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Abstract

We analyze the consequences of habit formation for income levels and long-term growth in an overlapping generations model with dynastic altruism and resource dependence. If the strength of habits is below a critical level, the competitive economy displays an altruistic (Ramsey-like) equilibrium where consumption sustainability obeys the Stiglitz condition, and habits yield permanent effects on output levels due to transitional effects on growth rates, capital profitability and speed of resource depletion. If the strength of habits is above the critical threshold, the economy achieves a selfish (Diamond-like) equilibrium in which habits increase growth rates and resource depletion even in the long run, sustainability conditions are less restrictive, consumption and output grow faster than in Ramsey equilibria, but welfare is much lower. Results hinge on resource dependence, as different depletion rates modify the intergenerational distribution of wealth and thereby the growth rate attained in either equilibrium.

Keywords Dynastic Altruism, Overlapping Generations, Capital-Resource Model, Habit Formation.

JEL Codes Q30, D91, E21.

1 Introduction

A growing body of empirical evidence shows that preferences are status-dependent. Economic agents form habits, and tend to assess present satisfaction on the basis of deviations from the standards of living enjoyed in the past (Osborn, 1988; Fuhrer and Klein, 1998; Fuhrer, 2000). At the theoretical level, the pioneering work of Ryder and Heal (1973) has been extended in various directions by the recent literature on comparison utility (Carrol et al. 1997). Habit formation generates relevant reallocation effects that influence capital accumulation through saving decisions, and may affect growth and income levels in different ways depending on the assumed technology (Alvarez-Cuadrado et al. 2008).

This paper studies the interactions between capital accumulation and desired living standards in dynastic economies where exhaustible resources are essential inputs in production. Agents have finite lifetimes, exhibit habit formation and, due to a positive degree of intergenerational altruism, may decide to leave bequests to successors. The general aim is to describe the implications of status-dependent preferences for welfare, income levels, and long-term growth when production possibilities are constrained by resource scarcity. In this regard, the analysis fills a gap as the effects of habit formation in capital-resource economies has not been analyzed so far. More specific questions include the characteristics of the accumulation regimes arising in competitive equilibria, and the operativeness of bequest motives. The relevance of these issues can be drawn from the results of two independent strands of literature.

I. An established result of the overlapping-generations (OLG) literature is that dynastic models may exhibit two types of accumulation regimes and long-run equilibria (Abel, 1987). If desired bequests are strictly positive, the equilibrium path of capital-labor economies is observationally equivalent to that of infinite-horizon models *à la* Ramsey-Cass-Koopmans. If altruism is not operative, the economy is observationally equivalent to the Diamond's (1965) OLG model with selfish agents, and the long-run equilibrium may display under-investment or over-accumulation (e.g. Thibault, 2000). Two recent contributions show that habit formation is relevant for the determination of bequests. Alonso-Carrera et al. (2007) demonstrate that excessive strength of habits may induce non-operative bequests and thereby selfish equilibria. Schäfer and Valente (2009) study endogenous population dynamics, and conclude that lower bequests induced by stronger habits decrease the growth rate through reductions in fertility.

II. The literature on capital-resource models, pioneered by Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974), shows that when both man-made capital and exhaustible resources - e.g. oil - are an essential input in production, preference parameters determine the long-run growth rate even if there are complete markets and constant returns to scale. This *endogeneity result*[1] is relevant in both Ramsey-type and OLG frameworks. With infinitely-lived agents, the long-run growth rate is proportional to the difference between the rate of resource-augmenting technical progress and the social discount rate (Stiglitz, 1974). In OLG economies with selfish agents, the long-run growth rate depends on private preferences and the other parameters determining the intergenerational distribution of wealth (Mourmouras, 1991).

This paper shows that the interactions between dynastic altruism, habit formation and resource dependence generate a peculiar mechanism that extends the results of previous literature in non-trivial ways. On the one hand, the analysis confirms the negative relation between habits and desired bequests: there exists a critical level of status desire below which altruism is operative in the long run, and above which bequests are zero. On the other hand, the endogeneity result mentioned above implies that altruistic and selfish equilibria exhibit substantial differences in terms of growth rates and intergenerational welfare. The underlying mechanism is induced by the coexistence of habit formation and resource dependence, and thus does not arise in capital-labor economies.

The main results can be summarized as follows. If the strength of habits is below a critical level, the dynastic economy converges towards an altruistic regime in which habits yield permanent effects on output levels and transitional effects on growth rates and capital profitability. In these Ramsey-type equilibria, consumption levels are non-declining if the Stiglitz (1974) sustainability condition is satisfied - i.e. the rate of altruism does not exceed the rate of resource-augmenting technical progress. If the strength of habits is above the critical threshold, instead, the economy achieves a selfish equilibrium in which habits increase growth rates, reduce capital profitability, and raise the speed of resource depletion even in the long run. Notably, in these Diamond-type equilibria, the growth rate is higher than in the altruistic regimes because of habit formation. On the one hand, this implies that the sustainability condition is less restrictive. On the other hand, the result that stronger habits imply faster growth must be interpreted with great care. Infact, the selfish economy does not satisfy PV-optimality - i.e. the condition that the present-value stream of discounted utilities is maximized (Pezzey and Withagen, 1998). In Diamond-type equilibria, desired bequests are actually negative - i.e. agents would like to receive transfers from the successors - and the non-enforceability of reversed transfers implies a corner solution with zero bequests. Consumption and output dynamics are observationally 'more favorable', but welfare may be substantially lower than in Ramsey-type equilibria because of the asymmetry between desired and observed bequests. A numerical simulation shows that, despite faster growth and higher income levels in the long run, welfare levels in a permanent Diamond equilibrium induced by excessive habit formation may be

[1] We use this terminology because the presence of preference parameters in the reduced-form expression of the growth rate signals the endogenous determination of the growth rate (Solow, 2000: p.119). In this respect, the capital-resource model with utility-maximizing agents of Dasgupta and Heal (1974) and Stiglitz (1974) differs from standard growth models with capital and labor. On the one hand, it differs from the Ramsey-Cass-Koopmans model because the long-run growth rate depends on preference parameters. On the other hand, it differs from endogenous-growth models with increasing returns because the aggregate technology satisfies constant returns to scale.

46% lower with respect to Ramsey-type equilibria.

The plan of the paper is as follows. Section 2 describes the aggregate economy, and defines the centralized allocation as the solution to a command-optimum problem. This analysis provides the benchmark for studying the issue of observational equivalence, and for describing the effects of habit formation in isolation from private decisions concerning bequests. Section 3 describes a dynastic competitive economy in which the lifetime welfare of each agent is linked to that of successors through a positive degree of intergenerational altruism. Section 4 describes the two accumulation regimes that may arise in the competitive economy, and derives the main results concerning the properties of altruistic regimes, selfish equilibria and the operativeness of bequests. Since the various results are represented by different Lemmas, section 5 collects the main conclusions in three Propositions, and summarizes the differences with respect to the previous literature. Section 6 offers some concluding remarks.

2 The Aggregate Economy

The general structure is represented by a two-period OLG model, extended to include habit formation in consumption according to the specifications used in Alonso-Carrera et al. (2007) and Schäfer and Valente (2009). The aggregate technology is borrowed from the capital-resource model with resource-augmenting technical progress pioneered by Stiglitz (1974).[2] This section characterizes the centralized allocation that a social planner would implement in order to maximize the present-value stream of utilities enjoyed by different generations, using a pre-determined social discount rate. This analysis is useful because it abstracts from competitive equilibria, dynastic altruism and bequests: centralized allocations allow us to study the 'six effects of habit formation' in capital-resource economies, in isolation from the constraints set by private decisions concerning bequests.

2.1 General Assumptions

Demographic structure. Time is discrete and indexed by $t = 0, ..., \infty$. The economy features overlapping generations of households with each agent living two periods $(t, t+1)$. In each period t total population equals $N_t \equiv N_t^y + N_t^a$, where N_t^a and N_t^y represent the number of adult and young agents, respectively. Each young agent generates $n \geq 1$ children at the end of the first period of life, implying the gross growth rates $N_{t+1}/N_t = N_{t+1}^y/N_t^y = N_{t+1}^a/N_t^a = n$.

Technology. Our assumptions regarding production possibilities reflect two basic features of capital-resource models. First, the economy is resource-dependent, in the sense that both man-made capital and extracted resources (e.g. oil) are essential inputs for production. Second, there exists a positive rate of resource-augmenting technical progress, i.e. a process of technological improvement by which the productivity of the extracted resource increases over time (Stiglitz, 1974). Moreover, the two-period demographic structure requires an active role for labor provided by young agents. These characteristics are formalized by means of the aggregate production function

$$Y_t \equiv F\left(K_t, m_t X_t, N_t^y\right) = K_t^{\alpha_1} \left(m_t X_t\right)^{\alpha_2} \left(N_t^y\right)^{\alpha_3}, \tag{1}$$

where Y_t is final output, K_t is man-made capital, X_t is the flow of extracted resource, m_t is an index of resource efficiency in production, N_t^y is aggregate labor (i.e. each young agent supplies inelastically one unit of labor), and parameters satisfy $\alpha_1 + \alpha_2 + \alpha_3 = 1$ with $\alpha_i \in (0, 1)$. Technology (1) implies that man-made capital and the extracted resource are both *essential* in the sense of

[2] Capital-resource models have been extensively used in more recent literature in addressing several issues regarding sustainability (Pezzey and Withagen, 1998), adjusted measures of aggregate income (Asheim, 1994), the characterization of constant consumption paths (Withagen and Asheim, 1998). The Stiglitz (1974) variant with resource-augmenting progress has been extended to include endogenous technical progress (e.g. Barbier, 1999) and directed technical change (Di Maria and Valente, 2008).

Dasgupta and Heal (1974) - that is, $F(., m_t X_t, .) = F(., m_t \cdot 0, .) = 0$. Input $m_t X_t$ will be called 'augmented resource', since the productivity index m_t grows over time according to

$$m_{t+1} = m_t (1 + \gamma), \qquad \gamma \geq 0, \tag{2}$$

where γ is the exogenous net rate of resource-augmenting technical progress.[3]

Productive stocks. Man-made capital K_t is homogeneous with the consumption good, and is fully depreciated after one period. The aggregate constraint of the economy thus reads

$$K_{t+1} = Y_t - N_t^y c_t - N_t^a e_t = F(K_t, m_t X_t, N_t^y) - N_t^y \left(c_t + \frac{e_t}{n}\right). \tag{3}$$

where consumption of young and adult agents is respectively denoted as c_t and e_t in per capita terms. The resource input X_t is extracted from a finite, non-renewable stock, and can thus be interpreted as an exhaustible natural resource like oil, minerals or fossil fuels. Denoting by Q_t the resource stock at the beginning of period t, the physical transition law of the resource is

$$Q_{t+1} = Q_t - X_t. \tag{4}$$

Direct Utility. For the moment, we abstract from dynastic altruism: as this section focuses on centralized allocations, we can limit our specification of preferences to the component of individual welfare that is directly related to personal consumption. For any agent born in period t, the direct utility index $U(c_t, e_{t+1})$ represents the private benefits from the consumption levels enjoyed over the lifecycle. We allow for the presence of habit formation by specifying a two-period additive index of the form

$$U(c_t, e_{t+1}) \equiv \frac{c_t^{1-\eta} - 1}{1-\eta} + \beta \frac{(e_{t+1} - \epsilon c_t)^{1-\eta} - 1}{1-\eta}, \qquad \eta > 0, \tag{5}$$

where $\beta \in (0,1)$ is the private discount factor between the two periods of life, and $\epsilon \geq 0$ is the crucial parameter representing the strength of habits. Setting $\epsilon > 0$, the benefits perceived in the second period are weighted on the basis of the previous consumption level c_t, and the higher is ε the stronger is the effect of historical consumption status. Higher values of ϵ thus generate stronger willingness to overcome previous standards of living. Setting $\epsilon = 0$ habits are inactive, as the second-period term of consumption utility only depends on e_{t+1}.[4]

Given this general structure, it is possible to study various allocation mechanisms. Below, we solve a centralized command-optimum problem. Laissez-faire regimes with dynastic altruism will be analyzed later in sections 3-4, by means of a decentralized version of the same model economy.

2.2 Centralized Allocation

In this section, we characterize the solution of a centralized problem in which a hypothetical social planner endowed with perfect foresight aims at maximizing the social welfare function

$$SW \equiv \sum_{t=0}^{\infty} N_t^y U(c_t, e_{t+1}) \Delta^t, \qquad \Delta \in (0,1), \tag{6}$$

where $U(c_t, e_{t+1})$ is given by (5), and Δ is the social discount factor. For future reference, we also define the social discount rate as $\delta \equiv \Delta^{-1} - 1$. The term in square brackets in (6) is the sum of

[3] We assume that γ is exogenously given because studying the role of endogenous technical change is beyond the aim of the present analysis. Nonetheless, the non-negativity of γ and the fact that the rate of technical progress tends to be resource-augmenting rather than capital-augmenting exhibits sound microeconomic foundations provided by the theory of directed technical change: see Di Maria and Valente (2008).

[4] Letting $\eta \to 1$, the utility index (5) reduces to $U(c_t, e_{t+1}) \equiv \log c_t + \beta \log(e_{t+1} - \epsilon c_t)$, which is the same specification adopted e.g. in Schäfer and Valente (2009). The more general case $\eta \gtreqless 1$ complies with the general properties of subtractive habits employed in e.g. Alonso-Carrera et al. (2007).

direct utilities of each agent born at the beginning of period $t = 0, ..., \infty$. The *centralized allocation* is defined as a sequence $\{c_t^*, e_t^*, X_t^*, K_t^*, Q_t^*\}_{t=0}^{\infty}$ that solves the dynamic problem

$$\max_{\{c_t, e_t, X_t\}_{t=0}^{\infty}} SW \text{ subject to (3), (4) and} \tag{7}$$

$$\begin{aligned} N_{t+1}^y / N_t^y &= n, \qquad m_{t+1} = m_t (1+\gamma), \\ X_t &\geq 0, \qquad Q_t \geq 0, \qquad K_t \geq 0, \qquad (K_0, Q_0) \text{ given.} \end{aligned}$$

Problem (7) can be solved using standard optimal control theory. As shown in the Appendix, the interior solution can by characterized as follows.

Lemma 1 *(Centralized Allocation) An interior solution of problem (7) is characterized by the intertemporal conditions*

$$c_{t+1}^*/c_t^* = \left[\Delta F_{K_{t+2}}^* \left(F_{K_{t+1}}^* + \epsilon \right) \left(F_{K_{t+2}}^* + \epsilon \right)^{-1} \right]^{1/\eta}, \tag{8}$$

$$e_{t+1}^*/e_t^* = \frac{\epsilon + \beta^{1/\eta} \left(F_{K_{t+1}}^* + \epsilon \right)^{1/\eta}}{\epsilon + \beta^{1/\eta} \left(F_{K_t}^* + \epsilon \right)^{1/\eta}} \left(\Delta F_{K_{t+1}}^* \frac{F_{K_t}^* + \epsilon}{F_{K_{t+1}}^* + \epsilon} \right)^{1/\eta}, \tag{9}$$

$$F_{X_{t+1}}^* / F_{X_t}^* = F_{K_{t+1}}^*, \tag{10}$$

$$e_{t+1}^*/c_t^* = \epsilon + \beta^{1/\eta} \left(F_{K_{t+1}}^* + \epsilon \right)^{1/\eta}, \tag{11}$$

where we have defined the partial derivative $F_{K_t} \equiv \partial F / \partial K_t$ and the chain derivative $F_{X_t} \equiv dF/dX_t$.

Lemma 1 is proved without exploiting the Cobb-Douglas form (1), so that the above intertemporal conditions hold for any well-behaved technology $F(K_t, m_t X_t, N_t^y)$. Expressions (8) and (9) govern the dynamics of consumption levels across generations, and are obtained from the Euler condition for the allocation of consumption across each agent's lifecycle. Both conditions show that, as long as the marginal product $F_{K_t}^*$ is time-varying, habits modify the growth rates of both types of consumption. Condition (10) is an intertemporal no-arbitrage condition over the use of productive stocks, and is a general-equilibrium variant of the so-called *Hotelling rule* - according to which the growth rate of resource prices must be equal to the prevailing interest rate on alternative investment. In the present context, this rule asserts that the marginal contribution of the raw resource to production, F_X, must grow at a rate equal to the marginal contribution of capital, F_K. Condition (11) determines the allocation of consumption across each agent's lifecycle, and reduces to the standard Euler condition $e_{t+1}/c_t = \left(\beta F_{K_{t+1}} \right)^{1/\eta}$ when habits are inactive.

The last expression in Lemma 1 deserves some further comments. In fact, condition (11) shows that habit formation induces a bias in favor of second period consumption: for any given level of the marginal product of capital, we have

$$\frac{\partial}{\partial \epsilon} \left(e_{t+1}^* / c_t^* \right) < 0. \tag{12}$$

Result (12) is the *consumption-bias effect* of habit formation (Schäfer and Valente, 2009). The intuition behind (12) is simple: stronger habits, i.e. higher values of ϵ, represent a stronger desire to overcome previous standards of living, and imply that second-period consumption must exceed first-period consumption to a greater extent in order to maximize the private utility index. Clearly, the consumption-bias effect has an impact on the rates of capital accumulation and of resource extraction. Since ϵ affects the allocation of consumption between youth and adulthood, the strength of habits modifies the time paths of capital, resource use, and output. This mechanism can be labelled as an 'intertemporal reallocation induced by habits', and will be studied in detail in section 2.4.

The conditions derived in Lemma 1 allow us to derive some important properties of the long-behavior of the economy. Following a standard procedure, we characterize the asymptotic values of the crucial variables by imposing that the marginal product of capital converges to a finite steady-state value.

Lemma 2 *(Steady-state centralized allocation) Provided that the marginal product of capital* $F^*_{K_t}$ *converges to a finite steady-state* $\lim_{t\to\infty} F^*_{K_t} = F^*_{K_{ss}}$*, the centralized allocation implies*

$$
\begin{aligned}
\lim_{t\to\infty} c^*_{t+1}/c^*_t &= \lim_{t\to\infty} e^*_{t+1}/e^*_t = \left(\Delta F^*_{K_{ss}}\right)^{1/\eta}, &(13)\\
\lim_{t\to\infty} c^*_t/e^*_t &= \left(\Delta F^*_{K_{ss}}\right)^{1/\eta}\left[\epsilon+\beta^{1/\eta}\left(F^*_{K_{ss}}+\epsilon\right)^{1/\eta}\right]^{-1}, &(14)\\
\lim_{t\to\infty} Y^*_{t+1}/Y^*_t &= \lim_{t\to\infty} K^*_{t+1}/K^*_t = n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}, &(15)\\
\lim_{t\to\infty} X^*_{t+1}/X^*_t &= n\Delta^{1/\eta}\left(F^*_{K_{ss}}\right)^{\frac{1-\eta}{\eta}}. &(16)
\end{aligned}
$$

This steady-state is unique, and represented by

$$F^*_{K_{ss}} = (1+\gamma)^{\frac{\alpha_2\eta}{\alpha_3+\alpha_2\eta}}\,\Delta^{-\frac{\alpha_3}{\alpha_3+\alpha_2\eta}}. \tag{17}$$

Lemma 2 incorporates three relevant results. The first is that, in the centralized allocation, the sign of consumption variations is determined by the Stiglitz (1974) condition for sustainability: individual consumption levels are sustained (i.e. non-declining) in the long run if and only if the social discount rate does not exceed the net rate of resource-augmenting technical progress. Indeed, substituting (17) in (13), the asymptotic growth rate of individual consumption levels is

$$\lim_{t\to\infty} c^*_{t+1}/c^*_t = \left[\Delta(1+\gamma)\right]^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}} = \left(\frac{1+\gamma}{1+\delta}\right)^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}}, \tag{18}$$

from which it follows that a necessary and sufficient condition for $\lim_{t\to\infty} c^*_{t+1}/c^*_t \geq 1$ is $\gamma \geq \delta$.

The second implication of Lemma 2 is that the long-run growth rates of consumption and output are independent of the strength of habit formation. From (13) and (15), growth rates are determined by the discount factor and the long-run value of the marginal product of capital, which is independent of ϵ. As discussed in section 5, this result is in line with the infinite-horizon Ramsey model, where habits do not modify the long-run growth rate as long as capital displays decreasing marginal returns (Ryder and Heal, 1973).

The third result contained in Lemma 2 is that variations in the degree of habit formation modify the distribution of aggregate consumption between young and adult agents. Expression (14) shows that stronger habits raise the share of adults in aggregate consumption (at least) in the long run:

$$\frac{\partial}{\partial\epsilon}\lim_{t\to\infty}\left(c^*_t/e^*_t\right) < 0. \tag{19}$$

The intuition behind result (19) is the consumption-bias effect (12) described above. Since habit formation induces a bias in favor of second-period consumption for each agent, the steady-state centralized allocation features a higher (lower) consumption share for the adult generation in association with stronger (weaker) habits.

2.3 Dynamic Stability

The intertemporal conditions (8)-(11) can be reduced to a two-by-two dynamic system (see Appendix)

$$
\begin{aligned}
\chi^*_{t+1} &= \left(\chi^*_t\right)^{\alpha_1}\left(1-\varphi^*_t\right)^{-\alpha_3}\left[\alpha_1^{-1}(1+\gamma)\right]^{\alpha_2} n^{\alpha_3}, &(20)\\
\varphi^*_{t+1} &= \Theta\left(\varphi^*_t, \chi^*_t, \chi^*_{t+1}, \chi^*_{t+2}\right), &(21)
\end{aligned}
$$

where $\chi_t \equiv Y_t/K_t$ is the output-capital ratio, $\varphi_t \equiv N_t^y\left(c_t + e_t n^{-1}\right)/Y_t$ is the aggregate consumption-output ratio, and $\Theta\left(\varphi_t^*, \chi_t^*, \chi_{t+1}^*, \chi_{t+2}^*\right)$ is a non-linear function. Since $F_{K_t}^* = \alpha_1 \chi_t$, the achievement of a stationary marginal product of capital, $\lim_{t\to\infty} F_{K_t}^* = F_{K_{ss}}^*$, is associated with the convergence of system (20)-(21) towards the fixed point $(\chi_{ss}^*, \varphi_{ss}^*)$. The simultaneous steady-state of (20)-(21) is unique, and characterized by[5]

$$\chi_{ss}^* = \alpha_1^{-1}(1+\gamma)^{\frac{\alpha_2\eta}{\alpha_3+\alpha_2\eta}} \Delta^{-\frac{\alpha_3}{\alpha_3+\alpha_2\eta}}, \tag{22}$$

$$\varphi_{ss}^* = 1 - n\alpha_1 \Delta^{\frac{\alpha_2+\alpha_3}{\alpha_2\eta+\alpha_3}} (1+\gamma)^{\frac{\alpha_2(1-\eta)}{\alpha_2\eta+\alpha_3}}, \tag{23}$$

Since $\Theta\left(\varphi_t^*, \chi_t^*, \chi_{t+1}^*, \chi_{t+2}^*\right)$ is highly non-linear, the presence of χ_{t+1}^* and χ_{t+2}^* in (21) implies that the stability properties of $(\chi_{ss}^*, \varphi_{ss}^*)$ must be studied by means of numerical simulations. It is nonetheless fair to argue that the usual stability result holds under several combinations of parameter values. This conjecture is corroborated by the fact that dynamic stability can be established analytically for various special cases of the model, e.g. when labor is excluded from the set of production factors:

Lemma 3 *Letting $\alpha_3 \to 0$, system (20)-(21) implies* $\lim_{t\to\infty}(\chi_t^*, \varphi_t^*) = (\chi_{ss}^*, \varphi_{ss}^*)$, *and therefore* $\lim_{t\to\infty} F_{K_t}^* = F_{K_{ss}}$.

In the more general case ($\alpha_3 > 0$), the characteristics of system (20)-(21) can be assessed by deriving the transitional dynamics of (χ_t, φ_t) numerically. An example is reported in the next section, where we use backward iteration in order to describe the intertemporal reallocation effects induced by habit formation.

2.4 Intertemporal Reallocation: The Six Effects of Habits

The numerical analysis of system (20)-(21) is relevant because it clarifies the intertemporal reallocation effect induced by habit formation. Although ϵ does not modify the long-run *growth rates* of endogenous variables (see Lemma 2), the strength of habit formation affects their *levels* along the entire the time paths. The reason, as mentioned before, is that the consumption-bias effect (12) influences the rates of capital accumulation and of resource extraction in the short-medium run, and thereby output and consumption levels through the whole time-horizon. We now address this point more precisely by simulating the transitional dynamics by means of (20)-(21), and compare the resulting time paths of output, capital, consumption and resource use, for two economies, labelled as I and II, with identical parameters and initial endowments, but different degrees of habit formation. In particular, economy I is the benchmark case with inactive habits, $\epsilon^I = 0$, whereas economy II exhibits a positive degree of status desire, $\epsilon^{II} = 0.375$. Both economies approach the asymptotic values[6]

$$\lim_{t\to\infty} Y_{t+1}^*/Y_t^* = 1.80,\ \lim_{t\to\infty} c_{t+1}^*/c_t^* = 1.33,\ \varphi_{ss}^* = 0.67,\ F_{K_{ss}} = 1.84. \tag{24}$$

Values (24) can be obtained by setting $n = 1.35$, $\Delta = 0.725$, $\gamma = 1.45$, $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$, and $\eta = 1$. Notice that the assumption of unit elasticity in preferences is particularly useful, as it allows

[5] Combining $F_K^* = \alpha_1\chi_{ss}^*$ with (17) yields (22). Imposing $\chi_{t+1}^* = \chi_t^* = \chi_{ss}^*$ and $\varphi_t^* = \varphi_{ss}^*$ in (20), solving the resulting expression for φ_{ss}^*, and substituting χ_{ss}^* with (22), we obtain - after some tedious but straightforward algebra - equation (23).

[6] Values (24) are chosen by standard reasoning: as the interval $(t, t+1)$ is generally meant to represent 30 years, the first two assumptions correspond to an average per-annum growth rate of 2% in aggregate output, and 1% in output and consumption per capita. The aggregate consumption-output ratio $\varphi_{ss}^* = 0.67$ corresponds to the average consumption share observed in the last fifty years in most industrialized economies. Given the equilibrium relations stated in Lemma 2, a consistent value of the marginal product of capital is $F_{K_{ss}} = 1.84$, which can be associated to an average annual interest rate, net of depreciation, equal to 2%.

us to study the transitional effects of ϵ on consumption propensities in isolation from additional transitional phenomena generated by $\eta \neq 1$.[7]

Solving system (20)-(21) by backward iteration (see Appendix for details), the results of the simulation are as follows. There exists a unique path (χ_t^*, φ_t^*) such that the convergence hypothesis $\lim_{t\to\infty}(\chi_t^*, \varphi_t^*) = (\chi_{ss}^*, \varphi_{ss}^*)$ is satisfied with a feasible consumption ratio, i.e. $\varphi_t^* \in (0,1)$ in each t. Both economies are in a neighborhood of $(\chi_{ss}^*, \varphi_{ss}^*)$ within 10 periods, and experience balanced growth thereafter. Given equal initial stocks $K_0 = 1$ and $Q_0 = 27$, the associated dynamics in consumption, capital, output and resource use, are described in Figure 1. The main results are summarized in the six effects of habit formation listed below.

i. (*Consumption-bias effect*) Economy II converges towards a lower consumption share for young agents ($c_{ss}^I/e_{ss}^I > c_{ss}^{II}/e_{ss}^{II}$) - see diagram (a) - and displays higher second-period consumption levels in the long run - see diagram (b).

ii. (*Accumulation effect*) The need to obtain higher second-period consumption is fulfilled by reallocating resources in disfavor of first-period consumption, and in favor of capital accumulation. Since economy II exhibits faster accumulation of capital in the short run, K_t^{II} exceeds K_t^I by nearly 1% in the long run - see diagram (c).

iii. (*Input-substitution effect*) More intense capital accumulation allows to substitute resource inputs in production, so that economy II displays a higher capital-resource ratio in the short run - see diagram (d).

iv. (*Transitional growth effects*) Faster accumulation and higher capital-resource ratio imply that economy II displays a lower marginal product of capital, a higher growth rate of output, and a higher growth rate of resource use in the short run. These 'growth effects' are purely transitional, as they disappear in the long run - see diagrams (e), (f), and (g), respectively.

v. (*Output level effect*) Faster short-run growth in economy II is obtained at the expense of lower output levels in the short run, but gaining higher output levels in the long run. The positive output gap for economy II lasts forever, as the growth rates of both economies approach the same asymptotic level - see diagram (h). Output levels are nearly 1% higher in economy II in the long run.

vi. (*Resource-use effect*) The combination of the previous effects - in particular, faster accumulation and input substitution - imply that economy II extracts lower amounts of the resource in the short run, and higher amounts in the long run - see diagram (j).

The above results can be briefly commented as follows. The consumption bias effect has been already described in section 2.2. The consumption bias induces higher investment in capital and thereby faster accumulation in the short run. This in turn yields higher growth rates in the short run and higher output levels in the long run. The resource-use effect is a consequence of input substitution: the fact that the time paths of resource use, X_t^I and X_t^{II}, must cross during the transition is due to the constraint

$$Q_0 = \sum_{t=0}^{\infty} X_t, \tag{25}$$

which has to be satisfied by both economies, given the same initial stock Q_0 (see Appendix). The concluding remark of this section is that, although ϵ does not matter for the long run growth rate of the centralized economy, habit formation generates permanent effects on the intertemporal

[7] From condition (79), it is easily shown that, when $\eta = 1$, inactive habits ($\epsilon = 0$) imply $c_t^*/e_t^* = \Delta/\beta$, whereas $\epsilon > 0$ implies transitional dynamics in the distribution of consumption between cohorts. When $\eta \neq 1$, instead, c_t^*/e_t^* exhibits transitional dynamics with or without habit formation. This general result implies that setting $\eta = 1$ is particularly useful to stress the role of habits: all the differences arising in the transitional dynamics observed in economies I and II are exclusively due to different values of ϵ, the effects of which are not distorted by peculiar values assumed by η.

allocation, and therefore modifies the long-run levels of output, capital and consumption. The discussion of competitive equilibria in dynastic economies will show that the rise of selfish equilibria modifies the above results because the transitional growth effects of habit formation (iv) become permanent.

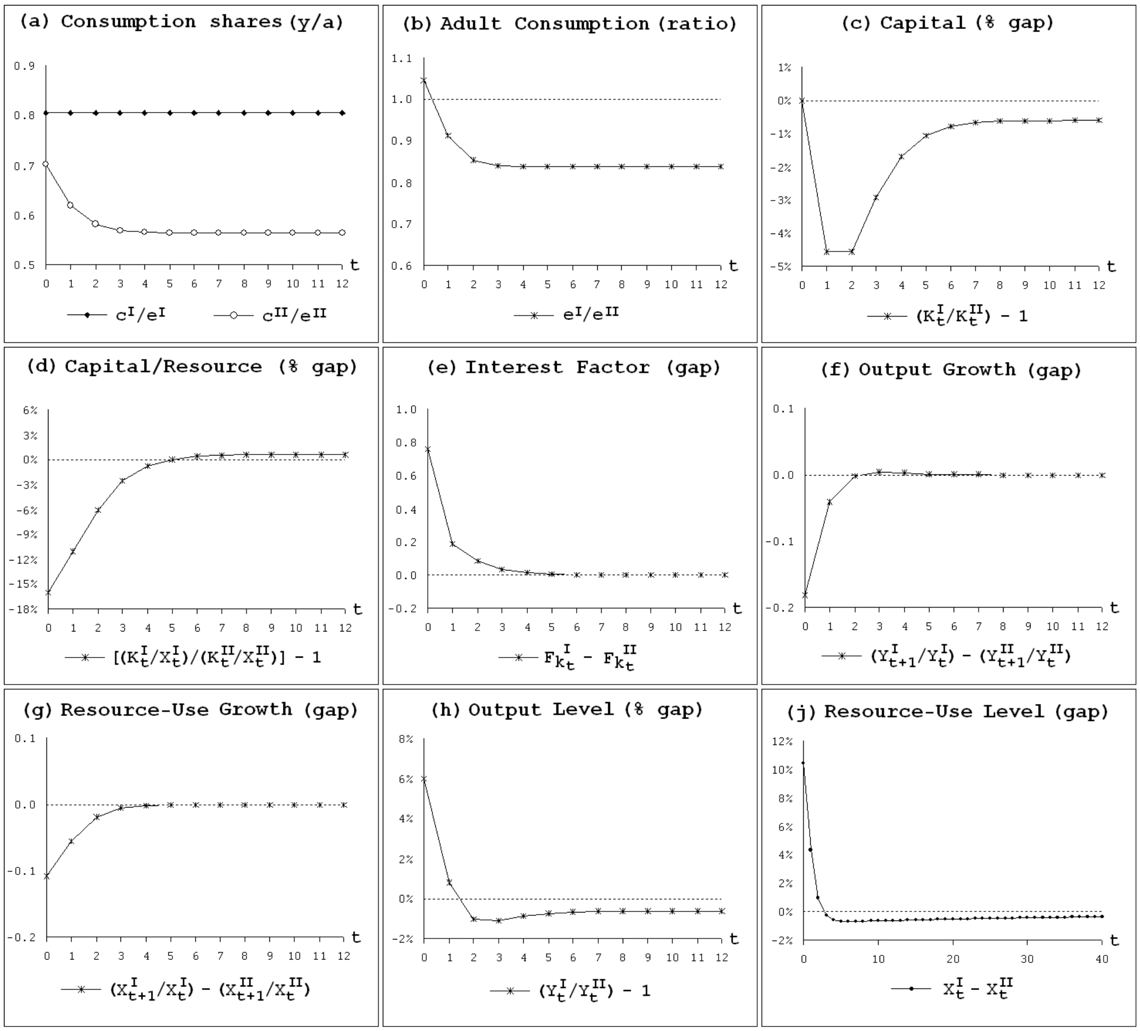

Figure 1: Transitional dynamics in the centralized allocation: simulation results. Parameter values are listed in section 2.4.

3 The Dynastic Competitive Economy

In this section, we study the behavior of *competitive dynastic economies* where private agents own the productive stocks, exhibit intergenerational altruism, and make consumption-saving choices in order to maximize private lifetime utility subject to individual budget constraints. The presence of altruism implies that agents may be willing to make transfers to successors. At the aggregate level, the economy satisfies the same basic relations (1)-(4) assumed in section 2.1.

A well-known result of the previous literature is that dynastic OLG models with one-way transfers - i.e. bequests defined as transfers in the father-to-son direction - exhibit two types of equilibria.

If transfers are strictly positive, the accumulation process is observationally equivalent to the optimal path of infinite-horizon models, with the altruism factor playing the same role as the social discount rate (Abel, 1985). If the bequest motive is not operative, the economy behaves as in the OLG model with selfish agents *à la* Diamond (1965): in capital-labor economies, the long-run equilibrium generally differs from the modified golden rule, and it may display dynamic inefficiency (Thibault, 2000; de la Croix and Michel, 2004).

A similar dichotomy applies to the present model. In the literature on capital-resource economies, the concepts of Ramsey-type and Diamond-type equilibria find their homologue in Stiglitz (1974) and Mourmouras (1991), respectively.[8] In the model of Stiglitz (1974), the resource-dependent economy is populated by infinitely-lived agents, and the optimal accumulation rule implies that the marginal product of capital converges asymptotically to the rate of resource-augmenting technical progress. In the model of Mourmouras (1991), the resource-dependent economy is populated by overlapping generations of selfish agents, and the laissez-faire equilibrium may exhibit over-exploitation of the resource stock. We refer to these results in defining the accumulation regimes that may arise in the dynastic economy:

Definition 4 *(Accumulation Regimes) If desired bequests are non-positive in each $t = t_0, ..., t_1$, the dynastic economy is said to exhibit a Diamond-Mourmouras temporary equilibrium in each $t = t_0, ..., t_1$. If desired bequests are strictly positive in each $t = t_0, ..., t_1$, the dynastic economy is said to exhibit a Ramsey-Stiglitz temporary equilibrium in each $t = t_0, ..., t_1$.*

The above definitions refer to definite intervals of time because, over an infinite time-horizon, the competitive economy may exhibit switchovers in accumulation regimes. In principle, the non-negativity constraint on bequests may be irrelevant up to a certain period, and become binding from that point onwards, or viceversa. In order to distinguish the situations in which there are no switchovers in the accumulation regime, we will exploit different definitions, that refer to permanent regimes:

Definition 5 *(Permanent Regimes) If desired bequests are non-positive in each $t = 0, ..., \infty$, the dynastic economy follows a Permanent Diamond-Mourmouras (PDM) equilibrium path. If desired bequests are strictly positive in each $t = 0, ..., \infty$, the dynastic economy follows a Permanent Ramsey-Stiglitz (PRS) equilibrium path.*

The remainder of this section lists the basic assumptions of the dynastic competitive economy, and derives the conditions that characterize dynastic competitive equilibria. Section 4 studies the characteristics of different accumulation regimes and the operativeness of bequests.

3.1 Assumptions

Private Preferences. In section 2.2, the analysis focused on centralized allocations with a social objective function represented by the discounted sum of direct utilities. In the competitive economy, a consistent specification of the private objective function is provided by altruistic preferences.[9] Formally, we define the *lifetime private utility* of each agent born in period t, denoted as W_t, as the sum of an individual term, represented by direct utility from consumption, and an altruistic term, related to the lifetime utility of the successors:

$$W_t = U(c_t, e_{t+1}) + \Psi n W_{t+1}, \qquad 0 < \Psi n < 1, \tag{26}$$

[8] The capital-resource model was pioneered by Solow (1974), Dasgupta and Heal (1974), and Stiglitz (1974) in the well-known *Symposium*. Actually, the 'homologue of the Ramsey model' with resource dependence and optimizing agents may well be considered the work of Dasgupta and Heal (1974), which provides a detailed characterization of the equilibrium path in the presence of exhaustible resources. In this paper, however, we refer our notion of equilibrium to the work of Stiglitz (1974) because it includes a positive rate of resource-augmenting technical progress (which is necessary to obtain sustained consumption in the long run).

[9] By 'consistent specification' we mean a private utility function that turns out to be structurally comparable with (5). Our claim that (26) is a consistent specification is intuitive. This point is clarified more rigorously in Lemma 8, which shows that, under some precise circumstances, the competitive equilibrium is observationally equivalent to the centralized solution studied in section 2.2.

where $U(c_t, e_{t+1})$ is given by (5), and Ψ is the altruism factor, i.e. the weight that each agent puts on the lifetime utility of each successor. For future reference, we also define the altruism rate as $\psi \equiv \Psi^{-1} - 1$.

Final producers. Given the assumption of constant returns to scale, the final sector can be represented as a unique competitive firm that exploits technology (1).The marginal rewards for capital use, resource use, and labor of young agents are respectively denoted as R_t, p_t^x, and w_t, and profit-maximizing conditions imply

$$R_t = F_{K_t} = \alpha_1 (y_t/k_t), \quad p_t^x = F_{X_t} = \alpha_2 (y_t/x_t), \quad w_t = F_{N_t^y} = \alpha_3 (y_t/n), \tag{27}$$

where F_{X_t} is the chain derivative $dF/dX_t = m_t \cdot \partial F/\partial (m_t X_t)$ as before.

Productive stocks. In the competitive economy, the productive stocks are privately owned by households. The mechanism by which the resource stock is allocated through different generations is purely market-based: at the beginning of period t, the whole stock of resources Q_t is held by adults. The extracted flow X_t is used for (and destroyed in) production, while the remaining stock constitutes *resource assets*, A_t, that are sold to the currently young. We thus have $Q_t = A_t + X_t$ and $Q_{t+1} = A_t = Q_t - X_t$, where the last expression follows from the natural transition law (4). Defining per-adult variables $q_t \equiv Q_t/N_t^a$, $x_t \equiv X_t/N_t^a$, and $a_t \equiv A_t/N_t^a$, the dynamic resource constraint can be re-expressed in individual terms as

$$q_t = a_t + x_t \text{ and } q_{t+1} = a_t/n = (q_t - x_t)/n. \tag{28}$$

Adults sell resource assets to the young at unit price p_t^a, and receive a marginal rent p_t^x for each unit of X_t supplied to firms. As regards capital, the whole stock K_t used for current production in t is owned by adult agents, so the whole stock K_{t+1} results from savings of young agents in period t.

Budget constraints. Following de la Croix and Michel (2004) and Thibault (2000), intergenerational transfers take the form of *inter-vivos* gifts, denoted by $b_t \geq 0$ and defined in the father-to-son direction. If bequest motives are operative, each young agent in period t receives b_t units of output and, in turn, will transfer b_{t+1} units of output to each of his n successors. Apart from bequests, young agents receive labor income w_t, and consume c_t units of the final good. The remaining income is either saved in the form of capital or used to purchase resource assets from the currently adult. In the second period of life, agents receive the rewards for capital and extracted resources sold to final producers, plus revenues from resource-assets sales to the young. Defining per-adult variables $k_t \equiv K_t/N_t^a$, $x_t \equiv X_t/N_t^a$, and $a_t \equiv A_t/N_t^a$, the budget constraints read

$$c_t = w_t + b_t - p_t^a (a_t/n) - k_{t+1}, \tag{29}$$

$$e_{t+1} = R_{t+1}k_{t+1} + p_{t+1}^x x_{t+1} + p_{t+1}^a a_{t+1} - nb_{t+1}. \tag{30}$$

Given the above assumptions, the problem of an agent born in period t consists of maximizing (26) - which contains the direct utilities of all the successors born after period t - subject to (28), (29), (30), and to the sequence of the same individual constraints that refer to all successors born after period t. This optimization structure is usually labelled as a *dynastic problem*, and can be solved in a recursive fashion, as shown below.

3.2 The Dynastic Problem

The dynastic problem can be reduced to a recursive consumer problem in which each agent born in period $t \geq 0$ chooses own consumption levels (c_t, e_{t+1}), the amount of resource assets to buy from the currently adult (a_t/n), the amount of capital to exploit in the subsequent period (k_{t+1}), the flow of extracted resource to sell to final producers (x_{t+1}), and the amount of bequests to transfer to each successor (b_{t+1}) under the assumption of perfect foresight. Given preferences (26), the utility-maximizing vector $\left(c_t^\ell, e_{t+1}^\ell, a_t^\ell, k_{t+1}^\ell, x_{t+1}^\ell, b_{t+1}^\ell\right)$ for each agent born in $t \geq 0$ is infact the vector that satisfies the Bellman equation

$$W_t^\ell (w_t + b_t) = \max_{\{c_t, e_{t+1}, a_t, k_{t+1}, x_{t+1}, b_{t+1}\}} \left[U(c_t, e_{t+1}) + \Psi n W_{t+1}^\ell (w_{t+1} + b_{t+1})\right] \tag{31}$$

where $W_t^\ell (w_t + b_t)$ is the value function of the individual problem - i.e. the maximum value of lifetime utility W_t for a given amount of income earned (w_t) and bequests received (b_t) - and the maximum is subject to the constraints (28), (29), (30), and to

$$b_{t+1} \geq 0. \tag{32}$$

In related literature, the non-negativity constraint (32) is usually interpreted as 'non-enforceability'. If adult agents desire negative bequests - i.e. they wish to receive transfers from the successors - they cannot oblige future generations to provide them with additional second-period income.[10]

Lemma 6 *(Dynastic Problem) The solution to the recursive dynastic problem (31) is characterized by the intertemporal conditions*

$$e_{t+1}^\ell / c_t^\ell = \epsilon + \beta^{1/\eta} (R_{t+1} + \epsilon)^{1/\eta}, \tag{33}$$
$$p_{t+1}^x = p_{t+1}^a, \tag{34}$$
$$p_{t+1}^x / p_t^x = p_{t+1}^a / p_t^a = R_{t+1}, \tag{35}$$
$$k_{t+1}^\ell n = y_t^\ell - n c_t^\ell - e_t^\ell, \tag{36}$$

and

$$c_{t+1}^\ell / c_t^\ell = c_{t+1}^{\ell 1} / c_t^{\ell 1} = \left[\Psi R_{t+2} \left(\frac{R_{t+1} + \epsilon}{R_{t+2} + \epsilon} \right) \right]^{1/\eta} \quad \text{if } b_{t+1}^\ell > 0, \tag{37}$$

where output per adult is defined as $y_t = Y_t / N_t^a$, and superscript '$\ell 1$' refers to the regime with strictly positive bequests.

Lemma 6 lists the intertemporal rules that characterize the temporary equilibria of the dynastic economy. Conditions (33)-(36) hold in the market economy under laissez-faire, independently of the operativeness of bequests. Equation (33) determines the allocation of consumption across each agent's lifecycle, and incorporates a consumption-bias effect that is analogous to the one described in section 2.2 - see (11) and (12); expression (34) is a no-arbitrage condition asserting that the market value of a resource unit must be the same in both uses; condition (35) is the usual Hotelling rule; and (36) is the aggregate constraint of the economy.

Expression (37) shows how the operativeness of bequests affects the time-profile of consumption across generations. When bequests are operative, the consumption levels of adjacent generations are linked by the Euler-type condition (37), which depends on future interest rates and the altruism factor Ψ. It must be stressed that rule (37) is not followed when bequests are not operative: if $b_{t+1}^\ell = 0$, adult agents consume all their incomes, while young agents choose their first-period consumption levels consistently with their own budget constraints. In order to distinguish between the two accumulation regimes, we implement the following notation: the utility-maximizing vector is denoted as $\left(c_t^{\ell 1}, e_{t+1}^{\ell 1}, ... \right)$ if $b_{t+1}^\ell > 0$, or as $\left(c_t^{\ell 2}, e_{t+1}^{\ell 2}, ... \right)$ if $b_{t+1}^\ell = 0$. Superscript '$\ell 1$' thus refers to Ramsey-Stiglitz temporary equilibria, whereas superscript '$\ell 2$' refers to Diamond-Mourmouras temporary equilibria.

4 Accumulation Regimes

On the basis of Lemma 6, it is possible to analyze the dynamics of the laissez-faire economy under the various accumulation regimes that may arise. In order to keep the analysis clear, we will focus on *permanent regimes* - that is, equlibrium paths where bequests are either permanently zero or permanently positive. The operativeness of bequests will be studied later in section 4.3.

[10] Alternatively, inequality (32) may interpreted as a self-constraint, i.e. a moral obligation that fathers have towards their children, which consists in 'not asking' transfers to successors. Choosing either interpretation may introduce differences in the description of the individual incentives that underly competitive equilibria. However, both reasonings induce to the same formal representation, that is (32).

4.1 Permanent Ramsey-Stiglitz Equilibrium

Suppose that, given the solution to the recursive problem (31), the bequest motive is operative in each $t = 0, ..., \infty$. In this case, the dynamics of the competitive economy are represented by an indefinite succession of Ramsey-Stiglitz temporary equilibria. The resulting time path of economic variables can be thus labelled as a *PRS equilibrium path* (Permanent Ramsey-Stiglitz). The main characteristic of the PRS equilibrium path is its observational equivalence with the centralized allocation studied in section 2.2. This result can be established as follows. From the solution to the dynastic problem, we obtain the intertemporal conditions that hold in the PRS equilibrium:

Lemma 7 *(PRS Equilibrium) If $b_t^{\ell} > 0$ in each $t = 0, ..., \infty$, the dynastic competitive economy follows a PRS equilibrium path characterized by*

$$c_{t+1}^{\ell 1}/c_t^{\ell 1} = \left[\Psi F_{K_{t+2}}^{\ell 1} \left(F_{K_{t+1}}^{\ell 1} + \epsilon \right) \left(F_{K_{t+2}}^{\ell 1} + \epsilon \right)^{-1} \right]^{1/\eta}, \tag{38}$$

$$e_{t+1}^{\ell 1}/e_t^{\ell 1} = \frac{\epsilon + \beta^{1/\eta} \left(F_{K_{t+1}}^{\ell 1} + \epsilon \right)^{1/\eta}}{\epsilon + \beta^{1/\eta} \left(F_{K_t}^{\ell 1} + \epsilon \right)^{1/\eta}} \left(\Psi F_{K_{t+1}}^{\ell 1} \frac{F_{K_t}^{\ell 1} + \epsilon}{F_{K_{t+1}}^{\ell 1} + \epsilon} \right)^{1/\eta}, \tag{39}$$

$$F_{X_{t+1}}^{\ell 1}/F_{X_t}^{\ell 1} = F_{K_{t+1}}^{\ell 1}, \tag{40}$$

$$e_{t+1}^{\ell 1}/c_t^{\ell 1} = \epsilon + \beta^{1/\eta} \left(F_{K_{t+1}}^{\ell 1} + \epsilon \right)^{1/\eta}, \tag{41}$$

together with the aggregate constraint (36). The dynamics of bequests are governed by

$$\frac{b_{t+1}^{\ell 1}}{y_{t+1}^{\ell 1}} = (\alpha_1/n) \frac{y_t^{\ell 1}}{k_{t+1}^{\ell 1}} \left\{ (\alpha_3/n) + \frac{b_t^{\ell 1}}{y_t^{\ell 1}} - \frac{c_t^{\ell 1}}{y_t^{\ell 1}} \left[1 + \frac{\epsilon + \beta^{1/\eta} \left(F_{K_{t+1}}^{\ell 1} + \epsilon \right)^{1/\eta}}{F_{K_{t+1}}^{\ell 1}} \right] \right\}. \tag{42}$$

In Lemma 7, the only result that is peculiar to the dynastic framework is equation (42), which describes the dynamics of the ratio between bequest (per young) and output (per adult). The remaining equations can be easily interpreted along the lines of section 2.2. Infact, comparing Lemma and Lemma 7, it is easy to show that

Lemma 8 *(Observational Equivalence) The PRS equilibrium path is observationally equivalent to the centralized allocation: given identical initial endowments (K_0, Q_0), and setting a social discount factor $\Delta = \Psi$, the two allocations coincide: $\{c_t^*, e_t^*, X_t^*, K_t^*\}_{t=0}^{\infty} = \{c_t^{\ell 1}, e_t^{\ell 1}, X_t^{\ell 1}, K_t^{\ell 1}\}_{t=0}^{\infty}$.*

Lemma 8 is in line with the results of related literature: if $b_t^{\ell} > 0$ in each $t = 0, ..., \infty$, the dynastic economy is observationally equivalent to a centralized economy where the stream of direct utilities is maximized using a social discount rate that coincides with the degree of intergenerational altruism in private preferences (Abel, 1987; see de la Croix and Michel, 2004). Indeed, the utility of the first agent born at $t = 0$ can be written as the discounted sum of direct utilities of all descendants: imposing the limiting condition $\lim_{j \to \infty} \Delta^{j-t} W_j = 0$, iteration of (26) gives

$$W_0 = \sum_{t=0}^{\infty} (\Psi n)^t U(c_t, e_{t+1}). \tag{43}$$

This indeed the same objective function as in section 2.2 - provided that we normalize $N_0^y = 1$, substitute $N_t^y = n^t$, and set $\Delta = \Psi$ in (6).

Lemma 8 implies that, under the assumption that bequests are always strictly positive, the analysis of sections 2.2-2.4 can be entirely applied to PRS equilibrium paths, including all the asymptotic results listed in Lemma 2. For the aims of this section, it is sufficient to recall those related to growth. Provided that the marginal product of capital $F^{\ell 1}_{K_t}$ converges to a finite steady-state $\lim_{t\to\infty} F^{\ell 1}_{K_t} = F^{\ell 1}_{K_{ss}}$, the economy achieves balanced growth in the long run, and equations (13) and (17) imply

$$F^{\ell 1}_{K_{ss}} = (1+\gamma)^{\frac{\alpha_2 \eta}{\alpha_3+\alpha_2\eta}}\, \Psi^{-\frac{\alpha_3}{\alpha_3+\alpha_2\eta}} \text{ and } \lim_{t\to\infty} c^{\ell 1}_{t+1}/c^{\ell 1}_t = \left(\Psi F^{\ell 1}_{K_{ss}}\right)^{1/\eta}. \tag{44}$$

By analogy with (45), the long-run growth rate is

$$\lim_{t\to\infty} c^{\ell 1}_{t+1}/c^{\ell 1}_t = \left[\Psi\left(1+\gamma\right)\right]^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}} = \left(\frac{1+\gamma}{1+\psi}\right)^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}}, \tag{45}$$

from which it follows that individual consumption levels are non-declining in the long run if and only if the net rate of technical progress is at least equal to the altruism rate, i.e. $\gamma \geq \psi$.

The PRS equilibrium path is an example of *PV-optimal path*. In this regard, we refer to the concept of PV-optimality used in Pezzey and Withagen (1998) - i.e. an allocation that maximizes the present-value of the stream of utilities enjoyed at different points in time using a pre-determined discount rate. The specific characteristic of the present model is that, along a PRS equilibrium path, individuals maximize the dynastic utility function (43) without being constrained by the non-negativity of bequests: parameters and initial conditions are such that (32) is never binding, and no asymmetries arise between desired and observed intergenerational transfers. This notion of PV-optimality will be useful in assessing the welfare properties of competitive equilibria in section 4.4.

The transitional dynamics of the PRS equilibrium path are obviously equivalent to those analyzed in section 2.4. The six effects of habit formation arise, and the only relevant extension concerns the dynamics of bequests. In this regard, the time path of b_t can be obtained from (42). In particular, equation (42) implies that $b^{\ell 1}_t/y^{\ell 1}_t$ is constant in the long run, and converges towards the steady-state value

$$\tau^{\ell 1}_{ss} \equiv \lim_{t\to\infty} \frac{b^{\ell 1}_t}{y^{\ell 1}_t} = \frac{F^{\ell 1}_{K_{ss}}}{F^{\ell 1}_{K_{ss}} - n\left(\Psi F^{\ell 1}_{K_{ss}}\right)^{1/\eta}} \left\{\Lambda\left(\epsilon\right) - \left(\alpha_3/n\right)\right\}, \tag{46}$$

where $\Lambda(\epsilon) > 0$ is a combination of constant parameters containing ϵ, and $F^{\ell 1}_{K_{ss}} > n\left(\Psi F^{\ell 1}_{K_{ss}}\right)^{1/\eta}$ (see Appendix). Figure 2 reports the time paths of bequests and the bequest-output ratio for different values of ϵ, all other parameters being equal. Diagrams (a) and (b) show that stronger habits induce lower bequest levels, as well as lower asymptotic values of the bequest-output ratio.

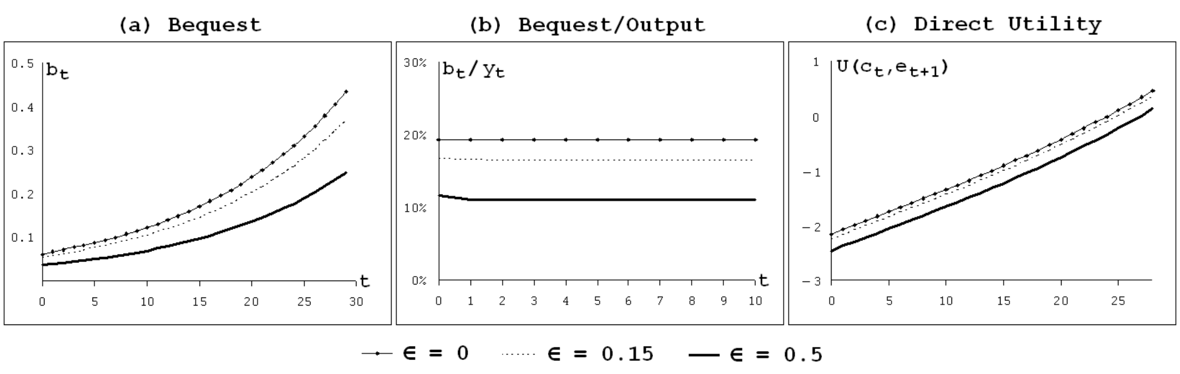

Figure 2: The time path of bequests and direct utility in the PRS equilibrium. Parameter values are $n = 1.1$, $\Psi = 0.65$, $\gamma = 1.45$, $\alpha_1 = 0.3, \alpha_2 = 0.1, \alpha_3 = 0.6$, and $\eta = 1$.

The negative relationship between ϵ and the bequest-output ratio in the long run is a result that can be established analytically. As shown in the Appendix, the derivative $\partial\Lambda(\epsilon)/\partial\epsilon$ is strictly negative, which implies the following

Lemma 9 *(The bequest-habit relation) In a PRS equilibrium path, stronger habits imply a lower bequest-output ratio in the long run:* $\partial\tau_{ss}^{\ell 1}/\partial\epsilon < 0$.

Lemma 9 is in line with Alonso-Carrera et al. (2007) and Schäfer and Valente (2009): in a dynastic framework, stronger habits reduce the propensity to leave bequests to successors. The economic intuition is provided by the consumption-bias effect already described. Habit formation prompts agents to seek higher second-period consumption, and this goes to the detriment of other outflows in the budget constraint. The desire to overcome previous living standards contrasts parental altruism, so that the propensity to leave bequests is lower the higher the strength of habits. As may be construed, this result will be useful in analyzing the operativeness of bequests (section 4.3).

Another remark relates to the welfare implications of habit formation. Welfare effects can be assessed in two ways. First, consider the direct components of private utility, $U(c_t, e_{t+1})$, the three simulations described in Figure 2 yield a negative relationship between utility levels and strength of habits: the time paths of direct utility associated with the three cases $\epsilon = 0$, $\epsilon = 0.15$ and $\epsilon = 0.5$ are reported in diagram (c). Second, in the case of logarithmic preferences $\eta = 1$, it is possible to calculate explicitly the lifetime utility W_t of the agent born at $t = 0$ as (see Appendix)

$$W_0 = \sum_{t=0}^{T-1} (\Psi n)^t U(c_t, e_{t+1}) + (\Psi n)^T \frac{U(c_T, e_{T+1})}{1-\Psi n} + (\Psi n)^{T+1} \frac{(1+\beta)\ln(g_{Y_\infty}/n)}{1-\Psi n}, \qquad (47)$$

where $T > 0$ is a sufficiently large time-index. Using (47), the three simulations yield $W_0 = (-6.8, -7.19, -7.93)$ for $\epsilon = (0, 0.15, 0.5)$. This means that, with respect to the habit-free case $\epsilon = 0$, raising the habit coefficient to $\epsilon = 0.5$ generates a welfare reduction of 15.7%. The welfare reduction can be interpreted as the counterpart of the gains induced by habit formation in terms of consumption, capital and output levels in the long run: these three key variables are permanently increased by stronger habits (cf. section 2.4), and the 'price' of this improvement is a lower level of welfare in present-value terms.

All the results of this section hinge on the implicit assumption that desired bequests are always strictly positive, but the question of whether, and under what circumstances, the economy actually follows a Ramsey-Stiglitz equilibrium path has not been addressed. Before dealing with this issue, we analyze the characteristics of the competitive equilibrium in the opposite case, when bequests are permanently zero.

4.2 Permanent Diamond-Mourmouras Equilibrium

Suppose that, given the solution to the recursive problem (31), the bequest motive is not operative in each $t = 0, ..., \infty$. In this case, the dynamics of the competitive economy are represented by an indefinite succession of Diamond-Mourmouras temporary equilibria. The resulting time path of economic variables can be thus labelled as a *PDM equilibrium path* (Permanent Diamond-Mourmouras). The main characteristics of the PDM equilibrium path can be summarized as follows. First, individual consumption levels are not determined by (37), but by the constraints (29)-(30) with $b_t = 0$ in each period. Second, the dynamics of resource use and capital accumulation change, and are affected by the degree of habit formation also in the long-run. This point can be addressed formally by defining the index z_t as the ratio between resource assets and resource units used in production, $z_t \equiv a_t/x_t$. This index is positively related to the degree of resource preservation observed in the economy: $N_t^a a_t$ is indeed the share of the resource stock that is sold to young generations in each period. As shown in the Appendix, the dynamics of the competitive in the PDM equilibrium path are described by the system

$$\chi_{t+1}^{\ell 2} = \left(\chi_t^{\ell 2}\right)^{\alpha_1} \left(1-\varphi_t^{\ell 2}\right)^{-\alpha_3} \left[\alpha_1^{-1}(1+\gamma)\right]^{\alpha_2} n^{\alpha_3}, \tag{48}$$

$$1-\varphi_t^{\ell 2} = \alpha_3 \Upsilon\left(\chi_{t+1}^{\ell 2}, \epsilon\right) - \alpha_2 z_t^{\ell 2}, \tag{49}$$

$$\alpha_1 z_t^{\ell 2} = \alpha_3 \Upsilon\left(\chi_{t+1}^{\ell 2}, \epsilon\right) \cdot \left(1+z_{t+1}^{\ell 2}\right) - \alpha_2 z_t^{\ell 2}\left(1+z_{t+1}^{\ell 2}\right), \tag{50}$$

where $\Upsilon\left(\chi_{t+1}^{\ell 2}, \epsilon\right)$ is a function of the output-capital ratio at period $t+1$, and of the degree of habit formation. As for the system studied in section 2.3, the dynamic stability of system (48)-(50) must be studied numerically, although convergence towards a symultaneous steady state $\left(\chi_{ss}^{\ell 2}, \varphi_{ss}^{\ell 2}, z_{ss}^{\ell 2}\right)$ may be established analytically for various special cases of parameter values. The basic differences with respect to centralized allocations and PRS equilibrium paths are that (i) there is no general explicit solution for $\chi_{ss}^{\ell 2}$, and most importantly, that (ii) the long-run equilibrium interest factor $F_{K_{ss}}^{\ell 2}$ does depend on the strength of habit formation.

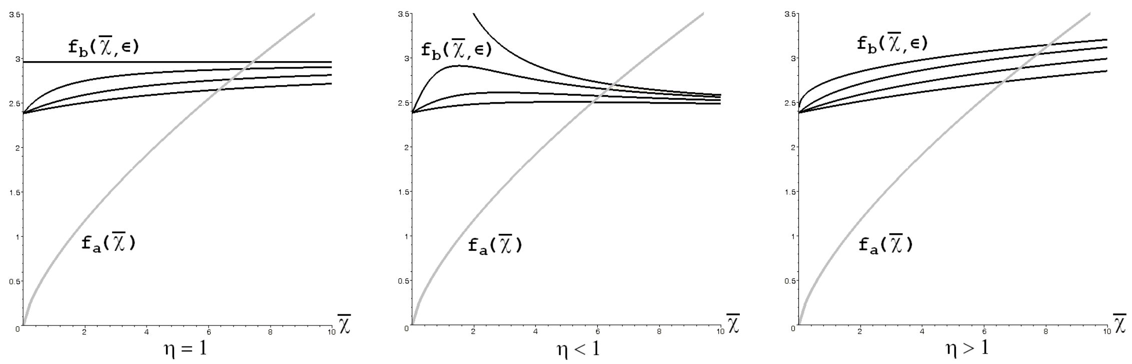

Figure 3: Determination of the long-run equilibrium of the PDM equilibrium path for different values of η. In all cases, higher values of ϵ imply downward shifts in $f_b(\bar{\chi}, \epsilon)$, and therefore lower equilibrium values of $\bar{\chi}$.

In order to simplify the notation, we will use $(\bar{\chi}, \bar{\varphi}, \bar{z})$ to denote the steady-state variables $\left(\chi_{ss}^{\ell 2}, \varphi_{ss}^{\ell 2}, z_{ss}^{\ell 2}\right)$ of the system (48)-(50). Imposing the simultaneous steady state $\left(\chi_t^{\ell 2}, \varphi_t^{\ell 2}, z_t^{\ell 2}\right) = (\bar{\chi}, \bar{\varphi}, \bar{z})$ for any t in system (48)-(50), we obtain

$$\bar{\chi} = (\bar{\chi})^{\alpha_1} (1-\bar{\varphi})^{-\alpha_3} \left[\alpha_1^{-1}(1+\gamma)\right]^{\alpha_2} n^{\alpha_3}, \tag{51}$$

$$1-\bar{\varphi} = \alpha_3 \Upsilon(\bar{\chi}, \epsilon) - \alpha_2 \bar{z}, \tag{52}$$

$$\alpha_1 \bar{z} = \alpha_3 \Upsilon(\bar{\chi}, \epsilon) \cdot (1+\bar{z}) - \alpha_2 \bar{z}(1+\bar{z}). \tag{53}$$

As shown in the Appendix, the simultaneous steady state is determined by an equilibrium condition of the type

$$f^a(\bar{\chi}) = f^b(\bar{\chi}, \epsilon), \tag{54}$$

where $f^a(\bar{\chi})$ is an increasing and strictly concave function satisfying $\lim_{\bar{\chi}\to 0} f^a(\bar{\chi}) = 0$ and $\lim_{\bar{\chi}\to\infty} f^a(\bar{\chi}) = \infty$, whereas the behavior of $f^b(\bar{\chi}, \epsilon)$ depends on the value assumed by the elasticity parameter η. Although $f^b(\bar{\chi}, \epsilon)$ assumes different shapes in the three cases $\eta < 1$, $\eta = 1$, $\eta > 1$, a general property is that this function is bounded, with

$$\lim_{\bar{\chi}\to 0} f^b(\bar{\chi}, \epsilon) = f_{\min}^b > 0 \text{ and } \lim_{\bar{\chi}\to\infty} f^b(\bar{\chi}, \epsilon) = f_{\max}^b < \infty, \tag{55}$$

and exhibits the necessary regularities to ensure the uniqueness of the steady state (see Appendix).

Given existence and uniqueness, the most important result is that function $f^b(\bar{\chi}, \epsilon)$ is negatively related to the strength of habit formation: independently of the value of η, it is shown that

$$\frac{df^b(\bar{\chi}, \epsilon)}{d\epsilon} < 0 \text{ for any } \epsilon > 0. \tag{56}$$

In graphical terms, result (56) implies that $f^b(\bar{\chi}, \epsilon)$ shifts downwards as ϵ increases, as shown in the parametric plots reported in Figure 3. Since $f^a(\bar{\chi})$ is increasing and independent of habits, it follows from this result that the steady-state value $\bar{\chi}$ is negatively affected by the strength of habit formation. The main implications are that we can define steady-state values as function of ϵ, with the following properties:

Lemma 10 *(Steady-state PDM equilibrium) Provided that $F^{\ell 2}_{K_t}$ converges to the steady-state value $\lim_{t\to\infty} F^{\ell 2}_{K_t} \equiv F^{\ell 2}_{K_{ss}} = \alpha_1 \bar{\chi}$, the PDM equilibrium path exhibits*

$$\lim_{t\to\infty} F^{\ell 2}_{K_t} = F^{\ell 2}_{K_{ss}} \equiv \alpha_1 \bar{\chi}(\epsilon) \;\; \textit{with } dF^{\ell 2}_{K_{ss}}/d\epsilon < 0, \tag{57}$$

$$\lim_{t\to\infty} \varphi^{\ell 2}_t = \varphi^{\ell 2}_{ss} \equiv \bar{\varphi}(\epsilon) \;\; \textit{with } d\varphi^{\ell 2}_{ss}/d\epsilon < 0, \tag{58}$$

$$\lim_{t\to\infty} z^{\ell 2}_t = z^{\ell 2}_{ss} \equiv \bar{z}(\epsilon) \;\; \textit{with } dz^{\ell 2}_{ss}/d\epsilon > 0. \tag{59}$$

In the long run, stronger habits imply higher rates of resource use and a higher growth rate of output:

$$\lim_{t\to\infty} g^{\ell 2}_{X_t} = g^{\ell 2}_{X_\infty}(\epsilon) \;\; \textit{with } dg^{\ell 2}_{X_\infty}/d\epsilon > 0, \tag{60}$$

$$\lim_{t\to\infty} g^{\ell 2}_{Y_t} = g^{\ell 2}_{Y_\infty}(\epsilon) \;\; \textit{with } dg^{\ell 2}_{Y_\infty}/d\epsilon > 0. \tag{61}$$

Lemma 10 shows that the PDM equilibrium path displays relevant differences with respect to centralized allocations and PRS equilibria. The coefficient of habit formation modifies asymptotic values and, in particular, stronger habits imply higher growth rates, a higher saving rate ($\varphi^{\ell 2}_{ss}$ is reduced), and a lower interest rate in the long run. Result (61) implies a flatter profile of extracted resource flows: as ϵ increases, $g^{\ell 2}_{X_\infty}$ becomes closer to unity from below. Given the resource constraint (25), this means that stronger habits imply lower resource use in the short run and higher resource in the long run.[11] This result is similar to the 'resource use effect' already encountered in section 2.4: the difference is that, in the PDM equilibrium, the long-run rate of resource depletion is modified by habits, whereas $\lim_{t\to\infty} g_{X_t}$ is only determined by discounting and technical progress in altruistic/centralized allocations - see (16). Figure 4 reports a numerical examples describing the results of Lemma 10.

With respect to Lemma 10, two remarks are in order. First, the long-run effects of habits in PDM equilibria are closely related to the transitional growth effects of habits studied in section 2.4 - result (iv). Recalling the previous analysis, stronger habits in centralized allocations - and also in PRS equilibria, given the observational equivalence - induce lower interest rates, faster accumulation, and higher growth rates in the short run. In the PDM equilibrium, these effects *become permanent*. This last result has a precise explanation which, for the sake of clarity, will be discussed later in section 5. In particular, it will be clarified (cf. Proposition 14) that the rise of permanent effects is peculiar to the present model, and hinges on the assumption of resource-dependence since it does not arise in capital-labor economies.

The second remark is that an important characteristic of the PDM equilibrium lies in its welfare properties. The PDM equilibrium path arises when parameters and initial conditions are such that desired bequests are negative, i.e. when agents would like to *receive* positive transfers from future generations, rather than supplying bequests to successors. Recalling our previous definition in section 4.1, this means that PDM equilibria do not satisfy PV-optimality. Since constraint (32) is binding, individuals act as if they maximize only direct utility $U(c_t, e_{t+1})$. The dynastic function (43) is not maximized due to the asymmetries between desired bequests and observed intergenerational transfers. Section 4.4 will show that the welfare implications of this asymmetry may be quite relevant.

[11] If we compare two selfish economies with different habit parameters - e.g. ϵ_A and ϵ_B with $\epsilon_A < \epsilon_B$ - and all other conditions being equal, result (59) implies that the resource use factor $z^{\ell 2}_{ss}$ will be higher in economy B. However, this does not mean that extracted resources will always be higher: since both economies have the same initial endowment Q_0, they both must fulfill the resource constraint (25). This implies that the economy B exhibits more intense resource extraction in the medium long-run, but this must be compensated by extracting less resource units in the short run.

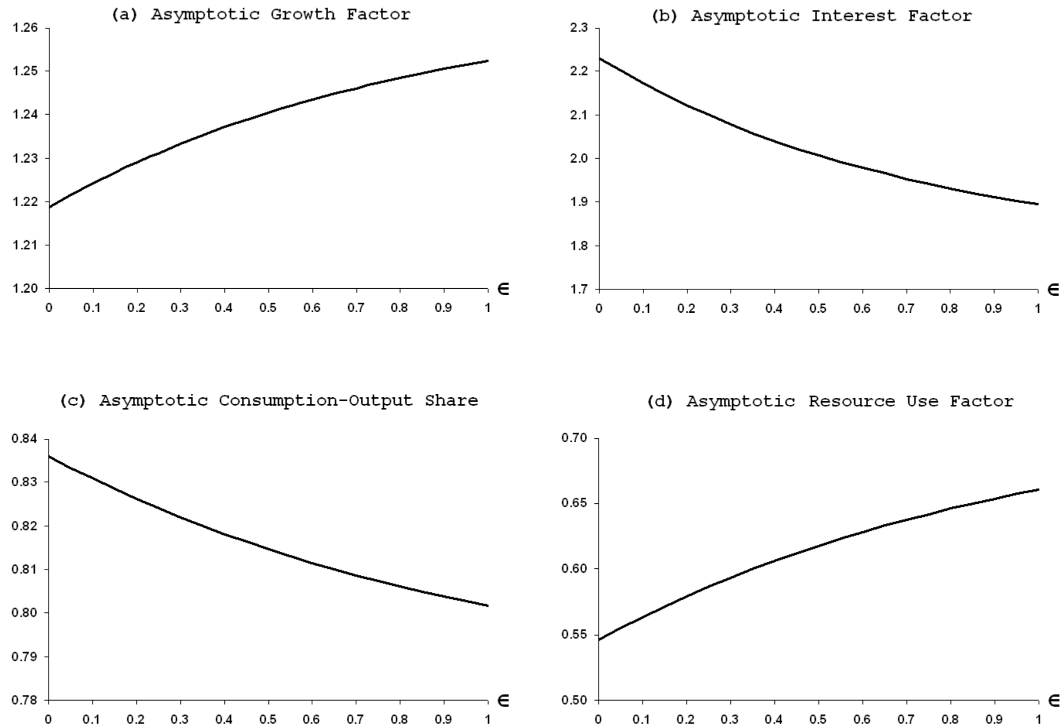

Figure 4: Asymptotic variables as functions of ϵ. Graph (a) reports $\lim_{t\to\infty} g^{\ell 2}_{Y_t} = g^{\ell 2}_{Y_\infty}(\epsilon)$. Graph (b) reports $\lim_{t\to\infty} F^{\ell 2}_{K_t} = F^{\ell 2}_{K_{ss}}(\epsilon)$. Graph (c) reports $\lim_{t\to\infty} \varphi^{\ell 2}_{t} = \varphi^{\ell 2}_{ss}(\epsilon)$. Graph (d) reports $\lim_{t\to\infty} g^{\ell 2}_{X_t} = g^{\ell 2}_{X_\infty}(\epsilon)$. Parameter values are $n = 1.2$, $\Psi = 0.5$, $\gamma = 1.45$, $\alpha_1 = 0.3, \alpha_2 = 0.1, \alpha_3 = 0.6$, and $\eta = 1$. Under this combinations of parameter values, bequests are not operative for any $\epsilon \in (0,1)$, and the steady-state values of $\bar{\chi}, \bar{\varphi}, \bar{z}$ associated with each value of ϵ in this range confirm the results of Lemma 10.

4.3 The Operativeness of Bequests

The permanent accumulation regimes studied in sections 4.1-4.2 arise only if parameters are such that the level of desired bequests is either strictly positive or non-positive in each instant. In general, there is no analytical way to test operativeness *in each period*, since the sign of b^{ℓ}_t at a generic t depends on the whole combination of values that the endogenous variables assume - see equation (42). The possibility of switchovers in the accumulation regime during the transition is concrete, but can only be studied numerically. What can be established analytically is which of the two accumulation regimes will be implemented in the long run.

The operativeness of bequests in the long run requires that equation (46) yield a positive value for the asymptotic bequest-output ratio $\tau^{\ell 1}_{ss}$, i.e. a strictly positive term inside the curly brackets. Recalling Lemma 9, function $\Lambda(\epsilon)$ is strictly decreasing in ϵ. A deeper analysis reveals that $\Lambda(\epsilon)$ can assume values that are sufficiently low to yield a negative right-hand side in (46). The consequence is that there exists a critical value of the strength of habits above which the economy does not display Ramsey-Stiglitz equilibria in the long run. More precisely,

Lemma 11 *(Operativeness of bequest motives) Provided that*

$$\lim_{\epsilon\to 0} \Lambda(\epsilon) < \alpha_3/n < \lim_{\epsilon\to\infty} \Lambda(\epsilon), \tag{62}$$

there exists a critical value $\bar{\epsilon} > 0$ of the strength of habits such that $\epsilon > \bar{\epsilon}$ implies that bequests are *necessarily zero from some period $t = t'$ onwards ($b_t^\ell = 0$ in each $t = t', ..., \infty$ with $0 \leq t' < \infty$). As a consequence, (i) a necessary condition for a PRS equilibrium path to arise is $\epsilon < \bar{\epsilon}$; (ii) if $\epsilon > \bar{\epsilon}$, the economy exhibits zero bequests in the long run (with either zero bequests in each period, or positive bequests for a limited number of periods during the transition).*

The intuition for Lemma 11 is similar to that behind Lemma 9. Habit formation prompts agents to seek higher second-period consumption, and this contrasts parental altruism. Lemma 11 establishes that, if habits are sufficiently strong, the desire to leave bequests it totally outweighed by the private willingness to overcome previous living standards, and this implies non-operative altruism towards the successors.

Numerical substitutions show that the pre-condition (62) can be easily met for several combinations of parameters. Given (62), the value of $\tau_{ss}^{\ell 1}$ is a decreasing function of ϵ and falls short of zero in correspondence of the critical threshold $\epsilon = \bar{\epsilon}$ - see Figure 5, diagram (a). Hence, similarly to Alonso-Carrera et al. (2007), the economy will exhibit a Ramsey-Stiglitz accumulation regime in the long run only for low degrees of habit formation ($\epsilon < \bar{\epsilon}$), whereas it will exhibit a Diamond-Mourmouras accumulation regime in the long run if the strength of habit is relatively high ($\epsilon > \bar{\epsilon}$). Differently from Alonso-Carrera et al. (2007), however, the present model exhibits permanent growth effects of habit formation in selfish equilibria, and this implies that capital profitability, output growth and the speed of resource depletion change depending on whether the strength of habits is above or below the critical threshold. The main consequences in this regard are summarized in the following

Lemma 12 *(The growth-habit relationship) The long-run growth rate of the dynastic economy is*

$$\lim_{t\to\infty} g_{Y_t}^\ell = \left\{ \begin{array}{lll} g_{Y_\infty}^{\ell 1} = n\left[\Psi\left(1+\gamma\right)\right]^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}} & \textit{if} & \epsilon < \bar{\epsilon} \\ g_{Y_\infty}^{\ell 2} = g_{Y_\infty}^{\ell 2}\left(\epsilon\right) & \textit{if} & \epsilon > \bar{\epsilon} \end{array} \right\}, \tag{63}$$

where $g_{Y_\infty}^{\ell 2}(\epsilon)$ is defined in Lemma 10. The growth rates coincide if $\epsilon = \bar{\epsilon}$, whereas $g_{Y_\infty}^{\ell 2}(\epsilon) > g_{Y_\infty}^{\ell 1}$ for any $\epsilon > \bar{\epsilon}$.

Lemma 12 describes the interactions between long-run growth rates and habit formation. This result is described graphically in Figure 5, diagram (b), which is obtained by combining the asymptotic values of g_{Y_t} obtained in Ramsey-Stiglitz equilibria and Diamond-Mourmouras regimes as ϵ ranges between zero and unity. For low degrees of habit formation, the economy converges towards altruistic long-run equilibria, where the asymptotic growth rate is independent of habits. For high degrees of habit formation, the long-run equilibrium is a selfish regime, in which the growth rate increases with ϵ. On the one hand, this conclusion implies that the conditions for obtaining sustained consumption and output levels in the long run are less restrictive in Diamond-Mourmouras regimes: when excessive habits ($\epsilon > \bar{\epsilon}$) induce a selfish regime in the long-run, growth rates are higher with respect to Ramsey-Stiglitz equilibria. On the other hand, this result should be interpreted with great care, as the welfare properties of the two accumulation regimes are fundamentally different. The PRS equilibrium path described in section 4.1 is PV-optimal, whereas the PDM equilibrium path described in section 4.2 is not. As shown below, the welfare attained in a PDM regime induced by excessive habit formation may be much lower than that obtained in a PRS regime characterized by weaker habits.

4.4 Welfare Comparisons

This section performs a numeric welfare comparison between PRS and PDM equilibrium paths. The aim is to describe the welfare properties of the two regimes when the only difference is represented by the coefficient of habit formation. Setting the parameters values $\alpha_1 = 0.3$, $\alpha_2 = 0.1$, $\alpha_3 = 0.6$, $\eta = 1$, $n = 1.1$, $\gamma = 1.45$, $\Psi = 0.55$, and $\beta = 0.9$, the critical value of the habit coefficient equals $\bar{\epsilon} = 0.36$. A first economy - denoted by superscript 'I' - exhibits a habit coefficient $\epsilon^I = 0.25$,

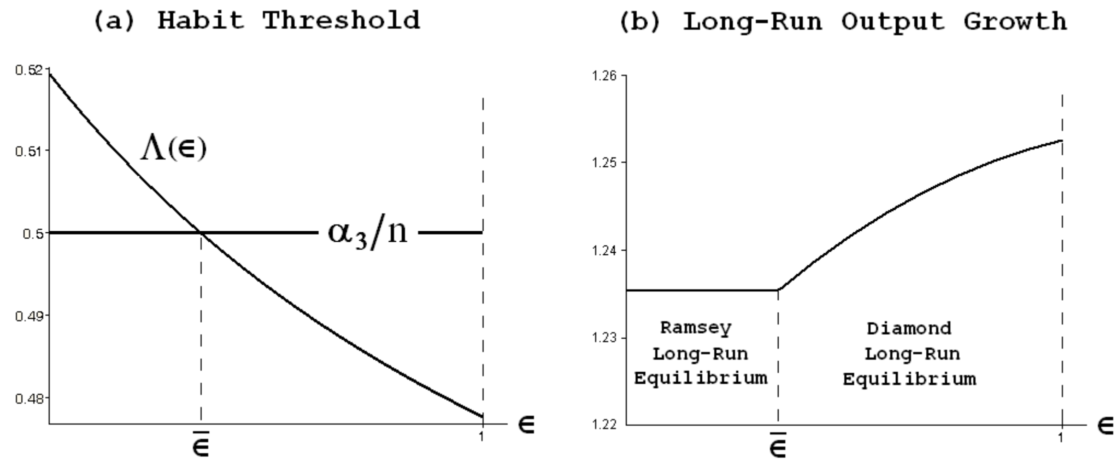

Figure 5: Diagram (a) reports the function $\Lambda(\epsilon)$ compared to the threshold level α_3/n. From (46), the intersection determines the critical value $\bar{\epsilon}$ below (above) which long-run desired bequests are positive (negative). Diagram (b) describes the growth-habit relationship established in Lemma 12. Parameter values are $n = 1.2$, $\Psi = 0.5$, $\gamma = 1.45$, $\alpha_1 = 0.3, \alpha_2 = 0.1, \alpha_3 = 0.6$, and $\eta = 1$.

which - for given initial conditions $K_0 = 1$ and $Q_0 = 8.85$ - implies a permanent Ramsey-Stiglitz equilibrium path. The asymptotic values for output growth, interest factor, and resource use are

$$g^I_{Y_\infty} = 1.148,\ F^I_{K_{ss}} = 1.897,\ g^I_{X_\infty} = 0.605.$$

A second economy - denoted by superscript 'II' - exhibits a habit coefficient $\epsilon^{II} = 0.6$, which - given the same initial endowments - implies a permanent Diamond-Mourmouras equilibrium path, with asymptotic values

$$g^{II}_{Y_\infty} = 1.155,\ F^{II}_{K_{ss}} = 1.825,\ g^{II}_{X_\infty} = 0.633.$$

The differences between the asymptotic values of the two economies confirm the results of Lemma 10. The transitional dynamics of the key variables are reported in Figure 6. Due to the consumption-bias effect, economy II exhibits lower first-period consumption in the short/medium run - diagram (a) - and a permanently higher second-period consumption - diagram (b). The reallocation effect generates faster capital accumulation, as reported in diagram (c). The bequest-output ratio in economy I converges to $\lim_{t\to\infty} b_t/y_t = 1.52\%$, whereas transfers are zero in economy II - diagram (d). Faster accumulation and higher growth rates generate a positive output gap for economy II already in period $t = 1$, which then exceeds 6% after ten periods - see diagram (e).

The welfare gap, however, is in favor of economy I. Considering only the direct components of private utility, $U\left(c^I_t, e^I_{t+1}\right)$ exceeds $U\left(c^{II}_t, e^{II}_{t+1}\right)$ by an almost steady value of 46%, as reported in Figure 6, diagram (f). The lifetime utility of the first newborn generation, W_0, can also be calculated on the basis of (47) for each time path $i = I, II$. The previous result is confirmed, as we obtain $W^I_0 = -5.4$ and $W^{II}_0 = -10.1$. This implies a 46.4% gap in present-value welfare in favor of the Ramsey-Stiglitz economy.

Recalling the previous analysis of PRS equilibria, this welfare gap appears huge. In section 4.1, using identical values of endowments and parameters - except for a slightly different altruism factor[12] - we have seen that the effect of an increase of 0.5 in the habit coefficient within the same

[12] The only difference in parameters between the simulations described in Figure 2 and the comparison reported in Figure 6 is that $\Psi = 0.65$ in the former case, whereas $\Psi = 0.55$ in the present case. This slight difference is however necessary because the two simulations hinge on different hypotheses. In Figure 2 we compare two PRS regimes, and the value $\Psi = 0.65$ guarantees that bequests are operative for all the values of ϵ considered there. In Figure 6, we are comparing a PRS with a PDM regime, and the value $\Psi = 0.55$ guarantees that $\epsilon = 0.25$ is associated with a PRS equilibrium and that $\epsilon = 0.6$ is associated with a PDM equilibrium.

(altruistic) regime could be quantified as a 15.7% welfare reduction. In the present simulation, an even smaller increase (from $\epsilon^I = 2.5$ to $\epsilon^{II} = 0.6$) yields a nearly fifty-percent reduction. This suggests that a major component of the welfare gap obtained here is determined by the regime shift: although part of $W_0^I - W_0^{II}$ may be due to rise in ϵ independently of the regime shift, the fundamental difference between economies I and II is that the PRS equilibrium is PV-optimal, whereas the PDM equilibrium is not. Before verifying this conjecture in a precise manner, we provide the economic intuition for this result.

In general, the presence of habit formation implies that personal satisfaction is not a simple function of consumption levels, but rather a preference index that induces a particular allocation of private incomes across the lifecyle. The welfare gap comes from the fact that, in PDM equilibria, there are asymmetries between desired and observed transfers. Agents are potentially altruistic as they wish to maximize lifetime welfare including the altruistic component, but they cannot achieve the combination of first-period income, first-period consumption, and second-period consumption that would fulfill this desire. The desired combination is achieved in PRS equilibria, instead, as confirmed by the fact that permanent altruistic regimes satisfy PV-optimality. The huge welfare gap observed in Figure 6 is a direct consequence of these circumstances.

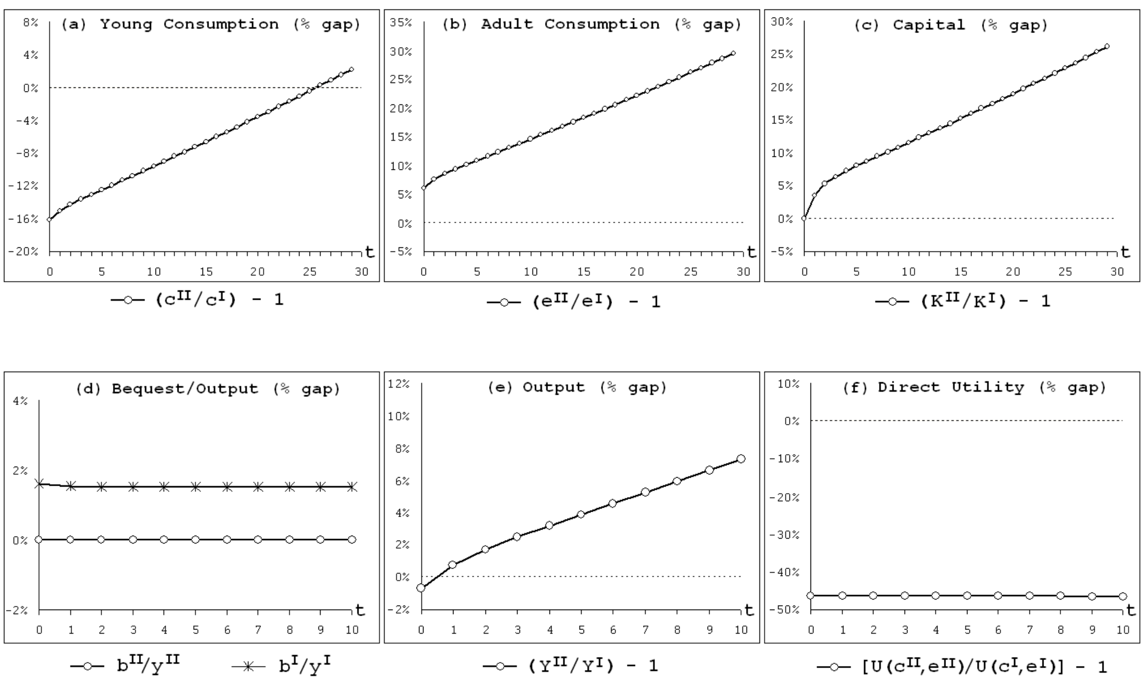

Figure 6: Simulation Results. Economy I exhibits $\epsilon < \bar{\epsilon}$ and experiences a PRS equilibrium path. Economy II exhibits $\epsilon > \bar{\epsilon}$ and experiences a PDM equilibrium path.

A concrete way to verify the above conclusion is to compare the PDM equilibrium path with a hypothetical *unconstrained path* that satisfies PV-optimality, while exhibiting identical parameters in every respect. The 'unconstrained path' is essentially the centralized allocation with $\Delta = \Psi$: under the parameters of economy II, this allocation is not achieved as a competitive equilibrium because the constraint $b_{t+1} \geq 0$ is binding. An equivalent interpretation is that the 'unconstrained path' is the competitive equilibrium that would arise in the competitive economy if agents were allowed to withdraw the desired amount of transfers from future generations. This path is characterized by the same intertemporal conditions holding in the PRS equilibrium, but in which negative bequests are allowed. Hence, the desired path can be evaluated in practice by running the simulation procedure of the PRS path, with the difference that the constraint $b_{t+1} \geq 0$ is relaxed in each period.

	ϵ	g_{Y_∞}	τ_{ss}	W_0	Welfare Gap
Economy I (PRS path)	0.25	1.148	$+1.52\%$	-5.45	$+46.4\%$
Economy II (PDM path)	0.60	1.155	0	-10.16	–
Economy III (unconstrained)	0.60	1.148	-2.74%	-5.92	$+41.7\%$

Table 1. Welfare comparison between PRS, PDM and unconstrained paths.

The results of this procedure are reported in Table 1, where the unconstrained path is denoted as 'Economy III'. The third column reports the welfare gap with respect to economy II, calculated as $\left(W_0^{II} - W_0^i\right)\left|W_0^{II}\right|^{-1}$ for $i = I, III$. The asymptotic growth rate g_{Y_∞} is obviously identical between economies I and III, as they are both PV-optimal. However, the strength of habit formation in economy III is above the critical threshold, implying that desired bequests are negative: the asymptotic bequest-output ratio $\tau_{ss} \equiv \lim_{t\to\infty} b_t/y_t$ is -2.74% . Welfare levels confirm our previous conclusion: the gap between the PDM equilibrium and the hypothetical unconstrained path is 41.7% in favor of the latter. Hence, the welfare gap of 46.4% arising between economies I and II comes to a minor extent (4.7%) from the direct effect of stronger habits, and to a major extent (41.7%) from the fact that stronger habits induce a shift to an accumulation regime that does not satisfy PV-optimality.

5 Main Propositions and Related Literature

This section summarizes the main results, and briefly discusses their connections with the previous literature.[13] The first conclusion that can be drawn from the previous analysis is

Proposition 13 *(Altruistic Regimes) If the strength of habits is below the critical level $\bar{\epsilon}$, the dynastic economy exhibits an indefinite sequence of Ramsey-Stiglitz equilibria in each $t = t', ..., \infty$, where $t' \geq 0$ is finite. Habit formation yields (i) permanent effects on output levels and resource use, and (ii) transitional effects on growth rates, capital profitability and speed of resource depletion. Moreover, (iii) the sustainability of consumption levels is determined by the Stiglitz (1974) condition with an intergenerational discount rate equal to the private degree of altruism.*

This Proposition summarizes the results of sections 2.2-2.4 and 4.1, and incorporates the main result regarding the operativeness of bequests in the long run (Lemma 11). The fact that habits do not affect long-run growth is a consequence of PV-optimality. The dynastic utility function (43) is maximized, and the intergenerational distribution of benefits is 'dictated' by the altruism factor Ψ - which determines the long-run growth rate of the economy together with the rate of resource-augmenting technical progress. The fact that consumption sustainability is determined by the Stiglitz (1974) condition is due to the observational equivalence between PRS equilibrium paths and centralized allocations (Lemma 8).

With respect to previous literature, the absence of permanent growth effects of habit formation in PV-optimal regimes is in line with the results of Ryder and Heal (1973) - who first implemented habits in the Ramsey model with infinite lives, showing that reallocation effects modify capital accumulation in the transition, but do not affect long-run growth. The same result applies to the OLG variant of Alonso-Carrera et al. (2007). The difference is that, in our model, the long-run growth rate is affected by preference parameters - the altruism factor Ψ and the intertemporal elasticity of substitution $1/\eta$ - in contrast to capital-labor economies where it is exogenously determined by

[13] For the sake of clarity, we will not draw explicit comparisons between the present analysis and the nonetheless important results obtained in the literature on habit formation in the presence of increasing returns and infinite lives. This literature, initiated by Carrol et al. (1997), shows that habits modify the long-run growth rate when the intertemporal elasticity of utility differs from unity. As our model is structurally different - in particular, it features resource dependence and does not exhibit increasing returns nor infinite lives - we avoid lengthening the discussion by adding the endogenous growth literature to the following considerations.

labor-efficiency. Nonetheless, the main differences with respect to the previous literature arise from our second main result, which is summarized below.

Proposition 14 *(Selfish Regimes/1) If the strength of habits is above the critical level* $\bar{\epsilon}$*, the dynastic economy exhibits an indefinite sequence of Diamond-Mourmouras equilibria in each* $t = t', ..., \infty$*, where* $t' \geq 0$ *is finite. A higher degree of habit formation yields (i) faster output growth, (ii) lower capital profitability, and (iii) higher rates of resource use in the long run.*

This Proposition summarizes the results of section 4.2. The fact that the economy exhibits different growth rates in different regimes is due to the presence of exhaustible resources, and hinges on the endogeneity result mentioned in the Introduction. In selfish regimes, agents accumulate capital and resource assets without relying on parental transfers. As the altruism factor Ψ does not play any role, the intergenerational distribution of wealth is entirely determined by the market conditions governing the profitability of (a) accumulating capital and (b) selling resource assets to the next generation. Since the intergenerational distribution of wealth influences the growth rate in capital-resource economies (cf. the endogeneity result), we obtain a different growth rate with respect to Ramsey-Stiglitz equilibria.

The fact that stronger habits imply faster growth is intimately linked to the above reasoning. The amount of resource assets that agents sell to the next generation is affected by their willingness to overcome previous living standards, and this modifies the intergenerational distribution. This results into higher growth rates because of the same reasons underlying the transitional growth effects studied in section 2.4 - that is, consumption-bias, accumulation and input-substitution effects. The difference with respect to Ramsey-Stiglitz equilibria is that the growth effects of habits become permanent, because the intergenerational distribution is not dictated by the altruism factor Ψ anymore.

It follows from the above remarks that the our main results in Proposition 14 hinge on the assumption of resource dependence. This is a new element with respect to the previous literatures on habit formation and on dynastic economies, and results are indeed quite different. The closest contributions make reference to standard capital-labor economies, in which there is either no permanent growth effect of habit formation (Alonso-Carrera et al. 2007), or a negative growth effect induced by the decrease of fertility rates generated by habits (Schäfer and Valente, 2009).

Proposition 15 *(Selfish Regimes/2) With respect to Ramsey-Stiglitz steady-state equilibria, the equilibrium path attained when* $\epsilon > \bar{\epsilon}$ *exhibits (i) a higher long-run growth rate, (ii) a less restrictive condition for sustained long-run consumption, but (iii) possibly huge welfare losses due to the violation of unconstrained PV-optimality.*

Proposition 15 summarizes the growth-habit relationship derived in Lemma 12 and the results of the welfare analysis in section 4.4. The fact that growth rates differ between altruistic and selfish regimes is new with respect to the previous literature. In capital-labor dynastic economies, there are no asymmetries in growth rates: since capital is only endogenously accumulated factor, the capital-labor ratio converges asymptotically to a steady state in both regimes. As a consequence, altruistic and selfish regimes in capital-labor economies may differ in long-run income levels, but exhibit the same long-run growth rate. This result holds with or without habit formation in capital-labor economies- see Alonso-Carrera et al. (2007) and Thibault (2000), respectively - but it does not hold in our model.

The present analysis yields furthermore different results as it shows that the selfish equilibrium implies faster growth than selfish regimes for any degree of habit formation above the critical threshold. This effect does not arise in Alonso-Carrera et al. (2007), and hinges on the fact that long-run growth in capital-resource economies is determined by the intergenerational distribution of resources, as explained before.

Statements (i)-(ii) in Proposition 15 are also new with respect to the sustainability literature. In capital-resource models, the role of habit formation has been neglected so far, and our results show that he interactions between habits and resources may be quite relevant for income levels and growth rates. At the same time, statement (iii) calls for interpreting growth-related issues in a precise manner. Habits raise the growth rate, but they do so in selfish equilibria that do not satisfy PV-optimality. Given the asymmetry between desired and observed intergenerational transfers, the welfare levels attained in a PDM regime induced by excessive habit formation may be much lower than in PRS regimes characterized by weaker habits.

6 Conclusions

This paper analyzed the implications of habit formation for welfare, income levels, and long-term growth when production possibilities are constrained by resource scarcity and finitely-lived agents exhibit one-sided altruism. If bequests are operative along the entire path, an increase in the strength of habits induces a bias in favor of second-period consumption that generates relevant reallocation effects. Faster capital accumulation in the short run and input-substitution between man-made capital and exhaustible resources yield permanent positive effects on output, capital, consumption levels and resource use. Habits also yield positive transitional effects on growth rates and capital profitability. However, this altruistic equilibrium may arise only if the coefficient of habit formation falls short of a critical level. If the degree of status desire exceeds the threshold, the economy achieves selfish equilibria in which habits increase growth rates, reduce capital profitability, and raise the speed of resource depletion in the long run. Long-run growth is higher than in the altruistic regime because of habit formation, and the condition for obtaining non-declining consumption is less restrictive. However, the selfish equilibrium is not optimal in the standard sense: although consumption and output dynamics are observationally more favorable, there are asymmetries between desired and effective intergenerational transfers. The consequence is that, despite substantial improvements in growth rates and income levels in the medium-long run, the welfare attained in selfish equilibria induced by excessive habit formation may be much lower than that obtained in altruistic regimes characterized by weaker habits.

It has been shown that these results differ from related findings in capital-labor economies because of the assumption of resource dependence: different rates of resource exploitation modify the intergenerational distribution of wealth, and thereby the growth rate attained in either equilibrium.

Appendix

Some useful relations. In the following derivations, we will exploit some basic relations implied by the assumptions of the model. First, assumption (5), implies

$$U_{c_t} = c_t^{-\eta} - \beta\epsilon \left(e_{t+1} - \epsilon c_t\right)^{-\eta} \text{ and } U_{e_{t+1}} = \beta \left(e_{t+1} - \epsilon c_t\right)^{-\eta}. \tag{64}$$

Second, denoting by $g_{p_t} \equiv p_{t+1}/p_t$ the gross growth rate of the generic variable p_t, technology (1) implies that the output growth is given by

$$g_{Y_t} = g_{K_t}^{\alpha_1} g_{X_t}^{\alpha_2} \left(1+\gamma\right)^{\alpha_2} n^{\alpha_3}. \tag{65}$$

Third, defining the output-capital ratio as $\chi_t \equiv Y_t/K_t$ and the aggregate consumption-output ratio as $\varphi_t \equiv \left(N_t^y/Y_t\right)\left(c_t + e_t n^{-1}\right)$, the accumulation constraint (3) can be written as $g_{K_t} = \chi_t \left(1-\varphi_t\right)$, which

$$g_{K_t} = \chi_t \left(1-\varphi_t\right). \tag{66}$$

Fourth, defining per-adult variables as $y_t \equiv Y_t/N_t^a$ and $k_t \equiv K_t/N_t^a$, the accumulation constraint (3) can be written as

$$k_{t+1} n = y_t - nc_t - e_t. \tag{67}$$

Fifth, by the previous definition of φ_t, we have $\varphi_t = \left(nc_t + e_t\right)/y_t$, so that the ratio between first-period individual consumption c_t and output per adult y_t, can be defined as $\nu_t \equiv c_t/y_t$, and written as

$$\nu_t = c_t/y_t = n^{-1}\left[\varphi_t - \left(e_t/y_t\right)\right]. \tag{68}$$

Proof of Lemma 1. The centralized problem can be solved by means of the Lagrangean

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ N_t^y U\left(c_t, e_{t+1}\right) \Delta^t + \lambda_t^k \left[F\left(K_t, m_t X_t, N_t^y\right) - N_t^y \left(c_t + \frac{e_t}{n} \right) - K_{t+1} \right] + \lambda_t^q \left(Q_t - X_t - Q_{t+1} \right) \right\},$$

where λ_t^k and λ_t^q are the dynamic multipliers associated with the transition laws of K_t and Q_t, respectively. The first-order conditions read

$$\begin{aligned}
\mathcal{L}_{c_t} &= 0 \quad \rightarrow \quad U_{c_t}\Delta^t = \lambda_t^k, && (69)\\
\mathcal{L}_{e_{t+1}} &= 0 \quad \rightarrow \quad U_{e_{t+1}}\Delta^t = \lambda_{t+1}^k, && (70)\\
\mathcal{L}_{X_t} &= 0 \quad \rightarrow \quad \lambda_t^k F_{X_t} = \lambda_t^q, && (71)\\
\mathcal{L}_{K_{t+1}} &= 0 \quad \rightarrow \quad \lambda_{t+1}^k F_{K_{t+1}} = \lambda_t^k, && (72)\\
\mathcal{L}_{Q_{t+1}} &= 0 \quad \rightarrow \quad \lambda_{t+1}^q = \lambda_t^q, && (73)
\end{aligned}$$

where F_{K_t} and F_{X_t} follow the definitions given in the main text. In addition, the centralized allocation must satisfy the transversality conditions

$$\lim_{t\rightarrow\infty} \lambda_t^q Q_t = 0 \text{ and } \lim_{t\rightarrow\infty} \lambda_t^k K_t = 0. \quad (74)$$

Equations (69), (70) and (72) imply

$$\begin{aligned}
U_{c_t}^* &= U_{e_{t+1}}^* F_{K_{t+1}}^*, && (75)\\
U_{c_t}^* \Delta &= U_{e_t}^*. && (76)
\end{aligned}$$

Substituting $U_{c_t}^*$ and $U_{e_{t+1}}^*$ by means of (64), conditions (75)-(76) respectively imply

$$\begin{aligned}
e_{t+1}^* &= c_t^* \left[\epsilon + \left(F_{K_{t+1}}^* \beta + \epsilon\beta \right)^{1/\eta} \right], && (77)\\
\left(c_t^*\right)^{-\eta} - \epsilon\beta \left(e_{t+1}^* - \epsilon c_t^* \right)^{-\eta} &= \left(\beta/\Delta\right) \left(e_t^* - \epsilon c_{t-1}^* \right)^{-\eta}. && (78)
\end{aligned}$$

From (77), we can substitute $e_{\cdot+1} - \epsilon c_{\cdot} = \left(F_{K_{\cdot+1}} \beta + \epsilon\beta \right)^{1/\eta}$ in (78) to obtain

$$\left(c_t^* / c_{t-1}^* \right)^{-\eta} = \Delta F_{K_{t+1}}^* \left(F_{K_t}^* + \epsilon \right) \left(F_{K_{t+1}}^* + \epsilon \right)^{-1}.$$

Setting this expression one period forward and solving for c_{t+1}^*/c_t^* yields (8). Using (77) to eliminate c_t from (8), and setting the resulting expression one period backward, we obtain

$$\frac{c_t^*}{e_t^*} = \frac{\left(F_{K_t}^* + \epsilon \right)^{1/\eta}}{\epsilon + \beta^{1/\eta} \left(F_{K_t}^* + \epsilon \right)^{1/\eta}} \left(\Delta \frac{F_{K_{t+1}}^*}{F_{K_{t+1}}^* + \epsilon} \right)^{1/\eta}. \quad (79)$$

Plugging (77) in (79) to eliminate c_t^*, we obtain condition (9) in the text. Since (73) implies that λ_t^q be constant, conditions (71)-(72) yield (10). Rearranging terms in (77) we obtain (11). □

Proof of Lemma 2. Setting $F_{K_t} = F_{K_{t+1}} = F_{K_{t+2}} = F_{K_{ss}}^*$ in (8)-(9), we obtain (13). The same procedure yields (14) from (79). From (1), the derivatives $F_{K_t} \equiv \partial F/\partial K_t$ and $F_{X_t} \equiv dF/dX_t$ equal

$$F_{K_t} = \alpha_1 \left(Y_t / K_t \right) \text{ and } F_{X_t} = \alpha_2 \left(Y_t / X_t \right). \quad (80)$$

From the first expression, $\lim_{t\rightarrow\infty} F_{K_t} = F_{K_{ss}}^*$ implies

$$\lim_{t\rightarrow\infty} \left(Y_t^* / K_t^* \right) = F_{K_{ss}}^* / \alpha_1. \quad (81)$$

Denote by $g_{p_t} \equiv p_{t+1}/p_t$ the gross growth rate of the generic variable p_t, and by $g_{p_\infty} \equiv \lim_{t\to\infty} (p_{t+1}/p_t)$ the asymptotic growth rate. Given (81), the accumulation constraint implies

$$g_{K_\infty} \equiv \lim_{t\to\infty} \frac{K^*_{t+1}}{K^*_t} = \frac{F^*_{K_{ss}}}{\alpha_1} - \lim_{t\to\infty} \frac{N^y_t \left(c^*_t + e^*_t n^{-1}\right)}{K^*_t}, \tag{82}$$

where, from (14) and $N^y_{t+1}/N^y_t = n$, the numerator of the last term grows at the constant rate $n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}$. By a standard argument, the cases in which the asymptotic growth rate of K^*_t differs from that of aggregate consumption can be excluded because the economy would either violate the non-negativity constraint on capital or the transversality conditions (74).[14] As a consequence, we have $g_{K_\infty} = n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}$. This in turn implies, from (81), equal growth rates between capital and output, $g_{K_\infty} = g_{Y_\infty} = n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}$, which proves result (15). Using (80) to eliminate F_{X_t} and $F_{X_{t+1}}$ from (10), the Hotelling condition can be written as

$$X^*_{t+1}/X^*_t = \left(Y^*_{t+1}/Y^*_t\right)\left(F^*_{K_{t+1}}\right)^{-1}. \tag{83}$$

Setting $F^*_{K_{t+1}} = F^*_{K_{ss}}$ in (83) and using (15), we obtain (16). The unique steady-state value of F_{K_t} that satisfies the balanced-growth conditions derived above can be obtained as follows. Taking the limit as $t \to \infty$ in (65), and substituting $g_{K_\infty} = g_{Y_\infty}$ and $g_{X_\infty} = g_{Y_\infty}/F^*_{K_{ss}}$ from (15) and (83), respectively, equation (65) implies

$$g^*_{Y_\infty} = \left[(1+\gamma)/F^*_{K_{ss}}\right]^{\alpha_2/\alpha_3} n. \tag{84}$$

Substituting $g^*_{Y_\infty} = n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}$ from (15), and solving the resulting expression for $F^*_{K_{ss}}$ gives (17) in the text. For future reference, notice that the transversality condition $\lim_{t\to\infty} \lambda^k_t K_t = 0$ is satisfied iff parameters satisfy

$$F^*_{K_{ss}} > n\left(\Delta F^*_{K_{ss}}\right)^{1/\eta}, \tag{85}$$

since this is necessary to have $\lim_{t\to\infty}\left(\lambda^k_{t+1}/\lambda^k_t\right) g_{K_\infty} < 1$. □

Derivation of (20)-(21). Equation (20) is derived as follows. Define the output-capital ratio as $\chi_t \equiv Y_t/K_t$. Since $F_{K_t} = \alpha_1\chi_t$ and $F_{X_t} = \alpha_2 (Y_t/X_t)$, the Hotelling rule (10) can be written as $g^*_{X_t} = g^*_{Y_t}\left(\alpha_1\chi^*_{t+1}\right)^{-1}$. Substituting this result in (65), we obtain

$$g^*_{Y_t} = \left(g^*_{K_t}\right)^{\frac{\alpha_1}{1-\alpha_2}}\left(\alpha_1\chi^*_{t+1}\right)^{-\frac{\alpha_2}{1-\alpha_2}}(1+\gamma)^{\frac{\alpha_2}{1-\alpha_2}} n^{\frac{\alpha_3}{1-\alpha_2}}, \tag{86}$$

where and superscript '*' is associated with the centralized allocation. Dividing both sides of (86) by $g^*_{K_t}$, and recalling that $g_{Y_t}/g_{K_t} = \chi_{t+1}/\chi_t$, we have

$$\chi^*_{t+1}/\chi^*_t = \left(g^*_{K_t}\right)^{-\frac{\alpha_3}{1-\alpha_2}}\left(\alpha_1\chi^*_{t+1}\right)^{-\frac{\alpha_2}{1-\alpha_2}}(1+\gamma)^{\frac{\alpha_2}{1-\alpha_2}} n^{\frac{\alpha_3}{1-\alpha_2}}.$$

Substituting $g^*_{K_t}$ by means of (66), we obtain

$$\chi^*_{t+1} = \left(\chi^*_t\right)^{\frac{\alpha_1}{1-\alpha_2}}\left(1-\varphi^*_t\right)^{-\frac{\alpha_3}{1-\alpha_2}}\left(\alpha_1\chi^*_{t+1}\right)^{-\frac{\alpha_2}{1-\alpha_2}}(1+\gamma)^{\frac{\alpha_2}{1-\alpha_2}} n^{\frac{\alpha_3}{1-\alpha_2}},$$

where $\varphi_t \equiv \left(N^y_t/Y_t\right)\left(c_t + e_t n^{-1}\right)$ is the aggregate consumption-output ratio. Solving the above expression for χ^*_{t+1} and re-arranging terms,we obtain equation (20) in the text.

Equation (21) is derived as follows. Since $F_{K_t} = \alpha_1\chi_t$, the right-hand side of (79) can be defined as a function of χ^*_t and χ^*_{t+1},

$$\frac{c^*_t}{e^*_t} = \theta^a\left(\chi^*_t, \chi^*_{t+1}\right) \equiv \frac{\left(F^*_{K_t}+\epsilon\right)^{1/\eta}}{\epsilon + \beta^{1/\eta}\left(F^*_{K_t}+\epsilon\right)^{1/\eta}}\left(\Delta\frac{F^*_{K_{t+1}}}{F^*_{K_{t+1}}+\epsilon}\right)^{1/\eta}. \tag{87}$$

[14] Proof.....

Using (87), the aggregate consumption-output ratio equals

$$\varphi_t^* \equiv \frac{N_t^y \left(nc_t^* + e_t^*\right)}{nY_t^*} = \frac{N_t^y e_t^*}{nY_t^*}\left[1 + n \cdot \theta^a \left(\chi_t^*, \chi_{t+1}^*\right)\right],$$

and its growth rate reads

$$\frac{\varphi_{t+1}^*}{\varphi_t^*} = n\frac{g_{e_t}^*}{g_{Y_t}^*}\left[\frac{1 + n \cdot \theta^a \left(\chi_t^*, \chi_{t+1}^*\right)}{1 + n \cdot \theta^a \left(\chi_{t+1}^*, \chi_{t+2}^*\right)}\right]. \tag{88}$$

From the right-hand side of (9), the growth rate of second-period consumption per adult, $g_{e_t}^*$, is a function of χ_t^* and χ_{t+1}^*,

$$g_{e_t}^* = \theta^b \left(\chi_t^*, \chi_{t+1}^*\right) \equiv \frac{\epsilon + \beta^{1/\eta}\left(F_{K_{t+1}}^* + \epsilon\right)^{1/\eta}}{\epsilon + \beta^{1/\eta}\left(F_{K_t}^* + \epsilon\right)^{1/\eta}}\left(\Delta F_{K_{t+1}}^* \frac{F_{K_t}^* + \epsilon}{F_{K_{t+1}}^* + \epsilon}\right)^{1/\eta}. \tag{89}$$

From (20) and (86), the growth rate of aggregate output is a function of $\left(\chi_t^*, \varphi_t^*, \chi_{t+1}^*\right)$,

$$g_{Y_t}^* = \theta^c \left(\chi_t^*, \varphi_t^*, \chi_{t+1}^*\right). \tag{90}$$

Combining (89)-(90) with (88), it follows that the dynamic equation of φ_t^* can be written as

$$\varphi_{t+1}^* = \varphi_t^* \cdot n\frac{\theta^b \left(\chi_t^*, \chi_{t+1}^*\right)}{\theta^c \left(\chi_t^*, \varphi_t^*, \chi_{t+1}^*\right)}\left[\frac{1 + n \cdot \theta^a \left(\chi_t^*, \chi_{t+1}^*\right)}{1 + n \cdot \theta^a \left(\chi_{t+1}^*, \chi_{t+2}^*\right)}\right]. \tag{91}$$

Defining the right-hand side of (91) as $\Theta\left(\varphi_t^*, \chi_t^*, \chi_{t+1}^*, \chi_{t+2}^*\right)$, we obtain expression (21) in the text. □

Proof of Lemma 3. Suppose that $\alpha_3 \to 0$. In this case, labor is not used as an input, and the output-capital ratio displays autonomous dynamics: equation (20) reduces to

$$\chi_{t+1}^* = \left(\chi_t^*\right)^{\alpha_1}\left[\alpha_1^{-1}\left(1+\gamma\right)\right]^{1-\alpha_1}, \tag{92}$$

which exhibits a globally stable steady-state $\chi_{ss}^* = \alpha_1^{-1}\left(1+\gamma\right)$. This result ensures that $\lim_{t\to\infty}\chi_t = \chi_{ss}^*$, and that the marginal product of capital converges to the steady-state $\lim_{t\to\infty} F_{K_t}^* = 1+\gamma = F_{K_{ss}}$.[15] From Lemma 2, the economy approaches balanced growth in the long run, which implies $\lim_{t\to\infty}\varphi_t = \varphi_{ss}^*$. □

Details on backward iteration.

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Derivation of (25). From (73), λ_t^q is constant. As a consequence, satisfying the first transversality condition in (74) requires $\lim_{t\to\infty} Q_t = 0$, i.e. exhausting the resource stock asymptotically. Iterating the dynamic resource constraint (4), we have

$$Q_t = Q_0 - \sum_{j=0}^{t} X_j.$$

Taking the limit of this expression as $T \to \infty$, and substituting the efficiency condition $\lim_{t\to\infty} Q_t = 0$, we obtain (25). Notice that, in order to verify (25) in practice, we can use the following approximation. Defining

$$z_t = \frac{Q_t - X_t}{X_t},$$

[15] Letting $\alpha_3 \to 0$ in (17), the steady-state value F_K^* indeed reduces to $1+\gamma$.

it can be shown that the expression

$$\frac{X_{t+1}}{X_t} = \frac{z_t}{1+z_{t+1}} \tag{93}$$

is valid at each point in time. From (16), we know that approaches a constant value z_{ss} in the long run: in the centralized allocation, in particular, this is equal to

$$z_{ss}^* = \frac{n\Delta^{1/\eta}\left(F_{K_{ss}}^*\right)^{\frac{1-\eta}{\eta}}}{1-n\Delta^{1/\eta}\left(F_{K_{ss}}^*\right)^{\frac{1-\eta}{\eta}}}. \tag{94}$$

In any case, since $z_t \to z_{ss}$, we can choose a sufficiently large T such that

$$X_{T+j} \simeq X_T \left(\frac{z_{ss}}{1+z_{ss}}\right)^j \text{ for any } j \geq 0. \tag{95}$$

From (95), we can re-write the infinite sum of extracted resources as

$$\begin{aligned}\sum_{t=0}^{\infty} X_t &= \sum_{t=0}^{T} X_t + X_T + X_{T+1} + ... = \sum_{t=0}^{T} X_t + X_T + X_T\left(\frac{z_{ss}}{1+z_{ss}}\right) + X_T\left(\frac{z_{ss}}{1+z_{ss}}\right)^2 + ... \\ &= \sum_{t=0}^{T} X_t + X_T \sum_{j=T}^{\infty}\left(\frac{z_{ss}}{1+z_{ss}}\right)^{j-T},\end{aligned}$$

where the fact that the term in round brackets is less than unity implies

$$\sum_{j=T}^{\infty}\left(\frac{z_{ss}}{1+z_{ss}}\right)^{j-T} = 1+z_{ss},$$

so that

$$\sum_{t=0}^{\infty} X_t = \sum_{t=0}^{T} X_t + X_T\left(1+z_{ss}\right). \tag{96}$$

Provided that z is approximately equal to z_{ss} from time T onwards, expression (96) equals the initial stock. □

Proof of Lemma 6. From (29) and (31), the lifetime budget constraint of an agent born in $t \geq 0$ reads

$$c_t + e_{t+1}R_{t+1}^{-1} = w_t + b_t - p_t^a\left(a_t/n\right) + R_{t+1}^{-1}\left[p_{t+1}^x x_{t+1} + p_{t+1}^a a_{t+1} - nb_{t+1}\right]. \tag{97}$$

Having eliminated k_{t+1}, we can solve the reduced problem

$$\max_{\{c_t, e_{t+1}, a_t, x_{t+1}, b_{t+1}\}} U\left(c_t, e_{t+1}\right) + \Psi n W_{t+1}^{\ell}\left(w_{t+1} + b_{t+1}\right)$$

subject to (97) and to $b_{t+1} \geq 0$. The Lagrangean at time t for this problem is

$$\begin{aligned}\mathcal{L}^r &= U\left(c_t, e_{t+1}\right) + \Psi n W_{t+1}^{\ell}\left(w_{t+1} + b_{t+1}\right) + \\ &\quad +\lambda_t\left\{w_t + b_t - p_t^a\left(a_t/n\right) + R_{t+1}^{-1}\left[p_{t+1}^x x_{t+1} + p_{t+1}^a a_{t+1} - nb_{t+1}\right] - c_t - e_{t+1}R_{t+1}^{-1}\right\},\end{aligned}$$

where λ_t is the Lagrange multiplier associated with (97). The first order conditions yield

$$\mathcal{L}_{c_t}^r = 0 \quad \to \quad U_{c_t}^{\ell} = \lambda_t, \tag{98}$$

$$\mathcal{L}_{e_{t+1}}^r = 0 \quad \to \quad U_{e_{t+1}}^{\ell} = \lambda_t R_{t+1}^{-1}, \tag{99}$$

$$\mathcal{L}_{x_{t+1}}^r = 0 \quad \to \quad p_{t+1}^x = p_{t+1}^a, \tag{100}$$

$$\mathcal{L}_{a_t}^r = 0 \quad \to \quad p_{t+1}^a = p_t^a R_{t+1}, \tag{101}$$

$$\mathcal{L}_{x_{t+1}}^r \leq 0 \quad \to \quad \Psi\frac{\partial W_{t+1}^{\ell}}{\partial b_{t+1}} - \lambda_t R_{t+1}^{-1} \leq 0 \quad \left(= 0 \text{ if } b_{t+1}^{\ell} \geq 0\right). \tag{102}$$

Conditions (98)-(99) imply $U^\ell_{c_t} = R_{t+1}U^\ell_{e_{t+1}}$, where we can substitute $U^\ell_{c_t}$ and $U^\ell_{e_{t+1}}$ by means of (64) to obtain

$$\left(c_t^\ell\right)^{-\eta} - \beta\epsilon\left(e_{t+1}^\ell - \epsilon c_t^\ell\right)^{-\eta} = R_{t+1}\beta\left(e_{t+1}^\ell - \epsilon c_t^\ell\right)^{-\eta}. \tag{103}$$

Rearranging terms and raising both sides to $1/\eta$ yields condition (33) in the text. The no-arbitrage conditions (34)-(35) directly follow from (100)-(101). The equation of motion of capital per-adult (36) coincides with (67), and can be re-derived by aggregating the consumption of both cohorts alive in t from the individual constraints (29) and (31).[16] Finally, we derive the condition for optimal bequests (37). Suppose that $b^\ell_{t+1} \geq 0$. Then, the strict equality in (102) together with (98) imply

$$\frac{\partial W^\ell_{t+1}}{\partial b_{t+1}} = \frac{U^\ell_{c_t}}{\Psi R_{t+1}}. \tag{104}$$

By definition, the value function at time $t+1$ is given by the maximum lifetime utility for the agent born at $t+1$ for given personal resources $w_{t+1} + b_{t+1}$, that is

$$W^\ell_{t+1} = U\left(c^\ell_{t+1}, e^\ell_{t+2}\right) + \sum_{i=t+2}^{\infty} (\Psi n)^{i-t}\, U\left(c^\ell_i, e^\ell_{i+1}\right).$$

Since with $c^\ell_{t+1} = w_{t+1}+b_{t+1}-p^a_{t+1}\left(a_{t+1}/n\right)-k_{t+2}$, it follows that $\partial W^\ell_{t+1}/\partial b_{t+1} = U^\ell_{c_{t+1}}$. Substituting this result into (104), and using superscript '$\ell 1$' to represent the Ramsey-Stiglitz accumulation regime, we obtain

$$U^{\ell 1}_{c_{t+1}}\Psi R_{t+1} = U^{\ell 1}_{c_t}. \tag{105}$$

Substituting $U^{\ell 1}_{c_t}$ and $U^{\ell 1}_{c_{t+1}}$ by means of (64), we obtain

$$\left[\left(c^{\ell 1}_{t+1}\right)^{-\eta} - \beta\epsilon\left(e^{\ell 1}_{t+2} - \epsilon c^{\ell 1}_{t+1}\right)^{-\eta}\right]\Psi R_{t+1} = \left[\left(c^{\ell 1}_t\right)^{-\eta} - \beta\epsilon\left(e^{\ell 1}_{t+1} - \epsilon c^{\ell 1}_t\right)^{-\eta}\right]. \tag{106}$$

From (33), we can substitute $e^{\ell 1}_{\cdot+1}/c^{\ell 1}_{\cdot} = \epsilon + \beta^{1/\eta}\left(R_{\cdot+1}+\epsilon\right)^{1/\eta}$ in both sides to eliminate second-period consumption levels, and obtain

$$\left(c^{\ell 1}_{t+1}\right)^{-\eta}\left[1 - \beta\epsilon\left[\beta\left(R_{t+2}+\epsilon\right)\right]^{-1}\right]\Psi R_{t+1} = \left(c^{\ell 1}_t\right)^{-\eta}\left[1 - \beta\epsilon\left[\beta\left(R_{t+1}+\epsilon\right)\right]^{-1}\right].$$

Rearranging terms and raising both sides to η, we obtain (37), which completes the proof. For future reference, notice that the lifetime budget constraint (97), the resource constraint (28), and the no-arbitrage conditions (100)-(101) imply

$$c^\ell_t + e^\ell_{t+1}R^{-1}_{t+1} = w_t + b^\ell_t - R^{-1}_{t+1}nb^\ell_{t+1}, \tag{107}$$

which is an equilibrium condition that holds in the dynastic competitive economy irrespective of the accumulation regime ($\ell 1$ or $\ell 2$). □

Proof of Lemma 7. From (27), we can substitute $F^\ell_{K_t} = R_t$ in equations (33) and (37), respectively obtaining (41) and (38). Combining these two equations and solving for $e^{\ell 1}_{t+1}/c^{\ell 1}_{t+1}$ yields

$$e^{\ell 1}_{t+1}/c^{\ell 1}_{t+1} = \left[\epsilon + \beta^{1/\eta}\left(F^{\ell 1}_{K_{t+1}}+\epsilon\right)^{1/\eta}\right]\left[\Psi F^{\ell 1}_{K_{t+2}}\left(\frac{F^{\ell 1}_{K_{t+1}}+\epsilon}{F^{\ell 1}_{K_{t+2}}+\epsilon}\right)\right]^{-1/\eta}. \tag{108}$$

[16] From (29) and (31), aggregation of consumption of both cohorts alibe in period t yields

$$N^y_t c_t + N^a_t e_t = w_t N^y_t - k_{t+1}N^y_t + R_t k_t N^a_t + p^x_t x_t N^a_t,$$

where we can substitute the profit-maximizing conditions (27) to obtain (3). Dividing both sides by N^a_t, and defining output per adult as $y_t \equiv Y_t/N^a_t$, we obtain (36).

Setting (108) on period backward and re-arranging terms, we obtain

$$c_t^{\ell 1} = e_t^{\ell 1} \left[\Psi F_{K_{t+1}}^{\ell 1} \left(\frac{F_{K_t}^{\ell 1} + \epsilon}{F_{K_{t+1}}^{\ell 1} + \epsilon} \right) \right]^{1/\eta} \left[\epsilon + \beta^{1/\eta} \left(F_{K_t}^{\ell 1} + \epsilon \right)^{1/\eta} \right]^{-1}, \tag{109}$$

which can be substituted back in (41) to obtain (39). From (27), we can substitute $F_{K_t}^{\ell} = R_t$ and $F_{X_t} = p_t^x$ in (35) to obtain the Hotelling rule (40). Equation (42) is obtained as follows. Solving (107) for b_{t+1}^{ℓ} and substituting condition (33), the dynamics of bequests along a PRS equilibrium path obey

$$b_{t+1}^{\ell 1} = \frac{R_{t+1}}{n} \left\{ w_t + b_t^{\ell 1} - c_t^{\ell 1} \left[1 + R_{t+1}^{-1} \left(\epsilon + \beta^{1/\eta} \left(R_{t+1} + \epsilon \right)^{1/\eta} \right) \right] \right\}. \tag{110}$$

Dividing both sides by output per adult $y_t = Y_t / N_t^a$, and substituting $w_t = \alpha_3 (y_t/n)$ and $R_{t+1} = \alpha_1 (y_{t+1}/k_{t+1})$ from (27), we obtain (42). □

Proof of Lemma 8. Setting $\Psi = \Delta$ in (38)-(39), the intertemporal conditions for the centralized allocation (11), (8), and (9) respectively imply $e_{t+1}^{\ell 1}/c_t^{\ell 1} = e_{t+1}^{*}/c_t^{*}$, $c_{t+1}^{\ell 1}/c_t^{\ell 1} = c_{t+1}^{*}/c_t^{*}$, and $e_{t+1}^{\ell 1}/e_t^{\ell 1} = e_{t+1}^{*}/e_t^{*}$. Since the Hotelling rule (40) coincides with (10)., and the aggregate constraint (36) is equivalent to (3) - see (67) above - it follows that, given identical initial endowments (K_0, Q_0), and setting a social discount factor $\Delta = \Psi$, the PRS equilibrium path coincides with the centralized allocation. □

Derivation of (46). Given the observational equivalence established in Lemma 8, results (13)-(17) in Lemma 2 imply

$$\begin{aligned}
\lim_{t\to\infty} F_{K_t}^{\ell 1} &= F_{K_{ss}}^{\ell 1} = (1+\gamma)^{\frac{\alpha_2 \eta}{\alpha_3 + \alpha_2 \eta}} \Psi^{-\frac{\alpha_3}{\alpha_3 + \alpha_2 \eta}}, \\
\lim_{t\to\infty} c_{t+1}^{\ell 1}/c_t^{\ell 1} &= \lim_{t\to\infty} e_{t+1}^{\ell 1}/e_t^{\ell 1} = \left(\Psi F_{K_{ss}}^{\ell 1} \right)^{1/\eta}, \\
\lim_{t\to\infty} c_t^{\ell 1}/e_t^{\ell 1} &= \frac{1}{\epsilon + \beta^{1/\eta} \left(F_{K_{ss}}^{\ell 1} + \epsilon \right)^{1/\eta}} \left(\Psi F_{K_{ss}}^{\ell 1} \right)^{1/\eta}, \\
\lim_{t\to\infty} Y_{t+1}^{\ell 1}/Y_t^{\ell 1} &= \lim_{t\to\infty} K_{t+1}^{\ell 1}/K_t^{\ell 1} = n \left(\Psi F_{K_{ss}}^{\ell 1} \right)^{1/\eta}, \\
\lim_{t\to\infty} X_{t+1}^{\ell 1}/X_t^{\ell 1} &= n \Psi^{1/\eta} \left(F_{K_{ss}}^{\ell 1} \right)^{\frac{1-\eta}{\eta}}.
\end{aligned} \tag{111}$$

Equation (46) is obtained as follows. Rewriting y_t/k_{t+1} as $(y_t/k_t)(k_t/k_{t+1}) = \chi_t / g_{k_t}$, and defining the convenient variables $\tau_t \equiv b_t/y_t$ and $\nu_t \equiv c_t/y_t$, expression (42) can be-rewritten as

$$\tau_{t+1}^{\ell} = (\alpha_1/n) \left(\chi_t^{\ell 1}/g_{k_t}^{\ell 1} \right) \left\{ (\alpha_3/n) + \tau_t^{\ell} - \nu_t^{\ell} \left[1 + \left(F_{K_t}^{\ell 1} \right)^{-1} \left(\epsilon + \beta^{1/\eta} \left(F_{K_t}^{\ell 1} + \epsilon \right)^{1/\eta} \right) \right] \right\}, \tag{112}$$

where τ_t is the bequest-output ratio. As the marginal product of capital $F_{K_t}^{\ell}$ converges to the finite steady-state $F_{K_{ss}}^{\ell}$, all the time-varying terms in (112) - that is, χ_t, g_{k_t}, ν_t and $F_{K_{t+1}}^{\ell}$ - achieve the stationary values[17]

$$\begin{aligned}
\chi_{ss}^{\ell 1} &= \lim_{t\to\infty} \chi_t^{\ell 1} = \alpha_1^{-1} F_{K_{ss}}^{\ell 1}, \\
g_{k_\infty}^{\ell 1} &= \lim_{t\to\infty} g_{k_t}^{\ell 1} = \left(\Psi F_{K_{ss}}^{\ell 1} \right)^{1/\eta}, \\
\nu_{ss}^{\ell 1} &= \lim_{t\to\infty} \nu_t^{\ell 1} = \lim_{t\to\infty} n^{-1} \left[\varphi_t^{\ell 1} - \left(e_t^{\ell 1}/y_t^{\ell 1} \right) \right], \\
F_{K_{ss}}^{\ell 1} &= \lim_{t\to\infty} F_{K_{t+1}}^{\ell} = (1+\gamma)^{\frac{\alpha_2 \eta}{\alpha_3 + \alpha_2 \eta}} \Psi^{-\frac{\alpha_3}{\alpha_3 + \alpha_2 \eta}},
\end{aligned}$$

[17] Justify the stationary values...

and the associated steady-state value of τ_t implied by (112) is

$$\tau_{ss}^{\ell 1} = \frac{F_{K_{ss}}^{\ell 1}}{F_{K_{ss}}^{\ell 1} - n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta}} \left\{\nu_{ss}^{\ell 1}\left[1 + \left(F_{K_{ss}}^{\ell 1}\right)^{-1}\left(\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right)\right] - (\alpha_3/n)\right\}, \quad (113)$$

where $\tau_{ss}^{\ell 1} = \lim_{t\to\infty} b_t^{\ell 1}/y_t^{\ell 1}$. Since $\nu_{ss}^{\ell 1}$ and the term in square brackets in (113) depend on ϵ (see the full derivation in eq.(120) below), we can define

$$\Lambda(\epsilon) \equiv \nu_{ss}^{\ell 1}\left[1 + \left(F_{K_{ss}}^{\ell 1}\right)^{-1}\left(\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right)\right], \quad (114)$$

and re-write (113) as (46). The fact that

$$F_{K_{ss}}^{\ell 1} > n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta} \quad (115)$$

follows by analogy with (85), which is necessary to satisfy the transversality condition on aggregate capital. □

Proof of Lemma 9. From definition (114), the term $\Lambda(\epsilon)$ can be explicitly determined as follows. From (68), the ratio $\nu_t^{\ell 1} = c_t^{\ell 1}/y_t^{\ell 1}$ equals $\nu_t^{\ell 1} = n^{-1}\left[\varphi_t^{\ell 1} - \left(e_t^{\ell 1}/y_t^{\ell 1}\right)\right]$. Substituting $e_t/y_t = (e_t/c_t)(c_t/y_t) = (e_t/c_t)\nu_t$, we have

$$\nu_t^{\ell 1} = \frac{\varphi_t^{\ell 1}}{n + \left(e_t^{\ell 1}/c_t^{\ell 1}\right)}, \quad (116)$$

From (111), and by analogy with (23), we respectively obtain

$$\lim_{t\to\infty} \frac{e_t^{\ell 1}}{c_t^{\ell 1}} = \left[\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right]\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{-1/\eta}, \quad (117)$$

$$\lim_{t\to\infty} \varphi_t^{\ell 1} = \varphi_{ss}^{\ell 1} = 1 - n\alpha_1 \Psi^{\frac{\alpha_2+\alpha_3}{\alpha_2\eta+\alpha_3}}(1+\gamma)^{\frac{\alpha_2(1-\eta)}{\alpha_2\eta+\alpha_3}}, \quad (118)$$

so that the limit of (116) as $t \to \infty$ reads

$$\nu_{ss}^{\ell 1} = \lim_{t\to\infty} \nu_t^{\ell 1} = \frac{\varphi_{ss}^{\ell 1}\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta}}{n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta} + \left[\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right]}. \quad (119)$$

Given (119) and (114), we can re-write $\Lambda(\epsilon)$ as

$$\Lambda(\epsilon) = n^{-1}\varphi_{ss}^{\ell 1}\frac{n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta}\left[1 + \left(F_{K_{ss}}^{\ell 1}\right)^{-1}\left(\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right)\right]}{n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta} + \left[\epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta}\right]}, \quad (120)$$

where, from (111) and (118), neither $\varphi_{ss}^{\ell 1}$ nor $F_{K_{ss}}^{\ell 1}$ depend on ϵ. Defining $h' \equiv n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta} > 0$, $h'' \equiv F_{K_{ss}}^{\ell 1} > 0$, and $\lambda(\epsilon) \equiv \epsilon + \beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta} > 0$, expression (120) reduces to

$$\Lambda(\epsilon) = n^{-1}\varphi_{ss}^{\ell 1}\frac{h' + (h'/h'')\lambda(\epsilon)}{h' + \lambda(\epsilon)}. \quad (121)$$

From (121), the derivative $\Lambda'(\epsilon) \equiv \partial\Lambda(\epsilon)/\partial\epsilon$ can be written as

$$\Lambda'(\epsilon) = \frac{n^{-1}\varphi_{ss}^{\ell 1}\lambda'(\epsilon)h'}{(h' + \lambda(\epsilon))^2 h''} \cdot (h' - h''). \quad (122)$$

Since $\lambda'(\epsilon) = 1 + (1/\eta)\beta^{1/\eta}\left(F_{K_{ss}}^{\ell 1} + \epsilon\right)^{1/\eta - 1} > 0$ and $h' - h'' = n\left(\Psi F_{K_{ss}}^{\ell 1}\right)^{1/\eta} - F_{K_{ss}}^{\ell 1} < 0$ (from (115) above), it follows from (122) that $\Lambda(\epsilon)$ is strictly declining in ϵ. From (46), the derivative $\partial\tau_{ss}^{\ell 1}/\partial\epsilon$ has the same sign as $\partial\Lambda(\epsilon)/\partial\epsilon < 0$, which completes the proof. □

Derivation of system (48)-(49)-(50). Notice that, from Lemma 6, conditions (33)-(36) hold in the PDM equilibrium. In particular, given the profit-maximizing conditions $p_t^x = F_{X_t}$ and $R_t = F_{K_t} = \alpha_1\chi_t$, the Hotelling rule (40), and the aggregate relations (65) and (66), we can follow the same steps as in the derivation of (20), to obtain the equivalent expression for the competitive economy,

$$\chi_{t+1}^{\ell} = \left(\chi_t^{\ell}\right)^{\alpha_1}\left(1 - \varphi_t^{\ell}\right)^{-\alpha_3}\left[\alpha_1^{-1}(1+\gamma)\right]^{\alpha_2} n^{\alpha_3}, \tag{123}$$

which holds independently of the accumulation regime (and hence in the PDM equilibrium as well). Setting superscripts $\ell = \ell 2$ yields (48). The peculiarities of the PDM equilibrium are given by (i) the dynamics of the consumption-output ratio $\varphi_t^{\ell 2}$, and (ii) the speed of resource depletion. In the first regard, equation (49) can be obtained as follows. Setting $b_t = 0$ in each $t = 0, ..., \infty$, the individual budget constraints (29)-(30) read

$$c_t^{\ell 2} = w_t^{\ell 2} - p_t^a\left(a_t^{\ell 2}/n\right) - k_{t+1}^{\ell 2} \text{ and } e_{t+1}^{\ell 2} = R_{t+1}k_{t+1}^{\ell 2} + p_{t+1}^a q_{t+1}^{\ell 2}, \tag{124}$$

where we used the no-arbitrage condition (34) and the resource constraint (28). Using the Hotelling rule (35) and substituting $q_{t+1} = a_t/n$ from (28), expressions (124) imply the standard present-value lifetime constraint $c_t^{\ell 2} + R_{t+1}^{-1}e_{t+1}^{\ell 2} = w_t$. Substituting condition (33) to eliminate e_{t+1}, we obtain

$$c_t^{\ell 2} = \frac{w_t}{1 + R_{t+1}^{-1}\left[\epsilon + \beta^{1/\eta}(R_{t+1} + \epsilon)^{1/\eta}\right]}. \tag{125}$$

From (27), the second expression in (124) implies $e_{t+1}^{\ell 2} = \alpha_1 y_{t+1}^{\ell 2} + \alpha_2\left(q_{t+1}^{\ell 2}/x_{t+1}^{\ell 2}\right)y_{t+1}^{\ell 2}$. Re-writing this expression at period t and substituting $q_t = x_t + a_t$, we have

$$e_t^{\ell 2}/y_t = \alpha_1 + \alpha_2 + \alpha_2\left(a_t^{\ell 2}/x_t^{\ell 2}\right). \tag{126}$$

Using $w_t = \alpha_3 y_t/n$ from (27), we can combine equations (125)-(126) to obtain

$$\varphi_t^{\ell 2} = \frac{nc_t^{\ell 2} + e_t^{\ell 2}}{y_t^{\ell 2}} = \frac{\alpha_3}{1 + R_{t+1}^{-1}\left[\epsilon + \beta^{1/\eta}(R_{t+1} + \epsilon)^{1/\eta}\right]} + \alpha_1 + \alpha_2 + \alpha_2\left(a_t^{\ell 2}/x_t^{\ell 2}\right). \tag{127}$$

Defining the convenient variable

$$\Upsilon_t \equiv \frac{R_{t+1}^{-1}\left[\epsilon + \beta^{1/\eta}(R_{t+1} + \epsilon)^{1/\eta}\right]}{1 + R_{t+1}^{-1}\left[\epsilon + \beta^{1/\eta}(R_{t+1} + \epsilon)^{1/\eta}\right]} < 1, \tag{128}$$

and the *depletion index* $z_t \equiv a_t/x_t$, equation (127) yields - after some rearrangements and using $\alpha_3 = 1 - \alpha_1 - \alpha_2$ - equation (49) in the text. Recalling that $R_{t+1} = F_{K_{t+1}} = \alpha_1\chi_{t+1}$, it is self-evident from (128) that Υ_t can be treated as a function of ϵ and χ_{t+1}, so that we use the notation $\Upsilon_t = \Upsilon\left(\chi_{t+1}, \epsilon\right)$ in the main text. The dynamics of the depletion index $z_t^{\ell 2} = a_t^{\ell 2}/x_t^{\ell 2}$ are obtained as follows. From (125) and the first-period constraint in (124), individual savings in the PDM equilibrium can be written as

$$p_t^a\left(a_t^{\ell 2}/n\right) + k_{t+1}^{\ell 2} = \Upsilon_t w_t, \tag{129}$$

where we have exploited definition (128). Using (27) to eliminate $p_t^a = p_t^x$ and w_t from (129), we obtain

$$n\frac{k_{t+1}^{\ell 2}}{y_t^{\ell 2}} = \Upsilon_t\alpha_3 - \alpha_2 z_t^{\ell 2}. \tag{130}$$

From (27), we can rewrite the Hotelling rule (35) as

$$\frac{k_{t+1}^{\ell}}{y_t^{\ell}} = \alpha_1 \frac{x_{t+1}^{\ell}}{x_t^{\ell}}, \tag{131}$$

and substitute it in (130) to obtain a dynamic equation for extracted resource per adult,

$$\frac{x_{t+1}^{\ell 2}}{x_t^{\ell 2}} = \Upsilon_t \frac{\alpha_3}{n\alpha_1} - \frac{\alpha_2}{n\alpha_1} z_t^{\ell 2}. \tag{132}$$

From the resource constraint (28), it follows that[18]

$$\frac{x_{t+1}}{x_t} = \frac{z_t}{n\left(1+z_{t+1}\right)}. \tag{133}$$

Combining (132)-(133), we have

$$\alpha_1 \frac{z_t^{\ell 2}}{1+z_{t+1}^{\ell 2}} = \alpha_3 \Upsilon_t - \alpha_2 z_t^{\ell 2},$$

which can be re-arranged to obtain (50). □

Derivation of (54)-(55) and existence conditions. From definition (128), recall that

$$\Upsilon\left(\bar{\chi},\epsilon\right) \equiv \frac{\left(\alpha_1\bar{\chi}\right)^{-1}\left[\epsilon+\beta^{1/\eta}\left(\alpha_1\bar{\chi}+\epsilon\right)^{1/\eta}\right]}{1+\left(\alpha_1\bar{\chi}\right)^{-1}\left[\epsilon+\beta^{1/\eta}\left(\alpha_1\bar{\chi}+\epsilon\right)^{1/\eta}\right]}. \tag{134}$$

In the steady-state system, equation (53) reduces to the second-order polynomial

$$\alpha_2\bar{z}^2 + \left[1-\alpha_3\left(1+\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right]\bar{z} - \alpha_3\Upsilon\left(\bar{\chi},\epsilon\right) = 0, \tag{135}$$

where the only positive root is

$$\bar{z} = \frac{\sqrt{\left[1-\alpha_3\left(1+\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right]^2+4\alpha_2\alpha_3\Upsilon\left(\bar{\chi},\epsilon\right)} - \left[1-\alpha_3\left(1+\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right]}{2\alpha_2} > 0. \tag{136}$$

Denote solution (136) as $\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)$. From (52)-(53), we have $1-\bar{\varphi} = \alpha_1\bar{z}\left(1+\bar{z}\right)^{-1}$. We can thus define the implicit function

$$1-\bar{\varphi} = \kappa\left(\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right) \equiv \alpha_1 \frac{\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)}{1+\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)}, \tag{137}$$

and substitute it in (51) to obtain

$$\frac{\bar{\chi}^{1-\alpha_1}}{\left[\alpha_1^{-1}\left(1+\gamma\right)\right]^{\alpha_2} n^{\alpha_3}} = \left(\frac{1}{\kappa\left(\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right)}\right)^{\alpha_3}. \tag{138}$$

defining the left-hand side of (138) as a function $f^a\left(\bar{\chi}\right)$, and the right-hand side as an implicit function

$$f^b\left(\bar{\chi},\epsilon\right) \equiv f^b\left(\kappa\left(\bar{z}\left(\Upsilon\left(\bar{\chi},\epsilon\right)\right)\right)\right), \tag{139}$$

[18] From (28), we have $a_{t+1} + x_{t+1} = a_t/n$. Dividing both terms by x_{t+1} and rearranging terms, we obtain

$$1 + z_{t+1} = n^{-1}\left(a_t/x_{t+1}\right).$$

Substituting $a_t/x_{t+1} = \left(a_t/x_t\right)\left(x_t/x_{t+1}\right) = z_t\left(x_t/x_{t+1}\right)$ and rearranging terms yields (133).

the steady-state condition can be written as $f^a(\bar{\chi}) = f^b(\bar{\chi}, \epsilon)$, which proves (54). The existence and uniqueness of the steady-state can be established as follows. From (138), it is easy to show that $f^a(\bar{\chi})$ exhibits the following properties:

$$\frac{\partial f^a(\bar{\chi})}{\partial \bar{\chi}} > 0, \quad \frac{\partial^2 f^a(\bar{\chi})}{\partial \bar{\chi}^2} < 0, \quad \lim_{\bar{\chi}\to 0} f^a(\bar{\chi}) = 0, \quad \lim_{\bar{\chi}\to\infty} f^a(\bar{\chi}) = \infty. \tag{140}$$

As regards $f^b(\bar{\chi}, \epsilon)$, we exploit the following

Claim 16 *Function $\Upsilon(\bar{\chi}, \epsilon)$ has the following properties:*

$$\begin{array}{lll}
\textit{(i) If } \eta = 1, \epsilon = 0: & \lim_{\bar{\chi}\to 0} \Upsilon(\bar{\chi}, \epsilon) = 1, \quad \lim_{\bar{\chi}\to\infty} \Upsilon(\bar{\chi}, \epsilon) = \frac{\beta}{1+\beta} < 1, & \partial\Upsilon/\partial\bar{\chi} = 0, \\
\textit{(ii) If } \eta = 1, \epsilon > 0: & \lim_{\bar{\chi}\to 0} \Upsilon(\bar{\chi}, \epsilon) = 1, \quad \lim_{\bar{\chi}\to\infty} \Upsilon(\bar{\chi}, \epsilon) = \frac{\beta}{1+\beta} < 1, & \partial\Upsilon/\partial\bar{\chi} < 0, \\
\textit{(iii) If } \eta > 1, \epsilon \geq 0: & \lim_{\bar{\chi}\to 0} \Upsilon(\bar{\chi}, \epsilon) = 1, \quad \lim_{\bar{\chi}\to\infty} \Upsilon(\bar{\chi}, \epsilon) = 0, & \partial\Upsilon/\partial\bar{\chi} < 0, \\
\textit{(iv) If } \eta < 1, \epsilon = 0: & \lim_{\bar{\chi}\to 0} \Upsilon(\bar{\chi}, \epsilon) = 0, \quad \lim_{\bar{\chi}\to\infty} \Upsilon(\bar{\chi}, \epsilon) = 1, & \partial\Upsilon/\partial\bar{\chi} > 0, \\
\textit{(v) If } \eta < 1, \epsilon > 0: & \lim_{\bar{\chi}\to 0} \Upsilon(\bar{\chi}, \epsilon) = 1, \quad \lim_{\bar{\chi}\to\infty} \Upsilon(\bar{\chi}, \epsilon) = 1, & 0 < \Upsilon(\bar{\chi}, \epsilon) < 1 \textit{ for } \forall\bar{\chi} \in (0, \infty)
\end{array}$$

where, in case (v), we have $\partial\Upsilon/\partial\bar{\chi} < 0$ for low values of $\bar{\chi}$, $\partial\Upsilon/\partial\bar{\chi} = 0$ in an intermediate level of $\bar{\chi}$, and $\partial\Upsilon/\partial\bar{\chi} > 0$ for high values of $\bar{\chi}$. It follows from (i)-(v) that $\Upsilon(\bar{\chi}, \epsilon)$ is bounded between 0 and unity for any $\bar{\chi} \in (0, \infty)$ and any $\epsilon \geq 0$. Moreover, it follows from (ii)-(iii) that $\eta \geq 1$ implies $\Upsilon(\bar{\chi}, \epsilon)$ be monotonically declining in $\bar{\chi}$ for any $\epsilon > 0$.

From (136), function $\bar{z}(\Upsilon)$ is monotonically increasing in Υ,

$$\frac{d\bar{z}(\Upsilon)}{d\Upsilon} = ... = \frac{\alpha_3 [(1/2) + 2\bar{z}]}{\sqrt{[1 - \alpha_3(1+\Upsilon)]^2 + 4\alpha_2\alpha_3\Upsilon}} > 0. \tag{141}$$

Given that $0 < \Upsilon(\bar{\chi}, \epsilon) < 1$ for $\forall\bar{\chi} \in (0, \infty)$, it follows from (136) and (141) that $\bar{z}(\Upsilon(\bar{\chi}, \epsilon))$ is bounded between finite and positive limits (respectively denoted as z' and z'') over the range of $\bar{\chi}$,

$$0 < z' \leq \bar{z}(\Upsilon(\bar{\chi}, \epsilon)) \leq z'' < \infty \text{ for any } \bar{\chi} \in (0, \infty) \text{ and any } \epsilon \geq 0. \tag{142}$$

Next consider the implicit function $\kappa(\bar{z}(\Upsilon(\bar{\chi}, \epsilon)))$ defined in (137), which is monotonically increasing in $\bar{z}$,

$$d\kappa(\bar{z})/d\bar{z} = \alpha_1(1+\bar{z})^{-2} > 0, \tag{143}$$

and is bounded by the boundedness of $\bar{z}$ in (142): there exists a couple (κ', κ'') such that

$$0 < \kappa' \leq \kappa(\bar{z}) \leq \kappa'' < 1 \text{ for any } \bar{z} \in [z', z'']. \tag{144}$$

The last implicit function $f^b(\kappa)$ defined in (139) is monotonically decreasing in κ,

$$df^b(\kappa)/d\kappa = -\alpha_3\kappa^{-\alpha_3 - 1} < 0, \tag{145}$$

and is bounded by the boundedness of κ in (143): there exists a couple $\left(f^b_{\min}, f^b_{\max}\right)$ such that

$$0 < f^b_{\min} \leq f^b(\kappa) \leq f^b_{\max} < \infty \text{ for any } \bar{z} \in [z', z'']. \tag{146}$$

Taking the total derivative with respect to $\bar{\chi}$ of the implicit function $f^b(\bar{\chi}, \epsilon)$ defined in (139), we obtain

$$\frac{df^b(\bar{\chi}, \epsilon)}{d\bar{\chi}} = \underbrace{\frac{df^b(\kappa)}{d\kappa}}_{\text{negative}} \cdot \underbrace{\frac{d\kappa(\bar{z})}{d\bar{z}}}_{\text{positive}} \cdot \underbrace{\frac{d\bar{z}(\Upsilon)}{d\Upsilon}}_{\text{positive}} \cdot \underbrace{\frac{\partial\Upsilon}{\partial\bar{\chi}}}_{\text{depends on } \eta}, \tag{147}$$

where the signs reported in (147) respectively follow from (145), (143) and (141). On the basis of the above results, it is possible to establish that

Lemma 17 *(Existence of PDM steady-state equilibrium) For any $\eta > 0$, there always exists a value $\bar{\chi}^*$ that satisfies the steady-state condition $f^a(\bar{\chi}^*) = f^b(\bar{\chi}^*, \epsilon)$. In this solution, $f^b(\bar{\chi}^*, \epsilon)$ cuts $f^a(\bar{\chi}^*)$ from above, i.e.*

$$\left.\frac{\partial f^b(\bar{\chi}, \epsilon)}{\partial \bar{\chi}}\right|_{\bar{\chi}=\bar{\chi}^*} < \left.\frac{\partial f^a(\bar{\chi})}{\partial \bar{\chi}}\right|_{\bar{\chi}=\bar{\chi}^*}. \tag{148}$$

Proof. Result (146) implies that

$$\lim_{\bar{\chi}\to 0} f^b(\bar{\chi}, \epsilon) = f^b(0, \epsilon) > 0 \text{ with } f^b(0, \epsilon) \text{ finite.} \tag{149}$$

Since $\lim_{\bar{\chi}\to 0} f^a(\bar{\chi}) = 0$, results (140) and (149) imply that, if there exists an intersection between $f^a(\bar{\chi})$ and $f^b(\bar{\chi}, \epsilon)$, then it must be an intersection where $f^b(\bar{\chi}, \epsilon)$ cuts $f^a(\bar{\chi})$ from above. The existence and uniqueness of this intersection is proved as follows.

Case $\eta = 1$, $\epsilon = 0$**.** From Claim 16, $\eta = 1$ with $\epsilon = 0$ implies that $\partial\Upsilon/\partial\bar{\chi} = 0$. From (147), this in turn implies that $f^b(\bar{\chi}, \epsilon)$ does not depend on $\bar{\chi}$, so that $f^b(\bar{\chi}, \epsilon)$ is a straight horizontal line with respect to $\bar{\chi}$. Since $f^a(\bar{\chi})$ is strictly increasing, there necessarily exists a unique intersection $f^a(\bar{\chi}^*) = f^b(\bar{\chi}^*, \epsilon)$.

Case $\eta \geq 1$ **with** $\epsilon > 0$. From Claim 16, $\eta \geq 1$ implies that $\Upsilon(\bar{\chi}, \epsilon)$ is strictly declining in $\bar{\chi}$ for any $\epsilon > 0$. From (147), this in turn implies that $f^b(\bar{\chi}, \epsilon)$ is monotonically increasing in $\bar{\chi}$. However, $f^b(\bar{\chi}, \epsilon)$ is bounded from above by a finite $f^b_{\max}$. Combining these properties, it must be that $f^b(\bar{\chi}, \epsilon)$ is strictly increasing, strictly concave, and converges asymptotically to a finite limit $f^b(\infty, \epsilon) < \infty$. The boundedness and monotonicity of $f^b(\bar{\chi}, \epsilon)$ guarantee the existence of at least one intersection $f^a(\bar{\chi}^*) = f^b(\bar{\chi}^*, \epsilon)$ in which $f^b(\bar{\chi}, \epsilon)$ cuts $f^a(\bar{\chi})$ from above. The strict concavity of $f^b(\bar{\chi}, \epsilon)$ guarantees that this intersection must be unique.

Case $\eta < 1$. From Claim 16, $\eta < 1$ implies that $\Upsilon(\bar{\chi}, \epsilon)$ is always below a straight line, determined by the coinciding limits $\lim_{\bar{\chi}\to 0} f^b(\bar{\chi}, \epsilon) = \lim_{\bar{\chi}\to\infty} f^b(\bar{\chi}, \epsilon) = f^b(0, \epsilon) > 0$ with $f^b(0, \epsilon)$ finite. As a consequence, there always exists a unique $\bar{\chi}^*$ such that $f^a(\bar{\chi}^*) = f^b(\bar{\chi}^*, \epsilon)$, and it satisfies property (148).

The above results complete the proof of Lemma 17. □

Derivation of (56). From definition (134), we have

$$\frac{\partial \Upsilon(\bar{\chi}, \epsilon)}{\partial \epsilon} = \frac{\epsilon + (1/\eta)\,\beta^{1/\eta}(\alpha_1\bar{\chi} + \epsilon)^{\frac{1-\eta}{\eta}}}{\alpha_1\bar{\chi}\{1 + ...\}^2} > 0. \tag{150}$$

Taking the total derivative with respect to $\bar{\chi}$ of the implicit function $f^b(\bar{\chi}, \epsilon)$ defined in (139), we thus obtain

$$\frac{df^b(\bar{\chi}, \epsilon)}{d\bar{\chi}} = \underbrace{\frac{df^b(\kappa)}{d\kappa}}_{\text{negative}} \cdot \underbrace{\frac{d\kappa(\bar{z})}{d\bar{z}}}_{\text{positive}} \cdot \underbrace{\frac{d\bar{z}(\Upsilon)}{d\Upsilon}}_{\text{positive}} \cdot \underbrace{\frac{\partial \Upsilon}{\partial \epsilon}}_{\text{positive}} < 0, \tag{151}$$

where the signs reported in (147) respectively follow from (145), (143), (141) and (150). This means that $f^b(\bar{\chi}, \epsilon)$ shifts downwards as ϵ increases. Since $f^a(\bar{\chi})$ is increasing and independent of habits, it follows from property (148) that the unique steady-state value $\bar{\chi}$ is negatively affected by the strength of habit formation:

$$d\bar{\chi}/d\epsilon < 0. \tag{152}$$

Proof of Lemma 10. By (27) and definition $\chi_t \equiv Y_t/K_t$, the interest factor is given by $F^{\ell 2}_{K_t} = \alpha_1\chi_t$. Result (152) implies $dF^{\ell 2}_{K_{ss}}/d\epsilon < 0$, which proves (57). From (51), the steady-state condition

$$\bar{\chi} = \left[\alpha_1^{-1}(1+\gamma)\right]^{\frac{\alpha_2}{1-\alpha_1}} n^{\frac{\alpha_3}{1-\alpha_1}} (1-\bar{\varphi})^{-\frac{\alpha_3}{1-\alpha_1}} \tag{153}$$

implies that

$$\frac{d\bar{\chi}}{d\epsilon}\cdot\frac{1}{\bar{\chi}}=-\frac{\alpha_3}{1-\alpha_1}\cdot\frac{1}{1-\bar{\varphi}}\cdot\frac{d\left(1-\bar{\varphi}\right)}{d\epsilon}. \tag{154}$$

Since $d\bar{\chi}/d\epsilon<0$, result (154) implies that $d\left(1-\bar{\varphi}\right)/d\epsilon>0$, and therefore

$$d\bar{\varphi}/d\epsilon=-d\left(1-\bar{\varphi}\right)/d\epsilon<0, \tag{155}$$

which proves (58). From (52)-(53), the steady-state condition $1-\bar{\varphi}=\alpha_1\bar{z}\left(1+\bar{z}\right)^{-1}$ implies that the sign of $d\bar{z}/d\epsilon$ is opposite to that of $d\bar{\varphi}/d\epsilon$, which proves $d\bar{z}/d\epsilon>0$ in (59). Since $g_{X_\infty}=\bar{z}\left(1+\bar{z}\right)^{-1}$, the sign of $dg^{\ell 2}_{X_\infty}/d\epsilon$ is the same as that of $d\bar{z}/d\epsilon$, which proves $dg^{\ell 2}_{X_\infty}/d\epsilon>0$ in (60). In the steady-state, the growth rate of output coincides with that of capital, i.e. $g^{\ell 2}_{Y_\infty}=\bar{\chi}\cdot\left(1-\bar{\varphi}\right)$. Differentiating this expression with respect to ϵ, and substituting (154), it is easily shown that $dg^{\ell 2}_{Y_\infty}/d\epsilon>0$, which proves (61). □

Derivation of (47). Assumption (26) implies that all generations are linked through dynastic altruism, as the utility of the first agent born at $t=0$ can be written as the discounted sum of direct utilities of all descendants: imposing the limiting condition $\lim_{j\to\infty}\Delta^{j-t}W_j=0$, iteration of (26) gives (47). In the simulation performed in section 4.2, we have assumed $\eta=1$, in which case the direct utility index (5) reduces to

$$U\left(c_t,e_{t+1}\right)=\ln c_t+\beta\ln\left(e_{t+1}-\varepsilon c_t\right). \tag{156}$$

In the long run, consumption levels grow at the same balanced growth rate of output per capita, g_{Y_∞}/n. As a consequence, we can use $c_{T+1}=c_T g_{Y_\infty}/n$ and $e_{T+2}=e_{T+1}g_{Y_\infty}/n$ to write

$$U\left(c_{T+1},e_{T+2}\right)=U\left(c_T,e_{T+1}\right)+\left(1+\beta\right)\ln\left(g_{Y_\infty}/n\right). \tag{157}$$

Choose a period T where T is large enough so that the economy is on the balanced growth path. From (47), we can write W_0 as

$$W_0=\sum_{t=0}^{T-1}\left(\Psi n\right)^t U\left(c_t,e_{t+1}\right)+\sum_{t=T}^{\infty}\left(\Psi n\right)^t U\left(c_t,e_{t+1}\right). \tag{158}$$

Using (157) to factorize the last term of (158), simple algebra shows that

$$\sum_{t=T}^{\infty}\left(\Psi n\right)^t U\left(c_t,e_{t+1}\right)=U\left(c_T,e_{T+1}\right)\sum_{t=T}^{\infty}\left(\Psi n\right)^t+\left(1+\beta\right)\ln\left(g_{Y_\infty}/n\right)\sum_{t=T+1}^{\infty}\left(\Psi n\right)^t.$$

Substituting this expression in (158), and recalling that $\sum_{t=T}^{\infty}\left(\Psi n\right)^t=\left(\Psi n\right)^T\left(1-\Psi n\right)^{-1}$ and $\sum_{t=T+1}^{\infty}\left(\Psi n\right)^t=\left(\Psi n\right)^{T+1}\left(1-\Psi n\right)^{-1}$, we obtain (47), which allows us to calculate the values of W_0^I and W_0^{II} mentioned in section 4.2. □

Proof of Lemma 11. The proof builds on the fact that $\bar{\epsilon}$ is the threshold value implying zero bequests in the long run by equation (46). Since $F^{\ell 1}_{K_{ss}}>n\left(\Psi F^{\ell 1}_{K_{ss}}\right)^{1/\eta}$ from (115), the sign of $\tau^{\ell 1}_{ss}$ is the same as that of the term in curly brackets in the right hand side of (46). From (121), we have

$$\lim_{\epsilon\to 0}\Lambda\left(\epsilon\right) = n^{-1}\varphi^{\ell 1}_{ss}\frac{h'+\left(h'/h''\right)\left(\beta F^{\ell 1}_{K_{ss}}\right)^{1/\eta}}{h'+\left(\beta F^{\ell 1}_{K_{ss}}\right)^{1/\eta}}>0, \tag{159}$$

$$\lim_{\epsilon\to\infty}\Lambda\left(\epsilon\right) = n^{-1}\varphi^{\ell 1}_{ss}\left(h'/h''\right)>0, \tag{160}$$

where (160) follows from L'Hospital rule. Since $\Lambda(\epsilon)$ is strictly decreasing in ϵ, it follows from (159)-(160) that condition (62) guarantees the existence of a critical value $\bar{\epsilon} > 0$ such that

$$\Lambda(\bar{\epsilon}) = \alpha_3/n \text{ and } \Lambda(\epsilon) \gtrless \alpha_3/n \text{ if } \epsilon \lessgtr \bar{\epsilon}. \tag{161}$$

Notice that condition (62) can be satisfied for a wide range of parameters because (160) can be written as[19]

$$\lim_{\epsilon\to\infty} \Lambda(\epsilon) = \left[1 - n\alpha_1 \Psi^{\frac{\alpha_2+\alpha_3}{\alpha_2\eta+\alpha_3}} (1+\gamma)^{\frac{\alpha_2(1-\eta)}{\alpha_2\eta+\alpha_3}}\right] \Psi^{1/\eta} \left(F^{\ell 1}_{K_{ss}}\right)^{\frac{1-\eta}{\eta}} > 0. \tag{162}$$

From (113)-(46), result (161) implies that

$$\tau^{\ell 1}_{ss} = \begin{cases} \geq 0 \text{ if } \epsilon \leq \bar{\epsilon} \\ < 0 \text{ if } \epsilon > \bar{\epsilon} \end{cases}, \tag{163}$$

where τ^{ℓ}_{ss} is asymptotic value towards which the ratio b_t/y_t would converge if the economy exhibited an indefinite sequence of Ramsey-Stiglitz temporary equilibria. Result (163) implies that, when $\epsilon > \bar{\epsilon}$, a succession of Ramsey-Stiglitz temporary equilibria would drive the economy towards a steady-state equilibrium in which $\lim_{t\to\infty} b_t < 0$, which is inconsistent with a PRS equilibrium path (i.e. a path characterized by strictly positive bequests in each period). As a consequence, a necessary condition for a PRS equilibrium path to arise is $\epsilon < \bar{\epsilon}$. When $\epsilon > \bar{\epsilon}$, we may have three cases. In the first case (A), parameters and initial endowments are such that desired bequests are non-positive in each $t = 0, ..., \infty$, and the economy follows a permanent Diamond-Mourmouras equilibrium path. In the second case (B), parameters are such desired bequests are non-positive for an initial interval $t = 0, ..., t_B$, and bequests become operative at some finite t_B. After t_B, the economy may exhibit a succession of Ramsey-Stiglitz temporary equilibria, but this leads towards a steady-state equilibrium in which $\lim_{t\to\infty} \tau^{\ell}_t = \tau^{\ell}_{ss} < 0$, so that there must be a finite time $t_0 > t_B$ in which bequests become zero again. Even if further switchovers in accumulation regimes arise, the fact that $\tau^{\ell}_{ss} < 0$ implies that there must be a finite time $t' < \infty$ after which bequests are zero. In the third case (C), parameters are such that $b^{\ell}_t > 0$ for a finite number of periods, $t = 0, ..., t_0$. In this case, the economy exhibits a succession of Ramsey-Stiglitz temporary equilibria in the short run, but this leads towards a steady-state equilibrium in which $\lim_{t\to\infty} \tau^{\ell}_t = \tau^{\ell}_{ss} < 0$. Following the same reasoning as in the previous case (B), there must be a finite time $t' < \infty$ after which bequests are zero. In all the three cases (A), (B), (C), there exists a period t' such that $b^{\ell}_t = 0$ in each $t = t', ..., \infty$, with $0 \leq t' < \infty$. $\square$

Proof of Lemma 12. The proof hinges on the fact that $\epsilon = \bar{\epsilon}$ implies equal long-run growth rates between the altruistic and the selfish regime. This can be proved as follows. From the solution of the dynastic problem (31), the condition for optimal bequests (105) in the steady-state of a Ramsey-Stiglitz equilibrium is

$$U^{\ell 1}_{c_{t+1}}/U^{\ell 1}_{c_t}\Big|_{F_K = F^{\ell 1}_{K_{ss}}} = \left(\Psi F^{\ell 1}_{K_{ss}}\right)^{-1}. \tag{164}$$

Equation (164) is valid whenever desired bequests are equal or greater then zero in the steady-state. Now suppose that $\epsilon = \bar{\epsilon}$. By construction, $\bar{\epsilon}$ is the value of ϵ implying that desired bequests are exactly zero in the steady-state of a Ramsey-Stiglitz equilibrium. As a consequence, condition (164) holds. At the same time, zero desired bequests in the steady-state imply that agents consume all their lifetime incomes in this steady-state, and this implies that the asymptotic growth rates of consumption and output are equivalently obtained from the steady-state conditions (51)-(53) of the

[19] From the definitions used in (121), we have $h'/h'' \equiv n\left(\Psi F^{\ell 1}_{K_{ss}}\right)^{1/\eta}/F^{\ell 1}_{K_{ss}}$, whereas from (23) with $\Delta = \Psi$, we have $\varphi^{\ell 1}_{ss} = 1 - n\alpha_1 \Psi^{\frac{\alpha_2+\alpha_3}{\alpha_2\eta+\alpha_3}} (1+\gamma)^{\frac{\alpha_2(1-\eta)}{\alpha_2\eta+\alpha_3}}$. Plugging these results into (160), we obtain (162).

dynamic system (48)-(50), which is obtained from setting zero bequests in the individual budget constraints. This implies that

$$\epsilon = \bar{\epsilon} \quad \Longrightarrow \quad U^{\ell 1}_{c_{t+1}}/U^{\ell 1}_{c_t}\Big|_{F_K=F^{\ell 1}_{K_{ss}}} = U^{\ell 2}_{c_{t+1}}/U^{\ell 2}_{c_t}\Big|_{F_K=F^{\ell 2}_{K_{ss}}},$$

and, since both regimes exhibit balanced growth in the respective steady-states,

$$\epsilon = \bar{\epsilon} \quad \Longrightarrow \quad Y^{\ell 1}_{t+1}/Y^{\ell 1}_{t}\big|_{F_K=F^{\ell 1}_{K_{ss}}} = Y^{\ell 2}_{t+1}/Y^{\ell 2}_{t}\big|_{F_K=F^{\ell 2}_{K_{ss}}}. \tag{165}$$

Since growth rate of altruistic and selfish regimes coincide in the steady-state for $\epsilon = \bar{\epsilon}$, the relationship between long-run growth and habit formation is as follows. For $\epsilon < \bar{\epsilon}$, the long-run equilibrium is characterized by strictly positive bequests, and therefore by the same asymptotic growth rate of PRS equilibria. From (44)-(45), we have

$$\epsilon < \bar{\epsilon} \quad \Longrightarrow \quad \lim_{t\to\infty} Y_{t+1}/Y_t = Y^{\ell 1}_{t+1}/Y^{\ell 1}_{t}\big|_{F_K=F^{\ell 1}_{K_{ss}}} = n\left[\Psi\left(1+\gamma\right)\right]^{\frac{\alpha_2}{\alpha_3+\alpha_2\eta}}. \tag{166}$$

For $\epsilon > \bar{\epsilon}$, the long-run equilibrium is characterized by zero bequests, and therefore by the same asymptotic growth rate of PDM equilibria. From Lemma 10, we have $\lim_{t\to\infty} Y^{\ell 2}_{t+1}/Y^{\ell 2}_{t} = g^{\ell 2}_{Y_\infty}(\epsilon)$, which completes the proof of expression (63). The fact that $g^{\ell 2}_{Y_\infty}(\epsilon) > g^{\ell 1}_{Y_\infty}$ for any $\epsilon > \bar{\epsilon}$ follows from the fact that (i) the two growth rates coincide for $\epsilon = \bar{\epsilon}$, and (ii) the growth rate $g^{\ell 2}_{Y_\infty}(\epsilon)$ is a strictly increasing function of ϵ as proved in Lemma 10. □

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