

The Calm Before the Storm? - Anticipating the Arrival of General Purpose Technologies*

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This paper presents a Schumpeterian quality-ladder model incorporating the impact of new General Purpose Technologies (GPTs). GPTs are breakthrough technologies with a wide range of applications, opening up new innovational complementarities. In contrast to most existing models which focus on the time *after* the arrival of a new GPT, the model developed in this paper focuses on the events *before* the arrival if R&D firms know the point of time and the technological impact of this drastic innovation. In this framework we can show that the economy goes through three main phases: First, the economy is in its old steady state. Second, there are transitional dynamics. These transitional dynamics are characterized by oscillating cycles: In the time immediately before the arrival of a new GPT there is an increase in R&D activities and growth going even beyond the old steady state levels. This is followed by a large slump in R&D activities using the old GPT. In a third phase, as the new GPT becomes active, the economy is in a new steady state with higher growth rates.

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1 Introduction

Ever since the pioneering paper of Romer (1990), technological change as the driving force of economic growth has been in the focus of economic growth theory. Early endogenous growth models assumed that innovation follows a smooth path in the course of time. This implies that economic growth is driven by a constant stream of small innovations. However, a look back in economic history shows that, in any given era, economic development was driven by a small number of breakthrough technologies: The industrial revolution was facilitated by major improvements on the design of steam engines by James Watt. Later, electricity not only shaped the way how and where goods are produced, but also deeply altered the lifestyle of consumers. Finally, today's economic landscape has been transformed by the introduction of the computer and modern communication technologies such as the internet and e-mail.

Bresnahan and Trajtenberg (1995) pioneered the concept of "General Purpose Technologies" (GPTs), thus formalizing the idea that economic growth is driven by such breakthrough technologies. Lipsey, Bekar and Carlaw (1998) present the following definition: "A GPT is a technology that initially has much scope for improvement and eventually comes to be widely used, to have many uses, and to have many Hicksian and technological complementarities".¹

We contribute to the GPT literature by analyzing the effect of a new GPT on R&D-activity and growth in a quality-ladder model based on the model on Schumpeterian growth by Barro and Sala-i-Martin (2004). Other examples where GPTs are considered within a quality-ladder framework are as follows: Petsas (2003) incorporates the idea of GPTs in the quality-ladder model of Dinopoulos and Segerstrom (1999) in order to model the diffusion of GPTs across industries. In an earlier paper, Cheng and Dinopoulos (1996), while not explicitly addressing GPTs, have built a quality-ladder model, where cycles arise due to a sequence of technological breakthroughs and subsequent incremental improvements. Smulders, Bretschger and Egli (2005) present how successive GPT generations within the quality-ladder framework of Grossman and Helpman (1991, chapter 4) can explain the long-term evolution of environmental quality.

In addition to incorporating the idea of GPTs in such a framework, we demonstrate that the mere anticipation of a new GPT induces cyclical behavior in the economy: Initially, the economy using the old GPT is in a steady state with constant growth rates in output and R&D expenditures. As the arrival of a new GPT draws nearer these growth rates start to oscillate around the steady state levels. This is followed by an increase in R&D activities and growth going beyond the old steady state levels. Immediately before the arrival of the new GPT, there is a large slump in R&D activities resulting in lower growth.

¹Both, the volume edited by Helpman (1998), especially the mentioned article by Lipsey, Carlaw and Bekar (1998) and the book by Lipsey, Carlaw and Bekar (2005), offer more background on the general concept of GPTs.

1.1 Anticipating the Arrival of General Purpose Technologies

Both main assumption of our model, the ability of firms to anticipate the arrival of new GPTs, and our result that this can influence R&D activities, can be observed in numerous instances in economic history. Again, the steam engine serves as a prime example:

- As described in Rosenberg and Trajtenberg (2004), as well as in Atack (1979), the United States adopted the steam engine as a source of power decades after it has been introduced in the United Kingdom.
- The invention of the Watt steam-engine was a technological milestone. Nevertheless only the invention of the high-pressure steam engine years later truly made steam-power a GPT (see Crafts, 2004).
- Furthermore, the steam engine was not a GPT from the beginning, but started off as a single purpose technology mainly used for pumping out water from mine shafts.

Likewise, the advent of the computer as a GPT did not come entirely un-predicted. As Lipsey, Carlaw and Bekar (1998) state: “For example, long before its full potential had been exploited (which is still in the future), it became apparent to many observers that electronic computers were on their way to becoming a pervasive GPT.” Furthermore Eriksson and Lindh (2000) state that “... the idea of computers was thoroughly explored by researchers, like Alan Turing, already in the 1930s, although practical designs were far in the future.” The information and communication technology revolution started in the United States before making its way to the rest of the world. Finally, the rise of the computer to a GPT was a process taking place over a considerable amount of time, as discussed by David (1990). Current examples for possible future GPTs are nanotechnology and nuclear fusion: While both technologies have been shown to work on a small scale with either limited applications or at relatively large costs, there exists a huge potential of future uses if a number of known technical problems can be solved.

Altogether the possibility of knowing about a future GPT before its actual arrival can stem from several sources: For instance a new GPT can be invented and widely used in a specific country before spreading across the world. Or the concept of a new GPT can be conceived in a theoretical context, but can only be used productively once technological advances allow the realization of these ideas. Moreover, a number of GPTs started off as single purpose technologies before spreading throughout the economy.

Not only is it reasonable to believe that the arrival of a new GPT can be anticipated, it has also been observed that such expectations can have an influence on the extent of R&D activities: In the course of the 19th century, the replacement of water power as the main source of inanimate power by the steam engine seemed to be virtually inevitable. Nevertheless, some of the greatest leaps in efficiency of water power have been achieved during this time: Both the invention of the breastwheel and of the water turbine allowed a significantly

larger amount of horsepower to be extracted from a given flow of water (see Lipsey, Carlaw and Bekar, 2005, chapter 6). Further examples for this type of development, as presented by Lipsey, Carlaw and Bekar (2005, chapter 6), are as follows: Initially, rail was not seen as a serious competition to canals for domestic cargo transportation in the United Kingdom, but rather as being complementary for short-distance transportation. As it became clear that railways would also be able to effectively compete in long-haul cargo transportation, the pressure to improve the canal system rose accordingly. As a final example, the impending spread of steam as a source of power for marine uses, sparked an increase in efficiency of sail (see for example Graham, 1956).

1.2 Previous Literature

Previous models of GPTs also focus on the effect of such breakthrough technologies on economic growth. The wave of models that followed the introduction of this concept by Bresnahan and Trajtenberg (1995) quickly showed that from a theoretical perspective GPTs are a double-edged sword: On one hand the idea that new GPTs provide a boost to long-term growth is generally agreed upon. In the model of Carlaw and Lipsey (2006), for example the arrival of new GPTs helps to sustain long-term growth, offsetting the loss of productivity due to the ongoing depreciation of applied knowledge. While this positive long-term effect ultimately prevails in all models, the introduction of a new GPT is usually considered to have a negative *short-term* impact on the economy.²

There exists a variety of explanations for such an initial slump in productivity and output following the introduction of a new GPT: Helpman and Trajtenberg (1998a) postulate that after the arrival of a new GPT, innovators first have to build up a critical mass of complementary components to this GPT (e.g. software for computers), before it can be usefully applied to produce final output. This causes an initial slump in growth, before it picks up a faster pace as soon as the new GPT becomes active in final output production. In their follow-up paper Helpman and Trajtenberg (1998b) model more precisely how a new GPT diffuses throughout the whole economy after its arrival. Building on the previous two models, Aghion and Howitt (1998a) present a model (based on Aghion and Howitt, 1998b) where the component-building phase is preceded by a stage where so-called “templates” for these components need to be discovered.

Greenwood and Yorukoglu (1997) offer another explanation for this phenomenon: They argue that new GPTs require the acquisition of specific skills before they can be put to a productive use. The related learning processes can take a considerable amount of time, as new GPTs are typically revolutionary and complex new technologies. This results once more in a productivity slowdown in the initial phase after the introduction of a new GPT. Similarly, Nahuis (2004) presents a model, where a new GPT sparks an initial phase of experimentation. He explains that when R&D workers are faced by such a rev-

²Our paper, as well as the subsequently presented papers, concentrate on theoretic modeling of the economic impact of GPTs. Nevertheless the notion that technological changes have contractionary effects in the short-run but positive long-term effects can also be found in empirical works, as for example in Basu, Fernald and Kimball (2006).

olutionary technology, they first have to explore the opportunities offered by it. Only afterwards the possibilities offered by the GPT can be usefully applied in a firm. Atkeson and Kehoe (2007) also postulate that the transition following a technological revolution is governed by “substantial and protracted” learning processes, thus delaying the positive impact on productivity of such a new technology. The fact that activities with a higher degree of complexity take a comparatively longer time to learn has also been demonstrated by Jovanovic and Nyarko (1995).

In business cycle theory, the effects of (changes in) expectations regarding the future behavior of macroeconomic fundamentals, such as total factor productivity, have been thoroughly studied (see for example Beaudry & Portier, 2007 or Jaimovich & Rebelo, 2006). Meanwhile, the literature on GPTs is mainly concerned with effects taking place *after* the arrival of a new GPT. Nevertheless the idea that agents can have an advance knowledge about the arrival of a new GPT is not entirely new: Eriksson and Lindh (2000) present a model, building on Helpman and Trajtenberg (1998a), where the time of arrival of a new GPT is endogenized, inasmuch as the accumulation of social knowledge leads to invention of new GPTs.

By focussing on the time *before* the arrival of a new GPT, our model has a different focus than the models previously described. Despite this difference, it is not meant as a rival explanation for the dynamics arising due to new GPTs. It rather presents a channel that applies in addition to the ones described in those other models.

The remainder of this paper will be organized as follows: In the following Section we will introduce our model of GPTs in a quality-ladder framework. As a benchmark case, the analysis of the steady state behavior will be explored in Section 3, especially comparing steady states of two successive GPT generations. In Section 4 the transitional dynamics of our model are shown, before we will offer an outlook and some conclusions in Section 5.

2 General Purpose Technologies in a Quality-Ladder Model

We present a quality-ladder model incorporating the arrival of new GPTs. In a quality-ladder model final goods are typically assembled of a number of intermediate goods, which in turn are produced in a fixed number of distinct varieties. The producers of these intermediate goods can invest in R&D to improve the quality of a specific intermediate good, thus moving it up the quality-ladder. We model the impact of new GPTs in terms of an increase in research efficiency in this sector. This approach is common in GPT-modeling. In the words of Jacobs and Nahuis (2002): “A GPT [...] affects the marginal productivity of research as it opens new opportunities for knowledge-creating activities throughout the economy.” Accordingly they model the arrival of a new GPT (in their case the computer revolution) as an increase in research productivity. Bresnahan and Trajtenberg (1995) also stress the role of GPTs as “enabling technologies”, which open up new opportunities instead of offering complete, final solutions.

In their view, the productivity of R&D in downstream sectors increases through “innovational complementarities” arising from innovations in GPTs.

Our model of GPT-driven growth is based on the Schumpeterian model of quality ladders as described in chapter 7 of Barro and Sala-i-Martin (2004). The economy consists of three sectors: First, there is consumption. Second, there is an R&D sector where firms on one side produce a fixed variety of intermediate goods: These firms, called R&D firms in the remainder of this paper, may also perform in-house R&D in order to improve the quality of those intermediate goods. Third, the final goods sector demands and assembles these intermediate goods.

The crucial features of our GPT model are as follows: First, we introduce the concept of GPTs in this quality-ladder framework, by modeling the effect of a new GPT taking the form of an improvement in research efficiency as described in the first part of this Section. Second, we assume that the agents know about the arrival of the GPT. This leads to transitional dynamics before the arrival of a new GPT, during which the economy exhibits non-stationary growth rates.

If an R&D firm is successful in improving the quality of an intermediate good it can sell this good to the final goods producer at the monopoly price, since it holds the exclusive right to produce this intermediate good of the respective quality level. Final goods producers only use the leading-edge quality of each variety of goods.³ Therefore the incumbent monopolist in a sector earns this monopoly profit in each period until another R&D firm succeeds in developing an even higher quality of this intermediate good. The probability of having a research success is determined by various factors: On one side the amount of R&D expenditures are endogenously chosen by the R&D firms. The efficiency of these expenditures in attaining a research success is determined by exogenous factors (such as the sector-specific difficulty of research and the current GPT level). The arrival of a new GPT increases, *ceteris paribus*, the probability of a research success.

2.1 Consumers and the Final Goods Sector

As in all quality-ladder models, quality improvements are the driving force of growth in our model, which of course take place in the R&D-sector. Nevertheless, we first present the final goods sector and the behavior of consumers.

The representative consumer maximizes the overall utility U derived from consumption c as given by:

$$U = \int_0^{\infty} u(c(t))e^{-\rho t} dt \quad (1)$$

where ρ stands for the time preference rate. Consumers earn the interest rate r on assets and a wage w per unit of labor. Consumers spend their income on

³The assumption that of each variety of intermediate goods only the highest quality grade is produced and used is also made in Barro and Sala-i-Martin, 2004. Additionally they show that the general nature of results is unchanged under an equilibrium with limit pricing.

consumption and savings, therefore the accumulation of assets is given by:

$$\frac{d(\text{assets})}{dt} = r(\text{assets}) + wL - C \quad (2)$$

Assuming a standard constant intertemporal elasticity of substitution (CIES) utility function and through a simple maximization exercise the following Euler equation for consumption growth can be derived:

$$\frac{\dot{C}}{C} = (1/\theta)(r - \rho) \quad (3)$$

with C as the aggregate consumption and θ as the elasticity of marginal utility, which is equivalent to the reciprocal of the elasticity of intertemporal substitution.

Apart from being consumed, the aggregate output Y is used in the production of aggregated intermediate goods X and total R&D investments Z . This is reflected by the following resource constraint for the economy:

$$Y = C + X + Z \quad (4)$$

The production function for a firm i in the final goods sector is given by:

$$Y_i = AL_i^{1-\alpha} \sum_{j=1}^N (\tilde{X}_{ij})^\alpha \quad (5)$$

where $0 < \alpha < 1$.

L_i is the labor input and \tilde{X}_{ij} is the quality-adjusted amount of intermediate good j used in the production by firm i . N is the constant number of varieties of intermediate goods while A is an exogenous technology parameter.

The quality adjusted amount of an intermediate good \tilde{X}_{ij} is determined by both the physical quantity of the respective intermediate X_{ij} and its current quality-level q^{κ_j} :

$$\tilde{X}_{ij} = q^{\kappa_j} X_{ij} \quad (6)$$

where $q > 1$ is a constant and a new invention raises κ_j by one. In accordance with the quality-ladder concept, a new invention takes the form of an improvement in the quality of an existing intermediate by a factor of q .

Final good firms maximize their profits, given that only goods of the highest available quality level in each sector are demanded. From this maximization, the aggregate demand function for good j can be derived:

$$X_j = L(A\alpha q^{\alpha\kappa_j} / P_j)^{1/(1-\alpha)} \quad (7)$$

This expression represents the demand that firms in the R&D sector face. P_j is the price of the intermediate good j .

2.2 R&D sector

R&D firms both produce and sell intermediate goods. Additionally, they can make R&D expenditures aiming at the invention of a higher quality good in a certain sector. In order to maximize their profits they have to choose the optimal amount of R&D expenditures Z . For this maximization they need to consider two phases: In a first phase they can perform research in order to attain a monopoly in the respective sector. The main trade-off here is that an increase in the probability of having a research success comes at the cost of an increase in research expenditures. In a second phase, after having made a successful invention, they start to hold the monopoly on the highest-quality good in this specific sector and decide on pricing and the amount produced: Accordingly they can derive a monopoly profit in each period until they are displaced by a competitor having the next research success in this sector. We assume free entry in the R&D sector and risk-neutral R&D firms.⁴ Therefore the R&D firms equalize their R&D expenditures with the expected pay-off they receive from these investments, which is in turn subject to discounting and the future probability of being driven out of the market by a competitor.

The choice variable of R&D firms is the amount of inputs they use for research in each sector $Z(\kappa_j)$. This input influences the probability of having a research success in a certain sector, i.e. $p(\kappa_j)$ in the following fashion:

$$p(\kappa_j) = Z(\kappa_j)\phi(\kappa_j)B_m \quad (8)$$

where $\phi(\kappa_j)$ captures the difficulty of research in respect to the quality-ladder position of the sector. The current GPT of generation m , B_m , enters positively. In accordance to the idea that new GPTs lead to an enhancement in efficiency of R&D the arrival of a new GPT increases the value of B_m to $B_{m+1} > B_m$.

The monopoly profit flow a R&D firm that has had a research success receives from selling the corresponding intermediate good is characterized by the following equation:

$$\pi(\kappa_j) = (P_j - 1)X_j \quad \text{with} \quad P_j = \frac{1}{\alpha} \quad (9)$$

where the marginal cost of production equals 1 and X_j is given by equation (7). P_j is the usual optimal monopoly price, where the monopolist charges the markup $1/\alpha$ on the marginal costs. The following expression describes the monopoly profit flow of an innovator possessing the leading-edge technology:

$$\pi(\kappa_j) = \bar{\pi}q^{\frac{\kappa_j \alpha}{1-\alpha}} \quad (10)$$

where $\bar{\pi}$ is given by:

$$\bar{\pi} = \left(\frac{1-\alpha}{\alpha} \right) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L \quad (11)$$

⁴Although risks in individual sectors are idiosyncratic, they are not on the aggregate level: Therefore R&D firms can also use a portfolio approach leading to results analogous to risk-neutrality.

$\bar{\pi}$ is the basic profit flow, which is constant over time given that the size of the labor force L is fixed. Due to the higher demand for goods of a higher quality level, the profits of inventors $\pi(\kappa_j)$ increase with the quality level of intermediates κ_j .

In a sector with a given quality level κ_j investments in R&D are attractive if the R&D costs $Z(\kappa_j)$ are at least covered by the resulting return on these expenditures $p(\kappa_j)E[V(\kappa_{j+1})]$. $E[V(\kappa_{j+1})]$ is the expected present value of profit an intermediate goods producer obtains for his good of quality $\kappa_j + 1$. The size of $E[V(\kappa_{j+1})]$ depends on three major factors: The size of the profit flow the monopolist receives in each period $\pi(\kappa_{j+1})$, the probability that it is displaced by a future competitor inventing a good of an even higher quality given by $p(\kappa_{j+1})$ and the discount rate r .

Assuming that there is free entry, the expected return on R&D expenditures at any given time must be equal to the R&D expenditures:

$$Z(\kappa_j) = p(\kappa_j)E[V(\kappa_{j+1})] \quad (12)$$

By inserting $p(\kappa_j)$ from equation (8) and rearranging we can derive the following equation, which determines the optimal amount of research expenditures under free entry:

$$Z(\kappa_j)(\phi(\kappa_j)B_m E[V(\kappa_{j+1})] - 1) = 0 \quad (13)$$

As can be easily seen, the optimal amount of R&D expenditures $Z(\kappa_j)$ crucially depends on the size of $E[V(\kappa_{j+1})]$, which will therefore be more thoroughly explored in the next section.

3 Comparing Steady States

3.1 The General Case

The ultimate goal of our model is to investigate transitional dynamics caused by the anticipation of the arrival of a new GPT. Let us first, however, take a closer look at the effects of the introduction of a new GPT on steady states.

Consider a R&D firm that makes the κ_j th quality improvement at time t_{κ_j} : From then on it receives a flow of monopoly profit until it is displaced by a competitor inventing an even higher quality good in this sector. Therefore the firm that has invented technology κ_j earns the following expected present value of profit:

$$E[V(\kappa_j)] = \int_{t_{\kappa_j}}^{\infty} \left[\int_{t_{\kappa_j}}^{\tau} \pi(\kappa_j) e^{-\int_0^s r_u du} ds \right] g(\tau) d\tau \quad (14)$$

where $g(\tau)$ is the probability density function that the monopoly position ends at time τ due to a research success by a competitor. In the steady state, where $p(\kappa_j)$ and the duration of the monopoly are constant over time, $g(\tau)$ is given by:⁵

$$g(\tau) = p(\kappa_j) e^{-p(\kappa_j)\tau} \quad (15)$$

⁵For details on the derivation, see Barro and Sala-i-Martin, 2004, p. 345f.

Since in the steady state not only $p(\kappa_j)$, but also the interest rate r are constant, the expression for the expected present value of profits as given by equation (14) simplifies to:

$$E[V(\kappa_j)] = \frac{\pi(\kappa_j)}{r + p(\kappa_j)} \quad (16)$$

By rewriting this equation we get the following no-arbitrage equation:

$$r = \frac{\pi(\kappa_j) - p(\kappa_j)E[V(\kappa_j)]}{E[V(\kappa_j)]} \quad (17)$$

The interpretation of this equation is straightforward: The rate of return on R&D must be equal to the interest rate r , representing an alternative investment. This equation does not only consider the profit flow at each point in time, but also the probability $p(\kappa_j)$ of being driven out of the market by a competitor. Accordingly the expected rate of return falls with the level of the probability of research success.

R&D firms consider the free-entry condition (13) and the expected profit given by equation (16) in order to calculate their optimal amount of R&D expenditures, thereby determining their probability of research success. This probability crucially depends on the difficulty of research as given by $\phi(\kappa_j)$:

$$\phi(\kappa_j) = \left(\frac{\epsilon}{\zeta}\right) q^{-(\kappa_j+1)\alpha/(1-\alpha)} \quad (18)$$

This equation captures several effects: First, there is a constant parameter ζ , reflecting the costs of performing R&D. Second the difficulty of R&D rises with the quality of the good the R&D firm wants to improve. Finally, the term ϵ is later used to capture decreasing returns to current R&D, but is for now being held constant: Applying this $\phi(\kappa_j)$ to the free entry condition (13) and the general equation for the probability of having a research success (8) leads to the following expression for the probability of research success:

$$p = \frac{\epsilon\bar{\pi}B_m}{\zeta} - r \quad (19)$$

As only variables that are independent of the quality-level appear in this expression this probability is constant across all sectors. Furthermore the arrival of a new GPT, reflected by an increase of B_m to B_{m+1} leads to an increase in p . This increase is of course due to the fact that, triggered by the increase in the efficiency of R&D, the amount of resources devoted to R&D in sector j also rises so that the no-arbitrage equation (17) is again fulfilled. This can be seen in the following equation:

$$Z(\kappa_j) = \frac{q^{\frac{(\kappa_j+1)\alpha}{1-\alpha}} (\epsilon\bar{\pi}B_m - r\zeta)}{\epsilon B_m} \quad (20)$$

Additionally, the sectors on a higher quality level attract higher R&D expenditures.

By aggregation of the R&D expenditures of individual sectors as given by equation (20), the total amount of R&D expenditures in the economy can be derived as follows:

$$Z = \sum_{j=1}^N Z(\kappa_j) = \frac{q^{\frac{\alpha}{1-\alpha}} Q (\epsilon \bar{\pi} B_m - r \zeta)}{\epsilon B_m} \quad (21)$$

where

$$Q \equiv \sum_{j=1}^N q^{\kappa_j \alpha / (1-\alpha)} \quad (22)$$

is defined as the aggregate quality index, which captures the overall technological level of the economy. Clearly, Z is linearly dependent on Q and positively dependent on B_m .

The aggregate output of the final goods sector and the total intermediate demand can be derived analogously and are given by the following equations:

$$Y = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} LQ \quad (23)$$

$$X = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} LQ \quad (24)$$

With a constant labor force, equations (23) and (24) imply that Y and X are linear functions of the aggregate quality index Q . Furthermore it follows from equation (21) that Z is a linear function of Q .

Just considering the steady state with a given GPT generation m , it follows from equation (4) that C is also a linear function of Q . This implies that the growth rates of all of these variables are equal to the growth rate γ of Q . To derive this growth rate, we need to know the expected change of Q per unit of time, which is given by:

$$E[\Delta Q] = \sum_{j=1}^N p \left(q^{\frac{(\kappa_j+1)\alpha}{1-\alpha}} - q^{\frac{\kappa_j \alpha}{1-\alpha}} \right) \quad (25)$$

which in turn leads to:

$$E \left[\frac{\Delta Q}{Q} \right] = p \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \quad (26)$$

Given a large number of sectors, the law of large numbers implies (despite the fact that technical progress in individual sector takes place in discrete steps) that the aggregated average growth rate of Q equals the expression on the right-hand side of equation (26). Inserting in equation (26) the expression for p in equation (19), we obtain the following growth rate for Q :

$$\gamma = \frac{\dot{Q}}{Q} = \left(\frac{\epsilon \bar{\pi} B_m}{\zeta} - r \right) \left(q^{\frac{\alpha}{1-\alpha}} - 1 \right) \quad (27)$$

In order to derive the steady state, we need to equalize the growth equation (27) with the optimal growth rate of consumption as given by the Euler equation

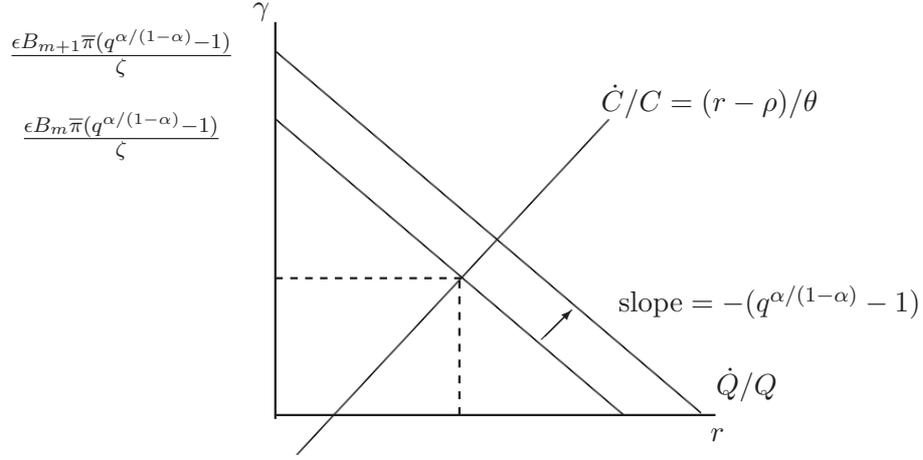


Figure 1: Determination of the Equilibrium Interest and Growth Rate

(3). Together these equations determine the steady state as plotted in Figure 1 for two consecutive GPT-generations, namely m and $m + 1$.

The arrival of a new GPT, as modeled by an increase in B , shifts the $\frac{\dot{Q}}{Q}$ line upwards. By utilizing the new GPT characterized by B_{m+1} the economy grows at a higher rate and has a higher interest rate than under the previous GPT generation: Remember that in the steady state with a constant GPT the growth rates of Y , X , Z and C all equal the growth rate of Q . Analytically, we can derive the following steady state values for the interest rate r and the growth rate γ , by equalizing the Euler equation (3) and equation (27):

$$r = \frac{\theta \epsilon B_m \bar{\pi} (q^{\frac{\alpha}{1-\alpha}} - 1) + \rho \zeta}{\zeta (1 + \theta (q^{\frac{\alpha}{1-\alpha}} - 1))} \quad (28)$$

$$\gamma = \frac{(q^{\frac{\alpha}{1-\alpha}} - 1) (\epsilon B_m \bar{\pi} - \rho \zeta)}{\zeta (1 + \theta (q^{\frac{\alpha}{1-\alpha}} - 1))} \quad (29)$$

To derive the probability of research success p , we can use (26) to get:

$$p = \frac{\gamma}{q^{\frac{\alpha}{1-\alpha}} - 1} \quad (30)$$

From the above equations we can easily derive the following expression:

$$p = \frac{\epsilon B_m \bar{\pi} - \rho \zeta}{\zeta (1 + \theta (q^{\frac{\alpha}{1-\alpha}} - 1))} \quad (31)$$

These steady state values for p , r and γ imply that the arrival of a new GPT not only leads to an increase in both growth and interest rates, but also in the probability of a research success.

Altogether, considering only steady states, the impact of the arrival of a new GPT on the economy in our model is entirely consistent with the long-term

effects observed in the majority of models in this field: First, the arrival of a new GPT makes investments in R&D more attractive, due to higher expected returns on research investments. Second, the new GPT generates a boost to the long-run growth rate of the economy.

3.2 Decreasing Returns to Current R&D

As has been described by Kortum (1993) and Stokey (1995) there are decreasing returns to current R&D efforts because of congestion externalities. This negative duplication externality is called “stepping on toes effect” by Jones and Williams (2000) and proposed to arise e.g. due to patent races. Applied to our model this implies that research does not only become more difficult the higher the quality of the good an R&D firms wants to improve upon, but also with rising R&D efforts at a point of time. We capture this effect in the following specification of $\phi(\kappa_j)$:

$$\phi(\kappa_j) = \left(\frac{1 - p(\kappa_j)}{\zeta} \right) q^{-(\kappa_j+1)\alpha/(1-\alpha)} \quad (32)$$

As in the general model presented in the previous section, research is more difficult the higher the new quality-ladder level of a new intermediate good is. In addition to this assumption, we model decreasing returns to current R&D by setting $\epsilon = 1 - p(\kappa_j)$. While any function where ϵ depends negatively on $p(\kappa_j)$ would result in similar qualitative results, we have chosen this specification for ease of computation of the transitional dynamics. Furthermore with this specification $0 \leq p(\kappa_j) \leq 1$ always holds as equation (8) and (32) imply

$$p(\kappa_j) = \frac{Z(\kappa_j)B_m q^{-(\kappa_j+1)\alpha/(1-\alpha)}}{\zeta + Z(\kappa_j)B_m q^{-(\kappa_j+1)\alpha/(1-\alpha)}} \leq 1 \quad (33)$$

Before the transition path using this specification is derived in the next Section, the implications of this specification of $\phi(\kappa_j)$ on steady state values are shown. The probability of research success is now given by equation (34) and the total amount of R&D expenditures by equation (35):

$$p = \frac{\bar{\pi}B_m - r\zeta}{\zeta + \bar{\pi}B_m} \quad (34)$$

$$Z = \frac{q^{\frac{\alpha}{1-\alpha}} Q(\bar{\pi}B_m - r\zeta)}{1 + r} \quad (35)$$

The steady state values of the interest rate, growth rate and the probability of research success are now given by the following three equations:

$$r = \frac{\theta \bar{\pi} B_m (q^{\frac{\alpha}{1-\alpha}} - 1) + \rho(\zeta + \bar{\pi} B_m)}{\theta \zeta (q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \bar{\pi} B_m} \quad (36)$$

$$\gamma = \frac{(q^{\frac{\alpha}{1-\alpha}} - 1)(\bar{\pi}B_m - \rho\zeta)}{\theta\zeta(q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \bar{\pi}B_m} \quad (37)$$

$$p = \frac{\bar{\pi}B_m - \rho\zeta}{\theta\zeta(q^{\frac{\alpha}{1-\alpha}} - 1) + \zeta + \bar{\pi}B_m} \quad (38)$$

These steady state values for p , r and γ imply that the arrival of a new GPT under this specification of $\phi(\kappa_j)$ leads qualitatively to the same implications as in the general case, i.e. to an increase in both growth rates and interest rates.

4 The Calm Before the Storm – Transition Paths

If researchers do not know about the arrival and the future course of a new GPT, but only know about the current marginal return on R&D-expenditures (as in Carlaw and Lipsey, 2006), the economy would simply jump from one steady state to another upon the arrival of a new GPT.

However, the arrival of a new GPT might very well be foreseen, in which case the arrival of a new GPT *does* give rise to transitional dynamics. Let us therefore assume that the time of arrival of the next GPT t^* is known in advance. Due to this information, R&D firms using the GPT of generation m adjust their R&D decisions in the time before the arrival of the GPT of generation $m + 1$. In the example of the United States of the late 19th century, it is obvious that firms could predict the rise of steam as a power source. However, the subsequent replacement of steam by electricity decades later could hardly have been taken into account. We therefore make the additional assumption that R&D firms are only concerned with the current and the next GPT generation. This seems reasonable because (apart from the fact that technological breakthroughs in the far future are virtually impossible to predict) the intervals between the arrival of new GPTs are typically very large compared to the lifetime of a single invention. In our model this leads to the simplification that the interest rate and the probability of research success will jump to the new steady state values at the time of arrival of the new GPT and will remain there forever.

4.1 Derivation of R&D Expenditures in the Transition Phase

Before simulating the transition path, we describe the derivation of the model equations for this phase, still assuming the specification $\phi(\kappa_j)$ with decreasing returns to R&D given by equation (32). Basically the same equations as before for the consumers and the R&D firms hold. However, one important change has to be taken into account: Due to the possible fluctuation of the probability of research success and the interest rate in the transition phase, the results regarding the expected payoff on R&D expenditures used in the previous Section (e.g. equations (16) and (17)) do not apply directly during the transition phase. The expected future profits of the incumbent monopolist are still discounted by the interest rate and by the probability of losing the monopoly. We define ω_t according to equation (39) as the overall multiplier of the profit flow in case of a successful invention that encompasses both the varying interest rates and the probabilities of being displaced as a monopolist:

$$\omega_{t+1} = \int_{t_{\kappa_j}}^{\infty} \int_{t_{\kappa_j}}^{\tau} e^{-\int_0^s r_u du} g(\tau) ds d\tau \quad (39)$$

Rewriting equation (14) by inserting both (10) and (39) we get the following expression for the expected payoff in case of a research success during the transition phase:⁶

$$E_t[V(\kappa_j + 1)] = \omega_{t+1} \bar{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} \quad (40)$$

Using the specification of $\phi(\kappa_j)$ in equation (32) the expression for p_t is now found by inserting equation (40) into the free-entry condition (13):

$$p_t = 1 - \frac{\zeta}{B_m \omega_{t+1} \bar{\pi}} \quad (41)$$

The amount of resources devoted to R&D in sector j and the aggregate amount of resources devoted to R&D can be calculated in the same manner as in the steady state and are given by:

$$Z_t(\kappa_j) = \frac{q^{\frac{(\kappa_j+1)\alpha}{1-\alpha}} (B_m \omega_{t+1} \bar{\pi} - \zeta)}{B_m} \quad (42)$$

and

$$Z_t = \sum_{j=1}^N Z_t(\kappa_j) = \frac{q^{\frac{\alpha}{1-\alpha}} Q_t (B_m \omega_{t+1} \bar{\pi} - \zeta)}{B_m} \quad (43)$$

Again, the higher the aggregate quality index is, the more research is performed. R&D input is also positively dependent on the current GPT level. Furthermore, the higher the future interest rate or the probability of research success, the smaller is the R&D investment. This is captured by the fact that Z_t is positively dependent on ω_{t+1} .

The general expression for the expected change in quality, equation (26), remains valid. By inserting equation (41) we derive the following growth rate for Q :

$$\frac{\dot{Q}_t}{Q_t} = \frac{B_m \omega_{t+1} \bar{\pi} - \zeta}{B_m \omega_{t+1} \bar{\pi}} (q^{\frac{\alpha}{1-\alpha}} - 1) \quad (44)$$

As can be easily seen in equation (43), the aggregate research expenditures Z_t are again linearly dependent on the quality index Q . Due to the fact that only the R&D sector is directly affected by changes in the current GPT, the optimization problems for the final goods sector and for the consumer remain the same as in the steady state. This means that the same equations (23) and (24) apply for X and Y in the transition phase, both of which are linearly dependent on Q . Again it follows that C is linearly dependent on Q as well.

⁶In the steady state case ω_t reduces to $\omega_t = \frac{1}{r+p(\kappa_j)}$ leading to equation (16).

Hence, we can take the same steps as before for the derivation of the steady state in order to calculate r_t , γ_t and p_t during the transition phase. This results in the following equations:

$$\gamma_t = \left(1 - \frac{\zeta}{B_m \omega_{t+1} \bar{\pi}}\right) (q^{\frac{\alpha}{1-\alpha}} - 1) \quad (45)$$

$$p_t = 1 - \frac{\zeta}{B_m \omega_{t+1} \bar{\pi}} \quad (46)$$

$$r_t = \theta \left(1 - \frac{\zeta}{B_m \omega_{t+1} \bar{\pi}}\right) (q^{\frac{\alpha}{1-\alpha}} - 1) + \rho \quad (47)$$

As in the steady state case, the growth rate is positively dependent on the current GPT level and q . B_m enters positively in the probability of research success p_t and the interest rate r_t .

4.2 Simulation Results

The time-frame for which R&D firms have to choose a time-path with varying research expenditures is restricted to the time before the arrival of a new GPT. This is due to the assumptions that the time of arrival of the next GPT t^* is known in advance and that afterwards the steady state values for p_t and r_t will apply from time until eternity. This allows us to solve the optimization problem of R&D firms and solve the transition path numerically using backward induction.

In our specific case, the intuition of this procedure is as follows: At the time of arrival t^* of the new GPT of generation B_{m+1} , the economy immediately jumps to the new steady state as described in the previous Section and remains there forever. Firms that take the decision on how much to invest in R&D in the last period before the arrival of the GPT of generation B_{m+1} , i.e. in the period $t^* - 1$, face the following problem: The payoff on their R&D expenditures is still governed by the old GPT which is less efficient than the next GPT generation. On the other hand, the rate by which their potential invention will be displaced in the future will be determined by the new, more efficient GPT. In other words, they have the disadvantage that they produce in a period where the expected probability of research success is *ceteris paribus* smaller than in all subsequent periods. This of course leads to a reduction in research expenditures in this period. In period $t^* - 2$, the outcome of period $t^* - 1$ and all subsequent periods is known to all firms and the maximization problem is solved conditional to these future constraints. We continue with this procedure until the level of the old steady state is approximately reached, i.e. when the time until the arrival of the next GPT is large enough that the impact on current R&D of this future development is negligible.

As in the previous Sections we are interested in the overall behavior of the economy, not of single firms or sectors. Therefore only aggregate values for the whole economy are taken. Accordingly we do not follow the profits of each R&D firm individually, since it is reasonable to assume that due to the law of large numbers a fraction $p(\kappa_j)$ of R&D firms is actually successfully innovating

and offers an intermediate product of a higher quality.⁷ Again the growth rate of the aggregate quality index determines aggregate growth.

The results of our simulation are shown in the subsequent figures, whereby a new GPT is supposed to arrive at time $t^* = 0$.⁸ From that time on, the economy remains in the new steady state. The dashed line in Figure 2 represents the expected present value of profits after a successful innovation $E_t[V]$, which of course rises at a constant rate in the new steady state. Immediately before the arrival of the new GPT in t^* though, there is a sharp reduction in these expected profits. This is the logical result of the fact that in the time immediately before the arrival of the new GPT R&D firms are most affected by the acceleration in R&D in the future due to the more efficient GPT of the next generation.

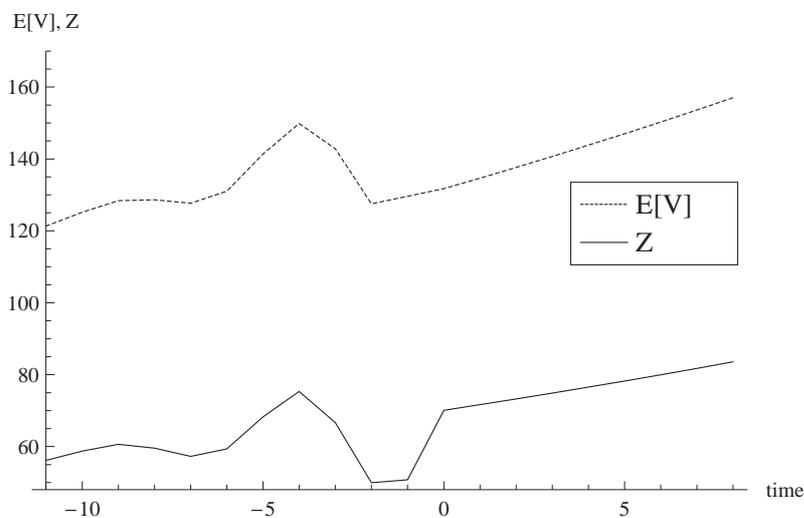


Figure 2: Expected Present Value of Profits after a Successful Innovation $E[V]$ and R&D Investment Z

The probability of research success is dependent on the amount of R&D expenditures and therefore exhibits the same behavior as the research expenditures.⁹ Since the expected profit falls at the end of the lifetime of the old GPT, R&D investments (see figure 2) become less attractive and therefore decrease. What happens *before* the slump in R&D expenditures and the related probability is more intriguing: The research activity in the economy rises even beyond the old steady state levels. This is due to the fact that R&D firms know about the slump in research activities during the last periods of the transition phase. As this leads to a lower chance of being displaced as a monopolist in this phase, research becomes relatively more attractive in the phases before the

⁷See Appendix A for further details on the aggregation of the profit flow.

⁸Please note that the simulation is performed in discrete time steps. Furthermore, we have calibrated our simulation to fit yearly data. This can be seen in Appendix B, where the parameters chosen for the numerical solution are presented.

⁹See Appendix C for a plot of the time path of p and of the expected present value of profit before the research success $p_t * E_t[V]$, i.e. the expected profit a firm faces making his research decision, not knowing if it is successful.

slump. Initially, R&D investment is departing from the steady state by minimal oscillation around the old steady state values. The amplitude of this oscillation is becoming bigger the nearer the arrival of the new GPT is.

The dynamics described above during the transition path can also be seen in the time path of the growth rate and the interest rate depicted in Figure 3. After leaving the old steady state the economy is characterized by cycles. The growth rate and the interest rate start to oscillate around the old steady state values. Four periods before the arrival of the new GPT the maximum values of this path are reached. In the periods ultimately before the arrival of the new GPT the growth rate and the interest rate fall. Altogether there is only a short time where the economy suffers from lower growth compared to the old steady state.

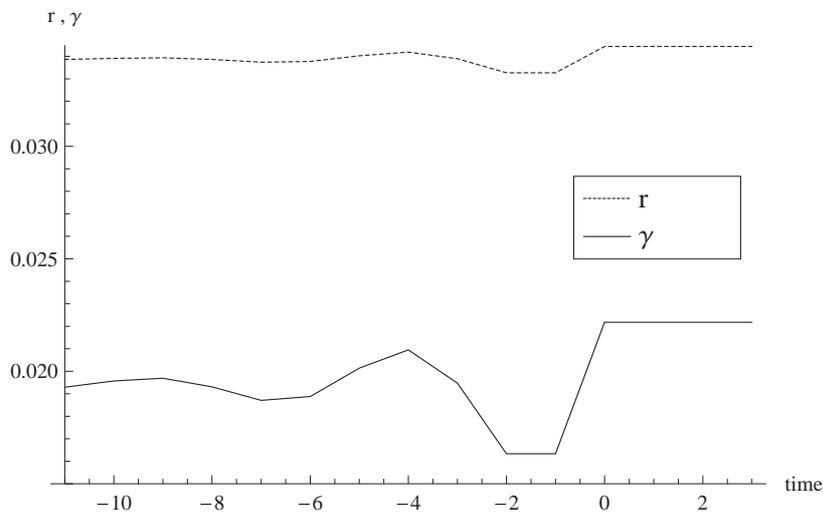


Figure 3: Interest rate and growth

Altogether, the dynamics in our model can be divided in three phases: First the economy is in the steady state using the old GPT. Because firms anticipate the arrival of the GPT of the next generation, a phase of transitional cycles is characterized by oscillation. Shortly before the arrival of a new GPT, R&D activity and growth rates increase beyond the old steady state levels before there is a slump. Finally, in a third phase, the economy is in the new steady state using the new GPT resulting in higher growth rates and interest rate levels. As has been shown in Section 1 the result of our model, that the introduction of a new GPT is preceded by a surge in R&D activities using the old GPT before its ultimate demise also has several examples in history.

5 Conclusions

In order to investigate the cyclical effects caused by GPTs we introduce the notion of GPTs in the quality-ladder model on Schumpeterian growth by Barro

and Sala-i-Martin (2004). In our model the arrival of a new GPT results an increase in R&D productivity: This increase in research efficiency makes investments in R&D more attractive. Regarding steady states, this leads to higher growth and interest rates.

Contrary to previous models on cycles induced by changes in GPTs, our analysis concentrates on the time *before* a new GPT arrives. In doing so, we can show within our model framework that a slowdown in output growth can also occur in the time immediately before the arrival of a new GPT. This slump is preceded by a period of increased R&D activity and oscillatory cycles.

While a surge in research activities in anticipation of a future rival technology has many examples in history, reality is of course more complex in many respects: For instance the time of arrival of a new GPT is usually not clear-cut. Furthermore, a number of GPTs can be active simultaneously. Both of these obvious limitations are shared with most existing models dealing with GPTs. Nevertheless, our model presents a channel that applies in addition to the ones described in other GPT models. While the vast majority of theories on GPTs explain an initial slump in productivity after the arrival of a new GPT, we present a channel on how a new GPT can induce cycles even before its arrival. This automatically opens possibilities for future research, as these two approaches could be combined in a single model: Such a model could incorporate both the cycles in research activities in anticipation of a new GPT *and* processes taking place after the new GPT has arrived, as described in other models.

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Appendix

A. Derivation of the aggregate profit

The profit flow per unit of time for an individual successful R&D firm is given by equation (10):

$$\pi(\kappa_j) = \bar{\pi} q^{\frac{\kappa_j \alpha}{1-\alpha}} \quad (\text{A.1})$$

By aggregating the profits, we get:

$$\pi = \int_{j=0}^N \pi(\kappa_j) = N \bar{\pi} Q \quad (\text{A.2})$$

By taking a continuum of firms going from 0 to 1, setting the limits on the integral accordingly and by using equation (11), we obtain the profit flow per unit of time for the aggregate of R&D firms:

$$\pi(\kappa_j) = \left(\frac{1-\alpha}{\alpha} \right) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L Q \quad (\text{A.3})$$

B. Parameters used for numerical solution of the transition path and the resulting steady state values

Parameter	GPT B_m	GPT B_{m+1}
B	1.3	1.69
α	0.3	0.3
θ	0.2	0.2
ρ	0.03	0.03
ζ	10	10
A	15	15
L	2	2
Q		10
q	1.1	1.1

Table B.1: Parameters used for numerical solution of the transition path

The parametrization of our model is shown in Table B.1. While the parameters are chosen to both fulfill basic assumptions and to yield sensible results for growth and interest rates, the resulting dynamics are robust across a wide range of parameters, as shown in Appendix D. Note that only the explicit value for Q at the point of time when the GPT of generation $m+1$ arrives is listed. The reason is that while the growth rates of the aggregate quality level Q are endogenous in our model, we had to define the explicit value of Q at a point of time in order to calculate the remaining values.

The evolution of the GPT parameter B_m is modeled to move along a quality-ladder which is similar to the dynamics in the R&D sector:

$$B_m = d^m \tag{48}$$

where $d > 1$. The increase in R&D efficiency due to the arrival of a new GPT is therefore modeled through an increase of B_m by the factor d . Our simulation starts with generation 0 where $m = 1$ and we assume that $d = 1.3$. The steady state values resulting from our simulation are listed in Table B.2.

Parameter	GPT B_m	GPT $B_m + 1$
r	3.39%	3.44%
p	46.5%	53.2%
γ	1.94%	2.22%

Table B.2: Steady state values of the numerical solution

Appendix C: Additional Figures of the Transition Path

The slump of the present value of expected profits before the arrival of the new GPT as described in the main text in Figure 2 can also be seen in Figure C.1 where the expected payoff to R&D expenditures Z , $p_t * E_t[V]$ (i.e. the expected present value of profits a firm faces making its research decision, since it gets the expected profit only with probability p_t) is depicted. The same applies to Figure C.2 where the probability of research success is plotted.

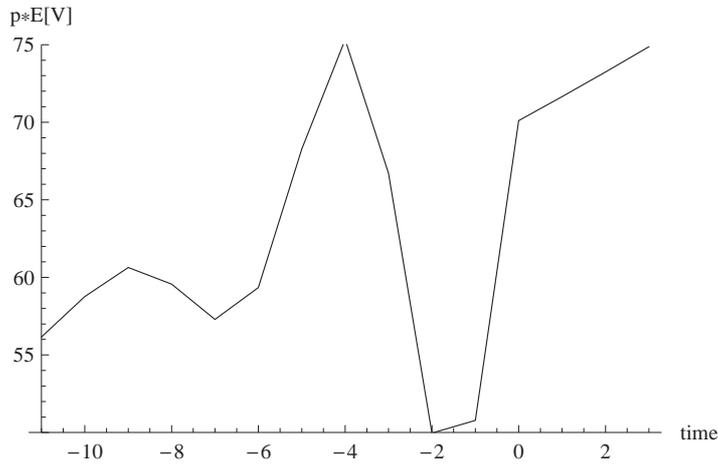


Figure C.1: Expected Return on R&D Expenditures $p_t * E_t[V]$

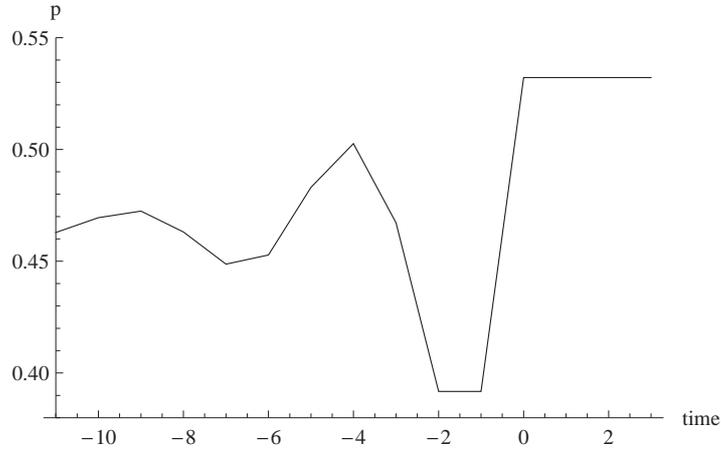


Figure C.2: Probability of Research Success p_t

Appendix D: Sensitivity Analysis

The qualitative results of the numerical simulation of the transition path presented in the main part of this paper are very robust to changes in the exogenous parameters (presented in Appendix B). This of course includes the cyclical behavior of the transition path. Meanwhile, in Table D.1 the impact of a ceteris paribus change of a single exogenous parameter on the resulting variables, namely p , γ and $E[V]$, is described.

Parameter	r	p	γ
α	↑	↑	↑
A	↑	↑	↑
L	↑	↑	↑
ζ	↓	↓	↓
θ	↑	↓	↓
ρ	↑	↓	↓
Q	-	-	-
q	↑	↓	↑

Table D.1: Sensitivity analysis with respect to a ceteris paribus increase of a parameter

As can be easily seen, an increase in the parameters which have a positive influence on the demand of intermediate goods, i.e. A , L and α have a positive impact on all endogenously determined variables. Furthermore a change in demand parameters does not influence the size of the oscillation during the transition phase.

Conversely an increase in the costs of performing R&D ζ lowers the exoge-

nous variables lowers p , r and γ but the expected profit $E[V]$ rises. Furthermore Z is lower in the new steady state, since the effect of the lower p dominates the effect of the higher $E[V]$. Additionally Z oscillates more during the transition phase, leading to temporarily higher levels than in the benchmark case previously presented. The lower steady state value of the R&D input Z is straightforward since ζ reflects the costs of R&D. Output is higher before, the same in the first period of the new steady state and then smaller.

Raising the preference parameters ρ and θ has a positive impact on the interest rate r , but also lowers p , $E[V]$ and Z which in turn leads to a lower growth rate γ . Output is only affected through the lower growth rate: It is higher during the transition phase, the same in the first period of the new steady state compared to the specification described in the main text, and then smaller due to the smaller growth rate in the new steady state.

The size of Q in the first period of the new steady state determines the level of output, Z and $E[V]$ but has no impact on p , r and γ .

If q is increased, both r and γ increase, whereas p decreases and $E[V]$, Z and output are higher in the new steady state but smaller before.