WIF - Institute of Economic Research

Innovative investments, natural resources, and intergenerational fairness: Are pension funds good for sustainable development?

Lucas Bretschger
Karen Pittel

Working Paper 05/36
February 2005

Economics Working Paper Series

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Innovative investments, natural resources, and intergenerational fairness: Are pension funds good for sustainable development?

Lucas Bretschger (ETH Zurich) and

Karen Pittel (ETH Zurich)*

February 2005

Abstract

We analyse long-term consumption paths in a dynamic two-sector economy with overlapping generations. Each young generation saves for the retirement age, both with private savings and pension funds. The productivity of each sector can be raised by sector-specific research while the essential use of a non-renewable natural resource poses a threat to consumption possibilities in the long run. Bonds, the two types innovations, and resource stocks are the different investment opportunities. We show that pension funds have a positive impact on long-term development, provided that individuals have a preference for own investments. In this case, sustainability is more likely to be achieved due to pension fund savings.

Keywords: Pension funds, sustainable development, financial investments, overlapping generations

JEL Classification: O4 (economic growth), Q01 (sustainable development), Q3 (non-renewable resources), G23 (pension funds)

*ETH Zurich, WIF - Institute of Economic Research, ETH Zentrum, WET D 3, CH-8092 Zurich, Tel. +41 (0)1 632 03 87, Fax +41 (0)1 632 13 62.
We thank Franck Amalric, Andre Burgstaller and Peter Buomberger for helpful comments on an earlier draft.

1 Introduction

Long-term investments have a major influence on economic development. Accordingly, they constitute an important channel through which the sustainability of development can be promoted. Sustainability means that later generations enjoy a level of welfare which equals or exceeds the welfare of the currently living generation. The quantity and the direction of long-term investments decide on issues which are crucial for welfare such as changes in natural resource abundance and increase of knowledge stocks. Regarding the decisions on investments, pension funds are among the most important actors. In many developed countries, the share of total savings managed by pension funds has reached respectable dimensions. An interesting example is Switzerland, where total assets of pension funds had a market value of 440 bn Swiss Francs by the end of 2002; approximately one fourth was held in shares, see Swiss Federal Statistical Office (2004). At the same time, total capitalisation of the Swiss market, including domestic and foreign shares and bonds, amounted to 644 bn Swiss Francs in shares and 435 bn Swiss Francs in bonds, see SWX (2002). This conspicuously underlines the important role of pension funds for Swiss asset allocation. Looking at investment strategies for professional portfolio managers, the endeavour to invest in a socially responsible manner is increasingly emphasised. It has been estimated for the United States, that in 2003, over 11 percent of total investment assets under professional management have been allocated according to this principle and that the share will be increasing in the future, see Social Investment Forum (2003). In the UK, an amendment to the Pensions Act requires trustees of occupational pension funds to declare the extent to which social, environmental, and/or ethical issues are taken into account in their investment policies, see Eurosif (2003). In addition, a number of large British insurance companies today report to invest according to social responsibility criteria.

Corresponding to the large and rising importance of pension funds, their specific investment behaviour, and the broad public debate on sustainability, the topic of this paper is to analyse the consequences of pension fund savings for the sustainability of long-term development. In particular, we analyse long-term consumption paths in a dynamic two-sector economy with overlapping generations and natural resource scarcity. We focus on the role of pension funds for overall savings and investment. Furthermore the consequences of formulating mandatory investment rules for pension funds – e.g. investment in modern or "clean" sectors only – are considered.

This paper is based on two strands of recent literature. The first considers intergenerational transfers and long-run investment within a dynamic OLG framework, where early contributions include Hammond (1975) and Kotlikoff et al. (1988). Specific subjects in the field are the debate about funding versus pay-as-you-go systems, see Sinn (2000), intergenerational risk sharing, see e.g. Thøgersen (1998), Barbie et al. (2000), and Wagener (2001) and (2003), and problems faced by aging societies, see e.g. Meijdam and Verbon (1996), OECD (1998), Lassila and Valkonen (2001) for Finland, Ecoplan (2003) for Switzerland, and Börsch-Supan et al. (2002) for Germany. Yet none of these papers considers the role that intragenerational transfers may play in an economy which, realistically, faces natural resource scarcity. The second strand deals with the impacts of natural resource use on economic and technological development but does not regard the role of intergenerational transfers. The literature has been dominated by continuous time approaches with indefinitely living agents (e.g. Bovenberg and Smulders 1995, Stokey 1998) that preclude the explicit analysis of intergenerational aspects from the outset. Papers that deal with environmental and resource aspects in a discrete time framework include the early approaches by Howarth and Norgaard (1992), John and Pecchenino (1994) and Marini and Scaramozzini (1995). More recently, the topic was approached by Quang and Vousden (2002), Seegmuller and Verchère (2004) and Papyrakis and Gerlagh (2004) who also consider the role of resource scarcity on long-run investment.

Furthermore, in recent theory, the relationship between social security and long run investments, e.g. in the environmental or in the education sector, has prominently been studied by Rangel (2003). He finds that social security plays a crucial role in sustaining investments favouring future generations, which is one of the keys to achieve sustainable development. To evaluate the total impact of forced savings, the extent to which private savings are crowded out must be also taken into consideration. Pension funds may (but need not) change the quantity and direction of aggregate investments in an economy. In the case of complete crowding out, nothing happens at the aggregate level, i.e. sustainability is not endorsed. Attanasio and Rohwedder (2003) show that in the case of the UK, the earnings-related tier of pension funds savings has a negative impact on private savings with relatively high substitution elasticities while the impact of the flat-rate tier is not significantly different from zero.

Modelling of the OLG setting and the inclusion of non-renewable resources draws on the contributions of Quang and Vousden (2002) and Agnani, Gutierrez, and Iza (2003), respectively. Technology assumptions are based on Romer (1990); the impact of natural resource use in this kind of framework is treated in Bretschger (2003). Pittel (2002) provides a broad survey on the impact of the natural environment on economic growth.

The most important elements of our approach are the following. Each young generation saves for the retirement age, both with private savings and pension funds. Savings are in the form of bonds, two types of innovations and resource stock. Pensions guarantee a statutory minimum consumption of the old generation in terms of their previous consumption. This set-up is aimed at depicting the institutional frame in developed economies. To derive the structural effects of long-term investments, we assume an economy consisting of two final goods sectors. The two sectors differ according to two characteristics: the intensity of using natural resources and the productivity gains which arise from diversification in production. More specifically, in the so-called "modern" sector of the economy, gains from diversification are assumed to be high and relative resource input is low. In the "traditional" sector, the opposite assumptions apply. In both sectors, positive externalities emerge from research raising the public stock of knowledge.

Thus the dynamic behavior of the economy is driven by two types of R&D and natural resource scarcity, which increasingly diminishes the resource input available for production. When investments in innovative activities are too low, consumption growth may become negative. In this case, later generations receive lower utility which violates the sustainability criterion. However, increasing the size of investments and the sectoral mix of investments towards the modern sector increase the chances of sustainability.

For the development in the long run, we distinguish between private optimum paths, chosen by firms and consumers under free market conditions, social optimum paths, and paths with an active pension fund. Optimal paths which exhibit non-decreasing individual utility over time are called "sustainable" paths. We study under which conditions pension fund activities support sustainability, that is bring development closer to a sustainable pattern. Three mechanisms could be working in this direction. First, pension funds have a different objective function compared to households. They aim at achieving a certain standard of living for the old, so that they take their own view regarding specific issues such as production externalities affecting consumption or individual discounting. Second, social responsibility criteria may play a role, either because of the long-term perspective and/or the political environment of pension funds. Third, household may perceive pension fund saving as an incomplete substitute to own saving. As a result of these different mechanisms, pension funds have the potential to affect the total amount of savings in the economy as well as the direction of the savings to different investment opportunities.

We show that the social optimum path yields higher consumption and innovation growth rates than the path in the market equilibrium. Moreover, pension funds are found to have an impact on the level and the direction of investments, once we assume consumers to have a preference for own investments. This is reasonable given the various uncertainties involved when consigning own savings to an independent institution. As a conclusion it emerges that pension funds are an important channel through which the chances of sustainable development can be increased.

The remainder of the paper is organised as follows. Section 2 describes the model in detail. In section 3, we take a look at the solution in the pure market economy in which consumers maximise lifetime utility and firms maximise profits. Section 4 discusses the social optimum and section 5 introduces pension funds whose task it is to provide a specified level of pensions (a percentage of first period consumption) to the consumers in their second period of life. Section 6 concludes.

2 The model

2.1 Overview

We distinguish between two primary inputs, labour and non-renewable natural resources, see figure 1. Both inputs are used to produce differentiated intermediate goods for two final goods sectors, which we label "modern" and "traditional" sector. The two sectors differ as the modern sector uses relatively few natural resources but exhibits relatively large gains from specialisation from the use of differentiated inputs. Labour is also used as an input into two types of research. Each research type is directed at innovating new blueprints for designs of additional intermediate goods. Research entails positive spill-overs to sector-specific public knowledge. The invention of additional designs is assumed to be relatively more expensive in the modern compared to the traditional sector.

** Figure 1 about here **

We consider an economy with overlapping generations. It is assumed that each generation consists of a continuum of consumers each of which lives for two periods. During the first period the agent supplies labour inelastically and works in either the production of intermediates or in the R&D sector. She consumes and saves for her retirement in the second period. Savings are either in form of bonds or natural resource stock, which means that the young can also invest in the resource stock which they buy from the old. At the end of the "working period", each parent gives birth to one offspring. In the second period of her life the agent consumes what she saved in the first period. She receives interest on her savings in bonds and sells the resources acquired when young either to firms or to the next generation of consumers. Capital markets are assumed to be perfect, such that individuals can borrow or lend money at the equilibrium interest rate.

Individuals maximise utility over their two-period lifetime, where second-period consumption is discounted as usual. Pension funds collect part of wage earnings from the young, invest the savings and pay pensions to the same generation when it is old. They provide a specified level of pensions (a percentage of first period consumption) to the consumers in their second period of life. Moreover, they are assumed to consider the effects of investments to society as a whole. As a benchmark scenario we also regard the social planner solution and the optimum paths exhibiting non-decreasing utility in the long run.

2.2 Production

In the considered economy, two final goods are produced from intermediate inputs under the restriction of CES-production functions. Specifically, the "modern" good X and the "traditional" good Z are assembled from a continuum of intermediate goods, x_{i_t}, $i \in [l_{t-1}, l_t]$, and z_j, $j \in [m_{t-1}, m_t]$, according to:

$$X_t = \left(\int_{l_{t-1}}^{l_t} x_{it}^{\beta} \, di \right)^{1/\beta} \qquad \text{and} \qquad Z_t = \left(\int_{m_{t-1}}^{m_t} z_{jt}^{\gamma} \, dj \right)^{1/\gamma} \tag{1}$$

where l and m denote the number of horizontally differentiated intermediate products in the respective sectors and t is the time index. Competition in the intermediates sectors is assumed to be monopolistic with one firm producing one type of intermediate. Intermediate goods are used in one generation, then they are assumed to be outdated. This is a simplifying assumption which does not alter the quality of the results.

The modern and traditional sectors differ with respect to the gains from specialisation; the gains are assumed to be higher in the modern sector ($\beta < \gamma$). This implies that *ceteris paribus* the effect of an additional variety of a modern intermediate on the productivity of all modern intermediates is higher than the effect of an additional variety in the other sector. Intermediates are produced from labour L and non-renewable resources R under the restriction of Cobb-Douglas production functions:

$$x_i = (L_{x_i t})^{\alpha} (R_{x_i t})^{1-\alpha} \qquad \text{and} \qquad z_j = \left(L_{z_j t}\right)^{\delta} \left(R_{z_j t}\right)^{1-\delta} \tag{2}$$

where L_{k_t} and R_{k_t}, $k = x_i, z_j$, denote the input of labour and resources in the production of x_i and z_j. The production of intermediates in the modern sector is assumed to be more labour and less resource intensive than in the traditional sector, that is we have $\alpha > \delta$.

To obtain the right (or the capability) to produce a specific type of intermediate, firms have to acquire (to invent) the according patent or blueprint for the design first. The patent for a new good lasts for one period, after that, the good is replaced by subsequent intermediates. The invention of new intermediates entails proportional positive spill-overs to sectoral public knowledge, which is in turn a free input in the research sector. The number of new designs in period t is determined by:

$$l_{t+1} - l_t = \frac{L_{lt}}{a_l} l_t \quad \text{and} \quad m_{t+1} - m_t = \frac{L_{mt}}{a_m} m_t \tag{3}$$

where L_l and L_m denote the input of labour in the production of blueprints for the two sectors and a_l and a_m the per-unit input factors of labour in research for the respective sector; l and m stand for the knowledge input. We assume that the invention of a new blueprint in the modern sector requires relatively more labour input (is more expensive) so that $a_l > a_m$.

The size of the population is constant and normalised to unity. Labour is used in four different sectors, so that the labour market equilibrium becomes:

$$1 = L_{xt} + L_{zt} + L_{lt} + L_{mt}. \tag{4}$$

On resource markets, supply equals demand, according to:

$$R_t = R_{xt} + R_{zt} \tag{5}$$

where R is the part of the resource owned by firms and used for current production. Finally, the non-depletion condition states that the whole resource Stock V_0 is used for production when integrating over time, that is:

$$\sum_0^\infty R_t = V_0 \tag{6}$$

where V_0 is predetermined and $V_0 \geq 0$. At any point in time we have:

$$V_t = V_{t-1} - R_{t-1} \tag{7}$$

2.3 Consumers

The representative consumer maximises lifetime utility U which is received from consumption C in both periods (young and old) and own savings S:

$$U_t = \ln C_{1t} + \varrho \ln S_t + \frac{1}{1+\rho}(\ln C_{2t+1}) \tag{8}$$

where ρ denotes the individual discount rate, ϱ determines the intensity of the preference for own investment (as in contrast to forced savings through a pension fund), 1 and 2 stand for young and old, respectively. ϱ can be positive because of portfolio considerations and/or incomplete information about the pension fund's activities; we will discuss its impact below, especially the case $\varrho = 0$. Consumption is determined by:

$$C_t = X_t^{\phi} Z_t^{1-\phi}. \tag{9}$$

In every period, C_t is consumed by the two currently living generations, i.e. $C_t = C_{1t} + C_{2t}$.

Individuals supply labour inelastically when they are young and are subject to the following budget constraints in the two periods:

$$p_{C_t} C_{1t} + p_{H_t} H_t + S_t = w_t \tag{10}$$

$$p_{C_{t+1}} C_{2t+1} = (1 + r_{t+1}) S_t + p_{R_{t+1}} R_{t+1} + p_{H_{t+1}} H_{t+1} \tag{11}$$

$$H_t = H_{t+1} + R_{t+1} \tag{12}$$

where p_C denotes the consumption price index, H is the stock of the resource which is owned by consumers (and therefore not used in current production), w is the labour wage, p_H the price of H, and p_R the price of R.

3 Decentralised solution

As a benchmark scenario we first derive the decentralised market solution without any governmental or pension fund's activities. Consumers are maximising lifetime utility

$$\max_{C_{1t}, C_{2t+1}, S_t, H_{t+1}, R_{t+1}, H_t} U_t(C_{1t}, C_{2t+1}, S_t) \tag{13}$$

subject to the budget constraints (10) to (12). With respect to young- and old-age consumption maximization yields the familiar first-order conditions. From the introduction of consumers' preferences for own investment, we additionally get a modified FOC for consumers' savings. Combining this first-order condition with the first-order conditions for consumption when young and old, we get, after rearranging:

$$\frac{p_{C_{t+1}} C_{2t+1}}{p_{C_t} C_{1t}} = \varrho \frac{p_{C_{t+1}} C_{2t+1}}{S_t} + \frac{1 + r_{t+1}}{1 + \rho}. \tag{14}$$

This condition yields a savings rule for consumers which will be discussed at the end of this section.

Furthermore it follows from the first-order conditions for H_{t+1} and R_{t+1} that the price of the non-renewable resources sold to firms, p_{R_t}, and the price of those resources sold to the next generation, p_{H_t}, have to be equal:

$$p_{R_t} = p_{H_t} \tag{15}$$

which is intuitive as in equilibrium consumers are indifferent between selling the resource to firms or to the next generation. With respect to the development of resource extraction and the price of the resource, the familiar Hotelling pricing rule for non-renewable resources follows directly from the FOC for the young consumers' investment in resources, H_t:

$$\frac{p_{R_{t+1}}}{p_{R_t}} = \frac{p_{R_{t+1}}}{p_{R_t}} = 1 + r_{t+1}. \tag{16}$$

Initial resource prices are chosen to satisfy (6). With respect to the extraction of resources along the balanced growth path (BGP), it is shown below that R_t decreases at the rate $\frac{1}{1+r_t}$:[1]

$$g_R = \frac{R_{t+1}}{R_t} = \frac{p_{H_t}}{p_{H_{t+1}}} = \frac{1}{1+r}. \tag{17}$$

With respect to the production side of the economy we get for the aggregate demands for the modern and the traditional good, X_t and Y_t:

$$X_t = \frac{\phi}{p_{X_t}} \qquad \text{and} \qquad Z_t = \frac{1-\phi}{p_{Z_t}} \tag{18}$$

where we made use of the normalisation $p_{C_t} C_t = 1$ which is adapted to facilitate calculations and is possible because the model has no other numeraire.

Research is conducted by R&D firms on a perfectly competitive market, such that in equilibrium prices are equalised to marginal costs

$$p_{l_t} = \frac{a_l}{l_t} w_t \qquad \text{and} \qquad p_{m_t} = \frac{a_m}{m_t} w_t. \tag{19}$$

R&D is financed by consumers' savings S_t which is either directed towards research in the traditional or in the modern sector, such that $S_t = S_{l_t} + S_{m_t}$. Research firms on the competitive market operate at zero profits which implies

$$\begin{aligned} S_{l_t} &= w_t L_{l_t} & \text{and} \qquad S_{m_t} &= w_t L_{m_t} \\ &= (l_{t+1} - l_t)\frac{a_l}{l_t} w_t & &= (m_{t+1} - m_t)\frac{a_m}{m_t} w_t. \end{aligned} \tag{20}$$

[1]Throughout this paper $g_k = \frac{k_{t+1}}{k_t}$ is referred to as the growth rates of a variable. Variables which do not carry time indices denote equilibrium values along the BGP.

The patents for the blueprints which are developed in the R&D sectors in period t are sold to intermediate producers which produce in period $t+1$. The demands for the individual x_i's, $i \in [l_{t-1}, l_t]$ and z_j's, $j \in [m_{t-1}, m_t]$ are given by

$$x_i = \frac{X^{\frac{1}{\beta}} p_{x_i}^{\frac{1}{\beta-1}}}{\left(\int\limits_{l_{t-1}}^{l_t} p_{x_i}^{\frac{\beta}{1-\beta}} \right)^{\frac{1}{\beta}}} \qquad \text{and} \qquad z_j = \frac{Z^{\frac{1}{\gamma}} p_{z_j}^{\frac{1}{\gamma-1}}}{\left(\int\limits_{m_{t-1}}^{m_t} p_{z_j}^{\frac{\gamma}{1-\gamma}} \right)^{\frac{1}{\gamma}}}. \tag{21}$$

Competition in the modern and traditional intermediates' sectors is assumed to be monopolistic with the number of firms in each intermediate sector being equal to $l_t - l_{t-1}$, resp. $m_t - m_{t-1}$. Each firm purchases one patent granting her the right to produce the respective intermediate. After one period patents are outdated. In the next period firms have to acquire another patent to obtain the right to produce an intermediate of the next product generation. Using (21), maximisation of profits $\Pi_{k_t} = p(k_t)k_t - w_t L_t - p_{R_t} R_{k_t}$, $k = x_i, z_j$, yields the familiar price-over-marginal-cost pricing rules[2]

$$\begin{aligned} \alpha\beta \frac{x_{i_t}}{L_{xi_t}} p_{x_{i_t}} &= w_t \qquad \text{and} \qquad & \delta\gamma \frac{z_{j_t}}{L_{zj_t}} p_{z_{j_t}} &= w_t \\ (1-\alpha)\beta \frac{x_{i_t}}{R_{x_{i_t}}} p_{x_{i_t}} &= p_{R_t} \qquad \text{and} \qquad & (1-\delta)\gamma \frac{z_{j_t}}{R_{z_{j_t}}} p_{z_{j_t}} &= p_{R_t}. \end{aligned} \tag{22}$$

We regard symmetric equilibria where the intermediate goods producers within a sector have identical production functions. Then, the prices as well as the amounts produced of each intermediate in either sector are equal. Aggregating over all produced varieties, the sectoral profits are given by

$$\Pi_x = \phi(1-\beta) \qquad \text{and} \qquad \Pi_z = (1-\phi)(1-\gamma). \tag{23}$$

Consumers are compensated for their R&D investment by the profits generated in the intermediate sector in period $t+1$. In equilibrium savings have to yield the same return as investment in resources. As patents are worthless after one period, the no-arbitrage conditions for the patent market read

$$\Pi_{x_{t+1}} = (1 + r_{t+1}) S_{l_t} \qquad \text{and} \qquad \Pi_{z_{t+1}} = (1 + r_{t+1}) S_{m_t}. \tag{24}$$

[2]Using (22) we can now prove (17): From (22) it follows that $\frac{1-\alpha}{\alpha} L_{x_t} w_t = R_{x_t} p_{R_t}$ and $\frac{1-\delta}{\delta} L_{z_t} w_t = R_{z_t} p_{R_t}$. As w_t, L_x and L_z are constant along the balanced growth path, the LHSs of these equations are constant over time. Taking the equations at time $t+1$ and time t and dividing them gives $g_R = \frac{R_{t+1}}{R_t} = \frac{p_{R_t}}{p_{R_{t+1}}} = g_{p_R}$.

Using (20), (22) and (23) we can now derive conditions for the equilibrium allocation of labour. From (20) and (23) it follows that

$$L_{l_t} = \frac{1-\beta}{1-\gamma}\frac{\phi}{1-\phi}L_{m_t} \tag{25}$$

where it should be noted that the allocation of labour between the two R&D sectors is independent of the productivity parameters a_l and a_m. We will show (section 4) that a socially optimal allocation of labour across research sectors depends on the relative productivity of R&D. The independency of a_l and a_m therefore reflects market failures arising in the pure market economy. With respect to the influence of the gains from specialisation, reflected by β and γ, as well as with respect to the elasticity of C_t with respect to X_t and Y_t, ϕ, the allocation of labour between the two research sectors follows economic intuition: The higher the relative gains from specialisation in modern intermediates compared to traditional intermediates, i.e. the higher $\frac{1-\beta}{1-\gamma}$, the more labour is allocated towards R&D in the modern sector. Along the same lines, a higher $\frac{\phi}{1-\phi}$ also results in a relatively higher input of labour in modern R&D.

Furthermore we get a relation between the aggregate input of labour in the two intermediate sectors by employing (22), (23) and (25):

$$L_{x_t} = \frac{\phi}{1-\phi}\frac{\alpha}{\delta}\frac{\beta}{\gamma}L_{z_t} \tag{26}$$

where $L_{x_t} = \int_{l_{t-1}}^{l_t} L_{x_{i_t}}$ and $L_{z_t} = \int_{m_{t-1}}^{m_t} L_{z_{j_t}}$. Again, the economic intuition follows straightforwardly: A higher value of the relative elasticity of X_t in C_t, relatively higher gains from specialisation in modern intermediates and a relatively higher productiveness of labour in the production of modern intermediates lead to more labour input in modern production.

From (20) and (23) we can finally derive a rule for the optimal allocation of labour in the production of traditional patents and the production of intermediates from these patents along the balanced growth path. Considering that along the BGP not only the labour shares in each sector are constant, but (due to our normalisation of consumer expentitures) also the wage and interest rate, we get

$$L_z = \frac{\gamma}{1-\gamma}\delta(1+r)L_m. \tag{27}$$

In contrast to (25) and (26) the allocation of labour inputs between the respective R&D sector and the labour input in the intermediates producing sector depends on the interest rate. This is due to the fact that the patents produced in period t are employed in production with a one-period lag. Furthermore, the labour allocation between R&D and production in, e.g., the traditional sector, solely depends on the gains from specialisation and labour productivity in production within this sector. Less labour is devoted to research if the gains from specialisation and productivity of labour in production are relatively low.

Combining (25), (26) and (27) with the equilibrium condition for the labour market (4) gives the share of labour employed in the production of traditional intermediates as a function of the interest rate:

$$L_m = \left[1 + \frac{1-\beta}{1-\gamma}\frac{\phi}{1-\phi} + \frac{1+r}{1-\gamma}\left(\alpha\beta\frac{\phi}{1-\phi} + \gamma\delta\right)\right]^{-1} \tag{28}$$

To obtain a second condition for L_m and r we turn back to the consumers' optimisation problem and consider (14). In order to express the expenditures for consumption in terms of labour, the budget restrictions (10) and (11) and the zero profit conditions (20) are employed. Substituting these into (14), taking into account that $p_{R_{t+1}}(R_{t+1} + H_{t+1}) = (1 + r_{t+1})p_{R_t}H_t$ and rewriting gives:

$$\frac{1}{1+\rho} + \varrho\frac{w_t(L_{l_t} + L_{m_t}) + p_{R_t}H_t}{w_t(L_{l_t} + L_{m_t})} = \frac{w_t(L_{l_t} + L_{m_t}) + p_{R_t}H_t}{w_t - w_t(L_{l_t} + L_{m_t}) + p_{R_t}H_t}. \tag{29}$$

Using (12) it can be shown that, along the balanced path, $H_t = \frac{R_t}{r} = \frac{1}{r}(R_{x_t} + R_{z_t})$ has to hold.[3] In order to express $p_{R_t}R_t$ in terms of labour we use the equilibrium conditions in intermediates' production (22). (29) can be rewritten in terms of L_m and r only:

$$\frac{E(r)}{(1-E(r))} = \varrho\frac{E(r)}{L_l + L_m} + \frac{1}{1+\rho} \tag{30}$$

with $E(r) = L_l - L_m - \left(\frac{1-\alpha}{\alpha}L_x + \frac{1-\delta}{\delta}L_z\right)$ where L_z and L_x are determined by (25), (26) and (27). Substituting (28) finally gives (30).

Regarding the functional forms of the LHS and RHS of (30) it can be shown that (30) determines one unique equilibrium interest rate (see Appendix A). Given this equilibrium rate the optimal allocation of labour follows from (25), (26), (27) and (28).

The consumption growth rate along the balanced path $g_C = \frac{C_{t+1}}{C_t}$ can be derived by substituting (1), (2) and (3) into (9) and considering that labour shares are

[3] From (12) it follows that $\frac{R_{t+1}}{H_{t+1}} = -1 + \frac{H_t}{H_{t+1}}$. As along the balanced growth path the growth rate of H_t is constant, this implies that $\frac{R_{t+1}}{H_{t+1}}$ also has to be constant along the BGP, such that R_t and H_t have to grow at the same rate. Knowing from above that $g_R = \frac{1}{1+r}$, equality of growth rates, i.e. $g_R = g_H$, gives $H_t = \frac{R_t}{r}$.

constant along the BGP. Taking the resulting expression for C_t at $t+1$ and t and dividing C_{t+1} by C_t gives:

$$g_C = \left((1+r)^{-(1-\alpha)\beta} g_l^{1-\beta}\right)^{\phi} \left((1+r)^{-(1-\delta)\gamma} g_m^{1-\gamma}\right)^{1-\phi}. \tag{31}$$

According to (31) consumption growth depends positively on the two rates of innovation growth. On the other hand a higher interest rate has a negative impact as it speeds up resource depletion diminishing intermediate goods' production.

Let us now take a short look on the special case in which consumers do not have a preference for own investment ($\varrho = 0$). It can easily be seen that, in this case, the first term on the RHS of (30) vanishes. Consumers equalise the relative expenditures for consumption in their first and second period of life to the inverse of their discount factor. Rearranging (14) for $\varrho = 0$ yields the savings rule for consumers without a preference for own savings:

$$S_t + p_{R_t} H_t = \frac{1}{2+\rho} w_t. \tag{32}$$

Consumers are indifferent between saving in order to invest in R& D, S_t, and purchasing non-renewable resources when young, H_t. Both activities constitute perfect substitutes as in equilibrium the increase in either price is equal to the interest rate.

Overall investment in this economy is determined by the consumers' discount rate and the wage rate only. Straightforwardly, the higher the wage rate and the lower the rate with which consumers discount future utility, the higher the savings.

From (14) a similar, savings rule can also be obtained for $\varrho \neq 0$. Proceeding as before we get after rearranging:

$$S_t + p_{R_t} H_t = \frac{1}{1 + (1+\rho) S_t - \xi (w_t - S_t - p_{R_t} H_t)} w_t. \tag{33}$$

An increase in savings rises the LHS and lowers the RHS of (33), establishing one unique equilibrium savings rate. It can be seen that the effects of w_t and ρ on savings are qualitatively the same as for $\varrho = 0$. An increase in either one increases the value of the RHS for any given savings, such that equilibrium savings also rise. Additionally a high preference for own investment ξ also induces a positive effect on savings.

We come back to the special case of $\varrho = 0$ when discussing the impact of pension fund activities. It can be shown that whether or not it is assumed that consumers have a preference for own investment matters crucially for the effects arising from pension fund investments.

4 Social planner solution

In the pure market economy we just presented, a number of different market failures arise that drive a wedge between the market solution and the socially optimal growth path. These market failures are well known form the standard Romer (1990) model in continuous time: Firstly, monopolistic competition in the intermediates sectors induces intermediates' prices to be on a suboptimally high level. Secondly, gains from diversification and knowledge spillovers arise that are not taken into account on the firm level. By using the concept of a social planner having perfect information about the economy and correcting for all market failures we can derive the socially optimal balanced growth path. For simplicity we consider the case in which consumers have no preferences for own investment ($\varrho = 0$).

The social planner's objective is to maximise welfare

$$W_t = \frac{1}{1+\rho} \ln C_{20} + \sum_{t=0}^{T-1} \left[\ln C_{1t} + \varrho \ln S_t + \left(\frac{1}{1+\rho} \right) \ln C_{2t+1} \right] \frac{1}{(1+\bar{r})^{t+1}} \tag{34}$$

of the present and future generations where $\bar{r}$ denotes the rate used by the social planner to discount utility of future generations. Maximisation is subject to the equilibrium conditions for the factor markets and production technologies, such that the optimisation problem of the social planner reads:

$$\max_{C_{1t}, C_{2t}, L_{l_t}, L_{m_t} L_{x_t}, L_{z_t}, R_{x_t}, R_{z_t}, m_t, l_t, S_{t+1}} W_t(C_{1t}, C_{2t+1}, V_t) \tag{35}$$

$$\text{s.t.} \quad (\lambda_t) \quad L_{l_t} + L_{m_t} + L_{x_t} + L_{z_t} = 1 \tag{36}$$

$$(\mu_t) \quad C_{1t} + C_{2t} = \left[(l_t - l_{t-1})^{1-\beta} L_{x_t}^{\alpha\beta} R_{x_t}^{(1-\alpha)\beta} \right]^{\phi} \cdot \left[(m_t - m_{t-1})^{1-\gamma} L_{z_t}^{\delta\gamma} R_{z_t}^{(1-\delta)\gamma} \right]^{1-\phi} \tag{37}$$

$$(\omega_t) \quad V_{t+1} = V_t - R_{x_t} - R_{z_t} \tag{38}$$

$$(\upsilon_t) \quad (l_{t+l} - l_t) = \frac{l_t}{a_l} L_{l_t} \tag{39}$$

$$(\theta_t) \quad (m_{t+l} - m_t) = \frac{m_t}{a_m} L_{m_t} \tag{40}$$

where λ_t, μ_t, ω_t, υ_t and θ_t denote the Lagrangian multipliers associated with the respective restriction. V_t denotes the stock of the non-renewable resource at time t.

From the first-order conditions of welfare maximisation it can be shown that, with respect to resource extraction, the growth rate of R_t is equal to the inverse of the intergenerational discount rate of the social planner (see Appendix B):

$$\frac{R_{t+1}}{R_t} = \frac{1}{1+\bar{r}}. \tag{41}$$

The analogy to this relationship in the decentralised economy is given by (17) which states that g_R is equal to the inverse of $1+r$. This already shows that, with respect to resource markets, no market failures are present in the decentralised setting. It also underlines the interpretation of the market interest rate as a discount rate between generations in comparison to ρ which gives the discount rate with which a single generation discounts its old-age consumption. Nevertheless, the timing of the extraction of resources is of course not optimal in the market case, as repercussions occur from other markets failures on the level of resource extraction.

The analogy to the market case is also obvious when considering consumption growth along the optimal balanced path. Taking the ratio of C_{t+1} and C_t as given in (37) and keeping in mind that the labour allocation does not change along the BGP gives, under consideration of (41):

$$g_C = \left((1+\bar{r})^{-(1-\alpha)\beta} g_l^{1-\beta}\right)^{\phi} \left((1+\bar{r})^{-(1-\delta)\gamma} g_m^{1-\gamma}\right)^{1-\phi}. \tag{42}$$

Comparing (31) to (42) shows that consumption growth is determined by the same functional relationship. Again, the market interest rate is replaced by the social planner's discount rate. As before, innovation growth exerts a positive effect on consumption growth while the discount rate affects it negatively.

We can furthermore derive the following rules for an optimal allocation of labour across sectors (see Appendix B):

$$L_{m_t} = \frac{1-\beta}{1-\gamma}\frac{\phi}{1-\phi}(L_{l_t}+a_l) - a_m \tag{43}$$

$$L_{x_t} = \frac{\phi}{1-\phi}\frac{\alpha}{\delta}\frac{\beta}{\gamma}L_{z_t} \tag{44}$$

$$L_z = \frac{\gamma}{1-\gamma}\delta(1+\bar{r})L_m. \tag{45}$$

Conditions (44) and (45) are already known from the market solution (substituting again $\bar{r}$ for r), indicating that due to the assumption of log utility the income and substitution effects from consumers savings cancel out, such that neither the allocation of labour between intermediates sectors' nor the allocation between production and research is distorted. Yet comparing (25) and (43), it can be seen that the allocation of labour between the two research sectors is distorted in the market economy, as the productivity of R&D is not taken into account by the allocation decision of firms.

From (43), (44) and (45) in combination with the equilibrium condition for the labour market (36), the equilibrium share of labour devoted to R&D in the traditional sector can be derived:

$$L_m^s = \frac{(1-\gamma) - [Ba_m - Ca_l]}{1 + (1-\beta)\frac{\phi}{1-\phi} + (1+\bar{r})\left(\alpha\beta\frac{\phi}{1-\phi} + \gamma\delta\right)} \tag{46}$$

with

$$\begin{aligned} B &= (1-\beta)\frac{\phi}{1-\phi} + (1+\bar{r})\left(\alpha\beta\frac{\phi}{1-\phi} + \gamma\delta\right) \\ C &= \frac{\gamma\delta}{\alpha\beta}\frac{1-\phi}{\phi}\left(1 - \frac{\alpha\beta}{1-\beta}(1+\bar{r})\right) \end{aligned}$$

where the superscript s denotes the value of a variable along the socially optimal balanced growth path. It can be shown that (46) reduces to (28) by setting $a_m = a_l = 0$, i.e. by neglecting for the influence of productivity in research on the labour allocation. Inspecting the extra terms on the RHS of (46) shows that whether more or less labour is devoted to research in the modern or the traditional sector, respectively, depends crucially on the productivities of research. The lower the productiveness of traditional research, i.e. the higher a_m, the less labour is allocated towards this sector.

The negative effect of the interest rate on the share of labour allocated towards research, which was also present in the market economy, is enhanced by the dependency of L_m on the productivities in R&D.

Inserting (46) into (44) gives the optimal share of labour in the modern research sector. The growth rate of consumption can be derived from (39), (40), (43) and (42):

$$g_C^s = (1+\bar{r})^{-[(1-\alpha)\beta\phi + (1-\delta)\gamma(1-\phi)}\left(\frac{1-\beta}{1-\gamma}\frac{\phi}{1-\phi}\frac{a_m}{a_l}\right)^{(1-\beta)\phi}\left(\frac{L_m^s}{a_m} + 1\right)^{(1-\gamma)(1-\phi)}. \tag{47}$$

5 Pension fund

Let us now assume that a pension fund exists whose task it is to assure for a minimum standard of living of the consumer in his retirement period. The pension system is assumed to be fully funded, i.e. the pension fund collects a share τ_t of the consumer's wage income in the working period, invests the collected revenues on the capital market and repays the revenues plus the interest to the consumer as a pension in the retirement period. To take an extreme assumption, we postulate that the pension fund has the statutory obligation to invest in R&D for modern intermediates' blueprints only. This issue will be discussed below. In fact, it will turn out that it has no impact on the results. A general investment rule for pensions funds would be equally possible in this model.

The pension that is to be paid to the consumer is defined in terms of expenditures for first period consumption $p_{C_t}C_{1t}$, whereby the share of $p_{C_t}C_{1t}$ to which the pension has to amount is politically determined. The budget constraint of the pension fund is therefore given by

$$P_{t+1} = \xi(1 + r_{t+1})p_{C_t}C_{1t}, \qquad 0 < \xi < 1 \tag{48}$$

with P denoting the pension paid to the consumer in the second period of his life and ξ is the politically determined consumption share.

Due to the introduction of the pension fund the consumers' budget constraints (10) and (11) are modified to

$$p_{C_t}C_{1t} + p_{H_t}H_t + S_t = w_t(1 - \tau_t) \tag{49}$$

$$p_{C_{t+1}}C_{2t+1} = (1 + r_{t+1})S_t + p_{R_{t+1}}R_{t+1} + p_{H_{t+1}}H_{t+1} + P_{t+1} \tag{50}$$

where $\tau_t w_t = \xi p_{C_t}C_{1t}$ and (48) have to hold. As in section 3, consumers are assumed to maximise their lifetime utility (8), now subject to (49), (50) and (12). It is assumed that the contributions to the pension fund as well as the pension payments are exogenous to the consumers, i.e. consumers do not consider (48) in their optimisation.

It can be shown that – assuming consumers have a preference for own investment – the pension fund's activities will have an effect on savings. Yet while the equilibrium allocation of factors between intermediates producers and research firms changes due to the increase in savings, the statutory requirement to invest in modern R&D does not affect the equilibrium labour allocation between modern and traditional sectors.

Optimisation of the agent gives the standard first order conditions plus the additional condition for the optimal level of savings. Again we get after rearranging (14)

$$\frac{p_{C_{t+1}}C_{2t+1}}{p_{C_t}C_{1t}} = \varrho\frac{p_{C_{t+1}}C_{2t+1}}{S_t} + \frac{1 + r_{t+1}}{1 + \rho}.$$

To derive an expression for S_t in terms of the allocation of labour is a little more complicated in the pension fund's scenario. Overall investment in the modern sector is now given by $S_{l_t} = w_t L_{l_t} + w_t \tau_t$ whereby $w_t \tau_t = \xi p_{C_t} C_{1t}$ has to hold. Inserting the budget constraint for young-age consumption, substituting (20) for savings in the traditional sector and (22) for investment in non-renewable resources, we get after solving for savings in the modern sector:

$$S_{l_t} = w_t \left[L_{l_t} - \xi(1 - E(r)) \right]. \tag{51}$$

Substituting this expression and (20) for S_{m_t} into (14) and expressing $p_{C_{t+1}} C_{2t+1}$ and $p_{C_t} C_{1t}$ again in labour shares we get

$$\frac{E(r)}{1 - E(r)} = \varrho \frac{E(r)}{L_l + L_m - \xi(1 - E(r))} + \frac{1}{1 + \rho} \tag{52}$$

for the pension fund case (remember $E(r) = L_l - L_m - \left(\frac{1-\alpha}{\alpha} L_x + \frac{1-\delta}{\delta} L_z\right)$).

As the only modification with respect to the model of the no-pension fund scenario concerns the investment of consumers and pension funds in modern R&D, the equilibrium zero-profit and no-arbitrage conditions of firms remain unaltered. Comparing the equilibrium conditions (30) and (52), it can be seen that the optimal allocation of labour is different in the presence of the pension fund. This change in the labour allocation affects savings as well as investment in R&D. Inserting (25), (26) and (27) into (52) gives the modified equilibrium condition in terms of the interest rate only. Again it can be shown (see Appendix C) that (52) determines one unique equilibrium interest rate.

To determine in which way the investment of the pension fund alters the optimal provision of R&D and how this affects consumption growth, let us take a closer look at (30) and (52). Rearranging (30) and (52) gives

$$\left(L_l^{np}(r) + L_m^{np}(r)\right)\left(\frac{E(r)}{1 - E(r)} - \frac{1}{1 + \rho}\right) - \varrho E(r) = 0 \tag{53}$$

$$\left(L_l^{p}(r) + L_m^{p}(r)\right)\left(\frac{E(r)}{1 - E(r)} - \frac{1}{1 + \rho}\right) - \varrho E(r) = \xi(1 - E(r))\left(\frac{E(r)}{1 - E(r)} - \frac{1}{1 + \rho}\right) \tag{54}$$

where superscripts np and p denote the no-pension fund and pension fund scenario. For positive values of young- and old-age consumption, the LHS of either equation is first decreasing and then increasing in r with a minimum value $r < 0$ and $\lim_{r \to r^*} = \infty$ and $\lim_{r \to \infty} = -0$ (see figure 5). r^{np} denotes the equilibrium interest rate for the no pension fund case. In the pension fund case the RHS of (54) is strictly decreasing in r with $\lim_{r \to 0} = \infty$ and $\lim_{r \to \infty} = \xi$. For the two curves to intersect at a positive equilibrium interest rate r^p, $r^p < r^{np}$ has to hold. As was to be expected, the introduction of the pension fund lowers the equilibrium interest rate. As consumers have a preference for own investment, the pension fund's investment does not induce a complete crowding out.

The decrease of the interest rate induces a rise in the share of labor allocated towards R&D in the traditional sector, see (28), as well as in the modern sector, see (25). Whether modern sector R&D expands more due to the investment of the pension fund, i.e. whether economic growth becomes less resource dependent, hinges upon the efficiency parameter (ϕ) and the gains from specialisation (β and γ, for which we assumed $\beta < \gamma$). The increase in modern sector R&D will be higher than in the traditional sector if the higher gains from specialisation are not overcompensated by a low elasticity of modern goods in the production of C.

The relative increase in sectoral R&D is independent from the investment rule of the pension fund: investment in the modern sector only or – as the other extreme – in the traditional sector only will not only yield the same decrease in the interest rate, but also the same allocation of labour and therefore growth rates of R&D.

Consumption growth rises due to the pension fund's activities as can be seen from (31). As the equilibrium interest rate is lower with the pension fund's activities while the growth rates of R&D are higher, consumption growth is faster when the pension fund is active. This is due to the fact that, by raising investment in R&D, the pension fund internalises part of the spill-overs generated by the increase in the available knowledge stock in the traditional as well as in the modern sector.

Let us now compare the effects of the pension funds' activities in the presence of consumers preferences for own investment to the case where they are indifferent between their own savings and the investment of the pension fund, i.e. $\varrho = 0$. In this case (52) simplifies to

$$\frac{E(r)}{1 - E(r)} = \frac{1}{1 + \rho}. \tag{55}$$

(55) is identical to (30) for $\varrho = 0$, i.e. to the equilibrium condition for the no-pension fund case. As we already know that the other equilibrium conditions that determine the optimal allocation of labour also remain the same, it can easily be seen that the introduction of the pension fund has in this case no effect on the BGP of the economy. The pension fund's investment is perfectly crowded out by a decrease in consumers' savings, such that overall investment remains unchanged.

By assuming that consumers have a preference for own investment, i.e. $\varrho \neq 0$, a wedge is driven between the marginal utility from own investment and the marginal utility from the pension funds' investment. Due to this wedge, a perfect crowding out does not take place and overall savings increase.

6 Conclusions

Pension funds are important to determine investments, both in size and sectoral composition, as they hold a substantial share of savings in many economies. The different types of investments, such as investments in innovative activities and knowledge build-up or disinvestments in natural resource stocks, govern long-term development and decide on the welfare of future generations. Investment criteria in this field differ from private households because pension funds have an undiscounted consumption target and, possibly, social responsibility considerations instead of the discounted utility target of households.

In this paper, we have introduced dynamics in a two-sector economy through endogenous innovations and non-renewable natural resource use. The results of the paper show that the long-term dynamic impact of pension funds crucially depends on saving preferences of households. In the case of positive preferences for own investments, pension fund savings are an incomplete substitute for private savings and the pension fund activities contribute to higher knowledge build-up and lower natural resource use. On the other hand, without such preferences, private savings are completely crowded out by pension funds. Then, there is no effect of social security on long-term economic development and sustainability.

The reasons and conditions of the preferences for own investments could be determined more explicitly in a future stage of the research programme. In addition, the implications of market failures in resource markets like pollution should not be neglected. Also, to scrutinise the consequences of various statutory investment rules and optimisation targets for pension funds is a rewarding research topic left for future research.

References

Agnani, B., M. Gutierrez, and A. Iza (2003), Growth in Overlapping Generation Economies with Non-Renewable Resources, mimeo 2003.

Attanasio, O.P. and S. Rohwedder (2003), Pension wealth and household saving: Evidence from pension reforms in the United Kingdom, American Economic Review, 93, 5, 1499-1521.

Barbie, M, M. Hagedorn and A. Kaul (2000), Dynamic Efficiency and Pareto-Optimality in a Stochastic OLG Model with Production and Social Security, Bonn Econ Discussion Papers 8/2000.

Börsch-Supan, A., F. Heiss, A. Ludwig and J. Winter (2002), Pension Reform, Capital Markets, and the Rate of Return, mimeo 2002.

Bovenberg, A.L. and S.A. Smulders (1995), Environmental quality and pollution-augmenting technical change in a two-sector endogenous growth model, Journal of Public Economics, 57, 369-391.

Bretschger, L. (2003), Natural Resource Scarcity and Long-Run Development: Central Mechanisms when Conditions are Seemingly Unfavourable, Economics Working Paper Series 03/29, ETH Zurich.

ECOPLAN (2003), Analyse der Finanzierungsquellen für die AHV. SWISSOLG - ein Overlapping Generation Modell für die Schweiz, Bericht im Rahmen des Forschungsprogramms zur längerfristigen Zukunft der Alterssicherung (IDA ForAlt), Bern.

Eurosif (2003), SRI among European Institutional Investors, 2003 Report, 2003.

Hammond, P. (1975), Charity: Altruism and cooperative egoism?, in: Phelps, E. (1975), Altruism, morality, and economics, New York: The Russel Sage Foundation, 81-92.

Howarth, R.B. and R.B. Norgaard (1992), Environmental valuation under sustainable development, American Economic Review, Papers and Proceedings, 82, 473-477.

John, A. and Pecchenino, R. (1994), An overlapping generations model of growth and the environment, Economic Journal, 104, 1393-1410.

Kotlikoff, L.B., T. Persson and L.E.O. Svensson (1988), Social contracts as assets: A possible solution to the time-consistency problem, American Economic Review, 78, 662-677.

Lassila, J. and T. Valkonen (2001), Pension prefunding, ageing, and demographic uncertainty, International Tax and Public Finance, 8, 573-593.

Marini, G. and P. Scaramozzino (1995), Overlapping generations and environmental control, Journal of Environmental Economics and Management, 29, 64-77.

Meijdam, L. and Verbon, H.A.A. (1996), Aging and political decision making on public pensions, Journal of Population Economics, 9, 141-158.

OECD (1998), Macroeconomic Effects of Pension Reforms in the Context of Aging Populations: Overlapping Generations Model Simulations for Seven OECD Countries, Economics Department Working Paper, No. 201.

Quang, D.H. and N. Vousden (2002), Unionisation, Unemployment, and Economic Growth, Asia Pacific School of Economics and Government, Working Paper of the Australian National University, 02-10, 2002.

Papyrakis, E. and R. Gerlagh (2004), Natural Resources, Investment and Long-Term Income, FEEM Working Paper 87.2004.

Pittel, K. (2002), Sustainability and Endogenous Growth, Cheltenham, UK and Northampton, MA, US: Edward Elgar.

Rangel, A. (2003), Forward and backward intergenerational goods: Why is social security good for the environment?, American Economic Review 93, 3, 813-834.

Romer, P.M. (1990), Endogenous technological change, Journal of Political Economy 98, 2, 71-102.

Seegmuller, T. and A. Verchère (2004), Pollution as a source of endogenous fluctuations and periodic welfare inequality in OLG economies, Economics Letters, 84, 363-369.

Sinn, H.-W. (2000), Why a Funded Pension System Is Useful and Why It Is Not Useful, NBER Working Paper, No. 7592.

Social Investment Forum (2003), 2003 Report on Socially Responsible Investing, Trends in the United States, 2003.

Stokey, N.L. (1998), Are there limits to growth?, International Economic Review, 39, 1-31.

Swiss Federal Statistical Office (2004), Press Communication 13 Social Security, April, Neuchatel 2004.

Thørgersen, Ø. (1998), A note on intergenerational risk sharing and the design of pay-as-you-go pension programs, Journal of Population Economics, 11, 373-378.

SWX (2002) Swiss Exchange, Swissindex, Zurich 2002.

Wagener, A. (2003), Pensions as a portfolio problem: fixed contribution rates vs. fixed replacement rates reconsidered, Journal of Population Economics, 16, 111-134.

Wagener, A. (2001), Equilibrium dynamics with different types of pay-as-you-go pension systems, Economics Bulletin, 8, 6, 1-12.

Appendix

A. Prove of unique equilibrium interest rate (section 3)

To show that there exists a unique equilibrium for the pure market economy consider (30)

$$\frac{E(r)}{(1-E(r))} = \varrho \frac{E(r)}{L_l + L_m} + \frac{1}{1+\rho}.$$

Inserting (25), (26), (27) and (28) into (30) gives an expression in terms of the interest rate only.

It can be shown that the LHS of (30) is monotonically decreasing for permissible values of r, i.e. for positive values of r for which the value of consumption is also positive. By inspecting $E(r)$ it follows immediately that $E(r) > 0$. So for the LHS to be positive the denominator has to be positive. It can be shown that this condition holds for

$$r > r^* = \frac{\gamma(1-\delta)(1-\phi) + \beta(1-\alpha)\phi}{\gamma\delta(1-\phi) + \beta\alpha\phi}. \tag{56}$$

For $r > r^*$ the LHS of (30) is monotonically decreasing with $\lim_{r \to r*} LHS = \infty$ and $\lim_{r\to\infty} LHS = 0$ (see figure 2).

On the RHS of (30) only the first term which is strictly positive depends on the interest rate. This part is also monotonically decreasing in r with $\lim_{r\to 0} RHS = \infty$ and $\lim_{r\to\infty} RHS = \frac{1}{1+\rho} + \varrho \frac{1-\gamma\delta(1-\phi)-\beta\alpha\phi}{1-\gamma(1-\phi)-\beta\phi} = c$ (see figure 2).

It can easily established by inspection of figure 2 that the two curves intersect at the equilibrium interest rate $r = r^{np}$.

** Figure 2 about here **

B. Derivation of the social planner solution

From the optimisation problem of the social planner the following first order conditions for the respective variables can be derived:

$$\mathbf{C_{1t}} : -\mu_t = \frac{(1+\bar{r})^{-(t+1)}}{C_{1t}} \tag{57}$$

$$\mathbf{C_{2t}} : -\mu_t(1+\rho) = \frac{(1+\bar{r})^{-(t)}}{C_{2t}} \tag{58}$$

$$\mathbf{L_{m_t}} : \lambda_t = \theta_t \frac{m_t}{a_m} \tag{59}$$

$$\mathbf{L_{z_t}} : \lambda_t = \mu_t \gamma \delta (1-\phi) \frac{C_{1t}+C_{2t}}{L_{z_t}} \tag{60}$$

$$\mathbf{L_{l_t}} : \lambda_t = \upsilon_t \frac{l_t}{a_l} \tag{61}$$

$$\mathbf{L_{x_t}} : \lambda_t = \mu_t \beta \alpha \phi \frac{C_{1t}+C_{2t}}{L_{x_t}} \tag{62}$$

$$\mathbf{R_{z_t}} : -\omega = \mu_t \gamma (1-\delta)(1-\phi) \frac{C_{1t}+C_{2t}}{R_{z_t}} \tag{63}$$

$$\mathbf{R_{x_t}} : -\omega = \mu_t \beta (1-\alpha) \phi \frac{C_{1t}+C_{2t}}{R_{x_t}} \tag{64}$$

$$\mathbf{m_t} : \theta_{t-1} - \theta_t \left(1 + \frac{L_{m_t}}{a_m}\right) = (1-\gamma)(1-\phi)\left(\mu_t \frac{C_{1t}+C_{2t}}{m_t} - \mu_{t+1} \frac{C_{1_{t+1}}+C_{2_{t+1}}}{m_t}\right) \tag{65}$$

$$\mathbf{l_t} : \upsilon_{t-1} - \upsilon_t \left(1 + \frac{L_{l_t}}{a_l}\right) = (1-\beta)\phi\mu_t \left(\frac{C_{1t}+C_{2t}}{l_t} - \mu_{t+1} \frac{C_{1_{t+1}}+C_{2_{t+1}}}{l_t}\right) \tag{66}$$

$$\mathbf{V_{t+1}} : \omega_t = \omega_{t+1} \tag{67}$$

Combination of (57) and (58) gives the optimal allocation rule between consumption of the young and old generation living at time t:

$$\frac{C_{2t}}{C_{1t}} = \frac{1+\bar{r}}{1+\rho}. \tag{68}$$

The optimal allocation rule for the value of consumption over time, i.e. $\frac{\mu_{t+1}C_{t+1}}{\mu_t C_t}$

$$\frac{\mu_{t+1}C_{t+1}}{\mu_t C_t} = \frac{1}{1+\bar{r}} \tag{69}$$

is in contrast to the intragenerational allocation of consumption in (68) independent of the intragenerational discount factor ρ and only depends on intergenerational discounting, represented by $\bar{r}$.

The optimal growth rate of resource extraction can be obtained by taking (63) at $t+1$ and t, dividing the two expressions and considering (67) and (69) which yields:

$$\frac{\mu_{t+1}C_{t+1}}{\mu_t C_t} = \frac{R_{z_{t+1}}}{R_{z_t}} = \frac{R_{t+1}}{R_t} = \frac{1}{1+\bar{r}}. \tag{70}$$

To derive three conditions necessary to determine the optimal allocation of labor first combine (60) and (62) to

$$L_{x_t} = \frac{\beta}{\gamma}\frac{\alpha}{\delta}\frac{\phi}{1-\phi}L_{z_t}, \tag{71}$$

then take (59) and (61) from which we get

$$\theta_t = \frac{a_m}{a_l}\frac{l_t}{m_t}\upsilon_t. \tag{72}$$

Inserting this expression into (65) gives after rearranging:

$$\upsilon_{t+1} = \upsilon_t\left(1+\frac{L_m}{a_m}\right)\frac{l_t}{l_{t-1}}\frac{m_{t-1}}{m_t} + \frac{1}{1+\bar{r}}(1-\gamma)(1-\phi)\frac{a_l}{a_m}\frac{m_{t-1}}{l_{t_1}}\frac{1}{m_t} \tag{73}$$

Substituting (73) into (66), rearranging and evaluating along the balanced growth path gives

$$\upsilon_t\frac{1}{g_m}(g_m g_l - g_m g_l) = \frac{1}{1+\bar{r}}\frac{1}{l_{t-1}}\left[(1-\beta)\phi\frac{1}{l_t} - (1-\gamma)(1-\phi)\frac{a_l}{a_m}\frac{1}{g_m}\right]. \tag{74}$$

where the RHS of (74) is equal to zero along the balanced path. This in turn implies

$$\begin{aligned} g_m &= \frac{1-\gamma}{1-\beta}\frac{1-\phi}{\phi}\frac{a_l}{a_m}g_l \\ \Leftrightarrow \quad L_m &= \frac{1-\gamma}{1-\beta}\frac{1-\phi}{\phi}(L_l + a_l) - a_m \end{aligned} \tag{75}$$

Finally we get from (65) and (69):

$$\upsilon_t\left(\frac{1}{g_{\theta_t}} - g_{m_t}\right) = \frac{(1-\gamma)(1-\phi)}{l_t}\frac{\bar{r}}{1+\bar{r}}\mu_t C_t \tag{76}$$

where it can be shown by using (59) and (60) that $g_\theta = \frac{1}{1+\bar{r}}\frac{1}{g_m}$, such that after rearranging (76) is modified to

$$\theta_t m_t = (1-\gamma)(1-\phi)\frac{1}{(1+\bar{r})g_m}\mu_t C_t. \tag{77}$$

Employing again (59) and (60) it can be shown that

$$L_{z_t} = \delta\frac{\gamma}{1-\gamma}(1+\bar{r})(L_{m_t} + a_m). \tag{78}$$

Equations (71), (75) and (78) give (43), (44) and (45).

C. Prove of unique equilibrium interest rate (section 5)

In order to show that there exists a unique equilibrium for the market economy when pension funds are present consider (52)

$$\frac{E(r)}{(1-E(r))} = \varrho \frac{E(r)}{L_{l_t} + L_{m_t} - \xi(1-E(r))} + \frac{1}{1+\rho}.$$

The LHS of (52) is identical to the LHS of (30) for which we have already established that it is monotonically decreasing for $r > r^*$ (see Appendix A).

For the denominator of the RHS of (52) to be positive, it can be shown that $r < r^{**}$ has to hold with

$$r^{**} = e + (e^2 + r^*)^{\frac{1}{2}} \tag{79}$$

where

$$e = \frac{r^*}{2} - \frac{1}{2}\left(1 - \frac{1}{\xi}\frac{(1-\gamma(1-\phi)-\beta\phi)}{\gamma\delta(1-\phi)+\alpha\beta\phi}\right). \tag{80}$$

Keeping this restriction in mind, the RHS is an increasing function of r for $0 < r < r^{**}$ with $\lim_{r\to 0} RHS = \frac{1}{1+\rho} + \frac{\varrho}{\xi} = d$ and $\lim_{r\to r^{**}} RHS = \infty$.

Furthermore it can be shown that $r^* < r^{**}$ holds for all parameter values: r^* is determined by $1-E=0$, while r^{**} can be obtained from $1-E = \frac{1}{\xi}(L_m+L_l)$. $1-E$ is strictly increasing in r with $\lim_{r\to 0} 1-E = -\infty$ and $\lim_{r\to\infty} 1-E = 1$. On the other hand $\frac{1}{\xi}(L_m+L_l)$ is a monotonically decreasing function with $\lim_{r\to 0}\frac{1}{\xi}(L_m + L_l) = \frac{1+\gamma(1-\theta)-\beta\phi}{1+\gamma(1-\delta)(1-\theta)-\beta(1-\alpha)\phi}$ and $\lim_{r\to\infty}\frac{1}{\xi}(L_m+L_l) = 0$. Consequently $1-E = \frac{1}{\xi}(L_m+L_l)$ lies always to the right of $1-E=0$ (see Figure 4), such that $r^* < r^{**}$ holds.

** Figure 4 about here **

Combining the results for the LHS and RHS of (52) gives figure 3 where it can be seen that (52) determines one unique equilibrium interest rate (see Figure 3).

** Figure 3 about here **

Figures

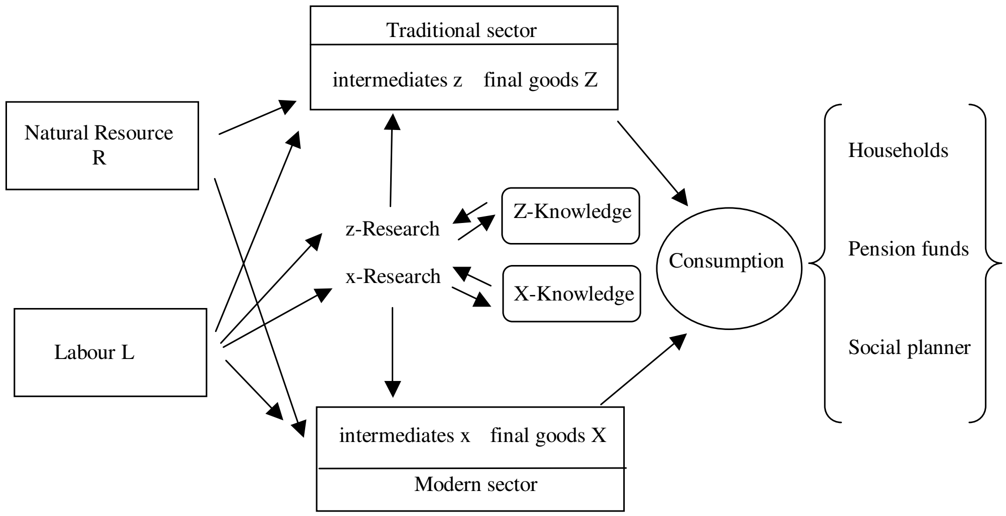

Figure 1: Model structure

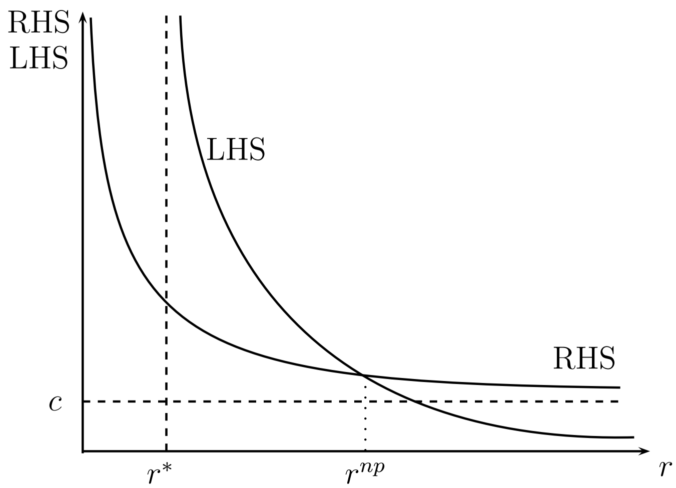

Figure 2: Equilibrium interest rate for no-pension fund scenario

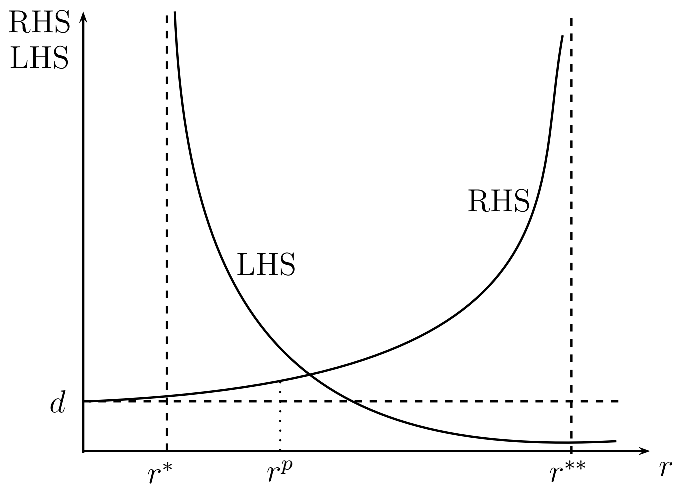

Figure 3: Equilibrium interest rate for no-pension fund scenario

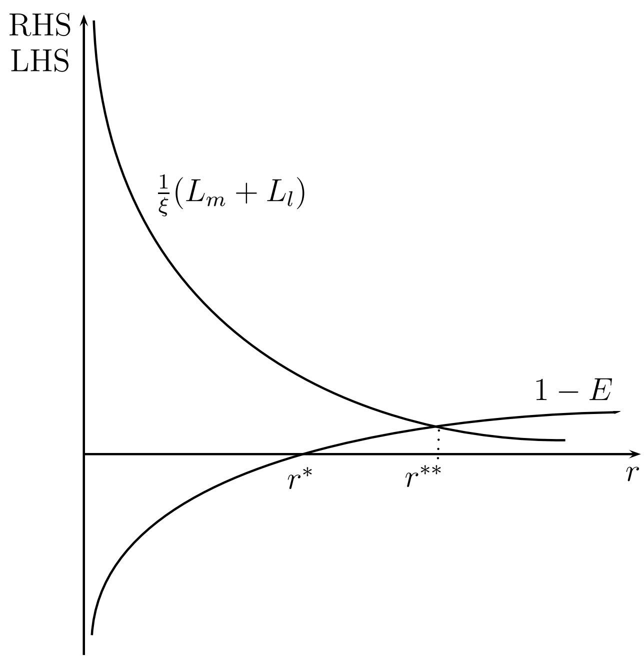

Figure 4: r^* and r^{**}

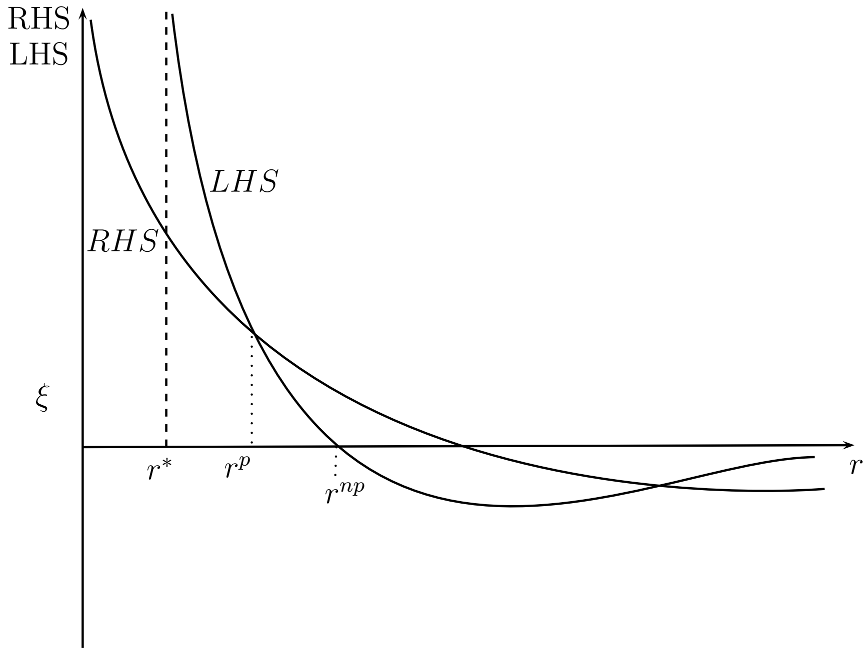

Figure 5: Equilibrium interest rate for pension fund and no-pension fund scenario