Private Insurance Against Systemic Crises?*

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Insurance contracts that are contingent on macroeconomic shocks or on average bank capital might act as insurance against banking crises. In a simple model, we illustrate how such contracts would work and achieve optimal insurance. We also characterize some critical issues of private insurance against banking crises: (i) insurance would have to be mandatory, (ii) the insurance capacity of an economy may be too limited, and (iii) insurance does not prevent excessive risk-taking (unobservable or observable). The insurers may also (iv) go bankrupt in crises. We suggest regulatory measures that could mitigate these critical aspects.

Keywords: Financial intermediation, banking crises, insurance contracts, automatic recapitalization, banking regulation

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1. Introduction

The literature and the current crisis provide ample evidence that in terms of GDP losses the costs of banking crises may be very large. It is a widely held view that traditional contractual arrangements in banking expose the banks to the risks associated with systemic or macroeconomic shocks, and that this may lead to a crisis (see e.g. Hellwig (1998)). Such shocks may be exogenous or they may result from many banks undertaking correlated investments and thereby increasing economy-wide aggregate risk.

A promising way of reducing the likelihood of banking crises might be to induce banks to buy insurance contracts contingent on macroeconomic events, such as GDP growth or other contractible macroeconomic indicators that are highly correlated with the financial health of the banking sector, such as the average bank equity of competing banks.

In this paper we discuss whether it is possible for a banking system to insure itself against a crisis. We consider a simple model of financial intermediation in which banks alleviate agency problems in financial contracting. Banks compete for deposits and equity and invest their funds. Investment returns are affected by macroeconomic and idiosyncratic shocks. A banking crisis occurs when a large number of banks become insolvent. Banks can buy insurance contracts that recapitalize banks when negative macroeconomic shocks occur. When a bank buys one unit of contract, it has to pay one Dollar to an investor. If a severe negative macroeconomic shock occurs, banks will receive a particular Dollar value that recapitalizes banks.

There are several ways of organizing crisis insurance. *Mandatory insurance* refers to the case where the regulator prescribes that a bank has to buy a specific number of insurance contracts. Under *voluntary insurance* banks decide themselves how many contracts they want to buy. Such insurance contracts can either be made contingent on macroeconomic indicators such as GDP growth, an index of real estate prices or interest rates,[1] or the average bank equity of the other banks in the industry. In the latter case one would construct an index by dividing total equity by the total assets of all other

[1] We use the terms systemic and macroeconomic shocks as synonyms.

banks in the industry before insurance contracts are executed. Overall, we obtain four cases represented by the four boxes (MMI, MBI, VMI, VBI) in Table 1.

Table 1: Scenarios for Banking System Insurance

	Mandatory Insurance	Voluntary Insurance
Insurance contingent on macroeconomic indicators (GDP growth, real estate prices, interest rate)	Mandatory macro-insurance (MMI)	Voluntary macro-insurance (VMI)
Insurance contingent on average bank equity	Mandatory bank equity insurance (MBI)	Voluntary bank equity insurance (VBI)

Our main insights are as follows: First, if the risk of the set of investment opportunities for banks is known to the regulator, mandatory insurance can fully insure the banking sector against crisis. By prescribing a sufficient number of insurance contracts a bank has to buy, avoidance of defaults by banks is guaranteed. Investors will recapitalize banks in the event of a downturn occurring that would trigger large-scale defaults in the banking industry. We identify a crucial condition for insurance called the *insurance capacity of an economy*. It measures the wealth of insurance-contract holders available for honoring their obligations in downturns. Second, the same insurance can be achieved by conditioning insurance contracts on average bank equity, as this index is highly correlated with the state of the economy if the number of banks is sufficiently large.

However, there are a number of potential pitfalls. In particular, the insurance capacity of the economy may be too limited if investors do not have sufficient wealth in a crisis. Furthermore, if insurance is voluntary, it will not be used by banks as it would decrease return on equity. Moreover, even mandatory insurance will fail to achieve complete insurance if banks find new risky investment opportunities and gamble. A further serious concern is that investors selling insurance contracts may go bankrupt in a crisis. Finally, managerial limits on a rising bank equity capital may make insurance impossible.

We then go on to discuss some complementary regulatory measures for fostering the effectiveness of crisis insurance, such as capital requirements. In particular, we suggest the mandatory purchase of insurance contracts against banking crises by bank managers. This would create incentives for bank managers not only to be concerned about their own institution, but also about the stability of the public good system.

The paper is organized as follows: In the next section we relate the paper to the literature. In section 3 we present a simple model. In section 4, we characterize the first-best allocation. In section 5 we discuss the case of no insurance to set a benchmark. In section 6 we analyze a variety of special cases to illustrate how insurance could work. Pitfalls are examined in section 7. Section 8 is a discussion of complementary regulatory measures. Section 9 concludes.

2. Relation to the Literature

There are two strands of literature that relate to the current paper. The first of them focuses on individual financial institutions. Highly leveraged financial institutions could be forced to issue securities that produce automatic recapitalization if the firm's value decreases. Wall (1989) has examined subordinated debt with an embedded put option. Doherty & Harrington (1997) and Flannery (2005) propose reverse convertible debentures to limit financial distress costs ex post.

On the subject of systemic risk and insurance against a collapse of the entire banking system, Gersbach (2004)[2] has investigated debt contracts contingent on macroeconomic events. In this paper we explore an alternative approach by separating financing and insurance. We use pure insurance contracts that do not finance banks but simply insure banks against an economic downturn or a systemic crisis.[3] We develop a model for the case of pure insurance and investigate the potentialities and limitations of this approach.

[2] An updated version can be found in Gersbach 2008.

[3] Kashyap, Rajan and Stein 2008 have outlined a policy proposal with insurance contracts.

3. Model

3.1. Macroeconomic Environment and Banking

We develop a model with three standard deviations from a frictionless Arrow-Debreu world. These frictions are well-known and have been substantiated in the literature (see e.g. Hellwig (1998)):

- Banks are necessary to finance a part of the investments in the economy.
- Deposit contracts cannot be conditioned on macroeconomic events.[4]
- Limited liability of bank equity holders induces excessive risk-taking.

More specifically, we consider a two-period setting with periods $t = 1$ and $t = 2$. The population of agents consists of a continuum indexed by $j \in [0, 1]$. Agents are divided into two classes. One fraction of agents, indexed by $j \in [0, \alpha]$ with $0 < \alpha < 1$, are investors. The other fraction, indexed by $j \in (\alpha, 1]$, are consumers. Each agent has individual wealth w in the first period. The main differences between potential investors and consumers are risk aversion and opportunities to invest.

There exist two technologies that convert investments in $t = 1$ into consumption goods in $t = 2$. There exists a technology (called 'banking technology') in which only banks can invest. There is a second technology in which all agents can invest directly and frictionlessly. This technology is called 'alternative technology'. The rationale for the need of financial intermediation is the need for monitoring.[5] We assume that banks offer a menu of equity and debt contracts. The amount of equity and debt contracts, and thus the capital structure of banks and the banking sector, will be determined endogenously in equilibrium.

[4]See Hellwig (1998). This assumption could be relaxed. It will turn out that depositors are identical to risk averse consumers, and shareholders are identical to risk-neutral investors in equilibrium. Hence, even if state contingent deposit contracts are allowed, risk averse consumers would only hold non-contingent deposit contracts.

[5]See Diamond (1984) and Hellwig (1998) for a survey and critical assessments of the foundations. Gersbach and Uhlig (2006) provide a foundation of the particular model we are using.

There are n banks indexed by $i = 1, \ldots, n$. At the beginning banks issue equity contracts and equity holders become the owners of the bank. Holders of equity contracts obtain the right to participate in the dividend payment in the second period depending on their share in the overall equity issued by the bank. Equity holders will therefore wish to maximize the return on equity. We use $e^{(1)}$ to denote the equity capital that bank i receives in $t = 1$. As banks are initially identical in the eyes of potential shareholders, all banks will receive the same amount of equity.[6] Aggregate equity is denoted by $E^{(1)} = ne^{(1)}$. Banks are operated by managers who act in the interest of owners.[7]

Investors are assumed to be risk-neutral and will only consume in the second period. The aggregate wealth of investors in the first period is denoted by W, where $W = \alpha w$. They have four possibilities for investing their money:

- bank equity capital
- bank deposits
- bank insurance contracts[8]
- alternative investment opportunity

The alternative investment opportunity may be thought of as an outside option, such as foreign government bonds or investments in other sectors of the economy that are not modeled explicitly. Investment in alternative technology is subject to decreasing returns, i.e. if an amount X is invested, the output is given by $f(X)$, where $f(0) = 0$, $f'(\cdot) > 0, f''(\cdot) < 0$, and $f'(0) = \infty, \lim_{X\to\infty} f'(X) = 0$. We assume that the residual output of investments in the alternative technology goes to a third class of agents. The third class of agents may represent another sector in the economy or owners of technologies in other countries. The existence of a third class of agents who collect the surplus on the riskless investment is a plausible situation when investments are undertaken abroad. All of our results hold if investors in the alternative technology

[6] Strictly speaking, we restrict the analysis to symmetric equilibria. This holds if one applies a suitable version of the law of large numbers and if investors who are potential shareholders randomize between banks.

[7] Bank managers can be viewed as a small subgroup of investors. Regarding investments and savings they can be neglected.

[8] Bank insurance contracts will be introduced in the next section.

receive the entire output and thus average instead of marginal returns matter. The endogenously determined return on the alternative investment opportunity is denoted by $1 + r_A$. The profit for the third class of agents is given by $f(X) - (1 + r_A)X$. We assume that the market for investments in the alternative technology is competitive and hence demand in competitive markets for funds is given by $f'(X) = 1 + r_A$ or $X = f'^{-1}(1 + r_A)$.

Regarding bank deposits, we assume that they are guaranteed implicitly by the government.[9] Throughout the paper we assume that the deposit rate is equal to the return of the alternative investment opportunity.[10]

Consumers are endowed with intertemporal preferences over consumption in the two periods of their lives. Let $u(c_1, c_2)$ be a standard intertemporal concave utility function of a consumer with decreasing marginal utility in consumption, where c_1, c_2 denote a consumer's first-period and second-period consumption, respectively. Accordingly, consumers are risk-averse. We assume that consumers only save via bank deposits.[11] Given the endowment w in the first period, each consumer saves the amount $s(r_A)$ in the first period if he maximizes utility subject to the budget constraint $\frac{c_2}{1+r_A} + c_1 = w$. The aggregate savings of all consumers are then denoted by $S(r_A)$ and given by $S(r_A) = (1 - \alpha)s(r_A)$. As a shortcut, we use $\overline{S} := S(r_A)$ for aggregate savings. As deposits are protected, banks are identical for consumers when they deposit their money, so each bank receives an amount $\frac{\overline{S}}{n}$ of deposits.[12]

[9] The current crisis has highlighted the fact that governments will ultimately protect depositors, independently of whether an explicit deposit insurance scheme has been put in place. The costs for bailing out current depositors does not affect their saving behavior if future guarantees have to put up the funds. Otherwise today's savings are reduced by expected future tax payments for bail-out and this feedback effect has to be taken into account.

[10] This assumption can be justified and derived endogenously as follows: The fact that the deposit rate cannot be smaller than r_A can be ruled out by arbitrage opportunities. Otherwise banks could simply collect funds from consumers and invest them into the alternative investment opportunity, thus making risk-free profits. If the deposit rate were larger than r_A, banks would absorb all the funds in the economy. The assumption $f'(0) = \infty$ rules out banks generating returns above r_A by investing all funds of depositors and investors.

[11] We could allow for other investment opportunities for consumers, but this makes the arguments substantially more complicated.

[12] This holds if a suitable version of the law of large numbers is applied and if consumers randomize between banks.

Banks use deposits and equity to finance investments in the banking technology. The amount of investment per bank without insurance is given by k with

$$k = e^{(1)} + \frac{\overline{S}}{n}. \tag{1}$$

Aggregate investment is denoted by $K = nk$. Investments by banks are risky, and returns to them are subject to two additive shocks. First, there can be idiosyncratic shocks ε_i for each bank. These shocks are uncorrelated and have an expected value of zero. In the basic version (until section 6.4) we neglect idiosyncratic risk. Second, there is a systemic shock η that affects the investment returns of all banks simultaneously. η can take three different values. The event $\eta = g$ represents good times, $\eta = 0$ characterizes moderate times, $\eta = -b$ $(b > 0)$ stands for a downturn.[13] We assume three aggregate states of the world to demonstrate later more convincingly that average equity can be used as macroeconomic indicator for insurance contracts. Of course, all of our conclusions a fortiori hold in a model with only a good and a bad state of the world.

The probabilities are denoted by p_G, p_0 and p_B respectively, where $p_G + p_0 + p_B = 1$ and $p_G g - p_B b = 0$, such that $\mathbb{E}(\eta) = 0$. Thus net returns on investment are $(\overline{r} + \eta)k$, where $\overline{r}$ is the given expected real return $(\overline{r} > 0)$ per unit of investment. Without insurance, the equity of bank i in the second period is

$$e_i^{(2)} = \max\left\{\left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n}\right)(1 + \overline{r} + \eta) - \frac{\overline{S}}{n} \cdot (1 + r_A), 0\right\}. \tag{2}$$

The objective of bank managers is to maximize the value of equity. As $e_i^{(1)}$ is given and investors are risk-neutral, bank managers maximize the return on equity, or equivalently, $\mathbb{E}[e_i^{(2)}]$.

3.2. Insurance

Banks can buy insurance contracts defined as follows:

[13] This may also represent a systemic crisis if b is large. As resources cannot be negative, b has to be smaller than $1 + \overline{r}$.

Definition 1 (Insurance Contracts)

If the bank buys one unit of insurance contracts it pays one Dollar. It receives nothing in good and moderate times and receives q in a crisis. Formally, the payoff R_I is given by

$$R_I = \begin{cases} 0 & if \quad \eta = g \text{ or } \eta = 0 \\ q & if \quad \eta = -b. \end{cases}$$

The payment q will be determined in equilibrium. There are several ways to organize insurance. First, as defined, the insurance contract is directly conditioned on the aggregate event. This is possible if the aggregate event can be described say by GDP growth or by a standardized index of real estate prices. In the basic version we follow this scenario. An alternative way might be to use an index of aggregate equity capital in the banking system. This will be discussed in section 6.5.Second, even if insurance contracts are available, banks might not be willing to buy them as this may not be in the interests of current shareholders.

In the basic version we assume that regulatory authorities stipulate that each bank has to buy an amount $\frac{\Omega}{n}$ of insurance contracts, where Ω is the aggregate amount of insurance contracts the government requires the banking system to buy. Stipulation of $\frac{\Omega}{n}$ per bank by the regulator is called insurance policy Ω. Note that the unit of an insurance contract is the price today which is normalized to one Dollar. The payment in a crisis of such a contract is determined endogenously.[14] In short, formally we will first focus on MMI. Later we will discuss the cases in which banks will voluntarily buy such contracts.

When banks buy insurance contracts, their investments in the first-period and the equity in the second-period change. In detail, with insurance equations (1) and (2) are given as follows:

[14]It is equivalent to fix the payment in a crisis per contract and let the price today to be determined in the market.

$$
\begin{aligned}
k &= e^{(1)} + \frac{\overline{S}}{n} - \frac{\Omega}{n}, \\
e_i^{(2)} &= \max\left\{ \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n} \right)(1 + \overline{r} + \eta) + R_I \frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1 + r_A), 0 \right\}.
\end{aligned}
$$

3.3. Equilibrium

We illustrate the economy by the following figure:

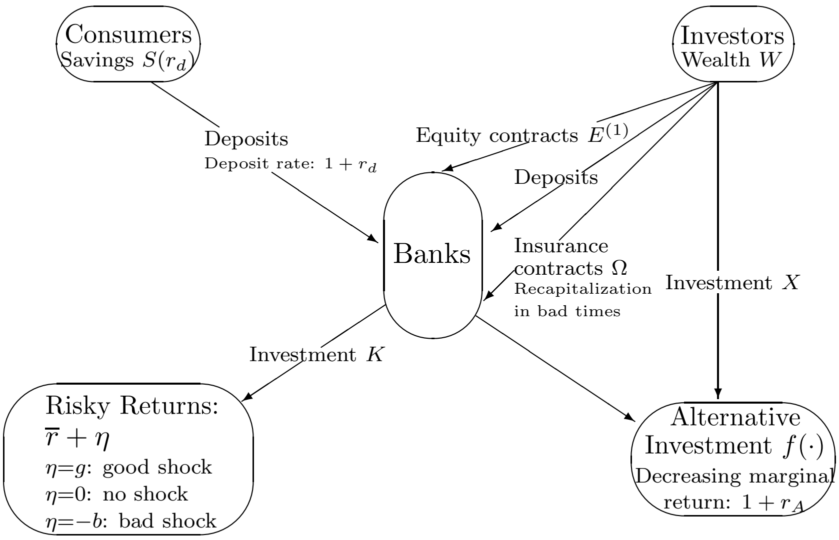

Figure 1: Flow of Funds

Two remarks on tie-breaking rules, which simplify the exposition, are in order. First, although investors can also take deposits, we assume that the invest in bank equity or insurance contracts if those assets generate at least the same expected repayment as deposits. The equilibrium definition takes this tie-breaking rule into account and in equilibrium investors do not deposit. As a consequence, $\overline{S}$ is the aggregate amount of deposits. Second, banks will not invest into the alternative technology if they expect at least the same return on their investment k. Again, in equilibrium banks will not invest in the alternative technology.[15]

An equilibrium is then defined as follows:

Definition 2 (Equilibrium)

Given an insurance policy Ω*, a competitive equilibrium is a quadruplet* $\left\{K, E^{(1)}, q, r_A\right\}$ *such that the following conditions hold*

$$\begin{aligned}
\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} &= \frac{K}{n} + \frac{\Omega}{n} && (3)\\
\mathbb{E}\left[e_i^{(2)}\right] &= (1 + r_A)\frac{E^{(1)}}{n} && (4)\\
\mathbb{E}\left[R_I\right] &= 1 + r_A && (5)\\
f'(W - E^{(1)} + \Omega) &= 1 + r_A. && (6)\\
(W + \Omega - E^{(1)})(1 + r_A) &\geq q\Omega && (7)
\end{aligned}$$

Condition (3) represents the savings/investment balance for each individual bank. By multiplying this equation by n, we obtain the aggregate balance sheet of the banking system $K + \Omega = \overline{S} + E^{(1)}$.

Condition (4) represents the equilibrium in the equity market. If the return on equity were below $1+r_A$, no investors would be willing to accept equity contracts. If the return on equity were larger than $1 + r_A$, investors would offer more equity capital because alternative investments yield lower returns. Aggregate equity in the second period, denoted by $E^{(2)}$, is given by $E^{(2)} = \sum_{i=1}^{n} e_i^{(2)}$. Aggregate equity in the second period in each state of the world is denoted by $E_g^{(2)}$, $E_0^{(2)}$ and $E_{-b}^{(2)}$, respectively.

Condition (5) is an equilibrium in the insurance market. If expected payment on insurance contracts were lower than $1 + r_A$, investors would invest as much as possible in insurance contracts.[16] If expected payment on insurance contracts were higher than

[15] If we assume small, but positive costs of banking, the definition of the equilibrium can be adapted accordingly. Then, banks have strict incentives not to invest in the alternative technology, as investors can do this cheaper.

[16] Suppose $\mathbb{E}\left[R_I\right] < 1 + r_A$ with $\mathbb{E}[R_I] = 1 + \hat{r}$ ($\hat{r} < r_A$). An investor can invest everything into bank equity and into the alternative investment opportunity. Then his expected wealth in the second period would be $w(1+r_A)$. Alternatively, he can sell the maximum possible amount $\hat{\Omega}$ of insurance contracts, so that he is able to pay banks $\hat{\Omega}\overline{q}$ if $\eta = -b$ with $\overline{q} = \frac{1+\hat{r}}{p_B}$. In the latter case, the investor would invest $w + \hat{\Omega}$ today and would earn $(w+\hat{\Omega})(1+r_A)$. Hence $\hat{\Omega}$ is determined by $(w+\hat{\Omega})(1+r_A) = \hat{\Omega}\overline{q}$,

$1 + r_A$, no investor would be willing to buy them.

Condition (6) is the equilibrium condition in the competitive market for investment in the alternative technology. The supply of funds is $W - E^{(1)} + \Omega$ and thus the equilibrium return is $f'(W - E^{(1)} + \Omega)$.[17] Note that condition (6) and the Inada conditions for $f(X)$ imply $\Omega + W > E^{(1)}$, as the outside investment opportunity is less attractive than investment by banks if investors put all their resources into the alternative investment project. We note that for investment in the alternative technology, only the net amount $-E^{(1)} + \Omega$ matters.

It is useful to summarize the payoffs of investors in different states

$$\begin{aligned} \eta = g: &\qquad (1 + r_A)(W - E^{(1)} + \Omega) + E_g^{(2)} \\ \eta = 0: &\qquad (1 + r_A)(W - E^{(1)} + \Omega) + E_0^{(2)} \\ \eta = -b: &\qquad (1 + r_A)(W - E^{(1)} + \Omega) + E_{-b}^{(2)} - \Omega R_I. \end{aligned}$$

In the bad state, investors face two sources of losses of their wealth. Bank equity may be small or even zero, and investors need to recapitalize banks.

Condition (7) is the repayment condition for investors, i.e. it describes whether the insurance capacity of the economy is adequate. In particular, if $\eta = -b$, investors must be able to honor their insurance promises, even if they face total losses on their investment in bank shares.

4. First-best Allocation

Before we explore the properties of the economy with and without insurance, it is useful to characterize the first-best allocation for the described economy when the three frictions introduced in 3.1. are absent. That is, direct financing of investment opportunities without frictions is possible, and financial contracts can be conditioned on each state

which leads to $\hat{\Omega} = \frac{w(1+r_A)}{\overline{q}-(1+r_A)}$. Then the expected wealth of an investor in the second period equals $(w + \hat{\Omega})(1 + r_A) - p_B\hat{\Omega}\overline{q}$, which is larger than $w(1 + r_A)$, since $r_A > \hat{r}$. Consequently, an investor would buy the maximum possible amount of insurance contracts if $\mathbb{E}[R_I] < 1 + r_A$.

[17] Recall that we have assumed that the residual output of investment goes to the third class of agents.

of the world in the Arrow-Debreu sense.[18] For this purpose, we define the level of investment in the alternative technology which produces marginal return $1 + \overline{r}$ by $\hat{X}$, i.e.

$$f'(\hat{X}) = 1 + \overline{r}.$$

Moreover, we denote by Y the aggregate output[19], given by

$$Y = f(X) + K(1 + \overline{r} + \eta).$$

Then we obtain

Proposition 1

Suppose

$$(W - \hat{X})(1 + \overline{r} - b) \geq S(\overline{r})b. \tag{8}$$

Then, a first-best allocation is characterized by

(i) An amount $\hat{X}$ is invested in the alternative technology, such that $r_A = \overline{r}$

(ii) Investments in the risky technology amount to

$$\overline{K} = S(\overline{r}) + W - \hat{X}$$

(iii) Expected aggregate output is given by

$$\mathbb{E}[Y] = f(\hat{X}) + \overline{K}(1 + \overline{r})$$

(iv) Consumers buy riskless securities with repayment $1+\overline{r}$ per unit of invested capital

(v) Investors buy risky securities which yield the following payoffs per unit of invested

[18] Then, limited liability does not play a role.

[19] The profits of the third class of agents are also included. The same results would hold if the profits of the third class of agents are excluded and $f'(X)X$ enters into the aggregate output.

capital

$$\begin{aligned} \textit{if}\, \eta = g: &\quad 1 + \overline{r} + g + \frac{S(\overline{r})g}{W - \hat{X}} \\ \textit{if}\, \eta = 0: &\quad 1 + \overline{r} \\ \textit{if}\, \eta = -b: &\quad 1 + \overline{r} - b - \frac{S(\overline{r})b}{W - \hat{X}}. \end{aligned}$$

Proposition 1 follows from the following considerations.

First, risk-averse consumers are fully insured against macroeconomic shocks. Condition 8 and the risky securities for investors ensure that the investors' insurance capacity is sufficient to achieve this purpose. Specifically, in the bad state, the output of the risky technology, given by

$$\overline{K}(1 + \overline{r} - b) = \left(S(\overline{r}) + W - \hat{X}\right)(1 + \overline{r} - b),$$

has to satisfy the repayment promises to consumers and investors. Those promises are given by

$$S(\overline{r})(1 + \overline{r}) + (W - \hat{X})\left(1 + \overline{r} - b - \frac{S(\overline{r})b}{W - \hat{X}}\right) = \left(S(\overline{r}) + W - \hat{X}\right)(1 + \overline{r} - b).$$

Hence, by construction of the security, the output of the risky technology in the bad state just suffices to produce the returns for consumers and investors. For this scheme to be feasible, the repayment to investors has to be non-negative, i.e. $1 + \overline{r} - b - \frac{S(\overline{r})b}{W - \hat{X}} \geq 0$, which yields the condition of the proposition: $(W - \hat{X})(1 + \overline{r} - b) \geq S(\overline{r})b$.
Third, in the neutral and the good state, the entire output of the risky technology is paid out to consumers and investors. Fourth, the expected repayment of the risky security is

$$\begin{aligned} & p_G\left(1 + \overline{r} + g + \frac{S(\overline{r})g}{W - \hat{X}}\right) + p_B\left(1 + \overline{r} - b - \frac{S(\overline{r})b}{W - \hat{X}}\right) + (1 - p_G - p_B)(1 + \overline{r}) \\ & = 1 + \overline{r} + \frac{S(\overline{r})}{W - \hat{X}}(p_G g - p_B b) = 1 + \overline{r}. \end{aligned}$$

Hence, investors are willing to buy these securities, as they yield the same expected repayment per unit of invested capital as the alternative opportunity.

Fifth, to maximize the expected aggregate output, given by $\mathbb{E}[Y] = f(\hat{X}) + \overline{K}(1+\overline{r})$, the expected return on investments in the alternative technology and in the risky technology have to be equalized.

Sixth, maximization of the aggregate output is socially desirable, as risk-averse agents can be fully insured and thus there is no risk/return tradeoff.[20]

In the following we return to the economy with frictions and with financial intermediation.

5. No Insurance

We examine first the conditions for the occurrence of a banking crisis, when insurance is not active, i.e. $\Omega \equiv 0$.

5.1. Equilibrium Conditions and Banking Crisis

We first note that the equilibrium equations (5) and (7) do not have to be considered. The aggregate balance of the banking system determines the aggregate amount of investment K, which itself depends on aggregate equity $E^{(1)}$. Therefore the determination of how much investors are willing to invest in bank equity capital is the crucial step needed to obtain the equilibrium. Equity in the second period is given by

$$e_i^{(2)} = \max\left\{ \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} \right)(1 + \overline{r} + \eta) - \frac{\overline{S}}{n} \cdot (1 + r_A), 0 \right\}. \tag{9}$$

Using equilibrium condition (4), i.e. equating $\mathbb{E}\left[e_i^{(2)}\right]$ with $(1 + r_A)\frac{E^{(1)}}{n}$, yields the equilibrium level of equity in the first period. Moreover, the instances in which $e_i^{(2)}$ is zero or negative characterize the default of bank i. When banks become insolvent and are not bailed out, the society encounters a banking crisis. As there are only macroeconomic

[20] A possibility to insure consumers even if condition (8) does not hold, is to issue risk securities on aggregate returns of the risky and the alternative technology. In this case the insurance capacity would be higher. Consumers can be insured as long as $(W - \hat{X})(1 + \overline{r} - b) + \hat{X}(1 + \overline{r}) \geq S(\overline{r})b$.

shocks, either all banks are solvent or insolvent.[21] We do not explicitly model the costs of a banking crisis occurring in subsequent periods. We assume that such costs are sufficiently large, such that it is socially desirable to avoid them.

We distinguish two cases.

Banking default

Suppose banks go bankrupt if $\eta = -b$ occurs. In such cases we obtain

$$\begin{aligned}\mathbb{E}\left[e_i^{(2)}\right] &= p_G \cdot \Big[\Big(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n}\Big)(1+\overline{r}+g) - \frac{\overline{S}}{n}(1+r_A)\Big] \\ &\quad + p_0 \cdot \Big[\Big(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n}\Big)(1+\overline{r}) - \frac{\overline{S}}{n}(1+r_A)\Big] + p_B \cdot 0 \\ &= \Big(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n}\Big)[(1+\overline{r})(p_G+p_0) + p_G \cdot g] - \frac{\overline{S}}{n}(1+r_A)(p_G+p_0).\end{aligned}$$

Using equilibrium condition (4) and $p_G g = p_B b$ yields

$$\begin{aligned}E^{(1)} &= \frac{\overline{S}((r_A-\overline{r})(p_G+p_0) - p_G \cdot g)}{(1+\overline{r})(p_G+p_0) + p_G \cdot g - (1+r_A)} \\ &= \frac{\overline{S}((r_A-\overline{r})(1-p_B) - p_B \cdot b)}{(1+\overline{r})(1-p_B) + p_B \cdot b - (1+r_A)}.\end{aligned} \tag{10}$$

No banking default

Let us suppose momentarily that $\frac{E^{(1)}}{n}$ is at levels where banks do not default. Then, using $\mathbb{E}[\eta] = 0$, equilibrium condition (4) can be written as

$$\mathbb{E}\left[e_i^{(2)}\right] = \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n}\right)(1+\overline{r}) - \frac{\overline{S}}{n} \cdot (1+r_A) = (1+r_A)\frac{E^{(1)}}{n}.$$

This yields the equilibrium level of equity in the first period

$$E^{(1)}(\overline{r} - r_A) = \overline{S} \cdot (r_A - \overline{r}). \tag{11}$$

There are two feasible solutions. First, $E^{(1)} = -\overline{S}$. This solution is economically meaningless as the initial equity capital would be negative. Second, equation (11) can be

[21] In practice this may happen if a small number of banks cannot fulfill their obligations.

solved by $\overline{r} = r_A$. Then, using condition (6), we have $f'(W - E^{(1)}) = 1 + \overline{r}$, which yields

$$E^{(1)} = W - (f')^{-1}(1 + \overline{r}). \tag{12}$$

where $(f')^{-1}$ denotes the inverse function of f'. The inverse exists as $f'' < 0$. Note that $E^{(1)}$ is positive as $W > (f')^{(-1)}(1 + \overline{r})$ by assumption. To simplify notation we use the following abbreviation:[22]

$$(f')^{-1}_{\overline{r}} := (f')^{-1}(1 + \overline{r}).$$

We can now summarize the results.[23]

Proposition 2

There exists a critical size of b, denoted by b^*, at which a banking crisis occurs if and only if $b > b^*$ and $\eta = -b$ is realized. This threshold is given by

$$b^* = (1 + \overline{r})\frac{W - (f')^{-1}_{\overline{r}}}{W - (f')^{-1}_{\overline{r}} + \overline{S}}.$$

(i) If $b = b^*$, both cases (default and no default of banks) are identical. The equilibrium is characterized by $r_A = \overline{r}$ and $E^{(1)} = W - (f')^{-1}_{\overline{r}}$.

(ii) If $b < b^*$, the equilibrium is characterized by $r_A = \overline{r}$ and $E^{(1)} = W - (f')^{-1}_{\overline{r}}$.

(iii) If $b > b^*$, the equilibrium is characterized by $r_A > \overline{r}$. $E^{(1)}$ is given by equation (10) and is larger than $W - (f')^{-1}_{\overline{r}}$.

Proposition 2 not only identifies the type of equilibria, it also illustrates the type of inefficiency that occurs when $b > b^*$. When b becomes larger and g rises correspondingly to ensure $\mathbb{E}[\eta] = 0$, it becomes more and more attractive to invest in banking equity as the downward risk is limited and gains become larger. Or put it differently: as macroeconomic volatility rises, the attractiveness of investment in bank equity rises, as it has a call-option-like payoff.

[22] We will also use $(f')^{-1}_{r_A}$ to denote $(f')^{-1}(1 + r_A)$.

[23] When we vary b, we assume that the probabilities p_B and p_G are fixed and g varies accordingly so that $p_G g = p_B b$.

As a consequence, fewer resources are invested in the alternative technology causing r_A to rise above $\overline{r}$. The return on equity is $1+r_A$ and thus larger than $1+\overline{r}$, as the downside risk of equity holders is limited and the higher return is purely at the cost of the deposit insurance fund. As shown in Proposition 1 this is inefficient, as the expected average return from aggregate investment could be increased by shifting some of the resources obtained from bank equity into the alternative technology. However, the amount of equity capital attracted is insufficient to buffer the more negative macroeconomic shocks. We summarize the results as follows:

Corollary 1

If $b > b^*$, too many resources are invested by banks, but too little equity capital is attracted to buffer negative macroeconomic shocks.

Throughout the paper we assume that $b > b^*$ and thus the probability that a banking crisis without insurance will occur is positive.

6. Complete Insurance

We next examine the scope of insurance. Again we assume that no idiosyncratic shocks can occur. We start with the equilibrium conditions.

6.1. Equilibrium Conditions

Equation (5) immediately determines repayment q in the bad states.[24]

$$q = \frac{1+r_A}{p_B}.$$

We note that equity in the second period with insurance is given by

$$e_i^{(2)} = \max\left\{ \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n} \right) (1+\overline{r}+\eta) + R_I \frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1+r_A), 0 \right\}.$$

[24] In the case of rare events it is often argued that focusing on expected returns, is sufficient to determine payments, even if investors are risk-neutral. It is straightforward to introduce a premium for ambiguity and to require a higher repayment than $\frac{1+r_A}{p_B}$ in the bad state.

Let us suppose momentarily that $\frac{E^{(1)}}{n}$ and Ω are at levels where banks will not default. Then, using $\mathbb{E}[\eta] = 0$, equilibrium condition (4) can be written as

$$\mathbb{E}\left[e_i^{(2)}\right] = \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n}\right)(1+\overline{r}) + p_B \cdot q\frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1+r_A) = (1+r_A)\frac{E^{(1)}}{n}.$$

This yields the equilibrium level of equity in the first period

$$E^{(1)}(\overline{r} - r_A) = \Omega(\overline{r} - r_A) + \overline{S} \cdot (r_A - \overline{r}). \tag{13}$$

Again there are two feasible solutions. First, $E^{(1)} = \Omega - \overline{S}$, which is economically meaningless. Second, equation (13) can be solved by $\overline{r} = r_A$. Then, using condition (6), we have $f'(W - E^{(1)} + \Omega) = 1 + \overline{r}$, which yields

$$E^{(1)} = W + \Omega - (f')^{-1}_{\overline{r}}. \tag{14}$$

We observe that the amount of equity with insurance is higher than without insurance, as investors have more funds available and invest more in bank equity.

6.2. Main Result

We first start with the simplest case, where complete insurance against any defaults is possible.

The condition for no bank going bankrupt is that $e_i^{(2)} \geq 0$ for $\eta = -b$ for all $i = 1, ..., n$, which implies

$$\left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n}\right)(1+\overline{r}-b) + q\frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1+\overline{r}) \geq 0,$$

where $E^{(1)}$ is given by equation (14).

The minimum number of insurance contracts for complete insurance is then given by

$$\underline{\Omega} := \frac{\overline{S}b - (W - (f')^{-1}_{\overline{r}})(1+\overline{r}-b)}{1+\overline{r}} p_B. \tag{15}$$

In addition, we obtain from condition (7) an upper bound for Ω:

$$(W + \Omega - E^{(1)})(1 + r_A) \geq q\Omega = \frac{1 + r_A}{p_B}\Omega$$

Using equation (14) yields

$$\Omega \leq p_B \cdot (f')^{-1}_{\overline{r}} := \overline{\Omega} \tag{16}$$

$\overline{\Omega}$ is the maximum number of insurance contracts investors can sell while still being able to recapitalize banks in a downturn even if they face total losses from their equity investments.

To sum up, we obtain

Proposition 3

Suppose $\overline{\Omega} > \underline{\Omega}$ and $\underline{\Omega} \leq \Omega < \overline{\Omega}$. The unique equilibrium with complete insurance is given by

$$\begin{aligned} r_A &= \overline{r} \\ E^{(1)} &= W + \Omega - (f')^{-1}_{\overline{r}} \\ \overline{S} &= S(\overline{r}) \\ q &= \frac{1 + \overline{r}}{p_B} \\ K &= W + \overline{S} - (f')^{-1}_{\overline{r}}. \end{aligned}$$

The corollary follows immediately:

Corollary 2

(i) Suppose $\overline{\Omega} < \underline{\Omega}$. Then no complete insurance against a banking crisis is possible.

(ii) Suppose complete insurance is possible for some value of p_B denoted by $\overline{p}_B$. Then insurance is feasible for any $p_B < \overline{p}_B$.[25]

The first point of the corollary refers to situations in which investors will not have sufficient wealth in a downturn to recapitalize banks, so the insurance capacity of the economy is too small. The second point indicates that rare events per se do not constitute

[25] When we vary p_B we assume that p_G varies accordingly so that $p_B b = p_G g$ holds.

a barrier to the insurability of banking crises. Both critical boundaries $\underline{\Omega}$ and $\overline{\Omega}$ scale with p_B.

6.3. An Example

As an example we use $f(x) = \ln(x)$. This implies $\frac{1}{W-E^{(1)}+\Omega} = 1 + \overline{r}$ and yields

$$\begin{aligned}
E^{(1)} &= W + \Omega - \tfrac{1}{1+\overline{r}} \\
\overline{\Omega} &= \tfrac{p_B}{1+\overline{r}} \\
\underline{\Omega} &= \tfrac{\overline{S}b-(W-\frac{1}{1+\overline{r}})(1+\overline{r}-b)}{1+\overline{r}} p_B \\
K &= W + \overline{S} - \tfrac{1}{1+\overline{r}}
\end{aligned}$$

6.4. Idiosyncratic Risk

We next allow for additional idiosyncratic risk. Then the second-period equity is given by

$$e_i^{(2)} = \max\left\{ \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n} \right)(1 + \overline{r} + \eta + \varepsilon_i) + R_I \frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1 + r_A), 0 \right\}.$$

We assume that ε_i is a random variable with two realizations: $\varepsilon_i = a$ or $\varepsilon_i = -a$ $(a > 0)$. Both events occur with probability $\frac{1}{2}$. If banks default without insurance in case that η is negative, independently of ε_i, the analysis is similar to subsection 5.1. We thus consider a situation in which, without insurance, a bank will default if and only if both shocks η and ε_i turn negative. The analysis from the preceding section can be easily adapted to this case. The non-default condition amounts to

$$\left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n} \right)(1 + \overline{r} - b - a) + q\frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1 + \overline{r}) \geq 0.$$

The level of equity is the same as in subsection 6.2, which yields

$$\underline{\Omega}^{IR} := \frac{\overline{S} \cdot (b + a) - (W - (f')^{-1}_{\overline{r}})(1 + \overline{r} - b - a)}{1 + \overline{r}} p_B\,.$$

This is the new lower boundary for insurance. The critical financing condition for investors is again given by $\Omega \leq \overline{\Omega}$, with $\overline{\Omega}$ given by equation (16). We note that the probability of an individual bank going bankrupt is $\frac{1}{2}p_B$ and on average only half of the banks would become insolvent without insurance. However, the insurance is not conditioned on bankruptcy but on the macroeconomic shock, i.e. it pays when $\eta = -b$. Hence the equilibrium value $q = \frac{1+\overline{r}}{p_B}$ still holds. Overall, we obtain the following result:

Corollary 3

Suppose $\overline{\Omega} \geq \underline{\Omega}^{IR}$. Then, if banks face idiosyncratic shocks and if $\Omega \in [\underline{\Omega}^{IR}, \overline{\Omega}]$, the same equilibrium as in Proposition 3 will obtain.

6.5. Alternative Indicators

The insurance contracts introduced in the preceding sections have been conditioned on the macroeconomic event, which itself could be measured by GDP growth.[26] However, as such data are frequently revised before a final estimate can be certified, insurance contracts conditioned on GDP growth could only be executed with some delay. Therefore it may be useful to consider other indicators that are highly correlated with the financial health of the banking sector. For instance, one might directly use the level of aggregate bank equity capital that would be realized if no insurance contracts were traded. This index is defined by the level of aggregate bank equity in relation to aggregate assets before crisis contracts are executed. We use

$$\kappa = \frac{\sum_{i=1}^{n} e_i^{(2)}}{\sum_{i=1}^{n} k_i(1 + \overline{r} + \eta + \varepsilon_i)}$$

to denote the ratio of aggregate equity to (realized) asset values. In contrast to GDP growth, the denominator and the nominator depend on the return of investments by banks. If the number of banks is sufficiently large and $b > b^*$, κ approximately takes three values:

$$\begin{aligned} \kappa^G &= \frac{ne_i^{(2)}}{nk(1+\overline{r}+g)} \\ \kappa^0 &= \frac{ne_i^{(2)}}{nk(1+\overline{r})} \\ \kappa^B &= 0 \end{aligned}$$

[26] One could use other standardized macroeconomic or housing prices as indices (see Shiller (2003)).

The probabilities that κ^G, κ^0 and κ^B will occur are p_G, p_0, and p_B, respectively. Hence, conditioning insurance contracts on κ yields the same results and thus provides the same scope of insurance against banking crisis.

7. Pitfalls

The insurance scheme works well under the conditions stated, but the time has now come to address the main challenges and pitfalls associated with this proposal.

7.1. Gambling

7.1.1. Unobservable Gambling

We examine whether banks have incentives to gamble when insurance is introduced, as collectively this could destabilize the banking system. Suppose we are in the situation in subsection 6.2: $\overline{\Omega} > \underline{\Omega}$ holds, so insurance is feasible. Moreover, suppose the government has required that each bank has to buy $(\underline{\Omega}/n)$ insurance contracts. Now suppose that one bank, say bank 1, has access to a risky technology called RT.

Definition 3 (Risky Technology (RT))
If bank 1 invests one unit of resources, it will obtain a repayment of $1+\overline{r}+\eta+\varepsilon_i$ *where* ε_i *is an idiosyncratic shock given by*

$$\varepsilon_i = \begin{cases} a-\delta & \text{with Prob } \frac{1}{2} \\ -a-\delta & \text{with Prob } \frac{1}{2}. \end{cases}$$

Here $a, \delta > 0, a-\delta > 0$, *so* $\mathbb{E}[\varepsilon_i] = -\delta$.

The other $n-1$ banks only have access to prudent technology with a return of $1+\overline{r}+\eta$ called PT. In addition, we assume that the RT investment opportunity is not observed by outsiders, in particular by equity holders.[27] As a consequence, we obtain the same values for $r_A, E^{(1)}, \overline{S}$, and q as in Proposition 3.

[27]Strictly speaking, outsiders attach a probability of zero to the availability of RT.

Regarding investment we have $n-1$ banks investing in PT with a return of $1+\overline{r}+\eta$. In case $\eta=-b$ these banks will just avoid bankruptcy. Bank 1 can either also invest in PT, generating an expected return on equity of $1+\overline{r}$, or it can invest in RT, which yields

$$\begin{aligned}\mathbb{E}\left[e_1^{(2)}\right] &= p_G\cdot\left[\Big(\frac{E^{(1)}}{n}+\frac{\overline{S}}{n}-\frac{\Omega}{n}\Big)(1+\overline{r}+g-\delta)-\frac{\overline{S}}{n}(1+r_A)\right]\\ &\quad + p_0\cdot\left[\Big(\frac{E^{(1)}}{n}+\frac{\overline{S}}{n}-\frac{\Omega}{n}\Big)(1+\overline{r}-\delta)-\frac{\overline{S}}{n}(1+r_A)\right]\\ &\quad + p_B\cdot\frac{1}{2}\cdot\left[\Big(\frac{E^{(1)}}{n}+\frac{\overline{S}}{n}-\frac{\Omega}{n}\Big)(1+\overline{r}-b+a-\delta)+q\frac{\Omega}{n}-\frac{\overline{S}}{n}(1+r_A)\right]\\ &\quad + p_B\cdot\frac{1}{2}\cdot 0\\ &= \Big(\frac{E^{(1)}}{n}+\frac{\overline{S}}{n}-\frac{\Omega}{n}\Big)\left[(1+\overline{r}-\delta)(1-\frac{1}{2}p_B)+p_G\cdot g+\frac{1}{2}p_B(a-b)\right]\\ &\quad -\frac{\overline{S}}{n}(1+r_A)(1-\frac{1}{2}p_B)+\frac{1}{2}p_Bq\frac{\Omega}{n}. \end{aligned} \tag{17}$$

as bank 1 goes bankrupt if and only if $\eta=-b$ and $\varepsilon=-a-\delta$. This implies

Proposition 4

For δ sufficiently small, i.e.

$$\delta<\overline{\delta}=\frac{\frac{1}{2}p_B\cdot a}{1-\frac{1}{2}p_B},$$

bank 1 engages in gambling under the insurance policy $\underline{\Omega}$.

The proof of Proposition 4 is given in the Appendix. Proposition 4 is a variant of the standard theme that maximization of return on equity under limited liability will induce banks to gamble.

The preceding argument can be applied when all banks face idiosyncratic risk as in section 6.4. But there are other possibilities for engaging in gambling by choosing risky technologies. Again banks have an incentive to load on more idiosyncratic risk.

7.1.2. Observable Gambling Opportunities

Next we investigate the same situation as in the last section, but assume that at the start all market participants observe the set of investment opportunities for banks. As a consequence of higher return expectations, investors might be willing to provide bank 1 with more equity, which might lower the incentive of bank 1 to gamble. To incorporate the investment choice of bank 1, we need to adapt our equilibrium concept. We exemplify this for bank 1, but the considerations can be applied to investment choices for all banks. We use K_{-1} and $E_{-1}^{(1)}$ to denote the aggregate investment and the aggregate equity by all banks except bank 1. Thus, we arrive at

Definition 4 (Equilibrium with Investment Choice)

Given an insurance policy Ω ($\Omega > 0$), a competitive equilibrium with investment choice for bank 1 is a tuple $\left\{K_{-1}, k_1, E_{-1}^{(1)}, e_1^{(1)}, q, r_A\right\}$ such that the following conditions hold:

$$\begin{aligned}
\frac{E_{-1}^{(1)}}{n-1} + \frac{\overline{S}}{n} &= \frac{K_{-1}}{n-1} + \frac{\Omega}{n} \\
e_1^{(1)} + \frac{\overline{S}}{n} &= k_1 + \frac{\Omega}{n} \\
\mathbb{E}\left[e_i^{(2)}\right] &= (1+r_A)\frac{E_{-1}^{(1)}}{n-1}, \ i = 2, \ldots, n \\
\mathbb{E}\left[e_1^{(2)}\right] &= (1+r_A)e_1^{(1)} \\
\mathbb{E}\left[R_I\right] &= 1 + r_A \\
f'(W - E^{(1)} + \Omega) &= 1 + r_A \\
(W + \Omega - E^{(1)})(1+r_A) &\geq q\Omega
\end{aligned}$$

The next proposition shows how observable gambling impacts on equity and default.

Proposition 5

Suppose $\overline{\Omega} > \underline{\Omega}$ and $\underline{\Omega} < \Omega < \overline{\Omega}$ so that all banks except bank 1 are insured against default.[28] *Let $0 < \delta < \overline{\delta}$. Bank 1 chooses among {RT, PT}.*

[28] $\underline{\Omega}$ has to be higher than $\underline{\Omega}$ since those banks with no access to RT will obtain somewhat less equity than in the equilibrium described by Proposition 3.

Then bank 1 will attract more equity than the other banks. Bank 1 chooses RT. In the event $\eta = -b$, only bank 1 defaults.

The proof of Proposition 5 is given in the Appendix. The intuition for Proposition 5 is as follows: Suppose bank 1 obtained the same level of equity as the other banks. Then the situation is the same as we would have if gambling were unobserved. According to Proposition 4 bank 1 would choose RT and would generate a higher return on equity than the other banks. Hence, this cannot be an equilibrium, as investors would supply more equity capital resources to bank 1 than to other banks.

Suppose that bank 1 attracted an amount of equity high enough to ensure that it would not go bankrupt even if it chose RT. As the gambling technology yields lower expected returns than $\overline{r}$, bank 1 could not offer the same expected return on equity $r_A = \overline{r}$ as the other banks. Accordingly, this cannot be an equilibrium either, as investors would not be willing to offer equity.

An equilibrium is thus characterized by an amount of equity for bank 1 that is higher than that of other banks but not high enough to avoid default by bank 1 if the realization of both the systemic and idiosyncratic shocks is negative. So if gambling opportunities are anticipated by market participants, the equity capital allocation across banks will change, but the risk-taking bank will still go bankrupt. The argument can be extended to investment choices made by all banks.

7.2. Voluntary Insurance

In this section we investigate whether banks themselves want to buy insurance contracts. Suppose we are in the equilibrium associated with $\Omega = 0$ and there are no idiosyncratic shocks. Suppose further that the downturn is sufficiently severe for all banks to go bankrupt if $\eta = -b$, i.e. $b > b^*$, and thus $r_A > \overline{r}$ holds in equilibrium. According to equation (10) the level of equity is given by

$$E^{(1)} = \frac{\overline{S}((r_A - \overline{r})(1 - p_B) - p_G \cdot g)}{(1 + \overline{r})(1 - p_B) + p_G \cdot g - (1 + r_A)}. \tag{18}$$

Second-period equity for all banks i is given by

$$e_i^{(2)} = max\left\{(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n})(1+\overline{r}+\eta) - \frac{\overline{S}}{n}(1+r_A), 0\right\}.$$

Now suppose that a bank, say bank 1, buys insurance amounting to Ω_1. We assume that the number of banks n is large and that Ω_1 is sufficiently small for the impact on return $1 + r_A$ to be negligible and hence neglected. Without insurance, the return on equity for bank 1 is $1 + r_A$. Suppose that bank 1 buys an amount Ω_1 so that it just avoids bankruptcy in state $\eta = -b$. Then bank 1's second-period equity changes to

$$e_1^{(2)} = \Big(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \Omega_1\Big)(1+\overline{r}+\eta) + R_I\Omega_1 - \frac{\overline{S}}{n}(1+r_A).$$

The necessary amount of insurance Ω_1 is given by $e_1^{(2)}(\eta = -b) = 0$. This yields

$$\Omega_1 = \frac{\overline{S}(r_A - \overline{r} + b) + E^{(1)}(b - (1+\overline{r}))}{n(q + b - (1+\overline{r}))}. \tag{19}$$

With these observations we obtain the following corollary:

Corollary 4
Suppose $b^* < b$. Then, if only one bank bought insurance, its return on equity $\mathbb{E}\left[e_1^{(2)}\right]$ would be below $\frac{E^{(1)}}{n}(1 + r_A)$.

The proof of Corollary 4 is given in the Appendix. An immediate consequence is that bank 1 has no incentive to buy insurance. A similar argument can be advanced for the scenario where $n - 1$ banks would buy insurance to avoid bankruptcy. Then the nth bank would be better off not to buy insurance. Overall, this analysis illustrates that voluntary actions will not insure the banking system against a crisis.

7.3. Bankruptcy of Insurers

Insurance only works if investors can and will pay the promised amount in event $\eta = -b$. Default can occur because investors do not have sufficient wealth to honor their promise or they may default strategically. Let us discuss each reason in turn. Two types of wealth constraints on investors acting as insurers have to be considered. First, investors risk total losses from their investments in bank shares. This repayment constraint is part of the equilibrium concept and is thus taken into account. It is important to note that complete insurance is feasible if wealth from investment in the outside opportunity is sufficiently large to recapitalize banks in a downturn.

Second, the outside investment opportunities may themselves be risky. In particular, suppose the returns in this alternative technology are correlated with the investments of banks. As a polar case, suppose that the return is given by

$$f'(X) + \eta$$

and thus returns on investments by banks and on outside investments are perfectly correlated. In such cases, the insurance capacity of an economy against a banking crisis shrinks substantially. The equilibrium repayment condition for investors (equation (7)) has to be replaced by

$$(W + \Omega - E^{(1)})(1 + r_A - b) \geq q\Omega.$$

The equilibrium properties $q = \frac{1+r_A}{p_B}$ and $E^{(1)} = W + \Omega - (f')^{-1}_{\overline{r}}$ still hold. Hence, the critical condition that investors can protect the economy against a crisis is given by

$$\Omega \leq (f')^{-1}_{\overline{r}} \frac{1 + r_A - b}{1 + r_A} p_B =: \overline{\Omega}^{BI}.$$

The new upper boundary for insurance $\overline{\Omega}^{BI}$ is strictly smaller than $\overline{\Omega}$. Moreover, the larger b is, the smaller the insurance capacity of an economy against banking crises will be. This observation indicates that insurance of an economy against a banking crisis crucially depends on the possibilities and the willingness of investors to protect their wealth in a downturn.

Even if investors have sufficient wealth they may default strategically as they may consume part of their wealth instead of honoring their insurance liabilities. Such risks of strategic defaults can be prevented by using investment in riskless technology as collateral against default on insurance commitments.[29]

[29]When insurance companies act as crisis insurers, the regulation of this industry has to be brought into the discussion. A comprehensive analysis of failures in the insurance industry and suitable regulatory

7.4. Private Benefits of Managers and Managerial Limits on Equity Capital

Managers may not be interested in obtaining the maximum amount of equity they could attract. Suppose managers can obtain private benefits from running banks.[30] Suppose further that such benefits are a function of the excess return on equity they achieve over the expected return or over the ex-post return of their rivals. If bank managers limit equity contract issuance, ex-ante equity returns and ex-post equity returns in the good state will increase, which may tend to increase the expected private benefits of managers.[31]

Limiting equity has severe consequences for the scope of insurance against banking crises, as the amount of contracts required to insure against a negative shock will increase and may exceed the insurance capacity of the economy. Moreover, incentives to gamble are reinforced. We have already observed how excessive risk taking undermines the scope of insurance against banking crises, as the number of contracts required to insure against banking crises will rise. Also, the incentives to load even more idiosyncratic risk on the balance sheet of banks remain.

8. Complementary Regulatory Measures

The inadequacy of an insurance scheme to offset banking crises may be at least partially remedied by complementary regulatory measures. In this section we discuss some of these measures.

8.1. Risk-sensitive capital requirement

In order to counteract the incentives of banks to engage in excessive risk taking, the regulator could impose risk-sensitive capital requirements in the spirit of Basel II. Sup-

responses is given in Plantin and Rochet (2008).

[30]Such benefits consist of both the pay package and non-monetary benefits, such as ego rents and empire building.

[31]The extent to which managers limit the share of bank equity depends on the precise relationship between private benefits and excess returns on equity.

pose we are in the scenario in section 7.1 where bank 1 has access to a risky investment RT while all other banks only have access to the prudent technology PT. Suppose the regulator can observe the choice of bank 1 and imposes either equity regulation e_1^R or e_1^P on bank 1 ($e_1^R > e_1^P$), depending on whether the bank selects RT or PT. All other banks are subject to regulation e_1^P. We obtain

Proposition 6

Let $\underline{\Omega} < \overline{\Omega}$. Suppose the regulator chooses an insurance policy $\Omega = \underline{\Omega}$. Suppose further that the regulator sets $e_1^P = \frac{E^{(1)}}{n}$ where $E^{(1)}$ is determined by equation (14).

Then, for e_1^R sufficiently high, bank 1 will refrain from gambling and complete insurance obtains.

The proof of Proposition 6 is straightforward. A sufficiently high level of e_1^R ensures that bank 1 will not choose RT, as bank 1 would not go bankrupt, so choosing RT would lower return on equity. As shown in Proposition 3, setting $\Omega = \underline{\Omega}$ would induce complete insurance if banks attract $\frac{E^{(1)}}{n}$. Hence, risk-sensitive capital requirements could in principle be a sensible complementary regulatory measure.

However, there are a number of serious drawbacks to such capital requirements, as discussed in Hellwig (2008). For instance, strictly enforcing capital requirements in each period makes it impossible to use bank equity as a buffer. Moreover, risk-sensitive capital requirements, the key principle of Basel II, reinforce downturns, as they amplify the procyclicality of banking and induce many banks to impose disproportional cuts on lending during a downturn. This is undesirable. These problems have to be resolved satisfactorily in order to use risk-sensitive capital requirements to counteract the undesirable consequences of insurance schemes against banking crises.

8.2. Insurance Contracts for Bank Managers

A sensible way of counteracting excessive risk-taking by managers is to require bank managers to sell a specified number of insurance contracts against systemic crises, either as part of their remuneration package or separately. Thus, bank managers act as insurers.

For example the number of insurance contracts a manager has to sell could be a fixed fraction of his salary. While insurance contracts for bank managers would only provide a small proportion of the financial resources required to bail out banks in a crisis, they would still have a number of advantages.

Such a scheme internalizes externalities and creates collective responsibility on the part of bank managers to be sufficiently prudent and to lower the likelihood of banking crises. Suppose, for example, that such contracts are based on average bank capital. While individual incentives to limit bank equity or gamble may still be present, bank managers engaging in such activities may face severe career risk by harming other colleagues. This creates incentives for managers to be concerned about systemic crises and the stability of the public goods system.[32]

Finally, collective responsibility on the part of bank managers[33] and the need to contribute to the recapitalization of banks in a crisis is useful as it will be perceived by citizens as fair burden-sharing. As a consequence, citizens my be more inclined to support policy measures drawing on taxpayer money to resolve banking crises if they know that bank managers have to shoulder a disproportionate share of the costs.

8.3. Other Extensions

The analysis allows for a variety of further extensions. For instance, our analysis can indirectly justify the use of public insurance schemes when the insurance capacity of investors is limited. Public insurance in the form of bank rescues through tax payers can supplement private insurance schemes. An analysis of such mixed schemes, by incorporating distortionary taxation as the cost of public insurance is left to future research.

Another useful extension is to consider a multiperiod framework, in which banks face repeated macroeconomic shocks. In the current two-period framework, banks have no

[32]One could even envision making the number of insurance contracts per bank manager risk-sensitive, i.e. a bank manager who chooses more risky investments has to sell a larger number of such contracts. In such cases, the scheme would counteract excessive risk-taking.

[33]In practical applications if is useful to require that only the managers of large banks or insurance companies need to act as insurers.

incentives to hold reserves, e.g. by investing into the alternative technology as this would (weakly) reduce return on equity. This is not necessarily the case at a longer time horizon when an early default prevents later and attractive gambling by banks. To avoid early defaults banks may want to hold reserves.

9. Conclusion

The current crisis is a painful illustration of the fact that sophisticated banking systems are vulnerable to systemic crises. Insurance schemes against banking crises are a sufficiently promising approach. However, it is neither an all-purpose panacea nor does it automatically guarantee a decline in the likelihood and the severity of banking crises. It will require complementary policy measures to ensure that its unintentional consequences will not outweigh its objectives.

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A. Appendix A: Proofs

Proof of Proposition 2.

(i) The critical threshold for the magnitude of the negative shock, denoted by b^*, is determined by $e_i^{(2)}(\eta = -b^*) = 0$. Moreover, for b^* the no-default case needs to coincide with the default case. The first condition yields $(E^{(1)} + \overline{S})(1 + \overline{r} - b^*) - \overline{S}(1 + r_A) = 0$, which is equivalent to

$$b^* = 1 + \overline{r} - (1 + r_A)\frac{\overline{S}}{E^{(1)} + \overline{S}}. \tag{20}$$

If banks do not default, we have $r_A = \overline{r}$ (see section 5.1), so $E^{(1)}$ is given by equation (12). Hence we obtain $b^* = (1 + \overline{r})\frac{W - (f')_{\overline{r}}^{-1}}{W - (f')_{\overline{r}}^{-1} + \overline{S}}$.

We next show that the default case is equivalent to the no-default case $b = b^*$. It is straightforward to show that for $b = b^*$ both expressions for $\mathbb{E}[e_i^{(2)}]$ (with and without default of banks) coincide. Furthermore, after some algebraic manipulation inserting $b = b^*$ as given in equation (20) into equation (10) and solving this equation for $E^{(1)}$ yields equation (11). Hence banking default coincides with no-default associated with $r_A = \overline{r}$.

(ii) Suppose $b < b^*$. Then $e_i^{(2)}(\eta = -b) > e_i^{(2)}(\eta = b^*) = 0$ if $E^{(1)}$ is given according to equation (12) and if $r_A = \overline{r}$. Hence, the no-default case constitutes an equilibrium. According to equilibrium definition, investments k are given by $k = \frac{1}{n}[W + \overline{S} - (f')_{\overline{r}}^{-1}]$.

(iii) Suppose $b > b^*$. We first show that default occurs for $\eta = -b$. Suppose, by contrast, that the no-default equilibrium obtains with $E^{(1)} = W - (f')_{\overline{r}}^{-1}$ and $r_A = \overline{r}$. In this case we observe that $e_i^{(2)}(\eta = -b) < e_i^{(2)}(\eta = -b^*) = 0$, i.e. default occurs.

Suppose therefore that default occurs if $\eta = -b$. Then first-period equity is given by equation (10) and depends on b, which we write as $E^{(1)} = E^{(1)}(b)$. We need to show that $e_i^{(2)}(\eta = -b) < 0$ and hence that default does indeed occur. To prove this assertion, suppose that, for some fixed b, first-period equity $E^{(1)}(b)$ is given according to equation (10). We calculate a critical shock value $\tilde{b}$ at which banks would indeed go bankrupt if they attracted equity $E^{(1)}(b)$. If $\tilde{b} < b$ for all $b > b^*$, the assertion is proved. $\tilde{b}$ is defined as

$$\begin{aligned}
& (E^{(1)}(b) + \overline{S})(1 + \overline{r} - \tilde{b}) - \overline{S}(1 + r_A) &=& \ 0 \\
\Leftrightarrow \quad & \overline{S}\Big(\tfrac{(r_A - \overline{r})(1-p_B) - p_B b}{(1+\overline{r})(1-p_B) + p_B b - (1+r_A)} + 1\Big)(1 + \overline{r} - \tilde{b}) - \overline{S}(1 + r_A) &=& \ 0 \\
\Leftrightarrow \quad & \tfrac{(r_A - \overline{r})(1-p_B) - p_B b + (1+\overline{r})(1-p_B) + p_B b - (1+r_A)}{(1+\overline{r})(1-p_B) + p_B b - (1+r_A)}(1 + \overline{r} - \tilde{b}) - (1 + r_A) &=& \ 0 \\
\Leftrightarrow \quad & \tfrac{(1+r_A)(1-p_B) - (1+r_A)}{(1+\overline{r})(1-p_B) + p_B b - (1+r_A)}(1 + \overline{r} - \tilde{b}) - (1 + r_A) &=& \ 0 \\
\Leftrightarrow \quad & (-p_B)(1 + \overline{r} - \tilde{b}) - ((1 + \overline{r})(1 - p_B) + p_B b - (1 + r_A)) &=& \ 0 \\
\Leftrightarrow \quad & (1 + r_A) - (1 + \overline{r}) + p_B(\tilde{b} - b) &=& \ 0 \\
\Leftrightarrow \quad & \tilde{b} &=& \ b + \tfrac{\overline{r} - r_A}{p_B}
\end{aligned}$$

Hence, given some fixed b and first-period equity $E^{(1)}(b)$, the negative shock sufficient to impose bankruptcy on banks is smaller than b, as $r_A > \overline{r}$. Hence, for $b > b^*$ banks will default.

In the following we show that $r_A > \overline{r}$. $E^{(1)}(b)$ represents the equilibrium equity level in the first period, and it also depends on r_A. Using the equilibrium condition (6) yields $W - (f')^{-1}_{r_A} = E^{(1)}(b)$. Inserting $E^{(1)}(b) = \overline{S}\frac{(r_A - \overline{r})(1-p_B) - p_B b}{(1+\overline{r})(1-p_B) + p_B b - (1+r_A)}$ yields the following implicit definition of r_A:

$$(W - (f')^{-1}_{r_A})\Big[(1+\overline{r})(1-p_B) + p_B b - (1+r_A)\Big] = \overline{S}\Big[(r_A - \overline{r})(1-p_B) - p_B b\Big]. \quad (21)$$

Suppose that $r_A = \overline{r}$ holds. Then the LHS is greater than the RHS if and only if $b > (1 + \overline{r})\frac{W - (f')^{-1}_{\overline{r}}}{W - (f')^{-1}_{\overline{r}} + \overline{S}} = b^*$, which is true by assumption. Furthermore, it is straightforward to show that the LHS is decreasing in r_A while the RHS is increasing in r_A. Accordingly, in equilibrium $r_A > \overline{r}$ must hold. Finally, we can calculate $k = \frac{1}{n}[E^{(1)} + \overline{S}]$ by using equilibrium condition (3).

To complete the proof, we observe that $E^{(1)}(b)$, as defined by equation (10) in the default case for $b > b^*$, is larger than $E^{(1)} = W - (f')^{-1}_{\overline{r}}$ in the non-default case with $b < b^*$. The observation follows from $f'(W - E^{(1)}(b)) = 1 + r_A$, $r_A > \overline{r}$ in the default case and $f'' < 0$. These properties imply $E^{(1)}(b) \geq W - (f')^{-1}_{\overline{r}}$.

□

Proof of Proposition 4.

Bank 1 engages in gambling if $\mathbb{E}\left[e_1^{(2)}\right] > (1+r_A)\frac{E^{(1)}}{n}$. Using equation (17), this is equivalent to

$$\delta < 1+\overline{r}+\frac{p_G\cdot g+\frac{1}{2}p_B(a-b)}{1-\frac{1}{2}p_B}-\frac{(1+r_A)(E^{(1)}+\overline{S}(1-\frac{1}{2}p_B))}{(E^{(1)}+\overline{S}-\Omega)(1-\frac{1}{2}p_B)}+\frac{p_B\cdot q\Omega}{2(E^{(1)}+\overline{S}-\Omega)(1-\frac{1}{2}p_B)}$$

Under unobservable gambling we obtain the equilibrium values as presented in section 6.2: $r_A=\overline{r}$, $E^{(1)}=W+\Omega-(f')^{-1}_{\overline{r}}$. Using these identities, we can transform the RHS of the above equation into

$$(1+\overline{r})+\frac{p_Gg+\frac{1}{2}p_B(a-b)}{1-\frac{1}{2}p_B}-\frac{(1+\overline{r})(W+\frac{1}{2}\Omega-(f')^{-1}_{\overline{r}}+\overline{S}(1-\frac{1}{2}p_B))}{(W+\overline{S}-(f')^{-1}_{\overline{r}})(1-\frac{1}{2}p_B)}$$

By assumption we have $\Omega=\underline{\Omega}$, as given by equation (15). Hence we can rewrite the third summand as

$$\begin{aligned}
&\frac{(1+\overline{r})(W+\frac{1}{2}\Omega-(f')^{-1}_{\overline{r}}+\overline{S}(1-\frac{1}{2}p_B))}{(W+\overline{S}-(f')^{-1}_{\overline{r}})(1-\frac{1}{2}p_B)}\\
=\;&\frac{(1+\overline{r})[W-(f')^{-1}_{\overline{r}}+\overline{S}(1-\frac{1}{2}p_B)]+\frac{1}{2}p_B[\overline{S}b-(W-(f')^{-1}_{\overline{r}})(1+\overline{r}-b)]}{(W+\overline{S}-(f')^{-1}_{\overline{r}})(1-\frac{1}{2}p_B)}\\
=\;&\frac{(1+\overline{r})[W-(f')^{-1}_{\overline{r}}+\overline{S}](1-\frac{1}{2}p_B)+\frac{1}{2}p_B[\overline{S}+W-(f')^{-1}_{\overline{r}}]b}{(W+\overline{S}-(f')^{-1}_{\overline{r}})(1-\frac{1}{2}p_B)}\\
=\;&1+\overline{r}+\frac{\frac{1}{2}p_Bb}{1-\frac{1}{2}p_B}
\end{aligned}$$

Overall we find that bank 1 will engage in gambling if and only if

$$\begin{aligned}
\delta &< (1+\overline{r})+\frac{p_Gg+\frac{1}{2}p_B(a-b)}{1-\frac{1}{2}p_B}-\left[1+\overline{r}+\frac{\frac{1}{2}p_Bb}{1-\frac{1}{2}p_B}\right]\\
\Leftrightarrow\;\delta &< \frac{p_Gg-p_Bb}{1-\frac{1}{2}p_B}+\frac{\frac{1}{2}p_Ba}{1-\frac{1}{2}p_B}\\
\Leftrightarrow\;\delta &< \frac{\frac{1}{2}p_Ba}{1-\frac{1}{2}p_B}=\overline{\delta}\,.
\end{aligned}$$

□

Proof of Proposition 5.

In the first steps we show the situations that do not constitute an equilibrium. In step 3 we provide the equilibrium.

Step 1:
Suppose that $e_1^{(1)} = \frac{E_{-1}^{(1)}}{n-1}$. Regarding equity and investments we are then in the same situation as in Proposition 4. Hence, $r_A = \overline{r}$, and bank 1 chooses RT to generate $\mathbb{E}[e_1^{(2)}]$ greater than $(1+r_A)e_1^{(1)}$. As a consequence, investors would offer bank 1 more resources as equity in order to benefit from higher return. Thus $e_1^{(1)} > \frac{E_{-1}^{(1)}}{n-1}$. Hence the initial situation does not constitute an equilibrium.

Step 2:
Suppose bank 1 obtains $e_1^{(1)}$ large enough for it not to default even by choosing RT. Suppose bank 1 chooses RT. Then, as $\mathbb{E}[\varepsilon_1] = -\delta < 0$, we obtain

$$\begin{aligned}\mathbb{E}[e_1^{(2)}] &= \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n}\right)(1+\overline{r}-\delta) + p_B \cdot q\frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1+r_A) \\ &< \left(\frac{E^{(1)}}{n} + \frac{\overline{S}}{n} - \frac{\Omega}{n}\right)(1+\overline{r}) + p_B \cdot q\frac{\Omega}{n} - \frac{\overline{S}}{n} \cdot (1+r_A) \\ &= \mathbb{E}[e_1^{(2)} \mid \text{bank 1 chooses PT}]\end{aligned}$$

Bank 1 is better off by choosing PT, so RT will not be chosen. If equity holders anticipate that PT will be chosen, bank 1 is identical to the other banks and would receive $e_1^{(1)} = \frac{E_{-1}^{(1)}}{n-1}$, i.e. the initial situation does not constitute an equilibrium.[34]

Step 3:
There exists an equilibrium in which $e_1^{(1)}$ is set at a level for which $\mathbb{E}[e_1^{(2)}] = (1+r_A)e_1^{(1)}$, bank 1 chooses RT and defaults whenever $\eta = -b$ and $\varepsilon_1 = -a-\delta$. Investors are willing to supply $e_1^{(1)}$. Such an amount of equity exists by reason of the intermediate value theorem: $\frac{\mathbb{E}[e_1^{(2)}]}{e_1^{(1)}}$ is decreasing in $e_1^{(1)}$, it is higher than $1+r_A$ for $e_1^{(1)} = \frac{E_{-1}^{(1)}}{n-1}$ and smaller than $1+r_A$ for $e_1^{(1)}$ as high as in step 2. □

[34]Note that we focus on symmetric equilibria if banks are identical from the perspective of investors.

Proof of Corollary 4.

We have to show that $\mathbb{E}\left[e_1^{(2)}\right] < \frac{E^{(1)}}{n}(1+r_A)$. Using $\Omega_1 = \frac{\overline{S}(r_A-\overline{r}+b)+E^{(1)}(b-(1+\overline{r}))}{n(q+b-(1+\overline{r}))}$ (see equation (19)) and $q = \frac{1+r_A}{p_B}$, we obtain

$$\begin{aligned}
n\mathbb{E}\left[e_1^{(2)}\right] &= (E^{(1)} + \overline{S} - n\Omega_1)(1+\overline{r}) + p_B q n \Omega_1 - \overline{S}(1+r_A) \\
&= (1+\overline{r})\Big[E^{(1)}(1 - \frac{b-(1+\overline{r})}{q+b-(1+\overline{r})}) + \overline{S}(1 - \frac{r_A-\overline{r}+b}{q+b-(1+\overline{r})})\Big] \\
&\quad + E^{(1)} \cdot (1+r_A) \cdot \frac{b-(1+\overline{r})}{q+b-(1+\overline{r})} + \overline{S} \cdot (1+r_A) \cdot \frac{r_A-\overline{r}+b}{q+b-(1+\overline{r})} - \overline{S}(1+r_A) \\
&= E^{(1)}\Big[\frac{q(1+\overline{r})}{q+b-(1+\overline{r})} + \frac{(b-(1+\overline{r}))(1+r_A)}{q+b-(1+\overline{r})}\Big] \\
&\quad + \overline{S}\Big[\frac{(q-(1+r_A))(1+\overline{r})}{q+b-(1+\overline{r})} + \frac{(r_A-\overline{r}+b)(1+r_A)}{q+b-(1+\overline{r})} - (1+r_A)\Big] \\
&= E^{(1)}\Big[\frac{(1+r_A)(1+\overline{r})}{(1+r_A)+p_B b - p_B(1+\overline{r})} + \frac{p_B(b-(1+\overline{r}))(1+r_A)}{(1+r_A)+p_B b - p_B(1+\overline{r})}\Big] \\
&\quad + \overline{S}\Big[\frac{(1-p_B)(1+r_A)(1+\overline{r})}{(1+r_A)+p_B b - p_B(1+\overline{r})} + \frac{p_B((r_A-\overline{r}+b)(1+r_A)}{(1+r_A)+p_B b - p_B(1+\overline{r}))} - (1+r_A)\Big] \\
&= E^{(1)}\,\frac{(1-p_B)(1+r_A)(1+\overline{r}) + p_B b(1+r_A)}{(1+r_A)+p_B b - p_B(1+\overline{r})} \\
&\quad + \overline{S}\,\frac{(1-p_B)(1+r_A)(\overline{r}-r_A)}{(1+r_A)+p_B b - p_B(1+\overline{r})} \\
&= \frac{1+r_A}{(1+r_A)+p_B b - p_B(1+\overline{r})}\Big[E^{(1)}[(1-p_B)(1+\overline{r}) + p_B b] + \overline{S}(1-p_B)(\overline{r}-r_A)\Big]
\end{aligned}$$

With this result we can now turn to the inequality that has to be proven:

$$\begin{aligned}
& n\mathbb{E}\left[e_1^{(2)}\right] < E^{(1)}(1+r_A) \\
\Leftrightarrow\ & \frac{E^{(1)}[(1-p_B)(1+\overline{r})+p_B b]+\overline{S}(1-p_B)(\overline{r}-r_A)}{(1+r_A)+p_B b-p_B(1+\overline{r})} < E^{(1)} \\
\Leftrightarrow\ & E^{(1)}[(1-p_B)(1+\overline{r}) + p_B b] + \overline{S}(1-p_B)(\overline{r}-r_A) < E^{(1)}[(1+r_A)+p_B b - p_B(1+\overline{r})] \\
\Leftrightarrow\ & E^{(1)}(1+\overline{r}) + \overline{S}(1-p_B)(\overline{r}-r_A) < E^{(1)}(1+r_A) \\
\Leftrightarrow\ & (\overline{r}-r_A)[E^{(1)} + \overline{S}(1-p_B)] < 0
\end{aligned}$$

The last inequality holds, as $r_A > \overline{r}$ in any equilibrium where $b > b^*$. □