Mathematical supplement

1 Cost of K

θs denote cost shares and λs factor shares, as in the main text. Then, $\hat{c}_K$ is calculated as:

$$\begin{aligned}
\hat{c}_K &= \theta_{PK}(\theta_{LP}\hat{w} + \theta_{EP}\hat{p}_E) + \theta_{NK}(\theta_{HN} \cdot (\theta_{LH}\hat{w} + \theta_{EH}\hat{p}_E) + \theta_{BN}(\theta_{LB}\hat{w} + \theta_{EB}\hat{p}_E)) \\
&= \theta_{PK}\theta_{LP}\hat{w} + \theta_{PK}\theta_{EP}\hat{p}_E + \theta_{NK}\theta_{HN}\theta_{LH}\hat{w} + \theta_{NK}\theta_{HN}\theta_{EH}\hat{p}_E \\
&+ \theta_{NK}\theta_{BN}\theta_{LB}\hat{w} + \theta_{NK}\theta_{BN}\theta_{EB}\hat{p}_E \\
&= [\theta_{PK}\theta_{LP} + \theta_{NK}\theta_{HN}\theta_{LH} + \theta_{NK}\theta_{BN}\theta_{LB}]\,\hat{w} \\
&+ [\theta_{PK}\theta_{EP} + \theta_{NK}\theta_{HN}\theta_{EH} + \theta_{NK}\theta_{BN}\theta_{EB}]\,\hat{p}_E \\
&= \theta_{LK} \cdot \hat{w} + \theta_{EK} \cdot \hat{p}_E.
\end{aligned}$$

with $\theta_{LK} = \theta_{PK}\theta_{LP} + \theta_{NK}\theta_{HN}\theta_{LH} + \theta_{NK}\theta_{BN}\theta_{LB} > 0$ and $\theta_{EK} = \theta_{PK}\theta_{EP} + \theta_{NK}\theta_{HN}\theta_{EH} + \theta_{NK}\theta_{BN}\theta_{EB} > 0$.

2 Proof of lemma 1

Differentiate (9) and (10) to obtain:

$$\begin{aligned}
\hat{E} &= \lambda_{EX}(\hat{a}_{EX} + \hat{X}) + \lambda_{EK}(\hat{a}_{EK} + \hat{g}_K) \\
0 &= \lambda_{LX}(\hat{a}_{LX} + \hat{X}) + \lambda_{LK}(\hat{a}_{LK} + \hat{g}_K) \\
0 &= \hat{c}_K + (\frac{g_K}{g_K + \rho})\hat{g}_K
\end{aligned}$$

With $\tilde{\rho} = (\rho + g_K)/g_K > 1$ we can write:

$$\hat{g}_K = -\tilde{\rho} \cdot \hat{c}_K = -\tilde{\rho}\theta_{EK}\hat{p}_E - \tilde{\rho}\theta_{LK}\hat{w}$$

Use this as well as:

$$\begin{aligned}
\hat{a}_{Eq} &= \theta_{Lq}\sigma_q(\hat{w} - \hat{p}_E) \\
\hat{a}_{Lq} &= -\theta_{Eq}\sigma_q(\hat{w} - \hat{p}_E)
\end{aligned}$$

for $q = X, K$ which yields:

$$\hat{E} = \lambda_{EX}\left[\theta_{LX}\sigma_X(\hat{w}-\hat{p}_E)+\hat{X}\right]$$
$$+\lambda_{EK}[\theta_{LK}\sigma_K(\hat{w}-\hat{p}_E)-\theta_{EK}\tilde{\rho}\hat{p}_E-\theta_{LK}\tilde{\rho}\hat{w}]$$
$$0 = \lambda_{LX}\left[-\theta_{EX}\sigma_X(\hat{w}-\hat{p}_E)+\hat{X}\right]$$
$$+\lambda_{LK}\left[-\theta_{EK}\sigma_K(\hat{w}-\hat{p}_E)-\theta_{EK}\tilde{\rho}\hat{p}_E-\theta_{LK}\tilde{\rho}\hat{w}\right]$$

Multiplying gives:

$$\hat{E} = \lambda_{EX}\theta_{LX}\sigma_X\hat{w}-\lambda_{EX}\theta_{LX}\sigma_X\hat{p}_E+\lambda_{EX}\hat{X}$$
$$+\lambda_{EK}\theta_{LK}\sigma_K\hat{w}-\lambda_{EK}\theta_{LK}\sigma_K\hat{p}_E-\lambda_{EK}\theta_{EK}\tilde{\rho}\hat{p}_E-\lambda_{EK}\theta_{LK}\tilde{\rho}\hat{w}$$
$$0 = -\lambda_{LX}\theta_{EX}\sigma_X\hat{w}+\lambda_{LX}\theta_{EX}\sigma_X\hat{p}_E+\lambda_{LX}\hat{X}$$
$$-\lambda_{LK}\theta_{EK}\sigma_K\hat{w}+\lambda_{LK}\theta_{EK}\sigma_K\hat{p}_E-\lambda_{LK}\theta_{EK}\tilde{\rho}\hat{p}_E-\lambda_{LK}\theta_{LK}\tilde{\rho}\hat{w}$$

Collecting for $\hat{w}$ and $\hat{p}_E$ gives:

$$\hat{E} = [\lambda_{EX}\theta_{LX}\sigma_X+\lambda_{EK}\theta_{LK}\sigma_K-\lambda_{EK}\theta_{LK}\tilde{\rho}]\,\hat{w}$$
$$-[\lambda_{EX}\theta_{LX}\sigma_X+\lambda_{EK}\theta_{LK}\sigma_K+\lambda_{EK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E+\lambda_{EX}\hat{X}$$
$$0 = [-\lambda_{LX}\theta_{EX}\sigma_X-\lambda_{LK}\theta_{EK}\sigma_K-\lambda_{LK}\theta_{LK}\tilde{\rho}]\,\hat{w}$$
$$+[\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}\sigma_K-\lambda_{LK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E+\lambda_{LX}\hat{X}$$

For a constant w the impact of $\hat{p}_E$ on $\hat{E}$ is $-\lambda_{EX}\theta_{LX}\sigma_X-\lambda_{EK}\theta_{LK}\sigma_K-\lambda_{EK}\theta_{EK}\tilde{\rho}<0$ that means it is negative as expected. Collecting further and solving the labour market for $\hat{w}$ gives:

$$\hat{E} = [\lambda_{EX}\theta_{LX}\sigma_X+\lambda_{EK}\theta_{LK}(\sigma_K-\tilde{\rho})]\,\hat{w}$$
$$-[\lambda_{EX}\theta_{LX}\sigma_X+\lambda_{EK}\theta_{LK}\sigma_K+\lambda_{EK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E+\lambda_{EX}\hat{X}$$

$$[\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}\sigma_K+\lambda_{LK}\theta_{LK}\tilde{\rho}]\,\hat{w}$$
$$=[\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}\sigma_K-\lambda_{LK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E+\lambda_{LX}\hat{X}$$
$$\hat{w} = \frac{[\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}(\sigma_K-\tilde{\rho})]\,\hat{p}_E+\lambda_{LX}\hat{X}}{\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}\sigma_K+\lambda_{LK}\theta_{LK}\tilde{\rho}}$$
$$=\frac{[\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}(\sigma_K-\tilde{\rho})]\,\hat{p}_E+\lambda_{LX}\hat{X}}{\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}(\theta_{EK}\sigma_K+\theta_{LK}\tilde{\rho})}$$

which shows that, for constant X, $\hat{w}$ and $\hat{p}_E$ have the same sign when $\sigma_K>\tilde{\rho}$ and that the opposite happens when $\lambda_{LX}\theta_{EX}\sigma_X+\lambda_{LK}\theta_{EK}(\sigma_K-\tilde{\rho})<0$ which requires large values of θ_{EK} and $\tilde{\rho}$. By inserting $\hat{w}$ in the energy equation and collecting we obtain:

$$\begin{aligned}\hat{E} = &- \{[\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}(\sigma_K - \tilde{\rho}] \cdot \\ &\frac{[\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})]}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} \\ &+ [\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho}]\}\hat{p}_E \\ &+ (\frac{\lambda_{LX}}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} + \lambda_{EX})\hat{X}\end{aligned}$$

or, respectively:

$$\begin{aligned}\hat{E} &= -\{\frac{[b_1\sigma_X + b_2(\sigma_K - \tilde{\rho}]\,[b_3\sigma_X + b_4(\sigma_K - \tilde{\rho})]}{b_5} + \tilde{b}\}\hat{p}_E + \Gamma \cdot (\hat{Y} - \gamma) \\ &= -\{\frac{b_1b_3\sigma_X^2 + \bar{b}\sigma_X(\sigma_K - \tilde{\rho}) + b_2b_4(\sigma_K - \tilde{\rho})^2}{b_5} + \tilde{b}\}\hat{p}_E + \Gamma \cdot (\hat{Y} - \gamma)\end{aligned}$$

with:

$$\begin{aligned}b_1 &= \lambda_{EX}\theta_{LX} > 0,\ b_2 = \lambda_{EK}\theta_{LK} > 0,\ b_3 = \lambda_{LX}\theta_{EX} > 0 \\ b_4 &= \lambda_{LK}\theta_{EK} > 0,\ b_5 = \lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho}) > 0 \\ \tilde{b} &= \lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho} > 0 \\ \bar{b} &= b_1b_4 + b_2b_3 > 0,\ \gamma = g_A + \frac{1-\beta}{\beta}g_K \geq 0 \\ \Gamma &= \frac{\lambda_{LX}}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} + \lambda_{EX} > 0\end{aligned}$$

which reveals that $\hat{p}_E$ has an unambiguously negative impact on $\hat{E}$ when $\sigma_K > \tilde{\rho}$. When $\sigma_K < \tilde{\rho}$ an ambiguity arises. In this (special) case, according to the expression $\lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})$ from above, wages decrease sharply after an energy price increase which causes a strong output effect in the capital sector possibly offsetting the direct energy price effect. This may happen even when taking into account the positive impact of $\tilde{b}$. In the short run, wages are not flexible; then, the impact of energy prices on energy use is unambiguous according to:

$$\hat{E} = -\tilde{b} \cdot \hat{p}_E$$

3 Proof of lemma 2

To evaluate $\hat{s}_i$ we write:

$$\hat{s}_i = \hat{k}_i - \hat{Y} = \hat{\theta}_{ki} - \hat{p}_{ki} + \hat{p}_X - \gamma$$

We use the optimum conditions in the capital sector, i.e. $\theta_{PK}/\theta_{NK} = [p_P/p_N]^{1-\sigma_{\tilde{K}}}$ and $\theta_{HN}/\theta_{BN} = [p_H/p_B]^{1-\sigma_N}$ to derive the cost shares θ for the different capital types i where $\sigma_{\tilde{K}}$ and σ_N are the elasticities of substitution between physical and non-physical capital and between human and knowledge capital, respectively. Moreover, we express $\hat{\theta}_{ki}$ as well as $\hat{p}_X$ and $\hat{p}_{ki}$ in terms of input prices $\hat{w}$ and $\hat{p}_E$, which yields for capital type P:

$$\begin{aligned}
\hat{s}_P + \gamma &= \theta_{NK}(1-\sigma_{\tilde{K}})(\hat{p}_P - \hat{p}_N) + (\theta_{LX} - \theta_{LP})\hat{w} + (\theta_{EX} - \theta_{EP})\,\hat{p}_E \\
&= \theta_{NK}(1-\sigma_{\tilde{K}})\,[(\theta_{LP} - \theta_{LN})\hat{w} + (\theta_{EN} - \theta_{EP})\hat{p}_E] \\
&\quad + (\theta_{LX} - \theta_{LP})\hat{w} + (\theta_{EX} - \theta_{EP})\,\hat{p}_E \\
&= \theta_{NK}(1-\sigma_{\tilde{K}})\,[(\theta_{EN} - \theta_{EP})\hat{w} + (\theta_{EN} - \theta_{EP})\hat{p}_E] + (\theta_{EP} - \theta_{EX})\,(\hat{w} - \hat{p}_E) \\
&= \theta_{NK}(1-\sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP})\,(\hat{w} - \hat{p}_E) + (\theta_{EP} - \theta_{EX})\,(\hat{w} - \hat{p}_E) \\
&= [\theta_{NK}(1-\sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP}) + (\theta_{EP} - \theta_{EX})]\,(\hat{w} - \hat{p}_E)
\end{aligned}$$

where we have used $\theta_{LX} = 1 - \theta_{EX}$, $\theta_{LP} = 1 - \theta_{EP}$ etc. Similarly, we obtain for H and B

$$\begin{aligned}
\hat{s}_H + \gamma &= [\theta_{BN}(1-\sigma_N)(\theta_{EB} - \theta_{EH}) + (\theta_{EH} - \theta_{EX})]\,(\hat{w} - \hat{p}_E) \\
\hat{s}_B + \gamma &= [\theta_{HN}(1-\sigma_N)(\theta_{EH} - \theta_{EB}) + (\theta_{EB} - \theta_{EX})]\,(\hat{w} - \hat{p}_E)
\end{aligned}$$

To find $\hat{w} - \hat{p}_E$ we use the factor market equlibria and the capital market equilibrium as well as $\hat{X} = -\theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E$ to get:

$$\begin{aligned}
\hat{E} &= \lambda_{EX}(\theta_{LX}\sigma_X(\hat{w} - \hat{p}_E) - \theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E) \\
&\quad + \lambda_{EK}\,[\theta_{LK}\sigma_K(\hat{w} - \hat{p}_E) - \theta_{EK}\tilde{\rho}\hat{p}_E - \theta_{LK}\tilde{\rho}\hat{w}] \\
0 &= \lambda_{LX}\,[-\theta_{EX}\sigma_X(\hat{w} - \hat{p}_E) - \theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E] \\
&\quad + \lambda_{LK}\,[-\theta_{EK}\sigma_K(\hat{w} - \hat{p}_E) - \theta_{EK}\tilde{\rho}\hat{p}_E - \theta_{LK}\tilde{\rho}\hat{w}]
\end{aligned}$$

so that:

$$\begin{aligned}
\hat{E} &= \lambda_{EX}\theta_{LX}\sigma_X\hat{w} - \lambda_{EX}\theta_{LX}\sigma_X\hat{p}_E - \lambda_{EX}\theta_{LX}\hat{w} - \lambda_{EX}\theta_{EX}\hat{p}_E \\
&\quad + \lambda_{EK}\theta_{LK}\sigma_K\hat{w} - \lambda_{EK}\theta_{LK}\sigma_K\hat{p}_E - \lambda_{EK}\theta_{EK}\tilde{\rho}\hat{p}_E - \lambda_{EK}\theta_{LK}\tilde{\rho}\hat{w} \\
0 &= -\lambda_{LX}\theta_{EX}\sigma_X\hat{w} + \lambda_{LX}\theta_{EX}\sigma_X\hat{p}_E - \lambda_{LX}\theta_{LX}\hat{w} - \lambda_{LX}\theta_{EX}\hat{p}_E - \lambda_{LK}\theta_{EK}\sigma_K\hat{w} \\
&\quad + \lambda_{LK}\theta_{EK}\sigma_K\hat{p}_E - \lambda_{LK}\theta_{EK}\tilde{\rho}\hat{p}_E - \lambda_{LK}\theta_{LK}\tilde{\rho}\hat{w}
\end{aligned}$$

and:

$$\begin{aligned}
\hat{E} &= [\lambda_{EX}\theta_{LX}\sigma_X - \lambda_{EX}\theta_{LX} + \lambda_{EK}\theta_{LK}\sigma_K - \lambda_{EK}\theta_{LK}\tilde{\rho}]\,\hat{w} \\
&\quad - [\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EX}\theta_{EX} + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E \\
0 &= [-\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{LK}\tilde{\rho}]\,\hat{w} \\
&\quad + [\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{EX} + \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{EK}\tilde{\rho}]\,\hat{p}_E
\end{aligned}$$

Written in matrix form we have:

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} \hat{w} \\ \hat{p}_E \end{bmatrix} = \begin{bmatrix} \hat{E} \\ 0 \end{bmatrix}$$

where

$$\begin{aligned} c_{11} &= \sum_q \lambda_{Eq}\theta_{Lq}\sigma_q - \lambda_{EX}\theta_{LX} - \lambda_{EK}\theta_{LK}\tilde{\rho} \\ c_{12} &= -\sum_q \lambda_{Eq}\theta_{Lq}\sigma_q - \lambda_{EX}\theta_{EX} - \lambda_{EK}\theta_{EK}\tilde{\rho} \\ c_{21} &= -\sum_q \lambda_{Lq}\theta_{Eq}\sigma_q - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{LK}\tilde{\rho} \\ c_{22} &= \sum_q \lambda_{Lq}\theta_{Eq}\sigma_q - \lambda_{LX}\theta_{EX} - \lambda_{LK}\theta_{EK}\tilde{\rho} \end{aligned}$$

By construction of the cs the determinant Δ of the system is maximum if $\sigma_q = 0$, which yields:

$$\begin{aligned} \Delta &= (\lambda_{EX}\theta_{LX} + \lambda_{EK}\theta_{LK}\tilde{\rho})(\lambda_{LX}\theta_{EX} + \lambda_{LK}\theta_{EK}\tilde{\rho}) \\ &\quad - (\lambda_{EX}\theta_{EX} + \lambda_{EK}\theta_{EK}\tilde{\rho})(\lambda_{LX}\theta_{LX} + \lambda_{LK}\theta_{LK}\tilde{\rho}) \\ &= \lambda_{EX}\theta_{LX}\lambda_{LX}\theta_{EX} + \lambda_{EX}\theta_{LX}\lambda_{LK}\theta_{EK}\tilde{\rho} + \lambda_{EK}\theta_{LK}\lambda_{LX}\theta_{EX} + \lambda_{EK}\theta_{LK}\lambda_{LK}\theta_{EK}\tilde{\rho} \\ &\quad - \lambda_{EX}\theta_{EX}\lambda_{LX}\theta_{LX} - \lambda_{EX}\theta_{EX}\lambda_{LK}\theta_{LK}\tilde{\rho} - \lambda_{EK}\theta_{EK}\lambda_{LX}\theta_{LX} - \lambda_{EK}\theta_{EK}\lambda_{LK}\theta_{LK}\tilde{\rho} \\ &= \lambda_{EX}\lambda_{LX}(\theta_{LX}\theta_{EX} - \theta_{EX}\theta_{LX}) + \lambda_{EX}\lambda_{LK}\tilde{\rho}(\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK}) \\ &\quad + \lambda_{EK}\lambda_{LX}(\theta_{LK}\theta_{EX} - \theta_{EK}\theta_{LX}) + \lambda_{EK}\lambda_{LK}\tilde{\rho}(\theta_{EK}\theta_{LK} - \theta_{EK}\theta_{LK}) \\ &= \lambda_{EX}\lambda_{LK}\tilde{\rho}(\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK}) + \lambda_{EK}\lambda_{LX}(\theta_{LK}\theta_{EX} - \theta_{EK}\theta_{LX}) \\ &= \lambda_{EX}\lambda_{LK}\tilde{\rho}(\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK}) - \lambda_{EK}\lambda_{LX}(\theta_{EK}\theta_{LX} - \theta_{LK}\theta_{EX}) \\ &= (\lambda_{EX}\lambda_{LK}\tilde{\rho} - \lambda_{EK}\lambda_{LX})(\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK}) \end{aligned}$$

Provided that intermediates production is relatively more intensive in energy use than capital accumulation, i.e. we have $\theta_{LX} < \theta_{LK}$, $\theta_{EK} < \theta_{EX}$ and $\lambda_{LX}/\lambda_{LK} < \lambda_{EX}/\lambda_{EK}$ so that $\lambda_{EX}\lambda_{LK}\tilde{\rho} - \lambda_{EK}\lambda_{LX} > 0$ and $\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK} < 0$, we get $\Delta < 0$. When intermediates production is relatively less energy-intensive than capital accumulation, i.e. we have $\theta_{LX} > \theta_{LK}$ and $\theta_{EK} > \theta_{EX}$ as well as $\lambda_{LX}/\lambda_{LK} > \lambda_{EX}/\lambda_{EK}$. In this case we have $\theta_{LX}\theta_{EK} - \theta_{EX}\theta_{LK} > 0$. When $\lambda_{EX}\lambda_{LK}\tilde{\rho} - \lambda_{EK}\lambda_{LX} < 0$ we get again $\Delta < 0$. For $\lambda_{EX}\lambda_{LK}\tilde{\rho} - \lambda_{EK}\lambda_{LX} > 0$ although $\lambda_{LX}/\lambda_{LK} > \lambda_{EX}/\lambda_{EK}$ we would, by the definition of $\tilde{\rho}$ and plausible values for the parameters, obtain the result that $\rho > g_K$. This, however, is not feasible in a growing economy and can be discarded. We infer that the determinant is negative, i.e. $\Delta < 0$. For the input prices we obtain:

$$\hat{w} = \frac{c_{22}}{\Delta}\hat{E} = \frac{1}{\Delta}\left[\lambda_{LX}\theta_{EX}(\sigma_X - 1) + \lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})\right] \cdot \hat{E}$$

$$\hat{p}_E = -\frac{c_{21}}{\Delta}\hat{E} = \frac{1}{\Delta}\left[\lambda_{LX}(\theta_{EX}\sigma_X + \theta_{LX}) + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})\right] \cdot \hat{E}$$

which means that:

$$\begin{aligned}
\hat{w} - \hat{p}_E = &\{[\lambda_{LX}\theta_{EX}(\sigma_X - 1) + \lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})] \\
&- [\lambda_{LX}(\theta_{EX}\sigma_X + \theta_{LX}) + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})]\} \cdot \frac{\hat{E}}{\Delta} \\
&= (\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{EX} + \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{EK}\tilde{\rho} \\
&- \lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{LK}\tilde{\rho}) \cdot \frac{\hat{E}}{\Delta} \\
&= (-\lambda_{LX}\theta_{EX} - \lambda_{LK}\theta_{EK}\tilde{\rho} - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{LK}\tilde{\rho}) \cdot \frac{\hat{E}}{\Delta} \\
&= [-\lambda_{LX}(\theta_{EX} + \theta_{LX}) - \lambda_{LK}\tilde{\rho}(\theta_{EK} + \theta_{LK})] \cdot \frac{\hat{E}}{\Delta} = \frac{\nu}{\Delta}\hat{E}
\end{aligned}$$

As we have $\Delta, \nu < 0$, it follows that a decrease in E causes an unambiguous decrease of the wage/energy price ratio. Inserting for the different capital types yields:

$$\begin{aligned}
\hat{s}_P &= [\theta_{NK}(1 - \sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP}) + (\theta_{EP} - \theta_{EX})]\,\frac{\nu}{\Delta} \cdot \hat{E} - \gamma \\
\hat{s}_H &= [\theta_{BN}(1 - \sigma_N)(\theta_{EB} - \theta_{EH}) + (\theta_{EH} - \theta_{EX})]\,\frac{\nu}{\Delta} \cdot \hat{E} - \gamma \\
\hat{s}_B &= [\theta_{HN}(1 - \sigma_N)(\theta_{EH} - \theta_{EB}) + (\theta_{EB} - \theta_{EX})]\,\frac{\nu}{\Delta} \cdot \hat{E} - \gamma
\end{aligned}$$