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Working Paper 08/82
March 2008

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

A Permit Allocation Contest for a Tradable Pollution Permit Market

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Abstract

In this paper we advocate a new initial allocation mechanism for a tradable pollution permit market. We outline a Permit Allocation Contest (PAC) that distributes permits to firms based on their rank relative to other firms. This ranking is achieved by ordering firms based on an observable 'external action' where the external action is an activity or characteristic of the firm that is independent of their choice of emissions in the tradeable permit market. We show that this mechanism efficiently allocates permits and, as a result, the tradeable permit market is cost-effective. We determine the symmetric equilibrium strategy of each firm in choosing their external action and find the choice is influenced by the firm's cost structure and the regulator's choice of permit allocation schedule (distribution of permits to the market). Furthermore, we investigate the factors that determine the regulator's choice of optimal permit allocation schedules.

Keywords: Rank-order contests; pollution permits; initial allocation

JEL Classification: D44; Q25

1 Introduction

The fundamental idea behind tradable permit markets allows a regulator to allocate pollution rights in a cost effective manner. As a consequence of competitive trading, theory dictates that firms' choice of abatement will be independent from the initial allocation of permits [22]. Indeed, the choice of allocation mechanism can be selected on equity criteria only. Yet, it is widely recognised that this independence rarely occurs in existing tradable permit markets due to the violation of various strong assumptions (see, for example, Hahn[11] and Stavins [32]). Consequently, an active debate has been established investigating the optimal choice of initial allocation mechanism. In particular, an important strand in this discussion has centered around whether grandfathering (a free allocation of permits based on historical emissions/output) or auctioning permits is the preferred form of initial allocation mechanism [4,8,26,27,29]. However, this debate has rarely ever considered the use of alternative allocation mechanisms and it is our aim in this paper to broaden the discussion of allocation design by outlining and advocating an alternative allocation mechanism that may be preferred to existing approaches.

In this paper we outline an alternative initial allocation mechanism in a tradeable permit market. Our mechanism, a Permit Allocation Contest (PAC), distributes permits to firms based on their rank relative to each other. The ranking is achieved by ordering firms based on an observable 'external action' where the external action is an activity or characteristic of the firm that is independent of their choice of emissions in the tradeable permit market. This ranking criterion is determined by the regulator who chooses this to meet a public policy objective. We show that this mechanism efficiently allocates permits and as a result the tradeable permit market is cost-effective. Moreover, we determine the symmetric equilibrium strategy of each firm to choose their external action and investigate the regulator's optimal choice of permit distribution.

As previously mentioned, in the existing literature, allocation types are usually distinguished into two broad categories: the grandfathering and auctioning of permits. The grandfathering of permits occurs when the regulator freely allocates allowances to each firm based on their historical emissions (perhaps output or some other proxy). Although a popular and frequently used mechanism, grandfathering is far from an ideal allocation mechanism as it is often viewed as politically cumbersome and inefficient [4,33]. Firms may have an incentive to lobby the regulator in favour of larger permit allocations which, due to the use of wasteful resources, may reduce social welfare in the economy. Moreover, when grandfathering is used with information that is updated over time—updated grandfathering— a link is created between a firm's current level of emissions and it's future permit allocation which may result in a distortionary incentive to increase emissions [2, 19]. In this case, grandfathering no longer produces a cost-efficient level of abatement on the part of firms.

The main alternative to grandfathering is generally considered to be auctioning. In an auction, permits are allocated to each firm based on their monetary bid relative to every other firm [4,6,12,17,18,25]. Auctions are often considered to be a 'lump-sum' allocation mechanism as permits are distributed to each firm independent of their historical emissions. Due to this characteristic, auctioning is viewed as a desirable and efficient method of allocating permits [4]. However, the main drawback, and as a result, the main reason for the infrequent use of auctions is the political difficulty in implementing such a mechanism. As the winners in the auction are obliged to pay for their permits, firms' resistance against implementing auctions have been a severe restriction on the implementation of such schemes.[1] It is possible to reduce firms' resistance to auctions by redistributing revenue to the participants (revenue neutral

[1] Auctioning, however, is slowly becoming an increasingly important and favoured initial allocation mechanism in existing tradeable permit markets, such as the US SO_2 'Acid Rain' Program and the European Union Emissions Trading Scheme (EU-ETS).

auction [12]) or to reduce distortionary taxes in the economy (the revenue recycling effect [26,27]), however, such schemes are very rarely implemented. With problems associated with both grandfathering and auctioning it is therefore desirable to try and find mechanisms that may be better suited for tradeable permit markets.

A frequently used allocation mechanism in areas such as labour markets and sporting competition analysis allows participants to be ranked in order of their effort or actions [16,34]. Within this general model, two main mechanisms exist: rank-order tournaments and rank-order contests. The distinction between the two rests on the relationship between agents' unobservable effort and observable actions. Rank-order tournaments are incentive schemes used in situations where firms' performance is observed with some exogenous noise. That is, in rank-order tournaments, it is generally assumed that each agent experiences a stochastic relationship between their effort and actions. In most cases, when the observation noise is common to all firms, rank-order tournaments typically outperform absolute, or individualistic, schemes [10, 13,16, 23, 24].

When there is no individual-specific noise involved in the observation of firms' actions, one can implement a rank-order contest, which is, in effect, a multi-prize all-pay auction [1,3,7,20,21]. This differs from tournaments as agents in rank-order contest models are generally assumed to have a deterministic relationship between effort and actions. In a rank-order contest, there is a finite number of prizes to be distributed among the participating agents, with the size of each prize known before the onset of the contest. Firms compete in this contest by submitting costly (monetary or non-monetary) "bids". Firms then are ranked in order of their bids, and the "prizes" are distributed to the firms according to firms' rankings. That is, a firm that submits the highest bid is ranked first, and thus gets the largest permit allocation ("first prize"), the firm that submits the second-highest bid is ranked second, and thus gets second-largest allocation ("second prize"), and so on, up to the firm that submits the lowest bid being ranked last, and thus receiving the smallest allocation (possibly nothing). Rank-order contests, like tournaments, tend to outperform alternative types of individualistic and contract based regulation.

The potential of rank-based mechanisms is clear when we consider the limited literature that focuses on environmental policy issues. By applying the seminal work of Lazear and Rosen [16], Govindasamy et al. [9] advocated the use of a tournament to control non-point pollution, whereby each polluting firm is ranked by its input use or pollution abatement effort. Govindasamy et al. [9] found that a tournament can work well as it can achieve the same efficiency conditions as a Pigouvian tax but with less costly information requirements. Shogren and Hurley [31] experimentally tested a tournament reward system to consider the implication for environmental policy (for example, they considered Coasian bargaining and environmental conflict) and found that using such a reward system made the experiment participants behave in a similar manner to theoretical predictions (for example, the Coasian bargaining outcome was achieved). They showed that tournaments reached the theoretical outcomes quicker than other "standard" mechanisms which suggests tournaments systems can provide robust incentives to effectively implement environmental regulation.

The above tournament studies have all assumed a probabilistic link between firm effort and observable action. However, it is also important to consider scenarios where firms' effort and observable action are deterministically linked. To the best of our knowledge, there has been no attempt at implementing a rank-order contest to environmental issues and, in particular, no attempt at implementing a rank-order contest as a mechanism for the initial allocation of pollution permits. This, then, is our main contribution to the literature.

Our partial equilibrium model attempts to reach a middle ground between grandfathering and auctioning. As our model is a type of 'all-pay auction' it has many similarities to a standard permit auction. Yet, as the ranking criterion in the PAC can be non-monetary, it is possible to have characteristics similar to a grandfathering mechanism. Our model has two stages. In the first stage, every firm is ranked in order of the size of external action. Each firm obtains a permit allocation which is directly related to their ranking in the PAC. In the second stage, firms obtain the permit allocation and choose a level of emissions to minimise the cost of participating in the tradeable permit market.

The papers that are most relevant to our argument are Glazear and Hassin [7] and Moldovanu and Sela [20]. Glazer and Hassin [7] study the design of a contest to try and maximise the expected aggregate output of a set of firms. They find that with identical firms, the prizes should be equal (apart from the lowest prize which should be zero). Moreover, when firms have different abilities, it is optimal to choose only one prize. In a similar vein, Moldovanu and Sela [20] study a rank-order contest with several risk neutral agents and a contest designer aiming to maximise the total expected effort. Moldovanu and Sela [20] separate their model into three distinct cases: when the costs of choosing effort are linear, concave or convex. They find it is optimal to allocate a single prize when contestants costs are linear or concave and to allocate multiple (possibly equal or unequal) prizes when costs are convex (again, apart from the lowest prize which should be zero). Our model uses a similar contest structure to the above studies by allowing a number of permit allocations to be allocated to several firms in a tradeable permit market. We study an allocation system in which there are no random errors present to alter firms' external action choices: in other words, the regulator can observe the external actions of each firm with no error (a perfectly discriminating contest).

The paper is organised as follows: section 2 introduces the concept of a PAC and explains the rationale for its use. Section 3 discusses the general properties of the model. Section 4 discuses the tradeable permit market. Section 5 details the PAC mechanism discuses the firm's problems and analyses the regulators optimal choice of permit allocations and Section 6 concludes.

2 An Alternative Approach to the Initial Allocation of Permits

This paper concentrates on a rank-order contest: a mechanism where agents are rank-ordered with respect to their (costly) bids [7,10,13,16,20,21,23,24]. In our model, we use a rank-order contest to allocate permits in a tradeable permit market, which we denote as a Permit Allocation Contest (PAC). To keep the ranking criterion as general as possible, we assume that firms will be ranked on their choice of an observable 'external action'. The observable 'external action' is an activity or characteristic of the firm which is independent from its choice of emissions and the permit market. It can be, at the extreme, an invariant characteristic of a firm. However, a more interesting case involves firms being able to decide upon their 'external actions', thus having the ability to alter their permit allocations. For example, possible 'external actions' include the improvement in noise reduction in firms' facilities, the record of health and safety incidents or some corporate and social responsibility criterion and so on. The regulator aims to select an appropriate criterion to rank all firms so that the action is independent of emission choices and where the aggregate action can fulfil an objective set by the regulator. We return to this issue in the next section.

A Permit Allocation Contest is a special type of auction in which every participating firm, regardless of the final outcome, incurs the cost of choosing a 'bid' or 'action' (an all-pay auction). It follows that a PAC has a number of properties similar to a standard permit auction (and some unique to itself).

In a PAC, the decisions regarding the number and size of permit allocations has a substantially different effect on the incentives of each firm compared to alternative mechanisms, such as a 'winner-pays' auction. The permit allocations in the PAC are not *directly* related to the firms' external actions, but instead they are determined by firms' rankings according to the size of their external actions. Thus, a small increase in the firm's external actions may result in a disproportionately large change in permit allocation. For example, a small increase in external action by the second-ranked firm could make this firm the winner of the contest, and thus lead to the largest permit allocation (which is typically made to be substantially larger than the "second prize").[2] Rank-order contests, and in particular our PAC, involve a clear rule of allocation of prizes (i.e. no regulator's subjective judgement is involved), and are easily adaptable to changing market and technological conditions. Moreover, as Krishna and Morgan [14] showed, all-pay auctions tend to generate higher aggregate bids than their winner-pay counterparts. In addition, as Moldovanu and Sela [20] showed, when the prize structure is suitably chosen, such a contest will tend to generate the largest aggregate bids. As the choice of external action at the margin can significantly alter a firm's permit allocation, the robust incentives created in the PAC system should induce all firms to maximise their external action.

As the ranking criterion need not be monetary in value, there may be a wide variety of possible external actions to choose from (*any* action that is independent of emissions choices is admissible). It follows that one may be chosen so that the scheme is politically acceptable for the regulator, market participants and the wider economy. Consequently, a PAC system has the possibility of being implemented in a wide variety of tradeable permit market contexts. For instance, a PAC could be implemented in an international permit market where the participating countries are allocated permits (or a burden is

[2] This frequently happens in sport tournaments where the difference between prizes (and notably between first and second prizes) is non-linearly increasing [34].

assigned to each country) based on their (country) external action, such as the proportion of recycling in that the country and so on. Yet, this system could also be adapted to smaller markets, such as firms choosing external actions based on their improvement in noise pollution. Every tradeable permit market has heterogeneous circumstances in which it operates and with a PAC, public policy objectives (and external actions) can be chosen to compliment the social 'norms' and prevailing political opinion in the specific emissions trading scheme. In contrast, although auctioning and grandfathering can be used in all tradeable permit markets, the only allocation criterion available is the comparison of firms' money 'bids' and historical emissions, respectively. The lack of other possible allocation criterion may make, especially for the case of auctions, implementation more difficult.

Using a PAC in a tradeable permit market may offer the (political) benefit of having a clear connection between permit allocations (including the differences between them) and some socially beneficial firm action. It is possible that a PAC system may actually appear 'fairer' to a larger number of groups in society than alternative mechanisms as it couples permit allocation (a reward to the firms) with some public policy objective. In contrast, grandfathering permits creates a perverse link between emissions and the permit rent each firm receives.[3] Therefore, large polluters are implicitly rewarded and small polluters are implicitly punished for their choice of emissions.

Similar to the auctioning of permits, a PAC takes an 'instrumentalist' perspective in that it ignores past and current permit holdings when determining permit allocations [28]. Therefore, this type of allocation approach treats all firms equally in that firms who invest early in pollution abatement are not implicitly punished (as would happen under a grandfathering scheme). However, unlike an auction, a PAC mechanism can

[3]The equitable issues associated with permit allocation are notoriously under researched in economics, mainly due to the normative aspects involved [28]. All allocation mechanisms can appear 'fair' as it very much depends on the attitude to property and the specific circumstances, i.e. an industry level or global emissions trading scheme.

be adapted so non-monetary criterion are used to rank the firms which may be more appealing to participating firms than an auction.

Although a PAC distribution mechanism appears to have a number of possible advantages over alternative mechanism a limitation of the PAC is that the external action must be defined in an appropriate manner. As noted above, an optimal external action has to be independent of emissions so that no distortions are created in the permit market whilst simultaneously being politically acceptable for all market participants and observable to the regulator. Unsurprisingly, the existence of an optimal external action may not necessarily occur. The ease with which an external action can be chosen crucially depends on the specific institutional context of the permit market. For instance, when the market participants are countries, such as in an international permit market, it may be relatively easy to find an external action that is both socially beneficial and independent of emissions. Countries in a carbon dioxide permit market, such as the EU-ETS, could be ranked on the proportional reduction of landfill waste from the non-trading sector (or the production of methane from it). As the market participants, become smaller in size (e.g. industries or firms), it may be more difficult to find an external action with the desirable qualities. Throughout this paper, for analytical simplicity, we discuss firms as the participating agents, however, the idea can be adapted to a wider institutional context.

3 General Properties of Model

Let $\Theta = \{1, 2, \ldots, n\}$ be a set of firms that participate in a competitive tradeable permit market to control a pollutant. In this tradeable permit market, firm i chooses a level of emissions e_i at a cost $c_i(e_i)$ with $\frac{dc_i(e_i)}{de_i} \leq 0$ and $\frac{d^2c_i(e_i)}{de_i^2} \geq 0$.

Aside from regulating emissions in a tradable permit market, the regulator also has a secondary (unrelated) objective. To allow for analysis, we restrict our attention to public policy scenarios in which the regulator aims to minimise a social 'bad' produced by *all* firms in the permit market, such as the improvement of: health and safety incidents, noise pollution, other pollutants, corporate responsibility and so on. Therefore, in our model, the regulator simply wants to minimise the aggregate social 'bad' (or maximise some social 'benefit') by using incentives in the form of permit allocations (without the need for standard command and control regulation). To adhere to the regulator's public policy objective, firm i chooses an *external* 'action' denoted by z_i, in which it bears a cost $v(z_i)$ with $\frac{dv(z_i)}{dz_i} \geq 0$ and $\frac{d^2v(z_i)}{dz_i^2} \geq 0$. In other words, the external action is an activity taken by each firm, independent of emissions choices, to comply with the regulator's goal of minimising some aggregate social 'bad'.[4]

The model is separated into two distinct stages. In the first stage, the regulator initially allocates the pollution permits to the market and in the second stage, firms are allowed to trade the pollution permits obtained in the first stage.

In stage one, the regulator chooses an ordered schedule (vector) of permit allocations, $\mathbf{s} = (s_1, s_2, \ldots, s_n) \in \mathbb{R}^n_+$ subject to $s_1 \geq s_2 \geq \ldots \geq s_n \geq 0$ and $\sum_{j=1}^{n} s_j = E$ where s_j is the j^{th} permit allocation and E is the absolute aggregate emissions cap for the tradeable permit market (the regulator's precise choice of permit allocations will be considered later in this paper).[5] Using the permit allocation schedule, the regulator distributes a (possibly unequal) permit allocation to each firm whilst ensuring the absolute emissions cap is binding. The specific permit allocation to a firm depends on each firm's size of

[4]In most permit markets, the participation of firms in the permit market is usually dependent on their inclusion in a product market e.g. a permit market may require participation of all energy producers. Given the permit market participants have similar product markets, it is possible that each firm in the permit market has a number of characteristics or 'actions' that are comparable amongst all participants, independently chosen from its emissions and socially beneficial, which can be used as the external factor.

[5]It is assumed that the regulator chooses the optimal level of aggregate level of emissions E to maximise the social welfare of emitting the pollutant. In other words, for the optimal level of aggregate emissions, the marginal benefit is equal to the marginal damage.

external action relative to every other firm, so that firms that have a larger relative size of external action obtain a larger permit allocation.[6] In a PAC, the regulator observes the external actions of all firms and ranks them in descending order of their external action where the firm with the highest level of external action is ranked first, the second highest firm is ranked second and so on until all firms are ranked.[7] Each ranked-ordered firm obtains a corresponding permit allocation so that the firm with the top ranking obtains the largest permit allocation (s_1), the second ranked obtains the second highest permit allocation (s_2) and so on until all individual permit allocations are distributed to the firms.

In stage two of the model, given a known permit allocation, each firm decides to choose a level of emissions to minimise the net cost of participation in the tradeable permit market.

As mentioned above, the regulator has two non-competing policy objectives.[8] Firstly, the regulator is motivated to choose a schedule of permit allocations to minimise the aggregate abatement cost in the tradeable permit market–the standard permit market regulatory objective. Second, the additional objective of the regulator is to provide incentives for the permit market firms to achieve some predetermined public policy target linked to the external actions of firms which we define as the maximisation of expected aggregate external actions. As such, the regulator is not a strict social cost minimiser since it is not concerned with the firms' costs of obtaining an external action (it simply wants to maximise the aggregate action). Following this approach shows the realistic

[6]The regulator must choose an external action that is feasible for the participating firms. In addition, if the industry exhibits increasing returns to scale, a regulator could allocate permits based on how each firm's present external action compares to its own past external action - e.g. based on the percentage reduction of noise pollution over time. However, scale effects will be captured by the form of the cost function described later.

[7]Other possible mechanisms are feasible, we could, for example, apply a yardstick competition mechanism to the external factor. Using a yardstick model would allow each firm to obtain a continuous expected allocation, instead of a discontinuous allocation, as experience in a PAC.

[8]Alternatively, the model can include two regulators with independent, non-competing policy objectives.

separation and independence between two legislative procedures which may commonly occur between a product and tradeable permit market. The regulator is not focusing on choosing an *efficient* level of aggregate external action for the second public policy objective, instead, the regulator wants to simple maximise aggregate actions.

We solve the model backwards by investigating the permit market in the following section and then focusing on the initial allocation of permits in the subsequent section.

3.1 Stage Two: The Permit Market

In this section, we investigate firm i's optimal choice of emissions in a permit market when the tradeable permits have been allocated using a PAC.

From stage one, let us assume that firm i chose a positive level of external action (z_i) and, as a result of the firm's ranking, the regulator distributed a permit allocation $\tilde{s}_i \in \mathbf{s}$ to the firm where $\tilde{s}_i$ is independent of e_i. With the endowment of tradeable permit obtained, firm i aims to minimise (maximise) the cost (profit) of participating in the permit market. Formally, given a market equilibrium permit price p, firm i's objective function is:

$$\min_{e_i} \qquad c_i(e_i) + p(e_i - \tilde{s}_j) \tag{1}$$

Solving for firm i's emissions gives us:

$$-\frac{dc_i(e_i)}{de_i} = p \tag{2}$$

From equation (2), each firm will choose a level of emissions to equate their marginal abatement cost with the permit price and it follows from standard theory that:

Proposition 1 *When a PAC distributes permits, the tradeable permit market is least-cost.*

A PAC is an efficient instrument to allocate permits as it is a 'lump-sum' mechanism—a mechanism by which permits are distributed independently of the choice variable (emissions). The criterion for allocating the permits (the external action) is independent of emission choices, therefore, no distortions exist in the tradeable permit market. Due to the 'lump-sum' characteristics of a PAC, efficiency in the tradeable permit market is independent from the schedule of permit allocations chosen by the regulator [22]. In contrast, Böhringer and Lange [2] and MacKenzie et al. [19] have shown that the efficient distribution of abatement reached in equation (2), does not necessarily hold when permits are allocated using updated grandfathering—the use of updated historical information for the allocation of free permits.

3.2 Stage One: The Initial Allocation of Permits (PAC)

In the last section, it was noted that when firms obtain tradeable permits through a PAC—a mechanism that ranks firms in order of their level of external action—the permit market can be least-cost. In this section, we investigate the PAC in more detail and, in particular, given an exogenously fixed permit allocation schedule, find the conditions that affect each firm's choice of external action. We then investigate the optimal choice of permit allocation schedule that can maximise the aggregate external action (public policy objective).

The Permit Allocation Contest in this paper, follows closely to the work of Barut and Kovenock [1], Glazer and Hassin [7] and Moldovanu and Sela [20,21]. For analytical simplicity, we assume throughout that every firm participates in the PAC. Therefore, we are implicitly assuming the cost involved in participating in the PAC is less than the cost of abatement and/or purchasing permits from the market.[9] We begin by discussing

[9]The results would still be maintained if we assumed that some firms did not participate in the PAC. The vector of permit allocations would be distributed to the firms that did participate in the PAC and the remaining firms would purchase permits, reduce emissions or a mixture of both.

the firm's problem.

3.2.1 Firm's Problem

Using the contest approach of Moldovanu and Sela [20], we represent the ability of each firm to produce an external action by the parameter α_i where the costs of producing the external action are $\alpha_i v(z_i)$, with $v(0) = 0$.[10] Firm i's own ability parameter α_i is privately observed before the PAC commences. We further assume that the ability parameter is separable from the external action and is independently drawn from a support $[a, b]$, $0 < a < b < \infty$ with the (commonly known) distribution function $G(\alpha_i)$ with density $G'(\alpha_i) > 0$.

Although each firm knows its own ability parameter and the distribution of ability parameters for its competitors, no firm knows the actual *realization* of its rivals' ability parameters. Similarly, although the permit allocation schedule is common knowledge, each firm's *actual* permit allocation is uncertain at the time of the decision-making. In other words, by participating in PAC, all firms engage in a *game of incomplete information.* Given its knowledge of own ability, of the distribution of abilities, and of schedule of permit allocations, each firm uses its expectations of permit allocation to choose an optimal level of external action.

We employ the techniques used for perfectly discriminating contests (e.g. Moldovanu and Sela [20]), and suppose that each firm i adopts a symmetric strictly monotonic differentiable strategy $z_i = h(\alpha_i)$ which is strictly decreasing in its ability parameter α_i. Notice that strict monotonicity of strategies implies that $1 - G(\alpha_i)$ is the probability that firm i's external action z_i is greater than the external action of another firm k with an ability parameter α_k randomly drawn from the common distribution $G(\cdot)$, i.e. that

[10] The cost function thus involves only variable cost of the external action. In general, our arguments still hold for positive fixed costs, yet the optimal smallest prize would have to be positive - see the discussion of the smallest prize in the text as well as Glazer and Hassin [7].

$z_i = h(\alpha_i) > z_k = h(\alpha_k)$. Thus, one can write the expected permit allocation of a firm i as:

$$B(G(\alpha_i)) = \sum_{j=1}^{n} s_j \frac{(n-1)!}{(n-j)!(j-1)!} (1 - G(\alpha_i))^{n-j} (G(\alpha_i))^{j-1} \tag{3}$$

The expected permit allocation in equation (3) is a linear combination of n order statistics where the probability of obtaining the j^{th} permit allocation is based on the probability of being ranked j^{th} in the PAC [5,7,21]. For example, the probability of winning the largest permit allocation is the probability of being ranked first $((1 - G(\alpha_i))^{n-1})$, alternatively, it is the probability of $n-1$ firms being ranked below this firm. Equation (3) is strictly decreasing in α_i as a larger α_i implies a 'lower' ability and choice of external action and thus a lower expected permit allocation.

We now proceed to derive the symmetric equilibrium strategy $h(\cdot)$. Given the expected allocation of permits (3) and the market equilibrium permit price p, firm i chooses the optimal level of its external action $z_i = h(\alpha_i)$ by maximizing its expected net payoff from permit allocation:

$$\max_{z_i} \quad p \cdot B(G(h^{-1}(z_i))) - \alpha_i v(z_i) \tag{4}$$

where $h^{-1}(\cdot)$ is the inverse function of $h(\cdot)$. We have the following proposition:

Proposition 2 *Given its objective function (4), firm i's optimal level of external action is:*

$$z_i = h(\alpha_i) = v^{-1}\left[p \int_{\alpha_i}^{b} -\frac{B'(\cdot)G'(t)}{t} dt\right]. \tag{5}$$

Proof. Given the common strategy $z_i = h(\alpha_i)$ which is strictly decreasing, monotonic and differentiable function, suppose firm i chooses a level of external action $\tilde{\alpha}_i$ so that $\tilde{z}_i = h(\tilde{\alpha}_i)$. Substituting this common strategy into equation (4) gives

$$p \cdot B(G(h^{-1}(\tilde{z}_i))) - \alpha_i v(\tilde{z}_i)$$

Differentiating with respect to $\tilde{z}_i$, we arrive at the first-order condition:

$$p \cdot \frac{dB}{dG} \cdot \frac{dG}{dh^{-1}(\tilde{z}_i)} \cdot \frac{dh^{-1}}{d\tilde{z}_i} - \alpha_i \cdot \frac{dv(\tilde{z}_i)}{d\tilde{z}_i} = 0$$

In equilibrium, firm i will choose $\tilde{z}_i = z_i$ so that

$$p \cdot \frac{dB}{dG} \cdot \frac{dG}{dh^{-1}(z_i)} \cdot \frac{dh^{-1}}{dz_i} - \alpha_i \cdot \frac{dv(z_i)}{dz_i} = 0$$

Multiplying by $\frac{dz_i}{dh^{-1}}$ and rearranging, we get

$$\frac{dv(z_i)}{dh^{-1}(z_i)} = \frac{pB'(\cdot)G'(h^{-1}(z_i))}{\alpha_i}$$

Again, using the common strategy $z_i = h(\alpha_i)$ gives

$$\frac{dv(z_i)}{d\alpha_i} = \frac{pB'(\cdot)G'(\alpha_i)}{\alpha_i} \tag{6}$$

Notice that, in a symmetric equilibrium, the firm with the worst possible ability $\alpha_i = b$ will get the lowest-ranked permit allocation s_n with certainty, and thus will choose an external action of zero. This gives us the upper boundary condition of $h(b) = 0$. Integrating (6) with respect to α_i, and utilizing the boundary condition $h(b) = 0$, we get that

$$v(z_i) = p \int_{\alpha_i}^{b} -\frac{B'(\cdot)G'(t)}{t} dt$$

and as required

$$h(\alpha_i) = z_i = v^{-1}\left[p \int_{\alpha_i}^{b} -\frac{B'(\cdot)G'(t)}{t} dt\right]$$

■

Proposition (2) implies that firms' optimal external actions are determined by a number of factors. First, the shape (or curvature) of the cost function $v(\cdot)$ is an important determinant of the level of external action chosen by each firm. Indeed, the 'less' convex a firm's cost function, the higher the optimal external action. Second, since $\frac{dz_i}{dp} > 0$, a higher market equilibrium permit price p would lead to each firm choosing a higher external action. Third, a general increase in the regulator's schedule of permit allocations s would increase the value of the marginal permit allocation $B'(\cdot)$, thus increasing the optimal external action. Furthermore, an increase in the number of firms, as well as certain changes in the distribution of abilities $G(\cdot)$ may also lead to higher optimal external actions. Despite all of the above factors tending to lead to more aggressive "bidding" for permit allocations – and, thus, to higher aggregate external actions, – only one of the factors is in the regulator's control – namely the schedule of permit allocations. We now look into how the regulator can maximize the aggregate external actions by choosing an appropriate permit allocation schedule.

3.2.2 Regulator's Problem

In the previous subsection, we looked at the decision problems of the firms participating in the permit allocation contest (PAC). Using the incomplete information game approach to PAC, we derived the symmetric strictly decreasing differentiable equilibrium strategies for each participating firm. In this subsection, we focus on the second policy objective of the regulator, namely, the regulator's motivation to maximise some public policy objective.

Up to this point, we have assumed that the schedule of permit allocations has been exogenously fixed and known to all firms. In this sub-section, we relax this assumption and allow the regulator to choose a schedule from the set of feasible permit allocations $\left\{(s_1, s_2, \ldots, s_n) \in \mathbb{R}^n_+ : s_1 \geq s_2 \geq \ldots \geq s_n \geq 0, \sum_{j=1}^{n} s_j = E\right\}$.

Suppose that the regulator has committed to some public policy objective, which involves the maximization of the aggregate choice of the external action z_i by all participating firms, and that the regulator chose a PAC as a suitable mechanism to achieve his objective. Given the symmetric equilibrium strategy of each firm (5), one can write the regulator's objective as:

$$T = n \int_a^b h(\alpha_i) \cdot G'(\alpha_i) d\alpha_i \tag{7}$$

subject to:

$$s_1 \geq s_2 \geq \ldots \geq s_n \geq 0, \text{ and } \sum_j^n s_j = E \tag{8}$$

where $h(\alpha_i)$ is the symmetric strategy for the external action given by equation (5). Therefore, T is the expected value of aggregate external actions given that each firm obtains an ability from the support $[a, b]$ and each firm follows the symmetric equilibrium strategy. Note that the regulator's problem (7)-(8) in its general formulation cannot be solved analytically. However, a number of key implications can be discussed.

One popular permit allocation schedule discussed in tradeable permit market literature involves an egalitarian distribution of permits across all firms or countries [28]. For example, allocating an equal number of permits per capita has been strongly advocated as a distributional rule for an international permit market [15,30]. While some form of egalitarian allocation may have a number of merits, it may not be desirable to achieve the second policy objective. To see this, consider an extreme egalitarian allocation where firms obtain identical number of permits independent of all firms' actions or characteristics – a 'pure' lump-sum approach. In such a scenario, the regulator's schedule of permit allocations is $\mathbf{s}^{egal} = (\frac{E}{n}, \frac{E}{n}, \ldots, \frac{E}{n})$, where each firm in the PAC obtains an identical share of the permit cap. If the regulator were to use such a schedule of permit allocations, then from equations (5) and (7), it is immediate that:

Corollary 3 *If the regulator chooses an egalitarian permit allocation schedule ('pure'* *lump-sum approach)* $\mathbf{s}^{egal} = (\frac{E}{n}, \frac{E}{n}, \ldots, \frac{E}{n})$ *then no second public policy objective is achievable.*

Proof. Given an egalitarian distribution $s^{egal} = (\frac{E}{n}, \frac{E}{n}, \ldots, \frac{E}{n})$, we have that $B(G(\alpha_i)) = \frac{E}{n}$ and consequently we have $B'(\cdot)G'(\alpha_i) = 0$. In other words, the distribution of permits is independent of firm's external actions, and there is no uncertainty about firm's permit allocation. Hence from equation (5) it follows that $z_i = v^{-1}\left[p \cdot \int_{\alpha_i}^{b} 0 \; dt\right] = v^{-1}[p \cdot (C - C)] = v^{-1}[0] = 0$. ■

In other words, an egalitarian approach provides no incentives for firms to take an external action, and thus is not suitable if the regulator wants to combine the permit allocation of a tradeable permit market with a public policy objective. For a regulator to succeed in a public policy objective, it must instead choose a schedule of permit allocations that discriminates in favour of firms with larger external actions and against the ones with smaller actions.[11]

As Barut and Kovenock [1] and Glazer and Hassin [7] showed, to maximise the aggregate external action, the lowest-ranked permit allocation s_n must involve zero permits. Otherwise, there would be an incentive for firms with 'weaker' abilities (i.e. with high α_i's) to reduce their level of external action and obtain a positive level of permit allocation. In other words, if there are n firms, a schedule of permit allocations must involve at least one zero permit allocation $s_n = 0$ and no more than $n-1$ non-zero permit allocations.

We now turn to the discussion of the permit allocation schedule that the regulator may want to implement in order to maximize the aggregate external action.

[11] The 'large' external action can be a proportional change in their own actions. Therefore, large firms, in the absolute sense, do not have 'size effect' advantages.

4 Discussion of the Permit Allocation Schedule

In general, optimal contest mechanisms are notoriously difficult to characterize analytically. To analyze the permit allocation schedule that maximizes the aggregate value of the external action, one needs to take the expectation of z_i as given in equation (7). The resulting expression does not have an analytic solution, and thus, even the simplest analysis of the optimal allocation schedule is a formidable task. Some general insights to the problem were provided by Moldovanu and Sela [20,21] and references therein, advocating for some discriminatory features of contests. In particular, Moldovanu and Sela [20] showed that when costs functions are linear or concave, it is optimal to allocated the prize "pie" to only a single "first" prize. They also showed when cost functions are convex, several positive prizes may be optimal.

To our knowledge, Moldovanu and Sela [20] is the only work that provides an adequate analysis of the contest with convex costs, and thus is tremendously important for our purposes. However, their work does not go beyond the situation when there are only two potentially non-zero prizes, and provides only a limited analysis for an arbitrary number of contestants n. In other words, to date, there is no analytical solution of the optimal allocation schedule involving more than two potentially non-zero allocations, and, thus, a calculation of an optimal allocation schedule by necessity has to be numerical. However, even in the situation of more than three firms it is, however, not clear how to tackle the situation even numerically.[12] Here, we will try to extend the intuition behind the analysis of Moldovanu and Sela [20] to the case of more than two potentially non-zero allocations with convex costs, and employ it for numerical

[12] We also would like to caution against doing an ad-hoc numerical analysis for an arbitrary sample of firms' cost parameters. The results of such analysis will depend on the actual sample of cost parameters, and thus may lead to misleading results. Furthermore, if one assumes that the sample of the cost parameters is known to the participants, the contest game would change from an incomplete information game to a complete information game. For work on contests in a complete information setup, see Barut and Kovenock [1].

estimations.

Suppose that the regulator chose the schedule (vector) of permit allocations with $s_1 \geq s_2 \geq \cdots \geq s_{j-1} \geq s_j \geq \cdots \geq s_n \geq 0$, such that $s_1 + s_2 + \cdots + s_{j-1} + s_j + \cdots + s_n = E$. If all firms follow a symmetric strictly monotonic equilibrium strategy, then, according to Proposition (2), firm i will choose the following level of external action given by equation (5). Let us write $z_i = v^{-1}(y(\alpha_i))$, where $y(\alpha_i)$ is the argument of the inverse cost function $v(\cdot)$ as given in equation (5). In other words, let

$$y(\alpha_i) = p \int_{\alpha_i}^{b} -\frac{B'(\cdot)G'(t)}{t} dt \tag{9}$$

Since $v(\cdot)$ is a strictly increasing function, an increase in $y(\alpha_i)$ would result in an increase in z_i. Thus, if for a particular α_i, a certain change in the allocation schedule results in an increase in $y(\alpha_i)$, that would also imply an increase in firm i's choice of external action z_i.

To understand how firm i's equilibrium choice of z_i depends on the allocation schedule, let us first derive $B'(\cdot)$, which is the marginal return to firm i's expected permit allocation $B(G(\alpha_i))$ from an increase in firm's rank $G(\alpha_i)$. Using equation (3), after some manipulations, one arrives at the following expression:

$$B'(G(\alpha_i)) = -(n-1) \sum_{j=2}^{n} \frac{(n-2)!}{(n-j)!(j-2)!} (s_{j-1} - s_j) (1 - G(\alpha_i))^{n-j} (G(\alpha_i))^{j-2} \tag{10}$$

In other words, the marginal return to firm i's expected permit allocation $B(G(\alpha_i))$ from an increase in firm's rank $G(\alpha_i)$ depends on the combination of incremental changes in permits a firm i could obtain by moving from the $(j-1)$th-ranked allocation s_{j-1} to one-rank higher allocation s_j, for all j allocations.

Combining the above expression (10) with the expression (9) for $y(\alpha_i)$, we get

$$y(\alpha_i) = p(n-1)\int_{\alpha_i}^{b}\sum_{j=2}^{n}\frac{(n-2)!}{(n-j)!(j-2)!}(s_{j-1}-s_j)\,(1-G(t))^{n-j}\,(G(t))^{j-2}\,G'(t)\frac{dt}{t} \quad (11)$$

As equation (11) indicates, only the "allocation distance" between neighbouring-ranked permit allocations $s_{j-1}-s_j$ is important for firms' incentives. That is, what is important is how much more permits a j-th ranked firm could have obtained from moving one rank up to rank $j-1$, rather than the absolute levels of permit allocations s_j and s_{j-1}. One can now immediately observe that Corollary 3 holds, as if all allocations are the same, i.e. $s_1 = s_2 = \cdots = s_{j-1} = s_j = \cdots = s_n$ (in other words, all "allocation distances" are zero), each firm would choose zero external action (since $v(0) = 0$).

Note that, by construction, the "top" allocation s_1 is non-zero, while all permit allocations are weakly ranked, i.e. for any rank $2 \geq j \geq n-1$, we have that $s_{j-1} \geq s_j \geq s_{j+1}$. Thus, one of the important questions to be addressed is whether an optimal allocation schedule involves consequently ranked allocations which are equal to each other. As it turns out, this is a difficult task. To see this, let us look at what happens to $y(\alpha_i)$ when an allocation s_j increases. Inspecting the expression (11), observe that, for $2 \geq j \geq n-1$, any unit increase in an allocation s_j has two effects. First, as s_j increases, this decreases the "upward distance" $s_{j-1} - s_j$, and thus has a negative "upward distance" effect. Second, as s_j increases, the "downward distance" $s_j - s_{j+1}$ increases, so that the "downward distance" effect is positive. Thus, we can write the expression (11) as:

$$
\begin{aligned}
y(\alpha_i) &= p(n-1)\int_{\alpha_i}^{b}\big[s_1(1-G(t))^{n-2} - s_n G(t)^{n-2} + \\
&+ \sum_{j=2}^{n-1} s_j \left[-\frac{(n-2)!}{(n-j)!(j-2)!}(1-G(t))^{n-j}(G(t))^{j-2} + \right. \\
&+ \left.\left. \frac{(n-2)!}{(n-j-1)!(j-1)!}(1-G(t))^{n-j-1}(G(t))^{j-1}\right]\right] G'(t)\frac{dt}{t} \qquad (12)
\end{aligned}
$$

Here, the first term in the sum is the negative "upward distance" effect, and the second term is the positive "downward distance" effect. Thus, one can write the total marginal effect of a change in s_1 (which consists only of the positive "downward distance" effect) as

$$
\frac{\partial y(\alpha_i)}{\partial s_1} = p(n-1)\int_{\alpha_i}^{b}\frac{(1-G(t))^{n-2}G'(t)}{t}dt > 0 \qquad (13)
$$

and the total marginal effect of a change in s_n (which consists only of the negative "upward distance" effect) as

$$
\frac{\partial y(\alpha_i)}{\partial s_n} = -p(n-1)\int_{\alpha_i}^{b}\frac{G(t)^{n-2}G'(t)}{t}dt < 0 \qquad (14)
$$

Finally, the total marginal effect of a change in $s_j, 2 \geq j \geq n-1$ as

$$
\begin{aligned}
\frac{\partial y(\alpha_i)}{\partial s_j} &= p(n-1)\int_{\alpha_i}^{b}\left[-\frac{(n-2)!}{(n-j)!(j-2)!}(1-G(t))^{n-j}(G(t))^{j-2} + \right. \\
&+ \left. \frac{(n-2)!}{(n-j-1)!(j-1)!}(1-G(t))^{n-j-1}(G(t))^{j-1}\right] G'(t)\frac{dt}{t} \qquad (15)
\end{aligned}
$$

The problem could have been very easy to solve if at least one of the following three strong conditions hold.

1. If the total response of each firm $\alpha_i \in [a, b]$ to an increase in some s_j is non-

positive (i.e. $\frac{\partial y(\alpha_i)}{\partial s_j} \leq 0$), this would be sufficient to require that a given allocation s_j has to be equal to its lower bound - which is the one-rank lower allocation s_{j+1} or zero.

2. Even if the marginal change $\frac{\partial y(\alpha_i)}{\partial s_j}$ is strictly positive for all α_i (as it is the case for s_1), this is not a sufficient condition for an allocation s_j to be distinct from the one-rank below allocation s_{j+1}. This is because, it is possible that the marginal effect of one-rank lower allocation s_{j+1} could potentially be even larger for all firms, in which case it would be optimal to increase s_{j+1}. However, as the upper bound for s_{j+1} is the one-rank higher allocation s_j, it could be optimal for the allocation s_j to be set equal to its lower bound s_{j+1}. That is, $s_j = s_{j+1}$. To rule out this situation, one needs, in addition, to verify that the marginal effect is bigger for s_j than for one-rank lower allocation s_{j+1}. Thus, a strong sufficient condition for $s_j > s_{j+1}$ to be optimal, requires that $\frac{\partial y(\alpha_i)}{\partial s_j} > 0$ and, in addition, $\frac{\partial y(\alpha_i)}{\partial s_j} > \frac{\partial y(\alpha_i)}{\partial s_{j+1}}$ for all $\alpha_i \in [a, b]$.

3. Finally, if the total marginal effect of an increase in s_j is smaller than the marginal effect of one-rank lower allocation s_{j+1} for all α_i, this is sufficient to require that s_j to be equal to its lower bound, which is the one-rank below allocation s_{j+1}, or zero. That is, for $s_j = s_{j+1}$ to be optimal, it is sufficient that $\frac{\partial y(\alpha_i)}{\partial s_j} < \frac{\partial y(\alpha_i)}{\partial s_{j+1}}$ for all $\alpha_i \in [a, b]$.

Clearly, one of these strong sufficient conditions always holds for the bottom-ranked allocation s_n. That is, as $\frac{\partial y(\alpha_i)}{\partial s_n} < 0$ for each firm $\alpha_i \in [a, b]$, this is sufficient to require that the bottom-ranked allocation s_n is equal to zero. However, we could not find a single instance of either of these three strong conditions to take place for the higher-ranked allocations. Given the difficulties of the analytical approach, we now turn to our estimation with specific cost parameters.

5 Estimations for Uniformly Distributed Cost Parameters

We will now turn to the estimation of an optimal allocation schedule using the example of uniformly distributed cost parameters α_i on $[a, b], 0 \leq a < b < \infty$, so that $G(\alpha_i) = \frac{\alpha_i - a}{b-a}$. Here, the expression (12) becomes:[13]

$$\begin{aligned} y(\alpha_i) &= \frac{p(n-1)}{(b-a)^{n-1}} \int_{\alpha_i}^{b} \left[E(b-t)^{n-2} + \sum_{j=2}^{n-1} s_j \left[-(b-t)^{n-2} - \right.\right. \\ &- \frac{(n-2)!}{(n-j)!(j-2)!}(b-t)^{n-j}(t-a)^{j-2} + \\ &+ \left.\left. \frac{(n-2)!}{(n-j-1)!(j-1)!}(b-t)^{n-j-1}(t-a)^{j-1} \right] \right] \frac{dt}{t} \end{aligned} \tag{16}$$

To get some understanding of the incentives that arise from each allocation s_j, we will now turn to more specific examples.

5.1 A Case of Three Firms

The case of three firms is, perhaps, the least complicated, and even yields to limited analytical exploration. For $n = 3$, the expression (16) becomes:

$$\begin{aligned} y(\alpha_i) &= \frac{2p}{(b-a)^2} s_1 \left[b(\ln b - \ln \alpha_i) - (b - \alpha_i) \right] + \\ &+ \frac{2p}{(b-a)^2} s_2 \left[2(b-\alpha_i) - (a+b)(\ln b - \ln \alpha_i) \right] \end{aligned}$$

Note that, by the mean value theorem, there exists some $\omega_i \in [\alpha_i, b]$ such that

[13] In dealing with this expression, the following formula was useful:

$$\int \frac{(c+dt)^m}{t} dt = c^m \ln t + md \sum_{k=0}^{m-1} \frac{(m-1)!}{(m-1-k)!k!} \frac{c^{m-1-k} d^k t^{k+1}}{(k+1)^2}.$$

$\ln b - \ln \alpha_i = \int_{\alpha_i}^{b} \frac{1}{t} dt = (b - \alpha_i)\frac{1}{\omega_i}$ (note that ω_i is firm-specific, i.e. it depends on the cost parameter α_i). Thus, it is easy to verify that the marginal effect of the top-ranked allocation s_1 is strictly positive:

$$\frac{\partial y(\alpha_i)}{\partial s_1} = \frac{2p}{(b-a)^2}(b - \alpha_i)\left[\frac{b}{\omega_i} - 1\right] > 0$$

Since $a < \alpha_i < \omega_i < b$, the effect of the top-ranked allocation is positive for all $\alpha_i \in [a, b]$. As we will see, this, however, is not sufficient for a distinct top allocation s_1. Observe that the effect of the second-ranked allocation s_2 is

$$\frac{\partial y(\alpha_i)}{\partial s_2} = \frac{2p}{(b-a)^2}(b - \alpha_i)\left[2 - \frac{a+b}{\omega_i}\right]$$

Thus, for firms with $\omega_i > \frac{a+b}{2}$, the effect of second-ranked allocation $\frac{\partial y(\alpha_i)}{\partial s_2}$ is positive. A conservative estimate suggests that the distinct second-ranked allocation s_2 would have a total positive effect at least on firms with $\alpha_i \geq \frac{a+b}{2}$, or those in the upper half of the cost distribution (because $\omega_i > \alpha_i \geq \frac{a+b}{2}$).

Furthermore, we also need to check the difference between the two marginal effects:

$$\frac{\partial y(\alpha_i)}{\partial s_1} - \frac{\partial y(\alpha_i)}{\partial s_2} = \frac{2p}{(b-a)^2}(b - \alpha_i)\left[\frac{a+2b}{\omega_i} - 3\right]$$

so that the marginal effect of the top-ranked allocation s_1 is smaller for firms with $\alpha_i > \frac{a+2b}{3}$ - i.e. for firms in the upper third of the cost distribution.

This case is instructive, as it shows some important features of contests. Here, in order to "lift off" the external action by the relatively high-cost firms, the regulator may need to award a positive middle allocation. However, on the margin, the middle allocation is less effective for the relatively low-cost firms. Thus, if the realized sample of firms' costs parameters consists only of relatively high cost parameters, the *ex-post* optimal schedule involves equal top and middle allocations. If, instead, the realized sample of firms' costs parameters consists only of relatively low cost parameters, the *ex-post* optimal schedule involves only a single top allocation. However, since here the cost parameters are firms' private information, one needs to look into the *ex-ante* regulator's problem, i.e. to look into the expected total external actions.

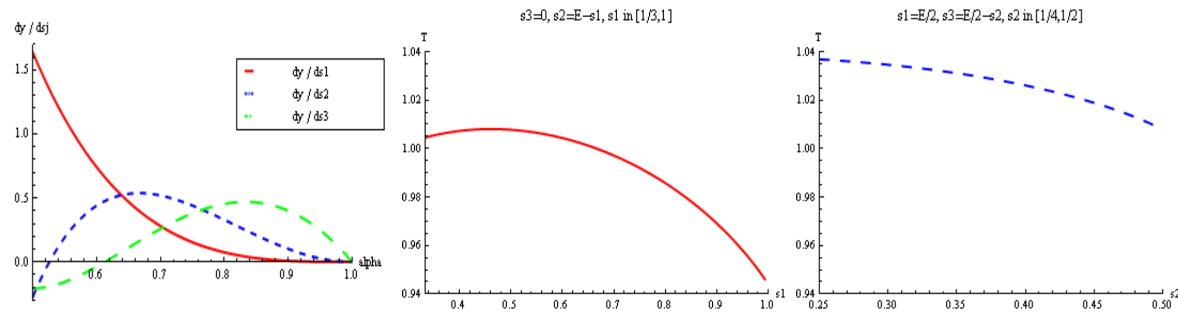
But because of the convexity of the cost function $v(\cdot)$ (see expression (5)), a firm i's external action z_i tends to exhibit diminishing returns, so that the marginal impact of relatively high-cost firms on the aggregate external action T tends to be substantial. Indeed, as Moldovanu and Sela [20] were able to show in a similar setup that an optimal allocation schedule involves two distinct non-zero allocations, so that $s_1 > s_2 > s_3 = 0$.

5.2 A Case of Four Firms

It turns out that for the higher number of firms, the situation is even more complicated. Rather than approaching the problem analytically, we turn now to numerical estimation. For $n = 4$, the expression (16) becomes:

$$\begin{aligned} y(\alpha_i) &= \frac{3p}{(b-a)^3}\left[s_1\left[b^2(\ln b - \ln \alpha_i) - 2b(b-\alpha_i) + \frac{b^2-\alpha_i^2}{2}\right] + \right. \\ &+ s_2\left[\frac{3}{2}(b-\alpha_i)^2 - (2a+b)\,b(\ln b - \ln \alpha_i) + (2a+b)\,(b-\alpha_i)\right] + \\ &+ \left. s_3\left[a(a+2b)(\ln b - \ln \alpha_i) - 2(2a+b)(b-\alpha_i) + \frac{3}{2}\left(b^2-\alpha_i^2\right)\right]\right] \end{aligned}$$

It is easy to confirm that neither of the three strong sufficient conditions hold here, so that we need to turn to numerical estimations (we used *Mathematica*). We consider a simple case of a uniform distribution on $\left[\frac{1}{2}, 1\right]$. Recall that, as usual, the bottom allocation s_n is set equal to zero. As Figure 1a shows, the marginal effect of the top allocation s_1 is the highest for the lowest-cost firms. On the other hand, the marginal effect of the second-ranked allocation s_2 peaks out for the mid-range costs, and the effect for the third-ranked allocation s_3 peaks out for the relatively higher-cost firms, and, moreover, the heights of the peaks are similar. This, together with the convexity of costs, suggests a possibility that it might be optimal to set the third-ranked allocation s_3 equal to the second-ranked allocation s_2.

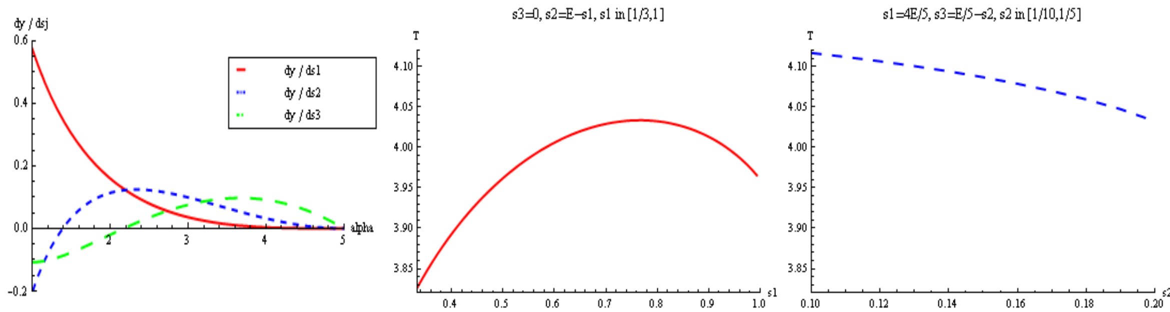


a) b) c)

Figure 1: Case of $n = 4$, $\alpha_i \sim U\left[\frac{1}{2}, 1\right]$: a) the total marginal effects of allocations s_1, s_2, and s_3 (the solid red curve, the narrowly dashed blue curve, and the widely dashed green curve, respectively), as a function of the cost parameter α_i; b) the aggregate external action T as a function of the top-ranked allocation s_1 for $s_3 = 0$, $s_2 = E - s_1$; c) the aggregate external action T as a function of the second-ranked allocation s_2 for $s_1 = \frac{E}{2}$, $s_3 = \frac{E}{2} - s_2$ (for $E = 1$, $p = 1$, $v(\cdot) = \sqrt{\cdot}$).

Indeed, let us first check what happens if we set the third-ranked allocation s_3 equal to zero. Setting $s_3 = 0$ allows us to express the middle allocation as $s_2 = E - s_1$, with $s_1 \in \left[\frac{E}{3}, E\right]$. In this case, as Figure 1b suggests, the expected aggregate external action (given by expression (7)) has a maximum around $s_1 \approx \frac{E}{2}$. In other words, here it is not optimal to allocate the entire "pie" of permit allocations only to a single top-ranked firm, i.e. we need that $s_1 < E$. Furthermore, if we now set the top-ranked allocation equal to a half of the pie (so that $s_3 = \frac{E}{2} - s_2$), the Figure 1c confirms that it would be optimal to set the third-ranked allocation to be equal to the second-ranked allocation, i.e. $s_2 = s_3$. Thus, the optimal allocation schedule for four firms and costs distributed uniformly on $\left[\frac{1}{2}, 1\right]$, will be approximately equal to $s_1 \approx \frac{E}{2}, s_2 \approx s_3 \approx \frac{E}{4}, s_4 = 0$.

Performing similar manipulations for costs distributed uniformly on $[1, 5]$, we find that the optimal allocation schedule for four firms will be approximately equal to $s_1 \approx \frac{4E}{5}, s_2 \approx s_3 \approx \frac{E}{10}, s_4 = 0$ (see Figure 2).



a) b) c)

Figure 2: Case of $n = 4$, $\alpha_i \sim U[1, 5]$: a) the total marginal effects of allocations s_1, s_2, and s_3 (the solid red curve, the narrowly dashed blue curve, and the widely dashed green curve, respectively), as a function of the cost parameter α_i; b) the aggregate external action T as a function of the top-ranked allocation s_1 for $s_3 = 0$, $s_2 = E - s_1$; c) the aggregate external action T as a function of the second-ranked allocation s_2 for $s_1 = \frac{4E}{5}$, $s_3 = \frac{E}{5} - s_2$ (for $E = 1$, $p = 1$, $v(\cdot) = \sqrt{\cdot}$).

Similarly, we found that for other uniform distributions, the pattern is similar, i.e. the highest expected aggregate external action T happens when there is a relatively large top allocation, followed by two equal allocations, with the bottom allocation being zero. For example, for $\alpha_i \sim U\left[\frac{1}{3}, 1\right]$, we have that $s_1 \approx \frac{3E}{5}, s_2 \approx s_3 \approx \frac{E}{5}, s_4 = 0$; for $U[1, 2]$ - $s_1 \approx \frac{E}{2}, s_2 \approx s_3 \approx \frac{E}{4}, s_4 = 0$; $U[1, 10]$ - $s_1 \approx \frac{9E}{10}, s_2 \approx s_3 \approx \frac{E}{20}, s_4 = 0$. Furthermore, we have not come across an example where three equal allocations were optimal.

While our numerical findings may not be robust with respect to the shape of the distribution and the number of firms, we are however able to show, similarly to Moldovanu and Sela [20], that the optimal allocation schedule in the presence of convex costs need not be very discriminatory, possibly exhibiting equal consecutively-ranked allocations.

However, our work also suggests a possibility that the optimal allocation schedules will tend to involve the top allocation s_1 to be larger than the lower-ranked allocations.

6 Conclusion

The objective of this paper was to outline a new type of permit initial allocation mechanism. In our model, the initial allocation of tradeable pollution permits is done via a Permit Allocation Contest (PAC). A PAC is a rank-order contest in which the firms are allocated permits according to the ordinal rank of the size of their external action (which is an activity or characteristic of the participating firms that is independent of emissions choices).

In our model, the regulator was assumed to have two policy objectives. First, by allocating permits based on the external action (rather than based on emissions), the regulator aims to minimise the aggregate cost of reducing emissions. Second, by choosing a suitable permit allocation schedule (i.e. the number of permits that firms can obtain by being ranked first, second, and so on), the regulator aims to fulfil a predetermined public policy objective, requiring maximisation of the aggregate actions, which are independent of emissions (i.e. "external" to the permit market) – e.g. improvements in health and safety policies, corporate and social responsibility, etc.

Since, by construction, the permit allocation schedule is independent of emissions, the allocation mechanism results in cost-effective permit market, in contrast to the outcome under updated grandfathering. We consider a symmetric strictly monotonic strategy equilibrium of a incomplete information game of PAC, where the permit allocation schedule as well as the cost distribution are publicly known, but where each firm's cost parameter of external actions is the firm's private information.

To obtain the public policy objective, the regulator must choose an optimal permit allocation schedule. We find that an egalitarian allocation schedule (whereby firms obtain identical permit allocations regardless of their external action) cannot achieve the public policy objective as an egalitarian allocation schedule leads to zero aggregate external actions. Instead, for a public policy objective to be achieved, the schedule must be discriminatory - at least for the lower-ranked permit allocations. Our analytical and numerical analysis is in accordance with earlier theoretical results. It shows that for the maximum aggregate external actions to be obtained, the lowest-ranked permit allocation has to be zero, and, when costs of external actions are convex, the higher-ranked permit allocations have to be less discriminatory. Although the regulator's optimal permit allocation schedule is difficult to solve analytically, a PAC is still an implementable mechanism. This paper provides guidance for policymakers on how to implement a PAC and select an optimal permit allocation schedule for a public policy objective. In particular, we have shown that the regulator's optimal permit allocation schedule will depend extensively on the structure and distribution of firms' costs and must be taken into consideration when implementing a PAC.

The PAC, at its simplest, has attempted to reach the middle ground between grandfathering and auctioning. On one hand, PAC creates similar incentives to an auction and could, in theory, efficiently allocate permits. On the other hand, it has features of grandfathering as it does not require politically unpopular monetary bids. While PAC does require other forms of expenditure, a suitably designed PAC may require expenditure on socially-beneficial activities which firms are already pursuing even in the absence of PAC, or which firms may find to attractive to pursue. Thus, a suitable designed PAC may be both politically feasible and efficient. In addition, PAC is a flexible mechanism as it allows ranking of firms using a wide variety of external actions, and thus could be adapted to a variety of industrial and regional circumstances.

One possible practical difficulty of implementation of PAC lies in the identification and implementation of a suitable external action. This is because in order for the PAC to achieve efficiency, the external action must be independent of emissions, and in addition it has to be politically agreeable to firms, the regulator and society. Given the current political climate, it might be difficult to identify an external action that satisfies all these requirements. However, we hope that further political process and public awareness will help to overcome these identification and implementation problems.

Acknowledgements

The authors would like to thank David Bell, Ed Hopkins and Matti Liski for useful comments and suggestions. The usual disclaimer applies.

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