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Legislative Process with Open Rules

Theresa Fahrenberger and Hans Gersbach

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Theresa Fahrenberger
CER-ETH
Center of Economic Research
at ETH Zurich
8092 Zurich, Switzerland
tfahrenberger@ethz.ch

Hans Gersbach
CER-ETH
Center of Economic Research
at ETH Zurich
8092 Zurich, Switzerland
hgersbach@ethz.ch

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Abstract

We examine the legislative game with open rules proposed by Baron and Ferejohn (1989). We first show that the three-group equilibrium suggested by Baron and Ferejohn does not always obtain. Second, we characterize the set of stationary equilibria for simple and super majority rules. Such equilibria are either of the three-group or four-group type. The latter type tends to occur when the size of the legislature becomes larger. Moreover, four-group equilibria imply large delay costs.

Keywords: Baron/Ferejohn model, bargaining in legislatures, open rules, three-group and four-group equilibria

JEL Classification: D7

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1 Introduction

Collective decision rules are often applied in distributing resources among individuals in a society. In order to achieve efficiency and fairness, various collective choice processes with endogenous agenda setting have been proposed.[1] In an important and influential paper, Baron and Ferejohn (1989) (henceforth BF) propose a model of legislative bargaining with an endogenous agenda setting. A body of legislators, each representing the voters of their district, decide about how to distribute some benefits. With an open amendment rule, the proposed allocation is either seconded or amended prior to vote. In the former case, it is voted up or down. In the latter case, there is a 'run-off' election between the original proposal and the amendment. The winner becomes the standing proposal in the next round.

BF characterize the stationary equilibria when the process repeats itself until an allocation is seconded and receives a majority of the votes. They show that a three-group equilibrium occurs. The agenda-setter proposes that the cake be distributed between himself and a subset of voters (second group) that form a majority of voters. The third group contains the remaining individuals who do not receive anything. Those members who obtain resources will move the proposal if they are recognized next. If, however, one individual in the third group is recognized, the member will offer an amendment chosen in such a way as to defeat the motion on the floor, and the legislature will move to the next period. Hence the initial proposal is not necessarily accepted in the first session, which implies a costly delay.

BF assume that the agenda-setter does not differentiate between individuals who will move his proposal if recognized next (and vote for it against status quo) and those who will not second his proposal if recognized next but will vote for the proposal if it is pitted against the status quo. We will analyze this problem and answer the question what happens, if the agenda-setter distinguishes between these two groups and thus forms a proposal consisting of four groups.

In addition, we show in this paper that the agenda-setter may want to offer four-group proposals, so BF's three-group equilibrium does not always hold in such circumstances.

[1] Baron and Ferejohn (1989), Harrington (1986), Harrington (1990), Banks and Gasmi (1987), Ferejohn, Fiorina, and McKelvey (1987), Epple and Riordan (1987).

We give a complete characterization of the set of stationary equilibria for simple and super majority rules. Such equilibria are either of the three-group or four-group type. We identify sets of parameters where three-group or four-group equilibria occur. As a rule, four-group equilibria obtain if the size of legislature is larger.

The paper is organized as follows: In the next section we outline the legislative process. In section three we identify the types of stationary equilibria. In section four we provide the general theorem that characterizes stationary equilibria and discuss the relationship to BF. In the fifth section, we identify numerically the parameter constellations where three-group or four-group equilibria occur. Section six concludes.

2 The Model

2.1 The Problem

We consider the problem of a group of N individuals (legislators) distributing one unit of benefits (N odd and $N > 1$) amongst themselves. Individuals are indexed by i, j, k, or l. All individuals are assumed to be risk-neutral and have utility functions

$$U_i = \sum_{t=0}^{\infty} \delta^t x_{it}$$

where x_{it} is the share of the cake individual i receives in period t. Since the cake can only be distributed once, x_{it} can only be positive for one period at most.[2] The discount factor δ is assumed to be identical across individuals. The distribution of the cake is governed by a collective choice process containing a voting rule and by a recognition rule that determines which individual can make a proposal. Of course, an allocation is Pareto-efficient if and only if the whole cake is distributed in the first period, i.e. in $t = 0$.

[2]Theoretically, it would be possible to distribute the cake partially over many periods. But this is weakly dominated for proposal-makers, so this possibility is neglected in our analysis.

2.2 The Amendment Process and Open Rules

We examine the legislative process with open rules introduced by Baron and Ferejohn (1989). With an open rule the floor is subject to an amendment. Under an open rule, a member who is in a position to make a proposal or an amendment has to take into account the fact that his proposal will be pitted against another proposal. The entire collective choice process under an open rule is given by

Period t=0

(i) The first agenda-setter A is recognized by fair randomization and sets a proposal $x_A = (x_A^1, ...x_A^N)$. This proposal is called the motion on the floor.

(ii) The second agenda-setter B is recognized randomly. He can move the proposal x_A to a vote against the status quo, or he makes an amendment x_B.

(iii) Voting takes place either between x_A and status quo or between x_A and x_B. If there is a vote between the status quo and a proposal, then an α-majority rule is applied (for a given $\alpha \in [0.5, 1]$). If there is a vote between two proposals, the proposal that receives most of the votes is accepted and the game moves to the next period.

(iv) If x_A competes against the status quo and is voted up, the benefits are distributed. If x_A is voted down, the game moves to the next period.

Period t=1

(v) Depreciation of the cake with discount factor δ.

(vi) If x_A has lost against the status quo in the previous period, the game continues with (i), i.e. with recognition of a first agenda-setter.

(vii) If x_A has been pitted against x_B, the winner will get the same status as proposal x_A in period t=0, i.e. the winner becomes the motion on the floor. The game continues with (ii), i.e. with recognition of an amendment-setter.

Note that the game ends if and only if a proposal is moved and wins a majority against the status quo.

Like BF, we simplify the analysis by the following tie-breaking rule: If an individual is indifferent between two proposals he will vote for the one proposed last.[3]

2.3 Equilibrium Concept

We adopt the concept of stationary equilibria introduced by BF. The idea is that an individual will act the same way if he is in the same situation. To capture the notion of stationary equilibria, BF introduce structurally equivalent subgames defined as follows:

Definition
Two subgames are structurally equivalent if

1. the extant agendas at the initial nodes of the subgames are identical;
2. the sets of individuals who may be recognized next are the same;
3. the strategy sets of individuals are the same.

An equilibrium is said to be stationary if the continuation values are the same for each structurally equivalent subgame. A consequence of this definition is that the strategies are stationary in a stationary equilibrium. Stationary strategies are necessarily history-independent, and members are assumed to believe that moves off the equilibrium path are accidents that will not reoccur in future play. As observed by BF, in any stationary equilibrium the second proposal is a permutation of the first proposal. The reason is as follows: Any legislator recognized as second agenda-setter will only propose a distribution of the cake if this proposal wins against the proposal on the floor. After winning the ballot against the first proposal, the second agenda-setter faces the same situation as the prior agenda-setter. Therefore, in a stationary equilibrium the amendment will be a permutation of the first proposal as a suitable amendment will win against the proposal on the floor because of our tie-breaking rule and the first proposal generates the maximal expected payoff for an agenda setter.

[3]Note that abstention is a weakly dominated strategy since in the case of indifference there is no gain from abstention. We can therefore neglect abstention.

3 Stationary Equilibria

3.1 Observations

In this section, we identify the types of stationary equilibria. We start with some simple observations that motivate the construction of the equilibria. A member who has gained initial recognition for making a proposal takes into account the fact that his proposal must be sufficiently attractive to a subset of members to be seconded if one of those members is recognized for making an amendment. The amount of benefits that the initial agenda-setter must offer other members so that his proposal will be moved is denoted by M. Furthermore, he has to garantuee that in a ballot against status quo a majority votes for his proposal. BF state that the agenda-setter offers M to $m \geq \frac{N-1}{2}$ individuals if the simple majority rule is applied. The number m determines the probability of a proposal being moved and therefore accepted.

We examine α-majority rules ($\frac{1}{2} \leq \alpha \leq 1$) and allow that the equilibrium number m of individuals receiving M could be smaller than $\alpha N - 1$ and in particular smaller than $\frac{N-1}{2}$. A fourth group may arise, consisting of individuals who are offered a certain amount called T, so that they do not move the proposal when recognized next but vote for a proposal against the status quo. In that case, the group with members obtaining T forms an α-majority with the group that obtains M and the agenda-setter. We summarize this observation in the following proposition.

Proposition 1

An optimal proposal structure is characterized by

- *A subset of size m obtains M which is defined as the minimal amount such that these individuals will move the proposal when recognized next.*
- *A subset of size $\max\{\alpha N - 1 - m, 0\}$ obtains T which is defined as the minimal amount such that an individual will vote for a proposal against status quo.*
- *A subset of size $\min\{N - \alpha N, N - m - 1\}$ obtains.*
- *The agenda-setter gets the leftover: $1 - mM - \max\{\alpha N - 1 - m, 0\}T$.*

We will later characterize precisely the values M, T and m. Two simple observations help to sharpen the intuition. First, an individual when recognized as second agenda-setter has - one period later - the same power as the first agenda-setter (if he offers an amendment). Therefore, M has to be the discounted continuation value of the agenda-setter.

Second, if a ballot against status quo occurs, all individuals are equal regarding future periods as no agenda-setter for the next period is determined. Hence, T will correspond to the ex ante continuation value of all individuals before any further recognition.

Note that two different types of equilibria can occur.

Three-group equilibrium

- The agenda-setter obtains $x_a = 1 - mM$.
- m individuals obtain M with $m \geq \alpha N - 1$.
- $N - 1 - m$ members receive zero.

Four-group equilibrium

- The agenda-setter obtains $x_a = 1 - mM - (\alpha N - 1 - m)T$.
- m individuals obtain M with $0 < m < \alpha N - 1$.
- $\alpha N - 1 - m$ members obtain T.
- $(1 - \alpha)N$ members receive zero.

The intuition for the existence of four-group equilibria runs as follows: The agenda-setter faces the following trade-offs: He must make a proposal sufficiently attractive to a subset of members such that these individuals will move the proposal if they are recognized next. The larger the subset of such members is, the more expensive it is, and the smaller is the share of the agenda-setter. The agenda-setter simultaneously has to ensure that there are enough votes if a proposal is voted up or down. For the latter purpose, a smaller amount of benefits is needed as these individuals do not count with the value of being recognized. Hence, it may be optimal for the agenda-setter to create four groups in his proposal.

3.2 Amendment Choice

We next examine the determination of an amendment. As stated in the last section, in equilibrium, any amendment will be a permutation of the proposal on the floor. We first discuss which type of permutation will win against a prior proposal characterized by Proposition 1. In the case where $m > \frac{N-1}{2}$, any permutation wins the ballot. Every individual who receives at least M under an amendment (except the first agenda-setter) and the amendment-setter will vote for it. If $m \leq \frac{N-1}{2}$, the amendment-setter cannot choose any permutation to win against the first proposal. It is not necessarily ensured that $\frac{N+1}{2}$ individuals obtain at least as much as under the proposal on the floor. This is illustrated by the following example.

Example: Suppose that a legislature consisting of $N = 11$ individuals uses an α-majority rule with $\alpha = 0.75$, i.e. a majority of 9 individuals is required to adopt a proposal. We consider a first proposal and an amendment that is a permutation of the first proposal. The proposals are illustrated in the following table.

Table A: First proposal and amendment.

	1	2	3	4	5	6	7	8	9	10	11
First proposal	x_a	M	M	M	M	T	T	T	T	0	0
Amendment	0	T	T	T	T	0	M	M	M	M	x_a

x_a denotes the share of the agenda-setter, i.e. we assume that individual 1 is the first agenda-setter and individual 11 is recognized as second agenda-setter. Furthermore, we assume that $m = 4 < \frac{N-1}{2} = 5$ individuals obtain M (individuals 2 to 5). Hence, a group of $4 = \alpha N - m - 1$ individuals (individuals 6 to 9) is offered T under the first proposal. We will show later that $0 < T < M < x_a$ which is assumed here. Hence, the amendment-setter and all individuals who obtain M under the amendment vote for it, i.e. $m + 1 = 5 = \frac{N-1}{2}$. All other individuals vote for the first proposal. Therefore, this amendment would not win.

In the following we assume that the second agenda-setter chooses the amendment in the spirit of the priority (or tie-breaking) rule of BF. This allows us to compare our results with BF and replicate their result. The priority rule is part of Proposition 2 and Proposition 3. We will discuss the significance of the priority rules and the work of Primo (2007) on this issue in section 4.4.

Since the equilibrium strategies differ depending on whether $m \geq \frac{N-1}{2}$ or $m < \frac{N-1}{2}$, we differentiate between these two cases. First, we provide the equilibrium in the case where $m \geq \frac{N-1}{2}$. The special case $\alpha = \frac{1}{2}$ and $m \geq \frac{N-1}{2}$ is the BF result.

3.3 The Case $m \geq \frac{N-1}{2}$

Proposition 2

Suppose that a society uses the α-majority rule ($\alpha \in [0.5, 1]$) and a random recognition rule within an open rule amendment process. The number m of individuals who receive enough to move a proposal is exogenously restricted to $m \geq \frac{N-1}{2}$. Then a stationary equilibrium exists and is given by the following:

- *The individual recognized first makes a proposal x: M to m members (where $N-1 \geq m \geq \frac{N-1}{2}$), T to $\max\{\alpha N - 1 - m, 0\}$ other members, $x_a = 1 - mM - \max\{\alpha N - 1 - m, 0\}T$ to himself, and 0 to the rest.*

- *If the member recognized next is one of those m individuals, he moves the previous proposal. The proposal x is accepted with a majority of $\max\{m+1, \alpha N\}$ votes.*

- *If the member recognized next is one of the $N-1-m$ individuals, he makes a proposal that is a permutation of x: x_a to himself, M to m other members, excluding the first agenda-setter and including all individuals who would have received zero or T in the first proposal. The other individuals who obtain M or T in this proposal are chosen randomly among the m members who were offered M in the initial proposal. This amendment defeats the prior proposal.*

- *An individual will vote for a proposal (against the status quo) if he receives at least T.*

The proof of Proposition 2 is given in the appendix. The equilibrium proposal is uniquely determined by m that depends (uniquely) on the discount factor δ, the required size of majority α and the size of the legislature N. In the following, we denote the equilibrium strategies in Proposition 2 by index L indicating the "large" m. The proposal is denoted by x^L, the strategies are called L-strategies.

According to the proof of Proposition 2, the equilibrium function $m^L(\delta, \alpha, N)$ is given by

$$\begin{aligned}
m^L(\delta,\alpha,N) &= \arg\max_m (V_a^m(x^L)) \\
& \text{with} \\
V_a^m(x^L) &= \frac{\frac{m}{N-1} + \Big[(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})A1\,\delta^2 - \frac{m}{N-1}\,h\Big]\,T}{1+\frac{m^2}{N-1}\,\delta - (1-\frac{m}{N-1})\Big(\frac{1}{N-1} + (1-\frac{m+1}{N-1})A2\Big)\,\delta^2} \\
& \text{where we use the abbreviations} \\
A1 &= \frac{1}{B}\Big(\frac{h}{N-1}(1-\frac{m}{N-1})\delta\Big) \\
A2 &= \frac{\delta}{B}\Big[\frac{m}{N-1} + \delta\big(1-\frac{m}{N-1}\big)\big(\frac{N-m-2}{m}\big)\big(\frac{1}{N-1}\big)\Big] \\
B &= 1 - \frac{2m-N+2}{m}\big(1-\frac{m}{N-1}\big)\delta - \frac{N-m-2}{m}\big(1-\frac{m+1}{N-1}\big)\big(1-\frac{m}{N-1}\big)\delta^2 \\
h &= \max\{\alpha N - 1 - m, 0\} \\
T &= \frac{m\delta}{N((1-\delta)(N-1)+m\delta)} \qquad (1)
\end{aligned}$$

$V_a^m(x^L)$ denotes the expected value of the individual recognized first if he offers the equilibrial proposal x^L.

In the next section we examine the case where m has to be smaller than $\frac{N-1}{2}$.

3.4 The Case $m < \frac{N-1}{2}$

In this section we derive an equilibrium for the same situation as in section 3.3, but we assume exogenously that m has to be smaller than $\frac{N-1}{2}$. An immediate result is that only four-group equilibria can occur.

Proposition 3

Suppose a society of N individuals using the α-majority rule with $\alpha \geq \frac{1}{2}$ in an open rule amendment process. The number m of individuals who are offered enough to move the proposal is exogenously restricted to $m < \frac{N-1}{2}$. Then a stationary equilibrium exists and is given by

- *The agenda-setter recognized first makes a proposal x: M to m members (where $1 \leq m < \frac{N-1}{2}$), T to $\alpha N - m - 1$ other members, $x_a = 1 - mM - (\alpha N - m - 1)T$ to himself, and 0 to the rest.*

- *If the individual recognized next is one of the m members, he will move the proposal to a vote against the status quo. The proposal is accepted with an α-majority.*

- *If the individual recognized next is one of the members who receive T or zero in the first proposal, he will offer an amendment that is a permutation of the first proposal:*
  x_a to himself, zero to the previous agenda-setter, M to m members chosen randomly from the $N-2-m$ individuals who received T or zero in the first proposal. All remaining members of the group that received zero or T under the first proposal obtain T in the amendment. The individuals who received M in the first proposal receive zero or T, which is again chosen randomly.

- *All members who receive at least T will favor the proposal if it is pitted against status quo.*

The proof of Proposition 3 is given in the appendix. Again, the equilibrium in this case is uniquely determined by m that depends on δ, α and N. In the following, we assign this equilibrium by index S (for "small" m), e.g. the proposal will be denoted by x^S.

The proof of Proposition 3 yields

$$m^S(\delta,\alpha,N) = \arg\max_m (V_a^m(x^S))$$

with

$$V_a^m(x^S) = \frac{\frac{m}{N-1}+\left[(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})\frac{D1}{E}\,\delta^2-\frac{m(\alpha N-m-1)}{N-1}\right]T}{1+\frac{m^2}{N-1}\,\delta-(1-\frac{m}{N-1})\left[\frac{1}{N-1}+(1-\frac{m+1}{N-1})\frac{D2}{E}\right]\delta^2}$$

where $D1$, $D2$ and E are given by

$$D1 := (1-\tfrac{m}{N-1})(\tfrac{m}{N-m-2})\tfrac{m}{N-1}p_k\,\delta + (1-\tfrac{m}{N-m-2})\tfrac{m}{N-1}p_l$$

$$D2 := (\tfrac{m}{N-m-2})\tfrac{m}{N-1}\delta + (\tfrac{m}{N-m-2})(1-\tfrac{m}{N-1})\tfrac{1}{N-1}\,\delta^2 + (1-\tfrac{m}{N-m-2})\tfrac{1}{N-1}\,\delta$$

$$E := 1-(\tfrac{m}{N-m-2})(1-\tfrac{m}{N-1})(1-\tfrac{m+1}{N-1})\,\delta^2 - (1-\tfrac{m}{N-2-m})(1-\tfrac{m+1}{N-1})\,\delta$$

with $p_k = 1-\min\{\frac{(1-\alpha)N}{m},1\}$ and $p_l = \min\{\frac{\alpha N-1-m}{N-2m-2},1\}$. Note that p_k denotes the probability of an individual who received M in the first proposal obtaining T (instead of zero) under an amendment, while p_l denotes the probability of an individual who received T or zero in the first proposal obtaining T under an amendment.

The value T is again given by equation (1). However, the values T^S and T^L differ since $T = T(m, \delta, N)$ and $m^S \neq m^L$. Moreover, T is increasing in m. We therefore obtain the result that $T^S < T^L$ because $m^S < m^L$.

4 The Overall Equilibrium

In the previous section we derived equilibria restricted by the assumptions that either $m \geq \frac{N-1}{2}$ or $m < \frac{N-1}{2}$, using some tie-breaking rules regarding the choice of amendments. We will now summarize these results to cover both cases and formulate our main result.

4.1 The Main Result

In equilibrium, an agenda-setter will always maximize his share of the cake. He will therefore compare the expected payoffs under both restrictions $m \geq \frac{N-1}{2}$ and $m < \frac{N-1}{2}$ and choose the alternative with the higher expected value.

Theorem 1

Suppose a society with N members using the α-majority rule with $\alpha \geq \frac{1}{2}$ and discount factor $\delta \in [0, 1]$ in an open amendment process. Then a stationary equilibrium exists and is given by

- *The agenda-setter recognized first calculates $V_a^{m^L}(x^L)$ and $V_a^{m^S}(x^S)$. He sets*

$$m^* = \begin{cases} m^L, & V_a^{m^L}(x^L) \geq V_a^{m^S}(x^S) \\ m^S, & V_a^{m^L}(x^L) < V_a^{m^S}(x^S) \end{cases} \tag{2}$$

- *If $m^* = m^L$ then all individuals play the L-strategies according to Proposition 2. If $m^* = m^S$ then all individuals play the S-strategies according to Proposition 3.*

Henceforth we denote the compounded equilibrium proposal by x^*.

4.2 Properties of the Equilibrium

It is useful to discuss some properties of the equilibrium.

Property 1: $x_a^* \geq M^*$

The property is proved by contradiction: Suppose $x_a^* < M^* = \delta V_a^{m^*}(x^*)$. Then using equation (3) in the proof of Proposition 2 we obtain

$$V_a^{m^*}(x^*) < \frac{N-1-m}{N-1-\delta\, m}\, \delta V_L^{m^*}(x^*) \leq V_L^{m^*}(x^*).$$

$V_a^{m^*}(x^*)$ denotes the expected value of the agenda-setter, while $V_L^{m^*}(x^*)$ denotes the expected value of an individual who receives 0 under the proposal x^*. Therefore this is a contradiction.

Property 2: *The probability that a proposal is accepted may be lower than in the BF model.*

Property 2 follows from the following consideration: The number of individuals receiving M represents the probability of a proposal being seconded and therefore accepted. If a four-group equilibrium is formed with $m^* = m^S$, this probability becomes lower than one half. The probability that a proposal will be accepted shrinks, also in $t = 0$. This implies higher costly delay than in the BF model.

Property 3: *Playing the S-strategies generates a less equal distribution than playing the L-strategies.*

A four-group equilibrium with $m^* = m^S$ generates a less equal distribution. The number of individuals who receive at least something remains the same, i.e. αN, but while $m^S + 1$ individuals receive more in comparison with the L-strategies, the third group consisting of $\alpha N - 1 - m^S$ individuals obtains less.

Taking derivatives of T with respect to N, δ, or m yields the following properties:

Property 4: *The amount T is decreasing in N.*

A consequence of this property is that a four-group equilibrium becomes more attractive for the agenda-setter, the larger N is. Our simulation results suggest that for all $\alpha \in [0.5, 1]$ and $\delta \in [0, 1]$ there is a critical value $\hat{N}$ such that a four-group equilibrium arises if $N \geq \hat{N}$. Furthermore, only equilibria with $m^* < \frac{N-1}{2}$ will occur for N sufficiently large.

Property 5: *The amount T is increasing in δ.*
Individuals are more patient if δ is large. The agenda-setter must offer a higher amount for individuals to vote for his proposal if it is pitted against status quo. Moreover, it is straightforward to show that $T \in [0, \frac{1}{N}]$.

Property 6: *The amount T is increasing in m.*
The intuition of this result is as follows: If the probability of a proposal being moved and therefore accepted rises, then the expected value of an individual before first recognition will rise, i.e. T will rise.

It is intuitively clear that we also have $T \leq M$. Individuals who receive M support the proposal in two ways. First, they will move the proposal if recognized to make an amendment, and second they will vote for a proposal if it is pitted against status quo. The amount T only covers the vote for a proposal against status quo. The agenda-setter has to offer more to an individual who is already recognized for making an amendment than to an individual who is not recognized.
Note that equality between M and T occurs if and only if $\delta = 1$ and $\alpha = 1$, i.e. when unanimity is required and individuals are infinitely patient. In this case, the agenda-setter recognized first will offer $\frac{1}{N}$ to everybody. This proposal is moved and accepted in the first period, i.e. in $t = 0$.

4.3 Relation to Baron-Ferejohn Model

We look at the special case $\alpha = \frac{1}{2}$. Baron and Ferejohn (1989) state that if the simple majority rule is applied within an open amendment process, then every stationary equilibrium involves the proposal $x^* = x^L$, i.e. $m^* \geq \frac{N-1}{2}$, and all individuals play the L-strategies. With the following proposition, which is proved in the appendix, we show that there are parameter constellations (given by $\alpha = 0.5$, $\delta \in [0, 1]$, $N \geq 3$) where four-group equilibria occur, and, moreover, where a stationary equilibrium does not necessarily include the L-strategies, i.e. $x^* \neq x^L$. This amends the theorem of BF.

Proposition 4

Suppose $\alpha = \frac{1}{2}$.

(i) If m *is constrained by* $m \geq \frac{N-1}{2}$*, only three-group equilibria arise.*

(ii) A three-group equilibrium may not exist.

(iii) There exist four-group equilibria.

The proof of Proposition 4 is given in the appendix.

As already noted, we obtain the same result as BF if we set $\alpha = \frac{1}{2}$ and restrict m to $m \geq \frac{N-1}{2}$, as we use the same priority rule regarding the type of amendments in equilibrium as BF. In the next section we will discuss alternative of priority rules.

4.4 Priority Rules for Amendment Choice

Our results have been derived for an amendment setting that reflects two considerations. First, it is never optimal to bribe the first agenda-setter to vote for the second proposal. The price is too high. Therefore, the prior agenda-setter obtains 0 under an amendment in equilibrium. Second, as the amendment-setter is indifferent about whom to give M, T or 0 among the remaining $N-$ individuals as long as his proposal wins, we have chosen a particular priority rule. This rule is in the spirit of BF and allows us to replicate their result. Furthermore, the priority rule reflects an element of 'fairness': The second agenda-setter gives M first to individuals who have received T or 0 under the first proposal. Primo (2007) has observed that an amendment can follow other randomization strategies when deciding whom to make offers to and each of those strategies can be sustained as a distinct equilibrium with slightly different payoffs. Our results can be extended in the spirit of Primo (2007) to other randomization strategies which produces distinct four-group and three-group equilibria.

In the next section we look in more detail at parameter constellations where L- or S-strategies are played and therefore three- or four-group equilibria occur.

5 L- versus S-Strategies

5.1 Graphic Examples

As we have shown in the previous section, there are situations where a four-group equilibrium leads to a higher expected value for an agenda-setter than a three-group equilibrium. To give an impression of the relation between the outcomes using the L-strategies and the S-strategies, we provide some numerical examples in this section. We choose $\alpha = 0.5$ and $\delta \in \{0.1, 0.5, 0.9\}$ and plot $V_a^{m^L}(x^L)$ and $V_a^{m^S}(x^S)$. The size of the legislature N runs from 3 to 150.

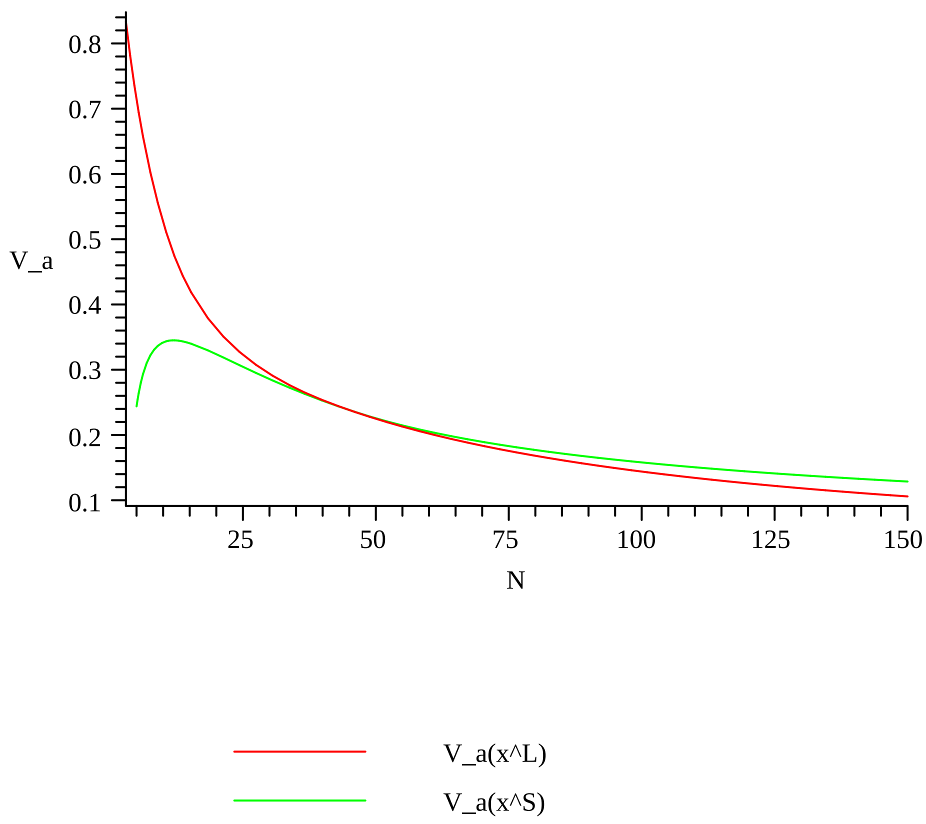

Figure 1: $\alpha = 0.5$, $\delta = 0.1$

The graphs show that the four-group equilibrium with $m < \frac{N-1}{2}$ becomes more attractive the larger N is. The figures suggests the existence of a threshold $\hat{N}$ such that only four-group equilibria arise if $N > \hat{N}$.

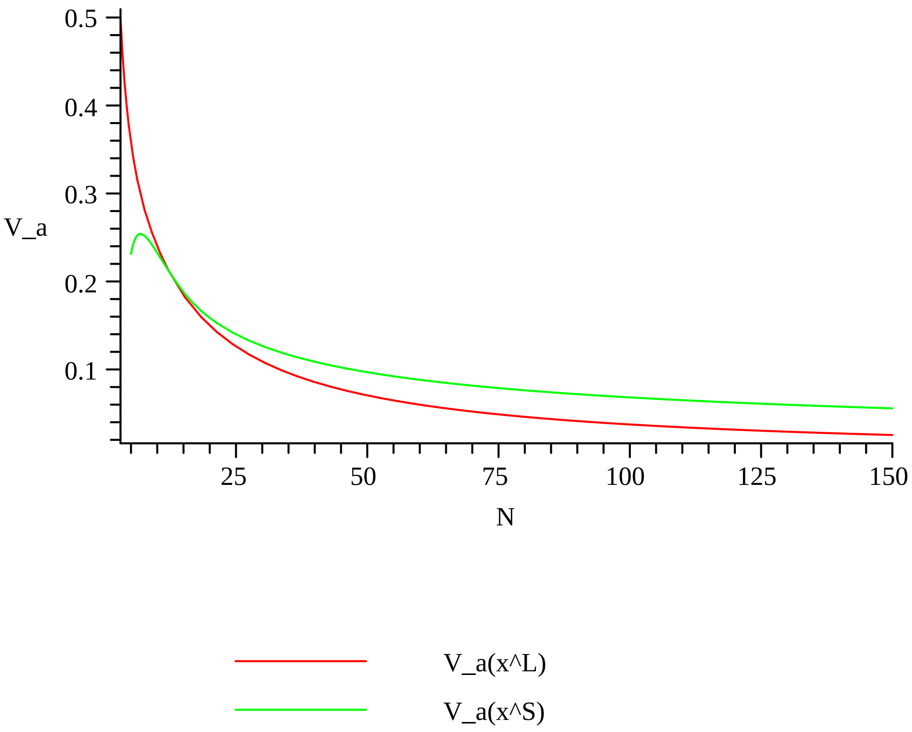

Figure 2: $\alpha = 0.5$, $\delta = 0.5$

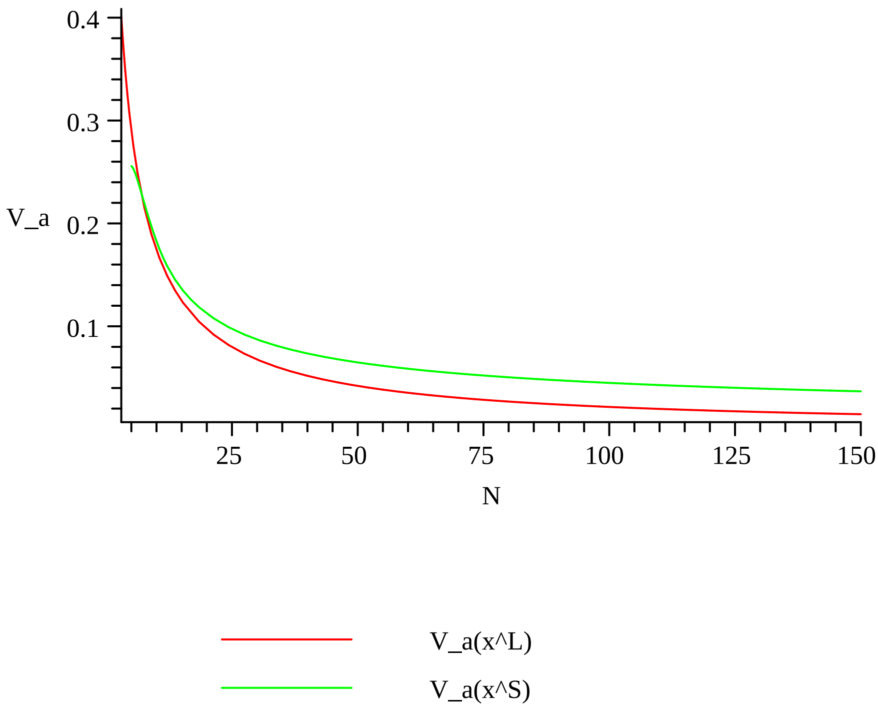

Figure 3: $\alpha = 0.5$, $\delta = 0.9$

5.2 Calculations

The following tables illustrate results of Proposition 2 and Proposition 3. In the first (Table 1a) we describe the L-strategies and indicate in the last column whether a four-group equilibrium occurs. In the second table (Table 1b) the S-strategies are described. Now m is always smaller than $\alpha N - 1$. The proposal consists of four groups: The agenda-setter receiving $x_a = 1 - mM - (\alpha N - 1 - m)T$, the individuals who receive $M = \delta V_a^{m^*}(x^*)$, the individuals who obtain T, and the other members who obtain zero. In the last column we indicate whether the S-strategies occur in equilibrium. The calculations are made for $\alpha = 0.75$, $\delta \in \{0.1, 0.5, 0.9\}$ and $N \in \{5, 11, 101, 1001\}$.

Further tables are given in the appendix. Table 2a (2b) describes the L-strategies (S-strategies) for $\alpha = 0.5$ and $\delta \in \{0.1, 0.5, 0.9\}$. Tables 3a and 3b delineate the L- and S-strategies, respectively, for $\alpha = 0.9$ and $\delta \in \{0.1, 0.5, 0.9\}$. The last tables 4a (4b) illustrate the L-strategies (S-strategies) for $\alpha = 1$, i.e. when unanimity is required, and $\delta \in \{0.1, 0.5, 0.9\}$.

Table 1a.

N	α	δ	m^L	M^L	T^L	$V_a^{m^L}(x^L)$	x_a	4-group-equil.
5	0.75	0.1	4	0.07	0.02	0.7	0.714	
11	0.75	0.1	10	0.05	0.009	0.5	0.5	
101	0.75	0.1	50	0.014	0.0005	0.14	0.28	X
1001	0.75	0.1	500	0.0019	0.00005	0.019	0.038	X
5	0.75	0.5	3	0.18	0.09	0.356	0.467	
11	0.75	0.5	5	0.105	0.03	0.21	0.4	X
101	0.75	0.5	50	0.017	0.0033	0.034	0.066	X
1001	0.75	0.5	500	0.0018	0.00033	0.0037	0.007	X
5	0.75	0.9	2	0.23	0.16	0.26	0.42	X
11	0.75	0.9	5	0.12	0.07	0.14	0.23	X
101	0.75	0.9	50	0.015	0.008	0.017	0.03	X
1001	0.75	0.9	500	0.0016	0.0008	0.0018	0.003	X

Table 1 b.

N	α	δ	m^S	M^S	T^S	$V_a^{m^S}(x^S)$	x_a	$V_a^{m^S}(x^S) > V_a^{m^L}(x^L)$
5	0.75	0.1	1	0.024	0.0054	0.242	0.966	
11	0.75	0.1	4	0.034	0.0039	0.34	0.85	
101	0.75	0.1	32	0.016	0.00034	0.156	0.49	X
1001	0.75	0.1	99	0.005	0.000011	0.05	0.5	X
5	0.75	0.5	1	0.11	0.04	0.22	0.82	
11	0.75	0.5	4	0.104	0.026	0.207	0.5	
101	0.75	0.5	13	0.033	0.0011	0.066	0.5	X
1001	0.75	0.5	44	0.011	0.00004	0.022	0.49	X
5	0.75	0.9	1	0.19	0.14	0.21	0.57	
11	0.75	0.9	4	0.12	0.07	0.14	0.28	X
101	0.75	0.9	9	0.035	0.0044	0.039	0.39	X
1001	0.75	0.9	29	0.013	0.0002	0.014	0.48	X

The tables confirm the relationship derived in section 5.1 between L- and S-strategies for $\alpha > 0.5$. The strategy $m = m^S$ yields a higher expected payoff for the agenda-setter if N is sufficiently large. The tables also illustrate examples where a four-group equilibrium occurs with $x^* = x^L$, i.e. $m^* \geq \frac{N-1}{2}$. One such parameter constellation is given by $\delta = 0.5$, $\alpha = 0.75$ and $N = 11$.

Note that the share x_a that the agenda-setter proposes for himself is much higher in the proposal x^S even if $V_a^{m^S}(x^S) < V_a^{m^L}(x^L)$. This is because the probability of a proposal being seconded and hence adopted is much smaller under S-strategies. The higher risk of being rejected when using the S-strategies is compensated by a higher payoff if accepted.

6 Conclusion

We have examined the BF model of an open amendment rule, using the concept of stationary equilibria. We have shown that the equilibrium consideration of BF need to be amended, as the possibility of a four-group equilibrium has to be taken into account. We have proved that there is a situation where the BF equilibrium is not an equilibrium. Moreover, calculations indicate that a four-group equilibrium is always better if the number of individuals N is sufficiently large.

Our results may have implications for two areas of research. First, the BF approach has been applied to a wide variety of questions and has been investigated experimentally (e.g. Frechette, Kagel, and Lehrer (2002)). Hence, as far as the role of open rules is concerned, our results call for some qualifications. Second, the comparison between closed and open rules in legislatures may be amended. On the one hand, the power of the member recognized under an open rule may be still considerably high. On the other hand, the costs of delay under open rules may be much higher, as the probability of a proposal being accepted is lower than one half in four-group equilibrium with $x^* = x^S$.

7 Appendix

Proof of Proposition 2

We would like to calculate the continuation value of the agenda-setter. Note, that his 'real' share of the cake, denoted by x_a, is the left-over of the cake after paying other individuals to move the proposal and / or to vote for his proposal against status quo, i.e. the whole cake is distributed and $x_a = 1 - mM - \max\{\alpha N - 1 - m, 0\}T$. We will calculate the amounts M and T in a first step.

- Calculation of M:
  Let V_a denote the continuation value of the agenda-setter. The amount needed to bribe legislators to move the proposal is given by the discounted continuation value of the agenda-setter. We obtain the condition

  $$M \geq \delta V_a \,.$$

  Since every agenda-setter a will maximize his own share x_a we can assume equality.

- Calculation of T:
  Using symmetry we have that conditional on acceptance in the next period the continuation value for each individual is $\frac{1}{N}$ as every agenda-setter will propose to distribute the whole cake and individuals are ex ante identical. The probability that a proposal is accepted in a particular period is given by $\mathbb{P}[A] = \frac{m}{N-1}$, if m is the number of legislators that are bribed to move the proposal. The critical value needed for an individual to vote for a proposal (against status quo) - without being recognized as agenda-setter - is denoted by T and is equal to the continuation payoff from rejecting a proposal (in a ballot against status quo). Thus, T is given by

  $$T = \mathbb{P}[A]\delta\frac{1}{N} + (1 - \mathbb{P}[A])\mathbb{P}[A]\frac{\delta^2}{N} + (1 - \mathbb{P}[A])^2\mathbb{P}[A]\frac{\delta^3}{N} + ...$$

  and accordingly solves the following equation

  $$T = \delta\Big[\mathbb{P}[A]\frac{1}{N} + (1 - \mathbb{P}[A])T\Big].$$

Isolating T yields

$$T = \frac{\delta \mathbb{P}[A]}{N[1-\delta(1-\mathbb{P}[A])]} = \frac{\delta m}{N[N-1-\delta(N-1-m)]}.$$

Now we can calculate the continuation value V_a of the agenda-setter. We obtain

$$V_a = \frac{m}{N-1} x_a + (1 - \frac{m}{N-1}) \delta V_L \tag{3}$$

where V_L denotes the continuation value of a member recognized next who is offered zero, e.i. a 'loser'. This covers the case where an amendment is made under which the prior agenda-setter receives zero.

The continuation value V_L is given by

$$V_L = \frac{m}{N-1} 0 + \frac{1}{N-1} \delta V_a + (1 - \frac{m+1}{N-1}) \delta V_W$$

With probability $\frac{m}{N-1}$ the proposal is moved and therefore accepted. With probability $\frac{1}{N-1}$ the individual will be recognized as amendment-setter and with probability $1-\frac{m+1}{N-1}$ an amendment is made by an other individual so that (using our tie-breaking rules) the indvidual under consideration obtains M under the amendment, i.e. becomes a 'winner'.

The continuation value V_W of an individual who is offered M is given by

$$V_W = \frac{m}{N-1} M + (1 - \frac{m}{N-1}) \delta V_S = \frac{m}{N-1} \delta V_a + (1 - \frac{m}{N-1}) \delta V_S$$

Again, the proposal will be accepted with probability $\frac{m}{N-1}$ (which includes recognizing the individual under consideration as amendment-setter). With probability $1 - \frac{m}{N-1}$ an amendment will be made under which he may obtain 0, T or M. V_S denotes the continuation value in this case, i.e. of a first proposal winner if an amendment is made. The amendment-setter will offer $2m - N + 2$ of the prior proposal winners M again. They are chosen randomly. If the equilibrial proposal is a four-group equilibrium the amendment-setter will also choose randomly $\alpha N - m - 1$ prior proposal winners to obtain T under the amendment. All other first proposal winners obtain zero. Note that individuals who receive T act in the same way as individuals who obtain 0 in the first proposal when they are recognized as amendment-setter, i.e. as second agenda-setter.

This yields to

$$
\begin{aligned}
V_S &= \tfrac{2m-N+2}{m}\Big[\tfrac{m}{N-1}\delta V_a + (1-\tfrac{m}{n-1})\delta V_S\Big] \\
&+ \tfrac{N-m-2}{m}\Big[\tfrac{m}{N-1}\Big(\tfrac{\max\{\alpha N-m-1,0\}}{N-2-m}T + (1-\tfrac{\max\{\alpha N-m-1,0\}}{N-2-m})0\Big) \\
&+\tfrac{1}{N-1}\delta V_a + (1-\tfrac{m+1}{N-1})\delta V_W\Big]
\end{aligned}
$$

With this equation we can calculate V_a (that depends on m) and then obtain

$$m^L(\delta,\alpha,N) = \max\{m \geq \frac{N-1}{2} \,:\, V_a(m) \geq V_a(m')\,\forall\, m' \in \{\frac{N-1}{2},...,N-1\}\}$$

Calculation of V_a:

Notation: $h := \max\{\alpha N - m - 1, 0\}$ = number of individuals who are offered T.

$$
\begin{aligned}
V_S &= \frac{(\frac{2m-N+2}{m})(\frac{m}{N-1}) + (\frac{N-m-2}{m})(\frac{1}{N-1})}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta}\,\delta V_a \\
&+\frac{(\frac{N-m-2}{m})(1-\frac{m+1}{N-1})}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta}\,\delta V_W + \frac{(\frac{N-m-2}{m})(\frac{h}{N-2-m})(\frac{m}{N-1})}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\,\delta}T \\
V_W &= \frac{m}{N-1}\,\delta V_a + (1-\frac{m}{N-1})\,\delta V_S \\
&= \frac{m}{N-1}\,\delta V_a + \frac{(1-\frac{m}{N-1})\Big[(\frac{2m-N+2}{m})(\frac{m}{N-1}) + (\frac{N-m-2}{m})(\frac{1}{N-1})\Big]}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta}\,\delta^2 V_a \\
&+\frac{(1-\frac{m}{N-1})(\frac{N-m-2}{m})(1-\frac{m+1}{N-1})}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta}\,\delta^2 V_W \\
&+\frac{\frac{h}{N-1}(1-\frac{m}{N-1})\delta}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\,\delta}T \\
\Leftrightarrow V_W &= \frac{\frac{m}{N-1} + (1-\frac{m}{N-1})(\frac{N-m-2}{m})(\frac{1}{N-1})\delta}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta - (\frac{N-m-2}{m})(1-\frac{m+1}{N-1})(1-\frac{m}{N-1})\delta^2}\,\delta V_a \\
&+\frac{(\frac{N-m-2}{m})(\frac{h}{N-2-m})(\frac{m}{N-1})(1-\frac{m}{N-1})\delta}{1-(\frac{2m-N+2}{m})(1-\frac{m}{N-1})\delta - (\frac{N-m-2}{m})(1-\frac{m+1}{N-1})(1-\frac{m}{N-1})\delta^2}T \\
&=: \frac{\frac{m}{N-1} + (1-\frac{m}{N-1})(\frac{N-m-2}{m})(\frac{1}{N-1})\delta}{B}\,\delta V_a + \frac{(\frac{h}{N-1})(1-\frac{m}{N-1})\delta}{B}T \\
&=: A2\,V_a + A1\,T
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow V_L &= \frac{1}{N-1}\,\delta V_a + (1-\frac{m+1}{N-1})\,\delta V_W \\
&= \frac{1}{N-1}\,\delta V_a + (1-\frac{m+1}{N-1})A2\,\delta V_a \\
&\quad +(1-\frac{m+1}{N-1})A1\,\delta\,T \\
&= \Big[\frac{1}{N-1} + (1-\frac{m+1}{N-1})A2\Big]\,\delta V_a + (1-\frac{m+1}{N-1})A1\,\delta\,T
\end{aligned}
$$

Inserting V_L into V_a we can calculate the continuation value of the agenda-setter, depending on the exogenous parameter N, δ, α and the (maximizing) variable m.

$$
\begin{aligned}
V_a &= \frac{m}{N-1}\,x_a + (1-\frac{m}{N-1})\,\delta V_L \\
&= \frac{m}{N-1}(1-m\,\delta V_a - \,hT) \\
&\quad +(1-\frac{m}{N-1})\Big[\frac{1}{N-1} + (1-\frac{m+1}{N-1})A2\Big]\,\delta^2 V_a \\
&\quad +(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})A1\,\delta^2\,T \\
\Rightarrow V_a &= \frac{m}{N-1} - \frac{m^2}{N-1}\,\delta V_a \\
&\quad +(1-\frac{m}{N-1})\Big[\frac{1}{N-1} + (1-\frac{m+1}{N-1})A2\Big]\,\delta^2 V_a \\
&\quad +\Big[(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})A1\,\delta^2 - \frac{m}{N-1}\,h\Big]\,T \\
\Leftrightarrow V_a &= \frac{\frac{m}{N-1} + \Big[(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})A1\,\delta^2 - \frac{m}{N-1}\,h\Big]\,T}{1+\frac{m^2}{N-1}\,\delta - (1-\frac{m}{N-1})\Big(\frac{1}{N-1} + (1-\frac{m+1}{N-1})A2\Big)\,\delta^2} \\
&= V_1^m(x^L)
\end{aligned}
$$

Note that the sum of the shares in the equilibrium proposal is 1:
$x_a + mM + \max\{\alpha N - 1 - m, 0\}T + \min\{(1-\alpha)N, N-1-m\}0 = 1.$

■

Proof of Proposition 3

The proof of Proposition 3 follows the same outline as the proof of Proposition 2. Again we have

$$
\begin{aligned}
M &= \delta V_a \\
T &= \frac{\delta m}{N[N-1-\delta(N-1-m)]}
\end{aligned}
$$

and obtain as continuation value of the agenda-setter

$$V_a = \frac{m}{N-1}\, x_a + (1 - \frac{m}{N-1})\, \delta\, V_L\,.$$

where V_L is again the continuation value of an individual who receives zero in a proposal. V_L is given by

$$V_L = \frac{m}{N-1}\, 0 + \frac{1}{N-1}\, \delta\, V_a + (1 - \frac{m+1}{N-1})\, \delta\, V_{\overline{W}}$$

where $V_{\overline{W}}$ denotes the continuation value of an individual who would have obtained 0 or T in the first proposal and therefore receives $M = \delta V_a$ (with probability $\frac{m}{N-m-2}$) or T or 0 (with probability $1 - \frac{m}{N-m-2}$) in an amendment, i.e. becomes a 'winner'. Recall that $p_l = \min\{\frac{\alpha N-1-m}{N-2m-2}, 1\}$ denotes the probability of an individual who would have received T or 0 under the first proposal obtaining T instead of 0 under an amendment. With this notation we obtain

$$\begin{aligned} V_{\overline{W}} \;\; = \;\; & \frac{m}{N-m-2}\Big[\frac{m}{N-1}\, M + (1 - \frac{m}{N-1})\delta\, V_{\overline{S}}\Big] \\ & + (1 - \frac{m}{N-m-2})(\frac{m}{N-1})\Big(p_l\, T + (1-p_l)\, 0\Big) \\ & + (1 - \frac{m}{N-m-2})\Big[\frac{1}{N-1}\, \delta\, V_a + (1 - \frac{m+1}{N-1})\delta\, V_{\overline{W}}\Big] \end{aligned}$$

$V_{\overline{S}}$ denotes the continuation value of an individual who was offered M in the first proposal and therefore receives 0 or T under an amendment. It is given by

$$\begin{aligned} V_{\overline{S}} \;\; = \;\; & \frac{m}{N-1}\Big[\min\{\frac{(1-\alpha)N}{m}, 1\}\, 0 + (1 - \min\{\frac{(1-\alpha)N}{m}, 1\})\, T\Big] \\ & + \frac{1}{N-1}\, \delta\, V_a + (1 - \frac{m+1}{N-1})\delta\, V_{\overline{W}} \end{aligned}$$

Recall that $p_k = 1 - \min\{\frac{(1-\alpha)N}{m}, 1\}$ denotes the probability of an individual who would have received M under the first proposal obtaining T under an amendment. With this notation and the equations obtained above we can calculate V_a.

$$
\begin{aligned}
V_{\overline{W}} &= \tfrac{m}{N-m-2}\cdot\tfrac{m}{N-1}\delta V_a + \tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})\tfrac{m}{N-1}\cdot p_k\cdot\delta T \\
&\quad + \tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})\tfrac{1}{N-1}\delta^2 V_a + \tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})(1-\tfrac{m+1}{N-1})\delta^2 V_{\overline{W}} \\
&\quad + (1-\tfrac{m}{N-m-2})\tfrac{m}{N-1}\cdot p_l\cdot T + (1-\tfrac{m}{N-m-2})\tfrac{1}{N-1}\delta V_a \\
&\quad + (1-\tfrac{m}{N-m-2})(1-\tfrac{m+1}{N-1})\delta V_{\overline{W}} \\
&= V_a\Big[\tfrac{m}{N-m-2}\cdot\tfrac{m}{N-1}\delta + \tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})\tfrac{1}{N-1}\delta^2 + (1-\tfrac{m}{N-m-2})\tfrac{1}{N-1}\delta\Big] \\
&\quad + V_{\overline{W}}\Big[\tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})(1-\tfrac{m+1}{N-1})\delta^2 + (1-\tfrac{m}{N-m-2})(1-\tfrac{m+1}{N-1})\delta\Big] \\
&\quad + T\Big[\tfrac{m}{N-m-2}(1-\tfrac{m}{N-1})\tfrac{m}{N-1}\cdot p_k\delta + (1-\tfrac{m}{N-m-2})\tfrac{m}{N-1}\cdot p_l\Big] \\
\Rightarrow V_{\overline{W}} &= \tfrac{D2}{E}\cdot V_a + \tfrac{D1}{E}\cdot T
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow V_L &= \tfrac{1}{N-1}\delta V_a + (1-\tfrac{m+1}{N-1})\delta\tfrac{D2}{E}V_a + (1-\tfrac{m+1}{N-1})\delta\tfrac{D1}{E}T \\
&= \delta V_a\Big[\tfrac{1}{N-1} + (1-\tfrac{m+1}{N-1})\tfrac{D2}{E}\Big] + (1-\tfrac{m+1}{N-1})\delta\tfrac{D1}{E}T \\
\Rightarrow V_a &= \tfrac{m}{N-1}(1-m\cdot\delta V_a - (\alpha N-1-m)T) \\
&\quad + (1-\tfrac{m}{N-1})\Big[\tfrac{1}{N-1} + (1-\tfrac{m+1}{N-1})\tfrac{D2}{E}\Big]\delta^2 V_a \\
&\quad + (1-\tfrac{m}{N-1})(1-\tfrac{m+1}{N-1})\tfrac{D1}{E}\delta^2 T \\
&= V_a\Big[-\tfrac{m^2}{N-1}\delta + (1-\tfrac{m}{N-1})(\tfrac{1}{N-1} + (1-\tfrac{m+1}{N-1})\tfrac{D2}{E})\delta^2\Big] \\
&\quad + T\Big[(1-\tfrac{m}{N-1})(1-\tfrac{m+1}{N-1})\tfrac{D1}{E}\delta^2 - \tfrac{m(\alpha N-1-m)}{N-1}\Big] \\
&\quad + \tfrac{m}{N-1} \\
\Leftrightarrow V_a &= \frac{\frac{m}{N-1} + \Big[(1-\frac{m}{N-1})(1-\frac{m+1}{N-1})\frac{D1}{E}\delta^2 - \frac{m(\alpha N-m-1)}{N-1}\Big]T}{1+\frac{m^2}{N-1}\delta - (1-\frac{m}{N-1})\Big[\frac{1}{N-1}+(1-\frac{m+1}{N-1})\frac{D2}{E}\Big]\delta^2} \\
&= V_a^m(x^S)
\end{aligned}
$$

■

Proof of Proposition 4

The first point is obvious. If we exogenously restrict $m \geq \frac{N-1}{2}$ and use our tie-breaking rules concerning the choice of amendments (that are in the spirit of BF), the resulting equilibrium is equivalent to the theorem of BF. The three groups are the agenda-setter, the individuals who receive zero, and the members who receive M.

Without the restriction to $m \geq \frac{N-1}{2}$ it is possible that the optimal strategy is to set $m^* = m^S$, even if $\alpha = 0.5$. This amends the theorem of Baron and Ferejohn (1989). To prove the second point of Proposition 4, we show that the L-strategies do not form a stationary equilibrium if

$$V_a^{m^S}(x^S) > V_a^{m^L}(x^L).$$

Suppose that $V_a^{m^S}(x^S) > V_a^{m^L}(x^L)$ for some N and δ. Despite the fact that the very first recognized agenda-setter is better off by proposing x^S, we assume that the L-strategies (i.e. BF strategies) are played but no proposal is accepted. The main question is if the agenda-setter recognized next has an incentive to deviate from the BF strategy, i.e. can he construct the four-group proposal with $m^S < \frac{N-1}{2}$ such that it defeats the prior proposal? Consider the following amendment:

The second agenda-setter offers x_a^S to himself, T^S to $\frac{N-1}{2} - m^S$ individuals who received zero under the first proposal (chosen randomly), zero to the first agenda-setter, and $\delta V_a^{m^S}(x^S)$ to $m^S < \frac{N-1}{2}$ individuals who are again chosen randomly between the other individuals. All other members receive no benefits.

This proposal defeats the prior proposal since the amendment-setter and all individuals who receive $T^S > 0$ or $M^S = \delta V_a^{m^S}(x^S) > M^L$ will vote for the second proposal. As we assumed that $V_a^{m^S}(x^S) > V_a^{m^L}(x^L) = V_a^{m^{BF}}(x^{BF})$, the second agenda-setter has an incentive to deviate from playing the L-strategyc.

For the last point of Proposition 4 we have to show that there exists a situation (given by $N \in \mathbb{N}$, $\alpha = \frac{1}{2}$ and $\delta \in [0, 1]$) such that $V_1^{m^S}(x^S) > V_1^{m^L}(x^L)$. We give an example. Set $\delta = 0.5$ and $N = 101$. We obtain

$$\begin{array}{rcl rcl} V_a^{m^L}(x^L) & = & 0.037 & V_a^{m^S}(x^S) & = & 0.068 \\ x_a^L & = & 0.07 & x_a^S & = & 0.48 \\ m^L & = & 50 & m^S & = & 14 \\ M^L & = & 0.019 & M^S & = & 0.034 \\ & & & T^S & = & 0.0012 \end{array}$$

The expected value of the first agenda-setter using the S-strategies is nearly twice as high as under the L-strategies.

■

Tables for $\alpha = 0.5$.

Table 2a

N	α	δ	m^L	M^L	T^L	$V_a^{m^L}(x^L)$	x_a	4-group equil.
5	0.5	0.1	4	0.07	0.02	0.714	0.714	
11	0.5	0.1	10	0.05	0.009	0.5	0.5	
101	0.5	0.1	50	0.014	0.0005	0.143	0.285	
1001	0.5	0.1	500	0.0019	0.000053	0.019	0.038	
5	0.5	0.5	3	0.178	0.086	0.356	0.5	
11	0.5	0.5	5	0.11	0.03	0.226	0.43	
101	0.5	0.5	50	0.02	0.0033	0.037	0.07	
1001	0.5	0.5	500	0.002	0.00033	0.004	0.0077	
5	0.5	0.9	2	0.26	0.16	0.29	0.477	
11	0.5	0.9	5	0.15	0.074	0.161	0.274	
101	0.5	0.9	50	0.019	0.008	0.021	0.037	
1001	0.5	0.9	500	0.002	0.0008	0.0022	0.0039	

Table 2b.

N	α	δ	m^S	M^S	T^S	$V_a^{m^S}(x^S)$	x_a	$V_a^{m^S}(x^S) > V_a^{m^L}(x^L)$
5	0.5	0.1	1	0.024	0.0055	0.244	0.97	
11	0.5	0.1	4	0.034	0.004	0.344	0.86	
101	0.5	0.1	32	0.016	0.00034	0.16	0.49	X
1001	0.5	0.1	100	0.005	0.000011	0.05	0.498	X
5	0.5	0.5	1	0.115	0.04	0.23	0.86	
11	0.5	0.5	4	0.11	0.026	0.22	0.54	
101	0.5	0.5	14	0.034	0.0012	0.068	0.48	X
1001	0.5	0.5	44	0.011	0.000042	0.022	0.498	X
5	0.5	0.9	1	0.23	0.14	0.26	0.7	
11	0.5	0.9	4	0.15	0.07	0.17	0.35	X
101	0.5	0.9	10	0.04	0.005	0.045	0.41	X
1001	0.5	0.9	31	0.0135	0.00022	0.015	0.48	X

Tables for $\alpha = 0.9$.

Table 3a.

N	α	δ	m^L	M^L	T^L	$V_a^{m^L}(x^L)$	x_a	4-group-equil.
5	0.9	0.1	4	0.07	0.02	0.7	0.714	
11	0.9	0.1	10	0.05	0.009	0.5	0.5	
101	0.9	0.1	50	0.014	0.00052	0.14	0.28	X
1001	0.9	0.1	500	0.002	0.000053	0.019	0.038	X
5	0.9	0.5	3	0.17	0.086	0.34	0.45	X
11	0.9	0.5	5	0.1	0.03	0.2	0.38	X
101	0.9	0.5	50	0.016	0.0033	0.032	0.06	X
1001	0.9	0.5	500	0.0017	0.00033	0.0034	0.007	X
5	0.9	0.9	3	0.21	0.17	0.23	0.29	X
11	0.9	0.9	5	0.1	0.074	0.12	0.19	X
101	0.9	0.9	50	0.013	0.008	0.015	0.025	X
1001	0.9	0.9	500	0.0013	0.0008	0.0015	0.0025	X

Table 3b.

N	α	δ	m^S	M^S	T^S	$V_a^{m^S}(x^S)$	x_a	$V_a^{m^S}(x^S) > V_a^{m^L}(x^L)$
5	0.9	0.1	1	0.024	0.0054	0.24	0.96	
11	0.9	0.1	4	0.034	0.004	0.34	0.85	
101	0.9	0.1	31	0.016	0.00033	0.16	0.5	X
1001	0.9	0.1	99	0.005	0.000011	0.05	0.5	X
5	0.9	0.5	1	0.1	0.04	0.2	0.79	
11	0.9	0.5	4	0.1	0.036	0.2	0.48	
101	0.9	0.5	13	0.032	0.0011	0.065	0.49	X
1001	0.9	0.5	43	0.011	0.00004	0.022	0.5	X
5	0.9	0.9	1	0.17	0.14	0.19	0.48	
11	0.9	0.9	3	0.106	0.066	0.117	0.29	
101	0.9	0.9	8	0.032	0.004	0.036	0.4	X
1001	0.9	0.9	28	0.012	0.0002	0.014	0.48	X

These tables give examples of equilibria with $x^* = x^L$ that consist of four groups. The equilibria are given by the parameter constellations $\alpha = 0.9$, $N \in \{5, 11\}$, and $\delta \in \{0.5, 0.9\}$.

Tables for $\alpha = 1$.

Table 4a.

N	α	δ	m^L	M^L	T^L	$V_a^{m^L}(x^L)$	x_a	4-group-equil.
5	1	0.1	4	0.071	0.02	0.71	0.714	
11	1	0.1	10	0.05	0.009	0.5	0.5	
101	1	0.1	50	0.014	0.00052	0.139	0.278	X
1001	1	0.1	500	0.002	0.00005	0.02	0.037	X
5	1	0.5	4	0.167	0.1	0.33	0.33	
11	1	0.5	5	0.096	0.03	0.19	0.37	X
101	1	0.5	50	0.016	0.0033	0.031	0.06	X
1001	1	0.9	500	0.0017	0.00033	0.0033	0.0064	X
5	1	0.9	4	0.196	0.18	0.22	0.217	
11	1	0.9	5	0.09	0.074	0.1	0.167	X
101	1	0.9	50	0.011	0.008	0.013	0.02	X
1001	1	0.9	500	0.0012	0.0008	0.0013	0.0022	X

Table 4b.

N	α	δ	m^S	M^S	T^S	$V_a^{m^S}(x^S)$	x_a	$V_a^{m^S}(x^S) > V_a^{m^L}(x^L)$
5	1	0.1	1	0.024	0.0054	0.24	0.96	
11	1	0.1	4	0.034	0.004	0.337	0.84	
101	1	0.1	31	0.015	0.00033	0.15	0.498	X
1001	1	0.1	99	0.005	0.000011	0.05	0.5	X
5	1	0.5	1	0.1	0.04	0.2	0.78	
11	1	0.5	4	0.096	0.026	0.19	0.46	
101	1	0.5	13	0.032	0.0011	0.064	0.49	X
1001	1	0.5	43	0.011	0.000041	0.0215	0.5	X
5	1	0.9	1	0.16	0.14	0.17	0.43	
11	1	0.9	3	0.094	0.066	0.105	0.25	X
101	1	0.9	8	0.03	0.004	0.033	0.38	X
1001	1	0.9	28	0.012	0.0002	0.013	0.47	X

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