant to *N*. The introduction of the pollution externality only requires a modification of the transition equation of atmospheric carbon to include emissions regarded as exogenous by each dynasty, F_t^{Exog} (cf. equations (1) and (2)):

$$E_{t+1}^P = E_t^P + \varphi_L (F_t + F_t^{Exog}),$$
$$E_{t+1}^T = (1-\varphi)E_t^T + \varphi_0 (1-\varphi_L)(F_t + F_t^{Exog}).$$

The introduction of the learning externality needs a modification of the transition equation of cumulative production to include production regarded as exogenous by each dynasty, R_t^{Exog} :

$$B_{t+1} = B_t + R_t + R_t^{Exog}.$$

In the BAU scenario all dynasties essentially play a dynamic non-cooperative game, which leads to a Nash equilibrium in which each agent forecasts the paths of emissions and renewable generation correctly and all agents take the same decisions. As all dynasties are identical, equilibrium requires $F_t^{Exog} = (N-1)F_t$ and $R_t^{Exog} = (N-1)R_t$. Under business as usual the decision maker only adjusts her controls to take into account the effects of her own decisions (i.e. 1/*N*th of the greenhouse gas and the learning externalities). If $N = 1$ the externalities are internalized and we obtain the social optimum. As $N \to \infty$, we obtain the “laissez faire” outcome characterized in section 2.

Following Rezai (2011), the numerical routine starts by setting the time path of emissions exogenous to the dynasty's optimization, ${}^0F_t^{Exog}$ and ${}^0R_t^{Exog}$, at an informed guess. (The left superscript denotes the index of iteration.) GAMS solves for the representative dynasty's welfare-maximizing investment, consumption, and energy use choices conditional on this level of exogenous emissions. $(N-1)$ times the representative dynasty's energy profile defines the time profile of exogenous fossil and renewable energy use in the next iteration, ${}^{i+1}F_t^{Exog} = (N-1)\cdot {}^iF_t$ and ${}^{i+1}R_t^{Exog} = (N-1)\cdot {}^iR_t$. The routine is repeated and F_t^{Exog} and R_t^{Exog} are updated until the difference in the time profiles between iterations meets a pre-defined stopping criterion. In the reported results iterations stop if the deviation at each time period is at most 0.001%.

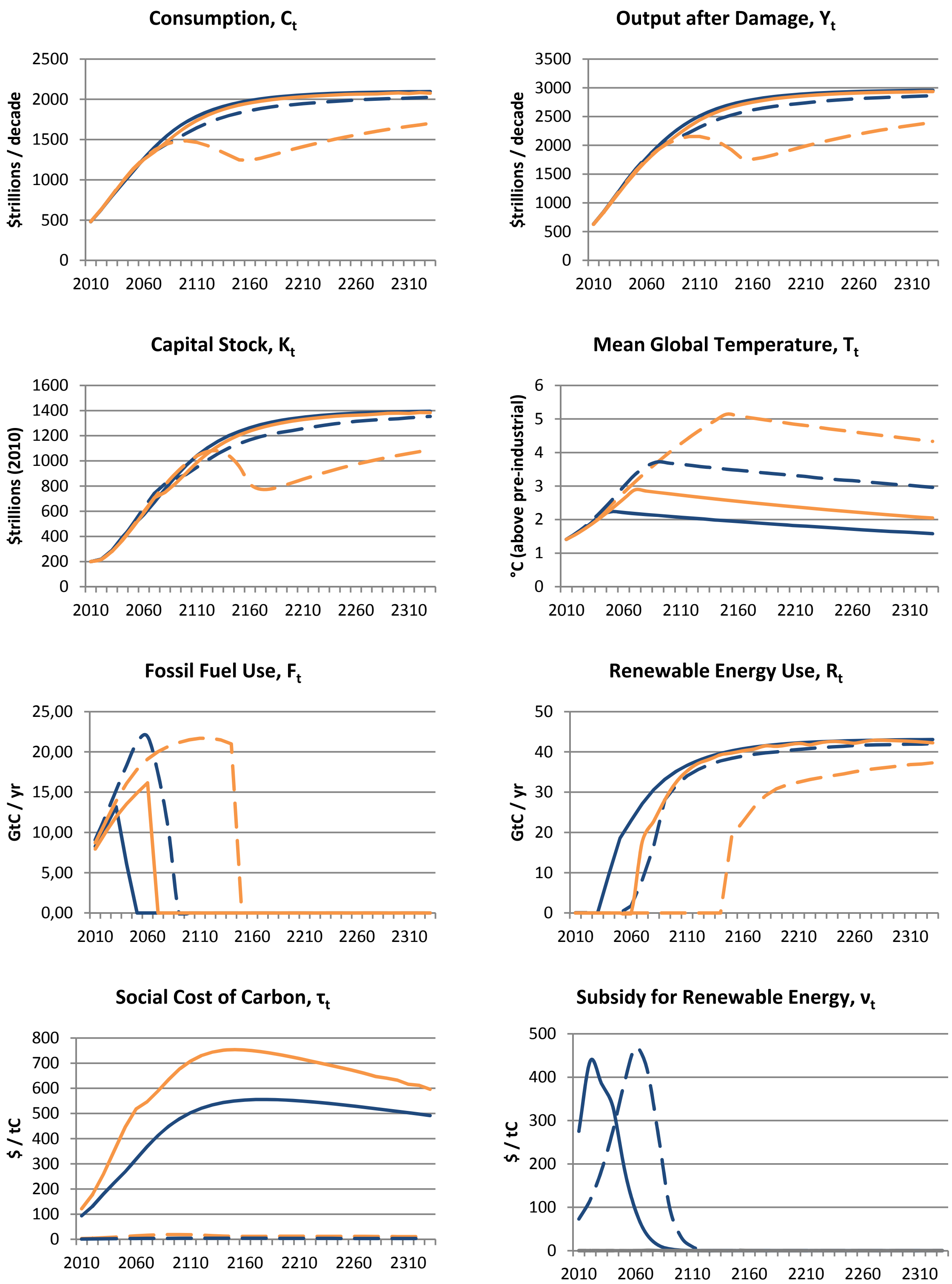
We set $N = 400$ to account for the fact that in the present world economy, the externality in the market of GHG emissions is already internalized to a very small extent through the imposition of carbon taxes or tradable emission permits and non-market *regulation* (e.g. through the Kyoto Protocol or the establishment of the European Union Emission Trading

Scheme). In our BAU simulations, the dynastic planner takes into account less than 0.25% of global emissions.

Appendix E: CES technology

With a CES technology the substitution possibilities between the capital-labour aggregate on the one hand and energy on the other hand are feasible, in contrast with the Leontief technology. This implies that energy demand is more sensitive to relative price changes. In the previous section we have already discussed the relationship between scenarios in economies with additive and multiplicative damages. The outcomes for the case of a CES production function are presented in fig. 4 and inspection confirms that the qualitative differences between additive and multiplicative production damages are unaffected. For the sake of brevity, we concentrate here on the differences brought about by allowing for a higher degree of substitutability. The patterns of optimal capital accumulation and consumption are hardly affected. With a higher degree of substitutability more fossil fuel is used initially to substitute for scarce capital. This also holds in the market economy. Therefore, substitutability helps poor economies to increase growth. As a result, more carbon is burnt, the social cost of carbon is consistently higher and the economy switches to renewable earlier. The same holds for the laissez faire economy: with better substitution possibilities more fossil fuel is used initially but for a shorter period of time, until 2090 instead of 2120.

Figure E.1: Simulation results with CES production technology



Key: no policy (– –), carbon tax only (——), renewable subsidy only (– –), social optimum (——)