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# Long-Run Growth and Recycling: A Material Balance Approach

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## Abstract

In this paper we analyze the implications of recycling for long-run economic development. In contrast to former approaches, we take explicit account of the circulation of matter in the economy. Building upon a Romer (1990) type growth model we consider virgin resources and recycled wastes as essential inputs to production. These material inputs either end up as waste after consumption or are bound in the capital stock – depending on the utilization of the produced output. As accumulating wastes can be recycled and again be employed in production, the waste stock serves as a source of valuable inputs in our model. We focus on the implications of recycling-related market failures and the integration of material balances on the dynamics of the economy. It is shown that, even in the absence of environmental policy, long-run development is sustainable. Yet, optimal environmental policy can serve as a means to raise growth.

**Keywords:** exhaustible resources, recycling, endogenous growth, sustainable development

**JEL-code:** O41, Q01, Q3, Q53

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# 1 Introduction

Until recently the old saying ‘the devil takes the hindmost’ proved to be very true for owners of junk cars. The last person to own a car often had to pay for its before scrapping, e.g. in Germany. Yet recently, the market for junk cars as a source of recyclable materials, especially steel, has been expanding at a considerable rate due to the fast rising demand for steel. Consequently lucky owners now get paid for their junk cars instead of having to pay for their disposal. This example shows clearly that the waste stock itself has to be considered as a resource pile which can constitute a substitute for virgin resources.

So far this aspect has, more often than not, been neglected within the environmental economics’ analysis of waste generation. Mostly, waste has only been associated with the generation of negative externalities. In this case, leaving the allocation of goods and factors to markets alone usually implies unsustainable long-run development. Consequently the policymaker is called for to ensure sustainability in the sense of non-decreasing utility.

Our approach is to treat the waste stock as a source of valuable inputs. We show that – even in the absence of environmental policy – long-run development will be sustainable in this case. The intuition behind this result is that, as waste is a valuable resource in our model, not recycling part of it cannot be optimal in the long-run.

The example of the market for scrap cars in Germany shows clearly that recycling becomes of more and more importance in a world in which virgin resources become increasingly scarce. Empirical data shows an increase in the global rates of recycling for many types of minerals, such as iron, aluminum and lead over the last three decades (van Beukering 2001). Also the recovery rate for household waste, e.g. paper and glass bottles, has increased considerably (van Beukering/Bouman 2001, OECD 2001, Fullerton/Kinnaman 2002). By using the examples of household waste and minerals, two reasons for promoting recycling can be exemplified. On the one hand recycling can be a means to reduce the negative environmental externalities associated with economic activities. In the case of household waste, for example, this includes deterring consumers from dumping their waste uncontrolled into nature. On the other hand recycling is a means to alleviate the scarcity of natural resources. In the case of household waste this concerns, for example, the scarcity of land available for landfills. With respect to minerals it is the scarcity of non-renewable inputs whose limited availability might put constraints to production. It is the latter aspect on which we focus our attention in this paper.

Often the observed increase in recycling rates can be attributed to environmental policy, i.e. recycling programmes as in the case of deposit systems for glass bottles (Ackerman 1997). In some cases – such as the scrap car market –, the increasing scarcity of resources leads to a development of markets, yet these self-emerging markets seem to be rather the exception than the rule. In this paper we show that even if such a market does not develop, environmental policy is not required for development to be sustainable provided that waste is perceived as a valuable resource. This by no means implies that an environmental policy is useless in the context of recycling. As we exemplify in Section 5, such a policy is still needed to correct for the market failures that may be arising in a market economy. By abstracting from negative

environmental externalities we follow Weinstein and Zeckhauser (1974) who also concentrate on the scarcity problem only and also come to the conclusion that in this case environmental policy is not a necessity for sustainability. Yet, in contrast to them we explicitly consider the consequences of part of the resources being bound in the capital stock of the economy.

In recent theory, there have been some attempts to integrate recycling into a macroeconomic dynamic framework (Mainwaring 1995, Huhtala 1999, Conrad 1999, di Vita 2001, Kuhn et al. 2003). Yet these approaches usually lack a sound material balance foundation. One exception to this rule is Huhtala (1999), who shows that by recycling, a steady state can be reached that is characterized by constant resource and waste stocks. However, her analysis is limited with regard to two aspects: first, she does not consider long-run growth.<sup>1</sup> Given that the economy does not grow in the long run, sustainability can thus be achieved by recycling without technological progress as a prerequisite. The picture changes considerably if long-run growth is considered. Although recycling reduces the scarcity of material inputs, the overall availability of materials nevertheless remains limited. Consequently long-run growth still has to be characterized by a continuing dematerialization of output. Long-run growth cannot be sustained by increasing the rate of recycling only, but has to rely on the capacity of humans to extract more and more value from a given quantity of material, i.e. growth still has to be driven by increases in the efficiency of factor employment.

The second limitation to Huhtala's analysis is that her model abstracts from the accumulation of capital and thereby ignores the fact that part of the resources used in production are at least temporarily bound in the capital stock, thereby reducing the amount of waste available for recycling.

Although a number of authors approached the problem of recycling over time, the substantial part of the literature on economic growth and sustainability still does not distinguish between the input and output side of environmental degradation.<sup>2</sup> Yet even if recycling is considered, it is often in a very simplistic manner. Musu and Lines (1995), for example, model recycling as one possibility to interpret a technological process that reduces the amount of waste disposed to the environment. However, the characteristic feature of recycling, that is the circulation of matter, is not modelled specifically.

In contrast to recycling, the role of exhaustible resources in endogenously growing economies has been addressed by a number of authors in recent years. The paper by Scholz and Ziemes (1999) is especially close to our approach as it also examines a Romer (1990) type economy with non-renewable resources. The authors, however, do not include material balance considerations as the resources are not traced through the system, i.e. they simply disappear after having been employed in production.<sup>3</sup>

In order to integrate the circulation of matter into our model, we explicitly derive and include

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<sup>1</sup>As also holds for the wider – and especially the older – range of papers dealing with the dynamics of recycling, e.g. Smith (1972), Lusky (1976), Hoel (1978) and more recently Dinan (1993) and Huhtala (1997). An exception to this rule are the endogenous growth approaches by e.g. Musu/Lines (1995) and Kuhn et al. (2003).

<sup>2</sup>For an extensive review of these approaches, see e.g. Pittel (2002).

<sup>3</sup>The same holds for a number of other papers from this strand of literature, see e.g. Aghion/Howitt (1998, chapter 5), Schou (2001, 2002), Groth/Schou (2002), and Grimaud/Rougé (2003, 2005).

the material balance condition for the considered economy. Following the so-called Lavoisier law, it is taken into account that material can only be converted but never be destroyed. Consequently, of the amounts of recycled materials and virgin resources employed in production, the part that is not bound in the accumulation of capital returns after consumption to the waste heap. The material which thus accumulates as waste can again be extracted for recycling processes at later points in time.

We consider a three sector economy of the Romer (1990) type which is enhanced by resource scarcity and recycling. A homogeneous final good is produced from labor, capital and differentiated intermediate products. These intermediates mirror the input of materials into final goods production at each point in time. They are composed of virgin resources and recycled waste only, whereby resources and waste constitute substitutes in production. The extraction of non-renewable virgin resources is the ultimate source of all material in the economy whether it is bound in capital, used in production, or accumulated as waste. After consumption, the material content of the consumed products ends up on the waste pile. A market for this raw waste does not exist. Households neither have to pay to discard their waste, nor are they paid for it, while recycling firms on the other hand can mine the waste stock at no cost.

Recycling firms as well as virgin resource producers do not take the effects of their production on the future availability of waste into account. Yet today's employment of material inputs in production exerts a positive feedback-effect on the stock of material available for recycling in the future. As after consumption part of the recycled waste and the virgin resources end up as waste, the waste pile 'regenerates' to a certain extent. This implies that the stock of waste available for recycling over time is endogenously determined and depends on the share of consumption in output.

With respect to long-run development, we distinguish between the decentralized solution, chosen by firms and consumers under free market conditions, and the socially optimal path. We show that although policy intervention is necessary for long-run development to be optimal, it is no prerequisite for growth to be sustainable. An optimal policy scheme in this economy not only has to correct for the 'standard' Romer market failures, but also has to take account of the recycling related market failures. The latter could, for example, be corrected by the introduction of a market for waste and the subsidization of intermediate goods producers.

The paper proceeds as follows: Section 2 introduces the model whereby special emphasis is put on the derivation of the material balance condition. An additional subsection is devoted to the sources of market failures in the considered economy. Section 3 then takes a look at the socially optimal solution which can be considered as a benchmark scenario. The market solution is derived in Section 4. It is shown that neither the socially optimal, nor the market growth rate depend on the relative productivity of the two material inputs. Section 5 finally derives optimal economic and environmental policies. Section 6 concludes the paper.

## 2 The Model

We consider a closed economy of the Romer (1990) type which is enhanced by a material balance approach. In introducing the model we proceed as follows: First we take a look at the economics side of our model by describing the production and household sectors, which are in many respects quite standard. The exception is the characterization of intermediates goods as differentiated types of material inputs, which are produced from virgin resources and recycled waste. Subsequently we turn to the ecological part of the model which is captured by the material balance condition. To facilitate its derivation, we consider a discrete time version first, which is then adapted to our continuous time framework. The section closes with a look at the sources of market failures in this economy.

### 2.1 Household sector

Households are assumed to derive utility solely from consumption  $C$  with the utility function satisfying the usual properties. We consider an economy with a constant population we normalize to unity. The representative household maximizes discounted lifetime utility with respect to his intertemporal budget constraint

$$\max_c \int_0^{\infty} U(C(t))e^{-\rho t} dt \quad \text{s.t.} \quad \dot{D} = dD/dt = w + rK + \Pi - C \quad (1)$$

with  $U_C > 0$ ,  $U_{CC} < 0$ .  $\rho$  is the discount rate and  $D$  denotes household wealth, that is the stock of bonds held by the household on the bond market, and  $\Pi$  is the sum of the instantaneous profits of the industry in the different production sectors. We normalize to one the price of the consumption good, that is all prices are measured in real terms. The wage rate is given by  $w$ , while  $r$  is the interest rate on the bond market. The representative household supplies one unit of labor inelastically. Household's preferences are of the CRRA-type, such that the instantaneous utility function reads  $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$  with  $\sigma \neq 1$  and  $\sigma > 0$ .

### 2.2 Production sectors

On the production side of our economy five different types of goods are produced. Besides a homogeneous final good, we consider an endogenous number of differentiated 'material' goods which are employed in the production of the final good. Blueprints for these intermediates are developed in the research sector. The inputs to the production of the material goods – virgin resources and recycled waste – are provided by two further sectors: one type extracts the virgin resource that is assumed to be non-renewable, the other specializes in the recycling of waste.

On the final goods market the homogeneous good  $Y$  is produced on a perfectly competitive market.  $Y$  can alternatively be used for investive or consumptive purposes. Its production function is assumed to be of the Cobb-Douglas type

$$Y = K^{\alpha_1} L^{\alpha_2} \tilde{X}^{\alpha_3}, \quad \alpha_1 + \alpha_2 + \alpha_3 = 1, \quad \alpha_i > 0 \quad (2)$$

where  $L$ ,  $0 < L < 1$ , is the amount of labor employed in final goods production, and  $\tilde{X}$  represents an index of horizontally differentiated intermediate inputs  $x_i$ ,  $i \in [0, H]$ .

Intermediates can be characterized as ‘material’ goods which are produced from virgin resources  $Z$  and recycled waste  $W_R$ . All intermediates are assumed to be produced by the same production technology  $x_i = z_i^\beta w_{R_i}^{1-\beta}$ ,  $0 < \beta < 1$ ,  $i \in [0, H]$ , where  $z_i$  and  $w_{R_i}$  are the amounts of virgin resources and recycled waste employed in the production of intermediate  $i$ , and  $H$  represents the ‘number’ of different intermediates. The parameter  $\beta$  shows how productive a unit of the virgin resource is relative to a unit of recycled materials. By choosing a Cobb-Douglas type production function we implicitly assume not only that virgin and recycled materials are essential inputs to production, but more specifically that inputs are perfectly substitutable, i.e. the substitution elasticity is equal to unity. The Cobb-Douglas function can be seen as a limit case of a more general technology. The choice of this particular functional form thereby points out that the production of intermediates could also be interpreted as part of a larger integrated production process, namely the production of final output.

With respect to the production of intermediates it is assumed that only one firm produces one type of intermediate, so that competition in the intermediates sector is monopolistic. To be eligible to produce a certain type of intermediate, a firm has to own the patent for the respective blueprint first. Patents are assumed to be of infinite duration.

The index of intermediates  $\tilde{X} = \left[ \int_0^H x_i^\gamma di \right]^{\frac{1}{\gamma}}$  with  $0 < \gamma < 1$  reflects the overall effect the input of a specific amount of intermediates has on the production of final output. An increase of  $\tilde{X}$  can be due to a quantity and/or quality effect. The quantity effect results from a sheer increase in the aggregate amount of intermediates employed, while the quality effect is due to a rise in the variety of available differentiated products (represented by an increase in  $H$ ). Following the approach of Dixit/Stiglitz (1977) and Ethier (1982), it is assumed that an increase in the variety of intermediates exerts a positive effect on the productivity of the overall input of intermediates.

As all intermediates are produced by the same technology, the amounts produced of each one are identical in equilibrium ( $x = x_i$ ). Consequently the index of intermediates simplifies to  $\tilde{X} = H^{\frac{1-\gamma}{\gamma}} X$ , where  $H^{\frac{1-\gamma}{\gamma}}$  represents the productivity effect which is due to an increase in variety, and  $X$  denotes the aggregate quantity of intermediates,  $X = Hx$ , employed in production.  $W_R$  and  $Z$  are the respective amounts of virgin resources and recycled materials used in the production of  $X$  ( $Z = \int_0^H z_i$  and  $W_R = \int_0^H w_{R_i}$ ).

Blueprints for new intermediates are developed in a perfectly competitive research sector where labor is assumed to be the only private input. Yet research also profits from experience gained in past research activities. As a proxy for this positive externality we take the number of already existing differentiated intermediates.  $H$  can therefore also be interpreted as the stock of public knowledge in this economy. The production function for new blueprints reads

$$\dot{H} = H(1 - L) \tag{3}$$

where  $1 - L$  is the amount of labor employed in research, and the number of new blueprints produced at a certain point in time is given by  $\dot{H}$ .

Finally we have to characterize the production of material inputs. On the one hand we consider the production of virgin materials  $Z$ , which are assumed to be extracted from a given,

known stock of a non-renewable resource  $S$  at no costs. The dynamics of  $S$  are given by  $\dot{S} = -Z$ . On the other hand, material inputs can be generated by recycling which is also assumed to bear unit costs of zero.<sup>4</sup> For virgin resource as well as recycling markets, we assume competition to be perfect. The amount of waste that is available for recycling purposes at each point in time is determined by the input mix and the consumption/investment ratio. To be able to determine the dynamics of the waste stock more specifically, let us now turn to the derivation of the material balance condition.

### 2.3 Material balance condition

All materials that are available for production purposes in our economy are originally stemming from a stock of non-renewable virgin resources  $S$ : virgin materials are extracted from the given stock and used in production. Residuals of production processes and/or consumption are then discarded as waste. The accumulated waste can again be considered as a kind of ‘ore pile’ from which secondary materials are taken for recycling.

As Lavoisier’s law of mass conservation states, matter (or energy – in the equivalent formulation of the first law of thermodynamics) can neither be created nor destroyed, but only transformed.<sup>5</sup> Consequently, the amount of materials that enter the production process have also to be part of the output produced: either in the shape of desired output, or in the shape of undesired, or – more general – involuntary, joint products referred to as pollution.

Abstracting from these joint products, all material inputs have to be bound in the final goods produced. Part of this final output is devoted to building up the economy’s capital stock while the rest is used for consumptive purposes. Assuming that capital is not depreciated in the course of production, this relation is given by the equilibrium condition for the capital market

$$\dot{K} = Y - C. \quad (4)$$

As we consider a very simple economy in which capital and consumption goods are produced by the same technology, the share of materials in newly created capital goods is identical to the share of materials in consumer goods, which facilitates the analysis considerably. The materials that are bound in consumption goods are completely discarded as waste. Over time the discarded residuals accumulate and can be used as a source of secondary materials for recycling. Alternatively, one could assume that only some given share of the waste flow from consumption actually returns to the waste pile, the other fraction being impossible to recover at reasonable costs, for example the carbon content from gasoline burnt by cars and emitted into the atmosphere. In this paper we abstract from this kind of dissipation, as such an assumption would not alter the main conclusions.<sup>6</sup>

<sup>4</sup>Alternatively – as already pointed out – the production of  $Y$  may be seen as an integrated production process employing virgin resources, recycled waste, labor and capital.

<sup>5</sup>Note that this implies the use of a double system of units in our model. The quantities  $Z$  and  $W_R$  are measured in atomic mass, that is in physical units, while  $X$ ,  $Y$ ,  $K$  or  $C$  correspond to the number of units used or produced, that is the number of forms that are generated and consumed in the economy.

<sup>6</sup>Ayres (1999), e.g., argues that complete recycling would be feasible if enough energy and storing space were



In (4) the depreciation rate is set equal to zero which simplifies the analysis considerably. Abstracting from depreciation implies that materials bound in capital are bound there forever – as long as investment is non-negative. Since it is not assumed that the accumulation of capital is irreversible, i.e. capital goods can be reconverted into consumption goods, the materials content of capital may become available for recycling again if  $\dot{K} < 0$ .

It is assumed that at  $t = 0$  there exists a stock of the virgin resource  $S_0$  as well as a stock of waste  $W_0$  that has already been accumulated during past production periods. At each point in time, the producers of intermediate goods decide upon the amounts of virgin resources and recycled waste to employ in production. The produced intermediate goods go into the production of final output. Intertemporal consumer preferences and technologies then determine the share of final products devoted to capital accumulation and consumption, and therefore also the amounts of waste generated at each point in time.

Given the above consideration, let us derive a discrete time version of the material balance condition first: on the input side of the material balance condition,  $Z_t + W_{R_t}$  enters the production process in each period  $t$ . On the output side the share of materials in final output is given by  $\frac{Z_t + W_{R_t}}{Y_t}$  whereby final output  $Y_t$  is allocated towards consumption  $C_t$  or investment  $I_t = K_{t+1} - K_t$ . Of the amount of materials contained in final output, only those which are bound in  $C_t$  are discarded onto the waste pile, such that the stock of waste at the beginning of period  $t + 1$ ,  $W_{t+1}$ , is determined by the stock  $W_t$ , the amount of materials taken from the pile for recycling  $W_{R_t}$ , and the materials that are residuals of consumption, i.e.  $W_{t+1} = W_t - W_{R_t} + \frac{Z_t + W_{R_t}}{Y_t} C_t$ . The natural degradation rate of waste is set equal to zero which facilitates the analysis, but has no impact on the general results. From these discrete time considerations, the continuous time version of the material balance condition follows straightforwardly:

$$\dot{W} = -W_R + \frac{Z + W_R}{Y} C. \quad (5)$$

As the share of materials in final output is determined endogenously and might consequently change over time, the stock of capital might not be uniform with respect to its material intensity.<sup>7</sup> Conventional wisdom suggests that the share of materials in output will decline over time if the economy is growing at a positive rate. Although a reuse of virgin as well as of recycled materials is possible, the overall stock of materials is nevertheless limited by the sum of  $S_0$  and  $W_0$  plus the materials bound in  $K_0$ . As long as capital is accumulated at a positive rate, more and more material will not be available for future recycling activities, leading to a continuous dematerialization of output.

As the waste stock is a source of productive inputs in our model, firms have an incentive to exploit the profit possibilities associated with this stock of materials. Consequently  $W$  decreases available. As the earth is an open system with respect to (solar) energy, we implicitly assume that the energy necessary for recycling is provided by the sun. One example for which the recycling rate is actually predicted to approach unity is copper (see Ayres et al. 2002).

<sup>7</sup>This changing composition of capital would have to be specifically considered if a positive depreciation rate were assumed. In this case a vintage capital model would be appropriate. Yet, as our basic results seem unperturbed by this assumption, we abstract from depreciation as it would complicate matters considerably without adding substantially to the economic intuition.

over time (even if no environmental policy is conducted) as not recycling part of the waste and leaving it on the waste pile cannot be optimal when recyclable materials are a scarce and essential input to production.

## 2.4 Sources of market failures

In the economy described in the previous sections several market failures may arise. On the one hand we have to deal with the typical market failures of this type of Romer economy, which result from monopolistic competition in the intermediate sector and the public good character of the knowledge stock  $H$ . On the other hand additional market failures can arise due to the consideration of recycling.

As waste is a valuable input in our economy, there should exist a market for (unrecycled) waste. Consumers who generate waste should be able to sell it on a market with the demand coming from the recycling sector. If no market for waste exists, consumers are not compensated for the provision of secondary materials to production. In the absence of such a market, the shadow price of consumption will be inefficiently high, inducing a suboptimal level of consumption together with dynamic inefficiency due to an ill-oriented capital investment policy. On the other hand, the price that recycling firms charge when selling their waste to the intermediate producers is too low in the absence of a market. In their pricing decision firms only take account of the fact that producing recyclables generates additional value by raising output, but neglect the additional value stemming from the generation of waste.

The second market failure arises as small recycling firms and resource extractors do not take into account that part of the material inputs they sell will again be available for future production through the reflux of materials from consumption. It is straightforward to assume that resource extracting firms do not consider this secondary usage in their pricing decision as they cannot appropriate the accruing profits. With respect to recycling firms we assume perfect competition, such that it is also plausible that a single firm does not internalize the effect of its recycling activity on the future availability of waste. Consequently, prices are set according to the basic Hotelling rule. Yet optimal pricing would require the internalization of the future revenues, such that firms should rather rent than sell resources to intermediate producers, as do consumers with respect to capital services in the conventional competitive growth models.

It will be shown that the social planner corrects for these potential market failures (Section 3). When analyzing the decentralized economy (Section 4) we assume the worst case scenario, i.e. neither a market for waste exists nor do recyclers and resource extractors internalize the effect of their supply on the reflux of materials. Since we want to focus on the sustainability issue in a decentralized context, we voluntarily adopt this pessimistic view of a market system. However, real life shows many examples of scrap markets, and the dynamic effects of secondary materials upon primary materials production plans and markets are well known features of many industries (aluminum, copper, gold, lead and steel production to cite a few). We show that even if the pessimistic view is adopted, the development of the economy is sustainable in terms of non-decreasing utility.

### 3 Socially Optimal Economy

#### 3.1 Optimality and efficiency conditions

The social planner maximizes the representative household's intertemporal utility subject to the production technologies as given by (1), the equations of motion for the capital stock  $K$ , the stock of public knowledge  $H$ , and the two stocks of materials  $S$  and  $W$ .

From the corresponding present value Hamiltonian  $H = U(C)e^{-\rho t} + \mu(\dot{K}) + \psi(\dot{H}) - \lambda Z + \theta(\dot{W})$  where  $\mu$ ,  $\psi$ ,  $\lambda$  and  $\theta$  denote the shadow prices of the respective stocks, the first order conditions for the state and control variables can be derived:

$$\begin{aligned}
 \text{a) } C : \quad \mu &= U' e^{-\rho t} && + \left[ \theta \frac{W_R+Z}{Y} \right] \\
 \text{b) } L : \quad \psi H &= \mu Y_L && - Y_L \left[ \theta \frac{W_R+Z}{Y} \right] \frac{C}{Y} \\
 \text{c) } W_R : \quad \theta - \theta \frac{C}{Y} &= \mu Y_{W_R} && - Y_{W_R} \left[ \theta \frac{W_R+Z}{Y} \right] \frac{C}{Y} \\
 \text{d) } Z : \quad \lambda - \theta \frac{C}{Y} &= \mu Y_Z && - Y_Z \left[ \theta \frac{W_R+Z}{Y} \right] \frac{C}{Y} \\
 \text{e) } K : \quad -\dot{\mu} &= \mu Y_K && - Y_K \left[ \theta \frac{W_R+Z}{Y} \right] \frac{C}{Y} \\
 \text{f) } H : \quad -\dot{\psi} &= \mu Y_H + \psi(1-L) && - Y_H \left[ \theta \frac{W_R+Z}{Y} \right] \frac{C}{Y} \\
 \text{g) } S : \quad 0 &= -\dot{\lambda} \\
 \text{h) } W : \quad 0 &\underbrace{\quad\quad\quad}_{\mathbf{2.}} = -\dot{\theta}. && \underbrace{\quad\quad\quad}_{\mathbf{1a.}} \quad \underbrace{\quad\quad\quad}_{\mathbf{1b.}}
 \end{aligned} \tag{6}$$

Furthermore the transversality conditions have to hold:

$$\lim_{t \rightarrow \infty} \mu K = 0, \quad \lim_{t \rightarrow \infty} \psi H = 0, \quad \lim_{t \rightarrow \infty} \lambda S = 0, \quad \lim_{t \rightarrow \infty} \theta W = 0. \tag{7}$$

(6g), (6h) and (7) show that it is socially optimal to completely exhaust the stocks of virgin and recyclable materials in the long run.

The first terms on the left hand side (LHS) and right hand side (RHS) of (6a) to (6f) are quite familiar from the standard growth literature. On the right hand side of (6a) to (6f), the first terms denote marginal utility, (6a), respectively the value of a marginal unit of the respective input in the production of an additional unit of capital, (6b) to (6f). In an economy without recycling, these would be equal to the first terms on the LHS: the marginal opportunity costs that arise, e.g., from foregone consumption, (6a), or inputs employed in final output production, (6b) to (6f).

Now consider the effect of integrating recycling: on the one hand, one further optimality condition arises, (6c), that reflects the marginal costs and revenues of recycling. On the other hand, additional terms show up in (6a) to (6f) that mirror the costs and benefits of recycling and its impact on the material balance condition. These additional terms (**1a.**, **1b.** and **2.**) reflect the sources of market failures introduced in the previous section.

The term **2.** on the LHS of (6c) and (6d)  $\theta c$  can be interpreted as the 'rental' element in the pricing of virgin resources and recycled materials that is due to the reflux of materials from consumption to the waste heap. It represents the reduction of the opportunity costs of resource extraction that arises as the share  $c$  of each unit of extracted materials can be used again in future production.

With respect to the optimality condition (6a) the additional term on the RHS (**1b.**) captures the value of the waste generated by consumption. It shows the necessary correction of the households' investment plans due to the reflux of materials to the waste pile. Due to the value generated by the reflux of waste from consumption, the value of a unit of output consumed is higher than in the absence of recycling.

When considering the marginal social revenue arising from a *ceteris paribus* increase of the input of any production factor in the  $Y$ -sector, one has to consider not only the direct effect of this increase on the level of output, but also the feedback effect on the generation of valuable waste. This latter effect is captured by **1a.**

**1a.** and **1b.** mirror the distortions arising in a market economy if a market for waste products does not exist. Without such a market, the additional value of production via the generation of waste will not be reflected by the price of waste. Consequently, the social value of consumption will be too low and the savings decision will be distorted. On the other hand firms will not consider the additional value created by employing an additional marginal unit of inputs in the production of  $Y$ , such that the input decision of firms will also be distorted.

To get a better understanding of the first-order conditions (6b) – (6f), let us take a closer look at (6c) which reads after rearranging

$$\theta(1-c) = Y_{W_R} [(1-c)\mu + cU'] \quad (8)$$

where  $c$  denotes the share of consumption in output,  $\frac{C}{Y}$ . On the LHS the net costs of one marginal unit of waste extracted from the waste pile and employed in production are given: of the one unit extracted,  $c$  flows back to the waste pile after consumption. Evaluating the net extraction  $(1-c)$  by the shadow price of waste  $\theta$  gives the net costs of extraction. The RHS shows the benefits from extracting one marginal unit of waste: of the  $Y_{W_R}$  produced from this marginal unit, the share  $1-c$  goes to capital accumulation while  $c$  ends up in consumption. Evaluating the shares by the shadow price of capital and marginal utility of consumption respectively, finally gives the benefits of extracting one marginal unit of waste.

From the ratio of marginal productivities

$$\frac{Y_{W_R}}{Y_Z} = \frac{\theta - \theta c}{\lambda - \theta c}. \quad (9)$$

it can be seen that the shadow price ratio between the two factors is distorted due to the reflux of materials to the waste pile  $\theta c$ . The ratio of marginal benefits accruing from the extraction of resources and waste is given by the ratio of marginal productivities as in the standard non-recycling model. This is due to the fact that the value of one marginal unit of output produced in consumption terms is of course identical over sectors.

Due to the modifications of the first-order conditions the Keynes-Ramsey rule (KKR) of this economy also takes a different shape. From (6a), (6c) and (6e) (see Appendix A), we get the KKR in real terms:

$$\sigma g_C + \rho = Y_K + Y_{W_R} \frac{\frac{W_{R+Z}}{Y}}{(1-c) \left(1 - \frac{W_{R+Z}}{Y}\right)} \left[ (1-c) Y_K + g \frac{W_{R+Z}}{Y} \right]. \quad (10)$$

Compared to the standard KKR, an additional composite term on the RHS appears that reflects the effects of introducing recycling and the material balance condition.

As with the KKR, the Hotelling rules for the two material inputs take a different shape (for the derivation see Appendix B):

$$\begin{aligned} \text{a)} \quad \frac{\dot{Y}_{WR}}{Y_{WR}} &= Y_K + \frac{Y_{WR}}{1-c} \left( \frac{W_R+Z}{Y} \dot{c} \right) - \frac{\dot{c}}{1-c} \\ \text{b)} \quad \frac{\dot{Y}_Z}{Y_Z} &= Y_K + \frac{Y_{WR}}{1-c} \left( \frac{W_R+Z}{Y} \dot{c} \right) - \frac{\dot{c}}{1-c} \frac{Y_{WR}}{Y_Z} \end{aligned} \quad (11)$$

The standard Hotelling rule that equalizes the growth rate of the physical rate of return to  $W_R$  (or  $Z$  respectively) to the marginal product of capital, is enhanced by two terms in order to incorporate the effects of the waste reflux. These terms reflect the quasi ‘renewability’ of the resource stocks that is due to recycling. Take (11a) for example: If an additional marginal unit were recycled, this would affect the development of the ‘regeneration rate’ of the waste stock. Firstly, the additional recycling would induce a change of the reflux rate and thereby the output producible in the future (second term on the RHS). Secondly, the time path of the savings rate,  $g_s = \frac{\dot{s}}{s} = \frac{\dot{c}}{1-c}$ , would be influenced (third term on the RHS). A positive  $g_s$  would, e.g., imply a lower waste reflux in the future and therefore an increase of the opportunity costs of extraction. The same line of reasoning holds for (11b).

### 3.2 Balanced growth path

From (2) and (3) and (6b) to (6f) and the equilibrium condition for the capital market we can derive the socially optimal balanced growth path of the economy (see Appendix C):<sup>8</sup>

$$g_Y^s = \frac{1}{\sigma} \left( \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1-\gamma}{\gamma} - \rho \right) \quad (12)$$

$$L^s = \frac{\alpha_2}{\alpha_3} \frac{\gamma}{1-\gamma} \left[ \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1-\gamma}{\gamma} - g_Y^s \right] \quad (13)$$

$$g_Z^s = g_Y^s - \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1-\gamma}{\gamma}. \quad (14)$$

with  $g_i = \frac{di/dt}{i}$  and the superscript  $s$  marking the socially optimal values of variables and growth rates. The term  $\frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1-\gamma}{\gamma}$  represents the value obtained from allocating a unit of labor to the R&D sector. Therefore the condition for  $L^s$  to be positive and  $g_Z^s$  to be negative ( $\frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1-\gamma}{\gamma} > g_Y^s$ ) states that balanced growth in output can only be maintained if the increases in efficiency obtained from increasing  $H$  can overcompensate the decrease in materials’ input over time. Growth rates as well as the allocation of labor are independent of the production elasticities of virgin resources and recycled waste, as the input of both materials develops at the same rate along the balanced path.

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<sup>8</sup>We define a balanced economic growth path or long-run equilibrium of the economy as a time path along which all variables grow at constant, possibly zero, rates.

## 4 Decentralized Economy

As we have already indicated, there are several market failures present in this economy, so that without optimal economic policy the decentralized growth path will be suboptimal. In the following we derive the decentralized solution under the assumption that none of these externalities are internalized.

### 4.1 Equilibrium in household and traditional production sectors

Households in our economy solve the standard optimization problem, which is given by (1) and which results in the familiar Keynes-Ramsey rule

$$\sigma g_C + \rho = r. \quad (15)$$

By comparing (15) to (10) the consequences of the missing market for waste can be seen. As no market for waste exists, households do not take account of the fact that the waste they generate in consumption has a positive value for future production. The first market failure reflected in **1b.** arises.

On the final goods market, for which perfect competition is assumed the following set of first-order conditions has to hold in equilibrium:

$$r = \alpha_1 \frac{Y}{K}, \quad w = \alpha_2 \frac{Y}{L}, \quad p_{\tilde{X}} = \alpha_3 \frac{Y}{\tilde{X}}. \quad (16)$$

In deriving these efficiency conditions, we assumed that firms cannot internalize the effect that their input decision has on the availability of waste for recycling. Consequently, the marginal revenues of the respective production factors in (16) do not reflect their true social value. In comparing the first-order conditions for the firms with the FOCs of the social planner in (6) it can easily be seen that the externality from the materials reflux, **1a.**, is not present in (16).

Intermediates' producers maximize their profit from the sale of intermediate products  $\pi_i = p_{x_i}x_i - p_Z z_i - p_{W_R} w_{R_i}$ , where  $p_Z$  and  $p_{W_R}$  denote the prices for virgin resources and recycled waste that are determined on the markets for virgin resources and recycled waste. Optimization gives the familiar result that the prices of all  $x_i$ s and also their quantities are identical in equilibrium ( $p_{x_i} = p_x$  and  $x_i = x$ ). Equilibrium profits are given by  $\pi = (1 - \gamma)p_x x$  while the price of  $X$  is equal to  $p_X = \alpha_3 \frac{Y}{X}$ .

Using Shepard's lemma, the sector demands for  $Z$  and  $W_R$  can now be derived:

$$Z = \beta \gamma \frac{p_x X}{p_Z}, \quad W_R = (1 - \beta) \gamma \frac{p_x X}{p_{W_R}}. \quad (17)$$

Before starting to produce, firms in the intermediate sector have to acquire a patent for the respective good. In equilibrium, intertemporal profits from production have to be equal to the patent price  $p_H$ , which gives the no-arbitrage condition for the patent market:

$$r = g_{p_H} + \frac{\pi}{p_H}. \quad (18)$$

Producers of patents act on a perfect market, so that in equilibrium marginal costs will be equalized to marginal revenues. As labor is the only private input to production the corresponding equilibrium condition reads  $w = p_H H$ . Equating this to the optimality condition for the input of  $L$  in  $Y$  from (16), gives the equilibrium condition for the labor market,  $p_H H = \alpha_2 \frac{Y}{L}$ .

Before turning to the equilibrium conditions for production on the markets for material inputs, we can already derive the equilibrium share of labor in  $Y$ -production and the growth rate of output as a function of the growth rates of the prices for  $Z$  and  $W_R$  (see Appendix D):

$$L = \frac{\alpha_3(1-\gamma)(\sigma-1) + \alpha_2\gamma(1+\rho)}{\alpha_3(1-\gamma)(\gamma+\sigma-1) + \alpha_2\gamma} - \frac{\alpha_3\gamma(\sigma-1)}{\alpha_3(1-\gamma)(\gamma+\sigma-1) + \alpha_2\gamma} (\beta g_{p_Z} + (1-\beta)g_{p_{W_R}}) \quad (19)$$

$$g_Y = \frac{\alpha_3(1-\gamma)(\alpha_3(1-\gamma) - \alpha_2\rho)}{\alpha_2(\alpha_3(1-\gamma)(\gamma+\sigma-1) + \alpha_2\gamma)} - \frac{\alpha_3\gamma((\alpha_3(1-\gamma) + \alpha_2))}{\alpha_2(\alpha_3(1-\gamma)(\gamma+\sigma-1) + \alpha_2\gamma)} (\beta g_{p_Z} + (1-\beta)g_{p_{W_R}}). \quad (20)$$

The second terms on the RHS of the growth rate and the equilibrium labor share reflect the impact of the development of material inputs' prices on the development of output.

To what extent  $g_{p_Z}$  and  $g_{p_{W_R}}$  influence the growth rate depends on the respective production elasticities of  $Z$  and  $W_R$ . The higher the production elasticity of a material input, i.e. the more productive this type is in the production of intermediates, the higher the impact of the development of its price is for economic growth.

## 4.2 Equilibrium in the material inputs producing sectors

With respect to the behavior of resource extracting firms, we assume that they do not internalize the positive effect their provision of virgin resources has on the reflux of materials onto the waste pile. Firms maximize their intertemporal profits from resource extraction, considering only the finiteness of the stock of virgin resources:

$$\max_Z \int_0^\infty p_Z Z e^{-\int_0^t r(s) ds} dt \quad \text{s.t.} \quad \dot{S} = -Z. \quad (21)$$

It has also been argued that the reflux of materials is likewise exogenous to the recycling firms, such that recycling firms solve the following profit maximization problem

$$\max_{\bar{W}_R} \int_0^\infty p_{W_R} \bar{W}_R e^{-\int_0^t r(s) ds} dt \quad \text{s.t.} \quad \dot{W} = -W_R + (\bar{W}_R + Z)c. \quad (22)$$

where  $\bar{W}_R$  is exogenous to the firms.

From the first-order and transversality conditions for (21) and (22), it can be shown that the prices of virgin resources and recycled waste follow the standard Hotelling path ( $g_{p_Z} = g_{p_{W_R}} = r$ ). This result already reflects the consequences of material input producers not considering the reflux of materials in their optimization. If they did, the standard Hotelling rules would be enhanced by additional terms that reflect the quasi-renewability stemming from recycling. Yet, since recyclers as well as virgin resource extractors do not internalize the 'regeneration' capacity of the waste stock, this regenerability has no effect on the growth rates of materials' prices – neither along the balanced path, nor, as can be shown, along the transitory path.

### 4.3 Balanced Growth Path

We can now derive the growth rate of output and the equilibrium allocation of labor in terms of the parameters of the model only (see Appendix D):

$$g_Y^m = \frac{\alpha_3(\alpha_2\rho + \alpha_3(1-\gamma)((1+\rho)\gamma - 1))}{-\alpha_2(\alpha_2\gamma - \alpha_3(1-\gamma)^2) - \alpha_3(\alpha_2 + \alpha_3(1-\gamma)\gamma)\sigma} \quad (23)$$

$$L^m = \frac{\alpha_2(\alpha_2\gamma(1+\rho) + \alpha_3(\gamma(1+\rho) + \sigma - 1))}{\alpha_2(\alpha_2\gamma - \alpha_3(1-\gamma)^2) + \alpha_3(\alpha_2 + \alpha_3(1-\gamma)\gamma)\sigma} \quad (24)$$

where superscript  $m$  denotes growth rates and variable values along the balanced path of the pure market economy.

As the prices for  $Z$  and  $W_R$  grow at the same rate,  $g_Y^m$  and  $L^m$  are independent of the production elasticity  $\beta$ , so that the relative productivity of the two material inputs has no effect on the growth rate of the economy. Yet with respect to optimal initial prices and quantities, the respective productivities of the inputs  $\beta$  and  $1 - \beta$  do matter, as we shall show in the next section.

Growth rates and labor allocation along the balanced growth path – in the market as well as in the socially optimal case – are furthermore independent of the initial stocks of virgin resources and waste, a well known feature of the standard resource economics models. The allocation of resources over time is solely determined by arbitrage considerations; initial stocks only matter with respect to which level the economy is operating at. So, initial stocks determine initial quantities and – in the case of the market economy – prices, as the next section will also show.

To sum up, we have seen that not only in the socially optimal case, but also in the market economy, the development of the economy will be sustainable along the balanced growth path – even in the absence of governmental policy. Nevertheless, it should be noted that due to the market failures output growth and R&D activities are lower in the decentralized scenario (see Appendix E).

## 5 Economic and Environmental Policy

As already discussed in Section 2, two classes of market failures arise in the economy under consideration. On the one hand, those associated with the standard Romer (1990) model and on the other hand those which are due to the integration of recycling into the model.

With respect to the first class of market failures, appropriate policy measures are well known from the literature (e.g. Barro/Sala-i-Martin 2004): the effects of monopolistic competition in the intermediate sector can, e.g., be compensated by subsidizing the production of intermediates. With respect to the positive knowledge spillovers from R&D, this can be cured by, for example, subsidizing R&D activities. In the basic Romer model, these two instruments would be sufficient to implement the socially optimal path. Yet, in our economy we still have to consider the recycling-related market failures.

The first failure arises from the fact that no market for unrecycled waste exists. Let us now introduce such a market for unprocessed waste. Households are assumed to be either unable or



not allowed to store their waste, such that their supply of waste is totally inelastic. Recycling firms compete for the right to buy the waste, thereby driving the price up to its equilibrium level.

Recycling firms are maximizing intertemporal profits subject to the waste stock restriction:

$$\max_{W_R, W_U} \int_0^\infty [p_{W_{R\tau}} W_{R\tau} - p_{W_{U\tau}} W_{U\tau}] e^{-\int_0^t r(s) ds} d\tau \quad \text{s.t.} \quad \dot{W}_t = -W_{Rt} + W_{Ut} \quad (25)$$

where  $p_{W_U}$  and  $W_U$  denote the price and quantity of the unprocessed waste. The first-order conditions of this optimization problem read

$$p_{W_R} e^{-\int_0^t r(s) ds} = p_{W_U} e^{-\int_0^t r(s) ds} = -\omega \quad \text{and} \quad \dot{\omega} = 0 \quad (26)$$

with  $\omega$  being again the shadow value of waste. From (26) it follows directly that the prices of unprocessed and recycled waste are equalized in equilibrium ( $p_{W_R} = p_{W_U}$ ). The intuition behind this result is that as recyclers do not perform any costly transformation of the waste and are operating on a perfect market, a difference between input and output prices cannot arise in equilibrium.

Furthermore, it can easily be seen from (26) that the growth rate of prices is again equal to the interest rate, so prices still follow the Hotelling rule. This might seem surprising, as the waste stock can be considered a renewable resource that regenerates due to the reflux of waste from consumption. Yet this reflux is external to the recyclers. It is households who decide how much to consume, and thereby also how much recyclable waste to produce. For the recycling firms, the supply decision is exogenous and they consequently do not take account of the waste reflux in their pricing decision.

If a market for waste exists, the net costs of consumption that arise to households are given by the difference between the price for output (which is normalized to unity) and the revenue from selling the waste generated per unit of consumption ( $\frac{W_R + Z}{Y}$ ):

$$1 - \frac{W_R + Z}{Y} p_{W_R}. \quad (27)$$

The second recycling-related market failure arises from the fact that neither recyclers nor virgin resource extractors consider the impact of their production decisions on the future availability of recyclable waste. As a way to correct for the distortions attributable to this externality, the government can subsidize the extraction of resources and the recycling of waste.

To be able to implement the correct subsidy rate, the policy maker has to compare market prices to the socially optimal prices implicitly given by the shadow values in (6). Yet, this requires that shadow values and market prices have to be expressed in the same units.

In the standard growth literature, a unit of output employed in consumption or capital accumulation is equally valued by households. As the marginal opportunity costs arising from consuming one unit of output or investing one unit are identical, shadow values are equalized. Consequently, it is irrelevant whether the shadow value of output, consumption or capital goods is chosen as the conversion unit. It is only by convention that the shadow value of consumption, i.e. the marginal utility of consumption, is usually employed.

Yet, in our model the shadow values of consumption, output and capital are not equal. The wedge between them is due to the fact that after consumption the material part of the consumed goods can again be employed in future production. On the other hand, the material which is bound in the share of output invested in capital accumulation does not become available again. This difference with respect to the future recycling possibilities has to be reflected by the wedge between the respective shadow values.

Considering the difference in the properties of consumption and capital goods, output, from which both are taken, is essentially a composite good. Its socially optimal shadow value

$$v = \mu - \theta \frac{W_R + Z}{Y} c = (1 - c) \mu + cU' \quad (28)$$

reflects this fact, as  $v$  is the weighted average of the value of the consumption share in output ( $cU'$ ) and the value of the investment share ( $(1 - c)\mu$ ).

When choosing an appropriate conversion unit to compare market prices and optimal shadow values, the just described differences in the shadow values have to be accounted for. In principle either shadow value could be employed for conversion. Yet, to allow for a direct comparison of market prices and socially optimal shadow values, it seems straightforward to express both in the same units. As we have so far expressed prices in the decentralized scenario in terms of the output price (e.g. in (27)), we now stick to this choice. Dividing (6a) to (6e) by  $v$  then yields the shadow values in terms of the value of output, that is the socially optimal prices  $w^s$ ,  $r^s$  and  $p_k^s$ ,  $k = C, W_R, Z$ .

To compute the optimal subsidy rates, we first have to determine the initial prices for the decentralized market case in the presence of a market for unprocessed waste. Equating the initial market prices to the socially optimal prices at time zero gives the optimal initial subsidy rates  $\tau_s$ .<sup>9</sup> Over time the development of the subsidy rate has then to reflect the rising scarcity of materials, such that the subsidized prices for virgin resources and recycled waste are given by  $p_k = (p_{k0} - \tau_s)e^{-\int_0^t r(s)ds}$ ,  $k = Z, W_R$ .

Having now corrected for the price level of recycled waste and virgin resources, in addition to the implementation of the market for waste and the correction of the Romer market failures, finally gives the socially optimal growth path for the decentralized economy.

Policy intervention through subsidies is of course not the only way to correct for this kind of externality. Since we are dealing with a dynamic externality, any asset market device able to give the correct incentives to resource processing firms would restore the optimality of the market economy. Such a device could be the creation of license markets.

Licenses would be issued by virgin resource producers for each resource unit extracted from the virgin resource stock. These licenses would correspond to infinite property rights on the processing of the resources after they first have been employed in production. In addition, the government would sell the rights on the processing of the initial stock of waste.

Waste recyclers would have to buy the licenses and an equilibrium price would equate the license price to the discounted sum of the flow of profits created by the use of the resources over

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<sup>9</sup>The derivation of the starting values is available from the authors upon request.

and over again. Government intervention would therefore be reduced to the assignment of rights via the licenses and the corresponding licensed processing of waste. Under rational expectations, the equilibrium price for the licenses would correctly reflect the marginal shadow value of the virgin and waste resources and the social optimum could be completely decentralized.

## 6 Conclusions

The purpose of this paper was to show that the treatment of accumulated waste as a source of valuable inputs has strong implications for the role of environmental policy as a means to ensure the sustainability of development. It was shown that, by introducing recycling and thereby the possibility to reuse virgin resources, environmental policy ceases to be a prerequisite for sustainable development. Due to the scarcity of materials – virgin resources and waste – the stocks of both types of materials have a positive value. As it would be suboptimal not to employ part of a valuable resource stock in production, the stock of virgin resources as well as the stock of recyclable waste go to zero in the long run, driven by the development of prices. Consequently, the stock of waste declines monotonically over time.

Although environmental policy is no condition for development to be sustainable in this economy, it may serve as a means to correct for the market failures which arise due to the introduction of recycling into the model. On the one hand, a market for unrecycled waste is missing in the unregulated economy. This market failure which characterizes the disposal and reuse of waste in many countries can be corrected for by the introduction of a market for recyclable waste. On the other hand, virgin resource extractors and recycling firms do not account for the effects of their output decision on the future availability of recyclable waste. By subsidizing recycling and resource extraction or introducing a license system, this externality can be internalized.

Within this paper we have so far restricted the analysis to the balanced growth path only. In a next step we shall focus on the transitional dynamics of the underlying model. Due to the dynamics of the prices for waste and virgin resources along the transition path, EKC-type dynamics might arise with respect to the development of the waste stock (see e.g. Pittel 2006). Furthermore the framework could be extended to include negative externalities of the waste stock. Yet, we do not expect this to alter the qualitative results. Integrating negative utility/productivity effects of waste generation into our model would possibly only lead to tighter optimal environmental policy and therefore a faster decrease of the waste pile over time.

## 7 Appendix

### Appendix A: Keynes-Ramsey rule

To ease the presentation we first introduce the composite variables  $\nu = c^{-\sigma} e^{-\rho t}$  and  $E = \frac{W_R + Z}{Y}$  for the discounted marginal utility and the material content per unit of  $Y$ , such that we can rewrite (6a) as  $\mu = \nu + \theta E$ . Inserting this condition into (6e) gives  $\dot{\mu} = -\nu Y_K - \theta Y_K E (1 - c)$ .

Differentiating  $\mu = \nu + \theta E$  and substituting for  $\dot{\mu}$  yields

$$\frac{\dot{\nu}}{\nu} = -Y_K - \frac{\theta}{\nu} \left[ Y_K E (1 - c) + \dot{E} \right] \quad (29)$$

where  $\frac{\theta}{\nu} = \frac{Y_{WR}}{(1-c)(1-Y_{WR}E)}$  can be obtained from dividing (6c) by  $\theta$  and solving for the shadow price ratio. Inserting  $\frac{\dot{\nu}}{\nu}$  into (29) gives (10) when taking into account that  $\frac{\dot{\nu}}{\nu} = -\sigma g_C - \rho$ .

## Appendix B: Hotelling rules

In order to derive the Hotelling rule for recycled waste, rewrite (6c) as  $Y_{WR} = \theta \frac{1-c}{\mu - \theta Ec}$  where  $E$  again denotes the material content of a unit of output,  $\frac{W_R + Z}{Y}$ . Differentiating this expression with respect to time gives

$$\frac{\dot{Y}_{WR}}{Y_{WR}} = -\frac{\dot{c}}{1-c} - \frac{1}{\mu - \theta Ec} \left( \dot{\mu} - \theta c \dot{E} - \theta E \dot{c} \right). \quad (30)$$

Rewriting (6c) as

$$\frac{1}{\mu - \theta Ec} = \frac{Y_{WR}}{\theta(1-c)} \quad (31)$$

and substituting into (30) together with  $\dot{\mu}$  from (6e) we get

$$\frac{\dot{Y}_{WR}}{Y_{WR}} = -\frac{\dot{c}}{1-c} - \frac{Y_{WR}}{(1-c)} \left( -\frac{\mu}{\theta} Y_K + Y_K Ec - c \dot{E} - E \dot{c} \right). \quad (32)$$

The ratio  $\frac{\mu}{\theta}$  can be obtained from (6c) as  $\frac{\mu}{\theta} = \frac{1 + Y_{WR}Ec - c}{Y_{WR}}$ . Inserting this expression into (32) gives the Hotelling rule (11a) after rearranging.

Proceeding similarly we obtain the Hotelling rule for virgin resources. From (6d) we know that  $Y_Z = \frac{\lambda - \theta c}{\mu - \theta Ec}$ . Differentiating this expression with respect to time, inserting (6e) and rearranging, we get

$$\frac{\dot{Y}_Z}{Y_Z} = Y_K + \frac{1}{\frac{\mu}{\theta} - c} \left[ -\frac{1}{Y_Z} \dot{c} + (\dot{E}c) \right]. \quad (33)$$

Substituting (31) into (33) gives the Hotelling rule for virgin resources (11b).

## Appendix C: Balanced growth in the social optimum

In order to derive the balanced growth path of the economy in the socially optimal case, we first solve (6c) for  $\theta$  which gives

$$\theta = \frac{\alpha_3(1-\beta)Y}{W_R(1-c) + \alpha_3(1-\beta)(Z + W_R)c} \mu. \quad (34)$$

Substituting (34) into (6a) and simplifying yields

$$-C^{-\sigma} e^{-\rho t} = \mu \left( -1 + \frac{\alpha_3(1-\beta)(W_R + Z)}{W_R(1-c) + \alpha_3(1-\beta)(Z + W_R)c} \right) \equiv \mu A. \quad (35)$$

From differentiating (35) with respect to time and dividing again by (35), we get

$$-(\sigma g_C + \rho) = g_\mu + g_A \quad (36)$$

whereby it can easily be shown that along the balanced path, where  $g_Y = g_C$  and  $g_Z = g_{W_R}$  have to hold, the growth rate of  $A$  is equal to zero. Moreover it can be derived from (6e) that  $g_\mu = g_Z - g_Y$  holds, such that (36) gives

$$g_Y = \frac{1}{\sigma - 1}(-g_Z - \rho). \quad (37)$$

Rearranging (6b) and making use of (34) again it can be shown that

$$\psi H = \mu \alpha_2 \frac{Y}{L} \left( \frac{(Y - C)W_R}{(Y - C)W_R + \alpha_3(1 - \beta)C(W_R + Z)} \right) \equiv \mu \alpha_2 \frac{Y}{L} B. \quad (38)$$

Differentiating (38) with respect to time, dividing the resulting expression again by (38) and considering that  $g_Y = g_C$  and  $g_Z = g_{W_R}$  which implies  $g_B = 0$  we get  $g_\psi + g_H = g_\mu + g_Y$ . With (3) and  $g_\mu = g_Z - g_Y$  this gives

$$g_\psi = g_Z - (1 - L). \quad (39)$$

In order to express  $g_Z$  as a function of  $L$  we make use of (6f). Dividing (6f) by  $\psi$  and substituting for  $\psi H$  from (38) some terms cancel and we get (by employing (39))

$$g_Z = -\frac{\alpha_3}{\alpha_2} \frac{1 - \gamma}{\gamma} L. \quad (40)$$

We can now express the growth rate of output (37) in terms of the labor input in the final goods sector:

$$g_Y = \frac{1}{\sigma - 1} \left( \frac{\alpha_3}{\alpha_2} \frac{1 - \gamma}{\gamma} L - \rho \right). \quad (41)$$

To solve for  $g_Y$  and  $L$  as functions of the parameter of the model only, a second condition is required that links  $g_Y$  to  $L$ . This expression can be obtained by consideration of the production function for final goods (2). Differentiating (2) with respect to time and rewriting the resulting expression in growth rates gives after slight rearranging

$$g_Y = \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1 - \gamma}{\gamma} (1 - L) + \frac{\alpha_3}{\alpha_2 + \alpha_3} g_Z \quad (42)$$

which yields under consideration of (40)

$$g_Y = \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1 - \gamma}{\gamma} - \frac{1 - \gamma}{\gamma} \frac{\alpha_3}{\alpha_2} L. \quad (43)$$

Solving (41) and (43) for  $g_Y$  and  $L$  finally gives (12) and (13).

## Appendix D: Decentralized balanced growth path

To derive the balanced growth rate of output, we first express the price for intermediates as a function of the prices for virgin resources and recycled waste:

$$p_{x_i} = \frac{1}{\gamma} \frac{p_{W_R}^{1-\beta} p_Z^\beta}{(1 - \beta)^{1-\beta} \beta^\beta}. \quad (44)$$

Differentiating (17) with respect to time, solving for  $g_{p_x} + g_X$  and equating the resulting expressions shows that the growth rates of the values of the two material inputs are equalized in equilibrium:

$$g_Z + g_{p_Z} = g_{W_R} + g_{p_{W_R}}. \quad (45)$$

Expressing (44) in growth rates shows that the growth rate of  $p_x$  is given by the weighted average of the growth rates of  $p_Z$  and  $p_{W_R}$ , whereby the weights correspond to the respective production elasticities ( $g_{p_x} = \beta g_{p_Z} + (1 - \beta) g_{p_{W_R}}$ ). From this result and (45) we get after rearranging:

$$g_Z = g_Y - g_{p_Z} \quad \text{resp.} \quad g_{W_R} = g_Y - g_{p_{W_R}}. \quad (46)$$

On the other hand we know from (2) and (3) that along the balanced path, on which  $g_Y = g_K$ ,

$$(\alpha_2 + \alpha_3)g_Y = \alpha_3 \frac{1 - \gamma}{\gamma} (1 - L) + \alpha_3 \beta g_Z + \alpha_3 (1 - \beta) g_{W_R} \quad (47)$$

holds. Inserting (46) into (47) yields:

$$g_Y = \frac{\alpha_3}{\alpha_2} \frac{1 - \gamma}{\gamma} (1 - L) - \frac{\alpha_3}{\alpha_2} (\beta g_{p_Z} + (1 - \beta) g_{p_{W_R}}). \quad (48)$$

A second expression for the relation between final output growth and  $L$  can be obtained from the following considerations: expressing the equilibrium condition for the labor market,  $p_H H = \alpha_2 \frac{Y}{L}$ , in growth rates and using (3) gives  $g_{p_H} = g_Y - g_H = g_Y - (1 - L)$ . Further information on the development of  $p_H$  stems from the equilibrium condition on the patent market (18) where  $\pi = (1 - \gamma) p_x x$ :

$$r = g_{p_H} + (1 - \gamma) \frac{p_x}{p_H} \frac{X}{H} = g_Y - (1 - L) + (1 - \gamma) \frac{p_x}{p_H} \frac{X}{H}. \quad (49)$$

$p_x$  and  $p_H$  can be obtained from  $p_X = \alpha_3 \frac{Y}{X}$  and  $p_H H = \alpha_2 \frac{Y}{L}$  while  $g_Y$  is given by (48). Inserting and rearranging the respective expressions yields

$$r = \frac{\alpha_3(1 - \gamma) - \alpha_2 \gamma}{\alpha_2 \gamma} (1 - L) + \frac{\alpha_3(1 - \gamma)}{\alpha_2} L - \frac{\alpha_3}{\alpha_2} (\beta g_{p_Z} + (1 - \beta) g_{p_{W_R}}). \quad (50)$$

Employing the Keynes-Ramsey rule (15) and (47) we finally get (19) and (20).

## Appendix E: Comparison of social optimal and market BGP

To compare the socially optimal and decentralized growth rates of output, consider the Hotelling rule, (50) and (15) which give after rearranging

$$g_Y^m = \frac{1}{\sigma} \left[ \frac{\alpha_3}{\alpha_2 + \alpha_3} \frac{1 - \gamma}{\gamma} - \frac{\alpha_3}{\alpha_2 + \alpha_3} \left( \frac{\alpha_2}{\alpha_3} (1 - L^m) + \frac{(1 - \gamma)^2}{\gamma} L^m \right) - \rho \right]. \quad (51)$$

Using (12) it is now straightforward to show that the equilibrium growth rate in the social planner case exceeds the growth rate in the decentralized economy

$$g_Y^s - g_Y^m = \frac{1}{\sigma} \left[ \frac{\alpha_3}{\alpha_2 + \alpha_3} \left( \frac{\alpha_2}{\alpha_3} (1 - L^m) + \frac{(1 - \gamma)^2}{\gamma} L^m \right) \right] > 0. \quad (52)$$

To show that R&D effort is lower in the market economy, we make use of the Hotelling rule and (48) which gives

$$g_Y = \frac{\alpha_3}{\alpha_2} \frac{1-\gamma}{\gamma} (1-L) - \frac{\alpha_3}{\alpha_2} r. \quad (53)$$

We can now show that

$$g_Y^i \left( 1 + \frac{\alpha_3}{\alpha_2} \rho \right) = \frac{\alpha_3}{\alpha_2} \frac{1-\gamma}{\gamma} (1-L^i) - \frac{\alpha_3}{\alpha_2} \rho, \quad i = m, s. \quad (54)$$

by substituting  $r$  from (15) into (53) or, analogously for the social planner case, combining (12) and (43). As we have already established that  $g_Y^m < g_Y^s$ , it follows directly from (54) that the R&D effort in the decentralized economy is below the socially optimal level.

## References

- Ackerman, Frank (1997), *Why Do We Recycle? Market, Values and Public Policy*, Washington: Island Press.
- Aghion, Philippe/Howitt, Peter (1998), *Endogenous Growth Theory*, Cambridge, Mass.: The MIT Press.
- Ayres, Robert U. (1999), The second law, the fourth law, recycling and limits to growth, *Ecological Economics* 29, 473-483.
- Ayres, Robert U./Ayres, Leslie W./Råde, Ingrid (2002), *The Life Cycle of Copper, its Co-Products and By-Products*, MMSD Working Paper, International Institute for Environment and Development.
- Barro, Robert J./Sala-i-Martin, Xala (2004), *Economic Growth*, Cambridge, MA: The MIT Press.
- Conrad, Klaus (1999), Resource and waste taxation in the theory of the firm with recycling activities, *Environmental and Resource Economics*, 14, 217-242.
- Dinan, Terry M. (1993), Economic efficiency effects of alternative policies for reducing waste disposal, *Journal of Environmental Economics and Management*, 25, 242-256.
- Dixit, Avinash K./Stiglitz, Joseph E. (1977), Monopolistic competition and optimal product diversity, *American Economic Review*, 67, 297-308.
- di Vita, Guiseppe (2001), Technological change, growth and waste recycling, *Energy Economics*, 23, 549-567.
- Ethier, Wilfred J. (1982), National and international returns to scale in the modern theory of international trade, *American Economic Review*, 72, 389-405.
- Fullerton, Don/Kinnaman, Thomas C. (2002), *The Economics of Household Garbage and Recycling Behavior*, Cheltenham, UK and Northampton, MA, US: Edward Elgar.
- Grimaud, Andre/Rougé, Luc (2005), Polluting non renewable resources, innovation and growth: Welfare and environmental policy, *Resource and Energy Economics*, 27, 109-129.
- Grimaud, Andre/Rougé, Luc (2003), Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies, *Journal of Environmental and Resource Economics*, 45, 433-453.

- Groth, Christian/Schou, Paul (2002), Can non-renewable resources alleviate the knife-edge character of endogenous growth?, *Oxford Economic Papers*, 54, 386-411.
- Hoel, Michael (1978), Resource extraction and recycling with environmental costs, *Journal of Environmental Economics and Management*, 5, 220-235.
- Huhtala, Anni (1999), Optimizing production technology choices: conventional production vs. recycling, *Resource and Energy Economics*, 21, 1-18.
- Huhtala, Anni (1997), A post-consumer waste management model for determining optimal levels of recycling and landfilling, *Environmental and Resource Economics*, 10, 301-314.
- Kuhn, Thomas/Pittel, Karen/Schulz, Thomas (2003), Recycling for sustainability - a long-run perspective?, *International Journal of Global Environmental Issues*, 3, 339-355.
- Lusky, Rafael (1976), A model of recycling and pollution control, *Canadian Journal of Economics*, 9, 91-101.
- Mainwaring, Lynn (1995), Primary resource use and voluntary recycling schemes: Dynamic issues in a global context, *Resource and Energy Economics*, 17, 341-357.
- Musu, Ignazio/Lines, Marji (1995), Endogenous growth and environmental preservation, in: G. Boero and A. Silberston, eds., *Environmental Economics: Proceedings of the European Economic Association at Oxford 1993*, Oxford: St. Martin's Press.
- OECD (2001), *OECD Environmental Outlook*, Paris: OECD.
- Pittel, Karen (2002), *Sustainability and Endogenous Growth*, Cheltenham, UK and Northampton, MA, US: Edward Elgar.
- Pittel, Karen (2006), *A Kuznets Curve for Recycling*, mimeo.
- Romer, Paul M. (1990), Endogenous technological change, *Journal of Political Economy*, 98, 71-102.
- Scholz, Christian M./Ziemes, Georg (1999), Exhaustible resources, monopolistic competition and endogenous growth, *Environmental and Resource Economics*, 13, 169-185.
- Schou, Paul (2002), When environmental policy is superfluous: Growth and polluting resources, *Scandinavian Journal of Economics*, 104, 4, 605-620.
- Schou, Paul (2001), Polluting non-renewable resources and growth, *Environmental and Resource Economics*, 16, 2, 211-227.
- Smith, Vernon L. (1972), Dynamics of waste accumulation: disposal versus recycling, *Quarterly Journal of Economics*, 86, 600-616.
- van Beukering, Pieter J.H. (2001), *Recycling, International Trade and the Environment: An Empirical Analysis*, Dordrecht: Kluwer.
- van Beukering, Pieter J.H./Bouman, Mathijs N. (2001), Empirical evidence on recycling and trade of paper and lead in developed and developing countries, *World Development*, 29, 1717-1737.
- Weinstein, Milton C./Zeckhauser, Richard J. (1974), Use patterns for depletable and recyclable resources, *Review of Economic Studies*, Symposium on the Economics of Exhaustible Resources, 41, 67-88.



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