Common and private ownership of exhaustible resources: theoretical implications for economic growth*

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Abstract

We develop two models of economic growth with exhaustible resources and consumers which are heterogeneous in their intertemporal preferences. The first model assumes private ownership of resource stocks. In the second model, resources are owned collectively and the resource rent is equally divided among all consumers, while the resource utilization rate is chosen by voting. We show that in the case of private resource stocks, the resource utilization rate and the steady state rate of growth in equilibrium are determined by the patience of the most patient consumers. In case of common resources these values are determined by the patience of the median consumer. If the median consumer is less patient than the most patient one, in the case of private resources the resource utilization rate is lower and the economic growth rate is higher than in the case of common resources.
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1 Introduction

We develop two Ramsey-type models of economic growth with heterogeneous consumers and exhaustible resources. Our goal is to describe possible effects of different ownership regimes on the resource utilization rate and the rate of growth. The first model assumes private ownership of resource stocks (mines, oilfields etc.) perceived by consumers as assets which they can buy and sell and in which they can invest their savings. That means the resource owners can earn a rent. The second model supposes common property, with resource stocks owned collectively (Heltberg, 2002; Ostrom and Hess, 2007), and the resource rent is equally divided among all the consumers.

Both models assume that consumers are heterogeneous in their inter-temporal preferences. Following (Becker, 1980) we describe this heterogeneity as a difference in consumers' discount factors which are higher for more patient consumers and lower for less patient ones. We show that in the first model, all the capital as well as all the resources belong to the most patient consumers. In the model with common property on the resources, consumer heterogeneity leads to variation in preferences on the rate of resource extraction, since less patient consumers wish the resources to be extracted faster than the more patient consumers. In this case, we assume that the consumers choose the resource utilization rate by voting and prove that the patience of the median voter determines the resource utilization rate which can be greater than

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in the model with private resource stocks. Since the resource utilization rate in equilibrium is related to the growth rate, we demonstrate that private ownership of exhaustible resources is characterized by lower equilibrium resource utilization rate and a lower rate of economic growth compared to the equilibrium resource utilization rate and the rate of economic growth in the case of common resources.

The paper is organized as follows. In section 2, we describe the model with private property on resources, define equilibrium paths, describe properties of steady-state equilibria, and show that in a steady-state equilibrium all exhaustible resources belong to the most patient consumers. In section 3, we develop the model with common resource stocks. We describe the model, define equilibrium paths, and analyze steady-state equilibria. After introducing the political economy consideration into the model (i.e., voting procedure), we narrow the set of equilibria under consideration to the steady state voting equilibria (we use the apporach proposed in Borissov *et al.*, 2010). We describe properties of the steady-state voting equilibria and formulate a version of the median voter theorem. The discussion of the results follows in section 4.

2 Economic growth model with private ownership of resource stocks

2.1 Consumers

Suppose that there are L consumers (we assume that there is an odd number of them). We presume that all the consumers live for infinite period of time and are identical in all respect except their discount factors. Each time each consumer supply one unit of labor force on the labor market. Thus the total labor supply at each time is L.

The utility function of consumer i is of the form

$$\sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}),$$

where β_i is the discount factor of this consumer and $C_{i,t}$ is his consumption at time t. We assume that $u(C) = \ln C$.

We suppose the households to be sorted in ascending order of their discount factors:

$$0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_L < 1.$$

By J we denote the set of agents with the highest discount factor:

$$J = \{i = 1, \ldots, L \mid \beta_i = \beta_L\}.$$

The budget constraints of consumer i are of the following form

$$C_{i,t} + S_{i,t} \leq (1 + r_t)\, S_{i,t-1} + W_t, \quad S_{i,t} \geq 0, \quad t = 0, 1, \ldots, \quad S_{i,-1} = \hat{S}_{i,-1} \tag{1}$$

Here r_t and W_t are the interest and wage rates at time t, and $S_{i,t}$ are the savings of consumer i at time t. The savings must be non-negative and can be invested in physical capital as well as in resource stocks. According to Hotelling's rule (Hotelling, 1931, see also Stiglitz, 1974) the return to the both investments in equilibrium must be equal. Hence, the resource price P_t in equilibrium grows at the rate r_t

$$P_t = (1 + r_t)\, P_{t-1}, \quad t = 0, 1, \ldots$$

Initially consumer i is supposed to be endowed with a quantity of the physical capital $\hat{K}_{i,0}$ and the natural resource $\hat{R}_{i,0}$ which are assumed to be given. Thus at the initial moment $t = 0$ the savings of consumer i is determined as

$$\hat{S}_{i,-1} = P_{-1}\hat{R}_{i,0} + \hat{K}_{i,0} \geq 0$$

where P_{-1} is the price of the natural resource at time $t = -1$.

2.2 Production

We assume that output Y_t at each time t is given by the Cobb-Douglas production function

$$Y_t = A_t K_t^{\alpha_1} L^{\alpha_2} E_t^{\alpha_3}, \quad \alpha_j > 0, \quad j = 1, 2, 3, \quad \sum_{j=1}^{3} \alpha_j = 1, \tag{2}$$

where A_t is the coefficient of total factor productivity, K_t is the physical capital stock at time t, L is the labor supply, and E_t is the volume of exhaustible resource extraction. Capital depreciates fully during one time period. The total factor productivity grows at an exogenously given rate λ

$$A_t = (1 + \lambda)^t .$$

The resource E_t expended for production decrease the available stock of the exhaustible resource

$$R_{t+1} = R_t - E_t, \quad t = 0, 1, \ldots$$

We denote by ρ_t the resource utilization rate: $\rho_t = E_t / R_t$, so that

$$E_t = \rho_t R_t.$$

2.3 Equilibrium paths and steady-state equilibria

The initial state is given by the initial distributions of the physical capital $\left(\hat{K}_{i,0}\right)_{i=1,\ldots,L}$ and the natural resource $\left(\hat{R}_{i,0}\right)_{i=1,\ldots,L}$ among all the consumers such that

$$\hat{K}_{i,0} \geq 0, \quad \hat{R}_{i,0} \geq 0, \quad i = 1, \ldots, L, \qquad \hat{K}_0 \equiv \sum_{i=1}^{L} \hat{K}_{i,0} > 0, \qquad \hat{R}_0 \equiv \sum_{i=1}^{L} \hat{R}_{i,0} > 0.$$

We define an *equilibrium path* starting from the initial state $\left(\hat{K}_{i,0}, \hat{R}_{i,0}\right)_{i=1,\ldots,L}$ as a sequence

$$\left\{K_t^*, R_t^*, 1 + r_t^*, W_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

such that

1. for each $i = 1, \ldots, L$ the sequence $\left(C_{i,t}^*, S_{i,t}^*\right)_{t=0,1,\ldots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{3}$$

$$C_{i,t} + S_{i,t} \leq (1 + r_t) S_{i,t-1} + I_t, \quad t = 0, 1, \ldots,$$
$$S_{i,t} \geq 0, \quad t = 0, 1, \ldots$$

at $r_t = r_t^*$, $\pi_t = \pi_t^*$, $I_t = W_t^*$, and

$$S_{i,-1} = \frac{P_0^* \hat{R}_{i,0}}{1 + r_0^*} + \hat{K}_{i,0};$$

2. capital is paid its marginal product:

$$1 + r_t^* = \frac{\alpha_1 A_t L^{\alpha_2} E_t^{*\alpha_3}}{K_t^{*1-\alpha_1}}, \quad t = 0, 1, \dots,$$

where $K_0^* = \hat{K}_0$;

3. labor is paid its marginal product:

$$W_t^* = \frac{\alpha_2 A_t K_t^{*\alpha_1} E_t^{*\alpha_3}}{L^{1-\alpha_2}}, \quad t = 0, 1, \dots;$$

4. the price of exhaustible resource is equal to its marginal product:

$$P_t^* = \frac{\alpha_3 A_t K_t^{*\alpha_1} L^{\alpha_2}}{E_t^{*1-\alpha_3}}, \quad t = 0, 1, \dots;$$

5. Hotelling's rule holds true

$$P_{t+1}^* = \left(1 + r_{t+1}^*\right) P_t^*, \quad t = 0, 1, \dots;$$

6. consumer savings are equal to investment into physical capital and exhaustible resource

$$\sum_{i=1}^{L} S_{i,t}^* = P_t^* R_{t+1}^* + K_{t+1}^*, \quad t = 0, 1, \dots;$$

7. the natural balance of exhaustible resource is fulfilled

$$R_{t+1}^* = R_t^* - E_t^*, \quad t = 0, 1, \dots,$$

where $R_0^* = \hat{R}_0$.

We shall make our emphasis on steady-state equilibria. They are defined as follows. The tuple

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1 + r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

is called a *steady-state equilibrium* if the sequence

$$\left\{K_t^*, R_t^*, 1 + r_t^*, W_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

given for $t = 0, 1, \dots$ and $i = 1, \dots, L$ by

$$\begin{aligned} K_t^* &= (1 + \gamma^*)^t K^*, & 1 + r_t^* &= 1 + r^*, \\ W_t^* &= (1 + \gamma^*)^t W^*, & P_t^* &= (1 + r^*)^t P^*, \\ R_t^* &= (1 - \rho^*)^t R^*, & E_t^* &= (1 - \rho^*)^t E^*, \\ C_{i,t}^* &= (1 + \gamma^*)^t C_i^*, & S_{i,t}^* &= (1 + \gamma^*)^t S_i^*, \end{aligned}$$

is an equilibrium path for some initial state.

Suppose that for some r, I, γ,

$$r_t = r, \quad I_t = (1 + \gamma)^t I, \quad t = 0, 1, \dots$$

We call the couple (C_i^*, S_i^*) a *balanced optimum of consumer* i if the sequence $\left(C_{i,t}^*, S_{i,t}^*\right)_{t=0}^{\infty}$ given by

$$C_{i,t}^* = (1+\gamma)^t C_i^*, \qquad S_{i,t}^* = (1+\gamma)^t S_i^*, \quad t = 0, 1, \ldots,$$

is a solution to problem (3) at $\hat{S}_{i,-1} = (1+\gamma)^{-1} S_i^*$.

It is clear that the tuple

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1+r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is a steady-state equilibrium if and only if

$$1 + r^* = \frac{\alpha_1 L^{\alpha_2} E^{*\alpha_3}}{K^{*1-\alpha_1}}, \qquad W^* = \frac{\alpha_2 K^{*\alpha_1} E^{*\alpha_3}}{L^{1-\alpha_2}}, \qquad P^* = \frac{\alpha_3 K^{*\alpha_1} L^{\alpha_2}}{E^{*1-\alpha_3}}, \tag{4}$$

$$(1+\gamma^*)\left(\frac{P^* R^*}{1+r^*} + K^*\right) = \sum_{i=1}^{L} S_i^*, \tag{5}$$

$$E^* = \rho^* R^*, \tag{6}$$

$$(1+\gamma^*)^{1-\alpha_1} = (1+\lambda)(1-\rho^*)^{\alpha_3}, \tag{7}$$

$$1 + r^* = \frac{1+\gamma^*}{1-\rho^*}, \tag{8}$$

and, for each $i = 1, \ldots, L$, the couple (C_i^*, S_i^*) is a balanced optimum of consumer i at $r = r^*$, $\gamma = \gamma^*$, and $I = W^*$.

To describe properties of steady-state equilibria we formulate the following simple lemma.

Lemma 1 *Given* r, I, *and* γ,

1. *a balanced optimum of consumer* i *exists if and only if*

$$\beta_i \leq \frac{1+\gamma}{1+r};$$

2. *if*

$$\beta_i = \frac{1+\gamma}{1+r},$$

then any couple (C_i^*, S_i^*) *such that*

$$C_i^* + S_i^* + = \frac{1+r}{1+\gamma} S_i^* + I, \quad C_i^* \geq 0, \quad S_i^* \geq 0$$

is a balanced optimum of consumer i*;*

3. *if*

$$\beta_i < \frac{1+\gamma}{1+r},$$

then there is a unique balanced optimum of consumer i, (C_i^*, S_i^*)*; it is given by* $C_i^* = I$, $S_i^* = 0$.

Now we can formulate an important proposition describing the structure of steady-state equilibrium. It follows from Lemma 1.

Proposition 1 *The tuple*

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1+r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if it satisfies conditions (4)–(8) and

$$\beta_L = \frac{1+\gamma^*}{1+r^*},$$

$$C_i^* + S_i^* = \frac{1+r}{1+\gamma} S_i^* + W^*, \quad C_i^* \geq 0, \quad S_i^* \geq 0, \quad i \in J,$$

$$C_i^* = W, \quad C_i^* \geq 0, \quad S_i^* = 0, \quad i \notin J.$$

To prove this proposition it is sufficient to repeat the well-known argument by (Becker, 1980, 2006). It allows us to note the following:

- The equilibrium resource utilization rate is determined by the patience of the most patient consumer
$$\rho^* = 1 - \beta_L.$$
The more patient is this consumer, the lower is the resource utilization rate.
- The growth rate is determined by the rate of technological change and the discount factor of the most patient consumer
$$1 + \gamma^* = \left[(1+\lambda)\, \beta_L^{\alpha_3}\right]^{\frac{1}{1-\alpha_1}}.$$
The higher is patience of the most patient consumer, the higher is the growth rate.

3 Economic growth model with common ownership of resources

In this section we propose a model of economic growth with heterogeneous agents and common resource stocks. The rent obtained from the sale of the exhaustible resource is equally distributeds among the consumers. Consumers choose resource utilization rate by voting. We describe equilibrium paths in the basic Ramsey-type model and after that introduce voting procedure and define voting equilibrium.

We maintain our earlier consumption and rewrite the budget constraints (1) of consumer i as follows

$$C_{i,t} + S_{i,t} \leq (1+r_t)\, S_{i,t-1} + W_t + \Omega_t, \quad S_{i,t} \geq 0, \quad t = 0, 1, \dots, \quad S_{i,-1} = \hat{S}_{i,-1}$$

Here r_t is the interest time t, and $S_{i,t}$ are the savings of consumer i at time t invested in physical capital. Consumer's income includes the wage W_t and resource income Ω_t, which is the per capita income from the sale of the extracted resource equally distributed among the consumers.

3.1 Equilibrium paths and steady-state equilibria

As in section 2.3, first we define equilibrium paths. Suppose that the sequence $\mathcal{R} = (\rho_t)_{t=0}^{\infty}$ of resource utilization rates is given. The initial state is given by a tuple of initial savings $\left\{\hat{S}_{i,-1}\right\}_{i=1,\dots,L}$ such that

$$\hat{S}_{i,-1} \geq 0, \quad i = 1, \dots, L, \qquad \sum_{i=1}^{L} \hat{S}_{i,-1} > 0,$$

and the initial stock of exhaustible resource $\hat{R}_0 > 0$. We define an *equilibrium path* starting from the initial state $\left\{ \left(\hat{S}_{i,-1} \right)_{i=1,\dots,L}, \hat{R}_0 \right\}$ as a sequence

$$\left\{ K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^* \right)_{i=1,\dots,L} \right\}_{t=0,1,\dots}$$

such that

1. for each $i = 1, \dots, L$ the sequence $\left(C_{i,t}^*, S_{i,t}^* \right)_{t=0,1,\dots}$ is a solution to problem (3) at $r_t = r_t^*$, $I_t = W_t^* + \Omega_t^*$, $S_{i,-1} = \hat{S}_{i,-1}$;
2. aggregate savings are equal to the capital stock
$$K_t^* = \sum_{i=1}^{L} S_{i,t-1}^*, \quad t = 0, 1, \dots;$$
3. capital is paid its marginal product:
$$1 + r_t^* = \frac{\alpha_1 A_t L^{\alpha_2} E_t^{*\alpha_3}}{K_t^{*1-\alpha_1}}, \quad t = 0, 1, \dots;$$
4. consumer income consists of the marginal product of labor
$$W_t^* = \frac{\alpha_2 A_t K_t^{*\alpha_1} E_t^{*\alpha_3}}{L^{1-\alpha_2}}, \qquad t = 0, 1, \dots$$
and the resource income
$$\Omega_t^* = \frac{P_t^* E_t^*}{L}, \qquad t = 0, 1, \dots;$$
5. the price of the exhaustible resource is equal to its marginal product
$$P_t^* = \frac{\alpha_3 A_t K_t^{*\alpha_1} L^{\alpha_2}}{E_t^{*1-\alpha_3}}, \quad t = 0, 1, \dots;$$
6. resource utilization is determined by
$$E_t^* = \rho_t R_t^*, \quad t = 0, 1, \dots;$$
7. the natural balance of exhaustible resource is fulfilled
$$R_{t+1}^* = R_t^* - E_t^*, \quad t = 0, 1, \dots,$$
where $R_0^* = \hat{R}_0$.

To describe steady-state equilibria suppose that the resource utilization rate is constant over time: $\rho_t = \rho$, $t = 0, 1, \dots$

The tuple

$$\left\{ \gamma^*, K^*, R^*, 1 + r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L} \right\}$$

is called a *steady-state equilibrium* if the sequence

$$\left\{ K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^* \right)_{i=1,\dots,L} \right\}_{t=0,1,\dots}$$

given for $t = 0, 1, \dots$ and $i = 1, \dots, L$ by

$$K_t^* = (1+\gamma^*)^t K^*, \qquad 1 + r_t^* = 1 + r^*, \qquad \tilde{W}_t^* = (1+\gamma^*)^t \tilde{W}^*$$
$$P_t^* = (1+\pi^*)^t P^*, \qquad R_t^* = (1-\rho)^t R^*, \qquad E_t^* = (1-\rho)^t E^*,$$
$$C_{i,t}^* = (1+\gamma^*)^t C_i^*, \qquad S_{i,t}^* = (1+\gamma^*)^t S_i^*,$$

is an equilibrium path. Here we do not suppose that Hotelling's rule holds true because it might be that $\pi^* \neq r^*$.

It is clear that for any constant over time resource utilization rate, ρ, the tuple

$$\left\{\gamma^*, K^*, R^*, 1 + r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if

$$1 + r^* = \frac{\alpha_1 L^{\alpha_2} E^{*\alpha_3}}{K^{*1-\alpha_1}}, \qquad W^* = \frac{\alpha_2 K^{*\alpha_1} E^{*\alpha_3}}{L^{1-\alpha_2}}, \tag{9}$$
$$\Omega^* = \frac{P^* E^*}{L}, \qquad P^* = \frac{\alpha_3 K^{*\alpha_1} L^{\alpha_2}}{E^{*1-\alpha_3}}, \tag{10}$$
$$(1+\gamma^*) K^* = \sum_{i=1}^{L} S_i^*, \tag{11}$$
$$E^* = \rho R^*, \tag{12}$$
$$(1+\gamma^*)^{1-\alpha_1} = (1+\lambda)(1-\rho)^{\alpha_3}, \tag{13}$$
$$1 + \gamma^* = (1+\pi^*)(1-\rho), \tag{14}$$

and, for each $i = 1, \dots, L$, the couple (C_i^*, S_i^*) is a balanced optimum of consumer i at $r = r^*$, $\gamma = \gamma^*$, and $I = W^* + \Omega^*$.

Lemma 1 can be readily applied to the model under consideration. The following proposition describing the structure of steady-state equilibrium follows from Lemma 1.

Proposition 2 *Let the constant over time resource utilization rate, ρ, be given. The tuple*

$$\left\{\gamma^*, K^*, R^*, 1 + r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if it satisfies conditions (9)–(14) and

$$\beta_L = \frac{1+\gamma^*}{1+r^*}, \tag{15}$$
$$C_i^* + S_i^* = \frac{1+r^*}{1+\gamma^*} S_i^* + W^* + \Omega^*, \quad C_i^* \geq 0, \quad S_i^* \geq 0, \quad i \in J, \tag{16}$$
$$C_i^* = W^* + \Omega^*, \quad C_i^* \geq 0, \quad S_i^* = 0, \quad i \notin J. \tag{17}$$

This proposition can be proved by the same way as proposition 2. The proposition says that only the most patient consumers make positive savings and own all the capital.

3.2 Voting equilibria

Here we introduce a voting procedure into the model described above. Consider an equilibrium path

$$\left\{K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

and ask each consumer i, weather he prefers to increase or decrease the resource utilization rate ρ_t at time t. We assume that when answering this question consumers take into account the fact that additional resource extracted at time t can be sold, used for production and possibly increase their consumption via corresponding part of the resource rent and via increase of factors income. On the other hand, the resource utilized at time t decrease the resource stock available in the future.

To describe consumers' decision-making procedure, first note that for each i, $(C^*_{i,t}, S^*_{i,t})_{t=0,1,\ldots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{18}$$
$$C_{i,t} + S_{i,t} \le A_t K_t^{*\alpha_1} L^{\alpha_2} E_t^{*\alpha_3} \left(\frac{\alpha_1 S_{t-1}}{K_t^*} + \frac{\alpha_2 + \alpha_3}{L} \right),$$
$$S_{i,t} \ge 0, \quad t = 0, 1, \ldots,$$

where $S_{i,-1} = \hat{S}_{i,-1}$. Recall that

$$E_t^* = \rho_t R_t^*, \text{ and hence } R_t^* = \hat{R}_0 \prod_{j=0}^{t-1} (1 - \rho_j), \quad t = 0, 1, \ldots. \tag{19}$$

Let us consider the value of (18), V_i, as a function of $\mathcal{R} = (\rho_t)_{t=0}^{\infty}$. Defining consumers' decision-making procedure we assume that the attitude of consumer i to a possible change in ρ_t is determined by sign of the derivative $\partial V_i / \partial \rho_t$, if it exists. Namely, if $\partial V_i / \partial \rho_t > 0$, consumer i *is in favor of increasing* ρ_t. If $\partial V_i / \partial \rho_t < 0$, consumer i is *in favor of decreasing* ρ_t.

Definition 1 *We call the sequence*

$$\left\{ \rho_t^*, K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C^*_{i,t}, S^*_{i,t}\right)_{i=1,\ldots,L} \right\}_{t=0,1,\ldots}$$

a voting equilibrium path if

$$\left\{ K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C^*_{i,t}, S^*_{i,t}\right)_{i=1,\ldots,L} \right\}_{t=0,1,\ldots}$$

is an equilibrium path at $\mathcal{R} = (\rho_t^*)_{t=0}^{\infty}$ *and for each* t *the number of consumers who are in favor of increasing* ρ_t *and the number of those who are in favor of decreasing* ρ_t *is less than* $L/2$.

Here we certainly do not mean that the individuals votes for any change in the resource utilization rate. This consideration merely reflects the idea that government controlling the resource stock responds somehow to the wishes of the majority when choosing the resource utilization rate.

We will not discuss an existence and properties of voting equilibria in a general form, but will describe voting steady-state equilibria.

Definition 2 *If the sequence*

$$\left\{ \rho_t^*, K_t^*, R_t^*, 1 + r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C^*_{i,t}, S^*_{i,t}\right)_{i=1,\ldots,L} \right\}_{t=0,1,\ldots}$$

given by

$$\rho_t^* = \rho^*, \qquad K_t^* = (1+\gamma^*)^t K^*, \qquad 1 + r_t^* = 1 + r^*,$$
$$W_t^* = (1+\gamma^*)^t W^*, \qquad \Omega_t^* = (1+\gamma^*)^t \Omega^*,$$
$$P_t^* = (1+\pi^*)^t P^*, \qquad R_t^* = (1-\rho)^t R^*, \quad E_t^* = (1-\rho)^t E^*,$$
$$C^*_{i,t} = (1+\gamma^*)^t C_i^*, \qquad S^*_{i,t} = (1+\gamma^*)^t S_i^*,$$
$$i = 1, \ldots, L, \quad t = 0, 1, \ldots$$

forms a voting equilibrium path, then the tuple

$$\left\{\rho^*, \gamma^*, K^*, R^*, 1+r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is called a voting steady-state equilibrium.

The following theorem describes voting steady-state equilibria. It reads that an important role in determining a steady-state equilibrium is played by the median consumer $m = (L+1)/2$.

Theorem 1 *The tuple*

$$\left\{\rho^*, \gamma^*, K^*, R^*, 1+r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

represents a voting steady-state equilibrium, if and only if it satisfies conditions (9)–(14) at $\rho = \rho^*$*, where* $\rho^* = 1 - \beta_m$.

Proof Let the tuple

$$\left\{\gamma^*, K^*, R^*, 1+r^*, W^*, \Omega^*, P^*, \pi^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

be a steady-state equilibrium constructed at $\rho = \rho^*$ and

$$\left\{K_t^*, R_t^*, 1+r_t^*, W_t^*, \Omega_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

be an equilibrium path corresponding to this steady-state equilibrium.

By the envelope theorem, taking into account (19), for all $i = 1, \ldots, L$ and all $t = 0, 1, \ldots$, we have

$$\frac{\partial V_i}{\partial \rho_t} = \beta_i^t u'\left(C_{i,t}^*\right) \frac{\alpha_3 \Phi_t^*}{\rho_t} - \sum_{k=t+1}^{\infty} \beta_i^k u'\left(C_{i,k}^*\right) \frac{\alpha_3 \Phi_k^*}{1-\rho_t},$$

where

$$\Phi_t^* = A_t K_t^{*\alpha_1} L^{\alpha_2} E_t^{*\alpha_3} \left(\frac{\alpha_1 S_{t-1}^*}{K_t^*} + \frac{\alpha_2 + \alpha_3}{L}\right).$$

Also we have

$$\Phi_{t+1}^* = (1+\gamma^*)\Phi_t^*, \qquad C_{i,t+1}^* = (1+\gamma^*) C_{i,t}^*, \qquad u'\left(C_{i,t+1}^*\right) = \frac{u'(C_{i,t}^*)}{1+\gamma^*}.$$

Therefore,

$$\frac{\partial V_i}{\partial \rho_t} = \alpha_3 \beta_i^t \Phi_t^* u'\left(C_{i,t}^*\right) \left[\frac{1}{\rho_t} - \frac{\beta_i}{(1-\beta_i)(1-\rho_t)}\right]$$

and hence

$$\text{sign} \frac{\partial V_i}{\partial \rho_t} = \text{sign}\left\{1 - \beta_i - \rho_t\right\}$$

To complete the proof it is sufficient to notice that

$$\frac{\partial V_i}{\partial \rho_t} \gtreqless 0 \quad \Leftrightarrow \quad \beta_i \lesseqgtr 1 - \rho_t.$$

That means:

- An equilibrium resource utilization rate is determined by the patience of the median consumer

$$\rho^* = 1 - \beta_m.$$

- Hotelling's rule (Hotelling, 1931), implying that the resource prices grow with the rate equal to the interest rate $\pi^* = r^*$ may be violated

$$\frac{1+r^*}{1+\pi^*} = \frac{\beta_m}{\beta_L}$$

  Indeed, if $\beta_m < \beta_L$ the resource price inflation rate π^* is larger than the interest rate r^*. See (Chermak and Patrick, 2002) for a discussion of Hotelling's rule applicability to observable price dynamics.

- The steady state rate of growth is determined by the rate of technological change and the patience of the median consumer

$$1+\gamma^* = \left[(1+\lambda)\,\beta_m^{\alpha_3}\right]^{\frac{1}{1-\alpha_1}}.$$

  It is interesting to note that the discount factor of the most patient consumers β_L does not influence the steady state rate of growth though it impacts the interest rate.

4 Discussion

In the case of common resources we demonstrate that in the voting steady-state equilibrium the resource utilization rate as well as the steady state rate of growth is determined by the patience of the median consumer

$$\rho^* = 1-\beta_m, \qquad 1+\gamma^* = \left[(1+\lambda)\,\beta_m^{\alpha_3}\right]^{\frac{1}{1-\alpha_1}}.$$

Contrary to that, in the case of private resource stocks the resource utilization rate and the steady state rate of growth are determined by the patience of the most patient consumers, which are the only owners of physical capital and resources in the economy at a balanced growth equilibrium

$$\rho^* = 1-\beta_L, \qquad 1+\gamma^* = \left[(1+\lambda)\,\beta_L^{\alpha_3}\right]^{\frac{1}{1-\alpha_1}}.$$

That means that if the median consumer is less patient than the most patient one, $\beta_m < \beta_L$, the resource utilization rate in the case of private resources is lower and the steady state rate of growth is higher than in the case of common resources.

Hence, private ownership of resource stocks is favorable for the economic growth. At the same time, it is well-known that the private exhaustible resources in developing countries are frequently exposed to risk of nationalization. One may speculate that the benefits of the private resources for the growth depend somehow on security of property rights. The violation of the property rights may lead to deterioration of investment incentives (Besley, 1995). In our framework this means effectively a decrease in the patience of the most patient consumers who own the resources (see, e.g., a consideration of Russian economy in Gaddy and Ickes, 2005). The owners of resources having in mind the possibility of nationalization become less patient. This results in lower values of β_L, higher equilibrium rate of resource utilization ρ^* and lower equilibrium growth rate γ^*.

An approach leading to this sort of effects is proposed by Borissov and Lambrecht (2009). They developed a model of endogenous growth where discount factors of heterogeneous consumers are determined endogenously and depend on economic inequality through the following two channels. On the one hand, they are positively related to individual consumer's relative wealth. On the other hand, they are negatively affected by a simple aggregate measure of social conflict. In this case, under some reasonable assumptions the growth rate dependence on the inequality index has an inverted U-shaped form.

Applying this idea to our model with exhaustible resources one may come to the following conclusions. The private ownership of resource stocks increases economic inequality and can have an ambiguous effect on economic growth. In the case of moderate inequality its influence can be positive, but in the case of substantial inequality, an increase in inequality may lead to a decrease in economic growth because of a decrease in the discount factors of capital owners induced by increased risk of social conflict at high level of inequality. One may conjecture that in this case there is a range of parameters where common ownership of resource stocks may lead to even larger economic growth rate then in the case of private resources.

5 Conclusion

We have developed two Ramsey-type models of economic growth with heterogenous consumers and exhaustible resources. The first model assumes private ownership of the resource stocks. The second model assumes the resource stocks to be collectively owned and the resource rent to be equally divided among all the consumers. Both models assume the consumers to be heterogeneous in their intertemporal preferences, which leads to different preferences on the rate of resource utilization. In the case of common ownership of resources, we suppose that the consumers choose the resource utilization rate by voting.

In both models we define equilibrium paths and analyze some properties of steady-state equilibria. In the model with voting we introduced the notion of voting equilibrium and describe properties of steady-state voting equilibria.

We found out that in the case of common resources in a voting steady-state equilibrium the resource utilization rate as well as the steady state rate of growth is determined by patience of the median consumer. Contrary to that, in the case of private resource stocks the resource utilization rate and the steady state rate of growth are determined by the patience of the most patient consumers. If the median consumer is less patient than the most patient one, in the case of private resources the resource utilization rate is lower and the economic growth rate is higher than in the case of common resources.

The developed models give the possibility to associate the economic growth rate with security of property rights. A threat of exhaustible resources nationalization can effectively decrease patience of the resource owners, leading to acceleration of resources extraction and decrease in economic growth rate.

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