# Voting Oneself into a Crisis*

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**Abstract**

We suggest that voters' lack of recognition of complex economic links may give rise to economic policies that eventually lead to a crisis. We consider a two-sector economy in which a majoritarian political process determines governmental regulation in one sector: a minimum nominal wage. If voters recognize general equilibrium feedbacks, workers across sectors form a majority and will favor market-clearing wages. If voters only take into account direct effects in the regulated sector, workers in the other sector are willing to vote for wage rises in each period since they also reckon with higher real wages for themselves. The political process leads to constantly rising unemployment and tax rates. The resulting crisis may trigger new insights into economic relationships on the part of the voters and may reverse bad times.

**JEL:** D72, D83, E24, J30

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# 1 Introduction

It happens frequently that governments follow fiscal policies that turn out to be unsustainable in the long-run. The recent examples of Greece or Spain are such cases. In this paper we provide an explanation for such phenomena. We argue that difficulties voters have in recognizing general equilibrium effects may yield economic policies that trigger a crisis. Moreover, a crisis may help to promote the understanding of general equilibrium effects on the voters' part, and this can reverse bad times.

The argument is developed for a two-sector economy with three types of workers. The first sector uses skilled and unskilled workers; the second sector uses a specific type of worker. The first sector is the "regulated" sector: there is a minimum wage and unemployment benefits. We consider the following democratic process for regulating sector 1: Two political parties propose a minimum wage for low-skilled workers in sector 1, where unemployment is financed by a tax on labor. As it will turn out, low-skilled workers in sector 1 will benefit from higher minimum wages while high-skilled workers in sector 1 are hurt. The workers in sector 2 essentially play the role of the median voter.

If workers take all direct and indirect effects into account when voting - called hereinafter General Equilibrium Voting (GEV) - they anticipate that raising low-skilled wages in sector 1 will affect not only labor demand, wages for high skilled workers, and prices in sector 1, but also wages in sector 2 and taxes to finance unemployed individuals. The latter general equilibrium effects imply that workers in sector 2 have single-peaked preferences regarding wages for low-skilled workers in sector 1, with market-clearing wages as their most preferred wage. Since high-skilled workers in sector 1 also prefer market-clearing wage over any other wage, the market clearing wage is the political equilibrium. As a consequence, there is no unemployment and hence no tax burden as the democratic process implements the free market solution.

Suppose, however, that individuals do not take into account general feedback effects in sector 2 connected with the minimum wage proposals in sector 1, whereas all effects in sector 1 are fully recognized. We refer to this as Partial Equilibrium Voting (PEV). This will be thoroughly justified in section 3. Voters taking this view assume that nothing will change in sector 2, including wages and output in this sector, and also that tax rates will remain constant.

With PEV, workers in sector 2 perceive that - from a certain wage level on - an increase in minimum wages will improve their utility. The logic can be understood by considering market clearing for the second good. Demand for good 2 of the low-skilled workers increases with a rising minimum wage because unemployed workers would receive unemployment benefits. Workers in sector 2 assume that their nominal wage, the tax rate and hence their demand for good 2 remains constant. They project that the relative price of good 1 has to decline with market clearing as demand for good 2 of high-skilled workers has to fall. As workers in sector 2 expect that the price in sector 1 declines they believe that their real wage increases. Therefore under PEV their preferred wage for low-skilled workers in sector 1 is higher than the market-clearing wage. Together with the low-skilled workers in sector 1, sector 2 workers will vote for an increase in wages, which results in a politically determined wage higher than the market-clearing wage.

Furthermore, the economic situation deteriorates over time. After the wage has been determined in a particular period tax rate adjust upwards. This causes workers in sector 2 to vote for further wage rises in the next period since on the basis of the new situation they perceive real wage gains for themselves and no tax rise. As a consequence, the political process will lead to perpetual incremental increases of minimum wages, unemployment, and taxes until the economy collapses. One of three situations may occur: First, individuals are not willing to accept high marginal tax rates and react by reducing labor supply or by moving into the shadow economy. Second, the tax burden approaches 100% and employed workers lapse into poverty due to the exploding welfare state. Third, at some time voters may recognize that their PEV view is incorrect and learn GEV.

Our main objective is to develop a coherent and well motivated political-economic model that simultaneously allows for awareness of direct effects and non-awareness of general equilibrium effects and which may explain how fiscal crises occur as we repeatedly observe even in industrial countries such as Spain or Greece. The general argument may be also applicable to unemployment in Europe. There is a large amount of literature on European unemployment that has stressed the interaction between shocks and labor market institutions (e.g. social protection, collective wage bargaining, minimum wages) as a potential cause of the unemployment problem. This literature also identifies large heterogeneities of unemployment performance and labor market institutions across European countries, which makes it impossible to single out one overarching cause for European unemployment (see e.g. Blanchard (2006)).

The non-awareness of general equilibrium effects offers a complementary explanation that may have contributed to the rise and persistence of high unemployment in some countries and that may have shown why such events can be reversed by a crisis. In some countries such as Sweden or the Netherlands, policy responses to a crisis have triggered a decline in unemployment, which we could interpret as a reversal of detrimental developments due to the emerging wisdom about economic relationships in crises.

Let us take the Netherlands as example. In the "Wassenaar Accord" in 1982, and in the face of high unemployment, the government, unions, and employers' organizations explicitly argued that a switch from an industrial perspective to an economy-wide approach, i.e., taking general equilibrium feedback effects into account, requires wage moderation and more labor market flexibility to stimulate job creation. A broad-based majority in parliament supported the corresponding policy measures and wage moderation took place in some sectors. This reversal of thinking and actions caused unemployment to fall below 5%, and this has been called the "Dutch unemployment miracle" (see Visser and Hemerijck (1997) and Nickell and van Ours (2000)).[1]

In this paper we compare three different awareness structures, one each for GEB, PEB and MB. Within each awareness structure, all agents have the same awareness, there is no uncertainty about the lack of awareness of the other agents, and one agent does not need to reason about lack of awareness of the other agent. It is nevertheless useful to relate our analysis to the unawareness models in the literature on how to model reasoning about unawareness consistently. Formalizing the concept of unawareness has turned out to be a difficult task. As shown in the seminal paper by Dekel, Lipman, and Rustichini (1998), the prevailing model representing uncertainty by a state space allows only for a trivial notion of unawareness: If an agent is unaware of anything then he is unaware of everything and thus knows nothing. A subsequent strand in the literature has developed important theories using multiple state spaces to model non-awareness (Li (2009)); Heifetz, Meier, and Schipper (2006); Galanis (2009), and using a mathematical logic perspective in Halpern and Rego (2006) and Board and Chung (2009)).

[1] See also Gersbach and Schniewind (2010) who consider the impact of awareness of general equilibrium effects on wage negotiations.

We relate our work on partial equilibrium voting to this literature in the following way. In the unawareness models by Heifetz, Meier, and Schipper (2006) and Li (2009), each agent has a subjective state space that is less detailed than the full state space and allows for multi-person unawareness. Ozbay (2008) applies these concepts of unawareness to incomplete contracts. Galanis (2009) showed how general unawareness structures by Heifetz, Meier, and Schipper (2006) can be used to model unawareness of theorems. The analogy of our model to the latter is as follows: Under partial equilibrium voting, voters assume that sector 2 is irrelevant for economic changes that occur in in sector 1 when the wage is changed. This means that when agents consider different wages in sector 1, they are oblivious of interdependencies with sector 2 when they evaluate a wage level. Agents may be aware of the existence of other economic sectors, but they do not recognize interdependencies with them.[2]

When the wage in sector 1 is changed, all changes in quantity, prices, and wages are attributed to changes in sector 1. Among other things, this means that the relationship between employment and wages in sector 1 is viewed as a function of wages and parameters. The parameters are objectively dependent on the outcomes in sector 2, but subjectively they are simply perceived by the agents as real numbers under partial equilibrium voting, as for them sector 2 appears irrelevant.

Many economists have suggested that departures from rationality may be important in macroeconomics (Akerlof (2002), Sargent (1993)). The notion that agents are unable to process all available information at once plays an important role in recent papers on the microfoundation of the Phillips curve (Woodford (2003), Ball (2000), and Mankiw and Reis (2002)). In Gersbach and Schniewind (2010) it is argued that non-awareness of general equilibrium effects by unions and employer associations may explain why unemployment is high in some European countries. These approaches view imperfect information acquisition as a device to capture the limited ability of agents to process information. We adopt a similar notion and assume that citizens may not be able to incorporate general equilibrium feedback effects when they cast their votes.

The paper is organized as follows: In section 2 we set up the model and derive the market equilibrium of the economy. In section 3 we motivate the political process and we specify GEV and PEV. In section 4 the utility functions depending on the minimum

[2]Sector 2 may be relevant for them in other economic activities. For instance, when they act as consumers, they may buy goods produced in sector 2.

wage of the low-skilled are derived for each view and for each group of workers. This yields the political equilibria in each time period and in the long-run. We compare the results from GEV with PEV and discuss how the political and economic system reacts to the emerging crisis under PEV. In section 5 we interpret the results. We shed some light on the robustness of our results in section 6, and set out our conclusions in section 7. The appendices contain proofs and supplementary material.

# 2 The Basic Economic Model

In this section we introduce the model of the economy on which we base our examination of the voting processes on minimum wages. There are two sectors producing good 1 and good 2 respectively. The only input into production is labor.[3] The production functions are given by:

$$q_1 = L_{1l}^{\beta} L_{1h}^{(1-\beta)} \tag{1}$$

with $\beta < 1$ and

$$q_2 = L_2 \tag{2}$$

Subscripts 1 and 2 denote the first and second sector, respectively. $h$ stands for the high-skilled workers of sector 1, $l$ for the low-skilled. In sector 2 we only have one skill level for the whole work-force.

We assume perfectly competitive goods markets and immobility of workers across industries, i.e., they can only work in one sector. Labor supply is assumed to be inelastic and is given by $\overline{L}_{1l} + \overline{L}_{1h}$ in sector 1 and $\overline{L}_2$ in sector 2. Firm owners are the high-skilled workers of sector 1 and the workers of sector 2. Each of them receives an equal share of the sum $\pi_1 + \pi_2$ of all the profits earned in both sectors.[4]

[3] In the long-run, there is no loss of generality associated with neglecting capital, provided that capacity constraints are not binding and the long-run capital stock is determined by equating the marginal product of capital with the real-world interest.

[4] This profit splitting assumption is inessential or the results. We note that we have constant returns to scale in both production technologies. Therefore we have zero profits as long as firms can satisfy their optimal labor demand.

Furthermore, we assume that all types of workers have the same symmetric Cobb Douglas utility function:[5]

$$u = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}} \tag{3}$$

where $c_1$ and $c_2$ denote the consumption levels of good 1 and good 2.

In the political process involving all workers as voters, the minimum nominal wage $w_{1l}$ for the low-skilled workers of sector 1 is set. In order that nominal wages have real effects, we need a further price rigidity and we assume that the price in sector 2 is constant.[6]

Thus, we can normalize $p_2$ to one:

$$p_2 = 1 \tag{4}$$

The appropriate consumer price index is:

$$p = p_1^{\frac{1}{2}} p_2^{\frac{1}{2}} = p_1^{\frac{1}{2}} \tag{5}$$

This price index guarantees that changes in prices do not affect household utility as long as real income remains constant.

As $p_2$ has been normalized to one, the real wage can exceed the market-clearing wage for the low-skilled workers.[7] As a result, unemployment can occur in this market. We assume that workers who have lost their jobs receive an exogenously given fraction $s \in (0, 1]$ of the minimum wage as unemployment benefits. In order to finance the benefits, labor is taxed by a fraction $\tau$ of the nominal wages they pay, i.e., $\tau$ is a payroll tax.

Finally, we assume that each of the three types of workers is a fraction of the population smaller than fifty percent:

[5]The symmetry assumption is made solely for ease of presentation. However, the assumption of constant and equal elasticities of substitution across all individuals is essential.

[6]Alternatively, we could assume that real minimum wages are set directly in the political sphere.

[7]Since $p_2 = 1$, $w_{1l}$ is the price of low-skilled labor in terms of good 2.

$$\frac{\overline{L}_f}{\overline{L}_{1l} + \overline{L}_{1h} + \overline{L}_2} < \frac{1}{2} \tag{6}$$

where $f = 1l, 1h, 2$.

## 2.1 Demand, Supply, and Government Budget

In the first step we derive demand and supply for goods and labor. By utility maximization for an individual worker we receive the following demand equations for consumption:

$$c_1^f = \frac{1}{2}\frac{b_f}{p_1} \tag{7}$$

$$c_2^f = \frac{1}{2} b_f \tag{8}$$

where $f = 1l, 1h, 2$ refers to the employed workers and $f = un$ refers to the unemployed. The budgets $b_f$ are $w_f + \frac{\pi_1 + \pi_2}{\overline{L}_{1h} + \overline{L}_2}$ for $f = 1h, 2$. For the employed low-skilled $b_{1l}$ equals $w_{1l}$ and for $f = un$ we have:

$$b_{un} = s w_{1l} \tag{9}$$

Profits of firms are sales minus costs and thus given as:

$$\pi_1 = p_1 q_1 - w_{1l}(1 + \tau) L_{1l} - w_{1h}(1 + \tau) L_{1h} \tag{10}$$

$$\pi_2 = q_2 - w_2(1 + \tau) L_2 \tag{11}$$

Firms are price-takers in both sectors. We obtain the first-order conditions for profit maximization in sector 1 and 2 as:

$$w_{1l}(1 + \tau) = p_1 \beta \Big(\frac{L_{1h}}{L_{1l}}\Big)^{(1-\beta)} \tag{12}$$

$$w_{1h}(1 + \tau) = p_1 (1 - \beta) \Big(\frac{L_{1l}}{L_{1h}}\Big)^{\beta} \tag{13}$$

$$w_2(1 + \tau) = 1 \tag{14}$$

Labor demand in sector 2 is perfectly elastic as long as gross wages do not exceed the value of 1.[8]

Both unregulated labor markets clear:

$$L_{1h} = \overline{L}_{1h} \tag{15}$$

$$L_2 = \overline{L}_2 \tag{16}$$

The governmental budget constraint is given by:

$$(w_{1l}L_{1l} + w_{1h}L_{1h} + w_2 L_2)\tau = \Delta b_{un} \tag{17}$$

where $\Delta$ denotes the unemployed work-force:

$$\Delta = \overline{L}_{1l} - L_{1l} \tag{18}$$

Using realized budgets we can apply Walras' law to the goods markets.[9] Therefore it suffices to clear one of the two goods markets:

$$L_{1l}c_2^{1l} + L_{1h}c_2^{1h} + L_2 c_2^2 + \Delta c_2^{un} = q_2 \tag{19}$$

## 2.2 The Market Equilibrium

We obtain a system of eight equations for the eight variables $\tau, w_{1h}, w_2, p_1, L_{1l}, L_{1h}, L_2, \Delta$. The system consists of the equations for labor demand ((12),(13), (14)), the governmental budget constraint ((17),(18)), and the market clearing conditions ((15),(16),(19)). Solving the system yields the following unique equilibrium solution $E(w_{1l})$:

[8] If gross wages do not exceed 1, profits are non-negative and independent of the employed labor force. If gross wages are higher than 1, profits are negative and the firm closes down.

[9] As workers adjust their demand for goods to their actual realized budgets, goods markets clear in spite of unemployment in one labor market.

$$
\begin{aligned}
\tau(w_{1l}) &= \frac{s(\beta\overline{L}_2 - w_{1l}\overline{L}_{1l})}{sw_{1l}\overline{L}_{1l} - 2\overline{L}_2} && (20)\\
w_{1h}(w_{1l}) &= \Big(\frac{1-\beta}{1+\tau}\Big)\frac{\overline{L}_2}{\overline{L}_{1h}} && (21)\\
w_2(w_{1l}) &= \frac{1}{1+\tau} && (22)\\
p_1(w_{1l}) &= \Big(\frac{\overline{L}_2}{\overline{L}_{1h}}\Big)^{1-\beta}\Big(\frac{w_{1l}(1+\tau)}{\beta}\Big)^{\beta} && (23)\\
L_{1l}(w_{1l}) &= \beta\overline{L}_2\frac{1}{w_{1l}(1+\tau)} && (24)\\
L_{1h}(w_{1l}) &= \overline{L}_{1h} && (25)\\
L_2(w_{1l}) &= \overline{L}_2 && (26)\\
\Delta(w_{1l}) &= \overline{L}_{1l} - \beta\overline{L}_2\frac{1}{w_{1l}(1+\tau)} && (27)
\end{aligned}
$$

We note that for a given $w_{1l}$, the associated tax rate $\tau$ and the equilibrium are unique. An important property is that $\tau$ strictly increases in $w_{1l}$.[10] In the absence of regulation, the low-skilled labor market in sector 1 also clears. Then we have $L_{1l} = \overline{L}_{1l}$ with $\tau = 0$ and from equation (24) we determine the lowest possible minimum wage as:

$$
w_{1l}^{min} = \beta\frac{\overline{L}_2}{\overline{L}_{1l}} \tag{28}
$$

For the maximum value of $w_{1l}$ we have:

$$
w_{1l}^{max} = \frac{2\overline{L}_2}{s\overline{L}_{1l}} \tag{29}
$$

For $w_{1l} > w_{1l}^{max}$ we can verify that $w_{1h}, w_2$ and $L_{1l}$ become negative and that $p_1$ becomes complex. Therefore they represent infeasible values. Furthermore, if $w_{1l}$ is smaller than $w_{1l}^{max}$ and $w_{1l} \to w_{1l}^{max}$, we obtain $\tau \to \infty$.

[10]The first derivative of $\tau$ with respect to $w_{1l}$ is $\frac{s\overline{L}_{1l}\overline{L}_2(2-s\beta)}{(sw_{1l}\overline{L}_{1l}-2\overline{L}_2)^2} > 0$.

# 3 Dynamics, Expectation Formation and the Political Process

In this section we motivate and model the political process. The two alternative views voters may follow are GEV and PEV. For that purpose we embed the static model from the last section in a discrete dynamic framework with time indexed by $t = 0, 1, 2 \ldots$. As we will see, under GEV, the outcome in the dynamic model is simply the repetition of the outcomes with PEV, however, the outcome in one period impacts on the outcome in the other period and thus dynamics is important.

GEV and PEV are connected with a certain kind of expectation formation concerning the economic effects of implementing a certain minimum wage. Workers vote accordingly. Under GEV voters hold completely rational expectations, while under PEV they follow a boundedly rational expectation formation scheme. In the following we explain these concepts in more detail.

## 3.1 Views

In each voting period, and based on the view they take, voters calculate their utility levels depending on the minimum wage $w_{1l,t}$. In their voting decision in a particular voting period $t$, they consider the level of $w_{1l,t}$, which maximizes their utility:

$$\operatorname*{argmax}_{w_{1l,t}} u(\tilde{E}_t^v(w_{1l,t}))$$

Here $\tilde{E}_t^v$ denotes the perceived short-run market equilibrium associated with a particular view, i.e., $v =$ GEV or $v =$ PEV. As discussed later in section 3.3, "Dynamics and Crisis", the political process generates the Median-voter's ideal wage as the short-run political equilibrium $\hat{w}_{1l,t}$. If this minimum wage is implemented, the economy reaches market equilibrium $E(\hat{w}_{1l,t})$.

### 3.1.1 Perceived Short-Run Political Equilibria under General Equilibrium Voting (GEV)

Under General Equilibrium Voting (GEV), voters consider all general equilibrium effects represented by equations (12)-(19). They form completely rational expectations. Therefore they correctly anticipate the market equilibrium $E(w_{1l,t})$. We denote the

Median-voter's ideal wage under GEV by $\hat{w}_{1l,t}^{GEV}$ and the equilibrium actually achieved under GEV by $E_t^{GEV} = E_t(\hat{w}_{1l,t}^{GEV})$. As the voters' perceived equilibrium $\tilde{E}_t^{GEV}$ equals the equilibrium $E_t^{GEV}$ actually achieved (rational expectation formation), the optimal wage before voting is still optimal after the new equilibrium has been achieved and voters have no reason to change their ideal wages after casting their votes the first time. Thus, under GEV, we have $\hat{w}_{1l,t}^{GEV} = ... = \hat{w}_{1l,1}^{GEV} = \hat{w}_{1l,0}^{GEV}$ as short-run political equilibria, as well as $E_t^{GEV} = ... = E_1^{GEV} = E_0^{GEV}$ as short-run market equilibria.

### 3.1.2 Perceived Short-Run Political Equilibria under Partial Equilibrium Voting (PEV)

Under Partial Equilibrium Voting (PEV), not all effects are taken into account by the voters. We assume that voters only consider changes in the regulated sector. They proceed on the assumption that the variables in sector 2 and the tax rate $\tau$ do not change, i.e. $w_2$, $L_2$, and $\tau$ are assumed to stay constant. Therefore under PEV voters anticipate that changing wages in sector 1 will affect prices and output in this sector, while they do not take into account general equilibrium repercussions from the economy on tax rate adjustments by the government. Thus PEV represents the plausible assumption that agents only consider the direct effects of regulatory changes when they cast their votes.

Moreover, we assume that non-awareness of general equilibrium effects persists. We motivate both levels of bounded rationality by empirical and experimental research.

*Non-Awareness of General Equilibrium Effects*

With reference to non-awareness of general equilibrium effects, there are a number of recent studies that support this assumption. Romer (2003) develops an important theory of misconceptions when voters individually obtain misleading but correlated signals about the outcome of a certain policy. New interesting studies have found that ideology plays an important role in the formation of beliefs about economic policies (see Caplan (2002) and Blinder and Krueger (2004)). The role of voter misconceptions regarding to tax policy is explored in Krupnikov, Levine, Lupia, and Prior (2006), Birney, Graetz, and Shapior (2006) and Slemrod (2003). Moreover, the non-awareness of general equilibrium effects may be the result of the fact that people tend to simplify decision problems - an observation which is experimentally well established (see e.g.

Rubinstein (1998)).

*Persistence of the Expectation Formation Scheme*

The persistence of voter misconceptions is well documented in Caplan (2002) and Blinder and Krueger (2004). We discuss some behavioral justifications for the persistence of non-awareness of general equilibrium effects. There is strong evidence suggesting that once people have formed an opinion they will maintain it for as long as possible. Barberis and Thaler (2002) identify two behavioral effects supporting this. "Belief perseverance" induces agents to refrain from searching for new evidence and adhering to an established opinion, even if they observe evidence to the contrary.

An even stronger behavioral phenomenon is "confirmation bias". People with a confirmation bias not only ignore contrary evidence, they even interpret that evidence as supporting their original hypothesis. This is in accordance with Kahneman, Slovic, and Tversky (1982), who observe that agents are conservative in updating their beliefs, or with Brenner (1996), who observes that people are sluggish and only change their behavior when feedback is extremely negative. In our context, this sluggishness may be supported by the fact that people do not know whether erroneous expectations are due to their own misconceptions or due to exogenous effects on the economy. For example, when unemployment is higher than expected, agents may presume that this is due to poor economic performance in other countries, leading to a fall in exports. They may not consider the fact that they have neglected general equilibrium effects.

One psychological explanation for conservatism may be "cognitive dissonance" in voting behavior. Mullainathan and Washington (2005) find empirical evidence that actual voting decisions may influence preferences and hence future voting decisions. The reason is that people feel a need for consistency, i.e., they want their behavior to be in line with their beliefs. Once people have cast their votes, they want to believe that their decision was correct, so they stick with their past decisions because otherwise they would feel uneasy.

*Formalization of PEV*

In period $t$ under PEV voters apply equations (12), (13), (15) and (18) which describe the behavior of agents in sector 1:

$$\begin{aligned}
w_{1l,t}(1+\tau_t) &= p_{1,t}\beta\Big(\frac{L_{1h,t}}{L_{1l,t}}\Big)^{(1-\beta)} \\
w_{1h,t}(1+\tau_t) &= p_{1,t}(1-\beta)\Big(\frac{L_{1l,t}}{L_{1h,t}}\Big)^{\beta} \\
L_{1h,t} &= \overline{L}_{1h} \\
\Delta_t &= \overline{L}_{1l} - L_{1l,t}
\end{aligned}$$

From the voters' point of view sector 2 is not affected at all. Therefore, they assume clearance of the market for good 2 (19):

$$L_{1l,t}c^{1l}_{2,t} + L_{1h,t}c^{1h}_{2,t} + L_{2,t}c^{2}_{2,t} + \Delta_t c^{un}_{2,t} = q_{2,t}$$

Voters base their considerations in period $t$ on the realization of the variables in sector 2 and on the tax rate in $t-1$ that are presumed to stay constant.

We use $\hat{w}^{PEV}_{1l,t}$ to denote the Condorcet winner under PEV in period $t$, which now depends on $E_{t-1}$, i.e. $\hat{w}^{PEV}_{1l,t}(E^{PEV}_{t-1})$, where $E^{PEV}_{t-1}$ is the equilibrium realized under PEV in period $t-1$. Since voters only partially anticipate the resulting equilibrium under PEV, we use $\tilde{E}^{PEV}_t(w_{1l,t})$ to denote the equilibrium perceived by voters when they determine $\hat{w}^{PEV}_{1l,t}$. To derive $\tilde{E}^{PEV}_t(w_{1l,t})$ we solve the system of 5 equations ((12),(13),(15),(18),(19)) for the perceived equilibrium values denoted by $\tilde{w}_{1h,t}, \tilde{p}_{1,t}, \tilde{L}_{1l,t}, \tilde{L}_{1h,t}$ and $\tilde{\Delta}_t$:

$$
\begin{aligned}
\tilde{\tau}_t^{PEV}(w_{1l,t}) &= \tau_{t-1}^{PEV} && (30)\\
\tilde{w}_{1h,t}^{PEV}(w_{1l,t}) &= (1-\beta)\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}} && (31)\\
\tilde{w}_{2,t}^{PEV}(w_{1l,t}) &= \frac{1}{1+\tau_{t-1}^{PEV}} && (32)\\
\tilde{p}_{1,t}^{PEV}(w_{1l,t}) &= (1+\tau_{t-1}^{PEV})\Big(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\Big)^{1-\beta}\Big(\frac{w_{1l,t}}{\beta}\Big)^{\beta} && (33)\\
\tilde{L}_{1l,t}^{PEV}(w_{1l,t}) &= \beta\frac{\epsilon_t(w_{1l,t})}{w_{1l,t}} && (34)\\
\tilde{L}_{1h,t}^{PEV}(w_{1l,t}) &= \overline{L}_{1h} && (35)\\
\tilde{L}_{2,t}^{PEV}(w_{1l,t}) &= \overline{L}_{2} && (36)\\
\tilde{\Delta}_t^{PEV}(w_{1l,t}) &= \overline{L}_{1l} - \beta\frac{\epsilon_t(w_{1l,t})}{w_{1l,t}} && (37)
\end{aligned}
$$

where

$$
\epsilon_t(w_{1l,t}) = \frac{\overline{L}_2 + \tau_{t-1}^{PEV} w_{2,t-1}^{PEV}\overline{L}_2 - s w_{1l,t}\overline{L}_{1l}}{1-s\beta} \tag{38}
$$

and $\tau_{t-1}^{PEV}$ and $w_{2,t-1}^{PEV}$ are the actual realized values of $\tau$ and $w_2$ under PEV in period $t-1$.

Note that $\epsilon_t(w_{1l,t})$ strictly decreases in $w_{1l,t}$ and that for the solution to be meaningful $\epsilon_t(w_{1l,t})$ has to be non-negative. Therefore, under PEV the perceived maximum wage for the low-skilled in sector 1 is:

$$
\tilde{w}_{1l,t}^{PEV,max} = \frac{\overline{L}_2 + \tau_{t-1}^{PEV} w_{2,t-1}^{PEV}\overline{L}_2}{s\overline{L}_{1l}} \tag{39}
$$

If $w_{1l,t} = \tilde{w}_{1l,t}^{PEV,max}$, then voters perceive that all low-skilled workers of sector 1 are unemployed, so output in this sector is zero.

As can be seen from equations (30) to (38) the perceived equilibrium $\tilde{E}_t^{PEV}(w_{1l,t})$ in period $t$ depends on the tax rate $\tau_{t-1}^{PEV}$ actually realized in the previous period. Consequently, the optimal minimum wage each voter group prefers to be implemented depends on the political equilibrium $\hat{w}_{1l,t-1}^{PEV}$ in the previous period.

## 3.2 Main Question

The main question we want to analyze is whether repeated PEV will converge to the equilibrium under GEV. As will be discussed in detail in section 4, if agents had completely rational expectations the high-skilled workers in sector 1 and the workers in sector 2 would always vote for the market-clearing wage as the minimum wage for the low-skilled workers in sector 1. As two worker groups always form a majority of voters, we can identify the free-market solution as a rational expectation equilibrium in the political process. But as we will see, the process involving PEV in each period does not lead to the free-market solution as two groups of workers vote for higher wages.

As a result, a crisis will occur in the long-run, since unemployment among the low-skilled workers will rise dramatically and the real wages of the high-skilled workers and workers in sector 2 will decline significantly.

## 3.3 Dynamics and Crisis

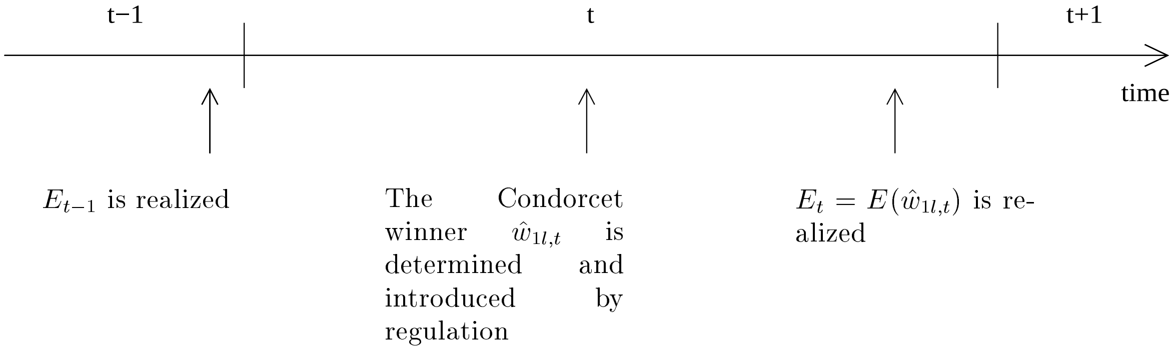
In this section we provide a detailed introduction to the political process itself. For this purpose we develop a dynamic framework. There are an infinite number of time periods, indexed by $t = 0, 1, ....$ In each period the static economy from the last section is at work and we use $E(w_{1l,t})$ or $E_t$ to denote the equilibrium realized in period $t$ after $w_{1l,t}$ has been determined. Within this framework the political process unfolds as follows: In each period each agent acts as a voter. Voters determine the minimum wage $w_{1l,t}$ through majority rule. Although we work directly with the Condorcet winner[11], we have the standard model of two-party competition in mind which generates the Median-voter result.[12] In every period, the preferred wage by the Median-voter, denoted by $\hat{w}_{1l,t}$ is introduced in the economy. We use $\hat{w}_{1l,t}$ to refer to the short-run political equilibrium. Since we have three different types of workers, we will in general also have three different ideal wage levels. The political and economic process is summarized in Figure 1.

The long-run behavior of the equilibrium can exhibit two patterns. First, if at some point in time a wage $\hat{w}_{1l,t}$ is larger than $\hat{w}_{1l,t} > w_{1l}^{max}$, the situation is no longer

[11] This is the minimum wage that defeats all other values of $w_{1l,t}$ in pairwise majority voting

[12] As we will see in the next section, the Median-voter corresponds to the Condorcet winner despite the fact that not all preferences are single-peaked.

Figure 1: The political and economic process




economically feasible, and the economy collapses. This means that output in sector 1 is zero and the tax rate infinitely large. This is bound to lead to a political crisis where voters as consumers and tax payers are no longer willing to accept the economic situation. Therefore they would wish to return to former values of the minimum wage, or they may recognize that their view has been mistaken (see section 4.4).

Second, no economic collapse occurs, i.e., $\hat{w}_{1l,t} \leq w_{1l}^{max}$ in all periods. If $\lim_{t\to\infty} \hat{w}_{1l,t}$ and $\lim_{t\to\infty} E(\hat{w}_{1l,t})$ exist, we denote them by $\hat{w}_{1l}^*$ and $E^*$ respectively and use $\hat{w}_{1l}^*$ to refer to the long-run political equilibrium of the process.[13]

On its path, the political process may generate a crisis or a reversal. There are three scenarios how crises can happen: In the first scenario, the sequence of $\hat{w}_{1l,t}$ converges to or reaches $w_{1l}^{max}$. Then, $\tau$ becomes infinitely large and we observe a political and economic crisis. This is because the real wages of the high-skilled of sector 1 and the workers of sector 2 are zero, as is output in sector 1. Furthermore, all low-skilled workers have lost their jobs. We call this a crisis with unlimited tax tolerance (CUTT), because voters then accept any tax rate imposed by the government.

In the second scenario, the latter is not the case and a crisis with limited tax tolerance (CLTT) occurs. In period $T$, the equilibrium tax rate exceeds a value $\tau_{max} < \infty$ that tax payers would accept.[14] The existence of an upper limit $\tau_{max}$ can be justified in several ways. For instance, tax payers may reduce labor supply or try to avoid taxes

[13] If $\hat{w}_{1l}^*$ is reached in finite time, the wages and the equilibrium of the economy remain constant thereafter.

[14] For example, if $\tau > 1$ more than fifty percent of the gross wage would be taxed as described in equation (22).

by moving into the shadow economy. Strictly speaking, to rationalize the reduction of labor supply one has to assume that workers receive utility from consuming leisure time. This can be integrated in our model in a simple way by assuming that the elasticity of labor supply is zero for $\tau \leq \tau_{max}$ and large for $\tau > \tau_{max}$. As a consequence, the state's budget constraint cannot be satisfied with a tax rate exceeding $\tau_{max}$ and a crisis emerges even before the equilibrium tax rate $\tau$ approaches infinity. While we do not explicitly model the reaction of individuals where $\tau > \tau_{max}$, it is obvious that the budget constraints will be violated if the amount of taxable labor income declines sufficiently.

Third, it could happen that voters, after experiencing a discrepancy between expected and realized utility levels for a certain time, recognize that the PEV view is incorrect and switch to GEV. Since that third scenario is qualitatively similar to the second scenario, we shall focus on the first two cases.

To summarize our concept of a crisis we start from the following condition:

*Suppose the sequence of short-run political equilibria $\hat{w}_{1l,t}$ converges to a long-run equilibrium $\hat{w}^*_{1l}$. Suppose further that all short-run equilibria are economically feasible, i.e., $\hat{w}_{1l,t} \leq w^{max}_{1l}$, where $w^{max}_{1l}$ denotes the maximal feasible wage level. Beyond this maximum wage level the economy collapses with output zero in sector 1.*

Then we define

**Definition 1 (Crisis with limited tax tolerance (CLTT))**
*In a period $T$ the short-run political equilibrium in this period exceeds a level $\tau_{max} < \infty$. Tax payers are not willing to accept a tax rate higher than $\tau_{max}$. Workers as tax payers will reduce labor supply or move into the shadow economy. The state's budget constraint cannot be satisfied any longer.*

**Definition 2 (Crisis with unlimited tax tolerance (CUTT))**
*The sequence $\hat{w}_{1l,t}$ of short-run political equilibria converges to $w^{max}_{1l}$. Voters accept any tax rate imposed by the government. The crisis realized in the long-run equilibrium $w^{max}_{1l}$ is characterized by the fact that all low-skilled workers in sector 1 have lost their jobs, so output is zero in sector 1.*

# 4 Long-Run Political Equilibria

On the basis of this conceptual framework we can now derive the political equilibria under GEV and PEV. For this, we need to identify the utility functions of voter groups, their optimal minimum wages and the Condorcet winners.

## 4.1 Long-Run Political Equilibria under GEV

Using a positive monotone transformation $U = 2\ln u$ of utility function $u$ (see equation (3)), we obtain for the workers of sector 2 in period $t$:[15]

$$\tilde{U}_{2,t}^{GEV} = \ln\Big(\frac{1}{2}\frac{\tilde{w}_{2,t}^{GEV}}{\tilde{p}_{1,t}^{GEV}}\Big) + \ln\Big(\frac{1}{2}\tilde{w}_{2,t}^{GEV}\Big) \tag{40}$$

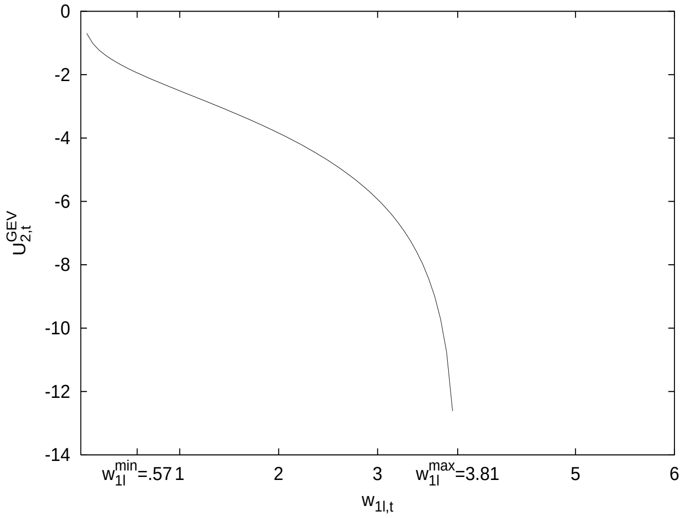
Given $\tilde{E}_t^{GEV} = E_t^{GEV} = E_t$, the perceived variables equal the actual realized variables and therefore, from now on, we dispense with the tilde for variables under GEV.

Using equations (22) and (23) and the fact that $\tau_t^{GEV}$ strictly increases in $w_{1l,t}$ we find that $w_{2,t}^{GEV}$ strictly decreases and $p_{1,t}^{GEV}$ strictly increases in $w_{1l,t} \in (0, w_{1l}^{max})$. Thus $U_{2,t}^{GEV}$ strictly decreases in $w_{1l,t} \in (0, w_{1l}^{max})$ and voters of sector 2 will prefer the lowest possible wage $w_{1l}^{min}$ for the low- skilled of sector 1.

To illustrate this fact, we plot in Figure 2 the utility functions of workers of sector 2 with the following parameter values for the economy: $s = 0.75, \beta = 0.4, \overline{L}_{1l} = 70,000, \overline{L}_{1h} = 50,000$ and $\overline{L}_2 = 100,000$. For these values we obtain $w_{1l}^{min} = 0.57$ and $w_{1l}^{max} = 3.81$. Furthermore, unless otherwise indicated, we use these parameter values for the illustrations of all other functions in this paper.

[15]Since production technologies exhibit constant returns to scale profits are zero and workers' budgets only consist of wages.

Figure 2: $U_{2,t}^{GEV}$ with $s = 0.75$ and $\beta = 0.4$




For the high-skilled of sector 1 we obtain:

$$U_{1h,t}^{GEV} = \ln\Big(\frac{1}{2}\frac{w_{1h,t}^{GEV}}{p_{1,t}^{GEV}}\Big) + \ln\Big(\frac{1}{2}w_{1h,t}^{GEV}\Big) \tag{41}$$

Because of equations (21) and (23) and the fact that $\tau_t^{GEV}$ strictly increases in $w_{1l,t}$, $w_{1h,t}^{GEV}$ strictly decreases and $p_{1,t}^{GEV}$ strictly increases in $w_{1l,t} \in (0, w_{1l}^{max})$. Thus $U_{1h,t}^{GEV}$ strictly decreases in $w_{1l,t} \in (0, w_{1l}^{max})$ and the high-skilled workers of sector 1 will also prefer $w_{1l}^{min}$.

We can summarize our observations in the following lemma:

**Lemma 1**

*$U_{2,t}^{GEV}(w_{1l,t})$ and $U_{1h,t}^{GEV}(w_{1l,t})$ have the following properties in $w_{1l,t} \in (0, w_{1l}^{max})$:*

*(i) $U_{2,t}^{GEV}(w_{1l,t})$ and $U_{1h,t}^{GEV}(w_{1l,t})$ strictly decrease in $w_{1l,t}$.*

*(ii) The workers of sector 2 and the high-skilled workers of sector 1 maximize their utilities $U_{2,t}^{GEV}(w_{1l,t})$ and $U_{1h,t}^{GEV}(w_{1l,t})$ if they choose the lowest possible wage $w_{1l}^{min}$.*

As two groups of workers always have a single majority of voters, the short-run political equilibrium under GEV in each period is given by:

$$\hat{w}_{1l,t}^{GEV} = w_{1l}^{min} = \beta\frac{\overline{L}_2}{\overline{L}_{1l}} \tag{42}$$

Furthermore, at $w_{1l}^{min}$ all values are economically feasible and $\tau = 0$. Thus, we can conclude:

**Proposition 1 (The Long-Run Political Equilibrium under GEV)**
*Under GEV, neither CLTT nor CUTT occurs and the long-run political equilibrium of the voting process equals the short-run equilibria in each period. It is given by:*

$$\hat{w}_{1l}^{GEV*} = \hat{w}_{1l,t}^{GEV} = \beta\frac{\overline{L}_2}{\overline{L}_{1l}}$$

*There is no unemployment and the equilibrium is equal to the unregulated economy.*

For completeness, in Appendix B we also examine the utility of low-skilled workers in sector 1 as a function of $w_{1l,t}$.

## 4.2 Long-Run Political Equilibria under PEV

In this subsection, we derive the technical results under PEV. In section 5 we provide intuitive explanations of the results.

Before we look at the utility functions themselves, it is useful to analyze $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ in its meaningful range, i.e. for $w_{1l,t} \in [0, \tilde{w}_{1l,t}^{PEV,max}]$:

$$\tilde{p}_{1,t}^{PEV} = (1+\tau_{t-1}^{PEV})\Big(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\Big)^{1-\beta}\Big(\frac{w_{1l,t}}{\beta}\Big)^{\beta}$$

The first derivative of $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ with respect to $w_{1l,t}$ is:

$$\frac{\partial\tilde{p}_{1,t}^{PEV}}{\partial w_{1l,t}} = \tilde{p}_{1,t}^{PEV}\Big((1-\beta)\frac{-s\overline{L}_{1l}}{\overline{L}_2 + \tau_{t-1}^{PEV}w_{2,t-1}^{PEV}\overline{L}_2 - sw_{1l,t}\overline{L}_{1l}} + \frac{\beta}{w_{1l,t}}\Big) \tag{43}$$

and for $w_{1l,t} \in [0, \tilde{w}_{1l,t}^{PEV,max}]$ we find one value of $w_{1l,t}$ that satisfies $\partial\tilde{p}_{1,t}^{PEV}/\partial w_{1l,t} = 0$ as expressed in the next lemma.

**Lemma 2**
*There exists a unique value $\tilde{w}_{1l,t}^{p_1}$ that maximizes $\tilde{p}_{1,t}^{PEV}$ for $w_{1l,t} \in [0, \tilde{w}_{1l,t}^{PEV,max}]$:*

$$\tilde{w}_{1l,t}^{p_1} = \beta \tilde{w}_{1l,t}^{PEV,max} = \beta \frac{\overline{L}_2 + \tau_{t-1}^{PEV} w_{2,t-1}^{PEV} \overline{L}_2}{s \overline{L}_{1l}} \tag{44}$$

The proof of Lemma 2 can be found in Appendix A.

Figure 3: The typical shape of $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$

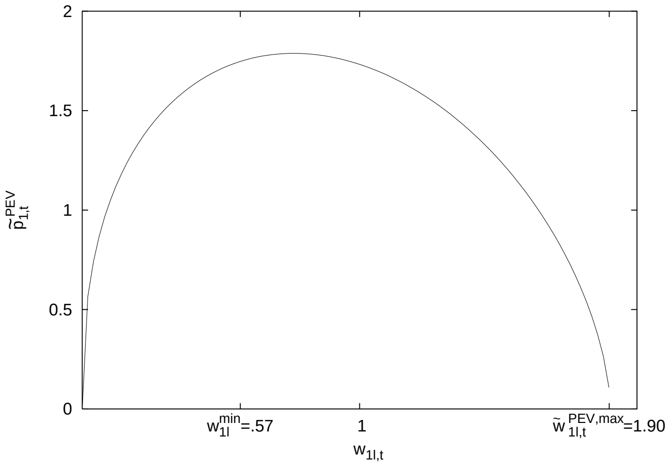



Figure 3 shows $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ for the case where $\tau_{t-1}^{PEV} = 0$ and thus $w_{2,t-1}^{PEV} = 1$ for the parameter values specified in subsection 4.1.[16] Then we have $\tilde{w}_{1l,t}^{PEV,max} = 1.90$ and $\tilde{w}_{1l,t}^{p_1} = 0.76$.

The utility of workers in sector 2 is:[17]

$$\tilde{U}_{2,t}^{PEV}(w_{1l,t}) = \ln\Big(\frac{1}{2}\frac{\tilde{w}_{2,t}^{PEV}}{\tilde{p}_{1,t}^{PEV}}\Big) + \ln\Big(\frac{1}{2}\tilde{w}_{2,t}^{PEV}\Big)$$

As under PEV people consider the wage of workers in sector 2 to be fixed, the characteristics of $\tilde{U}_{2,t}^{PEV}(w_{1l,t})$ depend on $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$.

**Lemma 3**

*$\tilde{U}_{2,t}^{PEV}(w_{1l,t})$ has the following properties:*

*(i)* $\lim_{w_{1l,t}\to 0} \tilde{U}_{2,t}^{PEV}(w_{1l,t}) = \infty$ *and* $\lim_{w_{1l,t}\to \tilde{w}_{1l,t}^{PEV,max}} \tilde{U}_{2,t}^{PEV}(w_{1l,t}) = \infty$.

[16] This is the case when there was no regulation in $t-1$.

[17] Also under PEV, profits of firms are zero since firms are assumed to be price takers and do not need to worry about equilibrium feedback effects.

*(ii) The local maximizer $\tilde{w}_{1l,t}^{p_1}$ for $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ is a local minimizer of $\tilde{U}_{2,t}^{PEV}(w_{1l,t})$ in $(0, \tilde{w}_{1l,t}^{PEV,max})$.*

*(iii) Workers in sector 2 maximize their utility $\tilde{U}_{2,t}^{PEV}(w_{1l,t})$ if they choose the largest possible wage $\tilde{w}_{1l,t}^{PEV,max}$.*

The last point follows from the fact that $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ is a continuous function in $[w_{1l}^{min}, \tilde{w}_{1l,t}^{PEV,max})$.

Lemma 3 is the key difference between GEV and PEV as workers in sector 2 desire higher wages under PEV than the market clearing wage. In Appendix C we examine the utility of high- and low-skilled workers in sector 1. High-skilled workers again maximize their utility at the lowest possible wage $w_{1l}^{min}$ while the maximal possible wage $\tilde{w}_{1l,t}^{PEV,max}$ maximizes the utility of the low-skilled workers.

Now we can determine the equilibria under PEV. In each round of voting workers in sector 2 and the low-skilled workers of sector 1 choose $\tilde{w}_{1l,t}^{PEV,max}$. Thus the short-run equilibrium in period $t$ is $\hat{w}_{1l,t}^{PEV} = \tilde{w}_{1l,t}^{PEV,max}$. It depends on the tax rate that actually satisfies the state's budget constraint of the previous voting period. To derive the long-run equilibrium we need a starting point for the economy characterized by $E(w_{1l,r})$ with the starting wage $w_{1l,r} \in [w_{1l}^{min}, w_{1l}^{max}]$ and the corresponding tax rate $\tau_r$. We obtain the following proposition (for proof see Appendix A):

**Proposition 2 (The Evolution of the Economy under PEV)**
*Under PEV, the economy evolves according to:*

$$\hat{w}_{1l,t}^{PEV} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)^t(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}} \tag{45}$$

$$w_{2,t}^{PEV} = \frac{1}{(2-s\beta)^{t+1}(1+\tau_r)} \tag{46}$$

$$\tau_t^{PEV} = (2-s\beta)^{t+1}(1+\tau_r) - 1, \tag{47}$$

*where $\tau_r < \infty$ is the tax rate that actually satisfies the state's budget constraint before period zero starts.*

We next determine whether a crisis will occur in the long-run under PEV.

For $w_{1l,r} \in [w_{1l}^{min}, w_{1l}^{max}]$, $\hat{w}_{1l,t}^{PEV}$ converges to $w_{1l}^{max} = \frac{2\overline{L}_2}{s\overline{L}_{1l}}$ as $t$ goes to infinity. As $\hat{w}_{1l,t}^{PEV}$ never exceeds the largest possible value $w_{1l}^{max}$, the variables $w_{1h,t}^{PEV}$, $w_{2,t}^{PEV}$, $L_{1l,t}^{PEV}$

and $p_{1,t}^{PEV}$ are always economically feasible, no economic collapse occurs, and we can determine an equilibrium $E^{PEV*}$. Nevertheless, we observe CUTT as $\lim_{t\to\infty} \hat{w}_{1l,t}^{PEV} = w_{1l}^{max}$.

Thus - starting with $w_{1l,r}$ - as $t$ increases, $\tau_t^{PEV}$ will become larger than the upper limit $\tau_{max}$. Therefore, with a finite upper limit on taxes $\tau_{max}$, CLTT will occur if:

$$(2 - s\beta)^{t+1}(1 + \tau_r) - 1 > \tau_{max}$$

or if:

$$t > \frac{\ln\left(\frac{1+\tau_{max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} - 1$$

Thus, the first voting period $T$ where $\hat{w}_{1l,t}^{PEV}$ "produces" an infeasible tax rate is:

$$T = \left\lfloor \frac{\ln\left(\frac{1+\tau_{max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} \right\rfloor \tag{48}$$

where $\lfloor\ \rfloor$ denotes the largest possible integer that is smaller than the expression under consideration.

We can summarize our results under the PEV view by the following proposition:

**Proposition 3 (The Long-Run Political Equilibrium under PEV)**

*(i) Under PEV and if CUTT holds, the long-run equilibrium for $w_{1l,r} \in [w_{1l}^{min}, w_{1l}^{max}]$ is given by*

$$\hat{w}_{1l}^{PEV*} = \lim_{t\to\infty} \hat{w}_{1l,t}^{PEV} = w_{1l}^{max}$$

*and all low-skilled workers lose their jobs:*

$$\Delta^{PEV*} = \lim_{t\to\infty} \Delta_t^{PEV} = \overline{L}_{1l}$$

*(ii) If the tax rate is not allowed to exceed $\tau_{max}$, CLTT occurs and the Condorcet winner of period $T$ in which the crisis emerges is*

$$\hat{w}_{1l,T}^{PEV} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)^T(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}}$$

*where*

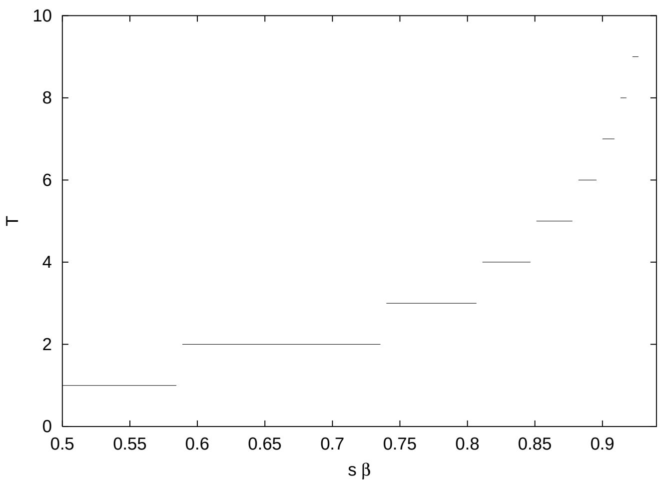
$$T = \left\lfloor \frac{\ln\left(\frac{1+\tau_{max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} \right\rfloor$$

*and the number of unemployed workers is:*

$$\Delta_T^{PEV} = \overline{L}_{1l} \frac{2(2-s\beta)^2 - 2\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)}}{2(2-s\beta)^2 - 2\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)} + s\beta\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)}},$$

*where $\tau_r < \infty$ is the tax rate that actually satisfies the state's budget constraint before period zero starts.*

Figure 4: The collapse period $T$ for $\tau_r = 0$ and $\tau_{max} = 1$




In Figure 4, $T$ is plotted as a function of $s\beta$ (see equation (48)) in a range of $s\beta = [0.50, 0.94]$ for the set of parameter values specified in subsection 4.1. We assume $\tau_{max} = 1$ and the market-clearing wage as starting wage, which implies $\tau_r = 0$. For $s\beta \leq 0.58$, $T$ equals 1, i.e. the implementation of the Condorcet winner in period 1 would require a tax rate that exceeds $\tau_{max}$. As $s\beta$ increases, $T$ also increases. The intervals for $s\beta$ in which $T$ stays constant become smaller. Eventually, $T$ goes to infinity as $s\beta$ approaches 1.

## 4.3 Comparing the Long-Run Political Equilibria under GEV and PEV

Proposition 4 summarizes our results and shows that in democracies where voters only take direct effects of regulations into account, strong negative effects from regulations will be experienced and eventually a crisis will occur.

**Proposition 4**

*The Condorcet winner wages satisfy:*

$$w_{1l}^{min} = \hat{w}_{1l}^{GEV*} < \hat{w}_{1l,T}^{PEV} < \hat{w}_{1l}^{PEV*},$$

*where $\hat{w}_{1l}^{GEV*}$ denotes the long-run political equilibrium under GEV, $\hat{w}_{1l,T}^{PEV}$ the long-run equilibrium under PEV with limited tax tolerance (CLTT), and $\hat{w}_{1l}^{PEV*}$ the long-run equilibrium under PEV with unlimited tax tolerance (CUTT). Accordingly, unemployment rates satisfy:*

$$0 = \Delta^{GEV*} < \Delta_T^{PEV} < \Delta^{PEV*},$$

*i.e., there is no unemployment under GEV whereas PEV produces unemployment both under CLTT and CUTT.*

## 4.4 Reaction to the Crisis

Under PEV, we assume first that voters do not learn that their view of the economy is incorrect although there is a discrepancy between their expected utility levels and those actually achieved. Nevertheless, at some point in time society enters a crisis because voters as tax payers will recognize that there are large negative general equilibrium effects: either $\tau_t$ approaches infinity or crosses $\tau_{max}$. As the gap between gross wages and net wages becomes too large and real wages become too small people will not be willing to accept this.

There are two conceivable reaction patterns to the crisis:

1. People perform ad hoc measures and - for the moment - give up their assumption of an unchanging tax rate and vote for previous values of $\hat{w}_{1l,t}$ or complementary policy actions (e.g. a reduction of $s$). They would expect a lower tax rate connected with these measures. But afterwards they return to their former beliefs or other mistaken views about the functioning of the economy. As a consequence, they could find themselves faced with the same crisis.

2. People learn that the principles of their former views are incorrect. They recognize the discrepancy between their beliefs and the actual realized values of the economy's variables. They adopt a new mental framework for thinking about the functioning of the economy and reverse their PEV view in favor of the GEV

view. In particular, sector 2 workers may switch to GEV as they become aware of their tax burden and real-wage decline. If this happens, parties offering market-clearing wages and a reduction in taxes will win and the wage in the unregulated economy will emerge as Condorcet winner.

# 5 Interpretation of the Results

In order to interpret our results it will be useful to discuss in detail the GEV view first. Then it will become transparent how PEV differs to GEV.

## 5.1 General Equilibrium Voting (GEV)

Under GEV, voters have equations (7) to (19) in mind when they contemplate about the consequences of the minimum wage's value $w_{1l,t}$ for their utility levels. To achieve an economic understanding of the effects of a changing minimum wage $w_{1l,t}$ on the variables of the model, they start with some $w_{1l,t}$ and consider what happens if $w_{1l,t}$ increases by a certain amount. From this they obtain $\tau_t^{GEV}$ and $p_{1,t}^{GEV}$, such that the market-clearing condition (19) and the governmental budget constraint (17) are fulfilled simultaneously:

$$L_{1l,t}^{GEV}\frac{b_{1l,t}^{GEV}}{2} + L_{1h,t}^{GEV}\frac{b_{1h,t}^{GEV}}{2} + L_{2,t}^{GEV}\frac{b_{2,t}^{GEV}}{2} + \Delta_t^{GEV}\frac{b_{un,t}^{GEV}}{2} = q_{2,t}^{GEV} \tag{49}$$

$$(w_{1l,t}L_{1l,t}^{GEV} + w_{1h,t}^{GEV}L_{1h,t}^{GEV} + w_{2,t}^{GEV}L_{2,t}^{GEV})\tau_t^{GEV} = \Delta_t^{GEV}b_{un,t}^{GEV} \tag{50}$$

where

$$b_{1l,t}^{GEV} = w_{1l,t}, b_{1h,t}^{GEV} = w_{1h,t}^{GEV}, b_{2,t}^{GEV} = w_{2,t}^{GEV} \quad \text{and} \quad b_{un,t}^{GEV} = sw_{1l,t}$$

In Appendix D, we explain in detail why workers of sector 2 and the high-skilled workers of sector 1 prefer the market clearing wage while low-skilled workers tend to prefer a higher wage. The main argument is as follows: An increasing minimum wage has two effects: a negative effect on total wealth and a redistributive effect in favor of the low-skilled. First, minimum wages increase unemployment, lower total output and therefore reduce the total wealth of society. This is represented by an increasing price for good 1 such that real wages become smaller and smaller not only for the high-skilled of sector

1 and workers of sector 2 but also - at least when $w_{1l,t}$ is big enough - for the low-skilled of sector 1. Second, setting a higher minimum wage increasingly redistributes the remaining wealth in favor of the low-skilled workers. This is represented by an increasing tax rate. In the extreme case where all wealth is allocated to the low-skilled workers, the tax rate must be infinitely large to ensure that all other groups channel all their gross earnings to the low-skilled via the state's tax regime.

The exact analytic result of voters' reasoning processes is given by equations (20) to (27). Clearly, workers of sector 2 and the high-skilled workers of sector 1 prefer the lowest possible minimum wage because an increase in $w_{1l,t}$ lowers their net wages and increases the price of good 1. The low-skilled have to consider a trade-off between a higher $p_{1,t}^{GEV}$ and increasing unemployment on the one hand, and higher net wages and unemployment benefits on the other. Therefore for some values of $s$ and $\beta$ they will prefer a minimum wage that exceeds $w_{1l}^{min}$.

## 5.2 Partial Equilibrium Voting (PEV)

Under PEV, the same reasoning process by agents occurs, but with two important differences. Both the nominal wage in sector 2 $\tilde{w}_{2,t}^{PEV}$ and the tax rate $\tilde{\tau}_t^{PEV}$ are assumed to stay constant.

Voters look at the second goods market and perform their computations concerning the price of good 1 such that goods market 2 clears. From these considerations they not only derive the price of good 1 but also their wages. This enables them to compute their Marshallian demand functions, which they assume will be satisfied. Thus, voters only indirectly observe output in sector 1 through the assumption that their Marshallian demand resulting from perceived prices and wages can be satisfied. But under PEV this assumption does not hold, since they do not take into account general equilibrium repercussions from the economy resulting from higher unemployment and the attendant change of the tax rate. This ignorance is represented by their assumption of a constant tax rate.

The key insight is the following: As voters assume that $\tilde{w}_{2,t}^{PEV}$ and $\tilde{\tau}_t^{PEV}$ remain constant, the demand of workers of sector 2 for the second good would also remain constant. If $w_{1l,t}$ rises, the demand of low-skilled workers for the second good must increase from a certain value of $w_{1l,t}$ on. In order to obtain market clearing in sector 2, the demand of high-skilled workers for the second good would have to decline in the eyes of the voters, which would require a decline of $\tilde{w}_{1h,t}^{PEV}$. A lower $\tilde{w}_{1h,t}^{PEV}$ would have to be in turn be accompanied by a lower price for good 1. This follows from the continuity of the price function and the arguments we present in the next paragraph. Since $\tilde{p}_{1,t}^{PEV}$ would decline under PEV, workers in sector 2 perceive that their utility increases with a rising $w_{1l,t}$ since their nominal net wages are expected to remain constant. We observe that workers in sector 2 do not anticipate that their own demand for sector 2 goods will decline since they assume $\tilde{w}_{2,t}^{PEV}$ and $\tilde{\tau}_t^{PEV}$ to be constant. This failure to recognize general equilibrium effects translates into a mistaken view about price reactions through the market clearing in sector 2 when $w_{1l,t}$ changes.

Under GEV, an increase in $w_{1l,t}$ leads to higher unemployment and therefore to an increasing tax rate. The increase in $\tilde{\tau}_t^{PEV}$ guarantees the necessary decrease in aggregate demand for good 2 by the high-skilled in sector 1 and workers of sector 2 while $w_{1l,t}$ increases and leads to a growing demand for good 2 by low-skilled workers. Since under PEV both $\tilde{\tau}_t^{PEV}$ and $\tilde{w}_{2,t}^{PEV}$ are perceived to remain constant, the necessary increase in aggregate demand in favor of the low-skilled could only be achieved by decreasing demand by the high-skilled of sector 1. In the extreme case where all of good 2 would be allocated to the low-skilled and the workers of sector 2, $\tilde{w}_{1h,t}^{PEV}$ would have to be zero. The corresponding minimum wage would be $\tilde{w}_{1l}^{PEV,max}$.

If we look at the political outcome under PEV we find that the crisis is self-enforcing: The higher the last period's equilibrium tax rate is the higher the minimum wage the median voters prefer in the present period. The short-run political equilibrium under PEV, $\hat{w}_{1l,t}^{PEV}$, strictly increases in the last period's tax rate $\tau_{t-1}^{PEV} = (2 - s\beta)^t(1 + \tau_r) - 1$ (see proposition 2) which in turn strictly rises in $t$. One possible interpretation is that with an increasing tax rate the perceived nominal wage in sector 2, $\tilde{w}_{2,t}^{PEV}$, decreases. Hence - in the perception of voters - more wealth can be redistributed to the low-skilled workers before their real demand for good 2 exceeds output in the second sector and the economy collapses. The maximum value for the minimum wage would increase and therefore the value of the Condorcet winner $\hat{w}_{1l,t}^{PEV}$ in the perspective period.

# 6 Robustness

The intuition behind PEV is that voters take a narrow standpoint: They assume that regulations in sector 1 do only affect this sector itself. Sector 2 and tax variables are perceived to stay constant.

There are a variety of alternative formulations of such a narrow viewpoint of voters which are briefly discussed in this section. First, instead of clearing the second goods market in their minds they could also clear the first goods market (View 2). Furthermore, it is conceivable that voters take PEV or View 2 but assume the price for good 1 to be fixed instead of the price for good 2 (View 3 and View 4 respectively).[18]

It can be shown that for all such partial equilibrium views at least two voter groups prefer a minimum wage as high as possible as long as it is economically feasible.[19] Therefore, as soon as voters take a narrower view than under GEV long-run equilibria can occur with high unemployment that are pareto-inefficient. Under GEV we always have full employment.

# 7 Conclusions

In this paper we give an explanation for the emergence of crises in democracies. In particular, we show that neglecting general equilibrium repercussions from the regulated sector on the rest of the economy (i.e., the unregulated sector and the tax rate) can make voters set regulations that are not only detrimental for the economy as a whole (total output) but also damage their own welfare. Even if a crisis occurs, reforms that result in efficient regulations can only take place if people anticipate general equilibrium effects correctly. However, crises can induce a better recognition of general equilibrium effects which will trigger a reversal of bad times. If this argument is significant enough, the question emerges whether it is possible for democracies to adopt GEV early on and thus avoid the painful cleansing effect caused by crises. Whether institutional frameworks for democracies exist that can trigger GEV is the fundamental and open question which we hope to answer in subsequent research.

[18] One could also imagine that voters take relative prices of good 1 and good 2 as constant and expect changes in both sectors.

[19] A detailed analysis is available on request. Economic feasibility means that markets clear and taxes are finite.

# Appendix A: Proofs

### Proof of Lemma 2

Because of the continuity of $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$, $\tilde{p}_{1,t}^{PEV}(w_{1l,t}) \geq 0$, $\tilde{p}_{1,t}^{PEV}(0) = 0$ and $\tilde{p}_{1,t}^{PEV}(\tilde{w}_{1l,t}^{PEV,max}) = 0$, $\tilde{w}_{1l,t}^{p_1}$ must be a local maximizer of $\tilde{p}_{1,t}^{PEV}(w_{1l,t})$ in $[0, \tilde{w}_{1l,t}^{PEV,max}]$. Moreover, since $\partial\tilde{p}_{1,t}^{PEV}/\partial w_{1l,t} = 0$ for $w_{1l,t} = \tilde{w}_{1l,t}^{p_1}$, we have:

$$\frac{\partial^2 \tilde{p}_{1,t}^{PEV}}{\partial(w_{1l,t})^2}(\tilde{w}_{1l,t}^{p_1}) = \tilde{p}_{1,t}^{PEV}\Big((1-\beta)\frac{-(s\overline{L}_{1l})^2}{(\overline{L}_2 + \tau_{t-1}^{PEV} w_{2,t-1}^{PEV}\overline{L}_2 - s w_{1l,t}\overline{L}_{1l})^2} - \frac{\beta}{(w_{1l,t})^2}\Big) < 0$$

$\square$

### Proof of Proposition 2

Equation (39) gives us the general connection between the Condorcet winner in one period and the previous period's realized tax rate and sector-2 wage values:

$$\hat{w}_{1l,t+1}^{PEV} = \frac{\overline{L}_2 + \tau_t^{PEV} w_{2,t}^{PEV}\overline{L}_2}{s\overline{L}_{1l}}$$

Thus the Condorcet winner in period zero is:

$$\hat{w}_{1l,0}^{PEV} = \frac{\overline{L}_2 + \tau_r w_{2,r}\overline{L}_2}{s\overline{L}_{1l}}$$

Using $w_2 = 1/(1+\tau)$ (see equation (22)) we obtain:

$$\hat{w}_{1l,0}^{PEV} = \frac{\overline{L}_2 + \frac{\tau_r}{1+\tau_r}\overline{L}_2}{s\overline{L}_{1l}} = \frac{\overline{L}_2 + \frac{\tau_r}{1+\tau_r}\overline{L}_2 - \frac{1+\tau_r}{1+\tau_r}\overline{L}_2 + \overline{L}_2}{s\overline{L}_{1l}} = \frac{2\overline{L}_2 - \frac{1}{1+\tau_r}\overline{L}_2}{s\overline{L}_{1l}}$$

With equations (20) and (22) we find in general:

$$w_{2,t}^{PEV} = \frac{2\overline{L}_2 - s w_{1l,t}^{PEV}\overline{L}_{1l}}{\overline{L}_2(2 - s\beta)}$$

and therefore:

$$w_{2,0}^{PEV} = \frac{2\overline{L}_2 - s\hat{w}_{1l,0}^{PEV}\overline{L}_{1l}}{\overline{L}_2(2 - s\beta)} = \frac{1}{(2 - s\beta)(1+\tau_r)}$$

Thus the tax rate in period zero is:

$$\tau_0^{PEV} = (2 - s\beta)(1 + \tau_r) - 1$$

Inserting $w_{2,0}^{PEV}$ and $\tau_0^{PEV}$ in (39) we have:

$$\hat{w}_{1l,1}^{PEV} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}}$$

and therefore:

$$\begin{aligned} w_{2,1}^{PEV} &= \frac{1}{(2-s\beta)^2(1+\tau_r)} \\ \tau_1^{PEV} &= (2-s\beta)^2(1+\tau_r) - 1 \end{aligned}$$

Continuing in this fashion we obtain Proposition 2.

□

# Appendix B: Utility of Low-skilled Workers under GEV

The utility of the low-skilled workers in sector 1 under GEV is given by the von Neumann-Morgenstern expected utility function:

$$U_{1l,t}^{GEV} = \frac{L_{1l,t}^{GEV}}{\overline{L}_{1l}}\Big\{\ln\Big(\frac{1}{2}\frac{w_{1l,t}}{p_{1,t}^{GEV}}\Big) + \ln\Big(\frac{1}{2}w_{1l,t}\Big)\Big\} + \frac{\Delta_t^{GEV}}{\overline{L}_{1l}}\Big\{\ln\Big(\frac{1}{2}s\frac{w_{1l,t}}{p_{1,t}^{GEV}}\Big) + \ln\Big(\frac{1}{2}sw_{1l,t}\Big)\Big\}$$

This can be simplified to:

$$U_{1l,t}^{GEV} = -2\frac{L_{1l,t}^{GEV}}{\overline{L}_{1l}}\ln(s) + 2\ln(w_{1l,t}) - \ln(p_{1,t}^{GEV}) + 2\ln(s) - 2\ln(2) \tag{51}$$

The properties of this utility function as a function of $w_{1l,t}$ is summarized in the following lemma.

**Lemma 4**

*$U^{GEV}_{1l,t}(w_{1l,t})$ has the following properties in $w_{1l,t} \in (0, w^{max}_{1l})$:*

*(i)* $\lim_{w_{1l,t}\to 0} U^{GEV}_{1l,t} = \infty$ *and* $\lim_{w_{1l,t}\to w^{max}_{1l}} U^{GEV}_{1l,t} = -\infty$.

*(ii) Depending on $s$ and $\beta$, the optimal wage for the low-skilled workers of sector 1 can exceed $w^{min}_{1l}$.*

**Proof of Lemma 4**

The proof of statement (i) is as follows:

Under GEV the utility function of the low-skilled in sector 1 is:

$$U^{GEV}_{1l,t} = -2\frac{L^{GEV}_{1l,t}}{\overline{L}_{1l}}\ln(s) + 2\ln(w_{1l,t}) - \ln(p^{GEV}_{1,t}) + 2\ln(s) - 2\ln(2)$$

From this we derive direct verification that $\lim_{w_{1l,t}\to w^{max}_{1l}} U^{GEV}_{1l,t} = -\infty$(by using equations (23) and (24) and equation (20) which implies $\lim_{w_{1l,t}\to w^{max}_{1l}} \tau = \infty$).

Furthermore, we have to show that $\lim_{w_{1l,t}\to 0} U^{GEV}_{1l,t} = \infty$. This is equivalent to showing that $\lim_{w_{1l,t}\to 0} u^{GEV}_{1l,t} = \infty$:

$$\begin{aligned}
u^{GEV}_{1l,t} &= \frac{L^{GEV}_{1l,t}}{\overline{L}_{1l}}\Big(\frac{1}{2}\frac{w_{1l,t}}{p^{GEV}_{1,t}}\Big)^{\frac{1}{2}}\Big(\frac{1}{2}w_{1l,t}\Big)^{\frac{1}{2}} + \frac{\Delta^{GEV}_{t}}{\overline{L}_{1l}}\Big(\frac{1}{2}\frac{sw_{1l,t}}{p^{GEV}_{1,t}}\Big)^{\frac{1}{2}}\Big(\frac{1}{2}sw_{1l,t}\Big)^{\frac{1}{2}} \\
&= \frac{1}{2}\frac{L^{GEV}_{1l,t}}{\overline{L}_{1l}}w_{1l,t}\frac{1}{\sqrt{p^{GEV}_{1,t}}}(1-s) + \frac{1}{2}sw_{1l,t}\frac{1}{\sqrt{p^{GEV}_{1,t}}} \\
&= \frac{1}{2}\beta\frac{\overline{L}_{2}}{\overline{L}_{1l}}\frac{1}{(1+\tau^{GEV}_{t})}\Big(\frac{\overline{L}_{1h}}{\overline{L}_{2}}\Big)^{\frac{1-\beta}{2}}\Big(\frac{\beta}{w_{1l,t}(1+\tau^{GEV}_{t})}\Big)^{\frac{\beta}{2}}(1-s) \\
&\quad + \frac{1}{2}sw_{1l,t}^{1-\frac{\beta}{2}}\Big(\frac{\overline{L}_{1h}}{\overline{L}_{2}}\Big)^{\frac{1-\beta}{2}}\Big(\frac{\beta}{(1+\tau^{GEV}_{t})}\Big)^{\frac{\beta}{2}}
\end{aligned}$$

Because $\lim_{w_{1l,t}\to 0}(1+\tau^{GEV}_{t}) = (1-(s\beta)/2)$, the first term goes to infinity and the second term goes to zero if $w_{1l,t}$ approaches zero. Therefore, $u^{GEV}_{1l,t}$ goes to infinity and consequently $U^{GEV}_{1l,t}$ does so too.

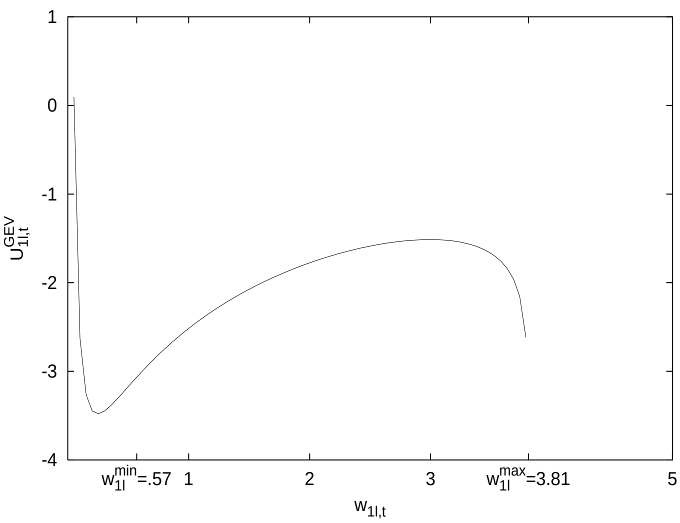
To illustrate (ii) we can make the following considerations and computations:

For $\partial U^{GEV}_{1l,t}/\partial w_{1l,t} = 0$ we obtain a polynomial of degree two in $w_{1l,t}$. Consequently, for $w_{1l,t} \in (0, w^{max}_{1l})$ there can be two or less values of $w_{1l,t}$ satisfying the necessary conditions for optimal points. They depend on the parameters $s$, $\beta$, $\overline{L}_{1l}$, and $\overline{L}_2$. [20] Considering the course of $U^{GEV}_{1l,t}$, which is a continuous and differentiable function for $w_{1l,t} \in (0, w^{max}_{1l})$, we can draw further conclusions: If there are two values satisfying the necessary and sufficient conditions for local optima, the smaller must be a local minimizer and the larger a local maximizer. In this case, if $w^{min}_{1l}$ is larger than the local minimizer and smaller than the maximizer, the low-skilled workers of sector 1 will prefer a minimum wage that exceeds $w^{min}_{1l}$. If $w^{min}_{1l}$ is smaller than both optimal points, it is possible that $w^{min}_{1l}$ will be the best choice. At all events, if $w^{min}_{1l}$ exceeds the local maximizer it is automatically the best choice. In all other conceivable cases $U^{GEV}_{1l,t}$ must depend negatively on $w_{1l,t}$ for $w_{1l,t} \in (0, w^{max}_{1l})$ [21] and the low- skilled choose $w^{min}_{1l}$.[22]

$\square$

Figure 5 shows $U^{GEV}_{1l,t}$ for the set of parameter values specified in subsection 4.1 with an optimal wage exceeding $w^{min}_{1l}$.

Figure 5: $U^{GEV}_{1l,t}$ with $s = 0.75$ and $\beta = 0.4$




[20]We used the software package MAPLE to solve $\partial U^{GEV}_{1l,t}/\partial w_{1l,t} = 0$ for $w_{1l,t}$. Whether the critical points are larger or smaller than $w^{min}_{1l}$ depends solely on $s$ and $\beta$.

[21]There are values of $w_{1l,t} \in (0, w^{max}_{1l})$ which are critical points but neither of them is a local minimizer or a local maximizer.

[22]Unfortunately, it is not possible to analyze $U^{GEV}_{1l,t}$ analytically.

# Appendix C: Utilities of Workers in Sector 1 under PEV

The utility function of the high-skilled workers of sector 1 under PEV is given by:

$$\tilde{U}^{PEV}_{1h,t}(w_{1l,t}) = \ln\Big(\frac{1}{2}\frac{\tilde{w}^{PEV}_{1h,t}}{\tilde{p}^{PEV}_{1,t}}\Big) + \ln\Big(\frac{1}{2}\tilde{w}^{PEV}_{1h,t}\Big)$$

Dividing $\tilde{w}^{PEV}_{1h,t}$ by $\tilde{p}^{PEV}_{1,t}$ we obtain:

$$\frac{\tilde{w}^{PEV}_{1h,t}}{\tilde{p}^{PEV}_{1,t}} = \Big(\frac{1-\beta}{1+\tau^{PEV}_{t-1}}\Big)\Big(\frac{\beta}{w_{1l,t}}\Big)^{\beta}\Big(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\Big)^{\beta} \tag{52}$$

from equations (31) and (52) we can conclude the following:

**Lemma 5**

$\tilde{U}^{PEV}_{1h,t}(w_{1l,t})$ *has the following properties:*

*(i)* $\tilde{U}^{PEV}_{1h,t}(w_{1l,t})$ *is strictly decreasing in* $w_{1l,t} \in (0, \tilde{w}^{PEV,max}_{1l,t})$.

*(ii) The high-skilled workers of sector 1 maximize their utility* $\tilde{U}^{PEV}_{1h,t}(w_{1l,t})$ *if they choose the lowest possible wage* $w^{min}_{1l}$.

Figure 6 illustrates the findings of Lemma 5 for $\tau^{PEV}_{t-1} = 0$ and for the set of parameter values specified in subsection 4.1.

The utility function of the low-skilled workers of sector 1 is given by:

$$\tilde{U}^{PEV}_{1l,t}(w_{1l,t}) = -2\frac{\tilde{L}^{PEV}_{1l,t}}{\overline{L}_{1l}}\ln(s) + \ln(w_{1l,t}) + \ln\Big(\frac{w_{1l,t}}{\tilde{p}^{PEV}_{1,t}}\Big) + 2\ln(s) - 2\ln(2) \tag{53}$$

We obtain the following lemma:

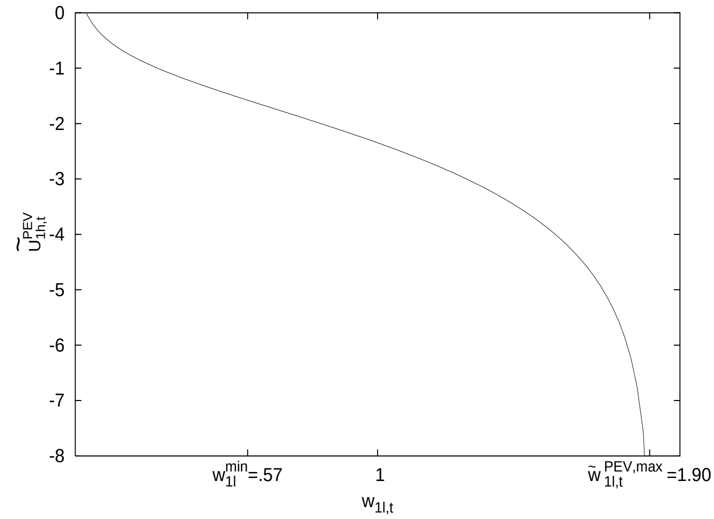
**Lemma 6**

$\tilde{U}^{PEV}_{1l,t}(w_{1l,t})$ *has the following properties:*

*(i)* $\lim_{w_{1l,t}\to 0}\tilde{U}^{PEV}_{1l,t}(w_{1l,t}) = \infty$ *and* $\lim_{w_{1l,t}\to \tilde{w}^{PEV,max}_{1l,t}}\tilde{U}^{PEV}_{1l,t}(w_{1l,t}) = \infty$.

*(ii) There is one local optimum - which is a minimum - in* $(0, \tilde{w}^{PEV,max}_{1l,t})$.

Figure 6: $\tilde{U}^{PEV}_{1h,t}$ with $\tau^{PEV}_{t-1} = 0$




*(iii) The low-skilled workers of sector 1 maximize their utility $\tilde{U}^{PEV}_{1l,t}(w_{1l,t})$ if they choose the largest possible wage $\tilde{w}^{PEV,max}_{1l,t}$.*

**Proof of Lemma 6**

The utility function of the low-skilled workers of sector 1 is given by (53). Furthermore, we obtain:

$$\frac{w_{1l,t}}{\tilde{p}^{PEV}_{1,t}} = \frac{\beta^{\beta}}{1+\tau^{PEV}_{t-1}} w^{1-\beta}_{1l,t} \Big( \frac{\overline{L}_{1h}}{\epsilon_t(w_{1l,t})} \Big)^{1-\beta}$$

It can be verified that $\lim_{w_{1l,t} \to \tilde{w}^{PEV,max}_{1l,t}} \tilde{L}^{PEV}_{1l,t} = 0$ (see equations (34),(38),(39)) and $\lim_{w_{1l,t} \to \tilde{w}^{PEV,max}_{1l,t}} (w_{1l,t}/\tilde{p}^{PEV}_{1,t}) = \infty$ (see equations (38),(39)). Thus, $\tilde{U}^{PEV}_{1l,t}(w_{1l,t})$ goes to infinity as $w_{1l,t}$ approaches $\tilde{w}^{PEV,max}_{1l,t}$. As $\tilde{U}^{PEV}_{1l,t}(w_{1l,t})$ is a continuous function in $[w^{min}_{1l}, \tilde{w}^{PEV,max}_{1l,t})$, the low-skilled cannot do better with any other wage level than $\tilde{w}^{PEV,max}_{1l,t}$.

To show that $\lim_{w_{1l,t} \to 0} \tilde{U}^{PEV}_{1l,t} = \infty$, is equivalent to show that $\lim_{w_{1l,t} \to 0} \tilde{u}^{PEV}_{1l,t} = \infty$ :

$$
\begin{aligned}
\tilde{u}_{1l,t}^{PEV} &= \frac{1}{2}\frac{\tilde{L}_{1l,t}^{PEV}}{\overline{L}_{1l}} w_{1l,t}\frac{1}{\sqrt{\tilde{p}_{1,t}^{PEV}}}(1-s)+\frac{1}{2}sw_{1l,t}\frac{1}{\sqrt{\tilde{p}_{1,t}^{PEV}}} \\
&= \frac{1}{2}\frac{1}{\overline{L}_{1l}}\beta\epsilon_t(w_{1l,t})\frac{1}{(1+\tau_{t-1}^{PEV})^{\frac{1}{2}}}\Big(\frac{\overline{L}_{1h}}{\epsilon_t(w_{1l,t})}\Big)^{\frac{1-\beta}{2}}\Big(\frac{\beta}{w_{1l,t}}\Big)^{\frac{\beta}{2}} \\
&\quad +\frac{1}{2}sw_{1l,t}^{1-\frac{\beta}{2}}\frac{1}{(1+\tau_{t-1}^{PEV})^{\frac{1}{2}}}\Big(\frac{\overline{L}_{1h}}{\epsilon_t(w_{1l,t})}\Big)^{\frac{1-\beta}{2}}\beta^{\frac{\beta}{2}}
\end{aligned}
$$

As $\tau_{t-1}^{PEV}$ is taken as given and $\epsilon_t(w_{1l,t})$ approaches a finite value for $w_{1l,t} \rightarrow 0$ (see equation (38)), the first term goes to infinity and the second to zero. Therefore, $\lim_{w_{1l,t}\rightarrow 0} \tilde{U}_{1l,t}^{PEV} = \infty$.

Since we obtain a polynomial of degree 2 in $w_{1l,t}$ for $\partial\tilde{U}_{1l,t}^{PEV}/\partial w_{1l,t} = 0$ there could be two local optima in $(0, \tilde{w}_{1l,t}^{PEV,max})$. We can, however, verify that there is only one local optimum - which is a minimizer - in this area because $\tilde{U}_{1l,t}^{PEV}(w_{1l,t})$ goes to infinity for $w_{1l,t} \rightarrow 0$ and $w_{1l,t} \rightarrow \tilde{w}_{1l,t}^{PEV,max}$, and $\tilde{U}_{1l,t}^{PEV}(w_{1l,t})$ is a continuous function in $(0, \tilde{w}_{1l,t}^{PEV,max})$.

□

Figure 7 illustrates the findings of Lemma 6 for $\tau_{t-1}^{PEV} = 0$ and for the set of parameter values specified in subsection 4.1.

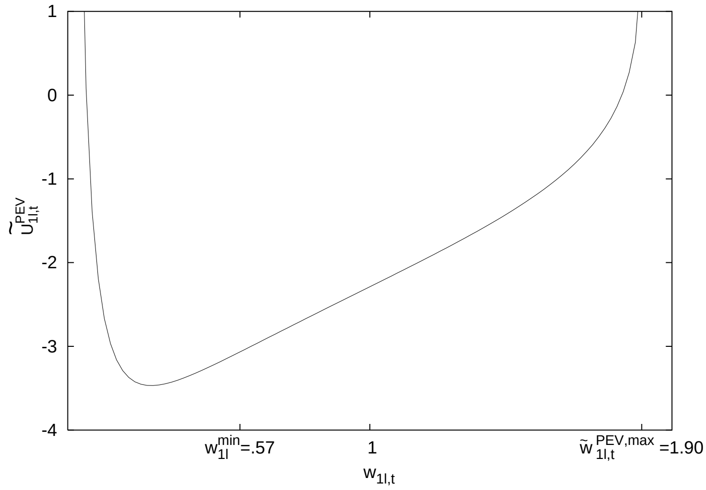
# Appendix D: Interpretation under GEV

To explain the results with GEV, we introduce relative labor costs, which will help to explain the functioning of the economy. The tax rate and the price for good 1 determine the relative labor costs $w_{1l,t}(1+\tau_t)/p_{1,t}$ and $w_{1h,t}(1+\tau_t)/p_{1,t}$ and therefore labor demand in sector 1.

Suppose, for example, that $w_{1l,t}(1+\tau_t)/p_{1,t}$ increases. Then labor demand for the low-skilled will decrease.[23] As the minimum wage is binding, the low-skilled labor force that will be employed also decreases. Furthermore, because low-skilled and high-skilled

[23] This follows from the profit maximization condition with respect to $L_{1l}$ (see equation (12))and the fact that the high-skilled labor market always clears and therefore $L_{1h,t} = \overline{L}_{1h}$ in all periods.

Figure 7: $\tilde{U}_{1l,t}^{PEV}$ with $\tau_{t-1}^{PEV} = 0$




labor are complementary inputs, the demand for high-skilled workers in sector 1 for a given wage level $w_{1h,t}$ decreases as well.[24] Consequently, as the high-skilled labor market in sector 1 is not regulated, the wage level $w_{1h,t}$ declines so that the labor market for high-skilled workers will clear. Of course, a change in $(1 + \tau_t)/p_{1,t}$ itself changes labor demand for the high-skilled. If $(1+\tau_t)/p_{1,t}$ goes down, $w_{1h,t}$ goes up and vice versa. Since $p_2 = 1$, relative labor costs in sector 2 are $w_{2,t}(1 + \tau_t)$. Again, this labor market is not regulated and thus relative labor costs remain constant, i.e., by the same proportion that $(1 + \tau_t)$ changes, $w_{2,t}$ too has to change, but in the opposite direction.

We can draw the conclusions of Proposition 1 mainly from equations (49) and (50) intuitively without explicitly computing the results.

In equilibrium, unemployment increases if the minimum wage $w_{1l,t}$ goes up. To see this, suppose that - starting from an equilibrium situation - unemployment would not increase if $w_{1l,t}$ increased. Then $L_{1l,t}^{GEV}$ would have to remain constant or increase. Hence, $(1 + \tau_t^{GEV})/p_{1,t}^{GEV}$ would have to fall by at least the same proportion as $w_{1l,t}$ increased. But if $(1+\tau_t^{GEV})/p_{1,t}^{GEV}$ declined while $L_{1l,t}^{GEV}$ did not fall, aggregate demand of the high-skilled for good 2 would increase and $w_{1h,t}^{GEV}$ would have to rise as $L_{1h,t}^{GEV} =$

[24]Note that $\partial^2 q_{1,t}/(\partial L_{1h,t} \partial L_{1l,t}) > 0$. If the use of $L_{1l,t}$ decreases, the marginal productivity of $L_{1h,t}$ also decreases. Because $\partial^2 q_{1,t}/\partial(L_{1h,t})^2 < 0$, the use of $L_{1h,t}$ has to decrease for a given wage level if firms want to maximize their profits.

$\overline{L}_{1h}$. To complete the argument we have to distinguish two cases: First, a constant or falling tax rate and second, an increasing tax rate. In the first case, i.e. in the case of a constant or decreasing tax rate, $w_{2,t}^{GEV}$ and therefore aggregate demand of sector 2 workers for good 2 would at least remain constant but never fall, because $w_{2,t}^{GEV} = 1/(1+\tau_t^{GEV})$. Furthermore, if an increasing $w_{1l,t}$ caused constant or decreasing unemployment, aggregate demand for good 2 of all low-skilled would go up. Hence, an increasing $w_{1l,t}$ would correspond to an increasing aggregate demand of all voter groups for good 2 as long as $\tau_t^{GEV}$ would not increase. Given that the right hand side of (49) always equals $q_{2,t}^{GEV} = \overline{L}_2$, it follows that a situation where unemployment decreases or remains constant while $w_{1l,t}$ increases and $\tau_t^{GEV}$ does not, cannot be an equilibrium. In the second case, i.e. if $\tau_t^{GEV}$ increased, $p_{1,t}^{GEV}$ also would have to increase since $(1+\tau_t^{GEV})/p_{1,t}^{GEV}$ would have to decline in the case of not increasing unemployment. If we look at the first goods market:

$$\left(L_{1l,t}^{GEV}\frac{b_{1l,t}^{GEV}}{2} + L_{1h,t}^{GEV}\frac{b_{1h,t}^{GEV}}{2} + L_{2,t}^{GEV}\frac{b_{2,t}^{GEV}}{2} + \Delta_t^{GEV}\frac{b_{un,t}^{GEV}}{2}\right)/p_{1,t}^{GEV} = q_{1,t}^{GEV} \tag{54}$$

we can recognize that an increasing $p_{1,t}^{GEV}$ together with an increasing or constant $q_{1,t}^{GEV}$ (non-decreasing employment of the low-skilled workers) would imply an increasing numerator on the left hand side of equation (54) to guarantee market-clearing in the first goods market. Since $q_{2,t}^{GEV}$ remains constant, equation (49) would not hold and goods market 2 would not clear. Thus, a situation where a rising $w_{1l,t}$ corresponds to non-increasing unemployment and an increasing tax rate cannot be an equilibrium, too. Therefore, independent of the changes in $\tau_t^{GEV}$, unemployment will always increase when $w_{1l,t}$ goes up.

If unemployment increases when the minimum wage goes up, then output in sector 1 will decrease (see equations (1) and (15)), i.e., good 1 will become scarcer. Hence, its price $p_{1,t}^{GEV}$ must rise if $w_{1l,t}$ increases.

Furthermore, since unemployment increases when $w_{1l,t}$ rises and thus $\Delta_t^{GEV}\frac{b_{un,t}^{GEV}}{2}$ also rises, the sum $L_{1l,t}^{GEV}\frac{b_{1l,t}^{GEV}}{2} + L_{1h,t}^{GEV}\frac{b_{1h,t}^{GEV}}{2} + L_{2,t}^{GEV}\frac{b_{2,t}^{GEV}}{2}$ has to fall to satisfy equation (49). But then $(w_{1l,t}L_{1l,t}^{GEV} + w_{1h,t}^{GEV}L_{1h,t}^{GEV} + w_{2,t}^{GEV}L_{2,t}^{GEV})$ also declines and therefore $\tau_t^{GEV}$ has to rise according to equation (50). Consequently, the tax rate increases monotonically in $w_{1l,t}$. Since relative labor costs $w_{2,t}^{GEV}(1+\tau_t^{GEV})$ in sector 2 have to remain constant as the labor market clears, this means that the nominal wage of sector 2 workers declines when $w_{1l,t}$ increases.

The question arises whether $w_{1l,t}$ can become infeasible. If we look at equation (49), we recognize that this must be the case from a certain value of $w_{1l,t}$ on, denoted by $w_{1l}^{max}$. The reason for this is that from this point on - as $w_{1l,t}$ is increased exogenously - the demand of the low-skilled will exceed $q_{2,t}^{GEV} = \overline{L}_2$ even if all low-skilled are unemployed since unemployed individuals receive $sw_{1l,t}$.[25] Then the market for good 2 could only clear if $L_{1l,t}^{GEV}$ was negative, which is not possible. Furthermore, at the critical level $w_{1l}^{max}$, the aggregate demand for good 2 of the high-skilled workers and workers of sector 2 has to be zero because the goods market in sector 2 clears. Thus, at $w_{1l}^{max}$, $w_{1h,t}^{GEV}$ and $w_{2,t}^{GEV}$ have to be zero. For a given non-negative value of $L_{1l,t}^{GEV}$, $w_{1h,t}^{GEV} = 0$ can only hold if $\lim_{w_{1l,t}\to w_{1l}^{max}}(1+\tau_t^{GEV})/p_{1,t}^{GEV} = \infty$ (see equation (13)). The result is that, because of equation (12), the employment of the low-skilled is also zero. We can conclude, therefore, that for $w_{1l,t} = w_{1l}^{max}$, where all low-skilled alone consume all of good 2, all low-skilled are unemployed and $(1+\tau_t^{GEV})/p_{1,t}^{GEV} = \infty$.

Thus, output in sector 1 is zero, and for clearance of this good market demand has to be zero, which implies $\lim_{w_{1l,t}\to w_{1l}^{max}} p_{1,t}^{GEV} = \infty$. Since $\lim_{w_{1l,t}\to w_{1l}^{max}}(1+\tau_t^{GEV})/p_{1,t}^{GEV} = \infty$, it follows that $\lim_{w_{1l,t}\to w_{1l}^{max}}(1+\tau_t^{GEV}) = \infty$. The latter can also been seen from the fact that $w_{2,t}^{GEV}$ has to be zero and according to equation (14) $w_{2,t}^{GEV} = 1/(1+\tau_t^{GEV})$.

Summarizing the analysis, we can say that an increasing minimum wage has two effects: a negative effect on total wealth and a redistributive effect in favor of the low-skilled.

[25] For $w_{1l}^{max}$ the demand of the low-skilled for good 2 is equal to $q_{2,t}^{GEV} = \overline{L}_2$.

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