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Population growth and natural resource scarcity: long-run development under seemingly unfavourable conditions

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Abstract

The paper considers an economy which is constrained by natural resource use and driven by knowledge accumulation. Resources are essential inputs in all the sectors. It is shown that population growth and poor input substitution are not detrimental but, on the contrary, even necessary for obtaining a sustainable consumption level. We find a new type of Hartwick rule defining the conditions for a constant innovation rate. The rule does not apply to capital but to labour growth, the crucial input in research. Furthermore, it relates to the sectoral structure of the economy and to demographic transition. The results continue to hold with a backstop technology and are extended for the case of minimum resource constraints.

Keywords: Population growth, non-renewable resources, poor input substitution, technical change, sustainability

JEL-Classification: Q32, Q55, Q56, O41

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1 Introduction

Population growth and natural resource scarcity are often perceived as severe threats to sustainable development. World population is currently growing fast and will continue to grow in the future. It is confronted with a natural resource supply that is ultimately limited. Declining oil production in several regions and reports about proven reserves which are lower than previously estimated are clear indications of the boundaries set by nature. Total use of natural resources and energies will have to shrink in future centuries, even when several energy and raw material deposits have been fully exploited so far. This is a fundamental change in economic history because up to now, the expanding world economy has relied on growing resource input.

The negative (Malthusian) perception of population growth is well represented in the literature, see e.g. Meadows et al. (1972) and Ehrlich and Ehrlich (1990). Similarly, in neoclassical growth and capital resource models, see Solow (1974), Stiglitz (1974), and Dasgupta and Heal (1974, 1979), population growth is unfavourable for development. Regarding resource constraints, finance ministers and central bank heads from the world's seven largest economies said that the high oil prices were a threat to global prosperity, see Financial Times (2008). Different and more positive indications stem from models using recent growth theory. Here it is emphasised that labour not only uses resources but also builds substitutes for resources. Moreover, the size of the labour force may determine the intensity of dynamic scale effects, notably in knowledge creation. Also, resource substitution is likely to promote technology development. But are these new theory elements powerful enough to change the general perception of population growth and resource scarcity? With humans constituting an important part of nature, a positive answer should please not only economists.

The present paper asks whether and how it is possible to obtain positive innovation and consumption growth under free market conditions even when population is growing and resource stocks are bounded. The model has the following features, which build on empirical regularities. First, unlike the majority of existing literature, the model does not postulate that population grows at a zero or a constant exponential rate. Instead, we assume a dynamic law reflecting a Malthusian perspective as well as demographic transition, see e.g. Tamura (2000) and de la Croix and Doepke (2003), which extends the standard framework in resource models. Endogenous population growth and knowledge are also treated in Kremer (1993), but there natural resources and the demographic transition are disregarded. In the resource context, population dynamics have recently been analysed in Asheim et al. (2007) who assume (exogenous) quasi-arithmetic population growth. Second, non-renewable resources are assumed to be an essential input in all sectors of the economy, including the innovation sector. This is usually not considered, with the exception of Groth and Schou (2002), who argue that resources are an important element in the technologies of present-day economies. Endogenous innovations drive the growth process but are severely constrained by natural resources; resource use restricts the emergence of productive knowledge and plays a similar role as the scarce investment funds in the theory of recombinant growth, see Weitzman (1998) and Tsur and Zemel (2007). The innovation sector supplements the rest of the economy by producing intermediates and final consumer goods, see especially Romer (1990) and Grossman and Helpman (1991) for the theoretical foundations and Bovenberg and Smulders (1995), Barbier (1999), Scholz and Ziemes (1999), Smulders (2000), Grimaud and Rougé (2003), and Xepapadeas (2006) for the combination with resource economics.

Third, the model assumes poor substitution between inputs in intermediates production. This reflects that, in empirical studies, the elasticity of substitution between natural resources and other inputs, specifically labour and capital, is estimated to be less than unity, see e.g. Christopoulos and Tsionas (2002) and Kemfert (1998). Poor input substitution is often disregarded in resource models because of its complexity; it has been used in Bretschger (1998) for renewable resources and in Bretschger and Smulders (2006) for exhaustible resources and a constant population. Fourth, sectoral change, which impacts resource use, will be reproduced by the model. Economies are undergoing a substantial structural change during long-run development. Between 1979 and 2002, the share of total employment in manufacturing decreased by 30 % in Europe and 34 % in the US, while employment in the research sector rose by 28 % in Europe and 40 % in the US, see GGDC (2004). In López, Anriquez and Gulati (2007) structural change is identified as a major topic in the sustainability debate. Fifth, physical capital has no impact on the growth rate in this model, because the scope for physical capital build-up is limited by material balance constraints, as emphasised by Cleveland and Ruth (1997). Population growth, non-renewable resources, and most of the model's assumptions might be called "unfavourable" conditions for development: they seem to limit both the scope for input substitution and the capacity to accumulate capital as a compensation for lower resource use. Nevertheless, the present paper shows that sustainable growth is feasible under these conditions.

We find that issues, which have been described as critical (or even lethal) before, turn out to be superable, neutral, or even positive under the assumptions of the model, which explains the qualification "seemingly unfavourable" in the title of the paper. In particular, it will turn out that population growth is not detrimental for growth but even needed to ensure enough innovation. This helps the economy during the transition phase and increases the chance of developing a backstop technology, which is favourable in the long run. Specifically, the capital-producing effect of labour is highly useful to compensate for fading resource use in research. In addition, poor input substitution fosters sectoral change, which turns out to be a central mechanism sustaining economic growth. These elements constitute a new condition for a constant innovation rate, which we interpret as a new form of the Hartwick rule (Hartwick 1977) for a knowledge-driven multi-sector economy.

Our general results are in line with earlier contributions, in particular with Simon (1981) who labelled labour – i.e. imagination coupled to the human spirit – as "ultimate resource", Boserup (1965) who finds a positive impact of population density on development in (poor) agrarian societies, and Johnson (2001) who emphasised the role of knowledge for development with a growing population. The present paper provides a coherent model-based foundation of their reasoning. When introducing non-renewable resources, we primarily think of fossil fuels and, in a somewhat broader sense, of energy supplies. However, one can interpret the resource input in a broader fashion, as the world as a materially closed economy is confronted with a fixed supply of raw materials needed for physical capital, housing etc. In addition, basic needs like food have an essential material component.

In accordance with Hartwick (1977) and many contributions of new growth theory, including Romer (1990), we focus on the market solutions of the model. The topic suggests doing so, as population policy is a very critical tool in general and sustainability problems are worldwide, where policy is difficult to implement. The combination of (i) a specific law of motion for population, (ii) the essential use of a non-renewable resource in all sectors of the economy, and (iii) poor input substitution in the intermediate goods sector entails that structural change becomes an important ingredient of development. Accordingly, besides characterising the steady state, we also focus on the nature of the transition phase. This is in contrast to most growth models, but is an appropriate procedure here as the adjustment period turns out to be an important part of development. In addition, the nature of the steady state depends on the characteristics of the transition phase, so that development becomes path-dependent. An element of long-run development with natural resources is the possible emergence of a so-called backstop technology, see Tsur and Zemel (2005). We will include this technology, although not as a central part and in a basic way, similar to Dasgupta, Heal and Majumdar (1977).

The remainder of the paper is organised as follows. Section 2 develops the model with natural resource use and endogenous innovations. Section 3 presents the results for transitional dynamics and for different scenarios regarding population growth. In section 4, the nature of the long-run equilibrium is analysed. Finally, section 5 concludes.

2 The model

The framework uses a standard expansion-in-varieties approach to model growth through innovations. Labour and non-renewable natural resources, which depict material inputs, are introduced as primary input factors. Differentiated intermediate services are the inputs for final goods production and knowledge capital is accumulated by endogenous R&D-activities through positive spillovers. Innovations are embodied in new intermediate goods varieties. They increase the productivity of the aggregate intermediate input. For the long run, a possible switch in technologies is evaluated to consider the effects of backstop technologies. Through this setting, the simplest case of a sectoral economy with endogenous innovations can be depicted in a very basic yet general way. The simultaneous motion of the three stocks knowledge, resources, and population drives the final results.

2.1 Firms

The model economy consists of three different sectors, which are R&D, intermediate services, and final goods, each with a different type of operating firm. R&D firms use labour L and non-renewable resources R as rival inputs and public knowledge κ as non-rival input to produce incremental technical change. Specifically, they generate the know-how for new intermediate goods in the form of designs. n denotes the number of intermediate goods at each point in time. With $\dot{n}$ denoting the derivative of n with respect to time and L_g and R_g the labour and resource inputs into R&D, the production of new designs $\dot{n}$ is given by:

$$\dot{n} = L_g^{\ \alpha} \cdot R_g^{\ 1-\alpha} \cdot \kappa \qquad (0 < \alpha < 1) \qquad . \qquad (1)$$

Time indices are omitted whenever there is no ambiguity. According to (1), *R* and *L* are both essential inputs into research. It reflects that labour can never become an inessential input in the long run and that research institutions always need some resources, like fossil fuels for heating and transportation or mineral products or other materials for machines and experiments. With positive spillovers from R&D to public knowledge, we get $\kappa = n^{\eta}$ where η denotes the intensity of the externalities; with proportional spillovers (see Romer 1990 and Grossman and Helpman 1991) we have $\eta = 1$ so that $\kappa = n$, which will be used below. Consequently, the growth rate of the number of designs (= the innovation rate) *g* becomes:

$$g = \frac{\dot{n}}{n} = L_g^{\ \alpha} \cdot R_g^{\ 1-\alpha} \qquad . \qquad (1')$$

With perfect competition in the research sector, the market value of an innovation p_n equals the per-unit costs of designs, which depend on the labour wage *w*, the resource price p_R and *n*:

$$p_n = \left(w/\alpha\right)^{\alpha} \cdot \left(p_R/(1-\alpha)\right)^{1-\alpha} / n \qquad . \qquad (2)$$

Y-firms assemble intermediate goods x_i on fully competitive markets to final output Y under a CES-production function restriction; *i* is used as an index with $i \in [0, n]$. Provided that the costs to produce x_i-goods are equal for all *x*-firms, we obtain $x_i = x \ (\forall x_i)$ so that *Y* is determined by:

$$Y = \left(\int_0^n x_i^{\beta} di\right)^{\frac{1}{\beta}} = n^{\frac{1-\beta}{\beta}} X \qquad (3)$$

$$(X = n \cdot x; \quad 0 < \beta < 1)$$

In (3), the gains from diversification, given by (1-β)/β, determine the impact of additional varieties *n* on output *Y* (and the effect of the innovation rate *g* on consumption growth). *n* has to be interpreted as a productivity index for total input of intermediates *X* in *Y*-production; it emerges from the symmetry assumption in the CES function so that (3) is clearly distinct from a Cobb-Douglas function. Intermediate goods firms use *L* and *R* as inputs to produce intermediate goods under the restriction of a CES production function; exploiting that the *x*-firms are all symmetrical we can write:

$$X = \left[\lambda \cdot L_X^{\ (\sigma-1)/\sigma} + (1-\lambda) \cdot R_X^{\ (\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \qquad (4)$$

$$(0 < \lambda, \sigma < 1)$$

with σ being the elasticity of substitution between *L* and *R*, assumed to be lower than unity. (4) reflects the relevant input substitution process governing the dynamic behaviour of the economy, while the relationship between *n* and *X* in (3) determines the efficiency of final goods production. Input substitution is supplemented by intersectoral substitution when inputs move from intermediates to research, which is the structural change modelled within this framework.

2.2 Inputs

Resources are owned by the households. The total stock of resource *R* at time τ is denoted by $S(\tau)$; its depletion occurs according to:

$$\dot{S} = -R\,, \qquad \text{with } S(0) \text{ given and } S(\tau) \geq 0 \text{ for all } \tau. \tag{5}$$

The population growth rate $\hat{L}$ is positive and assumed to reflect a Malthusian perspective as well as demographic transition; fertility and mortality have both to be considered. In particular, we postulate that $\hat{L}$ depends positively on productivity growth which increases living standards, decreases mortality (better health system), and may lower opportunity costs of parenting. In addition, $\hat{L}$ is assumed to be negatively affected by real wage growth because wages determine opportunity costs of parenting and the scope for retirement provision. Using p_Y for the price of output *Y*, w/p_Y for the real wage, and $\hat{A}$ for productivity growth we obtain:

$$\hat{L} = \xi \cdot \hat{\Gamma} = \xi\left[\hat{A} - (\hat{w} - \hat{p}_Y)\right] \geq 0 \quad \text{and} \tag{6}$$

$$L = \mu \cdot \Gamma^{\xi} \tag{6'}$$

$$(\xi, \mu > 0)$$

where ξ serves as "population response" parameter, μ is a constant representing population size, and hats denote growth rates. Below we set $\hat{A} = \left[(1-\beta)/\beta\right]g$ which seems to be a natural choice because it represents productivity growth in final goods production according to (3). It appears as an advantage to use a single parameter for population response; the alternative function $\hat{L} = \xi_1 \hat{A} - \xi_2(\hat{w} - \hat{p}_Y)$ is also feasible but does not yield additional insights, see the appendix. As we have $\hat{\Gamma} > 0$, a higher ξ means higher population growth. Equilibrium on labour and resource markets is given by:

$$L = L_X + L_g \tag{7}$$

$$R = R_X + R_g \tag{8}$$

2.3 Individuals

Let us denote individual consumption by c. Each agent born in *t* maximises the discounted stream of individual utility:

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \log c(\tau) d\tau \tag{9}$$

where ρ is the pure time preference rate. Individual financial wealth, denoted by z, equals the per capita value of assets representing R&D firms, i.e. $z = p_n n / L$. In addition, each agent owns a share $s = S / L$ of the resource stock, and sells $q = R / L$ units of extracted resource to producing firms, receiving a royalty p_R (extraction costs are set to zero for simplicity). The dynamic budget constraint for each individual reads:

$$\dot{z} = (r - \hat{L})z + p_R q + w - p_Y c \qquad (10)$$

where r the interest rate on financial wealth, while (5) implies the dynamic resource constraint:

$$\dot{s} = -q - \hat{L} \cdot s \qquad (11)$$

The consumer problem consists of maximizing (9) subject to (10) and (11) using c and q as control variables, and z and s as state variables. The current-value Hamiltonian reads:

$$H = \log c + \nu_1 \left[(r - g_L)z + p_R q + w - p_Y c \right] - \nu_2 \left(q + \hat{L} \cdot s \right) \qquad (12)$$

where ν_1, ν_2 denote the costate variables. Necessary conditions for an interior solution are given by the following first order and transversality conditions:

$$1/c = p_Y \nu_1 \qquad (13)$$

$$\nu_1 p_R = \nu_2 \qquad (14)$$

$$\dot{\nu}_1 = (\rho - r - \hat{L})\nu_1 \qquad (15)$$

$$\dot{\nu}_2 = (\rho + \hat{L})\nu_2 \qquad (16)$$

$$\lim_{\tau \to \infty} \nu_1(\tau) \cdot z(\tau) \cdot e^{-\rho(\tau - t)} = 0 \qquad (17)$$

$$\lim_{\tau \to \infty} \nu_2(\tau) \cdot s(\tau) \cdot e^{-\rho(\tau - t)} = 0 \qquad (18)$$

The transversality conditions (17) and (18) require that total firm and resource wealth each approaches a value of zero in the long run. Differentiating (13) logarithmically with respect to time and using (15) yields the Keynes-Ramsey rule:

$$\hat{c} + \hat{p}_Y = r - \hat{L} - \rho \qquad (19)$$

while differentiating (14) logarithmically with respect to time and using (15) and (16) gives the Hotelling rule:

$$\hat{p}_R = r \qquad (20)$$

The transversality conditions (17) and (18) yield:[1]

$$\lim_{\tau\to\infty} z(\tau)\cdot e^{-\int_t^\tau \left[r(u)-\hat{L}(u)du\right]} = L(t)^{-1}\lim_{\tau\to\infty} p_n(\tau)n(\tau)\cdot e^{-\int_t^\tau \left[r(u)du\right]} = 0 \qquad (21)$$

$$\lim_{\tau\to\infty} s(\tau)\cdot e^{\int_t^\tau \hat{L}(u)du} = L(t)^{-1}\lim_{\tau\to\infty} S(\tau) = 0 \quad . \qquad (22)$$

Condition (21) rules out Ponzi games, as it requires the value of financial assets not to grow asymptotically at a rate exceeding the rate of interest. Condition (22) is a standard efficiency requirement: the resource stock must be asymptotically exhausted, since leaving unexploited resources in the ground would imply an inefficient extraction plan. The clearing of goods markets requires:

$$Y/L = c \quad . \qquad (19')$$

As no resources are used to assemble differentiated goods to final output, expenditures can be expressed in terms of *Y* or *X*, i.e. $p_Y Y = p_x X$.

As innovations lead to new goods (i.e. there is no perfect substitute for them) the market form in the intermediate sector is monopolistic competition. The demand for an intermediate good can be derived from (3), see the appendix. Accordingly, the mark-up over marginal costs for the optimal price of an intermediate good is $1/\beta$, so that we get the per-period profit flow to each design holder:

$$\pi = (1-\beta)p_x X/n \qquad (23)$$

On capital markets, the return on innovative investments (consisting of the direct profit flow π and the change in value of the design) is equalised to the return on a riskless bond investment of size p_n:

$$\pi + \dot{p}_n = r\cdot p_n \qquad (24)$$

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[1] From (15), we can substitute $v_1(\tau) = v_1(t)e^{\rho(\tau-t)-\int_t^\tau\left[r(u)-\hat{L}(u)du\right]}$ in (17), obtaining $\lim_{\tau\to\infty} v_1(t)z(\tau)e^{-\int_t^\tau\left[r(u)-\hat{L}(u)du\right]} = 0$. For any initial value of the shadow price of individual wealth, $v_1(t) > 0$, this condition requires satisfying (21), where the central term is obtained by substituting the definition of individual wealth $z(\tau) = p_n(\tau)n(\tau)/L(\tau)$ and the demographic rule $L(\tau) = L(t)e^{\int_t^\tau \hat{L}(u)du}$. Similarly, plugging $v_2(\tau) = v_2(t)e^{\rho(\tau-t)+\int_t^\tau\left[\hat{L}(u)du\right]}$ in (18) implies $\lim_{\tau\to\infty} v_2(t)e^{\int_t^\tau\left[\hat{L}(u)du\right]s(\tau)} = 0.$ Taking $v_2(t)$ outside the limit and substituting *s(τ)=S(τ)/L(τ)* yields (22).

3 Steady state and transition

3.1 Long-run innovation rate

We label the cost share of labour in intermediate goods production with *d*; observing the fact that the mark-up factor in intermediates production is $1/\beta$ we have:

$$d \equiv \frac{wL_X}{\beta p_x X} \tag{25}$$

while 1- *d* denotes the resource share in intermediates production. Calculating relative factor demands of profit-maximising *x*-firms, we obtain from (4) for the relative share size:

$$\frac{d}{1-d} = \left(\frac{\lambda}{1-\lambda}\right)^{\sigma}\left(\frac{w}{p_R}\right)^{1-\sigma} \tag{26}$$

We are now ready to give the first result:

Proposition 1 *The steady-state innovation rate depends on the relative size of the population response parameter* ξ *and the output elasticity of the resource in the research sector* $1-\alpha$ *; it is given by:*

$$g(\infty) = 0 \qquad \text{if} \qquad 1-\alpha > \xi \tag{27a}$$

$$g(\infty) = \mu \cdot \left(\frac{\xi}{1-\xi}\right)^{\xi} \cdot \left(\frac{1-\lambda}{\lambda}\right)^{\frac{\xi\sigma}{1-\sigma}} \qquad \text{if} \qquad 1-\alpha = \xi \tag{27b}$$

$$g(\infty) = \infty \qquad \text{if} \qquad 1-\alpha < \xi \tag{27c}$$

Proof The appendix shows that the steady-state innovation rate amounts to:

$$g(\infty) = \mu \cdot d(\infty)^{\frac{1-\alpha-\xi}{1-\sigma}} \left[1-d(\infty)\right]^{-\frac{1-\alpha}{1-\sigma}} \tilde{\alpha}\tilde{\lambda} = \Omega(\infty) \tag{28}$$

$$\text{where } \tilde{\alpha} = \left[(1-\alpha)/\alpha\right]^{1-\alpha}, \ \tilde{\lambda} = \left[(1-\lambda)/\lambda\right]^{\frac{(1-\alpha)\sigma}{1-\sigma}}.$$

Given poor input substitution, *d* continuously decreases such that $d(\infty) = 0$, see the appendix. Then, $\left[1-d(\infty)\right]^{-\frac{1-\alpha}{1-\sigma}} = 1$ and long-run innovation is governed by the term $d(\infty)^{\frac{1-\alpha-\xi}{1-\sigma}}$. The drag from fading resource input is given by $d(\infty)^{\frac{1-\alpha}{1-\sigma}}$, the counterforce through increasing population is represented by $d(\infty)^{-\frac{\xi}{1-\sigma}}$. ■

According to (27a-c), the necessary condition for positive innovation and growth in the long run reads $\xi \geq 1-\alpha$. This is a challenging result: in a knowledge-driven economy, positive population growth turns out not to be detrimental but, on the contrary, to be needed in order to sustain economic growth. Put differently, low population growth is a curse because it limits innovation and therefore development. This will also hold true for the case with a backstop technology, see section 4. In reality, the output elasticity $1-\alpha$ is probably less than 2-3 percent so that an elasticity of ξ equalling this value appears to be realistic.

The model suggests that labour is indeed the ultimate resource, as it is highly productive in the accumulation of knowledge capital (and the development of the backstop). In (27a) and (27c), population response ξ governs the adjustment speed when α is given. The benchmark case with $\xi = 1-\alpha$ is interesting because it offers useful theoretical insights.[2] It can be interpreted as a new kind of "Innovation Hartwick rule" (see Hartwick 1977) for a multi-sector knowledge economy, with the following features:

(i) the growing input compensating for the fading resource is not capital but labour, which is central in the dynamic research sector,

(ii) labour growth reflects demographic transition, such that the response of population to macroeconomic conditions (ξ) decides on the nature of development with given technology (α),

(iii) resource use in research delivers the decisive technology parameter (1-α) in an economy which also comprises sectors which are not accumulating inputs,

(iv) the rule yields a constant innovation rate in the long-run steady state.

The innovation rate in (27b) depends positively on:

(i) the size of the population (μ),

(ii) the population response parameter (ξ),

(iii) the elasticity of substitution in intermediates production (σ).

In (27a) and (27c), these parameters determine the convergence speed. To confirm these statements we now analyse the transition phase.

3.2 Systems dynamics

The sectoral depletion rates are defined as:

$$v_X \equiv \frac{R_X}{S} \quad \text{and} \quad v_g \equiv \frac{R_g}{S} \tag{29}$$

We then state:

Lemma 1: *The dynamics of the system are fully given by the differential equations for d, g,* v_X*, and* v_g*, which read:*

[2] It is not studied because it has a high statistical probability; in this regard it is of the same quality as the Hartwick rule or the often-used assumptions of unitary substitution elasticities or proportional knowledge spillovers, see Romer (1990). The well-known criticism of Solow (1956) of the Harrod-Domar model was in fact not primarily on the knife-edge character of the result but on the assumed fixed input proportions causing it.

$$\dot{d} = (1-d)(1-\sigma)\left[\frac{dg}{\alpha} - \frac{(1-\beta)}{\beta}(\Omega - g)\right] \tag{30}$$

$$\dot{g} = \left[g - \frac{\alpha g + \Omega\left[(1-d)(\xi + 1 - \sigma) - 1\right]}{(1-d)(1-\sigma)}\right]\hat{d} + \rho(\Omega - g) \tag{31}$$

$$\dot{v}_X = v_X\left[-\frac{d}{(1-d)}\hat{d} + v_X + v_g - \rho\right] \tag{32}$$

$$\dot{v}_g = v_g\left[\hat{g} + \frac{\alpha}{(1-\alpha)(1-\sigma)}\hat{d} + v_X + v_g\right] \tag{33}$$

where: $\Omega \equiv \frac{gL}{L_g} = \mu\tilde{\alpha}\tilde{\lambda} \cdot d^{\frac{1-\alpha-\xi}{1-\sigma}} \cdot (1-d)^{-\frac{1-\alpha}{1-\sigma}} > 0$, see also (28).

Proof: See the appendix. ▪

The steady state of the system is given by $\dot{d} = \dot{g} = \dot{v}_X = \dot{v}_g = d = 0$. From (30) we can verify that then we have $g(\infty) = \Omega(\infty)$ as stated in the proof of proposition 1, see (28). In the same way, using (31) – and (A.40) from the appendix for $\hat{d}(\infty)$ – we can confirm that indeed $g(\infty) = \Omega(\infty)$. (32) says that $v_X(\infty) + v_g(\infty) = \rho$ which is confirmed when using (33) combined with (A.40). This certifies that the dynamic system is consistent with proposition 1. We use the fact that the system is decomposable: (30) and (31) constitute a system alone, providing final solutions for the innovation rate g and the sectoral labour share d.

We use phase diagrams of this subsystem for the different cases of (27). In each case we describe the transition phase and the associated long-term equilibrium. We use the assumption of poor substitution in the production of intermediate goods (σ < 1) throughout, avoiding the knife-edge assumption σ = 1 often used in literature. Under all scenarios regarding population growth, this entails a crowding out of labour from the intermediate sector, which supports economic dynamics through lower (nominal) wages. Goods prices decrease as well while the number of varieties increases during adjustment; thus, decreasing *nominal* wages are indeed compatible with constant or increasing well-being in this model. Consumption growth and backstop technologies are treated in section 4.

3.3 Population growth and transition

When population responds weakly to its determinants given in (6), i.e. when ξ takes a low value, population growth is low and innovation during transition is moderate because the fading resource input cannot be fully replaced by additional labour input. Indeed, it is that:

Proposition 2 *With low population growth, i.e. with* $\xi < 1-\alpha$, *the economy converges along a saddle-path to a state without innovation and production in the long run.*

Proof In figure 1, the dynamics are depicted with a phase diagram in the *d*-*g*-space for $\xi < (1-\alpha)$. The location and the shape of the $\dot{d}=0$ and the $\dot{g}=0$ curves are explained in the appendix. ▪

*** Figure 1 ****
about here

As becomes clear from figure 1, innovation steadily decreases and ultimately ceases in the long run. With given production technology (constant α), it is weak population growth (a low ξ) that leads to this outcome. A constant population unambiguously falls into this category. Provided that population growth is higher, the steady decline in the innovation rate can be avoided. We arrive at:

Proposition 3 *With intermediate population growth, i.e. when* $\xi = (1-\alpha)$, *the economy approaches on a saddle path a long-term equilibrium with constant positive innovation growth.*

*** Figure 2 ****
about here

Proof In figure 2, the dynamics are depicted in the *d*-*g*-space. The location and the shape of the $\dot{d}=0$ and the $\dot{g}=0$ curves are explained in the appendix. The innovation rate approaches a constant on the *Y* axis following a saddle path, which lies between the two loci for $\dot{d}=0$ and $\dot{g}=0$. The equilibrium satisfies the transversality conditions. Using logarithmic differentials, (21) reads $\lim_{\tau\to\infty} \hat{n}(\tau)+\hat{p}_n(\tau)-r(\tau)\le 0$ which becomes with (2) $\lim_{\tau\to\infty} \hat{w}(\tau)-r(\tau)\le 0$; this is satisfied for $\hat{d}<0$, see (26) and (A.7) in the appendix. Any path converging to *d*=1 must be ruled out since $\hat{d}>0$ would imply $\hat{w}(\tau)-r(\tau)>0$. $\lim_{\tau\to\infty} \hat{S}(\tau)\le 0$ is always satisfied with $R(\tau)>0$, see (22). Thus, the economy jumps on the saddle path and asymptotically approaches the equilibrium given by (27b). ▪

In the long-run steady state, the drag of decreasing resource input in the research sector must be exactly compensated by increasing labour input. Using realistic parameter values (see the appendix for the numbers) exhibits that the half-life of convergence is much longer than, for example, with the neo-classical growth model. High values of population response ξ and μ are positive for innovation growth. This clearly exhibits the importance of sufficient labour supply to support R&D-activities in the long run. The discount rate has two opposing effects on innovation: on the one hand, a high discount rate discourages investments; but on the other hand, it accelerates the price increase of natural resources and therefore sectoral reallocation of labour. According to (27b), the two opposing effects are of the same size so that the impact of ρ is exactly zero.

Following the discussion up to now, high population growth accelerates the innovation rate; we arrive at:

Proposition 4 *With high population growth, i.e. when* $\xi > 1 - \alpha$*, the economy follows a path with an increasing innovation rate as long as the backstop technology is not available.*

Proof During transition the innovation rate increases because the inflow of labour in the research sector, determined by the population growth parameter ξ, overcompensates the increasing scarcity of the resource input in the research sector. ▪

This super-exponential growth path is abandoned as soon as backstop technologies and/or minimum resource constraints emerge, which is treated in the next section, see figure 3.

4 Long-run development

In the long run, the growth rates of innovation and consumption depend on whether a backstop technology is available or not. Long-term income depends on the transition period, as it is a function of the number of varieties which result from cumulated research efforts in the past. A different income level may arise in the long run if the economy operates under a minimum resource constraint. This constraint says that a minimal resource input is needed to keep production running. Finally, in a world of structural change, adjustment costs affect the final results. These topics are treated in the following, with a focus on consumption growth.

4.1 No backstop technology

Following (3), aggregate consumption growth $\hat{Y}$ is determined by:

$$\hat{Y} = \tilde{\beta} g + \hat{X} \tag{34}$$

with $\tilde{\beta} = (1-\beta)/\beta$. $\hat{X}$ is negative because of the decreasing input of R into intermediate goods production. Labour gradually moves from the intermediate to the innovation sector, which increases R&D activities. In order to have positive consumption growth, the equilibrium innovation growth rate g must be big enough to compensate for the drag of R in the X-sector. In the (very) long run, d becomes (approximately) zero, resource use is given by $\hat{R}_X = -\rho$ (see A.33 in the appendix), which yields $\hat{X} = -\rho$. Inserting (27b) into (34) we thus obtain:

$$\hat{Y} = \left(\frac{\xi}{1-\xi}\right)^{\xi} \left(\frac{1-\lambda}{\lambda}\right)^{\frac{\xi\sigma}{1-\sigma}} \tilde{\beta}\mu - \rho \tag{35}$$

Whether consumption growth is positive in the long run depends on the parameters. High innovation growth, large gains from diversification and monopoly power (low β), and a large population size (high μ) favour positive (aggregate) consumption growth, whereas a high discount rate has a negative effect on consumption dynamics. Note that the negative effect of the discount rate stems from the negative effect of resource use on intermediates production and not from investment behaviour.

In order to get long-term per capita consumption growth we need to calculate $g_Y - g_L$. From time-differentiating (1') we get for constant g that $\alpha\hat{L} + (1-\alpha)\hat{R}_g = 0$. Noting that $\hat{R}_g = -\rho$ (see A. 41 in the appendix) in the long run we derive asymptotic population growth with a constant innovation rate according to:

$$\hat{L} = \frac{1-\alpha}{\alpha}\rho \tag{36}$$

so that we get for per capita consumption growth $\hat{c}$:

$$\hat{c} = \left(\frac{\xi}{1-\xi}\right)^{\xi}\left(\frac{1-\lambda}{\lambda}\right)^{\frac{\xi\sigma}{1-\sigma}}\tilde{\beta}\mu - \frac{1}{\alpha}\rho \quad . \tag{37}$$

Again, high innovation growth, large gains from diversification, and a large labour force are the best means to compensate for a positive discount rate, which is now weighted by $1/\alpha$ due to positive population growth. For positive (sustainable) growth in the long run, the discount rate must be bounded from above according to:

$$\hat{c} \geq 0 \quad \Leftrightarrow \quad \alpha\left(\frac{\xi}{1-\xi}\right)^{\xi}\left(\frac{1-\lambda}{\lambda}\right)^{\frac{\xi\sigma}{1-\sigma}}\tilde{\beta}\mu \geq \rho \tag{38}$$

A special case applies when the economy needs a minimum resource input to operate, see section 4.3.

4.2 With backstop technology

When interpreting R as fossil fuels, it is likely that a backstop technology will become available at some point in the future. This new technology could build on resources like solar, wind, and/or tidal power or similar energies, all of them being renewable (as long as the sun is shining). To fully endogenise the backstop would go beyond the scope of this paper. But we formulate necessary conditions for the emergence of a backstop along historical experience with the so-called “general purpose technologies”. We note that the largest innovations, like the moveable type printing by Gutenberg, the steam engine by Watt, and the computer, were achieved by actively combining a multitude of existing technologies. Thus we assume that a backstop technology can only emerge when a lot of knowledge is accumulated already and research activities in an economy are high in general. Accordingly, the successful development of the backstop is tied to two conditions. The first is that accumulated knowledge in the economy has to exceed a critical level, i.e. $\kappa(\tau) \geq \bar{\kappa}$. Second, it requires a critical research level in the economy, i.e. $g(\tau) \geq \bar{g}$. As soon as the backstop is available, it fully substitutes for the resource at current market prices. If it is never available, we have $\bar{\kappa} = \infty$. According to historical experience, general purpose technologies are not anticipated by individuals. We assume the transition to the backstop technology to be smooth (e.g. there are no price jumps) so that a modification of the households' intertemporal optimisation problem is not warranted. We now state results for low and high population growth. Following proposition 2 we have:

Corollary 1 *With low population growth, i.e. when* $\xi < (1-\alpha)$, *the backstop technology is never developed and the decline of economic activities becomes inevitable.*

Proof Figure 1 shows that for low population growth the innovation rate decreases over time. Accordingly, the condition $g(\tau) > \bar{g}$ for any τ is either fulfilled at the beginning of the optimisation or never. ▪

The critical level for knowledge, i.e. $\kappa(\tau) \geq \bar{\kappa}$, prevents the policy option of investing heavily in innovations during a short period of time. Moreover, as the backstop comes as an externality for individuals, there is no incentive for an agent to promote the backstop in any way. On the contrary, with high population growth innovation rate and knowledge stock become high so that the emergence of a backstop becomes possible. Following proposition 4 we have:

Corollary 2 *Higher population growth causes faster adjustment to the equilibrium with a backstop technology.*

Proof Figure 3 shows the corresponding dynamics in the *d*-*g*-space for the case $\xi > 1-\alpha$. On the saddle path, the innovation rate and the labour share reach point *C,* where we assume $\kappa(\tau) \geq \bar{\kappa}$ so that both variables *d* and *g* remain in *C* forever. ▪

*** Figure 3 ****
about here

The economy switches to a constant supply of *B,* fully replacing *R*, which will happen if not $\bar{\kappa} = \infty$. As we focus on market outcomes and the backstop comes as a pure externality, there is no specific pre-arrival activity by any agent in the economy. In the model, the backstop resource *B* replaces *R* in (1), (4), and (8), where we then postulate a fixed supply of *B* to be equal to the quantity demanded in the two sectors. The price is equal to a given c_B, which is the constant unit production cost of the backstop resource. A constant c_B results when the opposing effects of learning (causing a decreasing c_B) and of increasing scarcities (causing an increasing c_B) have the same size. Following (4) and (25), the share of the backstop resource in intermediates production can be expressed as:

$$c_B \cdot B_X = \beta(1-d) p_x X \tag{39}$$

From the profit maximisation of the research labs we have:

$$\frac{B_g}{L_g} = \frac{w(1-\alpha)}{c_B \alpha} \tag{40}$$

The labour share in intermediates production now evolves according to:

$$\hat{d} = (1-\lambda)(1-\sigma)\hat{w} \quad . \tag{41}$$

To find the equilibrium of the system with the backstop resource, hypothetically suppose that (nominal) wages decrease over time. With poor input substitution, this would imply that, following (41), d falls so that $1-d$ increases and B_X rises, following (39). With a given B this would decrease B_g, which harms research and growth. Obviously, this is not an optimum. On the other hand, a constant wage implies a constant d, a constant allocation of energy to the two sectors and a constant population, according to (A.10), see the appendix. The constant input of labour and energy in research yields constant innovation and consumption growth rates which is the optimum outcome in the case of a backstop technology. Individuals with rational expectation choose this development path. With a backstop technology, the model resembles the approach of Grossman and Helpman (1991, ch. 5) which provides constant growth rates due to constant returns to research. Summarising, we thus find that for any point in time after the backstop arrival:

(i) *d* becomes constant, i.e. sectoral change stops,
(ii) the innovation rate becomes constant,
(iii) the population growth rate becomes zero, and
(iv) per-capita consumption is constantly increased in the long run.

Result (iii) corresponds to the prediction that world population will be stable in the distant future. Implication (iv) is the consequence of a constant aggregate *X*-production and a positive innovation growth rate, as is the case in basic endogenous growth models.

Adopting a material interpretation of the resource *R*, recycling has a function which is similar to that of the backstop technology for energy. Regarding the minimum level, it is often assumed that a certain amount of material throughput is necessary to sustain economic activities in the long run. Recycling is the key to increasing the quantity of raw materials like metals etc. Assuming that a recycling technology becomes available at some point in time basically the same analysis as for the backstop applies. This holds true provided that it is possible to completely recycle the required (constant) quantity of material at a constant speed. If, however, it is not possible to recycle one hundred percent of the material, the minimum material requirement will not be met at some point in time and production in the model has to stop. Note that not all materials are non-renewable or predicted to be critical with regard to the minimum condition. For instance in food production, we primarily turn to the field of renewable natural resources.

Here, limited regeneration and complementary inputs like land and water are possible bottlenecks for production. Regarding housing, natural supplies of materials seem to be (relatively) more abundant and partly renewable (e.g. timber).

4.3 Minimum resource use

In the present model, the progressive exhaustion of the resource stock decreases the labour income share in the intermediate goods sector, while the labour share in the research sector remains constant. As a consequence, the relative value of labour decreases and workers move from the intermediate goods to the R&D sector with a parallel increase in total labour force. Without a backstop technology, the economy evolves toward a steady state where the knowledge stock grows to infinity, whereas natural resource use and the production of intermediate goods approach zero. The economy becomes "immaterial" in the long run because growth depends on increasing knowledge with an ever-decreasing input of intermediate goods and resources. In the long-run steady state, costs of innovations are (approximately) constant, that is the decreasing wages compensate for increasing resource prices.

During transition, resource use becomes very low and even converges to zero in the very long run. Note that final goods production in (3) states that a sufficiently increasing knowledge stock can compensate for fading intermediate services, which is independent of material use, so that (3) remains to be valid in the long run. But is this realistic? If a minimum resource input is needed for intermediates production we must write, instead of (4):

$$X = \left[\lambda \cdot L_X^{(\sigma-1)/\sigma} + (1-\lambda) \cdot R_X^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} - \overline{R} \qquad (4')$$

Now a path leading to $R_X(\tau) < \overline{R}$ causes consumption to fall back to zero as soon as the minimum resource input is reached. In principle, any path analysed in section 3 is a candidate for such a development. One might think about optimal strategies for agents anticipating this development, but this is beyond the scope of this paper. Just note that an unfavourable scenario including $R_X(\tau) < \overline{R}$ is not the consequence of excessive growth during convergence. In the model, growth results from research which is less resource intensive than intermediates production in the longer run so that moderation in the growth rate does not help the economy in any way. (Sufficiently) Increasing resource prices are the best way to get a smooth transition to backstop technologies. Zero production in the long run emerges as the model outcome from the combination of a minimum resource requirement and a lack of a backstop technology and/or incomplete recycling.

4.4 Adjustment costs

Two further issues could prevent the system from following the saddle paths depicted in figures 1-3. First, as structural change is the main mechanism driving the result, any deviation from zero adjustment cost can become critical for the outcome. Indeed, many causes for slow sectoral adjustments of labour, such as wage-setting procedures and efficiency wages, can be found in reality. Even more important, the research sector might require special skills which are not readily available in the economy. It becomes immediately clear from the results that, once we have too slow an inflow of labour into the R&D-sector, innovation growth rates will decrease. Specifically, equation (A.10) in the appendix gives the percentage change of labour input into R&D as a function of the change of the labour wage and the labour share in X-production. Provided that wages do not adjust as indicated on an equilibrium convergence path, the percentage change of labour input in R&D becomes smaller, which entails a lower innovation growth rate according to (1'). The same holds true for the world economy, where sectoral shifts are associated with international changes in the division of labour.

Also, several equations postulate perfect foresight of the agents that is we abstract from information costs. In addition to the usual assumptions regarding capital markets and the intertemporal budget constraint, this model includes optimisation of resource owners. When deviating from perfect information in the resource sector, it might be that price levels are too low or price increases are too slow (at least in a first phase), for instance due to myopia. As a consequence, too little knowledge is accumulated and, combined with adjustment costs on labour markets, the increase of labour in the innovative sector becomes too sluggish compared to the model solution.

Turning to the issue of optimal economic growth, the market equilibrium reached in the present economy does not correspond to a first best-solution. Due to the positive spillovers in R&D, research efforts are too weak in equilibrium. Activities in the intermediate goods sector are also too low compared to the optimum because of monopolistic competition. This would lead to a static distortion in consumer expenditures if there were another consumer sector with goods priced at marginal costs. However, there is only one consumer sector in this economy. Regarding the intermediate goods sector, relative prices between goods reflect relative marginal cost, so that no static distortion arises. Thus, depending on the size of positive spillovers, policy could restore optimum sector size and provide optimal growth by subsidising research. According to the assumption, this would also have an impact on population growth.

5 Conclusions

The paper presents a model in which population growth is endogenous and supports sustainable consumption in an economy under non-renewable resource constraints. An increasing labour force is positive for growth because it fosters knowledge capital substituting for natural resources. The knowledge creation effect of labour dominates the resource using effect because knowledge creation is labour intensive and knowledge is a public good, which can be equally used by all the agents even in case of population growth. It is also shown that increasing resource prices and low input substitution elasticities cause structural change, which helps innovation. Even a combination of several seemingly unfavourable conditions is thus not detrimental for long-run growth. On the contrary, the model suggests that labour and backstop technologies rather than natural inputs are the ultimate resources for an economy.

The results suggest that population policy is not only problematic because it affects families' welfare and becomes easily paternalistic toward less developed countries, but because it might be counterproductive with respect to economic dynamics. To foster innovation and increasing living standards turn out to be the more efficient way to obtain sustainable consumption. Thus the non-Malthusian results of this study do not suggest a laisser-faire policy; rather, by emphasising central mechanisms for development, the model indicates that the debate on population growth and the substitution of non-renewable resources should focus on issues like sectoral adjustment costs and the formation of long-term expectations. The results show that facilitating labour reallocation from knowledge-extensive to knowledge-intensive sectors is the best means to support sustainable development. The removal of subsidies to energy production (like the ones for coal in certain countries) and to shrinking and lagging sectors emerges as being desirable. The steady increase of resource prices is not seen as detrimental, quite to the contrary, it helps the economy to adjust in continuous small steps to a sustainable equilibrium. However, policies targeting at the population size cannot be advocated.

In an extended model, learning effects in the intermediates sector could sustain the incentive for a part of the labour force to remain in the intermediates sector in the long run, a straightforward extension of the present approach. The framework could also include that reallocating labour needs education efforts, which seems to be another possible direction for future research. In addition, or alternatively, one might assume that part of the population is not suited for employment in the research sector. A further extension of the model would be to introduce stock pollution, which represents another exhaustibility constraint similar to non-renewable resources.

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Appendix

This appendix explains the derivations of the equations in the main text in detail. If needed, more information is available from the author upon request.

Profits

To obtain (23), use the price index of final goods *Y*, which is written as:

$$p_Y = \left[\int_0^n \left(p_{xj} \right)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)} \tag{A.1}$$

With perfect competition in the *Y*-sector, this price equals the per-unit costs, so that differentiating (A.1) with respect to the price of intermediate good *i* yields, according to Shephard's Lemma, the per-unit input coefficient x_i / Y. Thus, the demand for intermediate good *i* becomes:

$$x_i = \frac{\left(p_{xi} \right)^{-\varepsilon}}{\int_0^n \left(p_{xj} \right)^{1-\varepsilon} dj} p_Y Y \tag{A.2}$$

For the case of many *x*-firms (large group case of Chamberlin), the denominator of (A.2) is given for the single firm so that the elasticity of demand for x_i is ε, and the optimum mark-up over marginal costs is indeed $1/\beta$, with $\beta = (\varepsilon - 1)/\varepsilon$. Hence, profits of *x*-firms used to compensate research are a share $1-\beta$ of total sales.

Innovation rate

To characterise the steady state we first derive the steady state innovation rate. (1') can be rewritten as:

$$g = L_g \cdot \left(\frac{R_g}{L_g} \right)^{1-\alpha} \tag{A.3}$$

The innovation growth rate depends on the labour input in R&D, L_g, and the relation of resource and labour input in the innovative sector. Cost minimisation in the R&D sector yields:

$$\frac{R_g}{L_g} = \frac{w}{p_R} \left(\frac{1-\alpha}{\alpha} \right) \tag{A.4}$$

Using (26) we derive:

$$\frac{w}{p_R} = \left(\frac{d}{1-d}\right)^{\frac{1}{1-\sigma}} \left(\frac{\lambda}{1-\lambda}\right)^{-\frac{\sigma}{1-\sigma}} \tag{A.5}$$

Combining (A.3), (A.4), and (A.5) yields:

$$g = L_g \left(\frac{d}{1-d}\right)^{\frac{1-\alpha}{1-\sigma}} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \left(\frac{\lambda}{1-\lambda}\right)^{-\frac{\sigma(1-\alpha)}{1-\sigma}} = L_g \left(\frac{d}{1-d}\right)^{\frac{1-\alpha}{1-\sigma}} \tilde{\alpha}\tilde{\lambda} \tag{A.6}$$

To determine the long-run values of L_g and d we log differentiate (26) and use (20) to write:

$$\hat{d} = (1-d)(1-\sigma)(\hat{w} - r) \quad . \tag{A.7}$$

Expressing the transversality condition (21) in growth rates and using (2) we have:

$$\lim_{\tau\to\infty} \hat{w}(\tau) \le r \qquad \text{for } \lim_{\tau\to\infty} g(\tau) > 0 \tag{A.8}$$

From (A.8) and (A.7) we conclude that – for $\sigma < 1$ – the intermediate labour share d unambiguously decreases over time and ultimately approaches zero; accordingly, labour steadily moves from intermediates production to research. Thus, the long-run steady state is characterised by

$$\lim_{\tau\to\infty} L_g(\tau) = L \text{ and } \lim_{\tau\to\infty} d(\tau) = 0 \, . \tag{A.9}$$

To determine $\hat{L}$ and L we use (6) and (6’). From (3), (4), (20), and (25) we know that $\hat{p}_Y = \hat{p}_x - \left[(1-\beta)/\beta\right]g \; = d\cdot\hat{w} + (1-d)r - \left[(1-\beta)/\beta\right]g$. Using (A.7) we thus derive

$$\hat{L} = \xi\left[(d-1)\hat{w} + (1-d)r\right] = -\frac{\xi}{1-\sigma}\hat{d} \tag{A.10}$$

and write for the population level:

$$L = \mu \cdot d^{-\frac{\xi}{1-\sigma}} \tag{A.11}$$

with $\mu, \xi > 0$. Using (A.9) in (A.11) and inserting into (A.6) gives for the long run

$$g(\infty) = L(\infty)\left(\frac{d(\infty)}{1-d(\infty)}\right)^{\frac{1-\alpha}{1-\sigma}} \tilde{\alpha}\tilde{\lambda} \quad . \tag{A.12}$$

which directly yields (28) and (27) in the main.

Alternative population response

Using $\hat{L} = \xi_1\hat{A} - \xi_2(\hat{w} - \hat{p}_Y)$ instead of (6) leads to:

$$\hat{L} = (\xi_1 - \xi_2)g - \frac{\xi_2}{1-\sigma}\hat{d} \tag{A.13}$$

which differs from (A.10) by the term $(\xi_1 - \xi_2)g$. Obviously, the assumption $\xi_1 > \xi_2$ would directly support population growth, the innovation rate, and economic growth. But as long as we have no empirical information about the difference between ξ_1 and ξ_2 we prefer to assume that they are of equal size such that $(\xi_1 - \xi_2)g = 0$.

Equations of motion

Starting with (A.7) for the dynamics of the labour share *d* we calculate $\hat{w} - r$ by first dividing (24) by p_n

$$\frac{\pi}{p_n} + \hat{p}_n = r \quad , \tag{A.14}$$

and use (23) and (25) to have

$$\frac{(1-\beta)p_x X/n}{p_n} + \hat{p}_n = \frac{(1-\beta)wL_x}{nd\beta p_n} + \hat{p}_n = r\,. \tag{A.15}$$

Calculating *w* as (value) marginal product from (1') gives

$$w = \alpha \cdot L_g^{\alpha-1} \cdot R_g^{1-\alpha} \cdot p_n \cdot n = \alpha \cdot p_n \cdot g \cdot n / L_g \tag{A.16}$$

which can be solved for p_n. Inserting (A.16) into (A.15) yields

$$\frac{\alpha g(1-\beta)L_x}{d\beta L_g} + \hat{p}_n = r \quad . \tag{A.17}$$

Using (2) and (20) to calculate $\hat{p}_n$ in (A.17) delivers

$$\frac{\alpha g(1-\beta)L_x}{d\beta L_g} + \alpha\hat{w} + (1-\alpha)r = r + g \tag{A.18}$$

so that by rearranging we get

$$\frac{\alpha(1-\beta)}{\beta}\left(\frac{\Omega - g}{d}\right) + \alpha(\hat{w} - r) = g \tag{A.19}$$

where $\Omega \equiv \dfrac{gL}{L_g}$ and

$$\hat{w} - r = \frac{1}{\alpha}\left[g - \frac{\alpha(1-\beta)}{\beta}\left(\frac{\Omega - g}{d}\right)\right] \tag{A.20}$$

Inserting (A.20) into (A.7) yields

$$\hat{d} = (1-d)(1-\sigma)\frac{1}{\alpha}\left[g - \frac{\alpha(1-\beta)}{\beta}\left(\frac{\Omega - g}{d}\right)\right] \tag{A.21}$$

and, after rearranging, (30) in the main text. To get the dynamics of the innovation rate in (31) we start from writing (7) in growth rates:

$$\hat{L} = \left(1 - \frac{L_g}{L}\right)\hat{L}_x + \frac{L_g}{L}\hat{L}_g \tag{A.22}$$

and

$$\hat{L}_g = \frac{L}{L_g}(\hat{L} - \hat{L}_X) + \hat{L}_X \quad . \tag{A.23}$$

From (A.3), (A.4), and (20) we derive, using differentials in logarithms:

$$\hat{L}_g = \hat{g} - (1-\alpha)(\hat{w} - r) \tag{A.24}$$

Employing (A.7) we get

$$\hat{L}_g = \hat{g} - \frac{1-\alpha}{(1-d)(1-\sigma)}\hat{d} \quad . \tag{A.25}$$

To get $\hat{L}_X$ we use (25) to write

$$\hat{w} + \hat{L}_X = \hat{d} + \hat{p}_x + \hat{X} \tag{A.26}$$

which yields, using (19) and (19'):

$$\hat{w} + \hat{L}_X = \hat{d} + r - \rho \quad . \tag{A.27}$$

Using (A.7) and rearranging gives

$$\hat{L}_X = \left[1 - \frac{1}{(1-d)(1-\sigma)}\right]\hat{d} - \rho \quad . \tag{A.28}$$

Inserting (A.25) and (A.28) into (A.23) and using (A.10) we get

$$\hat{g}=\frac{1-\alpha}{(1-d)(1-\sigma)}\hat{d}+\frac{L}{L_g}\left\{-\frac{\xi}{1-\sigma}\hat{d}-\left[1-\frac{1}{(1-d)(1-\sigma)}\right]\hat{d}-\rho\right\}+\left[1-\frac{1}{(1-d)(1-\sigma)}\right]\hat{d}-\rho$$

and, by simplifying:

$$\hat{g}=\left[1-\frac{g\alpha+\Omega\left[(1-d)(\xi+1-\sigma)-1\right]}{g(1-d)(1-\sigma)}\right]\hat{d}-\frac{\rho}{g}(\Omega-g) \qquad \text{(A.29)}$$

which directly yields (31) in the main text. To get the dynamics of resource extraction in (32) and (33) we first calculate $\hat{R}_X$, using the first order condition from profit maximisation in *X*-production (see also 26):

$$\frac{L_X}{R_X}=\left(\frac{\lambda}{1-\lambda}\right)^{\sigma}\left(\frac{w}{p_R}\right)^{-\sigma} \qquad \text{(A.30)}$$

to obtain with (25)

$$\hat{d}=(1-d)\left(\frac{1-\sigma}{\sigma}\right)(\hat{R}_X-\hat{L}_X) \quad . \qquad \text{(A.31)}$$

Inserting (A.28) into (A.31) yields

$$\hat{d}=(1-d)\left(\frac{1-\sigma}{\sigma}\right)(\hat{R}_X-\left[1-\frac{1}{(1-d)(1-\sigma)}\right]\hat{d}-\rho) \quad . \qquad \text{(A.32)}$$

Solving (A.32) for $\hat{R}_X$ and simplifying gives

$$\hat{R}_X=-\frac{d}{(1-d)}\hat{d}-\rho \qquad \text{(A.33)}$$

Note that in $\tau=\infty$ we have $d=0$ such that $\hat{R}_X(\infty)=-\rho$. Taking differentials in logs from (29) we get:

$$\hat{v}_X=\hat{R}_x-\hat{S} \qquad \text{(A.34)}$$

where with (5) and (8) we can express $\hat{S}=\dot{S}/S=-R/S=\left(-R_x-R_g\right)/S$ such that

$$\hat{v}_X=\hat{R}_x+v_x+v_g \quad . \qquad \text{(A.35)}$$

Inserting (A.33) into (A.35) gives (32) in the main text. Likewise we calculate $\hat{R}_g$ taking differentials in logs from (A.4)

$$\hat{R}_g - \hat{L}_g = \hat{w} - r \qquad . \qquad \text{(A.36)}$$

We further use (1') to write

$$\hat{g} = \alpha \hat{L}_g + (1-\alpha)\hat{R}_g \qquad \text{(A.37)}$$

which we solve for $\hat{L}_g$ to use it in (A.36). Further using (A.7) to eliminate $\hat{w} - r$ in (A.36) we obtain

$$\hat{R}_g = \hat{g} + \frac{\alpha}{(1-d)(1-\sigma)}\hat{d} \qquad . \qquad \text{(A.38)}$$

To get $\hat{R}_g(\infty)$ we know that $\hat{g}(\infty) = 0$ but need to calculate $\hat{d}(\infty) = \dot{d}(\infty)/d(\infty) \neq 0$. In (24) and (A.14) we can exploit that in a long run equilibrium both $\hat{p}_n$ and r must be constant so that π / p_n becomes constant as well. This says that $\hat{\pi} = \hat{p}_n$ or

$$\hat{p}_x + \hat{X} = \alpha \hat{w} + (1-\alpha) r \qquad \text{(A.39)}$$

where $\hat{p}_x + \hat{X} = r - \rho$ through (19) and (19'). So we get with (A.7)

$$d(\infty) = -\frac{(1-\sigma)\rho}{\alpha} \qquad \text{(A.40)}$$

and with (A.38) we have

$$\hat{R}_g(\infty) = -\rho \qquad . \qquad \text{(A.41)}$$

Taking differentials in logs from (29) we get:

$$\hat{v}_g = \hat{R}_g - \hat{S} \qquad \text{(A.42)}$$

where with (5) and (8) we can again express $\hat{S} = \dot{S}/S = -R/S = \left(-R_x - R_g\right)/S$ such that

$$\hat{v}_g = \hat{R}_g + v_x + v_g \qquad . \qquad \text{(A.43)}$$

Inserting (A.38) into (A.43) gives (33) in the main text.

Phase diagrams

The location and the shape of the $\dot{d} = 0$ locus in the *d*-*g*-plane is found by setting (30) equal to zero in order to derive, after rearranging:

$$g_d = \frac{\Omega}{\frac{d\beta}{\alpha(1-\beta)}+1} \tag{A.44}$$

with $\Omega = \mu\tilde{\alpha}\tilde{\lambda} \cdot d^{\frac{1-\alpha-\xi}{1-\sigma}} \cdot (1-d)^{-\frac{1-\alpha}{1-\sigma}} > 0$ as above and when g_d denotes the value of g determined by the $\dot{d}=0$ locus. With $d=0$ we get for Ω

$\Omega = 0$ if $1-\alpha > \xi$

$\Omega = \mu\tilde{\alpha}\tilde{\lambda}$ if $1-\alpha = \xi$

$\Omega = \infty$ if $1-\alpha < \xi$.

With $d=1$ we have $\Omega = \mu\tilde{\alpha}\tilde{\lambda}$ in all cases. Thus the $\dot{d}=0$ locus for $d=0$ yields

$g_d = 0$ if $1-\alpha > \xi$

$g_d = \mu\tilde{\alpha}\tilde{\lambda}$ if $1-\alpha = \xi$

$g_d = \infty$ if $1-\alpha < \xi$

To obtain g_d for $d=1$ we get from (A.44)

$$g_d = \alpha\tilde{\lambda}\tilde{\mu}\frac{1}{1+\frac{\beta}{\alpha(1-\beta)}} < \alpha\tilde{\lambda}\tilde{\mu} \quad . \tag{A.45}$$

The inequality sign in (A.45) makes clear that, for $1-\alpha \le \xi$, the $\dot{d}=0$ locus is always downward sloping when it is monotonous. For $1-\alpha > \xi$ it must have a maximum in the range $0 < d \le 1$. These claims are easiest confirmed by numerical simulation, which we do, using the parameter values $\alpha = 0.9$, $\sigma = 0.3$, $\rho = 0.01$, $\beta = 0.9$, and $\mu = 0.2$ (alternative values lead to the same finding). A separate supplement to this paper shows the results of the numerical simulations, including the behaviour of Ω.

For the $\dot{g}=0$ locus we conclude from (31) that, with $d=0$, we get $g_g = \Omega$. Thus we have:

$g_g = 0$ if $1-\alpha > \xi$

$g_g = \mu\tilde{\alpha}\tilde{\lambda}$ if $1-\alpha = \xi$

$g_g = \infty$ if $1-\alpha < \xi$

as it is for the $\dot{d}=0$ locus, which means that the two loci coincide on the vertical axis. To show that the $\dot{g}=0$ locus always lies below the $\dot{d}=0$ locus in the *d*-*g*-space, (and the two curves thus never intersect for $0 < d \le 1$), we build the difference $\Delta g = g_d - g_g$, for which we use (A.44) and the corresponding equation for g_g. We use the fact that Δg increases with Ω such that $\Omega = 0$ gives the minimum value of Δg. Calculating Δg with $\Omega = 0$ yields:

$$\Delta g = \frac{d\alpha\beta\rho}{\left(\alpha(1-\beta)+d\beta\right)(d(1-\sigma)+\alpha+\sigma-1)} \tag{A.46}$$

which is unambiguously positive for $0 < d \leq 1$, provided that $\alpha + \sigma > 1$. This is a safe assumption as α can be expected to be close to unity; most literature assumes $\alpha + \sigma = 2$, which is much higher. For $d = 0$ we get $\Delta g = 0$ as stated above. All these results are confirmed by numerical simulation with the same parameters values, see the separate supplement. Figures 1-3 are drawn accordingly.

Figures

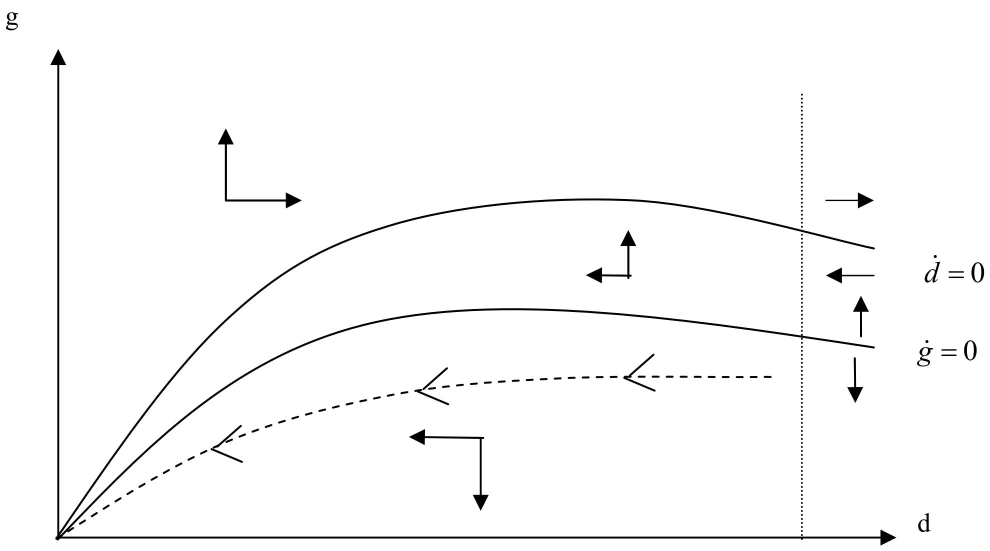

Figure 1

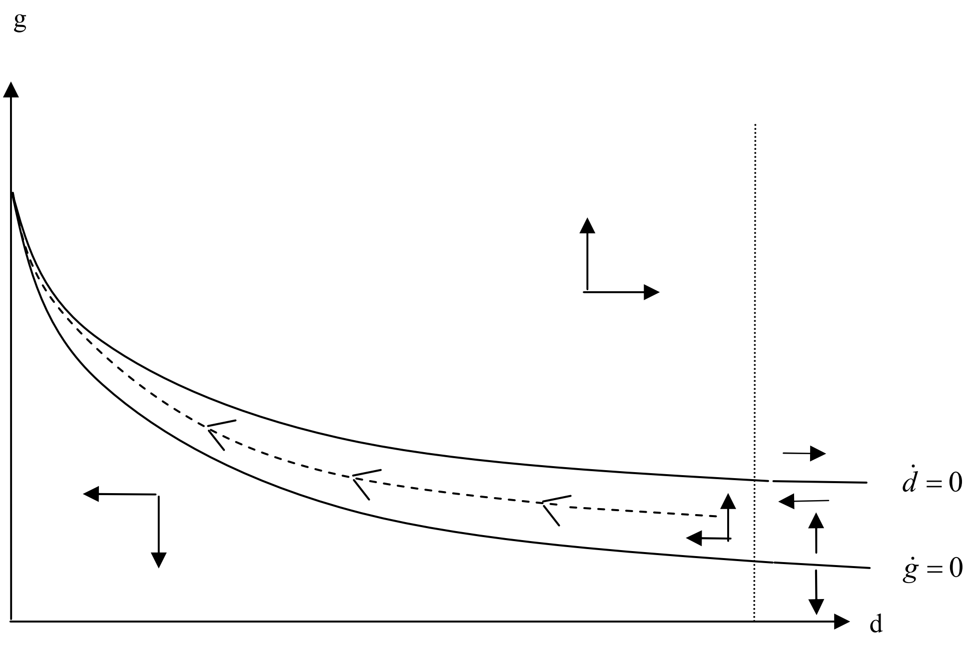

Figure 2

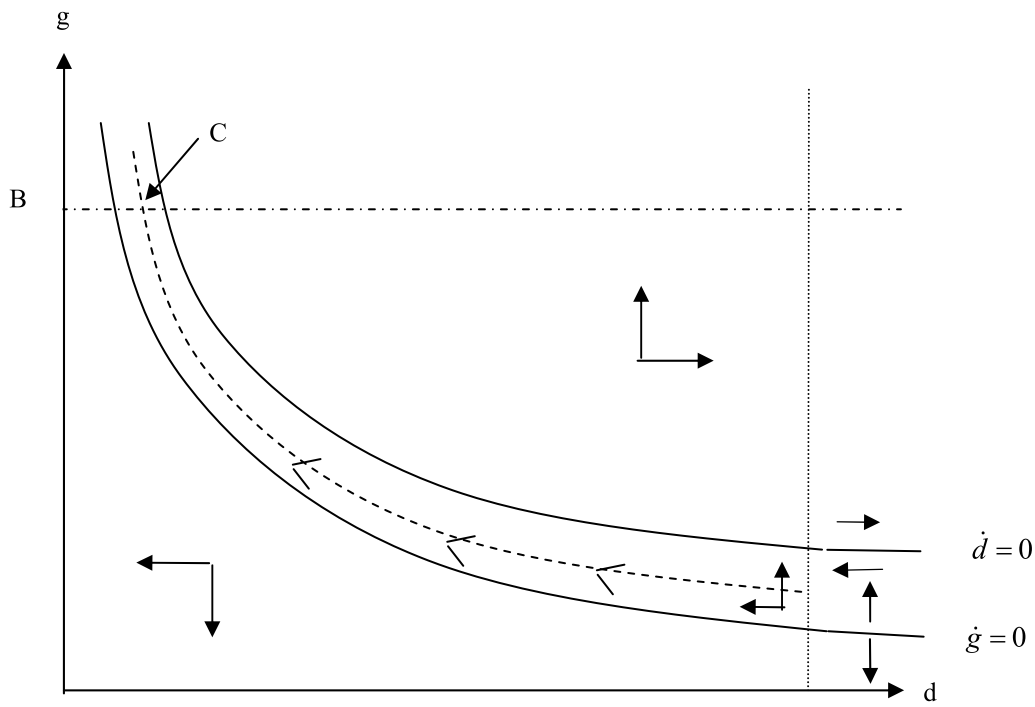

Figure 3