WIF - Institute of Economic Research

Economic Growth and Sectoral Change under Resource Reallocation Costs

Thomas M. Steger

Working Paper 03/30
December 2003

Economics Working Paper Series

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Economic Growth and Sectoral Change under Resource Reallocation Costs *

Thomas M. Steger[a]

December 2003

A general growth model with explicit resource reallocation costs is set up. A new feature is the property of hysteresis (i.e. a continuum of stationary equilibria) in closed-economy growth models. Employing a linear model the hysteresis range and the consequences for the long-run growth rate are determined analytically. The most important conclusions are the following: (1) An economy's long-run position may depend critically on the initial intersectoral allocation pattern as well as on the efficiency of the resource reallocation sector; (2) if we interpret the resource reallocation sector as a specific part of the education sector, there is a straightforward possibility for the government to reduce the range of hysteresis and hence the dependence on initial conditions; (3) international trade is an important device to overcome the negative consequences of high resource reallocation costs for long-run growth.

Keywords: Sectoral change; economic growth; resource reallocation costs; hysteresis; multiplicity of equilibria

JEL classification: O0; N1

* For helpful comments and suggestions I would like to thank Josef Falkinger, Volker Grossmann, Klaus Wälde and, especially, Sjak Smulders. Moreover, I am grateful to seminar participants at the University of Zurich for helpful discussions.

[a] ETH Zurich, WIF – Institute for Economic Research, ETH Zentrum, WET D 6, CH - 8092 Zurich, Tel. +41 (0)1 632 24 27, Fax +41 (0)1 632 13 62, steger@wif.gess.ethz.ch.

1. Introduction

The concept of hysteresis has become extraordinarily important when trying to explain a wide range of macroeconomic phenomena. Specifically, hysteresis has been fruitfully employed in labour economics to explain the persistency of the unemployment rate in response to adverse macroeconomic shocks (e.g. Roed, 1997) and in the theory of international trade to explain the consequences of exchange rate shocks on the trade pattern (e.g. Giavazzi and Wyplosz, 1984). Since hysteresis is associated with a continuum of stationary equilibria, the concept can also be applied to better understand the pronounced heterogeneity in growth experiences observable in the real world.[1] In the context of growth theory, hysteresis has in fact been derived from open-economy growth models by Grossman and Helpman (1991, chapter 8) and van de Klundert and Smulders (2001). Beside an international framework these models do, however, require a large number of fairly special assumptions. On the other hand, in this paper it is shown that hysteresis arises quite naturally from the standard closed-economy growth model when resource reallocation costs are plausibly taken into account.

There are two important observations which motivate the analysis conducted below. First, when taking (neoclassical or endogenous) growth models seriously to think about real-world economic growth, the transition process towards the balanced growth path must be taken into consideration. Moreover, the transition process is intrinsically characterised by a change in the sectoral allocation pattern (sectoral change). This becomes obvious by remembering that even the simplest growth model consists of two sectors, i.e. a consumption goods sector and an investment goods sector (this fact is sometimes hidden behind the one-sector formulation). Along the transition to the balanced growth path the relative importance of the different sectors (measured by the inputs allocated to these sectors) changes. Hence, standard growth models provide a simple baseline model of sectoral change. Second, it is quite plausible to assume that the reallocation of resources from one sector to another incurs substantial costs. In reality theses costs may take very different forms. To clarify the basic idea consider two specific examples of resources reallocation costs: (1) Suppose that the productivity of workers increases with the accumulated experience in production (e.g. on-the-job training). When an experienced worker is reallocated from one sector to another, productivity of this worker suddenly falls and subsequently rises again. The

[1] In fact, there are two concepts of hysteresis in the economics literature. The first form (being more frequent) relies on zero eigenvalues of the underlying dynamic system and a continuum of stationary equilibria, whereas

sudden fall in productivity may plausibly be interpreted as resource reallocation costs. (2) Assume that employment in specific sectors requires sector-specific skills. Crossing sectoral borders then necessitates to invest in human capital (e.g. occupational retraining). In this case, the resource reallocation costs consist in the initial human capital investment.

Standard growth models assume that resource reallocation is completely free of charge. At first glance, this simplifying assumption appears warranted as far as the long-run outcome should be explained. The reason might be seen in the fact that the economic structure usually remains constant along the balanced growth path. Hence, the widely employed, simplifying assumption of zero resource reallocation costs can be viewed as being uncritical with respect to the issue under study. It will be demonstrated, however, that this conjecture is wrong. The abstraction from resource reallocation costs turns out to be critical with respect to the long-run growth.

The present paper contributes to the literature on economic growth by investigating the consequences of resource reallocation costs for long-run growth. More specifically, a fairly general three-sector growth model with a single resource that can be accumulated is set up. The reallocation of this resource from the consumption goods sector to the investment sector (or vice versa) is costly in terms of inputs not being available for the production of investment goods. The resource reallocation sector captures the basic idea according to which the reallocation of resources is costly. Due to its generality the model can be considered as a generic growth model with explicit resource reallocation costs. This growth model should be viewed as a first step in understanding the nexus between economic growth and sectoral change under resource reallocation costs. The analysis is of major importance since sectoral change with the requirement of resource reallocation is an intrinsic property of real-world economic dynamics.[2]

There are two basic possibilities to accomplish sectoral change. According to the first and most obvious possibility, resources must explicitly be reallocated from one sector to another. In addition, sectoral change can be managed by the accumulation (or decumulation) of sector-specific inputs. One could argue that this second possibility does not require a

---

the second form is based on non-linearity of the underlying dynamic system, multiplicity of equilibria and bifurcation.

[2] Using a sectoral distinction which focuses on the production side, Maddison (1987) and, more recently, Temple (2001) demonstrate that sectoral change with the requirement of intersectoral resource reallocation represents a key aspect of real-world economic dynamics. Both authors find that the development process of now developed countries was for several decades characterised by massive intersectoral reallocations of labour.

reallocation of resources. This kind of reasoning, however, ignores the fact that the reinforced production of sector-specific inputs unambiguously requires a reallocation of resources to the sector producing those sector-specific inputs. Hence, sectoral change is inevitably associated with a reallocation of resources.

Of course, sectoral change can be analysed at different levels of aggregation. First, at the economy-wide level the consumption goods sector may be distinguished from the investment goods sector. Second, sectoral change within the sectors mentioned above can be considered. This kind of sectoral change may continue forever, i.e. even growth along the BGP may exhibit sectoral change (e.g. Kongsamut et al., 2001). The paper at hand is concerned with sectoral change between the consumption goods and the investment goods sector. From the perspective of growth theory this type of sectoral change is of major importance since the extend to which an economy adjusts its sectoral structure along this dimension may have important implications for the long run growth performance.

This paper builds on earlier work on resource reallocation costs. There are two previous studies dealing with resource reallocation costs within dynamic general equilibrium models. These papers have been published in the 1970s and belong to the international trade literature.[3] More specifically, Kemp and Wan (1974) study a dynamic two-sector model of international trade with labour as the single input. Since there is no resource that can be accumulated there is no growth in this model. The resource reallocation costs are formulated as forgone output due to resource reallocation (output-based formulation of resource reallocation costs). Moreover, Mussa (1978) studies the process of adjustment in a dynamic version of the Heckscher-Ohlin-Samuelson model. Capital is considered as quasi-fixed factor in the sense that the intersectoral reallocation entails costs, which consist in labour not being available for productive uses elsewhere (input-based formulation of resource reallocation costs). The author focuses on the role of expectations and the resource reallocation technology for the adjustment process to the new equilibrium in response to a shock in relative prices. Once more, this paper does not investigate the role of resource reallocation costs for economic growth either.

Strongly related and also highly relevant for the analysis of economic dynamics is the concept of capital adjustment costs, i.e. costs associated with the installation of new capital goods. There is a large literature on capital adjustment costs in dynamic models (e.g., Hayashi, 1982; Abel and Blanchard, 1983). The consideration of capital adjustment costs is highly plausible and its incorporation into standard growth models has led to valuable insights. Specifically, this approach lays the foundation for a self-contained demand for investment goods. In addition, an obvious flaw of the open-economy version of the neoclassical growth model (i.e. instantaneous international convergence of per capita income levels) can be avoided. Also the rate of convergence implied by the closed-economy version of the neoclassical model decreases significantly being in line with empirical evidence.

There are a number of important results derived from the analyses conducted below. A new feature is the property of hysteresis in closed-economy growth models. This finding bears strong positive and normative implications. Among these are the following: (1) An economy's long-run position may depend critically on the initial intersectoral allocation pattern as well as on the efficiency of the resource reallocation sector; (2) if we interpret the resource reallocation sector as a specific part of the education sector, there is a straightforward possibility for the government to reduce the range of hysteresis and hence the dependence on initial conditions; (3) international trade is an important device to overcome the negative consequences of high resource reallocation costs for long-run growth.

The paper is structured as follows. In Section 2 a generic growth model with explicit resource reallocation costs is set up. The implications of this general model are developed in Section 3. By focusing on a linear economy the hysteresis range and the consequences for long-run growth are determined analytically in Section 4. The economic interpretation together with the positive and normative implications are given in Section 5. Finally, Section 6 summarises and provides additional conclusions.

2. A generic growth model with resource reallocation costs

In this section we set up a basic growth model with explicit resource reallocation costs (RRC). On the one hand, the aim is to formulate a model which is general enough to serve as the framework for the analysis of specific growth models. In this sense the model can be considered as a generic growth model with (explicit) RRC.[4] On the other hand, the model is as

---

[3] Within this strand of the literature the term "adjustment costs" has been used to describe the costs associated with the intersectoral reallocation of resources. We prefer to label these costs as resource reallocations costs to distinguish it from capital adjustment costs associated with the installation of new capital goods.

[4] Implicit resource reallocation costs arise in two-sector models if the production possibility frontier is convex (e.g. due to decreasing returns to scale).

simple as possible to enable the derivation of unequivocal results. In addition, we focus on the social planner's problem and do not consider the underlying microeconomic structure. This procedure is justified since the aim is to understand the basic implications of RRC for sectoral change and lung-run growth. The formulation of the underlying microeconomic structure, which is especially important for the derivation of policy implications, is left for future research.

The model comprises three sectors, namely a consumption goods sector (c-sector), an investment goods sector (i-sector) and a resource reallocation sector (RR-sector). The output technology for the c-goods and the i-goods is denoted by $f(.)$ and is assumed to satisfy $f'(.) > 0$, $f''(.) \leq 0$ and $f(0) = 0$.[5] There is a single resource k which can be thought of as physical and/ or human capital. This single resource has three distinct uses. It can either produce c-goods, i-goods or it can be employed in the RR-sector. The intersectoral allocation of k across the c-sector and the i-sector is given by the intersectoral allocation variable θ. More specifically, θ gives the share of the single resource k allocated to the c-sector (consumption share). The production of c-goods accordingly is $c = f(\theta k)$. Similarly, provided that no resources are engaged in resource reallocation (RR), $1-\theta$ gives the share of the resource k devoted to the i-sector (investment share). The production of i-goods would then be given by $i = f[(1-\theta)k]$.

Whenever the social planner wants to change the intersectoral allocation pattern, resources must be devoted to the RR-sector. This formulation captures the basic idea that the reallocation of resources from the c-sector to the i-sector (or the other way round) incurs costs. The resource reallocation technology (RRT) is given by $\dot{\theta} = sign(\phi)g(|\phi|k)$, where $|\phi|$ is the share of the resource devoted to RR and a "dot" above a variable denotes its derivative with respect to time.[6] Consequently, with RR (implying $|\phi| > 0$) the production of i-goods is $i = f[(1-\theta-|\phi|)k]$.

As usual, the objective is to maximise the present value of an infinite utility stream. Instantaneous utility $u(c)$ with $u'(c) > 0$ and $u''(c) < 0$ is of the constant-intertemporal-

[5] The fact that both goods are produced with the same technology is not critical for the derived results. Nonetheless, this (simplifying) assumption allows a direct comparison of the model under study to the underlying model without RRC.

[6] The RRT is explained in more detail below.

elasticity-of-substitution (CIES) type. Let us now consider the model stated as the social planner's problem.

$$\max_{\{\phi\}} \int_0^\infty u(c)\, e^{-\rho t} dt \tag{1}$$

s.t. $\dot{k} = i - \delta k$ (2)

$$c = f(\theta k) \tag{3}$$

$$i = f[(1-\theta-|\phi|)\, k] \tag{4}$$

$$\dot{\theta} = sign(\phi) g(|\phi| k) \tag{5}$$

$$|\phi| \leq 1-\theta \tag{6}$$

$$\theta \in [0, 1] \tag{7}$$

$$k(0) = k_0\,;\ \theta(0) = \theta_0 \tag{8}$$

TABLE 1

For the readers convenience the notation is summarised in Table 1. Several aspects of the model are especially worth being noticed.

The single control variable is ϕ, which can be either positive or negative. Notice that $|\phi|$ gives the share of k devoted to RR. The restriction on the control variable (6) ensures that the sum of shares devoted to the three sectors cannot exceed unity. There are two state variables, namely the stock of the resource k and the intersectoral allocation variable θ. It should be observed that θ is the control variable within the underlying two-sector model without RRC.

Equation (5) shows the RRT. The function $g(|\phi| k)$ is assumed to satisfy the following conditions: $g'(|\phi| k) > 0$, $g''(|\phi| k) \leq 0$ and $g(0) = 0$. A change in the intersectoral allocation pattern can be accomplished only if costs are incurred. More specifically, the transfer of the amount $\dot{\theta} k$ during the (short) period dt from the c-sector to the i-sector (or vice versa) requires to allocate $g(|\phi| k) k$ to the RR-sector. Let us consider a specific example. Assume that the reallocation of $\dot{\theta} k$ requires the input of $B(|\phi| k)^\alpha$ with $0 < \alpha \leq 1$ in the RR-sector. In this case, the RRT may be expressed as $\dot{\theta} = sign(\phi) g(|\phi| k) = sign(\phi) B |\phi|^{\alpha} k^{\alpha-1}$. For the linear case ($\alpha = 1$) this RRT is simply given by $\dot{\theta} = sign(\phi) B |\phi|$.

The formulation of the RRT [equation (5)] might appear somewhat peculiar at first glance. Equation (5) together with the admissible control set (6) indicates that ϕ is allowed to become negative. In fact, this formulation comprises two decisions: First, the share of the resource devoted to RR is given by $|\phi|$ as appearing in equations (4) and (5). Second, the direction of RR is indicated by the sign of ϕ as expressed by $sign(\phi)$ in equation (5).

The RRC are formulated as input-based, i.e. the RRC consist in the amount of the resource devoted to RR not being available for production. Moreover, equation (4) shows that resources devoted to RR must be withdrawn from the i-sector. This formulation is fairly plausible economically since a change in the intersectoral allocation pattern can be considered as an investment in the desired allocation pattern as given by θ.[7]

The RR-sector of the social planner's formulation stated above need not describe a real sector. Instead it captures the basic notion that RR is not free of charge. Examples for a microeconomic set up comprise (1) firms which have to pay a premium over the wage rate if they want to attract labour previously employed in another sector; (2) a lower initial productivity of workers previously employed in another sector; or (3) workers who have to invest in human capital in order to cross sectoral borders.[8]

Finally, it should be noticed that there is one single resource within this model that can be productively employed in both the c-sector and the i-sector. With this formulation the intersectoral allocation pattern is unequivocally given by the intersectoral allocation variable θ. We accordingly abstract from sectoral change resulting from the accumulation (or decumulation) of sector-specific inputs. Instead we focus on sectoral change due to the explicit reallocation of resources. This perspective is justified by the fact that sectoral change resulting from the accumulation of sector-specific inputs necessitates the reinforced production of sector-specific factors, which unambiguously requires also a reallocation of resources to the sector producing those sector-specific inputs.

---

[7] In a more general framework, one could model the basic possibility that resources devoted to RR might be withdrawn from the i-sector as well as from the c-sector. The consequences for growth and convergence are probably different from the specification chosen here. This task is left for future research.

[8] Further microeconomic interpretations are given in Section 5.

3. Basic implications

In this section, the basic implications of economic growth and sectoral change under RRC are shown within the general model set up above. Analytical results are derived in Section 4.

3.1. First-order conditions

The current-value Hamiltonian for the above stated problem is given by

$$H(\phi,k,\theta,\mu_1,\mu_2) := u\left[f\left(\theta k\right)\right] + \mu_1\left\{f\left[\left(1-\theta-|\phi|\right)k\right]-\delta k\right\} + \mu_2\, sign(\phi)\, g(|\phi|k)\,. \tag{9}$$

Let ϕ^* denote the optimal choice of the control variable and H_x the derivative of the Hamiltonian with respect to x. The necessary first-order conditions may then be expressed as follows

$$H(\phi^*,k,\theta,\mu_1,\mu_2) \geq H(\phi,k,\theta,\mu_1,\mu_2)\,, \tag{10}$$

$$\dot{\mu}_1 = \rho\,\mu_1 - H_k = \mu_1\left[\rho+\delta-f'(.)(1-\theta-|\phi|)\right] - \mu_2\, sign(\phi)\, g'(.)|\phi| - u'(.)\, f'(.)\theta\,, \tag{11}$$

$$\dot{\mu}_2 = \rho\,\mu_2 - H_\theta = \rho\,\mu_2 + \mu_1\, f'(.)\,k - u'(.) f'(.)\,k\,. \tag{12}$$

Moreover, the equations of motion for k and θ [equation (2) and (5)] must hold.[9] Equation (10) applies to both corner and interior solutions. To determine the nature of the solution (corner vs. interior and $\phi=0$ vs. $\phi\neq 0$) we form the derivative of the Hamiltonian with respect to the control variable (to be employed below)

$$H_\phi = -\mu_1\, f'(.)\frac{\partial|\phi|}{\partial\phi}k + \mu_2\, sign(\phi) g'(.)\frac{\partial|\phi|}{\partial\phi}k \qquad \text{for} \qquad \phi\neq 0\,. \tag{13}$$

[9] In addition, the transversality conditions for k and θ must be satisfied, i.e. $lim_{t\to\infty} e^{-\rho t}\mu_1 k = 0$ and $lim_{t\to\infty} e^{-\rho t}\mu_2\theta = 0$. We further assume that the necessary conditions are also sufficient.

3.2. Corner solutions

A corner solution with either $\phi^* = \phi_{\min} = -(1-\theta) < 0$ or $\phi^* = \phi_{\max} = 1-\theta > 0$ is realised provided that the following condition holds (cf. appendix 7.1.)

$$\left|\mu_2\right| g'\left[(1-\theta)k\right] \geq \mu_1 f'\left[(1-\theta-|\phi|)k\right]\Big|_{(1-\theta-|\phi|)k=0} . \tag{14}$$

Whether $\phi^* = \phi_{\min} < 0$ or $\phi^* = \phi_{\max} > 0$ depends on whether $\theta(0) > \theta^*$ or $\theta(0) < \theta^*$ with θ^* denoting the optimal value of θ implied by the underlying model without RRC. It should be noticed that the shadow price of the allocation variable μ_2 is positive for $\theta(0) < \theta^*$ and negative for $\theta(0) > \theta^*$.

The economic intuition behind condition (14) is straightforward. The LHS shows the increase in overall welfare provided that a small amount of the resource is devoted to RR. This is given by the (absolute value of the) shadow price of the intersectoral allocation variable $|\mu_2|$ times the marginal product of the RR-sector $g'(|\phi|k)$; notice that $g'(|\phi|k)$ is evaluated at $|\phi|k = 1-\theta$. The RHS shows the increase in overall welfare provided that the same small amount of the resource is devoted to the i-sector. This is given by the shadow price of capital μ_1 times the marginal product of the i-sector $f'\left[(1-\theta-|\phi|)k\right]$. Notice that $f'\left[(1-\theta-|\phi|)k\right]$ is evaluated at $|\phi| = 1-\theta$. Considering condition (14) it becomes evident that it is rational to allocate, for example, the maximum feasible share of the resource to the RR-sector provided that (1) $\theta(0) < \theta^*$ and (2) the LHS (evaluated at $\phi_{\max} = 1-\theta$) exceeds the RHS (evaluated at $\phi_{\max} = 1-\theta$).

3.3. Interior solutions

Interior solutions result provided that $H_\phi = 0$. It must, however, be observed that equation (13) is defined for $\phi \neq 0$ only. This is due to the fact that $|\phi|$ is not differentiable at $\phi = 0$. More precisely, the crucial aspect is that $\lim_{\phi_- \to 0} \frac{\partial|\phi|}{\partial\phi} = -1$ and $\lim_{\phi_+ \to 0} \frac{\partial|\phi|}{\partial\phi} = 1$. This property gives rise to a continuum of stationary equilibria and, hence, hysteresis (in the sense of at least one eigenvalue equal to zero) results.

There are three types of interior solutions, which are distinguished by (1) $\phi^* < 0$; (2) $\phi^* > 0$ and (3) $\phi^* = 0$. It can be readily shown that interior solutions with $\phi^* \neq 0$ result provided that the following condition holds (cf. appendix 7.2.)

$$|\mu_2| g'\left(|\phi| k\right) = \mu_1 f'\left[\left(1-\theta-|\phi|\right) k\right]. \tag{15}$$

The sign of ϕ^* is determined by the initial allocation variable $\theta(0)$ in relation to θ^* (the unique optimal value of θ implied by the underlying model without RRC). If $\theta(0) > \theta^*$, we get $\phi^* < 0$. Similarly, for $\theta(0) < \theta^*$ it follows that $\phi^* > 0$.

Finally, and most importantly, there is a continuum of stationary equilibria with $\phi^* = 0$ whenever the conditions resulting from (1) $H_\phi = 0$, equation (13), $\lim_{\phi_- \to 0} \frac{\partial |\phi|}{\partial \phi} = -1$, $sign(\phi) = -1$ and $\phi = 0$ and (2) $H_\phi = 0$, equation (13), $\lim_{\phi_+ \to 0} \frac{\partial |\phi|}{\partial \phi} = 1$, $sign(\phi) = 1$ and $\phi = 0$ border a non-empty set in (k, θ)-space.[10] The condition which must hold for the optimal choice of the control variable being equal to zero ($\phi^* = 0$) reads as follows (cf. appendix 7.3)

$$|\mu_2| g'\left(|\phi| k\right)\Big|_{\phi=0} \leq \mu_1 f'\left[\left(1-\theta\right) k\right]. \tag{16}$$

Once more, the economic interpretation of condition (16) is straightforward. The LHS gives the increase in overall welfare provided that a small amount of the resource is devoted to the RR-sector. This is given by the (absolute value of the) shadow price of the allocation

[10] Condition (1) with $\phi < 0$ instead of $\phi = 0$ would give the necessary condition for interior solutions for adjustment from above (in θ-direction), while condition (2) with $\phi > 0$ instead of $\phi = 0$ would give the necessary condition for interior solutions for adjustment from below. By imposing $\phi = 0$ we determine the borders of the set with $\phi \neq 0$. These borders shape a set in (k, θ)-space where $\phi^* = 0$ must hold. For details the reader is referred to appendix 7.3.

variable $|\mu_2|$ (which might be negative in the case of $\theta > \theta^*$) times the marginal physical output of the RR-sector $g'(|\phi| k)$ (evaluated at $\phi = 0$). The RHS shows the increase in overall welfare if the same amount of the resource is devoted to the i-sector. This is given by the shadow price of capital μ_1 times the marginal product of the i-sector $f'[(1-\theta)k]$ (evaluated at $\phi = 0$). The preceding condition accordingly says that if the contribution of RR (i.e. investing in the desired allocation pattern) to overall welfare is smaller than the contribution of additional investment goods, then it is optimal to set $\phi = 0$ and leave the intersectoral allocation pattern unchanged.

Condition (16) is necessary and sufficient for the occurrence of hysteresis. It does not, however, prove that the hysteresis set is actually non-empty. A rigorous existence proof of hysteresis must show that, for some initial values $k(0)$ and $\theta(0)$, the initial shadow prices $\mu_1(0)$ and $\mu_2(0)$ [being endogenous jump variables which depend on $k(0)$ and $\theta(0)$] are such that condition (16) actually holds.

Nonetheless, using a specific model it is shown in Section 4 that the hysteresis set is indeed non-empty, i.e. there is a combination of $k(0)$ and $\theta(0)$ such that hysteresis actually occurs. For the specific model under study, the hysteresis set can be expressed as a hysteresis range in terms of θ only. This implies that the initial allocation of the resource $\theta(0)$ alone determines whether hysteresis does occur. Moreover, the hysteresis range in terms of θ will be determined analytically. It will also analytically be shown that this range of hysteresis (i.e. a continuum of stationary equilibria) covers the unique stationary equilibrium of the underlying model without RRC.

Provided that condition (16) holds, $\phi^* = 0$ and, by equation (5), $\dot{\theta} = 0$ as well. The intersectoral allocation pattern accordingly remains constant. The coefficient matrix of the relevant dynamic system [(2), (5), (11) and (12) together with $\phi = 0$] has one row equal to zero and, therefore, this dynamic system possesses at least one zero eigenvalue. As a result, there is a continuum of stationary equilibria, which constitute the range of hysteresis. Since the hysteresis range implies $\phi^* = 0$ the term "range of inaction" may also be appropriate. Within this range the adjustment dynamic is degenerate along the θ-dimension. Put differently, the stationary equilibria within the hysteresis range are characterised by indifferent stability properties (as opposed to stability vs. instability) along the θ-dimension.

Table (2) summarises the optimal solutions together with the necessary conditions and indicates the respectively relevant dynamic system.

TABLE 2

It should be noticed that for the input-based formulation of RRC (being fairly plausible) the occurrence of hysteresis does not depend on additional assumptions on the shape of the RRT. The reason lies in the fact that this formulation unambiguously implies a linear cost function. Indeed, from the input-based formulation it follows that the cost function is the identity transformation. The costs of allocating the amount ϕk to the RR-sector in terms of inputs withdrawn from the i-sector is given by the amount $|\phi| k$. Hence, the cost function must be non-smooth at the origin. This property gives rise to the necessary condition for hysteresis within this set-up, which is given by $\lim_{\phi_- \to 0} \frac{\partial |\phi|}{\partial \phi} \neq \lim_{\phi_+ \to 0} \frac{\partial |\phi|}{\partial \phi}$.[11]

4. A linear growth model with resource reallocation costs

It would be highly desirable at this stage to determine the hysteresis range in terms of θ analytically. Subsequently, the potential consequences of RRC with respect to the long-run growth rate could be investigated explicitly. Fortunately, this task can be accomplished by focusing on an almost linear economy.

The linear growth model with RRC results from (1) to (8) with $u(c) = log(c)$, $c = f(\theta k) = A\theta k$, $i = f[(1-\theta-|\phi|)k] = A(1-\theta-|\phi|)k$ and $\dot{\theta} = sign(\phi)B|\phi|$.[12] It should be noticed that there is always some non-linearity even with linear technologies since the final-output technology unambiguously contains the term θk (both θ and k being endogenous variables).

With this parameterisation one can readily derive the borders of the hysteresis range in terms of θ (cf. appendix 7.4.). The lower border is denoted as θ_l^*, while the upper border is

[11] This is in contrast to the output-based formulation of Kemp and Wan (1974). Here the cost function (mapping the amount resources devoted to RR into output losses) can be smooth at the origin.

[12] With this parameterisation the underlying growth model without RRC is essentially an AK model (Romer, 1986 and Rebelo, 1991).

labelled θ_u^*.[13] It turns out that $\theta_l^* = \frac{B\rho}{A(B+\rho)}$ and $\mu_{2,l}^* = \frac{A}{B\rho}$ (the steady state value of the shadow price of θ at $\theta = \theta_l^*$). Of course, since θ is restricted by (7), we get $\theta_l^* = 1$ for $B\rho \geq A(B+\rho)$ and $\theta_l^* = \frac{B\rho}{A(B+\rho)}$ provided that $B\rho < A(B+\rho)$.[14]

Similarly, the upper border of the hysteresis range turns out to read $\theta_u^* = \frac{B\rho}{A(B-\rho)}$. Since θ_u^* approaches infinity as $B-\rho$ converges to zero, θ_u^* hits the upper border of the set $[0,1]$ when B becomes sufficiently small. As θ is restricted by (7), $\theta_u^* = 1$ for $B\rho \geq A(B-\rho)$ and $\theta_u^* = \frac{B\rho}{A(B-\rho)}$ for $B\rho < A(B-\rho)$. The steady state value of the shadow price of θ at $\theta = \theta_u^* < 1$ is given by $\mu_{2,u}^* = -\frac{A}{B\rho}$.

At this stage it becomes obvious that the hysteresis range vanishes as B approaches infinity. More specifically, the upper and the lower bound of the hysteresis range both converge to the unique value of θ implied by the underlying growth model without RRC. This is readily recognised by applying the rule of L'Hôpital to θ_u^* and θ_l^*, which yields $\lim_{B\to\infty} \theta_l^* = \lim_{B\to\infty} \theta_u^* = \theta^* = \frac{\rho}{A}$. The economic reason for this observation lies in the fact that the RRC converge to zero as B approaches infinity and, hence, the underlying growth model without RRC represents the limiting case for $B \to \infty$.

The hysteresis range in terms of θ is defined by the set $\left[\theta_l^*, \theta_u^*\right]$ and has the following economic interpretation. Provided that the economy starts within this range, the intersectoral allocation remains unchanged. If the economy starts outside this range, it converges (either from below or from above) to the border of this range.

We are now ready to describe the magnitude of the hysteresis range (MHR) analytically. This magnitude may be simply expressed as $MHR = \theta_u^* - \theta_l^*$. Specifically,

---

[13] θ_l^* is the stationary solution for θ resulting from the dynamic system which governs the evolution of interior solutions for adjustments from below (in θ-direction), while θ_u^* is the stationary solution for θ resulting from the dynamic system which governs the evolution of interior solutions for adjustments from above.

[14] To keep things simple, we exclude the unlikely case $\theta_l^* = 1$, i.e. we assume that $B\rho < A(B+\rho)$ holds.

$MHR = 1 - \frac{B\rho}{A(B+\rho)}$ for $\frac{B\rho}{A(B-\rho)} \geq 1$ and $MHR = \frac{B\rho}{A(B-\rho)} - \frac{B\rho}{A(B+\rho)} = \frac{2B\rho^2}{A(B^2+\rho^2)}$ for $\frac{B\rho}{A(B-\rho)} < 1$. Stated more compactly the magnitude of the hysteresis range can be expressed as

$$MHR = \begin{cases} \frac{A(B+\rho) - B\rho}{A(B+\rho)} & for \quad B\rho \geq A(B-\rho) \\ \frac{2B\rho^2}{A(B^2+\rho^2)} & for \quad B\rho < A(B-\rho) \end{cases}. \tag{17}$$

FIGURE 1

Figure 1 illustrates the hysteresis range $\left[\theta_l^*, \theta_u^*\right]$ and the magnitude of the hysteresis range ($MHR = \theta_u^* - \theta_l^*$). The diagrams also demonstrate the sensitivity of the hysteresis range and hence the sensitivity of the MHR with respect to different parameters (B, A and ρ).[15]

Let us consider the consequences of RRC for heterogeneity in growth rates. It is well known that, within the endogenous growth framework, differences in the intersectoral allocation pattern (the investment share) translate into differentials of the long-run growth rate. For the linear growth model under study the long-run growth rate may simply be expressed as $\hat{y} = A(1-\tilde{\theta}) - \delta$, where a "hat" above a variable denotes its growth rate and $\tilde{\theta}$ the long-run consumption share.

In order to judge the extent of heterogeneity in long-run growth rates due to diverging initial conditions under RRC we consider the maximum growth rate differential (MGD). Assume that two economies share the same preferences and technologies but differ with respect to initial conditions. The MGD is then given by $\Delta\hat{y} := \hat{y}_1 - \hat{y}_2 = A\left|\tilde{\theta}_2 - \tilde{\theta}_1\right| = A(\theta_u^* - \theta_l^*)$. This differential in long-run growth rates would fully apply provided that $\theta_1(0) \leq \theta_l^*$ and

[15] The following set of parameters has been employed: $A = B = 0.1$, $\rho = 0.05$. For a detailed economic interpretation of Figure 1 see the next section.

$\theta_2(0) \geq \theta_u^*$ [or $\theta_1(0) \geq \theta_u^*$ and $\theta_2(0) \leq \theta_l^*$]. If the preceding condition fails, a growth rate differential between zero and $A(\theta_u^* - \theta_l^*)$ should be observed.

For a first assessment of the quantitative importance of the model with RRC, we employ the following parameter values: $A = B = 0.1$ and $\rho = 0.05$ (notice that the MGD is independent of δ). In this case, the MGD amounts to $MGD \cong 0.067$. The maximum differential in long-run growth rates under RRC due to diverging initial intersectoral allocations accordingly amounts to 6.7 percentage points. This rather drastic result indicates that RRC may be of major importance.

FIGURE 2

More generally, Figure 2 shows the MGD in response to the productivity of the RR-sector B. This diagram indicates that we can still explain a growth rate differential of up to 2 percentage points if we set $B = 0.3$. Indeed, RRC appear to be an important candidate explanation for the pronounced heterogeneity in long-run growth rates.

5. Economic interpretation, positive and normative implications

Economic interpretation

The economic interpretation of the hysteresis range (i.e. a range of inaction giving rise to a continuum of stationary equilibria) is straightforward. At first, it should be noticed that the underlying growth model without RRC implies a unique optimal long-run value for the allocation variable θ. This optimal value is determined such that the marginal present value (PV) of benefits equals the marginal PV of (implicit) RRC due to RR. The model with RRC adds an explicit cost component associated with RR. In this case, it might be optimal not to close the whole gap between the initial value of θ and the (hypothetical) long-run value implied by the underlying model without RRC. Put differently, if the economy would move into the hysteresis range, overall welfare decreases since the marginal PV of costs exceeds the marginal PV of benefits.

In order to fully understand the determinants behind the hysteresis range let us return to Figure 1, which illustrates the hysteresis range as well as the MHR in response to the model parameters (A , B and ρ).[16]

Let us start with the parameter B since this case is most instructive. The dashed line in Figure 1 (a) shows the unique long-run equilibrium value of θ for the underlying growth model without RRC, which is denoted as θ^* . Of course, θ^* is independent of B and hence is represented by a horizontal (dashed) line. Moreover, Figure 1 (a) also shows θ_l^* and θ_u^* in response to the productivity of the RR-sector B . The most interesting point concerns the fact that θ_u^* falls with B (ignoring border solutions), whereas θ_l^* rises with B . As a consequence, the MHR shrinks as B increases [Figure 1 (b)]; the kink is due to the initial border solution for θ_u^* . As has been stated above, the economic reason for this observation is due to the fact that the RRC converge to zero as B approaches infinity.

As B grows without bounds the MHR vanishes and the hysteresis range accordingly shrinks to a single point. This observation indicates that the underlying model without RRC (which could be a Ramsey-Cass-Koopmans model or an AK model) represents one limiting case of the model with RRC. Moreover, the second limiting case ($B = 0$) corresponds to the Solow (1956) model, where the investment share (saving rate) is fixed.[17] The model under study can therefore be viewed as a fairly general framework which encompasses a number of standard approaches as special cases.

Figure 1 (c) shows θ^* , θ_l^* and θ_u^* in response to the productivity of the investment goods sector A . Since an increase in A reflects a rise in the marginal and average product of capital the underlying linear growth model implies a falling θ^* as A rises (e.g. Barro and Sala-i-Martin 1995, p. 144). Moreover, provided that A is sufficiently low, θ^* , θ_l^* and θ_u^* equal unity (i.e. the return to investment in physical capital is too low to justify consumption renunciation). For the reason mentioned above, θ^* , θ_l^* and θ_u^* start to decrease as A increases beyond certain thresholds. Since θ_l^* starts to decrease before θ_u^* , the MHR increases initially as illustrated by Figure 1 (d). Behind a certain threshold value [which is $A = B\,\rho/(B-\rho)$], the MHR decreases and approaches zero as A converges to infinity.

[16] Restrictions on the set of parameters resulting from the applied optimisation method or the transversality condition are not taken into account. This would merely complicate the analysis and leave the basic results unchanged.

Figure 1 (e) depicts θ^*, θ_l^* and θ_u^* in response to the time preference rate ρ. As the time preference rate rises, a larger share of the resource is allocated to the c-sector since current consumption is valued stronger than future consumption. The diagram also illustrates that θ_u^* hits its upper boundary first. Therefore, the MHR increases initially and, beyond a certain threshold value, starts to decrease. Since θ_l^* eventually approaches the upper boundary of the set $[0,1]$ as ρ approaches infinity, the MHR eventually vanishes [Figure 1 (f)].

Positive implications

It has been demonstrated above that two economies with identical preferences and technologies may realise different long-run growth rates (or levels of per capita income, depending on the underlying growth model). Formally, the long-run outcome depends on the initial allocation of resources across sectors as expressed by $\theta(0)$. In this sense, initial conditions are crucial for long-run growth. As a consequence, the model with RRC predicts a higher variance in long-run growth rates compared to the underlying growth model without RRC. Moreover, the model with RRC reveals that long-run growth rates may additionally be determined by the efficiency of the resource reallocation sector.

What is the sensible interpretation of this result with respect to economic development? Typically, low-income countries allocate a lower share of resources to the i-sector. For instance, this follows unambiguously when subsistence consumption is taken into account (e.g. Steger, 2000). In the course of economic development, a reallocation of resources from the c-sector to the i-sector takes place. The model under study implies that economies with a more efficient RR-sector increase their investment rate further [i.e. a larger part of the gap between $\theta(0)$ and θ^* is closed] and hence do grow faster in the long run.

This story can be conveniently illustrated by Figure 3. Assume that an economy starts out with $\theta(0) > \theta_u^*$ and B is "very large". In this case, the economy converges towards $\theta_u^*(B = \infty)$; hence, we have an AK-type growth model with transitional dynamics like in Jones and Manuelli (1990). Consider now two economies which differ in the efficiency of RR B only (with $B_1 < B_2$). Both economies start with the same $\theta(0)$ and the initial growth rate of output is $\hat{y} = A[1 - \theta(0)] - \delta$. In the course of economic growth, the investment share $1 - \theta$ increases and $\hat{y}$ rises as well. The economy with high RRC (B_1 is low) adjusts the

[17] In this case, the hysteresis range is maximal and hence any (exogenous) change in θ is permanent.

intersectoral allocation pattern until $1-\theta_u^*(B_1)$ is reached, whereas the economy with low RRC (B_2 is high) adjusts its allocation further and ends up with an investment share of $1-\theta_u^*(B_2)$. The economy with the less efficient RR-sector accordingly realises the long-run growth rate $\hat{y}_1^* = A\left[1-\theta_u^*(B_1)\right]-\delta$, whereas the economy with the more efficient RR-sector ends up with $\hat{y}_2^* = A\left[1-\theta_u^*(B_2)\right]-\delta$ with $\hat{y}_1^* < \hat{y}_2^*$.

FIGURE 3

Normative implications

One obvious interpretation of the preceding model reads that the single resource is human capital and the RR-sector is a specific part of the education sector. There are at least two versions of this interpretation. First, employment in another sector requires specific skills, which can be formed within the RR-sector (e.g. occupational retraining). Second, the RR-sector produces general skills which are beneficial for RR. The more efficient this part of education (provided by the RR-sector), the higher is the level of these general skills (a component of human capital) and the lower are the costs of RR.[18]

In any case, the government can reduce the RRC and thereby increase the maximum growth rate by providing an efficient RR-sector. More concrete, it is plausible to assume that the efficiency of the education sector is positively related to public educational infrastructure. Expenditures on public educational infrastructure, therefore, increase the efficiency of the education sector, lower RRC, increase the long-run share of resources allocated to the i-sector and consequently foster long-run economic growth.[19]

The analysis also sheds new light on the role of international trade. The exchange of goods allows an economy to accumulate sector-specific inputs and hence to change the intersectoral allocation pattern without actually allocating resources to the sector producing those sector-specific inputs. Indeed, if an economy is technologically unable to produce

[18] According to a slightly different (complementary) interpretation, a reallocation of, say, a worker to another sector implies a lower productivity initially. The productivity of this worker then rises through on-the-job training. In this perspective, the RR-sector is not a real sector but captures the fact that there are costs (a lower initial productivity) associated with the reallocation of workers.

[19] Of course, a rigorous derivation of policy implications requires to set up the decentral economy and to identify potential market failures. This task is left for future research.

investment goods at all, this can be represented as $\theta(0) = 1$ and $B = 0$ within the present model. Hence, international trade is an important device to overcome the negative consequences of high RRC for long-run growth.

Moreover, international trade unfolds positive growth effects only when the intersectoral allocation variable θ hits the (upper) bound of the hysteresis range. This might help explaining the equivocal empirical findings on the nexus between openness and economic growth.[20]

6. Summary and conclusion

The starting point of this paper lies in the observation that sectoral change with the requirement of resource reallocation is an intrinsic property of real-world economic dynamics. It is clear that the reallocation of resources may incur significant costs. Therefore, the question arises whether the usual simplifying assumption according to which resource reallocation is free of charge is critical with respect to long-run growth.

In order to answer this question from a theoretical point of view, a generic growth model with explicit resource reallocation costs has been set up. The model comprises three sectors, namely a consumption goods sector, an investment goods sector and a resource reallocation sector. The resource reallocation sector accomplishes a reallocation of resources from the consumption goods to the investment goods sector (or the other way round). Based on this general set-up it has been shown that hysteresis in the sense of a continuum of stationary equilibria may result. Within this hysteresis range it is optimal to leave the allocation pattern unchanged. The economic significance of this implication lies in the fact that the long-run growth rate depends on initial conditions (initial allocation pattern) as well as on resource reallocation costs. Hence, divergence in long-run growth rates increases when resource reallocation costs are taken into account.

By employing a linear model the range of hysteresis (in terms of the allocation variable) and the consequences for long-run growth have been determined analytically. Moreover, using a plausible set of parameters it has been demonstrated numerically that resource reallocation costs are important for understanding differences in long-run growth rates.

The economic intuition for a pronounced diversity in long-run growth rates due to resource reallocation costs is as follows. At early stages of economic development the share of resources allocated to the investment goods sector (the investment share) is typically very low. Provided that the instantaneous growth rate is positive (i.e. the marginal product of capital exceeds the time preference rate) the investment share increases along the transition path. Standard growth models usually imply a unique optimal long-run allocation pattern and adjustment is complete in the sense that the investment share approaches this unique value. With positive resource reallocation costs, however, it might be optimal not to fully adjust the investment share simply because the reallocation of resources is costly.

It is quite instructive to notice that the model presented in this paper encompasses the Solow (1956) model and the Ramsey-Cass-Koopmans model (or, as the case may be, the AK model) as special (limiting) cases. Provided that the efficiency of the resource reallocation sector is zero, resource reallocation costs are infinite implying that a reallocation of resources is technically infeasible; this case corresponds to the Solow (1956) model. On the other hand, provided that the efficiency of the resource reallocation sector approaches infinity resource reallocation costs converge to zero. This second limiting case describes standard growth models (e.g. Ramsey-Cass-Koopmans model or AK model), where the reallocation of resources can be accomplished free of charge and hence instantaneously.

The model can also be used to better understand an important stylised fact of economic growth according to which, in a cross-section of countries, the variance of long-run growth rates is negatively correlated with the level of per capita income (Romer, 1989, p. 64). Assuming that the efficiency of resource reallocation increases with per capita income the model predicts that the variance of long-run growth rates is higher for low-income countries. The reason lies in the fact that the hysteresis range and hence the dependence on initial conditions is larger, the lower is the efficiency of the resource reallocation sector.

Moreover, the analysis bears also important implications for the convergence issue. First, the speed at which an economy converges to its balanced growth path can be clearly expected to fall as resource reallocation costs are taken into account; this consequence is similar to the implication of capital adjustment costs for the speed of convergence. Second, by making resource reallocation costly we include parts of the adjustment process which is a jump onto the stable manifold within the underlying model without resource reallocation costs

[20] The empirical evidence on the relationship between openness and long-run growth is mixed. Vamvakidis (2002) notes that, for a cross-section of countries, there is no correlation over the longer time period from 1870

into the (continuous) adjustment process along the stable manifold within the model with resource reallocation costs. This allows a more adequate representation of adjustment processes as occurring in the real-world.

The present paper indicates two important policy implications, which should be elaborated more explicitly in future research. First, one obvious interpretation of the growth model under study reads that the single resource is human capital and the resource reallocation sector is a specific part of the education sector. With this interpretation the government has the opportunity to reduce the hysteresis range (and hence the dependence of initial conditions) and increase the long-run growth rate by raising the efficiency of the relevant part of the education sector. Second, there is a new role for international trade with respect to economic growth. The exchange of goods allows an economy to accumulate sector-specific inputs and, therefore, to change the intersectoral allocation pattern without actually allocating resources to the sector producing those sector-specific inputs. Hence, international trade is an important device to overcome the negative consequences of high resource reallocation costs for long-run growth.

There are several issues for future research, which probably reveal further insights into the nexus between resource reallocation costs and economic growth. First, it is clearly indicated to set up the microeconomic structure of this growth model with resource reallocation costs. Subsequently, policy implications should be rigorously derived from the comparison of the social planner's solution and the market solution. Second, the efficiency of the resource reallocation sector was treated as exogenous. Treating this efficiency as an endogenous variable (which might be affected by government expenditures on education) may reveal further important policy implications. Third, the implications for the speed of convergence should be elaborated explicitly. If one understands to what extend resource reallocation costs reduce the speed of convergence, the diversity in growth rates can in part be explained as a transition phenomenon. Fourth, World Bank research on transition economies has focused on growth resulting from a more efficient allocation of resources (Selowsky and Martin, 1997).[21] So far, however, there is no explicit growth model which takes the costs and benefits of resource reallocations into account. The model presented in this paper can be used as a theoretical framework for this kind of empirical research.

---

until present, while a significantly positive correlation can be found for recent decades.

[21] Temple (2001) also investigates the contribution of sectoral change to economic growth.

7. Appendix

7.1. Conditions for a corner solution [condition (14)]

For $H_\phi \leq 0$ a corner solution with $\phi^* = \phi_{\min} = -(1-\theta) < 0$ is realised. The condition, which must hold for this corner solution to be optimal, results from $H_\phi \leq 0$, equation (13), $\phi_{\min} = -(1-\theta)$ and noticing that $\frac{\partial|\phi|}{\partial\phi} = -1$ and $sign(\phi) = -1$ in this case, which yields

$$\mu_1 f'\left[\left(1-\theta-|\phi_{\min}|\right)k\right] + \mu_2 g'\left(\phi_{\min} k\right) \leq 0. \tag{18}$$

Since $\phi_{\min} < 0$, $\theta > \theta^*$ (with θ^* denoting the optimal value of θ implied by the underlying model without RRC) implying that $\mu_2 < 0$ and, hence, the preceding condition may be expressed as

$$\mu_1 f'\left[\left(1-\theta-|\phi_{\min}|\right)k\right] \leq |\mu_2| g'\left(|\phi_{\min}| k\right). \tag{19}$$

Moreover, when we substitute $|\phi_{\min}|$ by $1-\theta$ we may express the preceding condition as

$$|\mu_2| g'[(1-\theta)k] \geq \mu_1 f'\left[(1-\theta-|\phi|)k\right]\Big|_{(1-\theta-|\phi|)k=0}. \tag{20}$$

Similarly, for $H_\phi \geq 0$ a corner solution with $\phi^* = \phi_{\max} = 1-\theta > 0$ is realised. The condition, which must hold for this corner solution to be optimal, results from $H_\phi \geq 0$, equation (13), $\phi_{\max} = 1-\theta$ and noticing that $\frac{\partial|\phi|}{\partial\phi} = 1$ and $sign(\phi) = 1$ in this case, which yields

$$-\mu_1 f'\left[\left(1-\theta-|\phi_{\max}|\right)k\right] + \mu_2 g'\left(\phi_{\max} k\right) \geq 0. \tag{21}$$

Since $\phi_{\max} > 0$, $\theta < \theta^*$ implying that $\mu_2 > 0$ and, hence, the preceding condition may be expressed as

$$\mu_1 f'\left[\left(1-\theta-|\phi_{\max}|\right)k\right] \leq |\mu_2| g'(\phi_{\max} k). \tag{22}$$

Moreover, when we substitute $|\phi_{\max}|$ by $1-\theta$ we may express the preceding condition as

$$|\mu_2| g'[(1-\theta)k] \geq \mu_1 f'\left[(1-\theta-|\phi|)k\right]\Big|_{(1-\theta-|\phi|)k=0}. \tag{23}$$

This is condition (14) in the main text.

7.2. The conditions for an interior solution with $\phi^* \neq 0$ [condition (15)]

Consider first the case of $\phi < 0$ and notice that $\left.\dfrac{\partial|\phi|}{\partial\phi}\right|_{\phi<0} = -1$ and $sign(\phi) = -1$ (adjustment from above in θ-direction). The condition, which must hold for $-(1-\theta) < \phi < 0$ to be optimal, results from $H_\phi = 0$, equation (13), $\dfrac{\partial|\phi|}{\partial\phi} = -1$ and $sign(\phi) = -1$, which yields

$$-\mu_2 g'(\phi k) = \mu_1 f'\left[\left(1-\theta-|\phi|\right)k\right]. \tag{24}$$

Since we started by assuming that $\theta > \theta^*$ (where θ^* denotes the optimal value of θ implied by the underlying model without RRC), $\mu_2 < 0$ and, hence, equation (24) may be expressed as

$$|\mu_2| g'(\phi k) = \mu_1 f'\left[\left(1-\theta-|\phi|\right)k\right]. \tag{25}$$

Consider next the case of $\phi > 0$ and notice that $\left.\dfrac{\partial|\phi|}{\partial\phi}\right|_{\phi>0} = 1$ and $sign(\phi) = 1$ (adjustment from below in θ-direction). The condition, which must hold for $0 < \phi < 1-\theta$ to be optimal, results from $H_\phi = 0$, equation (13), $\dfrac{\partial|\phi|}{\partial\phi} = 1$ and $sign(\phi) = -1$, which yields

$$\mu_2 \, g'(\phi k) = \mu_1 \, f'\left[(1-\theta-|\phi|)k\right]. \tag{26}$$

Since we started by assuming that $\theta < \theta^*$ (where θ^* denotes the optimal value of θ implied by the underlying model without RRC), $\mu_2 > 0$ and hence equation (26) may be expressed as

$$|\mu_2| \, g'(\phi k) = \mu_1 \, f'\left[(1-\theta-|\phi|)k\right]. \tag{27}$$

This is equation (15) in the main text, which determines the optimal value of ϕ, while the sign of ϕ is determined by $\theta(0)$ in relation to θ^*.

7.3. The condition for an interior solution with $\phi^* = 0$ [condition (16)]

The above discussion implies that there is a continuum of stationary equilibria with $\phi^* = 0$ whenever the conditions resulting from (1) $H_\phi = 0$, $\phi = 0$, $\lim_{\phi_- \to 0} \frac{\partial |\phi|}{\partial \phi} = -1$ and $sign(\phi) = -1$ as well as (2) $H_\phi = 0$, $\phi = 0$, $\lim_{\phi_+ \to 0} \frac{\partial |\phi|}{\partial \phi} = 1$ and $sign(\phi) = 1$ border a non-empty subset in (k, θ)-space.

From $H_\phi = 0$ together with equation (13), $\phi = 0$, $\lim_{\phi_- \to 0} \frac{\partial |\phi|}{\partial \phi} = -1$ and $sign(\phi) = -1$ we get

$$-\mu_2 \, g'(\phi k)\big|_{\phi=0} = \mu_1 \, f'\left[(1-\theta)k\right]. \tag{28}$$

Notice that $\mu_2 < 0$ since we are considering the left-hand limit implying that $\theta > \theta^*$. Hence, equation (28) can be expressed as

$$|\mu_2| \, g'(\phi k)\big|_{\phi=0} = \mu_1 \, f'\left[(1-\theta)k\right]. \tag{29}$$

Similarly, using $H_\phi = 0$, equation (13), $\phi = 0$, $\lim_{\phi_+ \to 0} \frac{\partial|\phi|}{\partial \phi} = 1$ and $sign(\phi) = 1$ we arrive at

$$\mu_2 \, g'(\phi k)\big|_{\phi=0} = \mu_1 f'\left[(1-\theta)k\right]. \tag{30}$$

In this case, $\mu_2 > 0$ since we are considering the right-hand limit implying that $\theta < \theta^*$. Hence, we can conclude that $\phi^* = 0$ provided that the subsequent relation holds

$$|\mu_2| \, g'(\phi k)\big|_{\phi=0} \le \mu_1 f'\left[(1-\theta)k\right]. \tag{31}$$

This is the hysteresis condition (16) in the main text.

7.4. The hysteresis range in terms of θ for the linear economy

The linear growth model with RRC results from (1) to (8) with $u(c) = log(c)$, $c = f(\theta k) = A\theta k$, $i = f[(1-\theta-|\phi|)k] = A(1-\theta-|\phi|)k$ and $\dot{\theta} = g(\phi k) = B\phi$; notice that for the linear case the RRT can be expressed more simple as $\dot{\theta} = g(\phi k)$.

Assuming further that the economy starts below its steady state in θ-direction (hence $\phi > 0$ and $\frac{\partial|\phi|}{\partial \phi} = 1$) the set of first-order conditions for interior solutions reads as follows:

$B\mu_2 = Ak\mu_1$	[from (13) and $H_\phi = 0$]	(32)
$\dot{\mu}_1 = \mu_1\rho - \frac{1}{k} - \mu_1[-\delta + A(1-\theta-\phi)]$	[from (11)][22]	(33)
$\dot{\mu}_2 = Ak\mu_1 - \frac{1}{\theta} + \mu_2\rho$	[from (12)]	(34)

[22] Notice that the second term on the RHS of (11), which is $\mu_2 \, g_k(.)$, vanishes since the RRT is independent of k.

$\dot{k} = A k(1-\theta-\phi) - k\delta$ [from (2)] (35)

$\dot{\theta} = B\phi$ [from (5)] (36)

This set of equations (together with appropriate border conditions) determines the dynamics for interior solutions with $\phi > 0$. Moreover, this set of equations also determines a stationary solution for θ, which is the bordering value of θ for adjustments from below (in θ-direction); this solution is labelled θ_l^*. Because the model is highly linear, θ_l^* can be determined analytically.

From (32) we have $\hat{\mu}_2 = \hat{k} + \hat{\mu}_1$, where a "hat" above a variable denotes its growth rate. Equation (33) together with the definition of a balanced growth path implies $\hat{k} = -\hat{\mu}_1$ and hence $\hat{\mu}_2 = 0$. By noting $k\,\mu_1 = \frac{B\,\mu_2}{A}$ [from (32)] (34) can be expressed to read $\hat{\mu}_2 = B - \frac{1}{\theta\,\mu_2} + \rho = 0$. Moreover, $\hat{k} = -\hat{\mu}_1$ together with (33) and (35) yields $\rho = \frac{A}{B\mu_2}$. Hence, we get $\theta_l^* = \frac{B\,\rho}{A(B+\rho)}$ and $\mu_{2,l}^* = \frac{A}{B\,\rho}$ (the steady state value of the shadow price of θ at $\theta = \theta_l^*$). Of course, since θ is restricted by (7), we get $\theta_l^* = 1$ for $B\,\rho \geq A(B+\rho)$ and $\theta_l^* = \frac{B\,\rho}{A(B+\rho)}$ provided that $B\,\rho < A(B+\rho)$.[23]

Provided that the economy starts with $\theta(0) > \theta_u^*$ (i.e. above the upper bordering value for θ), $\phi < 0$ and hence $\frac{\partial|\phi|}{\partial\phi} = -1$. Consequently, (32) becomes $-B\,\mu_2 = A\,k\,\mu_1$. Carrying out the same steps as before finally yields $\theta_u^* = \frac{B\,\rho}{A(B-\rho)}$. Since θ_u^* approaches infinity as $B-\rho$ converges to zero, θ_u^* hits the upper border of the set $[0,1]$ when B becomes sufficiently small. As θ is restricted by (7), $\theta_u^* = 1$ for $B\,\rho \geq A(B-\rho)$ and $\theta_u^* = \frac{B\,\rho}{A(B-\rho)}$ for $B\,\rho < A(B-\rho)$. The steady state value of the shadow price of θ at $\theta = \theta_u^* < 1$ is given by $\mu_{2,u}^* = -\frac{A}{B\,\rho}$.

[23] To keep things simple, we exclude the unlikely case $\theta_l^* = 1$, i.e. we assume that $B\,\rho < A(B+\rho)$ holds.

8. References

Abel, A. B. and O. J. Blanchard, An intertemporal model of savings and investment, Econometrica, 1983, 51, 3, 675-692.

Dixit, A., Investment and Hysteresis, Journal of Economic Perspectives, 1992, 6(1), 107-32.

Giavazzi, F. and C. Wyplosz, The Real Exchange Rate, the Current Account and the Speed of Adjustment, in: J. Bilson and R. Marston (eds.), Exchange Rates: Theory and Policy, Chicago University Press, Chicago, 1984.

Grossman, G. and E. Helpman, Innovation and Growth in the Global Economy, MIT Press, Cambridge Ma., 1991.

Hayashi, F., Tobin's Marginal q and Average q : A Neoclassical Reinterpretation, Econometrica, 1982, 50, 213-224.

Jones, L. E. and R. Manuelli, A Convex Model of Equilibrium Growth: Theory and Policy Implications, Journal of Political Economy, 1990, 98, 1008-1038.

Kemp, M. C. and H. Y. Wan, Hysteresis of Long-Run Equilibrium from Realistic Adjustment Costs, in G. Horwich and P.A. Samuelson (eds.): Trade, Stability and Macroeconomics, New York Academic Press, 1974, 221-242.

Klundert van de, T. and S. Smulders, Loss of technological leadership of rentier economies: a two-country endogenous growth model, Journal of International Economics, 2001, 54, 211-231.

Kongsamut, P., S. Rebelo and D. Xie, Beyond Balanced Growth, Review of Economic-Studies, 2001, 68, 869-82.

Maddison, A., Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment, Journal of Economic Literature, 1987, 649-698.

Mussa, M., Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model, Journal of Political Economy, 1978, 86, 5, 775-791.

Rebelo, S., Long-run policy analysis and long run growth. Journal of Political Economy, 1991, 99, 500–521.

Roed, K., Hysteresis in Unemployment, Journal of Economic Surveys, 1997, 11, 389-418.

Romer, P. M., Capital Accumulation in the Theory of Long-Run Growth, Robert J. Barro (ed.), Modern Business Cycle Theory, Basil Blackwell, Oxford, 1989, 51-127.

Romer, P. M., Increasing Returns and Long-Run Growth, Journal of Political Economy, 1986, 94, 1002-1037.

Selowsky, M. and R. Martin, Reform Boosts Growth and Foreign Investment, Transition Newsletter, 1997, 8, 3, May-June.

Solow, R., A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics, 1956, 70, 65-94.

Steger, T. M., Economic Growth with Subsistence Consumption, Journal of Development Economics, 2000, 62, 343-361.

Temple, J., Structural change and Europe's Golden Age, University of Bristol Discussion Paper No. 01/519, 2001.

Vamvakidis, A., How Robust is the Growth-Openness Connection? Historical Evidence, Journal of Economic Growth, 2002, 7, 57-80.

Table 1: Notation for the general growth model with resource reallocation costs.

$u(.)$: instantaneous utility function	θ : share of k allocated to c -sector
$f(.)$: output technology	$\lvert\phi\rvert$: share of k allocated to resource reallocation sector
$g(.)$: resource reallocation technology	μ_1 : shadow price of one unit of k
c : consumption goods	μ_2 : shadow price of one unit of θ
i : investment goods	ρ : time preference rate ($\rho > 0$)
k : stock of the single resource ("capital")	δ : depreciation rate of capital ($0 \le \delta \le 1$)

Table 2: Types of solution together with respective conditions and dynamic systems.

Type of Solution	**Necessary Conditions**	**Dynamic System**
Corner Solution $\phi^* = \phi_{\min} = -(1-\theta) < 0$	$-\mu_2\, g'[(1-\theta)k] \ge \mu_1 f'[(1-\theta-\lvert\phi\rvert)k]\big\rvert_{(1-\theta-\lvert\phi\rvert)k=0}$ with $\theta(0) > \theta^*$ and $\mu_2 < 0$	(2), (5), (11) and (12) with $\phi = -(1-\theta)$
Corner Solution $\phi^* = \phi_{\max} = 1-\theta > 0$	$\mu_2\, g'[(1-\theta)k] \ge \mu_1 f'[(1-\theta-\lvert\phi\rvert)k]\big\rvert_{(1-\theta-\lvert\phi\rvert)k=0}$ with $\theta(0) < \theta^*$ and $\mu_2 > 0$	(2), (5), (11) and (12) with $\phi = 1-\theta$
Interior Solution $\phi^* < 0$	$-\mu_2 g'(\phi k) = \mu_1 f'[(1-\theta-\lvert\phi\rvert)k]$ with $\theta(0) > \theta^*$ and $\mu_2 < 0$	(2), (5), (11) and (12) with ϕ^* determined by $H_\phi = 0$, $\frac{\partial\lvert\phi\rvert}{\partial\phi} = -1$, $sign(\phi) = -1$
Interior Solution $\phi^* > 0$	$\mu_2 g'(\phi k) = \mu_1 f'[(1-\theta-\lvert\phi\rvert)k]$ with $\theta(0) < \theta^*$ and $\mu_2 > 0$	(2), (5), (11) and (12) with ϕ^* determined by $H_\phi = 0$, $\frac{\partial\lvert\phi\rvert}{\partial\phi} = 1$, $sign(\phi) = 1$
Interior Solution $\phi^* = 0$	$\lvert\mu_2\rvert g'(\phi k)\big\rvert_{\phi=0} \le \mu_1 f'[(1-\theta)k]$	(2), (5), (11) and (12) with $\phi = 0$

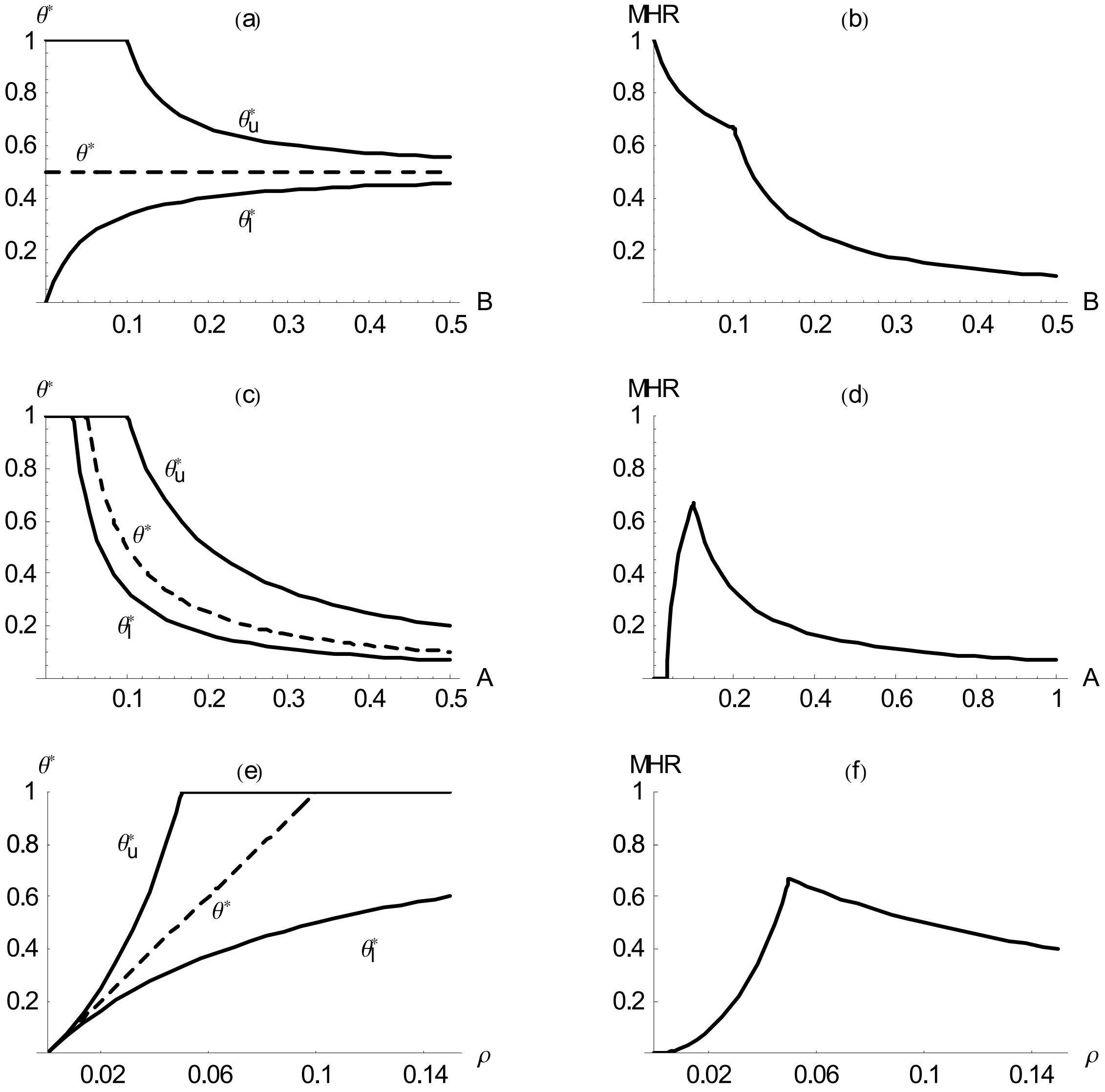

Figure 1: Bordering values for the allocation variable (θ_l^*, θ_u^*), steady state value for underlying model without RRC (θ^*) and the magnitude of the hysteresis range ($MHR = \theta_u^* - \theta_l^*$) for the linear growth model in response to model parameters.

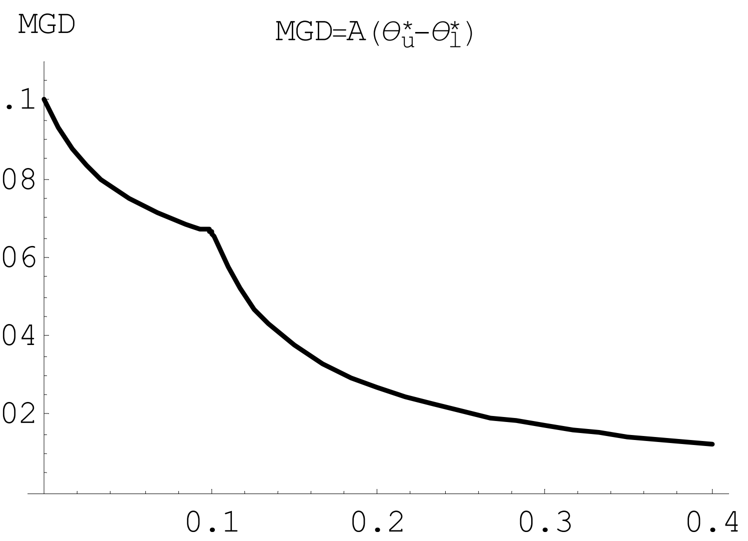

Figure 2: The maximum growth rate differential (MGD) in response to B.

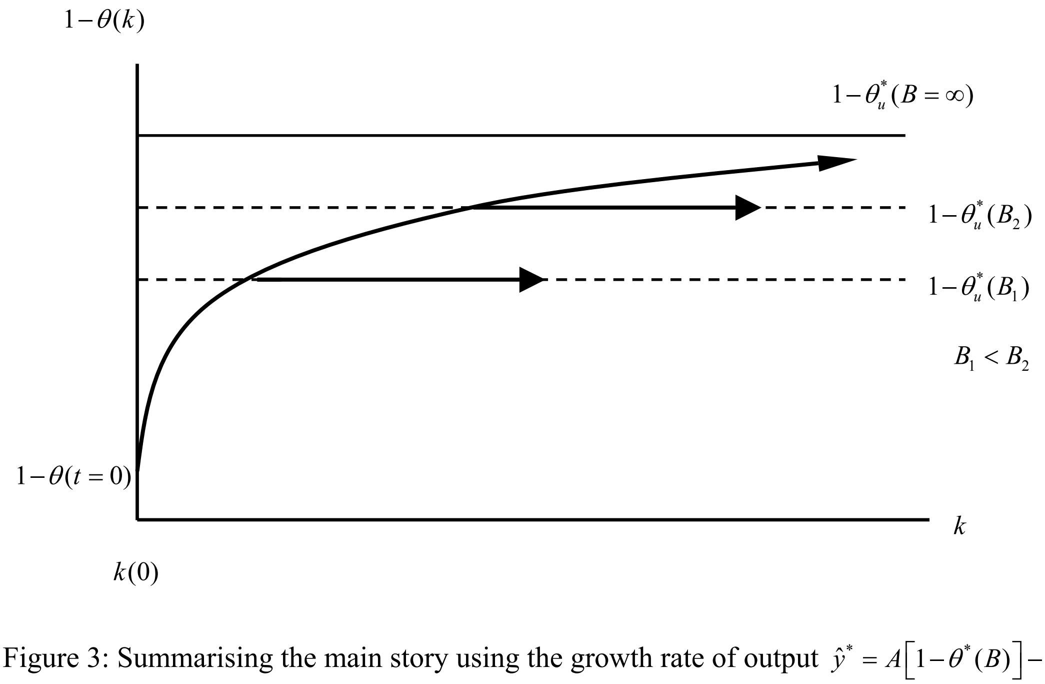

Figure 3: Summarising the main story using the growth rate of output $\hat{y}^* = A\left[1-\theta^*(B)\right]-\delta$.