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On the Design of Basic-Research Policy *

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Abstract

We augment a Schumpeterian growth model with a public basic-research sector to examine how much a country should invest in basic research. We find that the closer the country is to the world's technological frontier the more the government should invest in basic research. Basic-research expenditures are increasing with a country's degree of openness as long as innovation sizes are small. We provide possible explanations for the empirical evidence available on basic-research expenditures across countries.

Keywords: basic research, openness, distance to frontier, economic growth.

JEL: O31, O38

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1 Introduction

It is widely recognized in economic literature that basic research[1] plays a significant role in economic growth (e.g. Narin et al. 1997, Mansfield 1998, Martin 1998)[2]. However, large inequalities are observable across countries with respect to basic-research expenditures (e.g., Cole and Phelan 1999 and Schofer 2004). The majority of basic research is performed by a small number of highly industrialized countries. For example, the US and Japan together account for almost half of the global basic-research expenditures. By contrast, technologically backward countries such as China and Argentina invest very little in basic research (OECD 2006).

Basic research is usually provided publicly by the government (see OECD 2004). In this paper we build on Aghion et al. (2006) and incorporate publicly financed basic research into a Schumpeterian growth framework. We ask how much a country should invest in basic research depending on its degree of openness and its distance from the world's technological frontier.

We assume that basic research increases the innovation probability of private intermediate firms within a country.[3] Intermediate sectors differ with regard to the distance from the world's technological frontier. The country's degree of openness is reflected by the probability of market entry by a foreign firm. The country is exposed to the technological frontier, i.e. foreign firms always enter at the world's most advanced technological level and consequently drive domestic firms operating at a lower technological level out of the market. Only those domestic firms that are close to the technological frontier and have been successful with innovation are internationally competitive and can hence prevent a potential foreign entrant from encroaching on the domestic market.

We allow the government to allocate a share of labor to basic research. The workers in this sector have to be paid competitive wages. The government's decision on basic research is then characterized by the following considerations: First, by increasing

[1] A standard definition of basic research is given by the OECD: "Basic research is experimental or theoretical work undertaken primarily to acquire new knowledge of the underlying foundation of phenomena and observable facts, without any particular application or use in view" (OECD, 2002, p. 30).

[2] See Salter and Martin (2001) for an extensive review of this literature.

[3] Several empirical studies (e.g. Jaffe 1989, Katz 1994, Narin et al. 1997, Zellner 2003; see Salter and Martin (2001) for a detailed review of this literature.) support our assumption by indicating that basic research has a strong tendency to produce local effects. They suggest, for instance, that basic research increases the innovation chances of domestic firms by the education of problem-solvers and local informal face-to-face interactions.

the innovation probability, basic research helps to escape entry of foreign firms to technologically advanced sectors. Second, basic research helps to sustain the monopoly of backward firms, as the competitive fringe catches up with such a firm when it is not innovating. Third, basic research is a costly way of fostering technological progress in comparison with the free import of foreign high technology.

We solve the government's problem for polar cases - technologically advanced and backward countries - and we derive additional results by numerical simulations for intermediate cases. For large sets of parameter values, we find that the higher the share of high-technology sectors is the more a government should invest in basic research. The effect of openness on the optimal amount of basic research is ambiguous. With a small innovation size, the free productivity boost linked to the entry of foreign high-technology firms is small. Accordingly, the government will prefer to prevent foreign entry and keep the domestic intermediates' profits in the country. For this reason, optimal basic research increases with the country's openness. However, with a high innovation size, the opposite investment behavior is optimal. The entry of foreign firms is welcome as they import leading technology and effect large productivity increases in the domestic country. In other words, to achieve a higher level of technology, it is cheaper to allow the entry of foreign firms and to forgo intermediate profits than to draw labor from production for basic research.

As we will elaborate in detail in section 6.1, our theoretical results may provide possible explanations for two empirical patterns. First, basic-research expenditures and technology levels are positively correlated. Second, there is an ambiguous relationship between basic research and the degree of openness.

Our paper is organized as follows: In the next section we relate our work to the literature on economic growth. The model is presented in section 3. Section 4 contains a discussion of the effects of basic research in our model, followed by a comparative statics analysis in section 5. In section 6, we discuss our results and relate them to the empirical pattern of basic research. Our conclusions are presented in section 7.

2 Relation to the Literature on Economic Growth

Theoretical literature on economic growth has mostly focused on private R&D activities. In the standard innovation-driven growth models (Romer 1990, Aghion and

Howitt 1992) profit-maximizing firms engage in R&D activities to generate (partially) excludable knowledge or "blueprints" for new marketable intermediate goods.

There are a few attempts in literature to include basic research in an R&D-driven growth model. Contributions to the discussion mostly focus on the optimal basic-research investment for economic growth in closed countries. Shell (1967) presented the first model of this kind. However, his model does not consider applied research by private firms. Profit-maximizing firms conducting different kinds of research are the focus chosen by Osano (1992), who investigates how the composition of basic and applied research affects growth. He does not include a public basic-research sector. In Bailén (1994) the same issue is examined, this time with publicly financed basic research. Morales (2004) combines the approaches of Osano and Bailén in a model where both the firms and the government perform basic research. She analyzes what kind of research policy pertaining to public provision and subsidies for basic/applied research leads to optimal growth. In another article, Pelloni (1997) examines the government's task to generate the conditions for growth by financing higher education and/or scientific research, whereas both are perfect substitutes.

The papers closest to ours are those by Park (1998) and Carillo and Papagni (2007). The former analyzes cross-country spillovers of basic research in a growth model with two identical countries and determines the efficient size of a country's basic research sector in relation to the economy's openness. Here basic research is considered as a global public good, whereas we focus on basic research as an instrument in enhancing the innovation prospects of domestic firms. Additionally, we allow for different levels of technology across countries. The paper by Carillo and Papagni (2007) delivers an explanation for the high cross-country inequalities observed in basic research investments by examining the scientific reward system in relation to the size of the scientific sector. We focus on how openness and the state of technology impact on the size of the basic-research sector.

3 The Model

Building on the Schumpeterian growth model proposed by Aghion et al. (2006), we introduce a basic-research sector operated by the government. We assume that there is a continuum of identical households that live for one period, enjoy strictly increasing utility in consumption, inelastically supply one unit of labor, and receive an equal share of the final good and intermediate firms' profits. We consider a government maximizing domestic consumption over a single period by publicly providing basic research financed by an income tax.[4] Accordingly, we first describe the production side of the economy and derive the equilibrium for a given level of basic research. We then proceed to solve the government's optimization problem.

3.1 Final-Good Sector

In the final-good sector, a continuum of competitive firms produces the homogeneous consumption good y according to:

$$y = \int_0^1 A(i)^{1-\alpha} x(i)^\alpha \, di. \tag{1}$$

$x(i)$ stands for the amount of intermediate input of variety i and $A(i)$ is this variety's productivity factor. The parameter α determines the output elasticity of the intermediate goods or the level of technology. The price of the final consumption good is normalized to one. In the following we will operate with one representative final-good firm. Maximization of the final-good firm's profits π^y gives the inverse demand functions for intermediate goods $x(i)$:

$$\max_{x(i)} \pi^y = y - \int_0^1 p(i)x(i) \, di \quad \Longrightarrow \quad p(i) = \alpha \left(\frac{A(i)}{x(i)} \right)^{1-\alpha}, \tag{2}$$

where $p(i)$ is the price of good $x(i)$.

3.2 Intermediate-Goods Sectors

The intermediate goods $x(i)$ are produced by labor $L^x(i)$ only, using a linear technology:

$$x(i) = L^x(i). \tag{3}$$

Intermediate-goods firms act competitively in the labor market and compete à la Bertrand in their intermediate sector. The productivity leader is able to establish a monopoly position and perfect competition prevails if there is no technological leader.

[4] The model can be seen as a non-overlapping generations model in which each generation elects a government to provide public goods (here basic research) in order to maximize its well-being. The latter is equivalent to maximizing the consumption of the current generation. This, however, does not square with a social planner aiming at maximizing the utility of all generations. As we are only considering a single period, we omit the time index t.

Hence the intermediate firms are either monopolistic or fully competitive. A competitive intermediate firm sets prices equal to the marginal costs, $p^c(i) = w$, and profits vanish. Using (2), the labor demand of a competitive intermediate firm can be written as

$$L^{xc}(i) = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} A(i), \tag{4}$$

where w denotes the wage level.

The monopolistic intermediate firm asks a price $p^m(i) = \frac{w}{\alpha}$ for its goods, leading to a labor demand of

$$L^{xm}(i) = \left(\frac{\alpha^2}{w}\right)^{\frac{1}{1-\alpha}} A(i) \tag{5}$$

and profits

$$\pi^{xm}(i) = \frac{mA(i)}{w^{\frac{\alpha}{1-\alpha}}} \quad \left(m := (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\right). \tag{6}$$

3.3 Technological State, Innovation, and Foreign Entry

We assume that there is a world technological frontier which at the end of the observed period is given by $\bar{A}$ and grows exogenously over time in accordance with

$$\bar{A} = \gamma \bar{A}_{-1},$$

where $\bar{A}_{-1}$ denotes the technological frontier of the preceding period.[5] Further, we assume that $1 < \gamma \leq 2$.

At the end of the preceding period, each intermediate firm can be of three types:

Type 1 firms produce at the current technological frontier, $A_{-1}(i) = \bar{A}_{-1}$.

Type 2 firms are one step behind the technological frontier, $A_{-1}(i) = \bar{A}_{-2}$.

Type 3 firms are two steps behind the technological frontier, $A_{-1}(i) = \bar{A}_{-3}$.

By investing in research and development, each intermediate firm can enhance its probability of realizing a successful innovation. A successful innovation increases the firm's technology level by a factor γ, thus allowing it to retain its relative position vis-à-vis the technological frontier. We specify the probability of type 1 and type 2 firms innovating successfully as

$$\rho(i) = \min\left\{2\theta\sqrt{L^I(i)L^B}, 1\right\}, \tag{7}$$

[5] In general, the index $-j$ ($j \in \mathbb{N}$) indicates how many periods back the indexed term is.

where $\theta > 0$ is a parameter that captures the efficiency of research. $L^I(i)$ denotes the intermediate firm's labor employed for R&D and L^B the amount of labor in the basic-research sector financed by the government. Throughout the paper, we will look at economies where we have an interior solution for all $\rho(i)$. Note that basic research is a necessary input for innovation activities and constitutes a public good from which domestic intermediate firms that are technological leaders in their respective sector can benefit. Domestic intermediate firms lagging technologically have no incentive to invest in innovation, as such an investment would not enable them to get ahead of their rivals and earn profits. Like Aghion et al. (2006), we neglect the adoption costs of mature technologies, i.e. technologies two steps behind the world's frontier, and consequently assume that a type 3 firm's technology is automatically upgraded.

From a long term perspective, each intermediate sector is in one of three states at the beginning of the period considered:

State 1 Type 1 leader holding a monopoly

State 2 Type 2 leader holding a monopoly

State 3 Two (or more) type 3 firms acting competitively

We denote the fractions of the states by s_1 (state 1), s_2 (state 2), and s_3 (state 3), where $s_1, s_2, s_3 \geq 0$ and $s_1 + s_2 + s_3 = 1$.

In each sector i not producing at the world's technological frontier, either because the domestic intermediate firm has failed to innovate or has been lagging behind previously, the probability of a foreign competitor entering the domestic market is σ. We assume that the foreign intermediate firm enters with frontier technology $\bar{A}$ and consequently takes over the whole market.[6] There are many empirical studies indicating for industrialized countries that foreign direct investment by leading-edge companies can transfer the best production techniques to the host countries (e.g. Baily and Gersbach 1995, Keller and Yeaple 2003 or Alfaro et al. 2006). In sectors where the domestic intermediate firm produces at the highest possible level, foreign competitors will stay outside[7], so this can only be the case in sectors where a type 1 leader innovates successfully.

[6]This implies that there is a fourth state for the intermediate sectors, i.e. with a foreign type 1 leader holding a monopoly. For simplicity, we assume that at the beginning of the period considered no intermediate sector is in this state. Allowing for foreign firms from the outset would only lead to a downscaling of the effects and thus does not alter the results substantially.

[7]This statement can be justified by small entry costs preventing the foreign firm from entering the market under perfect competition (see Aghion et al. 2006).

3.4 R&D Decisions of Intermediate Firms

Given the state of their sector, the entry threat of foreign firms, and the level of basic research, the domestic intermediate firms maximize their expected profits with respect to the amount of labor employed in private R&D:

- State 1 leader

$$\max_{L^I(i)} \left\{ \left(\rho(i) \frac{m}{w^{\frac{\alpha}{1-\alpha}}} \bar{A} + \big(1 - \rho(i)\big)(1-\sigma) \frac{m}{w^{\frac{\alpha}{1-\alpha}}} \bar{A}_{-1} \right) - wL^I(i) \right\}$$

The state 1 leader will retain the market and make profits if it innovates successfully or if it does not innovate and there is no entry. The maximization problem leads to the following labor demand:

$$L^{I_1} = \frac{L^B}{w^{\frac{2}{1-\alpha}}} \bar{A}_{-1}^2 m^2 \theta^2 \big(\gamma - (1-\sigma)\big)^2. \tag{8}$$

This implies innovation probability and expected profits for the state 1 leader in accordance with

$$\rho^{I_1} = 2 \frac{L^B}{w^{\frac{1}{1-\alpha}}} \bar{A}_{-1} m \theta^2 \big(\gamma - (1-\sigma)\big) \tag{9}$$

$$\pi^{I_1} = \frac{L^B}{w^{\frac{1+\alpha}{1-\alpha}}} m^2 \theta^2 \big(\gamma - (1-\sigma)\big)^2 \bar{A}_{-1}^2 + (1-\sigma) \frac{m}{w^{\frac{\alpha}{1-\alpha}}} \bar{A}_{-1}. \tag{10}$$

- State 2 leader

$$\max_{L^I(i)} \left\{ \rho(i)(1-\sigma) \frac{m}{w^{\frac{\alpha}{1-\alpha}}} \bar{A}_{-1} - wL^I(i) \right\}$$

The state 2 leader will only make profits if it innovates successfully and there is no foreign entry. If it does not innovate, the type 3 rival will automatically catch up. The sector is then subject to perfect competition and the profits vanish. The solution to the problem yields

$$L^{I_2} = \frac{L^B}{w^{\frac{2}{1-\alpha}}} \bar{A}_{-1}^2 m^2 \theta^2 (1-\sigma)^2, \tag{11}$$

$$\rho^{I_2} = 2 \frac{L^B}{w^{\frac{1}{1-\alpha}}} \bar{A}_{-1} m \theta^2 (1-\sigma), \tag{12}$$

$$\pi^{I_2} = \frac{L^B}{w^{\frac{1+\alpha}{1-\alpha}}} m^2 \theta^2 (1-\sigma)^2 \bar{A}_{-1}^2. \tag{13}$$

- All the remaining domestic firms will not invest in R&D as they have no prospects of making profits.

3.5 Equilibrium

The economy comprises the market for the final consumption good with price unity, the labor market with wage rate w, and a continuum of intermediate-good markets with prices $\{p(i)\}_{i=0}^{1}$. It follows from section 3.2 that the market clearing conditions in the intermediate-good markets yield prices $p^m(i) = \frac{w}{\alpha}$ in the monopolistic sectors and $p^c(i) = w$ in the competitive ones. From (3), (4), and (5) we obtain the values for the supply of intermediate goods as

$$x^c(i) = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} A(i) \tag{14}$$

$$x^m(i) = \left(\frac{\alpha^2}{w}\right)^{\frac{1}{1-\alpha}} A(i) \tag{15}$$

in the monopolistic intermediate sectors and the competitive intermediate sectors, respectively.

In the labor market, labor $\bar{L}$ is supplied inelastically. Labor demand consists of the government's demand for basic researchers, the intermediate firm's demand for private R&D personnel, and the demand of workers for the production of the intermediate goods. Hence the labor market clears when

$$\bar{L} = L^B + \int_0^1 L^I(i)\, di + \int_0^1 L^x(i)\, di. \tag{16}$$

As we know from section 3.3, the demand for R&D personnel depends on the state of the intermediate firms' sector. Consequently, the first integral in equation (16) is given by

$$\int_0^1 L^I(i)\, di = s_1 L^{I_1} + s_2 L^{I_2}. \tag{17}$$

Note that the total demand for private researchers is determined by the number of sectors characterized by domestic monopolies at the beginning of the period. By contrast, the demand for workers in intermediate-goods production depends on the sector's technological level after innovation activities and foreign entry have occurred. This reflects our assumption that foreign intermediate firms bring leading technology with them from abroad but produce the intermediate goods within the country. Accordingly, in order to determine the second integral in (16) we need to know how sector states evolve during the period. The following scheme displays the probabilities for levels of technology achieved by an intermediate sector. The illustration also shows the resulting market structure in terms of the mode of competition and of whether intermediate firms are domestic or foreign.

$$s_1 \longrightarrow \begin{cases} \rho^{I_1} & : \bar{A}, \quad \text{local}, \quad \text{monopoly} \\ (1-\rho^{I_1})\sigma & : \bar{A}, \quad \text{foreign}, \quad \text{monopoly} \\ (1-\rho^{I_1})(1-\sigma) & : \bar{A}_{-1}, \quad \text{local}, \quad \text{monopoly} \end{cases}$$

$$s_2 \longrightarrow \begin{cases} \sigma & : \bar{A}, \quad \text{foreign}, \quad \text{monopoly} \\ (1-\sigma)\rho^{I_2} & : \bar{A}_{-1}, \quad \text{local}, \quad \text{monopoly} \\ (1-\rho^{I_2})(1-\sigma) & : \bar{A}_{-2}, \quad \text{local}, \quad \text{perfect competition} \end{cases}$$

$$s_3 \longrightarrow \begin{cases} \sigma & : \bar{A}, \quad \text{foreign}, \quad \text{monopoly} \\ (1-\sigma) & : \bar{A}_{-2}, \quad \text{local}, \quad \text{perfect competition} \end{cases}$$

Consequently, the total intermediates' demand for production workers is given by

$$\begin{aligned} \int_0^1 L^x(i)\,di = &\left(\sigma + s_1(1-\sigma)\rho^{I_1}\right) L^{xm}(\bar{A}) + \\ &\left(s_1(1-\sigma)(1-\rho^{I_1}) + s_2(1-\sigma)\rho^{I_2}\right) L^{xm}(\bar{A}_{-1}) + \\ &\left(s_2(1-\sigma)(1-\rho^{I_2}) + s_3(1-\sigma)\right) L^{xc}(\bar{A}_{-2}). \end{aligned} \tag{18}$$

Inserting (17) and (18) into (16) we obtain the equilibrium wage level. In general, the equilibrium wage is not unique. By assuming $\alpha = \frac{1}{2}$ and $\bar{L} = 1$, we can solve the market clearing condition for basic-research labor as a function of wage w:

$$L^B(w) = w^2 \frac{w^2 - \mathcal{A}}{w^4 + \mathcal{B} + \mathcal{C}} \tag{19}$$

with

$$\mathcal{A} = \frac{\bar{A}}{16\gamma^2}\left[\sigma\gamma^2 + (1-\sigma)\big(s_1\gamma + 4(s_2+s_3)\big)\right] \quad > \quad 0 \tag{20}$$

$$\mathcal{B} = \frac{\bar{A}^2}{256\gamma^2}\theta^2[s_1(\gamma-1+\sigma)^2 + s_2(1-\sigma)^2] \quad > \quad 0 \tag{21}$$

$$\mathcal{C} = \frac{\bar{A}^2}{128\gamma^3}\theta^2(1-\sigma)[s_1\gamma(\gamma-1)(\gamma-1+\sigma) + s_2(\gamma-4)(1-\sigma)]. \tag{22}$$

As the number of researchers can never be negative, the equilibrium wage must be higher than $\sqrt{\mathcal{A}}$. Further, we slightly restrict our parameter space in accordance with the following assumption:

Assumption 1
$\mathcal{B} + \mathcal{C} > -\mathcal{A}^2$.

We are now in a position to state

Lemma 1

Under Assumption 1 there exists a unique solution to the government's problem.

Proof: See Appendix A.1.

From the equilibrium wage we obtain the equilibrium prices for intermediate goods from which the equilibrium quantities and the firms' profits follow. To simplify notation, we will henceforth use w to denote the equilibrium wage associated with a particular level of basic research.

3.6 Government

The government chooses the amount of basic-research labor L^B required to maximize aggregate consumption c of the current generation. The expenditures wL^B are financed by a tax $\tau \in [0, 1]$ on household income. Households earn wages and obtain profits from final-good and domestic intermediate-goods production. Consequently, the budget constraint for the government is

$$wL^B = \tau(w\bar{L} + s_1\pi^{I_1} + s_2\pi^{I_2} + \pi^y), \tag{23}$$

where π^y denotes the profits of the final-good sector.[8] Aggregate consumption c equals total income after taxes:

$$c = (1 - \tau)\left(w\bar{L} + s_1\pi^{I_1} + s_2\pi^{I_2} + \pi^y\right). \tag{24}$$

With uniqueness of the equilibrium wage w for given L^B, the government's problem can also be solved via the control w. We will take this path as it permits an explicit solution for L^B as a function of w. Economically, this approach could be interpreted as a wage offer by the government for doing basic research in order to attract the corresponding (equilibrium) number of researchers.

Inserting $L^B(w)$ from equation (19) and budget constraint (23) into (24), we obtain overall consumption solely as a function of the equilibrium wage level w:

$$c(w) = \frac{w^4(2\mathcal{A} + 2\mathcal{D} + \mathcal{E}) + w^2(2\mathcal{B} + 3\mathcal{C} + \mathcal{F}) + (\mathcal{B} + \mathcal{C})(2\mathcal{D} + \mathcal{E}) - \mathcal{A}(\mathcal{C} + \mathcal{F})}{w(w^4 + \mathcal{B} + \mathcal{C})}, \tag{25}$$

[8] Note that the profits in the final-good sector amount to $\pi^y = (1 - \alpha)y$.

with

$$\mathcal{D} = \frac{\bar{A}}{16\gamma}s_1(1-\sigma) \quad > \quad 0 \tag{26}$$

$$\mathcal{E} = \frac{\bar{A}}{16}\sigma \quad > \quad 0 \tag{27}$$

$$\mathcal{F} = \frac{\bar{A}^2}{32\gamma^3}s_2(1-\sigma)^2\theta^2 \quad > \quad 0. \tag{28}$$

This is the objective function the government maximizes with respect to the wage w.

Lemma 2
Under Assumption 1 there exists a unique maximum consumption level.

Proof: See Appendix A.2.

However, it is impossible to derive an analytical solution to the problem in its entire generality. Accordingly, we will first focus our analysis on two particular settings and then discuss the effects of basic research more generally on the basis of numerical simulations.

4 Effects of Basic Research

Before turning to comparative statics, we now introduce the different direct and indirect effects of basic-research investment on aggregate consumption.

4.1 Direct Effects

Recall that aggregate consumption is defined by (24) and consists of total income after taxes, which includes labor income and the profits of the type 1 firms, the type 2 firms, and the final-good sector. If the government decides to increase basic research L^B, the direct effects on consumption are the following:

Escape Entry Effect: State 1 leaders (type 1 firms) will avoid foreign entry and retain the domestic market if they innovate successfully to keep up with the technological frontier. As basic research facilitates innovation, it helps state 1 leaders to retain domestic profits, which increases the consumption of households as shareholders.

Monopoly Effect: State 2 leaders (type 2 firms) can only preserve their technological advantage and retain their monopoly position if they innovate successfully (and no foreign entry takes place). Otherwise they will lose their competitive edge as type 3 firms will catch up technologically. In this way, basic research helps state 2 leaders to make profits, which has a positive effect on consumption. However, the monopoly of the state 2 leaders lowers the profits of the competitive final-good sector caused by higher intermediate-goods prices. The second effect dominates as a result of the well-known monopoly distortion factor. The net effect is what we call the *monopoly effect*.

Productivity Effect: Basic research increases the probability of successful innovation and thus enhances technology growth. Higher technology raises profits for both the intermediate firms and the final-good sector. Consequently this effect on consumption is positive.

Wage Income Effect: Higher labor employment in the basic-research sector reduces the labor supply for the intermediate firms. Consequently the equilibrium wage increases, which leads to higher consumption.

Direct Tax Effect: Enlarging the public sector implies a higher tax rate τ. This lowers consumption.

4.2 Indirect Effects

Indirect effects of basic research on consumption arise from changes in the equilibrium wage rate. The higher wage caused by an increase of basic research influences the profits and the tax rate in the following way:

Labor Price Effect: The higher price for labor increases the production and innovation costs of the intermediate firms and thus lowers their profits. This effect on consumption is negative.

Intermediate Price Effect: As the production costs of the intermediate firms increase with the wage rate, the price of the intermediate goods will also rise. This leads to lower profits in the final-good sector with negative effects on consumption.

Indirect Tax Effect: A higher wage means that the costs for one unit of labor in public basic research are higher. Therefore the tax has to be increased, which lowers consumption.

4.3 Magnitude of Effects

In general, the magnitude of the effects of basic research on domestic consumption depends on the values of the exogenous parameters. However, it is instructive to recall that the impact of basic research depends on the extent to which it is used, i.e. how much private research is done by the domestic intermediate firms. The total amount of private R&D depends on how many intermediate firms are investing in private research and the intensity with which they do so. The former is exogenous, whereas the latter is endogenously determined within the model.

The exogenously given use of basic research depends on the country's industry structure, i.e. shares s_1, s_2, s_3. Suppose that in each kind of intermediate sector the firms display certain research intensities. Then the benefits of basic research will increase in the shares of sectors with high research intensities.

However, the extent to which intermediate firms invest in private R&D is endogenously determined in the model. A type 1 firm will still retain a monopoly position without innovation, whereas a type 2 firm would lose all its profits if it fails to innovate. Hence, innovation incentives in a closed economy are higher for a type 2 firm than for a type 1 firm. This changes with higher degrees of openness, as for a type 1 firm the probability of retaining a monopoly position without innovating decreases as does the probability of retaining the monopoly with innovation for type 2 firms. Hence, the innovation incentive for type 1 firms increases with the degree of openness, whereas that of the type 2 firms decreases. These effects are similar to the escape entry effects identified in Aghion et al. (2006).

5 Comparative Statics

5.1 Corner Scenarios

First, we take a look at the corner scenarios. This will enable us to derive analytical results. We start by analyzing how the government should behave in an economy that only has technologically advanced intermediate firms ($s_1 = 1$), followed by the analysis of an economy that only has technologically backward intermediate firms ($s_2 = 1$). The scenario $s_3 = 1$ is of no interest, as we already know that in this state firms will not invest in R&D and consequently the government has no incentive to invest in basic research at all.

5.1.1 Technologically Advanced Country: $s_1 = 1$

The equations (19) and (25) remain the same, while (20)-(26), (27), and (28) simplify to

$$\mathcal{A} = \frac{\bar{A}}{16\gamma}\left[\sigma\gamma + (1-\sigma)\right] \quad > \quad 0 \tag{29}$$

$$\mathcal{B} = \frac{\bar{A}^2}{256\gamma^2}\theta^2(\gamma - 1 + \sigma)^2 \quad > \quad 0 \tag{30}$$

$$\mathcal{C} = \frac{\bar{A}^2}{128\gamma^2}\theta^2(1-\sigma)(\gamma-1)(\gamma-1+\sigma) \quad > \quad 0 \tag{31}$$

$$\mathcal{D} = \frac{\bar{A}}{16\gamma}(1-\sigma) \quad > \quad 0 \tag{32}$$

$$\mathcal{E} = \frac{\bar{A}}{16}\sigma \quad > \quad 0 \tag{33}$$

$$\mathcal{F} \quad = \quad 0. \tag{34}$$

We thus obtain

Lemma 3

(i) *$L^B(w)$, defined by (19), starts at zero for $w = 0$. It then declines and becomes zero again at the wage level denoted by $w_{zero} = \sqrt{\mathcal{A}}$. In the range $w > w_{zero}$ it is positive and strictly increasing. It finally converges to $\bar{L} = 1$ for $w \to \infty$.*

(ii) *For $w \geq w_{zero}$, $c(w)$ is either always decreasing and converging to zero or it increases first, reaches a unique maximum, and then monotonically declines and converges to zero for $w \to \infty$.*

Proof: (i) follows directly from equations (19) and (29)-(31). For the proof of (ii) see Appendix A.3.

It is clear from Lemma 3 (i) that for an eligible equilibrium $w \geq w_{zero}$ must hold. This accords with economic intuition, as w_{zero} is the equilibrium wage level resulting when the government does not intervene in the economy, i.e. when it does not invest in basic research. Only if the government offers a higher wage than w_{zero} can it attract labor for basic research.

Lemma 3 also illustrates that, in principle, the government can achieve any value $L^B < \bar{L} = 1$ by offering a wage rate that is high enough. Of course, a level of basic research arbitrarily close to $\bar{L}$ cannot be optimal. This follows directly from the decreasing marginal product of basic research and the production function of final goods satisfying the Inada conditions. Intuitively, as the high governmental wage will draw labor from production, the marginal increase in technology will not compensate for the marginal loss in the production of intermediates, and consequently final output will decline.

Proposition 1

(i) *The government's maximization problem has a unique solution w^*. $w^* > w_{zero}$ implies a positive level of optimal basic research, $L^B > 0$.*

(ii) *For all parameter values there exists a θ_{crit} such that for $\theta_{crit} \leq \theta < \theta_{max}$ a positive amount of optimal basic research L^B results. θ_{max} denotes the value of the research productivity coefficient that leads to $\rho^{I_1} = 1$.*

Proof: (i) follows directly from Lemma 3; (ii) see Appendix A.4.

In general, two characteristics of a country whose intermediate sectors are all of the state 1 variety favor basic research investments. First, as state 1 leaders will still retain a monopoly position even if they do not innovate (as long as there is no foreign entry), the negative implications of the *monopoly effect* on total consumption are absent. Second, the magnitude of the *escape entry effect* will reach its maximum, as basic research only enables s_1 firms to escape foreign competition. However, it is still possible that the negative effects described in section 4 will dominate, so that it would be favorable for the government to abstain from basic-research investments. From Proposition 1, this may occur when research productivity as reflected by θ is sufficiently low.

5.1.2 Technologically Backward Country: $s_2 = 1$

For $s_2 = 1$, the terms (20)-(26), (27), and (28) take the form

$$\mathcal{A} = \frac{\bar{A}}{16\gamma^2}\left[\sigma\gamma^2 + 4(1-\sigma)\right] > 0 \quad (35)$$

$$\mathcal{B} = \frac{\bar{A}^2}{256\gamma^2}\theta^2(1-\sigma)^2 > 0 \quad (36)$$

$$\mathcal{C} = \frac{\bar{A}^2}{128\gamma^3}\theta^2(\gamma-4)(1-\sigma)^2 < 0 \quad (37)$$

$$\mathcal{D} = 0 \quad (38)$$

$$\mathcal{E} = \frac{\bar{A}}{16}\sigma > 0 \quad (39)$$

$$\mathcal{F} = \frac{\bar{A}^2}{32\gamma^3}(1-\sigma)^2\theta^2 > 0. \quad (40)$$

Lemma 4

(i) $L^B(w)$, defined by (19), always increases in w.

(ii) $c(w)$, defined by (25), always falls in w in the relevant space $w > w_{zero}$.

Proof: (i) see proof of Lemma 1; (ii) see Appendix (A.5).

The curvature of the functions $L^B(w)$ and $c(w)$ indicate that for all L^B the negative effects of basic research on consumption will dominate.

Proposition 2

The government does not invest in basic research.

Proof: Follows directly from Lemma 4.

In contrast to the scenario where $s_1 = 1$, the positive *escape entry effect* does not exist when $s_2 = 1$. The domestic intermediate firms cannot avoid the entry of the foreign firms by innovating successfully, as they are too far away from the technological frontier. Further, the *monopoly effect* is maximal because the government could remove all monopoly distortions by forgoing basic research. Hence, the government's incentives to invest in basic research tend to be low. Thus we can state:

Proposition 3

$L^B(w^*) \mid_{s_1=1} \geq L^B(w^*) \mid_{s_2=1}$

Proof: Follows directly from Propositions 1 and 2.

According to Proposition 3, technologically advanced countries should always spend at least as much on basic research as technologically backward countries. More precisely, when comparing an economy characterized by $s_1 = 1$ with another characterized by $s_2 = 1$, only the advanced country is likely to conduct basic research. This result follows the intuition given with respect to Propositions 1 and 2. That is, in an economy where $s_1 = 1$, the negative *monopoly effect* of basic research on consumption has no effect. On the other hand, in the $s_2 = 1$ scenario the positive *escape entry effect* is irrelevant. Hence, basic research affects consumption more positively in the $s_1 = 1$ scenario than in the $s_2 = 1$ scenario.

5.2 Numerical Simulations

In this section we leave the corner scenarios to look at the comparative statics of more general settings that cannot be solved analytically.[9] Therefore we will use numerical simulations as the basis for the following discussion. The basic parametrization of the model will be: $\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $\sigma = 0.7$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$. We will then derive our comparative statics results by varying one parameter while holding the others fixed. This enables us to isolate the effects of different parameters on optimal basic-research expenditures. Our simulations indicate that these are valid for large parameter sets and are not specific to our parametrization. We will also address special cases.

5.2.1 Distance from Technological Frontier

We first consider the effect of a country's distance from the world's technological frontier on the optimal amount of basic research. The technological level of the economy varies across sectors and is thus characterized by the country's industry structure as reflected by the parameters s_1, s_2, and s_3. Since the shares of the different sector-types must add up to 1 the most, we will always vary the share of one sector type at the expense of the share of exactly one other sector type. In particular, we will discuss the following three cases: s_1 vs. s_2, s_1 vs. s_3, and s_2 vs. s_3.

[9] The optimal wage w^* is given by the derivative of equation (25), which is a polynomial of degree eight.

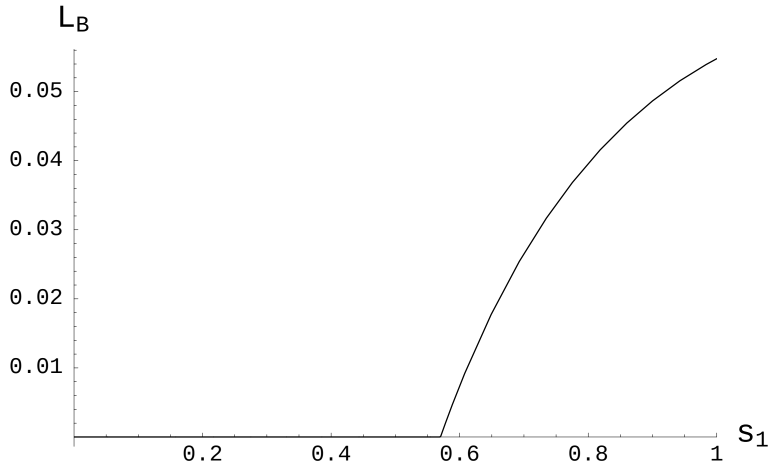

Figure 1: Effect of Distance from Frontier: s_1 vs. s_2 ($\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $\sigma = 0.7$, $s_2 = 1 - s_1$, $s_3 = 0$)

s_1 vs. s_2

This case generalizes the corner scenarios in the preceding section. Accordingly, we would expect the *escape entry effect* to monotonically increase in s_1, starting out from zero at $s_1 = 0$ and reaching its maximum at $s_1 = 1$. By contrast, the *monopoly effect* should decrease from its maximum value at $s_1 = 0$ to zero at $s_1 = 1$. Hence, optimal basic-research investment would increase in the share of technologically advanced sectors. As can be seen in Figure 1, the results are consistent with the intuition.

Robustness checks show that this result holds for a wide range of parameters. However, when openness σ is low, we may have the case where the optimal amount of basic research first increases in s_1 and then falls later on. When σ is low, the *escape entry effect* is smaller, and the remaining effects have relatively more weight. The costs of basic research increase in s_1 as type 1 firms have a higher demand for workers than type 2 firms due to their higher technological level and this leads to a wage increase. Where the escape entry effect is small due to a low degree of openness, the negative effect described becomes dominant for a high share of state 1 sectors. This induces the government to lower basic research when s_1 increases.

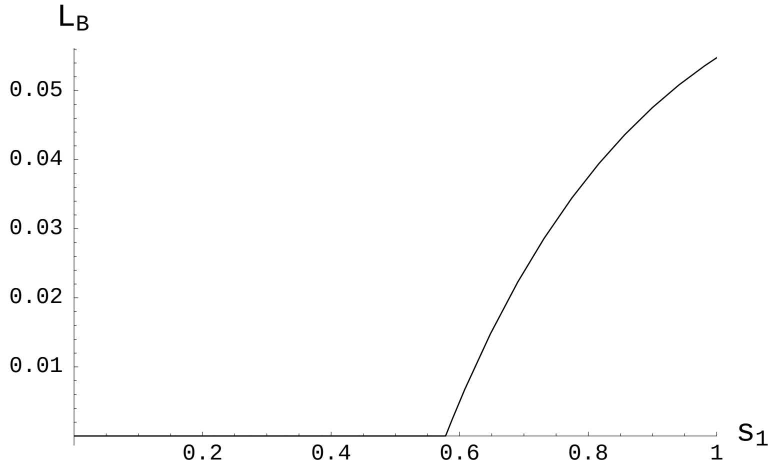

Figure 2: Effect of Distance from Frontier: s_1 vs. s_3 ($\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $\sigma = 0.7$, $s_2 = 0$, $s_3 = 1 - s_1$)

s_1 vs. s_3

The way in which the optimal amount of basic research is affected by a rise of s_1 coupled with a decline of s_3 is shown in Figure 2. Optimal basic-research investments increase in proportion with the share of state-1 sectors. The intuition is clear. The government has no incentive to invest in basic research to support the intermediate firms in state 3, because they will never invest in R&D. However, as we already know, there are positive incentives to provide basic research for state 1 leaders.

Nonetheless, robustness checks have shown that, as in the s_1 vs. s_2 analysis, when σ is low, optimal basic research may first increase with s_1, reach a maximum, and then fall. Again, the *escape entry effect* is small, and the costs of basic research, i.e. the wage rate increase in s_1, will assume a dominant role at some point. The reason is that the state 1 leaders' demand for labor in production and innovation for a given L^B is higher than the labor employment of intermediate firms in state 3. Hence, the larger the share s_1 is, the higher the wage and consequently the costs of basic research will be.

s_2 vs. s_3

Figure 3 shows that an increase of s_2 at the expense of s_3 has a positive effect on the optimal amount of labor for basic research. Simulations across all parametrical

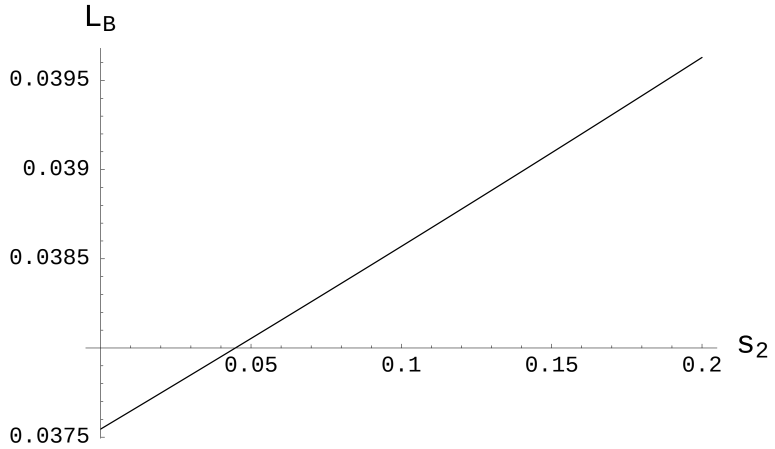

Figure 3: Effect of Distance from Frontier: s_2 vs. s_3 ($\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $\sigma = 0.7$, $s_1 = 0.8$, $s_3 = 0.2 - s_2$)

values confirm this result. Intermediate firms in state 3 are automatically upgraded in technological terms and thus do not need support from basic research. This explains the pattern in figure 3.

Summary

According to the analysis of these three distinct cases, the broad picture is that the closer a country is to the technological frontier, the more the government should invest in basic research. There are exceptions however, in particular when openness is small and research productivity is very high.

5.2.2 Openness

We next consider how variations in the degree of openness affects optimal basic research. The influence of openness on optimal basic research is ambiguous, as shown by the comparison of Figures 4 and 5. If we choose a lower innovation size γ as in Figure 4, we have a positive relationship between optimal basic research and openness. An increase of innovation size will cause the effect to go in the opposite direction, as illustrated by Figure 5. What explanation is there for this result?

The direction of the openness effect depends on the weight of the *productivity effect* relative to the *escape entry effect.* The latter is larger the more open a country is, because the threat of losing domestic profits increases. By contrast, the *productivity*

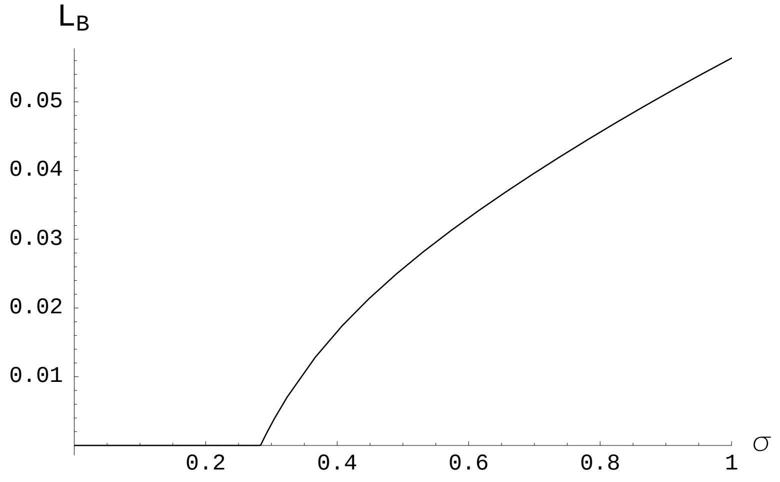

Figure 4: Effect of Openness ($\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

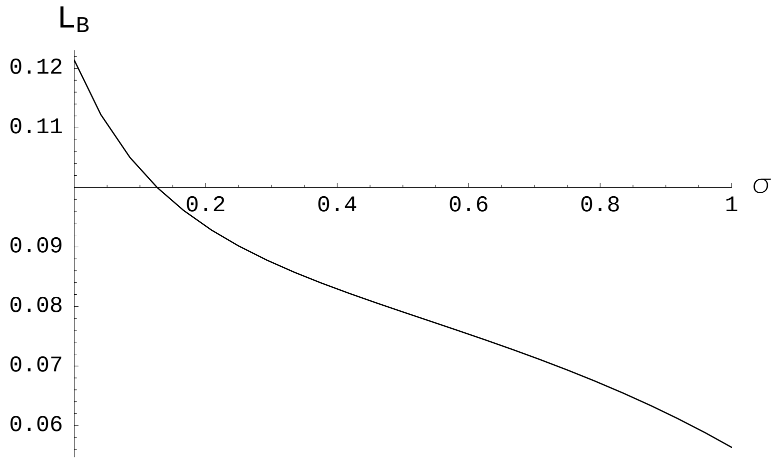

Figure 5: Effect of Openness ($\bar{A} = 100$, $\gamma = 1.5$, $\theta = 2.5$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

effect decreases with openness, as foreign intermediate firms transfer leading technology to the domestic country. Consequently, the government has less incentive to invest in basic research and hence in technological progress itself. As the innovation size γ scales the *productivity effect*, the *escape entry effect* dominates when γ is low, so that optimal basic-research expenditures increase with the country's degree of openness. Vice versa, a large innovation size implies that the *productivity effect* plays the more important role and optimal basic-research investments will decline with openness. Note that the escape entry effect is still present, but due to the higher magnitude of the *productivity effect*, optimal basic-research investments are in general higher with larger values of γ. With $\sigma = 1$ when the *productivity effect* reaches its minimum, optimal basic-research

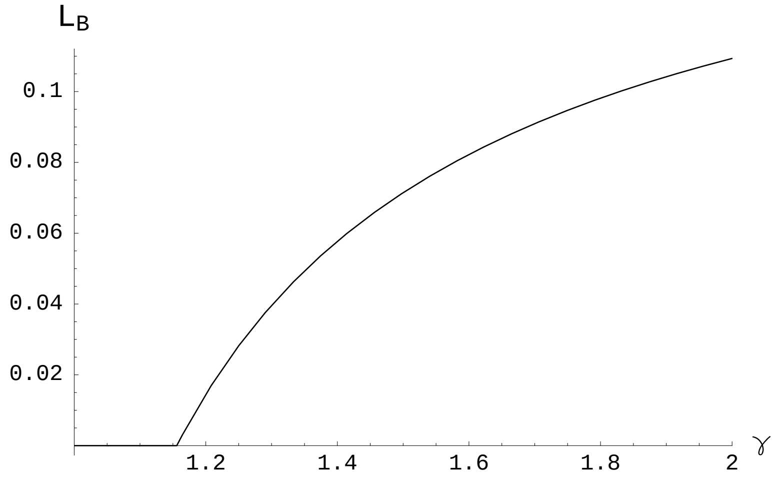

Figure 6: Effect of Innovation Size ($\bar{A} = 100$, $\theta = 2.5$, $\sigma = 0.7$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

expenditures are, in principle, independent of the innovation size.

Robustness checks support this result for all parameter values. However, we note that the presence of a certain number of state 1 sectors is needed to obtain a positive relationship between openness and basic research. The reason is that the magnitude of the *escape entry effect* depends positively on the share of state 1 sectors. This is clear, as only type 1 firms can compete with foreign intermediate firms. Additionally, with an increasing degree of openness, private research intensity also increases in state-1 sectors.[10]

5.2.3 Innovation Size

In this section we examine how the size of innovation γ influences optimal basic research. As expected and depicted in Figure 6, the higher the innovation size is, the more labor a government should employ for basic research. This is obvious, as basic research fosters innovation and so a higher innovation size will improve the efficiency of basic research. In other words, a larger innovation size implies a larger *productivity effect*. Our result is confirmed by simulations across all parametrical values.

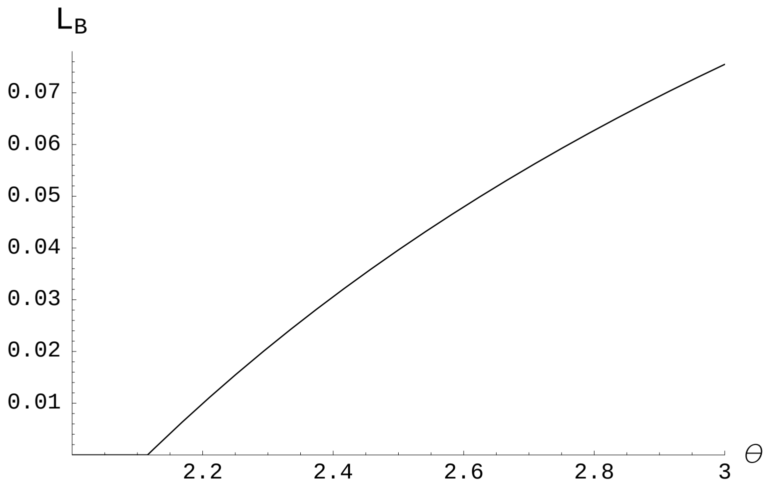

Figure 7: Effect of Research Productivity ($\bar{A} = 100$, $\gamma = 1.3$, $\sigma = 0.7$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

5.2.4 Research Productivity

We can alter the productivity of both company research and basic research by modifying θ. The higher θ is, the more probable a successful innovation becomes, so research productivity is higher. It is straightforward to show that the optimal amount of basic research increases with research productivity, as can be seen in Figure 7. Higher research productivity mainly increases the *escape entry effect* and the *productivity effect*.

6 Discussion of the Results

6.1 Empirical Patterns

The two main implications of the preceding comparative statics analysis are the following: First, the higher the share of technologically advanced sectors in a country is, the more the government should invest in basic research. Second, the relationship between openness and the optimal amount of basic research is ambiguous. If innovation sizes are low, open countries should invest more in basic research in order to compete with foreign intermediate firms. By contrast, if innovation sizes are large and the productivity effect dominates accordingly a country's policy might be to undertake less basic research the more open it is.

[10]This effect is similar to the escape entry effect identified in Aghion et al. (2006).

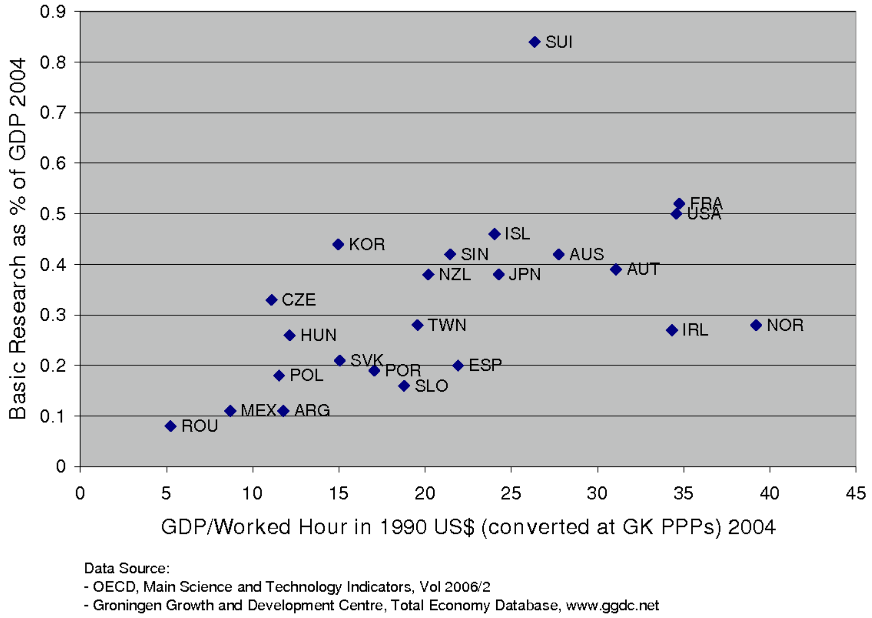

Figure 8: Relationship between Basic Research and Productivity

How do our results square with empirical patterns of basic research? In Figure 8 the relationship between basic research and productivity is shown based on country data. We observe that the technological level and basic-research expenditures are positively associated.[11] There is further empirical evidence supporting the positive relationship between economic development and basic research (e.g. Cole and Phelan (1999) and Schofer (2004)).

Figure 9 presents the data on openness and basic research. To quantify openness we construct an exposure index that measures to which degree domestic industries are exposed to foreign competition. The exposure index consists of the variables trade (percent of GDP), FDI flows (percent of GDP), FDI stocks (percent of GDP), hidden import barriers, mean tariff rate and taxes on international trade (percent of current revenue).[12] In figure 9 we consider only advanced countries, as the numerical simulations of section 5.2 indicate that positive basic-research investments require a certain proportion of technologically advanced sectors. The data indicate no clear correlation

[11] A more detailed empirical study that goes beyond a simple correlation would be desirable, but this is impeded by data availability.

[12] A broader economic globalization index is given in Dreher (2006) that takes into account international economic linkages beyond foreign competition. We refer to this work for a detailed description of the data sources and on how the variables are normalized. Dreher uses a principal components analysis to determine the weights of the variables. As equal weights produce approximately the same results, we choose the simple weighting scheme.

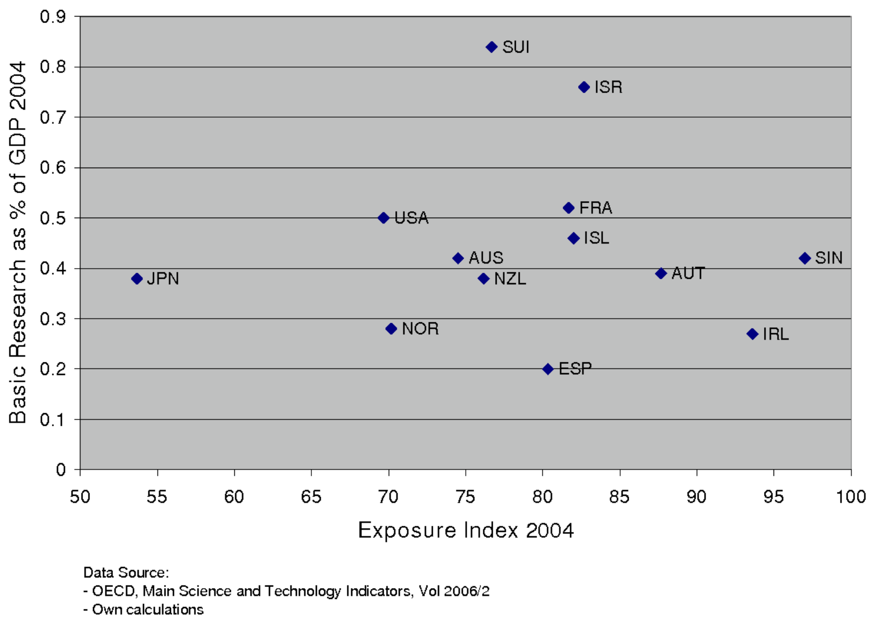

Figure 9: Relationship between Basic Research and Openness

between a country's degree of openness and its basic-research expenditures.[13] Taking the total factor productivity as a proxy for the total share of advanced sectors in the economy, the two figures 8 and 9 can be interpreted in different ways, as we exemplify for France and Ireland.

Consider the two countries France (FRA) and Ireland (IRL), which have about the same productivity but different levels of basic-research expenditures. Hence, we could explain the France/Ireland case as a negative relationship between basic research and openness. With Ireland being more open, it would invest less in basic research than France.

The France/Ireland case might also be consistent with a positive association between basic research and openness when we take into account the role of foreign direct investment. Ireland's policy over the last decades has been characterized by abandoning trade barriers and attracting foreign direct investment, which has helped to increase its total factor productivity considerably. Accordingly, the share of domestic high technology firms is substantially lower than in France as many leading sectors are populated by foreign firms.[14] Ireland may invest less in basic research for this reason despite be-

[13] An equivalent figure for the less developed countries leads to the same conclusion. Again, these results are only an initial impulse and should be considered with care.

[14] FDI data can be found in UNCTAD (2007) and World Bank (2007).

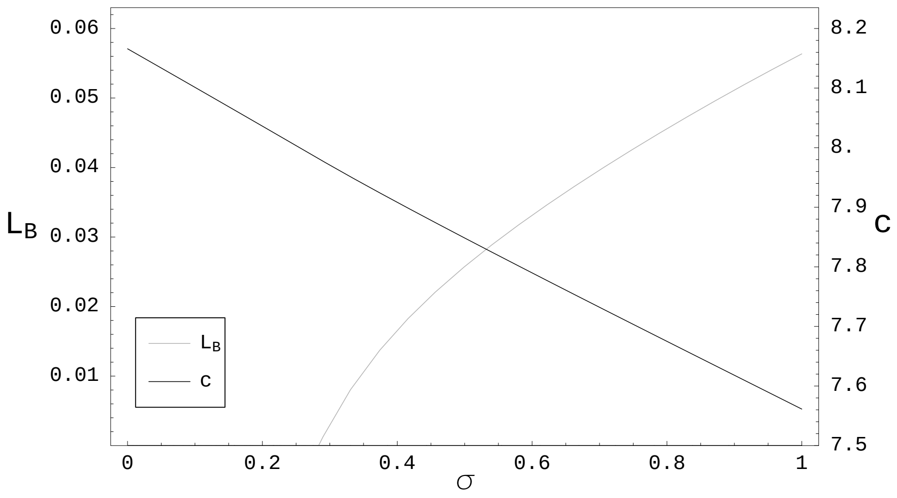

Figure 10: Effect of Openness on Consumption and Optimal Basic Research ($\bar{A} = 100$, $\gamma = 1.3$, $\theta = 2.5$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

ing more open. Which of these explanations is more accurate is left to detailed future empirical research.

6.2 Joint Policies: Openness and Basic Research

Are we able to say anything about optimal policies with respect to both basic research and openness from our previous examinations? Extending Figure 4 by including the graph of domestic consumption yields Figure 10. It shows a decrease of consumption with openness. Hence, the government's optimal policy is to set $\sigma = 0$ and not to invest in basic research in such a setting. On the one hand, low innovation size makes basic research less beneficial, and on the other, foreign firms do not greatly advance the technological level but take the profits away from some domestic firms.

As illustrated by Figure 11, if the innovation size is high, the government should open the borders to allow foreign firms to enter the market and bring in high technology. However, it should still invest in basic research, as is clear if we bear in mind that optimal basic-research expenditures with a high innovation size are always above those with a low innovation size.

As a hypothesis we can state that if the *productivity effect* dominates over the *escape entry effect*, the government should choose a high degree of openness and simultane-

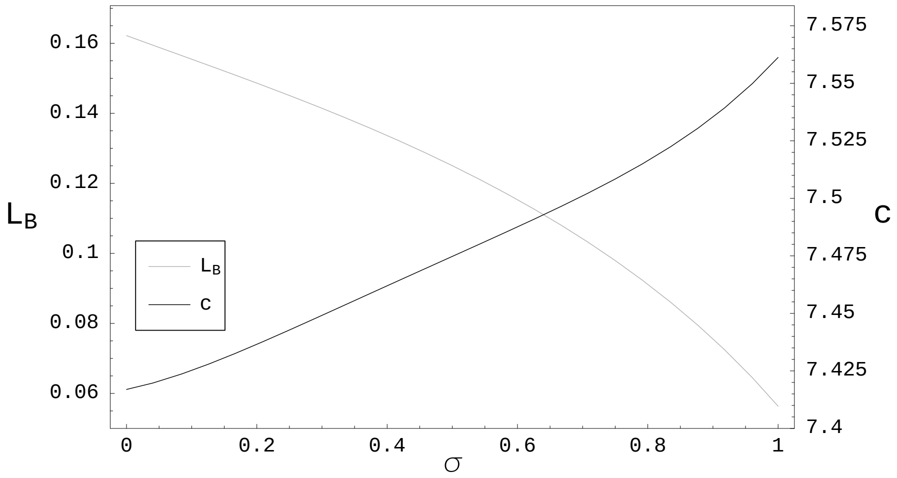

Figure 11: Effect of Openness on Consumption and Optimal Basic Research ($\bar{A} = 100$, $\gamma = 1.9$, $\theta = 2.5$, $s_1 = 0.8$, $s_2 = 0.2$, $s_3 = 0$)

ously a relatively low level of basic-research investments. In the opposite case, if the *escape entry effect* plays the central role, borders should be closed and basic-research expenditures should be very low or even zero.

7 Conclusion

In this paper, we have augmented the Schumpeterian growth model of Aghion et al. (2006) with a public basic-research sector which allows to determine how much a country should invest in basic research depending on its technological level and its openness to the world market.

Our model opens up a variety of avenues for future research. For instance, it might be useful to endogenize the behavior of foreign countries and to examine the problem of choosing basic research from a global perspective. It may also be interesting, and by no means trivial, to establish the dynamics of our model and to examine a social planner's solution for basic-research investment across many generations.

A Proofs

A.1 Proof of Lemma 1

We show that the function $L^B(w)$ strictly increases in w, hence the inverse function exists. As $L^B < 0$ is not feasible, wages lower than $\sqrt{\mathcal{A}}$ are not possible in equilibrium. Rewriting equation (19) as

$$L^B(w) = \frac{1 - \frac{\mathcal{A}}{w^2}}{1 + \frac{\mathcal{B}+\mathcal{C}}{w^4}} \tag{41}$$

reveals that L^B will be zero for $w = w_{zero} = \sqrt{\mathcal{A}}$ and converges to 1 for $w \to \infty$. No equilibrium exists for $w < \sqrt{\mathcal{A}}$. For $w \geq w_{zero}$, it is convenient to replace w by $\sqrt{x\mathcal{A}}$, where $x \geq 1$. We obtain

$$L^B(x) = \frac{1 - \frac{1}{x}}{1 + \frac{\mathcal{B}+\mathcal{C}}{x^2\mathcal{A}^2}}. \tag{42}$$

We now need to show that $L^B(x)$ is strictly increasing in x, i.e. that

$$\frac{\partial L^B(x)}{\partial x} = \frac{\frac{1}{x^2}\left(1 + \frac{\mathcal{B}+\mathcal{C}}{x^2\mathcal{A}^2}\right) + 2\frac{x-1}{x}\frac{\mathcal{B}+\mathcal{C}}{x^3\mathcal{A}^2}}{\left(1 + \frac{\mathcal{B}+\mathcal{C}}{x^2\mathcal{A}^2}\right)^2} > 0. \tag{43}$$

This condition can be rewritten as

$$1 - \frac{\mathcal{B}+\mathcal{C}}{x^2\mathcal{A}^2} + 2\frac{\mathcal{B}+\mathcal{C}}{x\mathcal{A}^2} > 0. \tag{44}$$

If $\mathcal{B}+\mathcal{C} > 0$, we can estimate the left-hand side from below by multiplying the last term with $\frac{1}{x}$. This gives us

$$1 + \frac{\mathcal{B}+\mathcal{C}}{x^2\mathcal{A}^2} > 0,$$

which is obviously satisfied.

We now consider the case where $\mathcal{B}+\mathcal{C} < 0$. As the left hand side of (44) is increasing in x, we know that if condition (44) is satisfied for $x = 1$, it will also be satisfied for $x > 1$. Inserting $x = 1$, we obtain

$$1 + \frac{\mathcal{B}+\mathcal{C}}{\mathcal{A}^2} > 0. \tag{45}$$

This holds under Assumption 1, i.e. if $\mathcal{B}+\mathcal{C} > -\mathcal{A}^2$.

A.2 Proof of Lemma 2

To prove that we have a unique maximum consumption level in the relevant space $w \geq w_{zero}$, we show that $c(w)$ is either always decreasing in w or increasing in w, reaching a local maximum, and then decreasing in w.

To analyze the slope of $c(w)$, we differentiate with respect to w:

$$\begin{aligned}\frac{\partial c(w)}{\partial w} = & \frac{-w^8(2\mathcal{A}+2\mathcal{D}+\mathcal{E}) - 3w^6(2\mathcal{B}+3\mathcal{C}+\mathcal{F}) + w^4(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{5w^4\mathcal{A}(\mathcal{C}+\mathcal{F}) + w^2(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{(\mathcal{B}+\mathcal{C})\big(\mathcal{A}(\mathcal{C}+\mathcal{F}) - (\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})\big)}{w^2(w^4+\mathcal{B}+\mathcal{C})^2}\end{aligned}$$

It is obvious that the denominator is positive. Hence, to determine the slope it is sufficient to focus on the numerator only. As we are interested in the relevant space $w \geq w_{zero} = \sqrt{\mathcal{A}}$, it is convenient to replace w by $\sqrt{x\mathcal{A}}$, whereas $x \geq 1$. The numerator takes the form

$$\begin{aligned}&\underbrace{-x^4\mathcal{A}^4(2\mathcal{A}+2\mathcal{D}+\mathcal{E})}_{U}\underbrace{-3x^3\mathcal{A}^3(2\mathcal{B}+3\mathcal{C}+\mathcal{F})}_{V}\underbrace{+5x^2\mathcal{A}^3(\mathcal{C}+\mathcal{F})}_{W}+ \\ &\quad\underbrace{x^2\mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E})}_{X}\underbrace{+x\mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F})}_{Y}+ \\ &\quad\quad\underbrace{(\mathcal{B}+\mathcal{C})\big(\mathcal{A}(\mathcal{C}+\mathcal{F})-(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})\big)}_{Z}.\end{aligned}$$

We note that $\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{E}, \mathcal{F} > 0$, $\mathcal{F} > -\mathcal{C}$, and $\mathcal{A} > \mathcal{D} + \mathcal{E}$. The analysis can be simplified by distinguishing four cases.

1. $\mathcal{B}+\mathcal{C} > 0$ and $(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E}) - \mathcal{A}(\mathcal{C}+\mathcal{F}) < 0$
   U and V, the terms with the highest exponents of x, are negative, while all the remaining terms, W, X, Y, and Z, are positive. Hence, it is obvious that in this case $c(w)$ either falls straightaway in x or w, or it rises first and falls after reaching its maximum.

2. $\mathcal{B}+\mathcal{C} > 0$ and $(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E}) - \mathcal{A}(\mathcal{C}+\mathcal{F}) > 0$
   U and V are negative, W, X, and Y are positive, and Z is again negative. If Y dominates Z for $x = 1$, it is always dominating and as in the preceding case the

exponents of x can be used to state the uniqueness of a maximum consumption level. Inserting $x = 1$ in $Y + Z > 0$ leads to

$$\begin{aligned}
\mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+4\mathcal{C}+2\mathcal{F}) - (\mathcal{B}+\mathcal{C})^2(2\mathcal{D}+\mathcal{E}) &> 0 \\
\mathcal{A}(2\mathcal{B}+4\mathcal{C}+2\mathcal{F}) &> (\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E}) \\
2\mathcal{A}(\mathcal{C}+\mathcal{F}) + 2\mathcal{A}(\mathcal{B}+\mathcal{C}) &> (2\mathcal{D}+\mathcal{E})(\mathcal{B}+\mathcal{C}).
\end{aligned}$$

As $\mathcal{A} > \mathcal{D}+\mathcal{E}$, the inequality holds, and the existence of a unique maximum is shown.

3. $\mathcal{B}+\mathcal{C} < 0$ and $2\mathcal{B}+3\mathcal{C}+\mathcal{F} > 0$
U and V are still negative, W is positive, and the remaining terms, X, Y, and Z, are negative. Thus it is sufficient to show that $W + X + Y + Z > 0$ holds for $x = 1$. Arguing with the exponents of x again, $c(w)$ is then either falling all along or rising before falling continuously. Next we prove that $W + X + Y + Z > 0$ for $x = 1$:

$$\begin{aligned}
5\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + \mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E}) + \mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F}) + \\
\mathcal{A}(\mathcal{B}+\mathcal{C})(\mathcal{C}+\mathcal{F}) - (\mathcal{B}+\mathcal{C})^2(2\mathcal{D}+\mathcal{E}) > 0
\end{aligned}$$

Estimating $W + X + Y + Z$ from below by using Assumption 1, the inequality reduces to

$$\begin{aligned}
4\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + \mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E}) + \mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F}) + \\
-(\mathcal{B}+\mathcal{C})^2(2\mathcal{D}+\mathcal{E}) &> 0 \\
4\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + \mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-2\mathcal{D}-\mathcal{E}) + \mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F}) &> 0 \\
3\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + \mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-2\mathcal{D}-\mathcal{E}) + 2\mathcal{A}(\mathcal{B}+\mathcal{C})^2 &> 0 \\
3\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + 2\mathcal{A}(\mathcal{B}+\mathcal{C})^2 - \mathcal{A}^2(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E}) + 6\mathcal{A}^3(\mathcal{B}+\mathcal{C}) &> 0.
\end{aligned}$$

The second and third terms are positive. Hence, again estimating the LHS from below by neglecting them gives us

$$\begin{aligned}
3\mathcal{A}^3(\mathcal{C}+\mathcal{F}) + 6\mathcal{A}^3(\mathcal{B}+\mathcal{C}) &> 0 \\
3(\mathcal{C}+\mathcal{F}) + 6(\mathcal{B}+\mathcal{C}) &> 0 \\
2\mathcal{B}+3\mathcal{C}+\mathcal{F} &> 0
\end{aligned}$$

This inequality holds by the definition of the case we are dealing with.

4. $\mathcal{B}+\mathcal{C}<0$ and $2\mathcal{B}+3\mathcal{C}+\mathcal{F}<0$

   In this case, U is negative, V and W are positive, X is negative, Y is positive, and finally Z is negative. It is thus slightly more complicated to show the existence of a unique maximum of $c(w)$. We have to take two steps. First we show that X dominates Y at $x=1$.

$$\begin{aligned}\mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E})+\mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F})&<0\\ \mathcal{A}(6\mathcal{A}-4\mathcal{D}-2\mathcal{E})+2\mathcal{B}+3\mathcal{C}+\mathcal{F}&>0\end{aligned}$$

   The fact that $\mathcal{A}>\mathcal{D}+\mathcal{E}$ allows us to reduce the inequality to

$$2\mathcal{A}^2+2\mathcal{B}+3\mathcal{C}+\mathcal{F}>0.$$

   Furthermore, omitting the positive term $\mathcal{C}+\mathcal{F}$ leads to

$$\begin{aligned}2\mathcal{A}^2+2(\mathcal{B}+\mathcal{C})&>0\\ \mathcal{A}^2&>-(\mathcal{B}+\mathcal{C}).\end{aligned}$$

   According to Assumption 1 the inequality holds. Consequently, $X+Y+Z$ is negative along the whole relevant interval because of $x\geq 1$ and X having the larger exponent of x. The next step is to prove that $V+W+Y+X+Z>0$ for $x=1$.

$$\begin{aligned}-3\mathcal{A}^3(2\mathcal{B}+3\mathcal{C}+\mathcal{F})+5\mathcal{A}^3(\mathcal{C}+\mathcal{F})+\mathcal{A}^2(\mathcal{B}+\mathcal{C})(6\mathcal{A}-4\mathcal{D}-2\mathcal{E})+&\\ \mathcal{A}(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}+\mathcal{F})+(\mathcal{B}+\mathcal{C})\big(\mathcal{A}(\mathcal{C}+\mathcal{F})-(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})\big)&>0\\ 2\mathcal{A}^3(\mathcal{C}+\mathcal{F})+2\mathcal{A}(\mathcal{B}+\mathcal{C})(\mathcal{B}+2\mathcal{C}+\mathcal{F})+&\\ -2\mathcal{A}^2(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})-(\mathcal{B}+\mathcal{C})^2(2\mathcal{D}+\mathcal{E})&>0\end{aligned}$$

   Making use of Assumption 1, we can reduce the inequity in the following way:

$$\begin{aligned}2\mathcal{A}^3(\mathcal{C}+\mathcal{F})+2\mathcal{A}(\mathcal{B}+\mathcal{C})(\mathcal{B}+2\mathcal{C}+\mathcal{F})-\mathcal{A}^2(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})&>0\\ 2\mathcal{A}(\mathcal{B}+\mathcal{C})^2-\mathcal{A}^2(\mathcal{B}+\mathcal{C})(2\mathcal{D}+\mathcal{E})&>0\end{aligned}$$

   As both terms are positive, the inequity is correct. Considering the exponents of x and the two facts that $X+Y+Z<0$ always holds and that $V+W+X+Y+Z>0$ at $x=1$, we can state the validity of $V+W+X+Y+Z>0$ along the whole relevant interval. Furthermore, we know that U is negative and has the highest exponent of x. Thus in this case too, $c(w)$ either falls continuously or rises first to reach a maximum and falls subsequently.

A.3 Proof of Lemma 3 (ii)

First we note that $\mathcal{A} = \mathcal{D} + \mathcal{E}$ and recall that $\mathcal{F} = 0$. Thus, equation (25) simplifies to

$$c(w)\mid_{s_1=1} = \frac{w^4(3\mathcal{A}+\mathcal{D}) + w^2(2\mathcal{B}+3\mathcal{C}) + (\mathcal{B}+\mathcal{C})\mathcal{D} + \mathcal{A}\mathcal{B}}{w(w^4+\mathcal{B}+\mathcal{C})}.$$

We know from equations (29)-(33) that $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E} > 0$. Hence, it is obvious that $c(w)$ is positive if $w > 0$. It starts at $+\infty$ for $w = 0$ and is converging to zero as w approaches infinity. To analyze the slope of $c(w)$, we differentiate it with respect to w

$$\begin{aligned}\left.\frac{\partial c(w)}{\partial w}\right|_{s_1=1} = & \frac{-w^8(3\mathcal{A}+\mathcal{D}) - 3w^6(2\mathcal{B}+3\mathcal{C}) - 2w^4\mathcal{D}(\mathcal{B}+\mathcal{C})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{w^4\mathcal{A}(4\mathcal{B}+9\mathcal{C}) + w^2(2\mathcal{B}+3\mathcal{C})(\mathcal{B}+\mathcal{C})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{-(\mathcal{B}+\mathcal{C})\big(\mathcal{A}\mathcal{B}+\mathcal{D}(\mathcal{B}+\mathcal{C})\big)}{w^2(w^4+\mathcal{B}+\mathcal{C})^2}.\end{aligned} \tag{46}$$

As the denominator is positive for $w > 0$, the numerator determines the sign of $\frac{\partial c(w)}{\partial w}$. It is clear that the numerator is negative for w close to zero and approaches $-\infty$ for $w \to \infty$. As we are interested in the interval $w \geq w_{zero} = \sqrt{\mathcal{A}}$, we insert $w = \sqrt{x\mathcal{A}}$ where $x \geq 1$. The numerator then takes the form

$$\begin{aligned}& -x^4\mathcal{A}^4(3\mathcal{A}+\mathcal{D}) - 3x^3\mathcal{A}^3(2\mathcal{B}+3\mathcal{C}) - 2x^2\mathcal{A}^2\mathcal{D}(\mathcal{B}+\mathcal{C}) + \\ & \quad x^2\mathcal{A}^3(4\mathcal{B}+9\mathcal{C}) + x\mathcal{A}(2\mathcal{B}+3\mathcal{C})(\mathcal{B}+\mathcal{C}) - (\mathcal{B}+\mathcal{C})\big(\mathcal{A}\mathcal{B}+\mathcal{D}(\mathcal{B}+\mathcal{C})\big).\end{aligned}$$

The last term is negative, but it is straightforward to show that it is dominated by the second to last term, i.e. $x\mathcal{A}(2\mathcal{B}+3\mathcal{C})(\mathcal{B}+\mathcal{C}) > (\mathcal{B}+\mathcal{C})\big(\mathcal{A}\mathcal{B}+\mathcal{D}(\mathcal{B}+\mathcal{C})\big)$. As the remaining negative terms possess the highest exponents of x, a negative value of the derivative of $c(w)$ at w_{zero}, i.e. at $x = 1$, implies that it will remain negative for the entire range, $w > w_{zero}$. On the other hand, if $\frac{\partial c(w)}{\partial w}$ at w_{zero} is positive, it will eventually change its sign once and stay negative. It follows that in the relevant set $w > w_{zero}$, $c(w)$ either rises first, then reaches a unique maximum, and declines later, or $c(w)$ is decreasing over the entire interval.

A.4 Proof of Proposition 1 (ii)

We show the existence of a unique θ_{crit} by the following line of argument: Investments in basic research are optimal if and only if

$$\left.\frac{\partial c(w)}{\partial w}\right|_{w=w_{zero}} > 0.$$

Since $\frac{\partial c(w)}{\partial w}\Big|_{w=w_{zero}}$ strictly increases in θ without bound and $\frac{\partial c(w)}{\partial w}\Big|_{w=w_{zero},\theta=0} < 0$, there must be a unique θ_{crit} where the derivative is exactly zero.

We already know from the proof of Lemma 2 (ii) that it is sufficient to analyze the numerator of (46) to determine the sign of $\frac{\partial c(w)}{\partial w}$. For $w = w_{zero}$ it takes the form

$$-\mathcal{A}^4(3\mathcal{A}+\mathcal{D}) - 2\mathcal{A}^3\mathcal{B} - 2\mathcal{A}^2\mathcal{D}(\mathcal{B}+\mathcal{C}) + (\mathcal{B}+\mathcal{C})\big(\mathcal{A}(\mathcal{B}+3\mathcal{C}) - \mathcal{D}(\mathcal{B}+\mathcal{C})\big).$$

Inserting the expressions for $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ yields

$$\begin{aligned}
&-\left(\frac{\bar{A}}{16\gamma}\right)^5 (\sigma\gamma+1-\sigma)^4\big(3\sigma\gamma+4(1-\sigma)\big) - 2\left(\frac{\bar{A}}{16\gamma}\right)^5 \theta^2(\sigma\gamma+1-\sigma)^3(\gamma-1+\sigma)^2+\\
&-2\left(\frac{\bar{A}}{16\gamma}\right)^5 \theta^2(\sigma\gamma+1-\sigma)^2(1-\sigma)(\gamma-1+\sigma)\big(3(\gamma-1+\sigma)-2\sigma\gamma\big)+\\
&\left(\frac{\bar{A}}{16\gamma}\right)^5 \theta^4(\gamma-1+\sigma)^2\big(3(\gamma-1+\sigma)-2\sigma\gamma\big)\Big(\sigma\gamma\big(7(\gamma-1+\sigma)-6\sigma\gamma\big)+4(1-\sigma)^2(\gamma-1)\Big).
\end{aligned}$$

Dividing by $\left(\frac{\bar{A}}{16\gamma}\right)^5$ and θ^2, the expression simplifies to

$$\begin{aligned}
&-\frac{1}{\theta^2}(\sigma\gamma+1-\sigma)^4\big(3\sigma\gamma+4(1-\sigma)\big) - 2(\sigma\gamma+1-\sigma)^3(\gamma-1+\sigma)^2+\\
&-2(\sigma\gamma+1-\sigma)^2(1-\sigma)(\gamma-1+\sigma)\big(3(\gamma-1+\sigma)-2\sigma\gamma\big)+\\
&\theta^2(\gamma-1+\sigma)^2\big(3(\gamma-1+\sigma)-2\sigma\gamma\big)\Big(\sigma\gamma\big(7(\gamma-1+\sigma)-6\sigma\gamma\big)+4(1-\sigma)^2(\gamma-1)\Big).
\end{aligned}$$

It is easy to see that the derivative of $c(w)$ is negative for small θ and strictly increases in θ without bound. This implies that there is a unique θ_{crit} with $\frac{\partial c(w)}{\partial w}\mid_{w=w_{zero}}= 0$ and $\frac{\partial c(w)}{\partial w}\mid_{w=w_{zero}}> 0$ for all $\theta > \theta_{crit}$.

We now show that the interval $\theta_{crit} < \theta < \theta_{max}$ is not empty by first verifying the existence of a θ_{max} such that $\rho(\theta_{max}) = 1$ and then establishing that $\theta_{crit} < \theta_{max}$.

The existence of a θ_{max} such that $\rho(\theta_{max}) = 1$ follows from the fact that public and private research expenditures are strictly increasing in θ, as is the innovation probability ρ. ρ is bound in L^B and L^I, as the latter cannot exceed the total labor supply, but it is not bound in θ. Consequently, there exists a unique θ_{max}.

We now show that $\theta_{max} > \theta_{crit}$. From Lemma 3 and the above considerations, we know that for $\theta \leq \theta_{crit}$, there is no basic research, $L^B = 0$. Hence according to equation (9), $\rho^{I_1}\mid_{\theta\leq\theta_{crit}}= 0$. It follows that θ_{max} must be larger than θ_{crit}.

A.5 Proof of Lemma 4 (ii)

First we note that, according to equations (36) and (37), $\mathcal{C} = 2\mathcal{B}\frac{\gamma-4}{\gamma}$ and consequently $2\mathcal{B} + \mathcal{C} = \mathcal{B}\frac{4\gamma-8}{\gamma} \leq 0$. Further, we know that $\mathcal{C} + \mathcal{F} = 2\mathcal{B}$. Taking the derivative of equation (25) with respect to w gives

$$\begin{aligned}\frac{\partial c(w)}{\partial w}\Big|_{s_2=1} = & \frac{-w^8(2\mathcal{A}+\mathcal{E}) - 6w^6(2\mathcal{B}+\mathcal{C}) - 2w^4\mathcal{E}(\mathcal{B}+\mathcal{C})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{2w^4\mathcal{A}(8\mathcal{B}+3\mathcal{C}) + 2w^2(2\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C})}{w^2(w^4+\mathcal{B}+\mathcal{C})^2} + \\ & \frac{(\mathcal{B}+\mathcal{C})\big(2\mathcal{A}\mathcal{B} - \mathcal{E}(\mathcal{B}+\mathcal{C})\big)}{w^2(w^4+\mathcal{B}+\mathcal{C})^2}.\end{aligned} \tag{47}$$

As the denominator is positive, it again suffices to analyze the numerator in order to determine the sign of $\frac{\partial c(w)}{\partial w}$. We verify that the numerator is negative in the relevant set $w \geq w_{zero}$ by showing that it is negative for w_{zero} and that it is decreasing in w for all $w \geq w_{zero}$.

Starting with the latter, we differentiate the nominator with respect to w to obtain

$$\begin{aligned}&\underbrace{-8w^7(2\mathcal{A}+\mathcal{E})}_{(V)}\underbrace{-36w^5(2\mathcal{B}+\mathcal{C})}_{(W)}\underbrace{+8w^3\mathcal{A}(8\mathcal{B}+3\mathcal{C})}_{(X)}+ \\ &\qquad\qquad\underbrace{-8w^3\mathcal{E}(\mathcal{B}+\mathcal{C})}_{(Y)}\underbrace{+4w(2\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C})}_{(Z)}.\end{aligned} \tag{48}$$

(V) is negative, while (W), (Y), and (Z) are positive. (X) is ambiguous.

In order to show that the derivative is negative, we proceed in two steps. First, we sum X and W to ξ and verify that ξ is positive in the relevant space by showing $\xi(w_{zero}) > 0$.[15] This enables us to argue that the derivative of polynomial (48) changes its sign once the most. As (48) will be negative for large values of w, we demonstrate in a second step that (48) is already negative for $w = w_{zero}$. This implies that it is negative for all $w > w_{zero}$.

1. Defining $\xi(w) := W(w) + X(w)$ and inserting the smallest wage of the relevant interval, which is $w = w_{zero} = \sqrt{\mathcal{A}}$, results in

$$\begin{aligned}-36\mathcal{A}^{2.5}(2\mathcal{B}+\mathcal{C}) + 8\mathcal{A}^{2.5}(8\mathcal{B}+3\mathcal{C}) &> 0 \\ -8\mathcal{B} - 12\mathcal{C} &> 0.\end{aligned}$$

[15] $\xi(w_{zero}) > 0$ is sufficient for $\xi(w) > 0 \forall w > w_{zero}$, as ξ will be positive for values of w larger than some threshold value due to the higher exponent of w in W.

Hence $\xi(w) > 0$ for $w \geq w_{zero}$. We conclude that equation (48) is always negative in the relevant set if it is already negative for the smallest value $w = w_{zero}$.

2. By inserting w_{zero} in equation (48) we obtain

$$\begin{aligned} -8\mathcal{A}^{3.5}(2\mathcal{A}+\mathcal{E}) - 36\mathcal{A}^{2.5}(2\mathcal{B}+\mathcal{C}) + 8\mathcal{A}^{2.5}(8\mathcal{B}+3\mathcal{C}) - 8\mathcal{A}^{1.5}\mathcal{E}(\mathcal{B}+\mathcal{C}) + \\ 4\mathcal{A}^{0.5}(2\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C}) < 0 \\ \Leftrightarrow -8\mathcal{A}^{3}(2\mathcal{A}+\mathcal{E}) - 4\mathcal{A}^{2}(2\mathcal{B}+3\mathcal{C}) - 8\mathcal{A}\mathcal{E}(\mathcal{B}+\mathcal{C}) + 4(2\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C}) < 0. \end{aligned}$$

Due to Assumption 1, $\mathcal{A} > \sqrt{-(\mathcal{B}+\mathcal{C})}$, we can estimate the expression from above by inserting $\sqrt{-(\mathcal{B}+\mathcal{C})}$ for $\mathcal{A}$:

$$\begin{aligned} -16(\mathcal{B}+\mathcal{C})^2 - 8\mathcal{E}\big(-(\mathcal{B}+\mathcal{C})\big)^{1.5} + 4(\mathcal{B}+\mathcal{C})(2\mathcal{B}+3\mathcal{C}) + 8\mathcal{E}\big(-(\mathcal{B}+\mathcal{C})\big)^{1.5} + \\ 4(2\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C}) < 0 \\ 16\mathcal{B} + 16\mathcal{C} - 8\mathcal{B} - 12\mathcal{C} - 8\mathcal{B} - 4\mathcal{C} = 0. \end{aligned}$$

Consequently equation (48) is smaller than zero, which implies that the numerator of equation (47) is declining in w in the relevant interval.

We will finish the proof by inserting the minimal wage $w = w_{zero} = \sqrt{\mathcal{A}}$ into the numerator of equation (47) and by showing that it is negative.

$$\underbrace{-2\mathcal{A}^5 + 4\mathcal{A}^3\mathcal{B} + 2\mathcal{A}(3\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C})}_{M} \underbrace{-\mathcal{E}\left(\mathcal{A}^4 + 2\mathcal{A}^2(\mathcal{B}+\mathcal{C}) + (\mathcal{B}+\mathcal{C})^2\right)}_{N} < 0 \qquad (49)$$

We now verify that both M and N are negative, which means that the whole term is negative. First we divide M by $\mathcal{A}$ and 2

$$-\mathcal{A}^4 + 2\mathcal{A}^2\mathcal{B} + (3\mathcal{B}+\mathcal{C})(\mathcal{B}+\mathcal{C}) < 0.$$

Differentiating with respect to $\mathcal{A}$ shows that it is decreasing in $\mathcal{A}$:

$$\begin{aligned} -4\mathcal{A}^3 + 4\mathcal{A}\mathcal{B} < 0 \\ -\mathcal{A}^2 + \mathcal{B} < 0. \end{aligned}$$

As $\mathcal{A} \geq \sqrt{-(\mathcal{B}+\mathcal{C})}$, we can insert $\sqrt{-(\mathcal{B}+\mathcal{C})}$ for $\mathcal{A}$, which results in

$$2\mathcal{B} + \mathcal{C} \leq 0.$$

This verifies that M is negative.

Turning to N, differentiating with respect to $\mathcal{A}$ gives

$$-4\mathcal{A}\mathcal{E}\big(\mathcal{A}^2 + (\mathcal{B} + \mathcal{C})\big) < 0.$$

The negativity is given as $\mathcal{A} \geq \sqrt{-(\mathcal{B} + \mathcal{C})}$. Consequently, N is falling in $\mathcal{A}$, and we can set $\sqrt{-(\mathcal{B} + \mathcal{C})} = \mathcal{A}$

$$-\mathcal{E}\left((\mathcal{B} + \mathcal{C})^2 - 2(\mathcal{B} + \mathcal{C})^2 + (\mathcal{B} + \mathcal{C})^2\right) = 0.$$

This proves that, for the special case $s_2 = 1$, $c(w)$ is decreasing in w in the relevant interval $w \geq w_{zero}$.

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