Supplementary Material to: Does the Adverse Announcement Effect of Climate Policy Matter? - A Dynamic General Equilibrium Analysis

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May 3, 2017

1 Some Notes on the matlab Code

We solve the models in matlab, using the bvp5c solver. The file Baseline.m is the main file for the baseline model, the file Backstop.m is the main file for the model with backstop technology. Both use additional files as inputs. All necessary files are in the folder 'CodesAnnouncement-Effect'.

Each model is made stationary by a different transformation. This is because the long-run behavior of the models with and without backstop technology differ. Details on both model versions and how we solve them are given below.

The coding tries to come up with a good guess, step-by-step, but this is not a guarantee for convergence for a suitable range of parameter values. Different approaches may be needed to obtain a good first guess.

2 The Model

The dynamic general equilibrium model combines an economic module with a climate module. Economic dynamics determine resource use and emissions, while the climate module translates emissions into temperature change. Temperature, in turn, affects output. The resource represents all fossil fuels lumped together, measured in terms of their carbon content. Due to the climate externality, the decentralized market equilibrium does not have an equivalent optimization representation. We therefore first derive the equilibrium conditions for the decentralized economy with an exogenous tax imposed on carbon use, subsequently deriving the tax rate leading to the welfare optimum.

With respect to notation, dotted variables (e.g. $\dot{X}$) denote time derivatives and hats denote growth rates ($\hat{X} := \dot{X}/X = \mathrm{d}\log X/\mathrm{d}t$). Latin letters denote variables that depend on time (with exemptions stated). Row vectors are written as $[\ldots]$, column vectors as transposed row vectors $[\ldots]^\top$.

2.1 The economic module

Firms and households interact on competitive markets. Firms produce the final output Y with capital K, resource flow R, and effective labor E as inputs according to a Cobb-Douglas technology

$$Y = \Omega(T_1) K^\alpha R^\beta E^{1-\alpha-\beta}, \tag{1}$$

with partial production elasticities $\alpha > 0$, $\beta > 0$, and $1 - \alpha - \beta > 0$. Output is scaled by the damage function $\Omega(T_1)$ based on Nordhaus (2016a) and Nordhaus (2014). The specification assumes the percentage loss of GDP to be a quadratic function of the global mean surface temperature T_1. Labor is constant, but its productivity grows exogenously such that effective labor E grows at the exogenous rate $\hat{E}$. The resource flow R represents a mixture of fossil fuels extracted from an exhaustible carbon resource stock S. As a common denominator, the fossil fuel inputs are measured in terms of their respective carbon content, which is emitted one-to-one into the atmosphere after having been burned in production. Hence, R stands for both input and emissions measured in `GtC/a`.[1] The carbon resource stock—'resource stock' in the following—also represents a mixture of fossil fuels converted into carbon and measured in `GtC`.

Due to perfect competition, input prices equal their respective marginal productivities. Taking the output good as numéraire, we obtain

$$\iota + \delta = \alpha Y/K, \tag{2}$$
$$q + r = \beta Y/R, \tag{3}$$

with resource price q, interest rate ι, depreciation rate δ, and specific tax rate r. Imposing the tax either on the resource input or on the emissions makes no difference because the carbon

[1] Units are printed in `teletype font`. `GtC` is Gigatons Carbon and `a` is annum. For notational consistency, we generally write `1/a` instead of the common `p.a.`. For a list of all units, see Table 2 in the Appendix.

extracted as a component of fossil fuel is emitted one-to-one as a component of CO_2. The tax is paid by the firms.

Input flow R is extracted from a continuous set of privately-owned carbon sources exploitable at costs per unit of flow that vary across sources. In their role as resource owners, households are price-takers selling the flow on a competitive market. It is well-known that under these conditions, sources are exploited in the order of ascending unit costs such that—provided the interest rate is positive—unit costs can be expressed as a non-increasing function $k(S)$ of the total resource stock S in all as yet-unexploited sources (see e.g. Herfindahl (1967); Laitner (1984); Solow and Wan (1976)).

Furthermore, resource owners collectively act like a representative owner of all sources who, at any time t, chooses an extraction path $R(\tau)$, $\tau \geq t$, so as to maximize the present (as of time t) value of future net revenues from selling the flows at prices taken as given. The asset value of the remaining stock $S(t)$ at time t is thus

$$v(t, S(t)) = \frac{1}{D(t)} \max_{R(\tau), \tau \geq t} \int_t^\infty D(\tau) R(\tau) \Big(q(\tau) - k(S(\tau)) \Big) \mathrm{d}\tau, \tag{4}$$

subject to

$$\dot{S} = -R. \tag{5}$$

The discount factor is D. Its law of motion is

$$\dot{D} = -\iota D, \tag{6}$$

with boundary condition $D(0) = 1$.

Optimum conditions[2] are

$$q = k(S) + p, \tag{7}$$

with co-state p of S displaying the marginal asset value per unit resource stock, and with

$$\dot{p} = \iota p + R k'(S), \tag{8}$$

the augmented Hotelling rule. The transversality condition[3] is

$$\lim_{t \to \infty} D(t) p(t) S(t) = 0. \tag{9}$$

The consumption side of the model is standard. An immortal representative household owns all assets a of the economy, inelastically supplies labor, and receives the tax revenue collected by the state as a lump sum. The state has no other role to play than collecting the tax and channeling it to the household's budget. The asset value is the value of the capital stock plus the value of the resource stock,

$$a = K + v. \tag{10}$$

As output is transformed one-to-one into investment, capital is measured in units of the numéraire.

The household receives utility from consumption C. As usual, instantaneous utility u has the constant intertemporal elasticity of substitution form with intertemporal elasticity of substitution $1/\theta$. Maximizing the discounted utility flow and taking into account the budget constraint displayed by the production balance of the economy[4]

$$\dot{K} = Y - \delta K - R k(S) - C \tag{11}$$

[2]Derived from the current-value Hamiltonian $\mathcal{H} = R\Big(q - k(S)\Big) - pR$.

[3]In addition, the following two conditions have to be fulfilled: (1) There exists a number m such that $|D(t)p(t)| \leq m$ for all $t \geq t_0$. (2) There exists a number t' such that $p(t) \geq 0$ for all $t \geq t'$ (Sydsæter et al. 2005). These conditions also have to be fulfilled in the case of all other transversality conditions.

[4]Appendix A gives a detailed derivation relating the household's budget constraint to the production balance of the economy.

yields the well-known Keynes-Ramsey rule

$$\hat{C} = (i - \rho)/\theta, \tag{12}$$

with time-preference rate ρ and the transversality condition

$$\lim_{t\to\infty} D(t)K(t) = 0. \tag{13}$$

Integrating the Keynes-Ramsey rule, we can write optimal consumption in levels as

$$C = (e^{\rho t} BD)^{-1/\theta}, \tag{14}$$

with an endogenous constant B. Note that, unlike the other variables written in Latin letters, B does not depend on time.

2.2 The climate module

The climate module is taken from the DICE model by Nordhaus (2016a).[5] It consists of two connected sub-systems. The first sub-system describes the carbon cycle, i.e. the evolution of the carbon masses between the three carbon reservoirs atmosphere M_1, upper ocean M_2, and lower ocean M_3. It reads

$$\dot{M} = \Gamma M + [R + \nu, 0, 0]^\top, \tag{15}$$

with $M := [M_1, M_2, M_3]^\top$, the carbon-transition matrix Γ and carbon emissions R from carbon used in the production process and emissions from land use change ν. Carbon stocks are measured in gigatons of carbon `GtC`, and carbon flows in gigatons of carbon per annum `GtC/a`.

The second sub-system describes the impact on temperature of the carbon concentration in the atmosphere. It reads

$$\dot{T} = \Lambda T + [\Pi(M_1, t), 0]^\top, \tag{16}$$

with $T := [T_1, T_2]^\top$. The global mean surface temperature T_1 dynamically interacts with the temperature of the lower oceans T_2, as described by the matrix Λ. Temperatures are measured as differences in °`C` from their level in the year 1900. The function $\Pi(M_1, t)$ describes radiative forcing. Figure 1 illustrates the interactions. The resulting system of equations is discussed in Appendix 2.3.

2.3 The system of equations

The equations that describe the model are summarized in Table 1, with algebraic and differential equations in the left-hand column and boundary conditions on the right. $\bar{M}$ and so forth denote initial stocks. If we plug in (7) for the resource price q, the model has thirteen unknown functions of time, namely three carbon masses stacked in the vector M, two temperatures stacked in the vector T, output Y, consumption C, resource flow R, resource stock S, the marginal asset value per unit of stock p, capital K, interest rate ι, and discount factor D. The model also contains the integration constant B. There are eight differential equations with initial boundary conditions corresponding to the variables M, T, D, S, and K. Furthermore, there are four algebraic equations for the variables Y, C, ι, and R. There is an additional differential equation for p. Finally, there are two terminal boundary conditions enabling us to determine both p and B, provided the differential equation system has two unstable eigenvalues. This turns out to be the case.

3 The optimal tax

So far, the tax rate has been taken as an exogenous policy instrument. To derive an optimal path for the tax rate, we solve the planner's problem of choosing a time path—starting at $t =$

[5]We have adapted his discrete time version to continuous time to match the economic module.

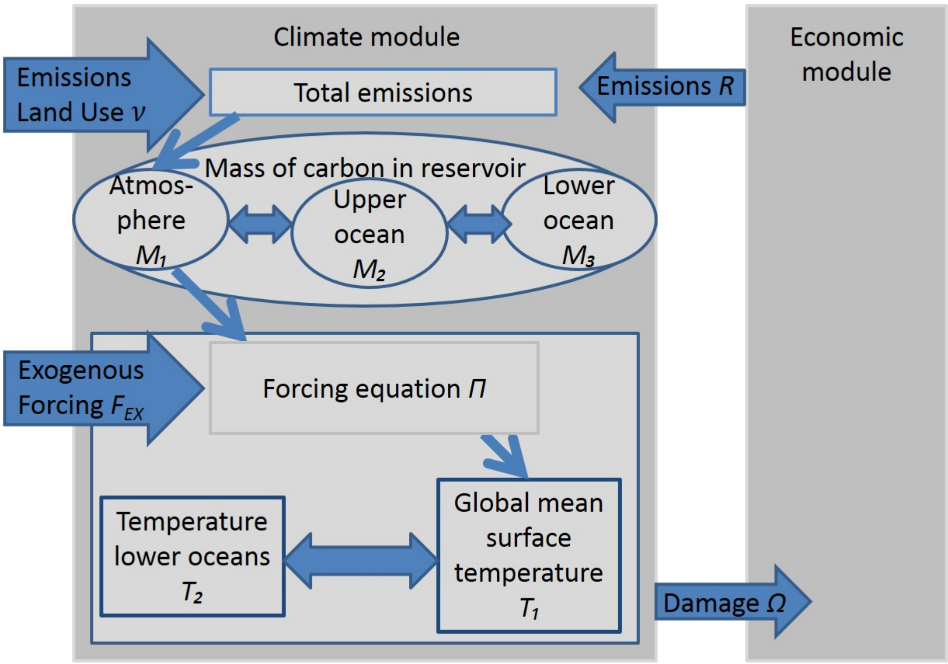

Figure 1: Interactions in the climate module.

0—for consumption C and resource extraction R to maximize the representative household's utility

$$U = \int_0^\infty u(C)e^{-\rho\tau}\mathrm{d}\tau,$$

subject to technological, resource and climate constraints

$$\begin{aligned}
\dot{K} &= \Omega(T_1)K^\alpha R^\beta E^{1-\alpha-\beta} - \delta K - C - Rk(S),\\
\dot{S} &= -R,\\
\dot{M} &= \Gamma M + [\nu + R, 0, 0]^\top,\\
\dot{T} &= \Lambda T + [\Pi(M_1, t), 0]^\top,
\end{aligned}$$

given inherited state variables K, S, M, and T at $t = 0$.

The present value Hamiltonian is

$$\begin{aligned}
\mathcal{H} =& u(C)e^{-\rho t} + \Big(\Omega(T_1)K^\alpha R^\beta E^{1-\alpha-\beta} - \delta K - C - Rk(S)\Big)P_K\\
&- RP_S + \Big(M^\top\Gamma^\top + [\nu + R, 0, 0]\Big)P_M + \Big(T^\top\Lambda^\top + [\Pi(M_1, t), 0]\Big)P_T. \qquad (17)
\end{aligned}$$

P_K and so forth are the costates associated with states K and so forth. P_M and P_T are column vectors. If $\mathcal{H}_R$ and so forth denote derivatives of the Hamiltonian with respect to R and so forth, the static and dynamic efficiency conditions read, respectively,

$$\mathcal{H}_R = 0 \quad \Rightarrow \quad 0 = P_K\frac{\beta Y}{R} - P_S + P_{M_1} - P_K k(S), \qquad (18)$$

$$\mathcal{H}_C = 0 \quad \Rightarrow \quad P_K = C^{-\theta}e^{-\rho t}, \qquad (19)$$

$$\mathcal{H}_K = -\dot{P}_K \quad \Rightarrow \quad -\dot{P}_K = P_K\left(\frac{\alpha Y}{K} - \delta\right), \qquad (20)$$

$$\mathcal{H}_S = -\dot{P}_S \quad \Rightarrow \quad -\dot{P}_S = -P_K k'(S)R, \qquad (21)$$

$$\mathcal{H}_M = -\dot{P}_M \quad \Rightarrow \quad -\dot{P}_M = \Gamma^\top P_M + \left[P_{T_1}\frac{\partial\Pi}{\partial M_1}, 0, 0\right]^\top, \qquad (22)$$

$$\mathcal{H}_T = -\dot{P}_T \quad \Rightarrow \quad -\dot{P}_T = \Lambda^\top P_T + \left[P_K\frac{\partial Y}{\partial T_1}, 0\right]^\top. \qquad (23)$$

Table 1: Overview equilibrium conditions market setting

$\dot{M} = \Gamma M + [\nu + R, 0, 0]^\top$	(15)	$M(0)$	$=$	$\bar{M}$	
$\dot{T} = \Lambda T + [\Pi(M_1, t), 0]^\top$	(16)	$T(0)$	$=$	$\bar{T}$	
$Y = \Omega(T_1) K^\alpha R^\beta E^{1-\alpha-\beta}$	(1)				
$C = \left(BDe^{\rho t}\right)^{-1/\theta}$	(14)	$\lim_{t\to\infty} D(t)K(t)$	$=$	0	(13)
$\dot{D} = -\iota D$	(6)	$D(0)$	$=$	1	
$\iota = \alpha Y/K - \delta$	(2)				
$R = \beta Y/(k(S) + p + r)$	(3)				
$\dot{p} = \iota p + Rk'(S)$	(8)	$\lim_{t\to\infty} D(t)p(t)S(t)$	$=$	0	(9)
$\dot{S} = -R$	(5)	$S(0)$	$=$	$\bar{S}$	
$\dot{K} = Y - \delta K - C - Rk(S)$	(11)	$K(0)$	$=$	$\bar{K}$	

The transversality condition is

$$\lim_{t\to\infty} Z(t)^\top P(t) = 0, \tag{24}$$

with $Z(t)$ and $P(t)$ denoting column vectors of states and costates in the corresponding order.

To relate the optimal solution to the market outcome, one has to relate the costates—the shadow prices—to the market prices. The product of the integration constant B and the discount factor D corresponds to the shadow price P_K, the marginal asset value per unit of resource stock in the decentralized outcome p corresponds to the shadow price of the resource stock transformed into units of the numéraire P_S/P_K.

The decentralized market will fulfill the optimality conditions if the tax rate is

$$r = -P_{M_1}/P_K. \tag{25}$$

To see that

$$r = -\frac{P_{M_1}}{P_K}$$

in the decentralized market leads to the optimal allocation, do the following. Insert BD for P_K and $\hat{D}$ for $\hat{P}_K$, $-r$ for P_{M_1}/P_K and $P_K p$ for P_S. Then (20) becomes (6) with ι from (2), (19) becomes (14), and (18) becomes (3) with $q = P_S/P_K + k(S)P_K$. Inserting $p = P_S/P_K$ into (8) leads to (21). Equations (13) and (9) are the transversality conditions in (24) for K and S, respectively.

The tax rule (25) has an obvious interpretation: $-P_{M_1}$ is the marginal utility loss from an extra unit of carbon in the atmosphere. It is translated into units of the numéraire by dividing it through P_K, the marginal utility of an extra unit of the numéraire.

4 Welfare evaluation

For the welfare evaluation of policies, we use two measures, a relative and an absolute intertemporal equivalent variation. The former is the constant percentage h by which consumption $\check{C}$ of the benchmark scenario must be changed to attain the utility level U of the policy scenario. A relative intertemporal equivalent variation $h > 0$ ($h < 0$) indicates a welfare gain (loss) compared to the benchmark. The relative intertemporal equivalent variation is thus implicitly defined by

$$U = \int_0^\infty \frac{\left((1+h)\check{C}\right)^{1-\theta}}{1-\theta} e^{-\rho\tau} \mathrm{d}\tau.$$

Using the definition of $\check{U}$,

$$\check{U} = \int_0^\infty \frac{\check{C}^{1-\theta}}{1-\theta} e^{-\rho\tau} \mathrm{d}\tau,$$

we obtain

$$h = \left(\left(\frac{U}{\check{U}} \right)^{1/(1-\theta)} - 1 \right).$$

The absolute intertemporal equivalent variation is the amount of the numéraire W one would have to give to the household in $t = 0$ to make it as well off as in the policy case,

$$W = h \int_0^\infty \check{D}(\tau)\check{C}(\tau)\mathrm{d}\tau. \tag{26}$$

The utility level U can be calculated comfortably by adding to the system an extra differential equation with an appropriate boundary condition. Define

$$V(t) := \int_t^\infty u(C) e^{-\rho\tau} \mathrm{d}\tau,$$

such that $U = V(0)$. Taking the time differential delivers the extra differential equation

$$\dot{V} = -u(C)e^{-\rho t}.$$

The transversality condition

$$\lim_{t\to\infty} V(t) = 0 \tag{27}$$

is obtained from

$$U = \lim_{t\to\infty} \left(\int_0^t u(C)e^{-\rho\tau}\mathrm{d}\tau + V(t) \right) = U + \lim_{t\to\infty} V(t).$$

Accordingly, we can treat V as a forward-looking variable that can be determined like the other variables in the system.

5 Solution Approach and Calibration

Two important issues arise when operationalizing the model. First, a solution concept has to be picked. Second, appropriate data are needed for calibration.

For our solution concept, we make use of the model's property that it comes arbitrarily close to a steady-state (or Balanced-Growth-Path) with constant growth rates in the long run. We then solve the model using the bvp5 two-point boundary-value solver in matlab. Due to numerical considerations, we solve the model in terms of *stationary transforms* of the variables. We give more details in Appendix B.

Appendix C gives details on calibration. In the following, we only state our general choices. The reference year for the model calibration and the starting point of the model ($t = 0$) is 2015. The calibration of the climate module is based on the 2016 version of the DICE model (Nordhaus 2016a), including initial values for carbon masses $M(0)$ and temperatures $T(0)$. Our choice for calibrating the damage function lies between values usually taken in integrated assessment models based on damage estimates and what is indirectly revealed by the international community's two-degree target. One could say that the two-degree target reflects a relatively save zone concerning impacts from climate change, i.e. for temperature changes beyond, damages will be large. The two-degree target can thus be seen as a statement on how much risk the international community is willing to take, having some welfare optimization in mind. We follow an approach explored by Nordhaus (2014). He modifies the damage function in the DICE model such that the optimal path implies that the two-degree threshold is never crossed. We use the same scaling factor as in Nordhaus (2014).[6]

[6] In our model without backstop technology, this still implies that it is optimal to cross the two-degree threshold of the global mean temperature. This is driven by the way we model the resource sector and by not including a backstop technology.

Calibration of consumption and of production, including the initial capital stock and the calibration of the resource sector, rely on several data sources. While we base the income shares and the depreciation rate direcly on data, we chose the time preference rate and the inverse of the intertemporal elasticity of substitution as well as the growth rate of effective labor to reflect certain values of the interest and the growth rate in steady state. Also, we calibrate the extraction cost function to reflect observed resource extraction in 2015, in terms of carbon emitted.

We calibrate the model in the laissez-faire set-up. To obtain correct welfare results, we keep the calibration in the alternative runs that include a tax.

6 A Backstop Technology

So far, the model does not allow for a backstop technology that would make it possible to switch to carbon-free production if the resource price becomes too high. A backstop technology enlarges the intervention possibilities, as climate policy can now affect not only the timing of emissions but also the total amount emitted by inducing a switch to the backstop. The introduction of a backstop technology would thus lead to a larger welfare gain from an instantaneously implemented emissions tax and could also make a difference to the critical lag.

Contrary to the model presented earlier—the 'baseline' model—, firms now produce output by using 'Energy' N instead of using the resource directly. Energy can be produced in three ways, (1) by transforming the resource to energy one-to-one, (2) by combining the resource and the output good as inputs to energy production, or (3) by exclusively transforming the output good into energy. Using output to produce energy inputs—i.e. using a non-polluting, non-exhaustible input to produce energy—represents the 'backstop technology' (as e.g. in van der Ploeg (2013), Hoel and Kverndokk (1996), and Tahvonen (1997)). Unit costs of producing the energy input from output are constant in terms of output, but high. Marginal extraction costs of the resource rise with the decreasing resource stock, so that a switch to the backstop occurs before the resource stock is physically depleted.[7]

Firms choose the minimal-cost technology. If the resource price, including tax, is low enough, they will choose the first option. In a medium range, they will choose the second. If the price is high enough, they will opt for the third.

The respective energy production function is constant returns to scale with unit cost function $g(\tilde{q})$, where $\tilde{q} := q + r$ denotes the resource price including tax, if there is any. The price of the other input (the output good) is suppressed, as it equals one (the output good is the numéraire). The inputs per unit of energy are thus $d_1 = g'$ and $d_2 = g - g'\tilde{q}$ for the resource and the output good, respectively.

Inputs are non-perfect substitutes. But differently from common specifications like CES, Inada is supposed not to hold. If $\tilde{q}$ is below a certain lower bound $\underline{q}$, only the resource is used. To make this case comparable to the baseline specification in Section 2, we assume this to be the case in the initial situation, i.e. $\underline{q} > \tilde{q}(0)$. If, on the other hand, $\tilde{q}$ is higher than a certain upper bound $\bar{q}$, the output good alone is used; this is the pure backstop case. Finally, in the intermediate case, both inputs are combined.

Formally, let $g(\tilde{q})$ be continuously differentiable with

$$\begin{aligned} d_1(\tilde{q}) = 1 \text{ and } d_2(\tilde{q}) = 0 \quad &\text{for} \quad \tilde{q} \leq \underline{q}, \\ g(\tilde{q}) \text{ quadratic in } \tilde{q} \quad &\text{for} \quad \underline{q} < \tilde{q} < \bar{q}, \\ d_1(\tilde{q}) = 0 \quad &\text{for} \quad \bar{q} \leq \tilde{q}. \end{aligned}$$

This uniquely specifies the unit cost function. Figure 2 shows the resulting unit costs ($g(\tilde{q})$) as well as the inputs per unit of energy ($d_1(\tilde{q})$ and $d_2(\tilde{q})$).

To introduce the backstop into the baseline model and to solve it, one has to modify some equations, calibrate the parameters describing the backstop technology, and take into account the fact that steady-state behavior differs from the baseline model.

[7] A common approach is to model the backstop technology with decreasing marginal costs, i.e. to reflect learning-by-doing. Modelling the backstop technology in this way is not likely to alter results because what matters is at which price the economy starts to switch to the backstop.

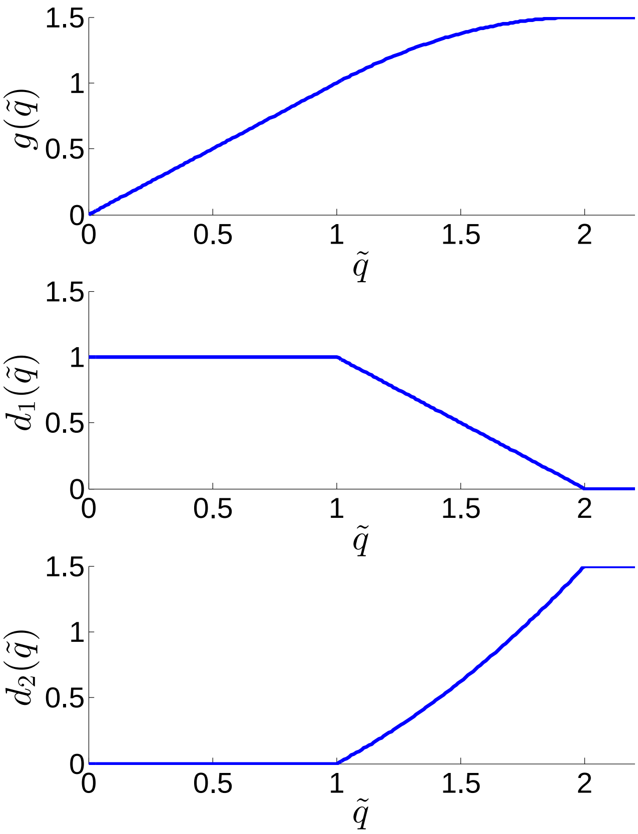

Figure 2: Illustration of the unit costs ($g(\tilde{q})$) as well as the inputs per unit of energy ($d_1(\tilde{q})$ and $d_2(\tilde{q})$), with $\underline{q} = 1$ and $\bar{q} = 2$.

With the new technology, the production balance of the economy changes to

$$\dot{K} = Y + rN - \delta K - Ng(\tilde{q}) - C.$$

In addition, (3) changes to

$$g(\tilde{q}) = \beta Y/N. \tag{28}$$

The decentralized market will fulfil the optimality conditions if the tax rate is

$$r = -P_{M_1}/P_K, \tag{29}$$

as before.

The calibration of the model version with a backstop technology follows the baseline setup. In addition, based on Popp (2006) and Nordhaus (2008b), the carbon prices limiting the transition phase to the backstop are chosen to be $\underline{q} = 2.5q(0)$ and $\overline{q} = 3.5q(0)$, respectively. The resulting backstop cost per unit of energy in the pure backstop case (i.e. for $\tilde{q} > \overline{q}$) is $(\overline{q} + \underline{q})/2 = 3\tilde{q}(0)$.

The long-term behavior of the economy in a model with backstop technology differs from the baseline version, as the switch to the backstop technology leads to $R = -\dot{S} = 0$ in the steady state. The resource loses its economic value, $p = 0$. The economic module reduces to the well-known Ramsey model. The Ramsey model becomes stationary when written in terms of 'per effective labor'. As before, the climate module is stationary in nature.

7 Solution Approach Model with Backstop

We write the economic system per effective labor with $y := Y/E$, $c := C/E$, $k := K/E$ and $n := N/E$. Then, the production function, the Keynes-Ramsey Rule and the production balance of the economy read

$$y = \Omega(T_1)k^\alpha n^\beta, \tag{30}$$

$$\hat{c} = (i - \rho)/\theta - \hat{E}, \tag{31}$$

$$\hat{k} = (1 - \beta d(q + r)/g(q + r))\, y/k - c/k - \delta - \hat{E}, \tag{32}$$

with $\beta y d(q + r)/g(q + r) = ng(q + r)$ from $g(\tilde{q}) = \beta Y/N$.

To write the shadow values of the climate module in stationary terms, we define

$$z_M := -(P_M/P_K)e^{-\hat{E}t},$$
$$z_T := -(P_T/P_K)e^{-\hat{E}t}.$$

The steady-state growth rate of the shadow values of the climate module is $-i^* + \hat{E}$. Isolating $-P_M$ and $-P_T$, taking the derivative with respect to time, and multiplying with $e^{-\hat{E}t}/P_K$ yields

$$\dot{z}_M = iz_M - \hat{E}z_M - \Gamma^\top z_M - \left[z_T(1)\frac{\eta_1}{1000M(1)}, 0, 0\right]^\top,$$
$$\dot{z}_T = iz_T - \hat{E}z_T - \Lambda^\top z_T + \left[\frac{\partial Y}{\partial T_1}, 0\right]^\top.$$

For utility, we get $\dot{V} = -c^{1-\theta} - \left((1-\theta)\hat{E} - \rho\right) V$.

We derive operational boundary conditions from the stationary state, denoted by *. The stationary state is described by growth rates that equal zero. Energy costs do not depend on the tax rate anymore, as in the long run, only the backstop is used for energy production with constant costs $d(q^*) = (\underline{q} + \overline{q})/2$ is used.

Writing equations (31) and $\iota + \delta = \alpha Y/K$ in terms of 'per effective labor' yields

$$\hat{c}^* = \theta(\alpha y^*/k^* - \delta - \rho) - \hat{E} = 0 \tag{33}$$

$$\Leftrightarrow y^*/k^* = 1/\alpha\left(\hat{E}/\theta + \delta + \rho\right). \tag{34}$$

From (33) and (32) follow

$$
\begin{aligned}
i^* &= \alpha y^*/k^* - \delta = \hat{E}/\theta + \rho, \\
\hat{k}^* &= y^*/k^* - c^*/k^* - \beta g(q^*)/d(q^*) y^*/k^* - \delta - \hat{E} = 0,
\end{aligned} \tag{35}
$$

respectively. Define s^* as the gross saving rate in stationary state, with

$$
s^* y^*/k^* = y^*/k^* - c^*/k^* - \beta g(q^*)/d(q^*) y^*/k^*
$$

and, from (35), $s^* = \alpha(\delta + \hat{E})/(i^* + \delta)$.

To obtain values for k^* and y^*, we define

$$
\begin{aligned}
A &:= y^*/k^* = (i + \delta)/\alpha, \\
B &:= \Omega T_1 \left(\beta/d(q)\right)^{\beta} = y^{1-\beta}/k^{\alpha}.
\end{aligned}
$$

The latter follows from (30) with $q + r = \beta Y/R$ written in 'per effective labor' and plugged in for n. In steady-state, $y^*/k^* = y/k$. Then, $k^* = \left(B/A^{1-\beta}\right)^{1/(1-\alpha-\beta)}$ and $y^* = Ak^*$.

Furthermore, $c^* = (1 - s^* - \gamma)y^*$. As the saving rate is defined as the gross saving rate, costs for the energy input also have to be substracted to obtain consumption.

The end condition of c is

$$
\log(c(t_{\text{final}}) = \log(c^*). \tag{36}
$$

For the stationary shadow prices of the climate module, we obtain

$$
\begin{aligned}
z_M^* &= ((i^* - \hat{E})I - \Gamma^{\top}) / \left[z_T^*(1) \frac{\eta_1}{1000M(1)}, 0, 0 \right]^{\top}, \\
z_T^* &= -((i^* - \hat{E})I - \Lambda^{\top}) / \left[\frac{\partial y^*}{\partial T_1}, 0 \right]^{\top},
\end{aligned}
$$

with identity matrix I.

Finally,

$$
V^* = -c^{1-\theta}/((1-\theta)\hat{E} - \rho),
$$

with $(1-\theta)\hat{E} - \rho$ being the steady-state growth rate of V.

To sum up the operational boundary conditions, for the state variables k, S, M, T and D, we use the initial boundary conditions. Instead of the transversality conditions, we require the climate shadow values to reach z_M^* and z_T^*, and for V to reach V^*, respectively. For consumption per effective labor, we require (36) to hold. For p, we use $\lim_{t\to\infty} D(t)p(t)S(t) = 0$ with $t = t_{\text{final}}$.

8 Some Results

To determine the critical lag for an announced emissions tax, we calculate the welfare differences between scenarios with an announced emissions tax and a do-nothing policy. The scenario with announcement is a scenario where, at the start of the time horizon, we have the announcement of the implementation of a tax in $t > 0$ that is optimal from the instant of implementation onward. In $t = 0$, agents correctly anticipate the introduction of the tax in $t > 0$. As set out before, the time path of the announced intervention starting in $t > 0$ cannot be chosen optimally from the present point of view, i.e. taking anticipation by the market participants into consideration, because such a policy would be time-inconsistent and thus not credible.

Prior to the implementation of the respective tax the announcement of the emissions tax causes a behavior change in comparison with the do-nothing scenario. This is the announcement effect. In the case with announcement emissions are initially higher than emissions in the case without intervention.

Despite the negative announcement effect, an optimal emissions tax implemented with a lag is still desirable if the overall welfare effect of the policy is positive. Figure 3 plots the welfare gain of the (delayed) intervention compared to no-intervention over the length of the implementation lag. A positive number indicates a welfare gain, a negative number indicates a welfare loss, which implies that, in welfare terms, no intervention would be better. For an announcement period of zero, the figure displays the welfare gain of the optimal policy. We find that the overall welfare effect depends on the length of the announcement period. A 'critical lag' for favorable, i.e. welfare-enhancing, climate policy arises. It is defined such that a shorter implementation lag is welfare-increasing compared to no-intervention, but a longer implementation lag is welfare-reducing. The critical lag is 69 years. If one looks at the Kyoto-Protocol with an implementation lag of 11 years, the implementation lag is shorter than the critical lag: policy-makers do not need to worry about the adverse announcement effect turning welfare negative. Still, as Figure 3 shows, earlier implementation is a welfare gain compared to later implementation.

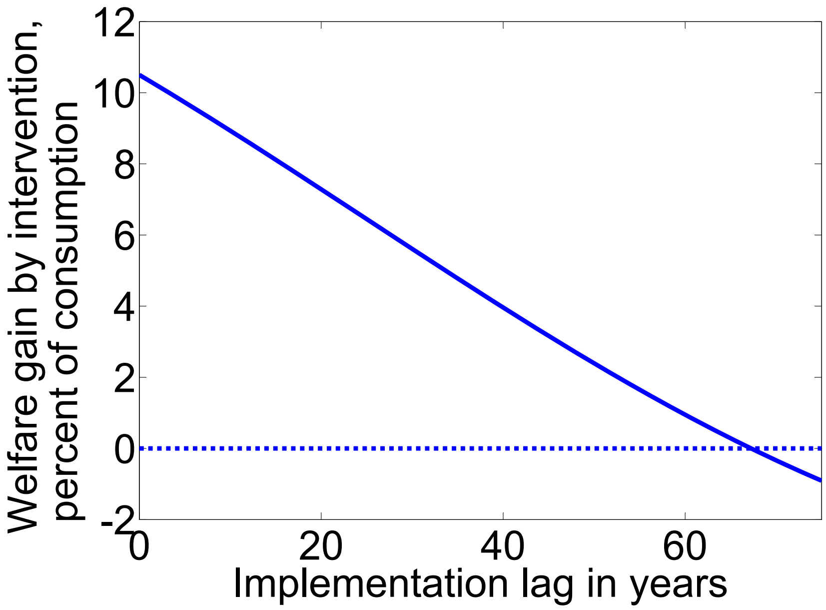

Figure 3: Critical lag: the welfare effect of lagged policy implementation depending on the length of the implementation lag.

With an instantaneously implemented optimal emissions tax, one gains a relative intertemporal equivalent variation of $h = 10.5$ % over laissez-faire corresponding to an absolute intertemporal equivalent variation of $W = 831.8 \times 10^{12}$ \$, nearly 8 times the 2015 GDP. This effect is large compared to many results from integrated assessment models, but not surprisingly large compared to models with threshold effects and tipping points (see e.g. Golosov et al. (2014) or Bretschger and Vinogradova (2016)). The tax corresponds to the 'Social costs of carbon'. Nordhaus (2016a) states estimates of 676.2 and 723.9 \$ per tC for 2015 when damages are scaled to reflect a maximum of 2.5 degree temperature increase and when discounting is low, respectively.[8] Our estimate of 1,282 \$ per tC is comparably larger, but we consider high damages *and* low discounting. Also, Golosov et al. (2014) report an even higher estimates of 4,263 \$ per tC for a low discount rate and 'catastrophic' damages.

[8] We state Nordhaus (2016a)'s estimates in terms of \$ per tC instead of \$ per CO_2 by multiplying with 3.667 based on Nordhaus (2014).

How does allowing for a backstop technology change the previous results? As expected, the welfare effect of an instantaneous implementation is larger in the case with a backstop technology than in the model without backstop (h=27.8 % compared to 10.5 % in the baseline case). Also, the introduction of a backstop technology leads to a larger critical lag. Unlike the baseline case, for an implementation lag of 70 years, the welfare effect is still positive compared to no-intervention.

Further results are discussed in: Riekhof, M.-C. and Broecker, J. (2017): Does the adverse announcement effect of climate policy matter? - A dynamic general equilibrium analysis. *Climate Change Economics*, forthcoming.

For results based on an older model version with staring year 2005, a different damage function and a higher steady state interest rate, see the ETH working paper version: Riekhof, M.-C. and Broecker, J. (2016): Does the adverse announcement effect of climate policy matter? - A dynamic general equilibrium analysis. CER-ETH Working Paper 16/254.

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Table 2: Overview units.

Abbreviation	Explanation
`$`	2010 US dollars
`GtC`	gigatons carbon
`kgC`	kilograms carbon
`Gtoe`	gigatons oil equivalent
`kgoe`	kilograms oil equivalent
`a`	annum
$^\circ$`C`	degrees Celsius
Δ°`C from 1900`	degrees Celsius difference from temperature level in the year 1900

Note that $\$/\texttt{kgC} = 10^{12}\$/\texttt{GtC}$.

A The Household's Optimization Problem

The household chooses consumption C to maximize utility

$$U = \int_0^\infty u(C)e^{-\rho\tau}\mathrm{d}\tau$$

subject to the budget constraint, with time-preference rate ρ. As usual, u has the constant intertemporal elasticity of substitution form

$$u = \frac{C^{1-\theta}}{1-\theta},$$

with intertemporal elasticity of substitution $1/\theta$. The flow budget constraint is

$$\dot{a} = \iota a + (1-\alpha-\beta)Y + rR - C, \tag{37}$$

stating that saving $\dot{a}$ equals income minus consumption. Income has three components: interest on the asset, labor income, and tax income collected by the state and paid to the household. The market prevents chain-letter credit financing so that for $t \to \infty$, the present value of the asset must be non-negative (**?**, p. 92),

$$\lim_{t\to\infty} D(t)a(t) \geq 0.$$

The optimality conditions are the Keynes-Ramsey Rule $\hat{C} = (\iota - \rho)/\theta$ and the transversality condition

$$\lim_{t\to\infty} D(t)a(t) = 0. \tag{38}$$

It is convenient to write the budget constraint (37) in a different equivalent way by inserting (2), (3), and (10),

$$\dot{K} + \dot{v} = \iota(K+v) + Y - (\iota+\delta)K - qR - C. \tag{39}$$

Taking the time derivative of (4), using (6) yields

$$\dot{v} = \iota v - R(q - k(S)). \tag{40}$$

Substituting this for $\dot{v}$ in (39), we obtain the production balance of the economy (11),

$$\dot{K} = Y - \delta K - Rk(S) - C.$$

As both components of $D(t)a(t) = D(t)K(t) + D(t)v(t)$ are non-negative, for (38) to hold, both components have to approach zero as $t \to \infty$. The transversality condition (38) thus implies (13)

$$\lim_{t\to\infty} D(t)K(t) = 0.$$

B Solution concept

In the long run, we make use of the model's property that it comes arbitrarily close to a steady-state (or Balanced-Growth-Path) with constant growth rates. To calculate the steady state, we make the following two assumptions. First, as emissions asymptotically tend towards zero, we consider the steady-state system that would prevail if no more emissions entered the climate module. Second, we assume that in the long term the extraction cost component in the production balance (11) grows more slowly than the capital stock, i.e.

$$\hat{R} - \epsilon\hat{S} < \hat{K}, \tag{41}$$

with ϵ denoting the elasticity of extraction costs with respect to the resource stock. With the assumption of constant steady-state growth rates, $\hat{Y}^* = \hat{K}^* = \hat{C}^* = g$ and $\hat{S}^* = \hat{R}^* = g - \iota^*$.

To obtain the model solution, we directly solve the dynamic market equilibrium conditions with $r = 0$ for the market outcome and $r = -P_M/P_K$ for the planner's solution. We use the bvp5 two-point boundary-value problem solver in matlab. Due to numerical considerations, we solve the model in terms of *stationary transforms* of the variables (see Bröcker and Korzhenevych (2013) for more details) such that the whole model becomes stationary. A stationary model means growth rates that equal zero. A stationary transform is defined by $\tilde{X} = Xe^{-\hat{X}^*t}$, with $\hat{X}^*$ denoting the steady-state growth rate. The stationary transforms are solved in log-deviations from the non-transformed values in 2015, obtained from data. For the shadow values, no data for 2015 are available. In this case, we use log-deviations from a 'proxy steady-state'. The proxy steady state is defined like the steady state, but with climate as in $t = 0$. Two variables are not solved in log-deviations: V is used directly, while T is solved in absolute deviations from the respective 2015 data.

As boundary conditions, we require a subset of the variables to reach constant growth rates at a final point in time. This is similar to requiring some variables to reach their steady-state values at a finite horizon, a method known from the literature (see Bröcker and Korzhenevych (2013) for a discussion). As there is some arbitrariness in fixing the finite horizon, we vary the horizon to ensure that the choice of the finite horizon does not affect results. We also run a model version using the stable manifold approach proposed by Bröcker and Korzhenevych (2013). The results do not change. The procedure with final boundary conditions is justified as discounting makes the present value welfare for one scenario with a finite and another scenario with an infinite horizon come arbitrarily close to each other.

C Calibration

The growth rate of effective labor is $\hat{E}$ =1.74 % `1/a` to obtain a steady-state growth rate of 1.5 % `1/a` (similar to Nordhaus (2016b)). Furthermore, the depreciation rate is $\delta = 0.05$ `1/a` (?). The resource's cost share is $1 - \alpha - \beta = 0.058$, calculated from from IEA (2007) and EIA (2010) based on data for 2005. Remaining income is divided between capital (1/3) and labor (2/3) (?). Total income is $Y(0) = 105.5 \times 10^{12}$ `$/a` (Nordhaus 2016b)

For the consumption side of the model, the elasticity of intertemporal substitution $1/\theta$ =1/1.3, and the time-preference parameter ρ =0.5 % `1/a` to yield a real interest rate in steady state of ι^* =2.46 % `1/a`.

There are no reliable data available for the initial capital stock $K(0)$. As capital dynamics are fast compared to the climate system, so that the steady-state relation is quickly reached, we use the steady-state capital-output ratio $K/Y = \alpha/(i^* + \delta)$ to calibrate $K(0)$. This gives an initial capital stock of $K(0)$ =443.99×10^{12} `$`.

To calibrate the resource sector, we assume ϵ to be constant, i.e. we specify extraction costs as

$$k(S) = \gamma S^{-\epsilon}, \tag{42}$$

similar to Laitner (1984). For condition (41) to hold in steady state,

$$\epsilon < \frac{\iota^*}{\iota^* - g} \tag{43}$$

is required, with steady-state interest rate ι^* and steady-state growth rate of the economy g. To see why, use the fact that the assumption of constant steady-state growth rates leads to $\hat{Y}^* = \hat{K}^* = \hat{C}^* = g$ and $\hat{S}^* = \hat{R}^* = g - \iota^*$. The variables of the climate module are constant in the long tun, which implies $\hat{\Omega}(T_1)^* = 0$. Condition (41) thus reads $(1 - \epsilon)(g - \iota^*) < g$, which one can rearrange to get (43). Extraction costs are zero for $\gamma = 0$ and constant for $\epsilon = 0$. For $\epsilon = 1$, unit extraction costs double if the stock is halved.

For the initial resource stock $S(0)$—the sum of oil, gas, and coal stocks converted into carbon—, we use data on the carbon resource base (the sum of reserves and resources)[9] and set $S(0) = 3000$ `GtC` (McGlade and Ekins 2015; UNDP 2000). As 'Resources' in particular are only recoverable with technological developments, we assume rising extraction costs and use $\epsilon = 1$. Condition (43) holds. The parameter γ is chosen to ensure an initial resource flow of $R(0)$ =9.78 `GtC`, which results in an initial resource price of $q(0)$ =0.63 `$/kgC`.[10] Note that the resource price has a cost component, $k(S) = \gamma S^{-\epsilon}$, and a rent component, p. The rent component is endogenous and driven by the abundance of the resource. It turns out to be 0.22 `$/kgC` in the base calibration.

Given $q(0)$, we can calculate rent-to-price ratios—the rent share in the price—to compare the model result with available data. Based on the model results, the rent-to-price ratio is 35% for 2015 Bauer et al. (2013) present data for 2010, with rents close to zero for natural gas and coal in Russia, but rent-to-price ratios around 50% for crude oil in the Middle East and North Africa, natural gas in EU27, and coal in China, and around 30% for crude oil in the USA. The average rent-to-price ratio for some European countries—namely the Netherlands, Denmark, the United Kingdom, and Norway—in 1999 for oil and gas was 34% (calculation based on information from European Commission (2002)). If coal were included, the rent-to-price ratio might decrease. On the one hand, coal with a reserve-to-production ratio[11] of 224 `a` is relatively more abundant than oil and natural gas with reserve-to-production ratios

[9] UNDP (2000, p. 481) defines 'Reserves' and 'Resources' as follows: 'Reserves: those occurrences of energy sources or mineral that are identified and measured as economically and technically recoverable with current technologies and prices' and 'Resources: those occurrences of energy sources or minerals with less certain geological and/or economic/technical recoverability characteristics, but that are considered to become potentially recoverable with foreseeable technological and economic development'.

[10] Note that $/`kgC` = $10^{12}$$/`GtC`.

[11] The reserve-to-production ratio gives the period the resource stock will last at the current extraction rate.

Table 3: Comparison of the predicted global mean temperature increases for the year 2100.

Scenario	our model	B1*	A1B*	A2*	RCP2.6**	RCP8.5**
Δ°C from 1900	3.5	2.4	3.4	4.2	1	4.1

*based on IPCC (2007):
A1B assumes a converging world with very rapid economic growth and balanced technological change;
B1 also assumes a converging world, but with a rapid change towards a service and information technology;
A2 considers a fragmented world with increasing population and slower technological change;
**based on Collins et al. (2013):
RCP2.6 is the low-emission scenario, RCP8.5 is the high-emission scenario.

of 40 and 62 a, respectively (Feygin and Satkin 2004).[12] On the other, deposits vary, e.g. in quality and extraction costs, so that high rents also occur for coal. Summarizing available studies, an average rent-to-price ratio is difficult to determine. It seems that our result is comparable to available estimates.

Overall, the set-up and the calibration of our model leads to results comparable to those found in other studies. An important measure is the predicted global mean surface temperature in the laissez-faire setting. Our model calculates a global mean surface temperature for the year 2100 that is in the range of the IPCC predictions (see Table 3).

Table 4: Overview variables

Symbol	Explanation	Value	Unit	Source
C	consumption		10^{12} \$/a	endogenous
ν in 2015	emissions from land use change	0.709	GtC/a	Nordhaus (2016b), see Note 1
R in 2015	extracted carbon resource flow = emissions	9.7764	GtC/a	Nordhaus (2016b), to convert from CO_2 into C: divide by 3.667
M_1 in 1750	carbon mass in atmosphere	596.4	GtC	Nordhaus (2008b), Nordhaus (2010)
M_1 in 2015	carbon mass in atmosphere	851	GtC	Nordhaus (2016b)
M_2 in 2015	carbon mass in upper ocean	460	GtC	Nordhaus (2016b)
M_3 in 2015	carbon mass in lower ocean	1740	GtC	Nordhaus (2016b)
P_M	shadow prices of carbon stocks		utils/GtC	endogenous
P_T	shadow prices of temperatures		utils/Δ°C from 1900	endogenous
P_K	shadow price of capital stock		utils/ 10^{12} \$	endogenous
P_S	shadow price of resource stock		utils/GtC	endogenous
q in 2005	resource flow price	0.3415	\$/kgC	EIA (2010), see Note 3
ι^*	steady-state interest rate	0.0246	1/a	see Note 4
S in 2015	resource stock	3000	GtC	McGlade and Ekins (2015); UNDP (2000)
T_1 in 2015	global mean surface temperature	0.85	Δ°C from 1900	Nordhaus (2016b)
T_2 in 2015	temperature lower ocean	0.0068	Δ°C from 1900	Nordhaus (2016b)
U	overall utility		present value utils	endogenous

[12]The 'resource base'-to-production ratio in the model is 387 a.

u	instantaneous utility		`utils/a`	endogenous
Y in 2015	GDP	105.5, PPP	10^{12}`$/a`	Nordhaus (2016b)
r	tax rate		`$/kgC`	endogenous
h	relative intertemporal equivalent variation		1	endogenous
W	absolute intertemp. equiv. variation		10^{12}`$`	endogenous

Table 5: Parameters taken from literature

Symbol	**Explanation**	**Value**	**Unit**	**Source**
Γ	carbon transition matrix	$\begin{bmatrix} -0.0240 & 0.0392 & 0 \\ 0.0240 & -0.0406 & 0.0003 \\ 0 & 0.0014 & -0.0003 \end{bmatrix}$	`1/a`	Nordhaus (2016b) see Note 5
δ	depreciation rate	0.05	`1/a`	**?**
α	income fraction capital	0.314	`1`	$(1-\beta)/3$, **?**, p. 58
θ	inverse of the intertemporal elasticity of substitution	1.3	`1`	see Note 4
Λ	temperature interaction matrix	$\begin{bmatrix} -0.0256 & 0.0018 \\ 0.0050 & -0.0050 \end{bmatrix}$	`1/a`	Nordhaus (2016b), see Note 6
ρ	time preference rate	0.005	`1/a`	see Note 4
$\underline{q}$, $\overline{q}$	prices limiting the transition phase to the backstop	$2.5q(0)$, $3.5q(0)$	`1`	based on Popp (2006) and Nordhaus (2008b)

Table 6: Functions

Symbol	**Explanation**	**Value**	**Unit**	**Source**
$\Pi(M_1, t)$	radiative forcing	$\eta_1 \log\left(\frac{M_1(t)}{M_1 \text{in} 1750}\right) + \eta_2 F_{EX}$	°`C/a`	Nordhaus (2010), see Note 8
$\Omega(T_1)$	damage function	$b(1 - \omega\xi_1 T_1 + \omega\xi_2 T_1^{\xi_3})$		Nordhaus (2016b), see Note 9
$k(S)$	extraction cost function	$\gamma S^{-\epsilon}$	`$/kgC`	see Section 5

Table 7: Calibrated parameters

Symbol	**Explanation**	**Value**	**Unit**	**Source**
K in 2015	capital stock		10^{12}`$`	$K(2015) = \alpha Y(2015)/(\iota^* + \delta)$
β	income fraction resource	0.058	`1`	Calibration: $\frac{q_{2005} R_{2005}}{Y_{2005}}$, see Note 2
$\hat{E}$	growth rate of effective labor	0.0174	`1/a`	Nordhaus (2016b), see Note 7
γ	scaler extraction costs			see Section 5
ϵ	exponent extraction costs	1	`1`	see Section 5

Notes

1. Total emissions in `GtC/a` are $R + \nu$, with $\nu = \nu(0)e^{-.0101t}$. The value of carbon emissions from land-use change in $t = 0$, i.e. in 2015 is 0.709 `GtC/a` (Nordhaus 2016b) The yearly shrinking rate of the emissions from land-use change is also taken from (Nordhaus 2016b) and transformed into its continuous time yearly counterpart, i.e. from $(1 - 0.115)^{t-1}$ to $\log(1 - 0.115/5) = -.0233$.

2. As we assume constant income shares, we base the estimates on 2005 data. Data on resource flows and stocks as well as on emissions are based on IEA (2007). Because the model only uses one carbon resource, we merge the data on oil, coal, and gas and convert units into gigatons of carbon, `GtC`. We use emissions in 2005 over resource extraction in 2005 to calculate a conversion factor. The derivation of the resource price is given in note 3.

3. The resource price q in 2005 is a weighted average of the average 2005 prices for oil, steam coal, and natural gas in `$/kgoe`, 0.3562, 0.0964 and 0.3109, respectively. The coal and gas prices are roughly estimated average prices 2005 from IEA (2007), while the oil price is the average of the 'Weekly All Countries Spot Price FOB Weighted by Estimated Export Volume' (EIA 2010). We take the shares of each resource in total resource consumption 2005 as weights and convert the units into `$/kgC`. The calculation yields $q = 0.3415$ in `$/kgC`.

4. The choices of θ and ρ lead to a steady state interest rate of $\iota^* = \rho + \theta g = 2.46\%$ `1/a`. Piketty and Zucman (2014) report rates of return on private wealth for eight rich countries of the world in 2010 ranging from 4% to 8%, with median 5%.[13] The global median of safe funds rates adjusted for inflation, however, presented in a smoothed long time series by Hamilton et al. (2015), is currently around zero. We take the middle between both as a pragmatic estimate of an average certainty equivalent of real asset returns.

5. We calculate the carbon transition matrix according to $\Gamma = \frac{B_1 - I}{5}$, using its discrete time counterpart, say B_1, in Nordhaus (2016b), with

$$B_1 = \begin{bmatrix} 0.8800 & 0.1960 & 0 \\ 0.0120 & 0.7970 & 0.0015 \\ 0 & 0.0070 & 0.9985 \end{bmatrix},$$

   and the identity matrix I. $B_1 - I$ is divided by 5 because one period in the DICE model corresponds to five years.

6. We calculate the temperature interaction matrix according to $\Lambda = \frac{B_2 - I}{5}$, using its discrete time counterpart, say B_2, in Nordhaus (2016b), with

$$B_2 = \begin{bmatrix} .8718 & .0088 \\ .0250 & 0.9750 \end{bmatrix}.$$

   As before, we divide by 5 because one period in the DICE model corresponds to five years.

7. The rate of technological progress is calibrated to match a steady-state growth rate of 1.5 % `1/a`. For g =0.0151= $\frac{(1-\alpha-\beta)\hat{E}-\rho\beta}{1-\alpha-\beta+\beta\theta}$ based on $\hat{q}^* = \iota^*$, meaning that $k(S)$ grows more slowly than p in the long term, the rate of technological progress is $\hat{E} = (0.0151(1-\alpha-\beta+\beta\theta)+\rho\beta)/(1-\alpha-\beta) =$ 1.74 % `1/a`.

8. F_{EX} is the exogenous radiative forcing in `watt/m`^{2}. It increases linearly from 0.5 to 1 `watt/m`^{2} within 90 years and stays constant afterwards. The parameter values $\eta_1 = 0.1068$ `°C/a` and $\eta_2 = 0.0201$ `(°C/a)(m`^{2}`/watts)` are based on Nordhaus (2016b).

9. We take the parameters $\xi_1 = 0$, $\xi_2 = 0.00236$ and $\xi_3 = 2$ from Nordhaus (2016b) and use the scaler b to ensure $Y(0) = \bar{Y}$. The parameter ω serves for sensitivity analysis and equals 4.4 based on Nordhaus (2014) to have damages more align with the two-degree target of the international community.

[13] The detailed data can be found in their Online-Appendix Piketty and Zucman (2013), Table A145.