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Working Paper 17/272
June 2017

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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This Version: June 2017

Abstract

We introduce a two-stage, multiple-round voting procedure where the thresholds needed for approval require a qualified majority and vary with the proposal on the table. We apply such a procedure to instances of public-good provision where the citizens' valuations can take two values and are private. We show that the procedure elicits and aggregates the information about the valuations and implements the utilitarian optimal public good level. This level is chosen after all potential socially optimal policies have been considered. We also develop a compound procedure to ensure utilitarian optimality when there are arbitrarily finitely many types of citizen.

Keywords: voting; utilitarianism; implementation

JEL Classification: C72, D70

*We would like to thank Matthew Jackson and Vincent Lohmann for valuable comments and suggestions. All errors are our own.

1 Introduction

It is well known that in general voting procedures based on fixed majority thresholds, and on the majority rule in particular, are unable to implement the decision that is utilitarian optimal for the citizenry. Although such voting schemes present numerous advantages—Black (1948), May (1952), Maskin et al. (1995), and Moulin (2014) are some of the canonical references—, they are not without their drawbacks—see e.g. Arrow (1950), Plott (1967), Gibbard (1973), Satterthwaite (1975), and McKelvey (1976). For one thing, fixed majority rules typically cannot elicit the intensity of preferences, especially without the help of redistribution schemes. This generally renders the utilitarian optimum unattainable via such voting procedures.[1]

To combine voting and utilitarian welfare maximization, one approach is to use flexible majority rules, which allow approval thresholds to depend on the proposal on the table. Such schemes are sometimes applied in practice and have been recently surveyed in Gersbach (2017). In Nebraska, for instance, increases of the property tax above 5% cannot by decided by the legislature directly but have to be approved in a referendum (Mullins and Cox, 1995).[2] In a public-good provision framework, Gersbach (2017) shows, in particular, that a sequential (or successive) procedure based on appropriately designed flexible majority rules implements the welfare-optimal level of the public good.[3] Instead of voting on the final level of the public good immediately, a series of votes on small increments are taken starting from the status quo, and voting goes on until a higher threshold cannot be met.[4] Also recently, Gershkov et al. (2016) have provided a mechanism design foundation of successive voting procedures by showing that every unanimous and anonymous dominant-strategy incentive-compatible mechanism is outcome-equivalent to a successive procedure with decreasing thresholds, and vice versa. Among such procedures, Gershkov et al. (2016) have further singled out those that are ex-ante utilitarian optimal.

Sometimes, the above procedures mean that the policy eventually adopted *(i)* is approved only by a minority of the citizenry and *(ii)* is not pitted against all potential socially optimal

[1]More recent papers that offer foundations for the majority rule are Aşan and Sanver (2002), Woeginger (2003), and Miroiu (2004). The Median Voter Theorem is the subject of an extensive body of literature, see e.g. Barberà et al. (1993), Sprumont (1991), Ching (1997), and Chatterji and Sen (2011).

[2]This example and others can be found in Gershkov et al. (2016). More generally, the principle that larger changes require larger majorities, *ad maiore ad minus*, is embedded in the constitutions of many countries. This is, for instance, the case in Spain (see `http://www.constitutionnet.org/files/constitutional_amendment_procedures.pdf`, retrieved 18 January 2017).

[3]Other procedures considered in the literature that aim to resolve the tension between mean and median voter (Rosar, 2015) and to discern the intensity of preferences include (Casella, 2005; Fahrenberger and Gersbach, 2010; Hortalà-Vallvé, 2010). Sequential procedures with fixed majority rules are broadly used in European parliaments (Rasch, 2000).

[4]Incremental voting dates back to Bowen (1943).

alternatives. The first property is in conflict with the majoritarian logic of collective decisions and may prompt cycling in policy-making if the procedure is applied repeatedly. What is more, by requiring majorities as large as possible for policy implementation, consensus within the society can be fostered more efficiently. The second property is desirable if the extent to which each citizen feels included in the decision process increases when the set of alternatives considered along the voting process includes all alternatives that are welfare-optimal for a certain distribution of the citizens' valuations.

The main object of this paper is to show that it is possible to find a democratic procedure that reconciles the implementation of the policy that is optimal from a utilitarian perspective with two requirements that are remedies to the problems we have just mentioned. First, the final policy decision has to be approved by at least half of the citizens—this requirement is called *consensus*. Second, the status quo as a default option is adopted only after all potential socially optimal alternatives have been considered—this requirement is called *inclusiveness*. We impose the first condition in the design of the procedure, while the second feature obtains in equilibrium.

To elaborate, we consider a (continuous) public-good provision problem where citizens are of two types (low and high) regarding their valuation of the public-good level. For such a setting, we introduce a two-stage, multiple-round voting procedure, which builds on the natural order within the finite set of potential socially optimal public-good levels: higher public-good levels are labeled with higher indices. The procedure is based on so-called *semi-flexible majority rules* and consists of two stages of successive voting rounds with varying thresholds, the outcome of the first stage being taken as the status quo in the second stage. In the first stage, the decision on which policy should be set as the status quo for the second stage may be taken by more or by less than half of the citizens, depending on the alternative being considered, and works as follows: Starting from a zero public-good level, moving to positive public-good levels requires the support of a small share of citizens. Moving to larger levels, however, requires the support of a much bigger share of citizens until it reaches the entire society. In the latter case, the highest possible public good level serves as the status quo for the second stage.

In the second stage, by contrast, the approval of any alternative requires an alternative-specific qualified majority, which is never lower than half plus one of the citizens—hence the term semi-flexible majority rule as opposed to flexible majority rules (Gersbach, 2017). Since it is the purpose of the second to select the alternative that will be eventually adopted, the entire procedure is in line with the first of our democratic requirements, viz. consensus. That is, the

final decision will be always taken by a (simple or qualified) majority of citizens. Only when all alternatives have been discarded against the status quo chosen in the first round will the latter be adopted. While the first stage considers policies from low public-good levels to higher ones, the second reverses the order.

The main result of the paper (see Theorem 1) shows that if citizens iteratively eliminate weakly dominated strategies—and this fact is common knowledge—, any perfect Bayesian equilibrium outcome of the game describing the procedure outlined will maximize ex-post utilitarian welfare.[5] Moreover, in equilibrium, the policy that is eventually adopted will always be chosen after consideration of all potential socially optimal alternatives. As a consequence, our procedure also meets the second of our democratic requirements, viz. inclusiveness.

Overall, our paper contributes to the literature on the implementation of social choice functions by providing a new procedure (or mechanism) that implements the utilitarian social choice function. However, since we consider a procedure with additional restrictions based on democratic considerations, our model departs from the standard mechanism design literature. Yet, the procedure that we suggest is anonymous, unanimous, and non-manipulable (in strategies that are not iteratively weakly dominated). We also investigate how to extend our procedure to accommodate the case where citizens are of more than two types. This we do in the second result of the paper, Theorem 2, where it is shown that these more general cases can be handled by repeating a variation of our two-stage procedure a certain number of times in order to match the total number of citizen types.

Two features of our analysis connect our contribution to other strands of the literature. First, in a context of local public goods with deadweight costs of redistribution, Gersbach et al. (2016) show that it is beneficial for society to always give the initiative in making proposals to the minority, leaving the majority the opportunity of counter-proposing and voting together with the minority on the two alternatives. Our main finding complements this result. In contrast to making the first proposal, our setting enables a minority of the citizenry to set the status quo that will be used in the final voting stage. Second, when citizens are of two types, the problem which decision to adopt can be seen as a bargaining problem between both sets of agents. There is an extensive body of literature on dynamic bargaining models where the outcome of one round is taken as a disagreement point in the next round—see e.g. Fershtman (1990), John and Raith (2001), or Diskin et al. (2011). Our paper adds to this strand of the literature by studying a democratic procedure that fulfills the consensus and inclusiveness requirements, where both

[5] In Section 3 we discuss the extent of player rationality that ensures the optimality of our procedure.

sets of agents jointly set the status quo (or disagreement point) for the second stage.

The paper is organized as follows: In Section 2 we set out the model and introduce our two-stage voting procedure. In Section 3 we prove the main result of the paper. In Section 4 we discuss how the procedure needs to be adapted if there are three or more types of citizen. Section 5 concludes. The proofs are given in the Appendix.

2 Model

2.1 Set-up

We consider a society with n individuals who decide about the level x of a public good. We let $n > 2$ and for ease of presentation assume that n is odd. Individuals are indexed by i or j, with $i, j \in \{1, \ldots, n\}$. Investment levels are denoted by x or y, with $x, y \in [0, \infty)$. The aggregate marginal cost of any unit of investment in the public good is $c > 0$. Costs are distributed equally among individuals. There are two types of individual, drawn from the type space $\mathcal{T} = \{t^L, t^H\}$, with $0 < t^L < t^H$. The type of individual i is denoted by t_i, with $t_i \in \mathcal{T}$. If an investment x is made, individual i derives utility from the public good that is equal to

$$v(x, t_i) = t_i \cdot f(x) - \frac{c}{n} \cdot x, \tag{1}$$

with $f(\cdot)$ being a real-valued, twice-differentiable function satisfying the Inada conditions, i.e., $f(0) = 0$, $f'(x) > 0$ and $f''(x) < 0$ for $x > 0$, $\lim_{x \to 0^+} f'(x) = +\infty$, and $\lim_{x \to \infty} f'(x) = 0$. Hence, individuals of type t^H benefit more from the implementation of any level of the public good than individuals of type t^L. An immediate consequence of Equation (1) is that preferences of individual i are single-peaked in x, with peak $x_i > 0$ defined by

$$f'(x_i) = \frac{c}{n \cdot t_i}.$$

The type of individual i is private information, and all citizen types are drawn from a joint distribution, with the property that the number of low-type (high-type) citizens has full support in $\{0, \ldots, n\}$. We do not specify the joint distribution since the properties of the procedure will not depend on it.[6]

Finally, we determine the level of investment that maximizes utilitarian welfare. The utilitarian

[6] The validity of Theorem 1 does not hinge on the assumption that the prior type distribution has full support. Nevertheless, the voting procedure we consider in Section 2.2 consists of several rounds, some of which may not be needed if the prior type distribution does not have full support. We discuss this issue further in Section 3.

level of investment is denoted by x^{soc} and can be computed from Equation (1) as follows:

$$f'(x^{soc}) = \frac{c}{\sum_{i=1}^{n} t_i}. \tag{2}$$

It will be opportune to introduce the notation $t^{soc} = \frac{1}{n}\sum_{i=1}^{n} t_i$. The value t^{soc} can be interpreted as the socially optimal (virtual) type.

2.2 A voting procedure

To decide about the public-good level x to be carried out, we develop a two-stage, multiple-round voting procedure. The object of the first stage is to set a status quo, denoted by $\bar{x}$, for the second stage, where the final outcome will be chosen. For all $j \in \{0, \ldots, n\}$, let y^j be the investment level defined by

$$f'(y^j) = \frac{c}{(n-j) \cdot t^L + j \cdot t^H}.$$

That is, y^j is the preferred level of investment for a society consisting of imaginary citizens of type $\frac{n-j}{n} \cdot t^L + \frac{j}{n} \cdot t^H$. The voting procedure will choose one outcome out of the following discrete set of alternatives:

$$\mathcal{Y} := \{y^0, \ldots, y^n\}.$$

The set $\mathcal{Y}$ consists of all the public-good investment levels that are utilitarian optimal for different combinations of individual types. Sometimes we will refer to the elements of $\mathcal{Y}$ as *feasible alternatives*. The (maximum) number of rounds in each of the two stages will directly depend on the cardinality of $\mathcal{Y}$ and hence indirectly on n. Citizens cannot abstain in any voting round.

We are now in a position to introduce the voting procedure, which, as already mentioned, will be based on so-called *semi-flexible majority rules*. The sequence of events is as follows:

Stage 1

Round 1.1: A vote is held between moving to the next round and setting $\bar{x} = y^0$. At least one vote is required to move to Round 1.2. Otherwise the procedure jumps to Stage 2 with $\bar{x} = y^0$.

$\vdots$

Round 1.$(n-1)$: A vote is held between moving to the next round and setting $\bar{x} = y^{n-2}$ and jumping to Stage 2. At least $n-1$ votes are required to move to Round 1.n.

Round 1.n: A vote is held between setting $\bar{x} = y^{n-1}$ and setting $\bar{x} = y^n$. Unanimity is required to set $\bar{x} = y^n$ and move to Stage 2 with such a status-quo policy. Otherwise Stage 2 starts with $\bar{x} = y^{n-1}$.

Stage 2 (Let $\bar{x} = y^k$, with $k \in \{0, \ldots, n\}$, be the outcome of Stage 1.)

Round 2.1: A vote is held between moving to the next round and choosing y^n as the final outcome. Unanimity is required to choose y^n, in which case the procedure ends.

$\vdots$

Round 2.$(n - r + 1)$ (with $r \geq k$): A vote is held between moving to the next round and choosing y^r as the final outcome. At least $\max\left\{r, \frac{n+1}{2}\right\}$ votes are required to choose y^r, in which case the procedure ends. Otherwise the procedure moves to Round 2.$(n - r + 2)$.[7]

Round 2.$(n - r + 1)$ (with $r < k$): A vote is held between moving to the next round and choosing y^r as the final outcome. Unanimity is required to choose y^r, in which case the procedure ends. Otherwise the procedure moves to Round 2.$(n - r + 2)$.

$\vdots$

Round 2.$(n + 2)$: If this round is reached, $\bar{x}$ is chosen as the final outcome.

Several remarks are in order. First, while we assume that voting is simultaneous in every round of the procedure, we do not make any assumption about the extent to which the results of each voting round are disclosed. As a matter of fact, for our results to hold, it will not be necessary for the voting outcome to always become public knowledge before the next round starts. It will suffice if all citizens understand that either they have moved to the next voting round (because the previous one was not successful) or that they have reached the end of the procedure.[8] Second, all public-good levels in $\mathcal{Y}$ can be the outcome of the suggested voting scheme for certain voter behaviors. Third, in Stage 1, increasing thresholds have to be met to set higher levels of the public good as the status quo for Stage 2. This conveys the idea that higher levels of the public good as the status quo for subsequent voting necessarily call for stronger support from the citizens. This property is illustrated by Figure 1.

[7] If we set $h = n + 1 - r$, the majority threshold considered in Round h, with $h \in \{1, \ldots, n + 1 - k\}$, is $f^k(h) := \max\{n + 1 - h, (n + 1)/2\}$. It turns out that for Theorem 1 to hold—see Section 3—, it will suffice to consider that $\{f^k(\cdot)\}_{k=0}^n$ is a collection of non-increasing, onto functions $f^k : \{1, \ldots, n + 1 - k\} \to \{\max\{(n + 1)/2, k\}, \ldots, n\}$ such that $f^{k+1}(\cdot) \leq f^k(\cdot)$. This is discussed in the Appendix.

[8] Gershkov et al. (2016) share with our model the property that the results hold independently of the disclosure policy about the results of successive voting rounds.

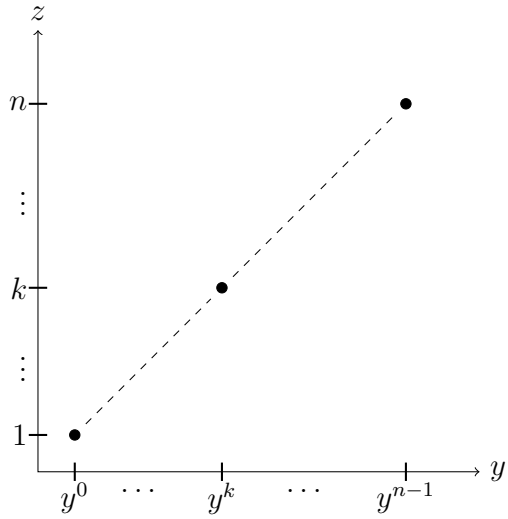
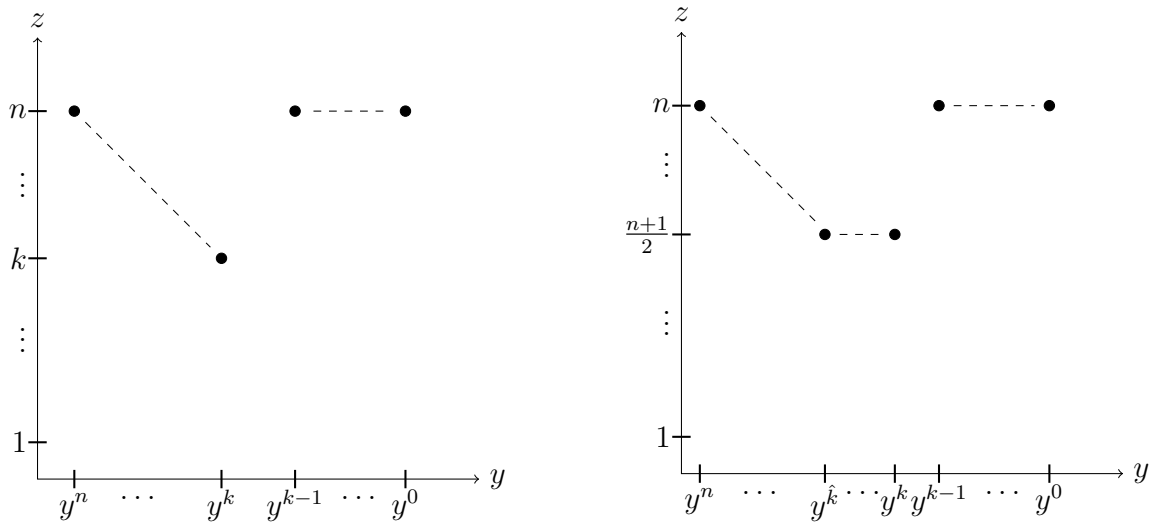


Figure 1: Stage 1—Number of votes (z) required to move from one project level to the next one. The procedure starts with y^0.

Fourth, in every voting round of Stage 2, a particular qualified majority of votes is compulsory to adopt any alternative from the set $\mathcal{Y}$ as the final decision. The required threshold is minimal in the case of the status quo set out in Stage 1 (viz., $\bar{x} = y^k$ for some $k \in \{0, \ldots, n\}$), and becomes higher as the level of the public good deviates from $\bar{x}$. For levels that are lower than the status quo, in particular, unanimity is required. The specific evolution of the vote thresholds along the different rounds is shown in Figure 2.



(a) Case $k \geq \frac{n+1}{2}$.

(b) Case $k < \frac{n+1}{2}$ (with $\hat{k} = \frac{n+1}{2}$).

Figure 2: Stage 2—Number of votes (z) required to approve a policy, with $\bar{x} = y^k$ the status quo set out in Stage 1. The procedure starts with y^n.

Fifth and last, only if no majority threshold is met in all the voting rounds of Stage 2 will the status quo chosen in Stage 1 be adopted by default.

3 Equilibrium Results

In this section we present and discuss the main result of the paper. We apply the concept of perfect Bayesian equilibrium and assume that citizens eliminate weakly dominated strategies iteratively by moving backwards in the two-stage voting procedure. This fact is considered to be common knowledge.[9] We obtain

Theorem 1

The outcome of the two-stage voting procedure is x^{soc}.

Several comments are helpful in fully grasping the relevance and implications of the theorem.[10] First, as already mentioned, the procedure we have suggested requires qualified majorities in Stage 2 for changes to the outcome of Stage 1 and unanimity, in particular, for public levels lower than the one specified in the status quo chosen therein. For a range of rounds, however, a whole family of qualified majority thresholds ensures utilitarian optimality, which includes the thresholds set out in Section 2. This is shown in the Appendix—see also Footnote 7. More specifically, it will suffice for the majority thresholds of Round 2.1 to Round 2.$(n+1-k)$ (where the proposal on the table is the status quo chosen in Stage 1) to be non-increasing, ranging from unanimity to a certain qualified majority (never lower than half plus one of the votes). This guarantees that high-type citizens will not be able to impose a public-good level that is higher than the socially optimal one. By contrast, the unanimity rule required in any voting round after Round 2.$(n+1-k)$ grants any individual the veto power to impose the status quo as the final outcome, which is essential for optimality in the general case.

The voting thresholds of Stage 2 can nonetheless be lowered in specific cases where the possible number of high- or low-type individuals is further bounded, and in particular when individual types are not drawn independently from each other. For instance, suppose the number of high-type individuals has support $\{0, \underline{n}, \underline{n}+1, \ldots, n\}$ for some $\underline{n}$ with $1 < \underline{n} < n$. Then the unanimity thresholds that apply to public-good levels lower than the status quo can be lowered to $\max\{n-\underline{n}+1, \frac{n+1}{2}\}$. The reason is that situations where only a few high-type individuals are

[9] Within any individual voting round we look for a sequence of elimination of weakly dominated strategies. It turns out that any such sequence will yield the same outcome.

[10] We note that while there is not a unique equilibrium, beliefs along the equilibrium path are the same for all perfect Bayesian equilibria (even under the most strict disclosure policy regarding voting outcomes).

present cannot occur. In these circumstances, each high-type individual will enjoy a *de facto* veto power enabling him to block the approval of any alternative that requires more than the support of $n - \underline{n}$ citizens of the low type. Knowing this, low-type citizens will also always vote to proceed to the next round, since their vote will only make a difference where there are no citizens of the high type at all.

Another example is a situation where the number of low-type individuals has support $\{\overline{n} - 1, \overline{n}, \ldots, n\}$ for some $\overline{n}$ with $1 < \overline{n} < n$. In this case, the second-stage thresholds for the approval of public-good levels that are higher than the one prescribed by the outcome of Stage 1 can be lowered to $\max\{n - \overline{n} + 1, \frac{n+1}{2}\}$. The reason is that there will never be $n - \overline{n} + 1$ or more individuals of the high type, and voting in favor of any alternative that proposes a public level y^r, with $r \geq n - \overline{n}$, will be weakly dominated for low-type individuals by voting to proceed to the next round. When either $\underline{n} = 1$ or $\overline{n} = 1$, the same logic explains the main mechanisms behind Theorem 1.

Second, to achieve utilitarian optimality, a number of proposals equal to the number of voters plus one would have to be considered in general. This is not always necessary. On the one hand, if the joint distribution of types has rather small support—e.g. because the preferences of different individuals are highly correlated—all those rounds can be skipped that cannot qualify for a potential socially optimal alternative. On the other hand, an approximate socially optimal solution may suffice when the number of citizens is considerable. If n is large and the citizens' types are i.i.d., in particular, the Central Limit Theorem guarantees that the socially optimal type will be distributed normally, with a mean μ and a variance σ that could be estimated. Then one could add a criterion to our procedure in order to exclude the tails of the distribution and hence the most extreme policies. For instance, for a given $\alpha \geq 0$, the set of feasible policies

$$\left\{ y : f'(y) = \frac{c}{n \cdot t}, t = \frac{n-j}{n} \cdot t^L + \frac{j}{n} \cdot t^H, \mu - \alpha \cdot \sigma \leq \frac{n-j}{n} \cdot t^L + \frac{j}{n} \cdot t^H \leq \mu + \alpha \cdot \sigma \right\}$$

could be considered. The procedure would then be run over them instead of the entire set $\mathcal{Y}$. Parameter α determines the loss of efficiency that such a procedure would induce. The larger α, the lower the loss.

Third, the procedure we have analyzed in Theorem 1 is by construction dependent on the citizens' utility function.[11] However, if types are constant (though private) but only the function f changes from problem to problem, the procedure can be adapted to hinge on types rather than on policies, thereby expanding its applicability to a wider range of problems. This follows

[11] This feature is common to Bowen (1943), Gershkov et al. (2016), and Gersbach (2017). In our setting, in particular, it follows from the dependence of the elements of $\mathcal{Y}$ on f.

from the trivial fact that a one-to-one correspondence exists between the set of potential socially optimal types and the set of feasible alternatives (see also the discussion in the next section). Of course, for such a procedure to be implementable, it should be possible, i.e. legal, to base a democratic procedure for public-good provision on types such as income, for example, rather than on the policies themselves.

Fourth, our notion of equilibrium is demanding in terms of players' rationality, since it requires it be common knowledge that all citizens iteratively eliminate weakly dominated strategies. However, there is only one particular range of voting rounds where this elimination process is crucial. Let us consider the voting rounds of Stage 2, where the proposal on the table specifies a public-good level lower than, or equal to, the status quo set in Stage 1. The unanimity requirement grants any high-type citizen the veto power to impose the adoption of the status quo, a power which he will use by voting to proceed to the next round independently of other individuals. By contrast, for low-type individuals is essential that *(i)* they do not take a voting decision in any round that is weakly dominated, and that *(ii)* they are able to eliminate iteratively by moving backwards in the procedure any weakly dominated voting decision *of their own*. Common knowledge about these two features guarantee that, in all voting rounds of Stage 2, low-type citizens will also vote to proceed to the next round (if there is one). The reason is that their decision will only be relevant when all citizens are low-type.

Finally, it is important to note that the same equilibria could also be obtained by using cut-off strategies in Stage 2, according to which an individual can only change his vote (*proceed* or *stop*) once at most along the different voting rounds of the stage. Iterative elimination of weakly dominated strategies can accordingly be seen as a foundation for such cut-off (behavioral) rules.[12]

4 Multiple Types

In our analysis thus far, we have assumed that individuals are of two types, low and high. Theorem 1 then demonstrates that the suggested two-stage, multiple-round procedure is able to elicit the information about how many citizens are of either type. This is possible because, as discussed in the previous section, there is a one-to-one correspondence between the number

[12]Monotone (or cut-off) strategies have been used by Gershkov et al. (2016), which have also been justified by an iterative process of elimination of weakly dominated strategies. The subtleties, however, are different here. One reason is that the last step of the procedure considered by Gershkov et al. (2016) consists of a vote between the last two alternatives, while our procedure consists of a vote between the status quo of Stage 1, $\bar{x} = y^k$, and the ex-ante status quo, y^0.

of individuals of the high (low) type and the alternative eventually chosen from the set $\mathcal{Y}$. One ensuing question is whether a democratic procedure also exists for the case where citizens can be of more than two types and is able to elicit information about how many individuals of each type there are. If this information can be elicited, the procedure can implement the utilitarian optimal public-good level. This will be addressed next.

Suppose now that there are $T > 2$ possible types and that the type of individual i is denoted by t_i, where $t_i \in \mathcal{T} = \{t^1, \ldots, t^T\}$ and $0 < t^1 < \ldots < t^T$. The utilitarian optimum outcome is still given by Equation (2). This means that any procedure intending to implement such an outcome should account for a set of potential (or feasible) alternatives that contains the set $\mathcal{Y}$, where now

$$\mathcal{Y} := \left\{ y : f'(y) = \frac{c}{t}, t = \sum_{v=1}^{T} n_v \cdot t_v, n_1, \ldots, n_T \geq 0, \sum_{v=1}^{T} n_v = n \right\}.$$

We next introduce the set

$$\mathcal{I} = \left\{ (n_1, \ldots, n_T) : n_1, \ldots, n_T \geq 0, \sum_{v=1}^{T} n_v = n \right\}.$$

Trivially, there exists a one-to-one correspondence between sets $\mathcal{Y}$ and $\mathcal{I}$. Hence, for any particular problem we can imagine a decision about set $\mathcal{Y}$ as a decision about set $\mathcal{I}$. For all $v \in \{1, \ldots, T-1\}$ and $j \in \{0, \ldots, n\}$, it will be useful to define the level of public good y_v^j by

$$f'(y_v^j) = \frac{c}{(n-j) \cdot t^v + j \cdot t^{v+1}},$$

as well as the set

$$\mathcal{Y}_v = \{y_v^0, \ldots, y_v^n\}.$$

We note that y_v^j is the optimal public good level when $n - j$ individuals are of type t^v and j individuals are of type t^{v+1}. By construction,

$$\bigcup_{v=1}^{T} \mathcal{Y}_v \subsetneq \mathcal{Y}.$$

Next, consider the following procedure, which we call the *T-compound procedure*:

Step 1: Apply our two-stage procedure to set $\mathcal{Y}_1$. The outcome of this step is denoted by $y_1^{n-N^1}$, with $N^1 \in \{0, \ldots, n\}$.

$\vdots$

Step $T-1$: Apply our two-stage procedure to set $\mathcal{Y}_{T-1}$. The outcome of this step is denoted by $y_{T-1}^{n-N^{T-1}}$, with $N^{T-1} \in \{0, \ldots, n\}$.

Step T: The final outcome is the allocation $y \in \mathcal{Y}$ such that

$$f'(y) = \frac{c}{\sum_{v=1}^{T} n_v \cdot t_v},$$

where $N^T = n$ and

$$\begin{cases} n_1 & = N^1 \\ \dots & \\ n_1 + \ldots + n_{T-1} & = N^{T-1} \\ n_1 + \ldots + n_{T-1} + n_T & = N^T. \end{cases} \tag{3}$$

Note that for $v \in \{1, \ldots, T-1\}$, the decision about N^v is taken according to the same procedure as for two types only, the case examined in the previous section. Hence it is subject to the same democratic concerns.[13] Moreover, it can be easily verified that the system of linear equalities in (3) has a unique solution, which is denoted by $(n_1^*, \ldots, n_T^*)$ and given by

$$n_v^* = N^v - N^{v-1} \quad (v \in \{1, \ldots, T-1\}),$$

with $N^0 = 0$.

We next examine the preference relation of citizen i of type $t_i = t^j \in \mathcal{T}$ with regard to any pair of elements $(n_1, \ldots, n_T), (n_1', \ldots, n_T') \in \mathcal{I} \cong \mathcal{Y}$, which we denote by $\succ_i$. Although these preferences are not fully specified, for $v, v' \in \{1, \ldots, T\}$ such that $n_v = n_{v'}' > n_{v'} = n_v'$ and $n_{v''} = n_{v''}'$ with $v'' \neq v, v'$, we must have

$$[j \geq v > v' \text{ or } v' < v \leq j] \Rightarrow (n_1, \ldots, n_T) \succ_i (n_1', \ldots, n_T'). \tag{4}$$

That is, *ceteris paribus*, individual i will prefer the number of individuals of type t^j that the procedure takes as input to be higher for types t^v close to t_i than it is for those types that are farther away, the measure of distance being $|t_i - t^v|$.

Consider now Step 1 of the T-compound procedure: Clearly, citizens of type t^1 and t^2 will behave exactly as they do in the case with two types examined in Section 3. The reason is that according to the preference property given by (4), the utility of citizens of type t^1 is increasing in N^1 and decreasing in N^2, with these statements reversed for individuals of type t^2. But, what about individuals of type t^3 and higher? Again, because of the preference property given by (4) they will all behave in Step 1 as if they were of type t^2. The same logic can now be applied to all steps, obtaining the following result:

[13] In the compound procedure, inclusiveness has to be considered with the following caveat: While the set of policies that can arise as outcome is the entire set $\mathcal{Y}$, not all the elements of this set will be considered along the different rounds of the procedure for a given realization of preferences. This contrasts with the case $T = 2$ analyzed in Section 3.

Theorem 2

The outcome of the T-compound voting procedure is x^{soc}.

Finally, it is instructive to explore the outcomes that could be achieved with limited use of the T-compound voting procedure. In particular, using the logic behind the above theorem, the following can be easily deduced: Suppose the two-stage procedure of Section 2 is applied to the multiple-type case by only considering, say, the set of alternatives $\mathcal{Y}_j$, with $j \in \{1, \ldots, T-1\}$. Then the final outcome would maximize the utility of an imaginary citizen of type $m_j \cdot t^j + (n - m_j) \cdot t^{j+1}$, where m_j is the number of individuals with types in the set $\{t^1, \ldots, t^{j-1}, t^j\}$.[14] This means that the compound procedure can be used to produce outcomes in which some types receive a particular weight in the collective decision—and, more particularly, only some of those types matter. For instance, this could occur when a group of citizens whose type belongs to a certain class (of types) form parties or factions and a representative individual, say the median member of the coalition, casts the votes for this group.

5 Conclusions

We have presented a new procedure implementing the utilitarian optimum in a standard problem of public-good provision. Unlike other procedures described in the literature that use multiple-round voting with varying thresholds, we have imposed the property that such thresholds require more than half of the votes for the policy finally approved. This is a plausible restriction that reflects the majoritarian logic of democracy. Our procedure also displays the property that, in equilibrium, all potential socially optimal proposals will be considered at some point in time. Both properties, which we have called consensus and inclusiveness respectively, may be desirable in the real world. Voting schemes based on our procedure could thus be introduced on an experimental basis in democracy. Like other voting procedures based on varying thresholds, however, the procedure that we have suggested is not universally applicable (unlike the majority rule) and has to be tailored to particular collective-choice decisions.

A variety of extensions of our model and further applications of our procedure could be considered for future research. For example, we could consider collective decisions in which citizens are ambiguous about the underlying distribution of types and then develop robust semi-flexible majority rules. Then again, the suggested scheme could be used in committee decisions about

[14] Suppose, by contrast, that the procedure of Section 2 were run on the (erroneous) assumption that all citizens are of types t^1 and t^T. In this case, besides the exact number of citizens of each actual type, the outcome would also depend on the values between t^1 and t^T.

any issue that can be translated into a numerical scale. In particular, the voting scheme could also be applied to the selection of a candidate from a pool of aspirants, provided that the differences among all candidates can be ordered along one dimension such as, say, the candidates' degree of conservatism in a procedure for choosing the head of a central bank.

Finally, our model and results are also relevant from a purely positive perspective. It has been stressed that reference points may have a crucial impact on the outcome of voting procedures. We have shown in particular that the utilitarian optimal outcome can be attained when the reference point (i.e., the status quo) together with the majority requirements for changing this reference point are chosen appropriately.

References

Arrow, K. J. (1950). A difficulty in the concept of social welfare. *Journal of Political Economy*, 58(4):328–346.

Aşan, G. and Sanver, M. R. (2002). Another characterization of the majority rule. *Economics Letters*, 75(3):409–413.

Barberà, S., Gul, F., and Stacchetti, E. (1993). Generalized median voter schemes and committees. *Journal of Economic Theory*, 61(2):262–289.

Black, D. (1948). On the rationale of group decision-making. *Journal of Political Economy*, 56(1):23–34.

Bowen, H. R. (1943). The interpretation of voting in the allocation of economic resources. *Quarterly Journal of Economics*, 58(1):27–48.

Casella, A. (2005). Storable votes. *Games and Economic Behavior*, 51(2):391–419.

Chatterji, S. and Sen, A. (2011). Tops-only domains. *Economic Theory*, 46(2):255–282.

Ching, S. (1997). Strategy-proofness and median voters. *International Journal of Game Theory*, 26(4):473–490.

Diskin, A., Koppel, M., and Samet, D. (2011). Generalized Raiffa solutions. *Games and Economic Behavior*, 73(2):452–458.

Fahrenberger, T. and Gersbach, H. (2010). Minority voting and long-term decisions. *Games and Economic Behavior*, 69(2):329–345.

Fershtman, C. (1990). The importance of the agenda in bargaining. *Games and Economic Behavior*, 2(3):224–238.

Gersbach, H. (2017). Flexible Majority Rules in Democracyville: A Guided Tour. *Mathematical Social Sciences*, 85:37–43.

Gersbach, H., Imhof, S., and Tejada, O. (2016). First proposal to minorities or how to channel the say in politics. *Mimeo.*

Gershkov, A., Moldovanu, B., and Shi, X. (2016). Optimal voting rules. *Review of Economic Studies, forthcoming.*

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica*, 41(4):587–601.

Hortalà-Vallvé, R. (2010). Qualitative voting. *Cuadernos de economía*, 33(92):5–43.

John, R. and Raith, M. G. (2001). Optimizing multi-stage negotiations. *Journal of Economic Behavior & Organization*, 45(2):155–173.

Maskin, E., Basu, K., Pattanaik, P., and Suzumura, K. (1995). Majority rule, social welfare functions, and games forms. *Choice, Welfare, and Development (essays in honor of Amartya Sen).*

May, K. O. (1952). A set of independent necessary and sufficient conditions for simple majority decision. *Econometrica*, 20(4):680–684.

McKelvey, R. D. (1976). Intransitivities in multidimensional voting models and some implications for agenda control. *Journal of Economic Theory*, 12(3):472–482.

Miroiu, A. (2004). Characterizing majority rule: From profiles to societies. *Economics Letters*, 85(3):359–363.

Moulin, H. (2014). *The Strategy of Social Choice*, volume 18. North Holland, Amsterdam.

Mullins, D. R. and Cox, K. A. (1995). *Tax and expenditure limits on local governments*, volume 19. Center for Urban Policy and the Environment, Indiana University, Indiana, US.

Plott, C. R. (1967). A notion of equilibrium and its possibility under majority rule. *American Economic Review*, 57(4):787–806.

Rasch, B. E. (2000). Parliamentary floor voting procedures and agenda setting in Europe. *Legislative Studies Quarterly*, 25(1):3–23.

Rosar, F. (2015). Continuous decisions by a committee: median versus average mechanisms. *Journal of Economic Theory*, 159:15–65.

Satterthwaite, M. A. (1975). Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217.

Sprumont, Y. (1991). The division problem with single-peaked preferences: A characterization of the uniform allocation rule. *Econometrica*, 59(2):509–519.

Woeginger, G. J. (2003). A new characterization of the majority rule. *Economics Letters*, 81(1):89–94.

Appendix

As discussed in Section 2—see Footnote 7—, the result of Theorem 1 holds not only for the procedure described in Section 2, but also for a broader family of procedures containing the former as an instance. Each procedure of this family is contingent on the choice of certain non-increasing sequences of majority thresholds to be applied to the first rounds of Stage 2, more specifically, to the rounds involving policies y^n, y^{n-1},..., and $y^k = \bar{x}$ (the status quo chosen in Stage 1). Formally, for each $k \in \{0, \ldots, n\}$ let

$$A^k := \{1, 2, \ldots, n+1-k\}$$

and

$$B^k := \left\{ \max\left\{\frac{n+1}{2}, k\right\}, \max\left\{\frac{n+1}{2}, k\right\} + 1, \ldots, n \right\}.$$

While we will interpret A^k as round indices, the elements of B^k will be majority thresholds—see below. Note in particular that $B^k = \{k, \ldots, n\}$ if $k \geq \frac{n+1}{2}$. Then for each $k \in \{0, \ldots, n\}$, with $\bar{x} = y^k$ being the status quo chosen in Stage 1, let

$$\begin{aligned} f^k : A^k &\longrightarrow B^k \\ h &\longrightarrow f^k(h). \end{aligned}$$

The only difference with regard to the description of the two-stage voting procedure defined in Section 2 is that now, for Round 2.h, with $h = 1, \ldots, n+1-k$, the majority threshold required to adopt the proposal on the table will be $f^k(h)$. Hence, each choice of $\left\{f^k(\cdot)\right\}_{k=1}^{n}$ defines a (democratic) procedure. We assume that $f^k(\cdot)$ is non-increasing and onto.[15] It follows, in particular, that $f^k(1) = n$ and

$$f^k(n+1-k) = \max\left\{\frac{n+1}{2}, k\right\}. \tag{5}$$

Moreover, we assume that

$$f^k(h) \geq f^{k+1}(h), \text{ for all } h \in \{1, \ldots, n+1-k\}. \tag{6}$$

For each $k \in \{0, \ldots, n\}$, we let $\mathcal{F}^k$ be the set of all such functions. It follows directly that if $k \geq \frac{n+1}{2}$, there is only one possible element of $\mathcal{F}^k$, namely

$$f^k(h) = n+1-h.$$

Finally, let

$$\underline{f}^k(h) := \max\left\{n+1-h, \frac{n+1}{2}\right\},$$

and

$$\overline{f}^k(h) = \min\left\{n, \frac{3(n+1)}{2} - k - h\right\}.$$

It is easy to verify that both functions belong to $\mathcal{F}^k$. Note, in particular, that $\underline{f}^k(\cdot)$ is the threshold function considered in the main body of the paper. It is also straightforward to check that for each $f^k(\cdot) \in \mathcal{F}^k$ and $h \in A^k$,

$$\underline{f}^k(h) \leq f^k(h) \leq \overline{f}^k(h). \tag{7}$$

The shape of the possible functions $f^k(\cdot) \in \mathcal{F}^k$ is illustrated in Figure 3 for the case where $k < \frac{n+1}{2}$. There it is shown that $\mathcal{F}^k$ is the convex hull of $\underline{f}^k(\cdot)$ and $\overline{f}^k(\cdot)$, which are the supremum and the infimum of the set of functions, respectively.

For each status-quo proposal $y = x^k$ chosen in Stage 1, $\mathcal{F}^k$ has been defined as the set of possible majority thresholds for voting rounds in Stage 2 where the proposal on the table specifies a public-good level higher than, or equal to, the one specified by the status quo chosen in Stage 1. This set is reminiscent of the class of decreasing thresholds considered by Gershkov et al. (2016). Indeed, when $\bar{x}$ coincides with the ex-ante status-quo, namely y^0, our class of thresholds coincides with the subclass of thresholds considered by Gershkov et al. (2016)

[15] A mapping $f : A \to B$ is *onto* if for all $y \in B$ there is $x \in A$ such that $y = f(x)$.

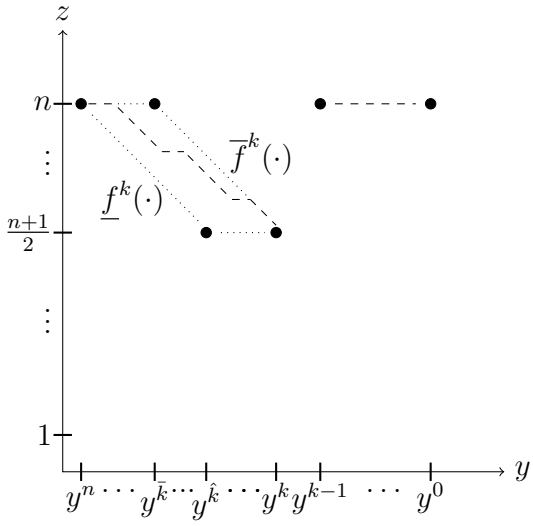


Figure 3: Generalized Stage 2 of the two-stage procedure—Number of votes (z) required to approve a policy, with $\bar{x} = y^k$ the status quo set out in Stage 1. Case $k < \frac{n+1}{2}$ (with $\hat{k} = \frac{n+1}{2}$ and $\bar{k} = \frac{n+1}{2} + k$). The procedure starts with y^n. Functions $\underline{f}^k(\cdot)$ and $\overline{f}^k(\cdot)$ are depicted by dotted lines.

satisfying the additional property that no threshold can be below half plus one of the number of citizens. Such a class of thresholds can be rationalized from a mechanism-design viewpoint when the majority requirement is imposed beyond standard properties such as anonymity, unanimity, and incentive compatibility.

In the remaining part of this appendix we solve the game induced by the procedure described above, and, in particular, the procedure considered in the main body of the paper. We analyze it backwards, taking into account whenever necessary the beliefs about the presence of high- and low-type citizens. We thus start with the analysis of Stage 2.

Proof of Theorem 1

Analysis of Stage 2

Let $\bar{x} = y^k$ be the outcome of Stage 1, with $k \in \{0, \ldots, n\}$. Next we consider Round 2.h of Stage 2, with $h = n+1, \ldots, 1$. We distinguish three cases.

Case I: $h = n+1$

The decision in Round 2.$(n+1)$ consists in choosing either y^0 or $\bar{x}$ as the final outcome. If $k = 0$, there is no real choice, and $\bar{x} = y^0$ will be adopted regardless of the citizens' vote. Hence, let $k > 0$ and thus $y^0 < \bar{x}$. In this case, unanimity is required for approval of y^0. Then it is in

any citizen's best interest to vote *truthfully* for the alternative that yields higher utility, either y^0 or $\bar{x}$, because doing so will make a positive difference when the vote is pivotal (i.e., when all other citizens vote for $\bar{x}$) and will make no difference otherwise. In other words, voting for the proposal on the table is weakly dominated for high-type citizens by voting to proceed to Round $2.(n+2)$, while the weak-domination relation is reversed for low-type citizens. Hence, if citizens play no strategies that are weakly dominated, low-type citizens will vote for y^0 and high-type citizens will vote to proceed to the last round, where $\bar{x}$ is automatically adopted.

Case II: $n-k+1 < h < n+1$

The decision in Round $2.h$, with $n-k+1 < h < n+1$, consists in choosing between y^{n-h+1} as the final outcome and proceeding to Round $2.(h+1)$. Unanimity is required for y^{n-h+1} to be adopted. This implies that any individual has the (veto) power to ensure that the procedure continues to the next round and ultimately to Round $2.(n+2)$, a round in which $\bar{x}$ will be adopted—if such a citizen votes in favor of this possibility in all these rounds.

Let us first consider an individual of type t^H. Such an individual will vote to proceed to the next round until Round $2.(n+2)$ is eventually reached. The reason is that he prefers $\bar{x} = y^k$ over y^{n-h+1} (recall that $n-k+1 < h$) and he anticipates that weakly undominated strategies (in particular, his own strategies) will be eliminated in future rounds. Indeed, if at least one other citizen votes to proceed to the next round, his vote will make no difference. However, if all other citizens vote in favor of the proposal on the table, his vote will be pivotal, in which case he benefits from moving to the next voting round. Since this property holds for all rounds until Round $2.(n+1)$ is reached—due to the unanimity rule necessary to adopt the proposal on the table—, voting to proceed to the next round weakly dominates, for any high-type citizen, voting in favor of the proposal on the table in all rounds $h = n-k+2, \ldots, n$.[16] This will result in adoption of $\bar{x}$ regardless of the vote of the remaining citizens whenever there is at least one individual of the high type.

We now consider an individual of type t^L. In Round $2.h$, he faces a subtle choice. If he votes in favor of the alternative on the table, y^{n-h+1}, his vote may theoretically help to guarantee that such a policy will be chosen. If, by contrast, he votes for the procedure to continue to the next round, the risk he takes is that the status quo $\bar{x}$ will be eventually chosen if all alternatives $y^{n-h}, \ldots, y^0$ are subsequently rejected. Because he prefers y^{n-h+1} to $\bar{x} = y^k$ (recall again that $n-k+1 < h$), the choice is not immediately obvious. Nevertheless, we next show that for an

[16]Note that high-type individuals do not need to assume anything in particular about the behavior of other citizens.

individual of type t^L, it will be a best response to always vote to proceed to the next voting round, provided that citizens play no weakly dominated strategies in subsequent rounds and that this is common knowledge.

We distinguish two cases. First, suppose a low-type individual believes that at least one high-type individual is present. Then this low-type individual expects that no unanimity will be reached in all subsequent rounds $h+1, \cdots, n$, which, in turn, would implement a public-good level different from the status quo $\bar{x}$. In particular, he expects that his vote will not change anything and that $\bar{x}$ will be eventually adopted. Now suppose the low-type individual, say citizen i, believes that no other citizen is of the high type. In this case, individual i expects that his vote to proceed to the next round will eventually lead to Stage $n+1$ whenever such a vote is pivotal. In Stage $n+1$, citizen i believes that all citizens will vote for y^0, which is i's preferred alternative. Thus, regardless of the beliefs that he holds at the node representing Round $2.h$, a low-type citizen will vote to proceed to the next round.

Case III: $1 \leq h \leq n-k+1$

Again, the decision in Round $2.h$, now with $1 \leq h \leq n-k+1$, consists in choosing y^{n-h+1} as the final outcome or proceeding to the next stage (note that $r = n+1-h$ is the index used in Section 2 to describe policy y^r). This time, however, a majority of $f^k(h)$ of votes is required for y^{n-h+1} to be adopted.[17] If such a majority does not materialize, the procedure will continue, yielding some outcome $y \in \mathcal{Y}$ satisfying the property that $y \leq y^{n-h+1}$, with $y^{n-h+1} \geq \bar{x} = y^k$ (recall that $n-k+1 \geq h$). This means that if the procedure does not stop at Round $2.h$ the outcome will leave high-type individuals worse off and low-type individuals better off (if $h < n-k+1$), or will leave high-type individuals *weakly* worse off and low-type individuals *weakly* better off (if $h = n-k+1$). We distinguish two cases. First, assume that $h < n-k+1$. Then, due to the fact that a vote can be pivotal for certain voting decision profiles, all citizens will vote truthfully: high-type individuals will vote in favor of y^h and low-type individuals will vote to proceed to the next round. The reason is that, for any high-type citizen, voting in favor of the proposal on the table weakly dominates voting to proceed to the next round. Similarly, for any low-type citizen, voting to proceed to the next round weakly dominates voting in favor of the proposal on the table. Note that these two properties hold independently of the majority required in the voting round, viz. $f^k(h)$.

Next, let us examine the case $h = n-k+1$. Consider first a citizen of type t^H. If he votes for the proposal on the table, i.e. for y^k, he may contribute to the adoption of such a proposal

[17] We stress that in the main body of the paper we have considered $f^k(h) = \max\left\{\frac{n+1}{2}, n+1-h\right\}$.

if his vote is pivotal. If, on the other hand, he votes for the procedure to continue to the next round, he knows that he will be able to ensure that $\bar{x} = y^k$ will also be adopted in subsequent voting rounds thanks to his veto power. Hence, any high-type citizen will be indifferent between either possibility—and we can assume arbitrary voting behavior in this round. Consider now a citizen of type t^L. Voting to proceed to the next round will produce an outcome $y \leq y^k = \bar{x}$, which in the case where $y < y^k$ (an outcome that is attainable for certain voting behavior) is strictly preferred over y^k. That is, for a low-type citizen, voting to proceed to the next round weakly dominates voting for the proposal on the table, y^k. Again, this holds independently of the majority required in the voting round, viz. $f^k(n+1-k) = \max\{\frac{n+1}{2}, k\}$. Note that in Stage 2, low-type citizens will accordingly *always* vote to proceed to the next voting round, with the exception of the last voting round, namely Round $2.(n+1)$, where they will vote for the proposal on the table.

Having established the citizens' behavior in all voting rounds of Stage 2, we now let k (with $\bar{x} = y^k$) vary from 0 to n and obtain the outcome of Stage 2 for different distributions of citizen types. We do so by building on the insights provided in Cases I–III above. Recall that the socially optimal level of public good is denoted by $t^{soc} = \frac{n-j}{n} \cdot t^L + \frac{j}{n} \cdot t^H$, with $j \in \{0, \ldots, n\}$. That is, t^{soc} denotes the utilitarian solution when the society is made up of j citizens of type t^H and $n-j$ citizens of type t^L.

First, if $j = 0$, all citizens (who are of the low type) will always vote to proceed to the next voting round, until Round $2.(n+1)$ is eventually reached, where all of them will vote in favor of the proposal on the table, y^0. Second, if $0 < j \leq \frac{n-1}{2}$, all low-type citizens, who constitute a majority of the electorate, will block the approval of any proposal of Rounds 2.1 to $2.(n+1-k)$. The reason is that a (qualified) majority is needed in all these rounds for the approval of the proposal on the table. However, since there is at least one citizen of the high type, any such individual will be able to guarantee that the status quo $\bar{x}$ will eventually be chosen. Third, if $\frac{n+1}{2} \leq j \leq n$, a majority of the electorate is made up of citizens of the high type. If $j = n$, in particular, all citizens, who are of type t^H, will vote for y^n, and the procedure will end just after Round 2.1.

Accordingly, we are left with the constellation

$$\frac{n+1}{2} \leq j < n. \tag{8}$$

Note that because there is always one citizen who is of type t^H, the procedure will yield some proposal y with the property that $y \geq y^k = \bar{x}$. We distinguish two cases, depending on the status quo chosen in Stage 1, viz. $\bar{x}$.

Case A: $\frac{n+1}{2} \leq k \leq n$

In this case, for $h = 1, \ldots, n+1-k$, we have $f^k(h) = n+1-h$ as the majority threshold required for the approval of the proposal on the table of Round $2.h$, namely y^{n+1-h}. The reason is that for $k \geq (n+1)/2$, the only non-increasing, onto function belonging to $\mathcal{F}^k$ is $f^k(h) = n+1-h$. Because in each of these rounds all individuals of the high type will vote in favor of the proposal on the table and all individuals of the low type will vote to proceed to the next voting round, the outcome of the procedure will be $\max\{y^j, \bar{x}\}$, where $\bar{x} = y^k$ is the status quo and j is the number of citizens of type t^H. In particular, if $\bar{x} = y^j$, the outcome will be the status quo proposal chosen in Stage 1.

Case B: $0 \leq k \leq \frac{n-1}{2}$

For $h = 1, \ldots, n+1-k$, we again have $f^k(h)$ as the majority threshold required for the approval of the proposal on the table of Round $2.h$, namely y^{n+1-h}. As in Case A, in each of these rounds all individuals of the high type will vote in favor of the proposal on the table and all individuals of the low type will vote to proceed to the next voting round. Accordingly, the final outcome of the procedure will be $\max\{y^{r^*(k,j)}, \bar{x}\}$, where $\bar{x} = y^k$ is the status quo, j is the number of citizens of type t^H, and[18]

$$r^*(k,j) = \max\{r : f^k(n+1-r) \leq j\},$$

That is, $y^{r^*(k,j)}$ is the highest level of the public good that the j citizens of the high type can guarantee as an outcome throughout Rounds 2.1 to $2.(n+1-k)$, given $\bar{x} = y^k$, which is the status quo chosen in Stage 1. Using Equations (5) and (8), we obtain

$$j \geq \frac{n+1}{2} = \max\left\{\frac{n+1}{2}, k\right\} = f^k(n+1-k).$$

It then follows that $r^*(k,j)$ is well-defined and that $r^*(k,j) \geq k$. What is more, $r^*(k,l)$ is non-decreasing in k, due to Equation (6), and non-decreasing in j, by construction. We further note that also due to Equations (5) and (8), we have

$$f^{(n-1)/2}\left(n+1-\frac{n-1}{2}\right) = \max\left\{\frac{n+1}{2}, \frac{n-1}{2}\right\} = \frac{n+1}{2} \leq j.$$

This implies

$$r^* := r^*\left((n-1)/2, j\right) \geq \frac{n-1}{2}. \tag{9}$$

Trivially, Equation (8) also implies that

$$j \geq \frac{n-1}{2}. \tag{10}$$

[18]For notational convenience, we have suppressed the dependence of $r^*(k,j)$ on the function $f^k(\cdot)$.

We next claim that

$$r^* \leq j. \tag{11}$$

Since $f^{(n-1)/2}(\cdot)$ is non-increasing and since Equations (9) and (10) guarantee that $n+1-r^*$ and $n+1-j$ belong to the domain of function $f^{(n-1)/2}(\cdot)$, Equation (11) can be rewritten as

$$f^{(n-1)/2}(n+1-j) \geq f^{(n-1)/2}(n+1-r^*).$$

Finally, it remains to note that

$$f^{(n-1)/2}(n+1-j) \geq \underline{f}^{(n-1)/2}(n+1-j) = \max\left\{\frac{n+1}{2}, j\right\} = j \geq f^{(n-1)/2}(n+1-r^*).$$

The above chain of inequalities can be explained as follows: The first inequality is due to Equation (7) (note that Equation (10) guarantees that $n+1-j$ belongs to the domain of $f^{(n-1)/2}(\cdot)$ and $\underline{f}^{(n-1)/2}(\cdot)$). The first equality is a direct consequence of the definition of $\underline{f}(\cdot)$. The second equality holds because $j \geq \frac{n+1}{2}$ (see Equation (8)). Finally, the second inequality holds by definition of $r^* = r^*\left((n-1)/2, j\right)$.

To sum up, building on all our previous analysis we can arrange the outcome of Stage 2 for different status-quo choices in Stage 1 in the following table:

$\bar{x}$ \ t^{soc}	t^L	$\frac{n-1}{n}t^L + \frac{1}{n}t^H$	...	$\frac{n+1}{2n}t^L + \frac{n-1}{2n}t^H$	$\frac{n-1}{2n}t^L + \frac{n+1}{2n}t^H$	...	$\frac{1}{n}t^L + \frac{n-1}{n}t^H$	t^H
y^n	y^0	y^n	...	y^n	y^n	...	y^n	y^n
y^{n-1}	y^0	y^{n-1}	...	y^{n-1}	y^{n-1}	...	y^{n-1}	y^n
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
$y^{\frac{n+1}{2}}$	y^0	$y^{\frac{n+1}{2}}$	...	$y^{\frac{n+1}{2}}$	$y^{\frac{n+1}{2}}$	...	y^{n-1}	y^n
$y^{\frac{n-1}{2}}$	y^0	$y^{\frac{n-1}{2}}$	...	$y^{\frac{n-1}{2}}$	$y^{r^*((n-1)/2,(n+1)/2)}$	...	$y^{r^*((n-1)/2,n-1)}$	y^n
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
y^1	y^0	y^1	...	y^1	$y^{r^*(1,(n+1)/2)}$	...	$y^{r^*(1,n-1)}$	y^n
y^0	y^0	y^0	...	y^0	$y^{r^*(0,(n+1)/2)}$	...	$y^{r^*(0,n-1)}$	y^n

Table 1: Outcome of Stage 2 as a function of the status quo $\bar{x} = y^k$ (rows) and the optimal utilitarian type $t^{soc} = (n-j)\cdot t^L + j \cdot t^H$ (columns).

That is, as we are considering a larger value of $\bar{x}$, the final outcome cannot be worse (or better) for high-type (low-type) citizens, whatever the type composition of the electorate. Note that together with the fact that $f^k(\cdot)$ is non-increasing and onto Equation (11) is crucial for Table 1. Moreover, the outcome is strictly better (worse) in some cases.

Analysis of Stage 1

Next we consider Round 1.h of Stage 1, with $h = n, \ldots, 1$. Let us focus on high-type citizens first. From Table 1, it immediately follows that in Round 1.n, voting for y^n (as status quo) yields higher expected utility than voting for y^{n-1}, regardless of the beliefs held by such individuals at this node of the game.[19] At any prior round of Stage 1, by contrast, voting to proceed to the next round yields higher utility than voting to stick with the current status quo policy. For low-type individuals, the optimal decisions need to be reversed.

It then follows that the elements of the off-diagonal of Table 1 will be chosen in Stage 1. Thus we obtain the following table:

t^{soc}	t^L	$\frac{n-1}{n}t^L + \frac{1}{n}t^H$	$\ldots$	$\frac{n+1}{2n}t^L + \frac{n-1}{2n}t^H$	$\frac{n-1}{2n}t^L + \frac{n+1}{2n}t^H$	$\ldots$	$\frac{1}{n}t^L + \frac{n-1}{n}t^H$	t^H
y	y^0	y^1	$\ldots$	$y^{\frac{n-1}{2}}$	$y^{\frac{n+1}{2}}$	$\ldots$	y^{n-1}	y^n

Table 2: Outcome y of Stage 2 as a function of the optimal utilitarian type t^{soc}.

Hence the utilitarian optimum is always implemented.

□

[19] If the prior type distribution has full support, any beliefs (for high-type citizens) obtained from Bayesian updating cannot distinguish between the cases where all citizens are of high type or all but one of them are.

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