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Working Paper 17/270
April 2017

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

# A Dynamic Model of Electoral Competition with Costly Policy Changes*

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This version: April 2017

**Abstract**

We consider an infinite-horizon model of elections where policy changes are costly for citizens and parties. The so-called costs of change increase with the extent of the policy shift and make policy history-dependent. First, we provide a detailed description of the equilibrium dynamics and analyze how policies are influenced by history, costs of change, party polarization, and the incumbent's ability. We show that policies converge to a stochastic alternation between two states and that in the long run costs of change have a moderating effect on policies. Second, we analyze welfare as a function of the marginal cost of change. If the initial level of policy polarization is low, welfare is highest for intermediate marginal costs of change. Moreover, any positive level of costs of change will benefit society if the future is sufficiently valuable. If the initial level of policy polarization is high, however, welfare will be highest for low or zero costs of change.

**Keywords:** democracy; dynamic elections; political polarization; costs of change; Markov perfect equilibrium

**JEL Classification:** C72, C73, D72, D78

*We would like to thank Ronny Razin, Guido Tabellini, Marco Battaglini, Volker Britz, Faruk Gul, Volker Hahn, Helios Herrera, Niall Hughes, Aniol Llorente-Saguer, César Martinelli, Rebecca Morton, Klaus Nehring, Marco Ottaviani, Thomas Palfrey, Mattias Polborn, Ronny Razin, Maik Schneider, Francesco Squintani, Christoph Vanberg, Bauke Visser, and Jan Zapal for helpful comments and suggestions. Feedback from participants at the 13th Meeting of the Society for Social Choice and Welfare and the Fifth World Congress of the Game Theory Society is also gratefully acknowledged. All errors are our own.

**Philippe Muller has been funded by SNF (Project no. 100010_153482/1).

# 1 Introduction

### Motivation

Standard models of electoral competition proceed on the assumption that any policy choice made in one term can be reversed in the next term without costs to political parties and citizens. Often, however, policy changes are costly, the costs being borne by the entire electorate.[1] First, smoothing the shift from the status-quo policy to a new policy may involve substantial transition costs. For instance, policy changes may require effort from the government to design new policies or to overcome resistance from groups wanting to preserve the status quo. Second, policy shifts may render original investments in human and physical capital obsolete. In the case of a nuclear phase-out, for example, the devaluation of existing assets generates large costs. At all events, the costs of change are borne by taxpayers and thus by the entire citizenry, including politicians.

In the presence of costs of change, office-holders face a trade-off. Changing the policy towards their own bliss point is desirable, but the benefits associated with the new policy may be outweighed by the costs of carrying out the policy change. Depending on the level of those costs, politicians may thus engineer major or minor policy changes, or none at all.

### Model

We develop an infinite-horizon model for elections with a continuum of citizens and two political parties, in which policy changes are costly. We consider a one-dimensional policy space over which citizens have quadratic preferences. In every election, one candidate from each party competes for office, with the winning candidate being elected to choose a policy for that period. Following an electoral defeat, the losing candidate is replaced by a new candidate from the same party in the next election. Political parties also have quadratic preferences over the unidimensional policy space, which represent the interests of the median party member, a citizen. A candidate's bliss point is that of the party he runs for, so the candidate's and the party's objective are perfectly aligned.

Once in office, each candidate is characterized by his capacity to efficiently carry out the usual governmental tasks. The *capacity* of an elected politician can be interpreted in a narrow sense as pure ability to perform such tasks or, in a broader sense, as the ability and propensity of key employees and political partners to perform the tasks involved in governing and administration.

[1] See Gersbach et al. (2015) for a detailed review of such costs.

During his tenure, exogenous events may negatively affect an incumbent's capacity as perceived by the voters. For instance, a corruption scandal involving the incumbent or some of his governmental partners might be understood as a proxy for a bad politician; a failed public project might taken to reveal the incumbent's inability as a policy-maker; a worldwide crisis might affect the incumbent's perceived ability—and that of his administration—, to name only a few examples. We assume that in every period, there is a constant probability that the current incumbent's capacity will decline. Such a capacity decrease is significant enough for a majority of voters to always vote for the challenger in the next election, regardless of any other consideration.

Unlike most models of electoral competition, we make the crucial assumption that, in any given period, there is a cost for voters and politicians alike that is proportional to the absolute difference between the policy choice in the current period and the status quo, i.e. the policy choice in the previous period. Accordingly, implementing the status quo is costless, and the more the policy choice in the current period deviates from it, the higher the costs will be. These *costs of change* thus establish a dynamic link between policy choices across periods, making preferences and policy history-dependent. Importantly, such costs will enable politicians to commit to policies, albeit in a partial way.[2]

Together with costs of change—expressed by a *marginal-cost* parameter—, three other parameters play a crucial role in our model. The second parameter, *party polarization*, captures the distance between the two parties' preferred policies. Potentially, the extent of party polarization has a large impact on policies and welfare whenever costs of policy changes are not negligible. For instance, high party polarization could induce large and costly policy switches after an incumbent has been deselected. Throughout the paper, we take party polarization as a given parameter and study how it influences policy choices. Due to the existence of costs of change, the policies implemented need not coincide with the preferred policies of political parties. The third crucial parameter is the initial level of *policy polarization*, which corresponds to the distance between the historically chosen policy and the median voter's current bliss point. Because we consider the initial level of policy polarization to be an exogenous parameter, our model embeds the possibility that preferences of parties and citizens have suffered an arbitrary shock.[3] Fourth, we also analyze how the *capacity-shock probability* influences turnover and policy choices. We say that the capacity of office-holders who have not yet suffered any shock is *normal*.

[2]We stress that costs of change have to be borne independently of whether the office-holder has held office in the previous period or not.

[3]We refer to Gersbach and Tejada (2016) for a model where the costs of change depend on the identity of the office-holder and the preferences of parties, and in which citizens also experience an arbitrary shock.

### Our goal

The main object of the paper is to study the short-term and long-term impact of these four parameters on policy choices and office-holder turnover and hence on *welfare*, which we define as the expected discounted sum of aggregate instantaneous utilities across all periods.[4] The main drivers of welfare turn out to be the distance of the implemented policies from the median voter's bliss point and the expected policy change in each period. The latter crucially depends on office-holder turnover.

We proceed as follows: To disentangle the impact of the costs of change from voter sophistication, we assume in the main body of the paper that voters and office-holders are myopic and choose strategies that maximize their current but not necessarily their long-term utility. An alternative interpretation is to see equilibrium outcomes as the result of political competition in a non-overlapping generation framework in which voters and policy-makers live for one period and where, after each period, power shifts with some probability to the other party. For our game-theoretic analysis we thus introduce the concept of a *Myopic Stationary Markov Perfect Equilibrium (MSMPE)*.[5] Subsequently, we increase the sophistication of voters and parties (office-holders) and examine whether our insights on the role played by costs of change remain as sophistication increases. It transpires that assuming shortsightedness on the part of voters and parties has no critical bearing on our results.

### Results

Our results can be classified into two categories. The first category has to do with the characterization of equilibria. We show that, for any marginal cost of change, any level of party polarization, any capacity-shock probability, and any level of initial policy polarization, the game described above has a unique MSMPE. Additionally, we characterize this equilibrium by using Markov transition diagrams.

Two findings from our analysis stand out. On the one hand, normal-capacity office-holders are always re-elected. This implies that costly policy changes are a potential source of the commonly observed incumbency advantage. On the other hand, candidates' equilibrium policy choices are

[4]Note that as long as the distribution of the citizenry's preferred policies is symmetric around the median voter—as we will assume in our paper—, maximizing the median voter's utility is equivalent to maximizing aggregate utility.

[5]In using stationary Markov equilibria we follow the literature on dynamic political economy; see e.g. Duggan and Kalandrakis (2012).

more moderate (i.e., closer to the median) in the long run than their bliss points. In other words, *policy polarization*—i.e. the distance between the policies implemented by candidates of different party affiliation—tends to be lower than party polarization. This is a consequence of the partial policy persistence that arises due to policy changes being costly.

The above properties imply that, in the long run, policies will converge to a stochastic alternation between two states. The transition probability between the two states is equal to the probability of a negative capacity shock. However, the policy paths display strong history-dependence. In particular, the transition phase from the initial policy to the long-term sequence of moderate policies depends crucially on the level of initial policy polarization. If initial policy polarization is high, the equilibrium policy path starts with a short sequence of extreme policies followed by an infinite sequence of less extreme policies. For low initial policy polarization, on the other hand, the moderate policy stage is reached in the first period already.

The second category of results tells us about the relationship between welfare and the marginal cost of change. From our analysis we gain the following insights: If, on the one hand, initial policy polarization is low, *(i)* welfare is highest for intermediate marginal costs of change and *(ii)* any non-zero marginal cost of change yields weakly higher social welfare than electoral competition without costs of change, provided that the capacity-shock probability is low and the social discount factor is large. If, on the other hand, initial policy polarization is high, *(i)* welfare is highest for low or zero marginal costs of change and *(ii)* high marginal costs of change are always welfare-reducing compared to the absence of such costs.

The intuition behind the latter results can be summarized as follows: Intermediate marginal costs of change are beneficial for the given society because they lead to moderate policies in the long run and to moderate actual costs of policy changes. However, if initial policy polarization is high, low marginal costs yield even more benefits to the citizenry, as they make the transition from extreme to moderate policies less costly while still generating moderate policies in the long run. By contrast, very high marginal costs of change bring about the implementation of the initial policy in each period, which is not desirable if initial policy polarization is high.

### Broader implications

Our results can be interpreted in a broader sense. Costs of change may be related to the legislative hurdles in the policy-making process of a political system. Our paper thus offers a rationale for a system of checks and balances. Moderate legislative hurdles can help stabilizing policies.

### Organization of the paper

The rest of the paper is organized as follows: In Section 2 we review the different strands of literature related to our paper. In Section 3 we present our model and introduce the equilibrium concept. Section 4 contains the equilibrium analysis. In Section 5 we analyze welfare as a function of the marginal cost of change. In Section 6 we study three extensions of our model that enable us to relax the assumption on voters' and parties' shortsightedness, on the one hand, and to study the case of non-linear costs of change on the other. We show that the main thrust of our results remains valid in all extensions. In Section 7 we discuss some of the predictions of our model. Section 8 concludes. The proofs can be found in the Appendix.

## 2 Related Literature

The paper is related to several strands of literature.

### Dynamic electoral competition

Our paper contributes to the literature on dynamic elections with endogenous state variables (see e.g. Battaglini et al. (2012)). In contrast to many papers with this approach, the state variable in our case is political (and not economic) in nature, i.e. the previous policy choice. Related papers are Kramer (1977), Wittman (1977), Forand (2014), and Nunnari and Zápal (2014), who also study dynamic models of electoral competition that are based on a static model of partisan competition. While Kramer (1977) considers purely office-motivated policy-makers, Wittman (1977) and Forand (2014) assume that candidates are policy-motivated, while Nunnari and Zápal (2014) consider candidates with a mixed motivation, i.e. policy-motivated candidates obtaining benefits from holding office (see e.g. Duggan, 2006). Forand (2014) and Nunnari and Zápal (2014) assume that parties are farsighted and that voters behave in accordance with some rule. We differ from these models by considering positive costs of change that reduce but do not eliminate the flexibility of office-holders ino engineering policy changes.

With its focus on shortsighted candidates, our model is similar to Kramer (1977) and Wittman (1977), who consider candidates basing their policy choices on the expected outcome of the upcoming election. As already stated, however, the assumption that voters and parties (candidates) are myopic is made for exposition purposes only and is not crucial for our results.

### Costs of change and policy persistence

Few models in the literature examine the costs associated with policy changes. In Gersbach et al. (2015), the impact of such costs is studied in a two-period model. Here we are interested in the long-term behavior of political competition with costs of change. Glazer et al. (1998) considers fixed costs of reversing policies in a two-period model. Both papers share with ours the feature that an incumbent will never reverse his previous policy choice when he is not in his first term. Hence, persistence in policy-making arises endogenously as a consequence of costs of change, and only newly-elected candidates will perform policy changes. Another model with endogenous policy persistence is presented in Alesina (1988). This model enables candidates to announce a policy rule before the elections and builds on the assumption that deviations will cause reputation losses. This differs significantly from our set-up.

A different, recently developed strand of literature on dynamic models of elections imposes persistence in the incumbent's policy choices as a feature of politicians' behavior. We refer again to Forand (2014) and Nunnari and Zápal (2014). In these models, policy-makers commit to implementing the same policy in all future periods once they are in office. Such commitments are made because of the huge reputation losses associated with policy changes (see e.g. Tavits (2007) or Miller and Schofield (2003)). Our cost-of-change approach differs significantly from this literature. While the latter restricts the current incumbent's choices, costs of change affect present and future office-holders equally and do not rule out policy changes, but merely make some policies less appealing than others.

The implications of dynamic links between policies across periods is the subject of other recent papers. Chen and Eraslan (2015) assume that a change in one policy dimension precludes the possibility of another change, at least in the short term. Bowen et al. (2014) and Bowen et al. (2017) consider an exogenous probability according to which the incumbent will remain in power in the next period. Callander and Raiha (2014) assume that policy decisions are durable. Our paper adds to these by studying the long-term effects of the costs of changing dynamically linked policies.

### Political polarization

Party and policy polarization are important variables in our model. A large literature has examined the causes (see e.g. Roberts and Smith, 2003; Theriault, 2006; Heberlig et al., 2006) and

consequences (see e.g. Jones, 2001; Binder, 2003; Fiorina et al., 2005; Testa, 2012; Hetherington, 2001) of both political phenomena. Our paper adds to this knowledge by investigating the way in which policy polarization is determined by parametrically varied levels of party polarization and costs of change.

**Incumbency advantage**

There is a large literature on the existence and the causes of incumbency advantage. Gelman and King (1990) and Alford and Brady (1989) empirically measure incumbency advantages in congressional elections. Levitt and Wolfram (1997) and Cox and Katz (1996) go beyond measurement and decompose its sources. In our paper, we find that normal-capacity candidates are always re-elected, which is in line with the existence of an incumbency advantage. Moreover, we add to the existing literature by identifying costs of change as a potential source of incumbency advantages.

# 3 The Model

## 3.1 General set-up

We examine an infinite-horizon model ($t = 1, 2, \ldots$) of electoral competition. In each period, there is an election by which a society elects an office-holder who will be responsible for policy-making in that period. The society consists of a continuum of voters of mass 1, with each voter indexed by $i \in [0, 1]$. There are two political parties: party $L$ and party $R$. One candidate from each party competes in every election. A party in $\{L, R\}$ is denoted by $K$, with $-K$ such that $\{K, -K\} = \{L, R\}$. Similarly, a candidate in $L \cup R$ is denoted by $k$, with $-k$ such that $k \in K$ and $-k \in -K$. The candidate defeated in one election is replaced by another candidate from the same party in the next election.

**Office-holder's tasks**

Once in office, a politician $k$ can be of *normal capacity* (i.e. his capacity is $a_k = 0$) or of *low capacity* (i.e. his capacity is $a_k = -A$). As discussed in the Introduction, capacity is a broad term designed to capture all aspects of the ability of an office-holder to perform well in policy-making. By default, an office-holder in his first term has normal capacity. At the end of each period $t$ in which the incumbent is still in office, he suffers from a negative capacity shock with probability

$\lambda \in [0,1]$. More precisely, with probability $\lambda$, he will have low capacity in any upcoming period $t' \in \{t+1, t+2, \ldots\}$. The capacity of a politician is always common knowledge. For a given period $t$, office-holder $k$ faces issues in two different dimensions:

- *Business-as-usual $\mathcal{B}$:* He undertakes the usual governmental tasks to provide basic public administration services. For simplicity, we assume that the output of all these tasks, which we denote by $g_{kt}$, is directly proportional to the capacity of the office-holder $k$ in period $t$, i.e.

$$g_{kt} = a_{kt},$$

  where $a_{kt}$ is either equal to zero or equal to $-A$. Normalizing the output to zero for normal capacity is simply done for convenience.[6] Note that in relation to $\mathcal{B}$, the office-holder's capacity is all that matters. Hence, a low-capacity politician can be interpreted simply as incapable of performing basic governmental tasks correctly.

- *Policy $\mathcal{I}$:* The office-holder chooses a policy $i_{kt}$, with $i_{kt} \in \mathcal{I} = [0,1]$. Note that we use $i_t$ instead of $i_{kt}$ if no confusion arises, i.e. if the office-holder's identity does not matter. We interpret $\mathcal{I}$ as the usual one-dimensional policy space that ranges from liberal ($i_t = 0$) to conservative ($i_t = 1$) positions. We assume that candidates (or parties) cannot commit to a policy before the election.

**Elections**

In each election, every citizen casts a vote for one of the two candidates that run for office. We distinguish two cases. The first election is an open race. Accordingly, the candidate who receives more votes enters office, with ties being broken according to the following rule: If $i_0 > \frac{1}{2}$ ($i_0 < \frac{1}{2}$), the right-wing (left-wing) candidate wins the election and if $i_0 = \frac{1}{2}$, each candidate wins the election with a probability of 50%, where $i_0 \in \mathcal{I}$ is the *status-quo policy* in $t = 1$. In the $t$-th election, with $t \geq 2$, ties are broken in favor of the incumbent.[7] The sequence of events in period $t$, for $t \in \{1, 2, \ldots\}$, is summarized in Figure 1.

[6] Any linear production function $g_{kt} = \tau a_{kt}$, with $\tau > 0$, would yield the same results, as would any increasing function. To reduce notational complexity, we set $\tau = 1$.

[7] Qualitatively our results also hold if ties are broken in favor of the challenger.

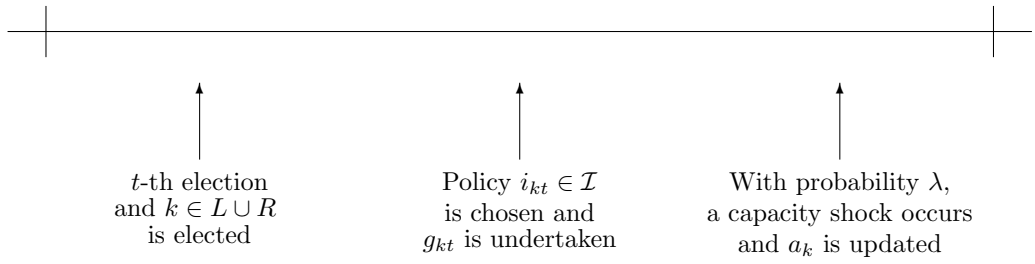



Figure 1: Sequence of events in period $t$, for $t \in \{1, 2, \ldots\}$.

**Instantaneous utilities**

First, voters and parties derive utility from both $\mathcal{B}$ and $\mathcal{I}$. On the one hand, the utility from the business-as-usual tasks in period $t$ is the same for all voters and for both parties:

$$U^{\mathcal{B}}(g_t) = g_t.$$

On the other hand, citizens have differing preferences on $\mathcal{I}$. More specifically, voter $i \in [0, 1]$ has bliss point $i$ and derives utility

$$U_i^{\mathcal{I}}(i_t) = -(i_t - i)^2$$

from the policy choice $i_t \in \mathcal{I}$ in period $t$. Thus, $i$ refers to both the voter $i$ and his ideal point regarding policy $\mathcal{I}$. We assume that citizens' preferences are uniformly distributed on $[0, 1]$, so the median voter is $m = \frac{1}{2}$. Similarly, in period $t$, party $K \in \{R, L\}$ derives utility

$$U_K^{\mathcal{I}}(i_t) = -(i_t - \mu_K)^2$$

from the policy choice $i_t \in \mathcal{I}$, where $\mu_R$ and $\mu_L$ are the parties' ideal points regarding $\mathcal{I}$. Party $R$ is the right-wing and party $L$ is the left-wing party. Accordingly, we assume that

$$\frac{1}{2} < \mu_R \leq 1 \quad \text{and} \quad \mu_L = 1 - \mu_R.$$

Throughout the paper, $\Pi = \mu_R - \frac{1}{2}$ denotes the level of *party polarization*. Accordingly, the higher the value of $\Pi$, the more opposed the two parties' interests are. Any party's candidate will inherit the instantaneous utility of the party he represents, so their interests are fully aligned at any given period. In the rest of the paper we will often slightly abuse language and refer indistinctly to a party's and a politician's preferences. This equivalence of interests no longer holds true if parties are farsighted—see Section 6.2.

Second, we assume that policy changes are costly for all voters and parties. More precisely, given the policy choice $i_{t-1} \in \mathcal{I}$ in period $t-1$, we assume that the policy choice $i_t \in \mathcal{I}$ in period $t$ imposes a utility loss in period $t$ on voters and parties alike, given by[8]

$$U^c(i_{t-1}, i_t) = -c \cdot |i_{t-1} - i_t|.$$

The parameter $c \geq 0$ corresponds to the *marginal cost of policy change.*[9] In the first period, costs of change are equal to $U^c(i_0, i_1) = -c \cdot |i_0 - i_1|$, where $i_0 \in \mathcal{I}$ is the status-quo policy in $t = 1$. We use $\Gamma^{i_0} = \left|i_0 - \frac{1}{2}\right|$ to denote the *initial level of policy polarization.* Accordingly, the higher the value of $\Gamma^{i_0}$, the more distant the initial policy will be from the interests of the median voter.

Finally, for each voter $i \in [0, 1]$ and each period $t \in \{1, 2, \ldots\}$, we define

$$U_i(i_{t-1}, i_t, g_t) = U^{\mathcal{B}}(g_t) + U_i^{\mathcal{I}}(i_t) + U^c(i_{t-1}, i_t). \tag{1}$$

Hence, $U_i(i_{t-1}, i_t, g_t)$ is the instantaneous utility of voter $i$ in period $t$. Similarly, for party $K \in \{L, R\}$ and each period $t \in \{1, 2, \ldots\}$, we define

$$U_K(i_{t-1}, i_t, g_t) = U^{\mathcal{B}}(g_t) + U_K^{\mathcal{I}}(i_t) + U^c(i_{t-1}, i_t). \tag{2}$$

We denote the above-described game by $\mathcal{G}^{i_0}$, with the set of players being made up of all voters and the two political parties, and the initial status-quo policy being $i_0 \in \mathcal{I}$.

## 3.2 Equilibrium concept

We assume that voters and parties (candidates) are *myopic*, i.e. they only care about their own utility in the current period. More precisely, voters base their voting decisions on the utility they expect from both candidates in the current period, while the office-holder does not care about re-election. These assumptions may be quite restrictive, but they enable us to study long-term consequences of marginal costs of change, party polarization, initial policy polarization, and capacity-shock probability on welfare. Later we shall relax these assumptions.[10] We consider stationary equilibria in pure Markov strategies, which is standard in the literature on dynamic political economy (see e.g. Duggan and Kalandrakis (2012) or the recent survey by Duggan and Martinelli (2014)). We introduce the following two definitions:

[8] In Section 6.3 we analyze the robustness of our results to costs of change that are convex (instead of linear) in the extent to which policies change.

[9] Politicians and voters all have the same parameter $c$. Nevertheless, our main results hold for small cost differences between politicians and voters. This follows from standard continuity arguments.

[10] In Sections 6.1 and 6.2, we analyze the robustness of our results with regard to voters or parties being non-myopic.

**Definition 1**

*A Stationary Markov Strategy for voter $i \in [0,1]$ is a function $\sigma_i : \{\emptyset, L, R\} \times \mathcal{I} \to \{L, R\}$ that maps the identity of the incumbent's party and the status-quo policy into the decision on which candidate to elect. A Stationary Markov Strategy for party $K \in \{L, R\}$ is a function $\sigma_K : \mathcal{I} \to \mathcal{I}$ that maps the status-quo policy into the policy of the current period.*

Throughout the paper, we write $\sigma_v = (\sigma_i)_{i\in[0,1]}$ to denote the voters' strategy profile. Next, we define the notion of equilibrium that we will be using in the main part of the paper.

**Definition 2**

*A Myopic Stationary Markov Perfect Equilibrium (MSMPE) of $\mathcal{G}^{i_0}$ is a profile of Stationary Markov Strategies $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ such that for each $t \in \{1, 2, \ldots\}$, each previous policy $i_{t-1} \in \mathcal{I}$, and incumbent $k \in K$ in period $t$,*

$$\sigma_K^*(i_{t-1}) \in \operatorname*{argmax}_{i_t \in \mathcal{I}} U_K(i_{t-1}, i_t, a_{kt}) \tag{3}$$

*and, for all $i \in [0,1]$,*

$$\sigma_i^*(K, i_{t-1}) = K \Leftrightarrow U_i\left(i_{t-1}, \sigma_K^*(i_{t-1}), a_{kt}\right) \geq U_i\left(i_{t-1}, \sigma_{-K}^*(i_{t-1}), a_{-kt}\right). \tag{4}$$

To facilitate the analysis, we assume henceforth that $A \gg 0$. This implies that for all $i_{t-1}$, $i_t$, $i'_{t-1}$, and $i'_t \in \mathcal{I}$, we have

$$U_i\left(i_{t-1}, i_t, 0\right) > U_i\left(i'_{t-1}, i'_t, -A\right) \text{ for all } i \in [0,1]. \tag{5}$$

On the one hand, (3) guarantees that candidates will choose the policy that maximizes their instantaneous utility and hence that of the party they represent. On the other hand, according to (4), voters always vote—there is no abstention in our model—and they do so for the candidate/party from whom they expect the higher instantaneous utility from policy-making, provided that both candidates are of normal capacity. Otherwise, i.e. if one candidate (the challenger) has normal capacity and the other candidate (the incumbent) has already suffered a negative shock, the former is always elected because he will yield higher instantaneous utility.

# 4 Analysis of the Game

In this section we prove that an MSMPE of the dynamic electoral competition game $\mathcal{G}^{i_0}$ exists and is unique. Since we require sequential rationality in each period, we start with the analysis of the policies chosen by the incumbents and then find the election outcomes.

## 4.1 Equilibrium policy choices

First we solve the problem for the incumbent who chooses his policy to maximize his instantaneous utility.

**Proposition 1**

*Let $k \in K$ be the office-holder in period $t$, with $t \in \{1, 2, \ldots\}$ and $K \in \{R, L\}$. In any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$, $k$'s policy choice in period $t$ is given by*

$$\sigma_K^*(i_{t-1}) = \min\left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}, \mu_K + \frac{c}{2}\right\}, \tag{6}$$

*where $i_{t-1} \in \mathcal{I}$ is the policy implemented in period $t-1$.*

Note that $\sigma_L^*(i_{t-1}) \leq \sigma_R^*(i_{t-1})$ for all $i_{t-1} \in \mathcal{I}$, i.e., a left-wing office-holder will always choose a more leftist policy than a right-wing office-holder. The policy choice given in (6) is illustrated by Figure 2.

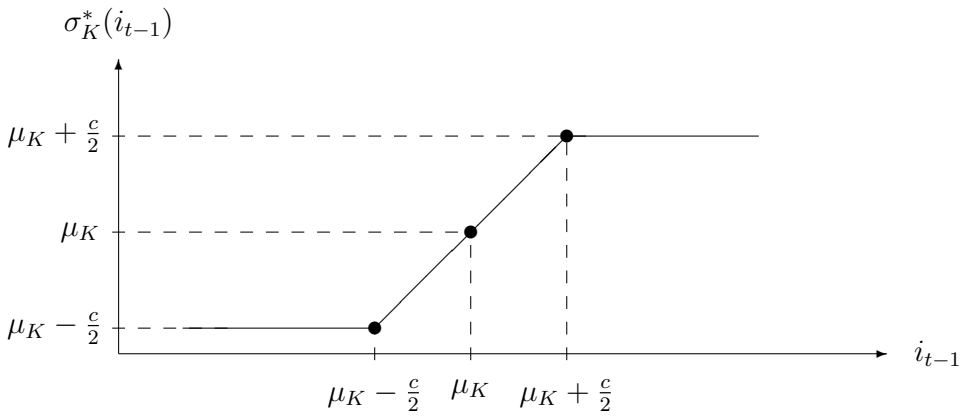



Figure 2: Best response of office-holder $k \in K$ to policy $i_{t-1}$.

It will be convenient to introduce a measure of policy persistence. For each party $K \in \{L, R\}$, let

$$\mathcal{Y}_K = \min\left\{\frac{c}{2}, \max\{1 - \mu_K, \mu_K\}\right\} = \min\left\{\frac{c}{2}, \frac{1}{2} + \Pi\right\}.$$

According to Proposition 1, the best response of the office-holder's party has the following properties: If the status-quo policy $i_{t-1} \in \mathcal{I}$ is sufficiently close to its bliss point, i.e. $|i_{t-1} - \mu_K| \leq \mathcal{Y}_K$, the best response of party $K$—through candidate $k \in K$—is to maintain the status-quo policy. By contrast, if the status-quo policy $i_{t-1}$ is far away from its bliss point, i.e. $|i_{t-1} - \mu_K| > \mathcal{Y}_K$, the best response of $K$—through candidate $k \in K$—is to choose a more moderate policy than the status quo, which is exactly at a distance $\mathcal{Y}_K$ from its bliss point and is situated between $\mu_K$

and $i_{t-1}$. Since $\mathcal{Y}_L = \mathcal{Y}_R$, we define $\mathcal{Y} := \mathcal{Y}_R = \mathcal{Y}_L$ and interpret $\mathcal{Y}$ as the *level of (endogenous) policy persistence.*[11] Indeed, $\mathcal{Y}$ corresponds to the range of policies that induce the office-holder not to change the status-quo policy. In particular, note that if $\mathcal{Y} = \frac{1}{2} + \Pi$, or equivalently, if $\frac{c}{2} \geq \frac{1}{2} + \Pi$, the initial status-quo policy $i_0$ is implemented in every period $t \geq 1$, independently of the office-holder's identity. Therefore the interesting policy dynamics occur for low values of $c$ relative to $\Pi$, i.e. for $c < 1 + 2\Pi$.

Finally, we note that the best response of parties in $t = 2$ is the same for all status-quo policies close to the respective party's bliss point and, more generally, that such a best response does not respond locally to changes in the status-quo policy. These properties follow from the assumption that costs associated with policy changes are linear in the extent of the change. As is discussed in Section 6.3, however, such assumption is not knife-edged, and the validity of our results extends to convex specifications of such costs. With linear costs of change, however, a full-fledged analytical solution of the political game is possible. It also enables us to define the important concept of persistency, which will turn out to be particularly useful in the case where parties are not myopic—see Section 6.1.

## 4.2 Equilibrium voting decisions

Second, we analyze the voters' behavior. In the first election, ties are broken according to the following rule: If $|i_0 - \mu_K| = |i_0 - \mu_{-K}|$, both candidates will win the election with equal probability. Otherwise party $K$ with $|i_0 - \mu_K| < |i_0 - \mu_{-K}|$ will win the election. In any other election, the incumbent is re-elected if and only if he obtains a vote-share of at least $\frac{1}{2}$. As a consequence, we need to treat separately *(i)* the elections that take place in periods $t \in \{2, 3, \ldots\}$ and *(ii)* the first election.

To characterize the election outcomes, it will suffice to consider the case where both candidates have the same capacity. Due to (5), a low-capacity incumbent will never be re-elected in equilibrium. Thus, we build on the implicit assumption that the loss of support for a candidate will affect the entire party, meaning that the latter will not be able to counteract the negative shock by simply replacing the bad candidate by another candidate of the same party in the first election

[11] Note that all candidates of one party have the same persistence, so we can speak of persistence as a feature at party level.

after the shock.[12] Let the status-quo policy be $i_{t-1} \in \mathcal{I}$, the incumbent $K \in \{\emptyset, L, R\}$ have normal capacity, and the citizens vote according to the strategy profile $\sigma_v = (\sigma_i)_{i\in[0,1]}$. We denote the outcome of the $t$-th election in this case by

$$E(\sigma_v, K, i_{t-1}) \in \{L, R\}. \tag{7}$$

When the policy outcome is stochastic, we write $E(\sigma_v, K, i_{t-1}) = pL(1-p)R$ to denote the following outcome: party $L$'s candidate is elected with probability $p$ and party $R$'s candidate is elected with probability $1-p$, with $p \in [0,1]$. Recall that the probability that a normal-capacity incumbent will have low capacity at the end of his current term is $\lambda$. We obtain the following result:

**Proposition 2**

*Let $J \in \{L, R\}$ such that $|\mu_J - i_{t-1}| \leq |\mu_{-J} - i_{t-1}|$. For any period $t \in \{1, 2, \ldots\}$, any status-quo policy $i_{t-1} \in \mathcal{I}$, and any incumbent $K \in \{\emptyset, L, R\}$, in any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$ the mapping $E(\sigma_v^*, K, i_{t-1})$ satisfies the following properties:*

*(i) If $t = 1$, then* $E(\sigma_v^*, \emptyset, i_{t-1}) = \begin{cases} J & \text{if } 0 < \Gamma^{i_0} \leq \frac{1}{2}, \\ \frac{1}{2}L\frac{1}{2}R & \text{if } \Gamma^{i_0} = 0. \end{cases}$

*(ii) If $t > 1$, then $E(\sigma_v^*, K, i_{t-1})$ is given by*

$$\begin{cases} -K & \text{if } \max\left\{\frac{c}{2} - \Pi, 0\right\} < \left|i_{t-1} - \frac{1}{2}\right| < \Pi + \frac{c}{2} \text{ and } |\mu_{-K} - i_{t-1}| < |\mu_K - i_{t-1}|, \\ K & \text{otherwise.} \end{cases}$$

Together with the fact that low-capacity incumbents are always deselected, the above proposition describes the optimal choices of the electorate. Part *(i)* refers to the first election, in which there is no incumbent. Since the candidates from the two parties both have normal capacity and their bliss points are in symmetric positions with respect to the median voter, the electorate will select the politician who will carry out the smaller policy shift. If the initial policy is already biased towards the bliss point of one party, it is the candidate of that party who will be elected—see Proposition 1. If there is no bias, the winner of the election will be decided according to a fair coin toss. Part *(ii)* refers to any subsequent election in which there is always an incumbent and a challenger. In this case, the challenger $-K$ will win the election if either the incumbent $K$ has suffered a negative shock or if the challenger will carry out a smaller policy change than

[12] There are numerous instances in which the loss of a candidate's credibility has dragged the entire party down into certain electoral defeat. On some occasions, the candidate is very powerful within the party and is able to impose his own agenda, on other occasions the electorate likes to punish the entire party as also being accountable for bad policy-making. Formally, however, our model would not change if we were to assume that parties can replace bad candidates with a certain probability. In such a case, it would suffice to lower the exogenous probability with which a candidate is hit by a negative shock.

the incumbent while still choosing a moderate policy. This latter feature is guaranteed when $\max\left\{\frac{c}{2}-\Pi, 0\right\} < \left|i_{t-1}-\frac{1}{2}\right| < \Pi + \frac{c}{2}$ and $|\mu_{-K} - i_{t-1}| < |\mu_K - i_{t-1}|$.

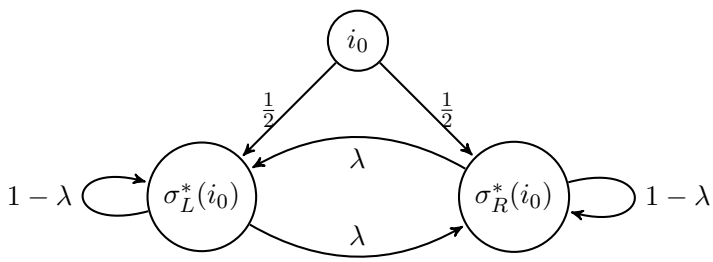
## 4.3 Unique MSMPE

From the combination of Proposition 1 and Proposition 2 it follows that there exists a unique MSMPE of $\mathcal{G}^{i_0}$. The characterization of this unique equilibrium is given in our first main result:

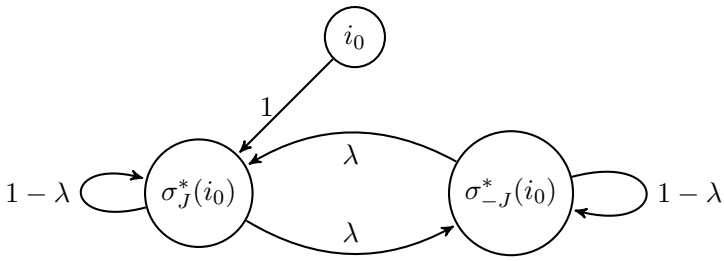
**Theorem 1**

*Let $i_0 \in \mathcal{I}$ be the status-quo policy in $t = 1$. There exists a unique MSMPE of $\mathcal{G}^{i_0}$, referred to as $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$. Depending on $\Gamma^{i_0}$, the initial level of policy polarization, $\sigma^*$ is characterized by one of the following Markov transition diagrams, where the equilibrium policy choices are determined according to the best response given in Proposition 1 and $J \in \{L, R\}$ is a party that satisfies $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$:*

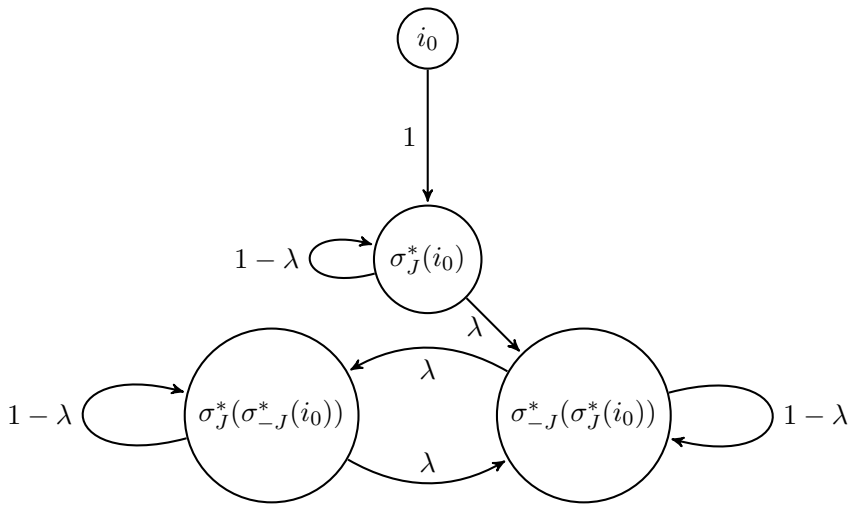
*(i) If $\Gamma^{i_0} = 0$, then $\sigma^*$ is characterized by*




*(ii) If $0 < \Gamma^{i_0} \leq \min\left\{\left|\Pi - \frac{c}{2}\right|, \frac{1}{2}\right\}$, then $\sigma^*$ is characterized by*




*(iii) If $\min\left\{\left|\Pi - \frac{c}{2}\right|, \frac{1}{2}\right\} < \Gamma^{i_0} \leq \frac{1}{2}$, then $\sigma^*$ is characterized by*




In all three cases described in Theorem 1, the infinite sequence of policies converges to a set of two policies, each implemented by a candidate from one of the two parties. The outcome switches randomly from one policy to the other. More specifically, the shift from one long-term policy to the other only occurs when the respective office-holder is deselected, which happens in each period with probability $\lambda$. Whenever an office-holder is re-elected, he maintains the status-quo policy. That is to say, persistence in the incumbent's policy choices arises endogenously in our model. We summarize these insights in the following corollary:

**Corollary 1**

*For any initial policy $i_0 \in \mathcal{I}$, the political outcomes converge to a stochastic fixed point with two policies and transition probability $\lambda$ in each period.*[13]

If $c \geq 2\Pi$, the policies chosen by the two parties are identical in the long run. That is, only the office-holder's party affiliation is stochastically alternating in this case, but not the actual policy choices. If $c < 2\Pi$, however, the infinite sequence of policies consists of a stochastic alternation between $\mu_L + \frac{c}{2}$ (implemented by a left-wing office-holder) and $\mu_R - \frac{c}{2}$ (implemented by a right-wing office-holder). Importantly, these long-term policies are more moderate than the parties' bliss-points. Theorem 1 thus implies that, in the long run, costs of change may have a moderating effect on policies. Theorem 1 also demonstrates that although the long-term sequence of policies is independent of the initial level of policy polarization, the transition path that describes how to get there crucially depends on $i_0$ (see the diagrams). More precisely, if $\Gamma^{i_0}$ is large, the infinite sequence of alternating policies is only reached after an initial phase of more extreme policies. For low $\Gamma^{i_0}$ such a sequence is immediately reached in $t = 1$.

[13]For the concept of stochastic fixed points, see e.g. Bharucha-Reid et al. (1976).

Furthermore, from the above characterization of $\sigma^*$ follows that, in each period, the incumbent is deselected with probability $\lambda$, which corresponds to the probability of office-holders being hit by a negative capacity shock. The reasoning, based on Proposition 2, is as follows: The best-response policy choices $\sigma^*_J(i_0)$, $\sigma^*_{-J}(\sigma^*_J(i_0))$, and $\sigma^*_J(\sigma^*_{-J}(i_0))$ are always in the subset of $\mathcal{I}$ for which normal-capacity incumbents are re-elected. We summarize these insights in a second corollary:

**Corollary 2**

*In any period $t > 1$, the incumbent will always be elected if he has normal capacity.*

The fact that normal-capacity incumbents are always re-elected in equilibrium may be interpreted as a manifestation of the commonly observed incumbency advantage (see Section 7). Our paper identifies costs of change as a potential source of incumbency advantage. The reason is that an office-holder with normal capacity can secure re-election by making it too costly for a majority to switch to the equally capable challenger.

It is important to point out that the stark feature identified in Corollary 2 depends on the assumption that the median voter's preferred policy is 1/2, which is equidistant from $\mu_L$ and $\mu_R$. In real settings, this may not always be the case. For one thing, parties may have difficulty adapting to a change in the electorate's preferences. Or there may exist a systemic bias in favor of or against the incumbent, e.g. due to differential abstention or redistricting. While Corollary 2 may not hold in these circumstances, the equilibrium dynamics described in Theorem 1 would still remain valid, but with possibly different (and asymmetric) transition probabilities. Note that if there were a sudden, symmetric change in the parties' platforms that would result in an increase (decrease) of party polarization, the above results could be immediately applied, notably if actual policies had already converged to the long-term stochastic alternation between two states.[14] Under this assumption, the new situation would be equivalent to Case *(ii)* or Case *(iii)* of Theorem 1, the only difference being that now the convergence would be towards the stochastic alternation between the two states calculated with the new policy platforms.

# 5 Welfare Analysis

In this section we build on the results of Section 4 to investigate how changes in $c$, $\Pi$, $\Gamma^{i_0}$, or $\lambda$ influence the society's well-being in the long run. We use $\beta \in (0, 1)$ to indicate the *social discount factor* and start by defining welfare and various other concepts.

[14] Similar reasoning can be applied if policies have not yet reached the long-term stochastic alternation.

## 5.1 Definitions

We measure *welfare* by the expected discounted sum of aggregate instantaneous utility throughout all periods. Equivalently, since citizenry preferences are symmetric with respect to the median voter, welfare can be measured by the expected discounted sum of the median voter's instantaneous utility throughout all periods. To obtain an explicit expression for welfare, it is convenient to introduce a number of additional concepts.

First, we use $\mathcal{S} = (s_1, s_2, \ldots)$ to denote an arbitrary realization of the stochastic process containing the capacity shocks to incumbents throughout all periods, i.e., for each $t \in \{1, 2, \ldots\}$, $s_t = 1$ with probability $\lambda$ and $s_t = 0$ with probability $1 - \lambda$. Since the probability of a negative shock is independent of all other variables of the model, the above stochastic process can be defined independently of policy choices and incumbents' identities. In particular, $s_t = 1$ indicates that the incumbent in period $t$ had a capacity shock at the end of his term, while $s_t = 0$ indicates that he did not. Second, given a strategy profile $\sigma = (\sigma_v, \sigma_L, \sigma_R)$ and $\mathcal{S}$, we use $\mathcal{P}(\mathcal{S}) = \mathcal{P}(\mathcal{S}, \sigma, i_0) = (i_0, i_1, i_2, \ldots)$ to denote a path of policies in $\mathcal{I}$ generated when voters and parties decide according to strategy profile $\sigma$, the initial status-quo policy is $i_0 \in \mathcal{I}$, and candidates are hit by capacity shocks in accordance with $\mathcal{S}$. Third, we use $\mathcal{A}(\mathcal{S}) = \mathcal{A}(\mathcal{S}, \sigma, i_0) = (g_1, g_2, \ldots)$ to denote a path of outputs in $\mathcal{B}$ generated for given $\sigma$ and $i_0 \in \mathcal{I}$, when candidates are hit by capacity shocks in accordance with $\mathcal{S}$. When there is no possible confusion, we write $(i_0, i_1, i_2, \ldots)$ and $(g_1, g_2, \ldots)$ without explicitly referring to $\mathcal{S}$, $\sigma$, and $i_0$.

From Section 3.1 we know that, for each period, the expected utility of the median voter $m = \frac{1}{2}$ consists of three terms: the expected value of the utility from $\mathcal{B}$ ($U^{\mathcal{B}}$), the expected value of the utility from policies ($U^{\mathcal{I}}_{\frac{1}{2}}$), and the expected value of the utility from costs of change ($U^c$). Accordingly, welfare at the beginning of the period $t = 1$ is given as a function of $c \in [0, \infty)$ by[15]

$$W(c) = EU^{\mathcal{B}}(c) + EU^{\mathcal{I}}_{\frac{1}{2}}(c) + EU^c(c), \tag{8}$$

where, for $(g_1, g_2, \ldots) = \mathcal{A}(\mathcal{S}, \sigma^*, i_0)$ and $(i_0, i_1, i_2, \ldots) = \mathcal{P}(\mathcal{S}, \sigma^*, i_0)$,

$$EU^{\mathcal{B}}(c) = \mathbb{E}_{\mathcal{S}}\left[(1-\beta) \cdot \sum_{t=1}^{\infty} \beta^{t-1} \cdot U^{\mathcal{B}}(g_t)\right],$$

$$EU^{\mathcal{I}}_{\frac{1}{2}}(c) = \mathbb{E}_{\mathcal{S}}\left[(1-\beta) \cdot \sum_{t=1}^{\infty} \beta^{t-1} \cdot U^{\mathcal{I}}_{\frac{1}{2}}(i_t)\right],$$

[15] We stress that $EU^{\mathcal{B}}(c)$ does not actually depend on $c$.

and

$$EU^c(c) = \mathbb{E}_{\mathcal{S}}\left[(1-\beta)\cdot\sum_{t=1}^{\infty}\beta^{t-1}\cdot U^c(i_{t-1},i_t)\right]$$

are the median voter's discounted expected lifetime utilities from the output of governmental tasks, from policies, and from costs of change, respectively.

## 5.2 Welfare as a function of the marginal cost of change

We next analyze welfare as a function of $c$, the marginal-cost-of-change parameter. Our results show how variations in this parameter affect welfare in standard elections.

**Theorem 2**

*Let $\lambda \in (0,1]$ and $\beta \in (0,1)$, and let $i_0 \in \mathcal{I}$ be the status-quo policy in $t=1$. In $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$, the unique MSMPE of $\mathcal{G}^{i_0}$, the following holds:*[16]

*(i) If $0 \leq \Gamma^{i_0} < \Pi$, then*[17]

- $\operatorname{argmax}_{c\geq 0} W(c) \cap [0, 2\Pi) = \emptyset$.
- $\operatorname{argmax}_{c\geq 0} W(c) \cap [2\Pi, 2(\Pi+\Gamma^{i_0})] \neq \emptyset$.
- *If $\lambda < \frac{1}{4}$, $\exists \beta^* < 1$ such that $W(c) \geq W(0)$ for any $c > 0$ and $\beta \geq \beta^*$.*

*(ii) If $\Pi < \Gamma^{i_0} \leq \frac{1}{2}$, then*

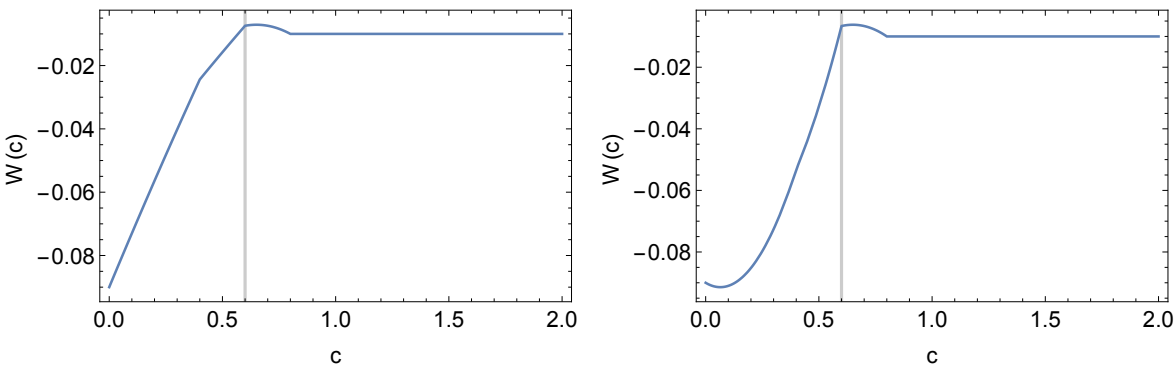
- $\operatorname{argmax}_{c\geq 0} W(c) \subseteq [0, 2\Pi]$.
- *$\exists \beta^{**} > 0$ such that, for $\beta < \beta^{**}$,* $\operatorname{argmax}_{c\geq 0} W(c) = \{0\}$.
- *$W(c) < W(0)$ for any $c \geq 2(\Gamma^{i_0} + \Pi)$.*

The results of Theorem 2 can be interpreted as follows: *(i)* If initial policy polarization is low, welfare is highest for intermediate marginal-cost-of-change levels. If, additionally, $\lambda$ is low and $\beta$ high, then any value $c > 0$ will yield a weakly higher welfare than $c = 0$. *(ii)* If, on the other hand, initial policy polarization is high, then welfare will be highest for low marginal costs of change. If $\beta$ is low, in particular, only $c = 0$ is welfare-maximizing. Moreover, large marginal-cost-of-change levels yield lower welfare than $c = 0$.

[16] Statement *(ii)* remains valid if $\lambda = 0$, but statement *(i)* is not entirely satisfied. More precisely, if $0 \leq \Gamma^{i_0} < \Pi$ and $\lambda = 0$, then $\operatorname{argmax}_{c\geq 0} W(c) \cap [0, 2\Pi) \neq \emptyset$, but we still have $\{0\} \notin \operatorname{argmax}_{c\geq 0} W(c)$.

[17] If $\Gamma^{i_0} = \Pi$, statement *(i)* holds with the exception that 0 might be an element of $\operatorname{argmax}_{c\geq 0} W(c)$.

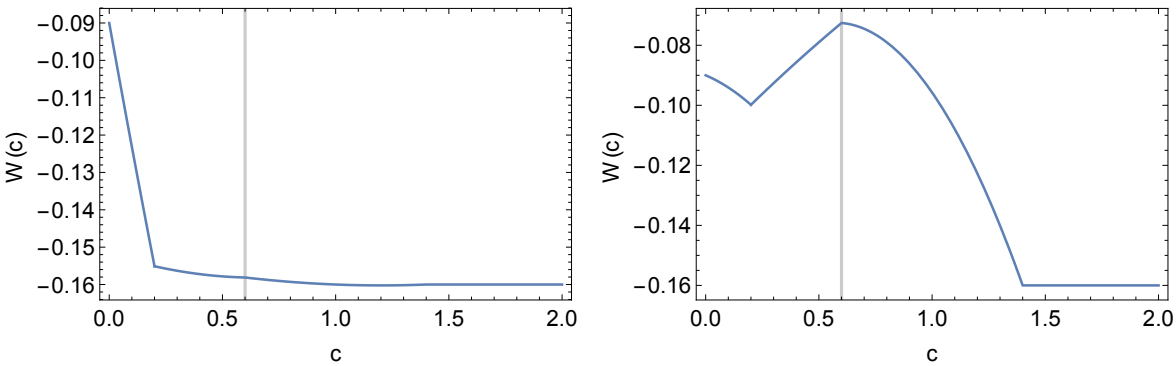
Next we provide an intuitive explanation why large cost levels, namely marginal-cost-of-change parameters $c$ such that $c \geq 2(\Gamma^{i_0} + \Pi)$, can be socially beneficial (compared to $c = 0$) if initial policy polarization is low, while this is not the case for high levels of $\Gamma^{i_0}$. As noted earlier, if $c \geq 2\Pi$ the sequence of long-run policies converges to one specific policy chosen by both parties. This implies that for $c \geq 2\Pi$ there will be no costs of change in the long run, just as for $c = 0$. This can be beneficial for voters when $\Gamma^{i_0}$ is low because the long-run policy, which is equal to $i_0$, is moderate (i.e., close to the median). However, if initial policy polarization is high and $c$ is large, the long-term policy is far away from the median voter's preferred policy, thus $c = 0$ yields higher welfare than large values of $c$.




(a) $W(c)$ as a function of $c$, for $\lambda = 0.2$.

(b) $W(c)$ as a function of $c$, for $\lambda = 0.6$.

Figure 3: Welfare as a function of $c \geq 0$, for $\Gamma^{i_0} = 0.1$. The other parameter values are $\Pi = 0.3$, $\beta = 0.9$, and $\lambda = 0.2$ (resp. $\lambda = 0.6$).




(a) $W(c)$ as a function of $c$, for $\beta = 0.4$.

(b) $W(c)$ as a function of $c$, for $\beta = 0.9$.

Figure 4: Welfare as a function of $c \geq 0$, for $\Gamma^{i_0} = 0.4$. The other parameter values are $\Pi = 0.3$, $\lambda = 0.2$, and $\beta = 0.4$ (resp. $\beta = 0.9$).

Theorem 2 can be better understood with the help of Figures 3(a)–3(b) and Figures 4(a)–4(b).

In all graphs, a vertical gridline marks the value $c = 2\Pi$. The former two plots show comparative-statics results for a low level of $\Gamma^{i_0}$. They illustrate that welfare is highest for intermediate marginal-cost-of-change levels if initial policy polarization is low. Moreover, Figure 3(a) illustrates that if $\lambda$ is low and $\beta$ is large, any $c > 0$ will yield higher welfare than $c = 0$. However, as can be seen from Figure 3(b), this is not the case if $\lambda$ is large. Indeed, for $\lambda$ large and $c$ close to 0, the increase in $EU^{\mathcal{I}}_{\frac{1}{2}}(c)$ that follows from marginally increasing $c$ does not outweigh the corresponding decrease in $EU^{c}(c)$. Figures 4(a)–4(b) in their turn illustrate statement *(ii)* of Theorem 2. That is, if initial policy polarization is high (i.e. $\Gamma^{i_0}$ is large), welfare is highest for low levels of marginal costs. The upper bound of the range of low marginal-cost-of-change levels is at $c = 2\Pi$. If $\beta$ is sufficiently low, in particular, welfare will be highest only for $c = 0$.

## 5.3 Welfare as a function of party polarization

We next assess the behavior of $W(c)$ as a function of party polarization $\Pi$. As noted earlier, the most interesting parameter constellations are such that $c < 2\Pi$. In that case, the infinite sequence of policies converges to the stochastic alternation between two policies, viz. $\mu_R - \frac{c}{2}$ and $\mu_L + \frac{c}{2}$. We thus focus on $\Pi \in \left(0, \frac{1}{2}\right]$ and $c \geq 0$ such that $c < 2\Pi$. Then a marginal increase in $\Pi$ implies a decrease in $W(c)$ if $\beta$ is sufficiently large. To see why this is the case, note that changes in $\Pi$ affect welfare only via the policy choices (and the corresponding costs of change) and not via the transition probabilities between these policies. Indeed, the incumbents' re-election probabilities are not affected by a rise in $\Pi$. Accordingly, if $\Gamma^{i_0} \leq \left|\Pi - \frac{c}{2}\right|$, raising $\Pi$ obviously has a negative effect on $W(c)$, due to more extreme policy choices. For $\Gamma^{i_0} > \left|\Pi - \frac{c}{2}\right|$, a potential decrease in costs of change (due to an increase in $\Pi$) is outweighed by utility losses if $\beta$ is sufficiently large.

## 5.4 Welfare as a function of capacity-shock probability

We have already analyzed how variations in the marginal cost of change and party polarization affect welfare for different levels of initial policy polarization. We now assess the impact of $\lambda$, i.e. the probability that an incumbent will experience a negative shock in some period during his tenure. From the previous analysis it immediately follows that higher values of this parameter will always decrease welfare in the long run, as they make policy changes more likely without rendering policies more moderate. This means that if we were to interpret $\lambda$ as the degree of instability of a political system, we would obtain the (unsurprising) result that in the long run more instability has negative consequences on welfare. In the short term, however, more instability may yield

welfare gains, but only if the status-quo policy, $i_0$, is very extreme—or, equivalently, the initial level of policy polarization is large—and the social discount factor $\beta$ is low. In this case, the long-term negative implications are less important.

# 6 Extensions

In the following we explore the robustness of Theorems 1 and 2 when some features of the baseline model are changed. First, we assume that voters (but not parties) are farsighted. If initial policy polarization is not too large, the description of the unique equilibrium remains the same as under the assumption of myopic voters. What is more, the welfare analysis of Theorem 2 is robust to voters being farsighted, provided that voters' discount factor is not too high. Second, we consider that parties (but not voters) are farsighted. For this variation of the baseline model, however, we restrict the parties' strategy space in a plausible way. It turns out that, except for a small range of intermediate values of $c$, the main thrust of our findings also remains the same if parties are not too farsighted.[18] Third and last, we consider the case where in the extent of the policy change costs of change are not linear but convex. Since this extended model cannot be solved analytically in full, we numerically compute all possible equilibrium policy paths and the corresponding transition probabilities for some (representative) examples. We generate numerical simulations of welfare as a function of $c$. Our analysis suggests that the assumption of costs of change being linear is not critical for the validity of our findings.[19]

## 6.1 Non-myopic voters

This section is devoted to the analysis of how crucial voter shortsightedness is for the validity of the results in Theorems 1 and 2. For that purpose we assume that voters (but not parties) are non-myopic and we have them discount future payoffs with a common discount factor $\theta \in (0, 1)$. This parameter refers to the sophistication of voters' behavior regarding future outcomes, so it may differ from the social discount factor $\beta$. Note that since parties are still assumed to be myopic, they do not care whether their candidates are re-elected or not in the next election when choosing their policies. Given the status-quo policy $i_0 \in \mathcal{I}$, we us $\mathcal{G}_\theta^{i_0}$ to denote the modification

[18] The parties' strategy space with an infinite horizon is very large. Accordingly, we look for equilibria in which both parties choose the same level of policy persistence.

[19] Note that imposing costs of change to be linear is a first-order approximation of the general case where costs associated with policy changes increase arbitrarily in the extent of the policy shift.

of the game $\mathcal{G}^{i_0}$ where voters' discount factor is $\theta$. In the following we modify the notion of equilibrium that we have used for our baseline model.

**Definition 3**

*A Party-Myopic Stationary Markov Perfect Equilibrium (P-MSMPE) of $\mathcal{G}_\theta^{i_0}$ is a profile of Stationary Markov Strategies $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$ such that, for each $t \in \{1, 2, \ldots\}$, each $i_{t-1} \in \mathcal{I}$, and $k \in K$ denoting the incumbent in period $t$, we have*

$$\sigma_K^*(i_{t-1}) \in \operatorname*{argmax}_{i_t \in \mathcal{I}} U_K(i_{t-1}, i_t, a_k)$$

*and, for all $i \in [0, 1]$,*

$$\begin{aligned}\sigma_i^*(K, i_{t-1}) = K \Leftrightarrow U_i\left(i_{t-1}, \sigma_K^*(i_{t-1}), a_k\right) + \mathbb{E}_{\mathcal{S}}\left[\sum_{t' \geq t+1} \theta^{t'-t} \cdot U_i\left(i_{t'-1}, i_{t'}, g_{t'}\right)\right] \\ \geq U_i\left(i_{t-1}, \sigma_{-K}^*(i_{t-1}), a_{-k}\right) + \mathbb{E}_{\mathcal{S}}\left[\sum_{t' \geq t+1} \theta^{t'-t} \cdot U_i\left(i'_{t'-1}, i'_{t'}, g'_{t'}\right)\right],\end{aligned}$$

*where*

$$\begin{aligned}&\mathcal{P}(\mathcal{S}, \sigma^*, \sigma_K^*(i_{t-1})) = (\sigma_K^*(i_{t-1}), i_{t+1}, i_{t+2}, \ldots) = (i_t, i_{t+1}, i_{t+2}, \ldots) \quad \textit{resp.} \\ &\mathcal{A}(\mathcal{S}, \sigma^*, \sigma_K^*(i_{t-1})) = (g_{t+1}, g_{t+2}, \ldots)\end{aligned}$$

*and*

$$\begin{aligned}&\mathcal{P}(\mathcal{S}, \sigma^*, \sigma_{-K}^*(i_{t-1})) = (\sigma_{-K}^*(i_{t-1}), i'_{t+1}, i'_{t+2}, \ldots) = (i'_t, i'_{t+1}, i'_{t+2}, \ldots) \quad \textit{resp.} \\ &\mathcal{A}(\mathcal{S}, \sigma^*, \sigma_{-K}^*(i_{t-1})) = (g'_{t+1}, g'_{t+2}, \ldots)\end{aligned}$$

*are the paths of policies in $\mathcal{I}$ (resp. outputs in $\mathcal{B}$) that follow the decision of the incumbent or the challenger when $\mathcal{S} = (s_{t+1}, s_{t+2}, \ldots)$ is the realization of the stochastic process containing the capacity shocks and voters and parties decide in accordance with $\sigma^*$.*

As in the baseline model, we assume henceforth that $A \gg 0$, so that incumbents whose capacity is low are never re-elected. This greatly facilitates the analysis. As in (7), we denote the outcome of an election when the incumbent has zero capacity by $E(\sigma_v, K, i_{t-1}) \in \{L, R\}$. Since candidates are myopic, Proposition 1 still holds in this modified setting. However, the behavior of farsighted voters is no longer given by Proposition 2. Along the same lines as the proof of this latter proposition, the following result with respect to the robustness of Theorem 1 can be shown:[20]

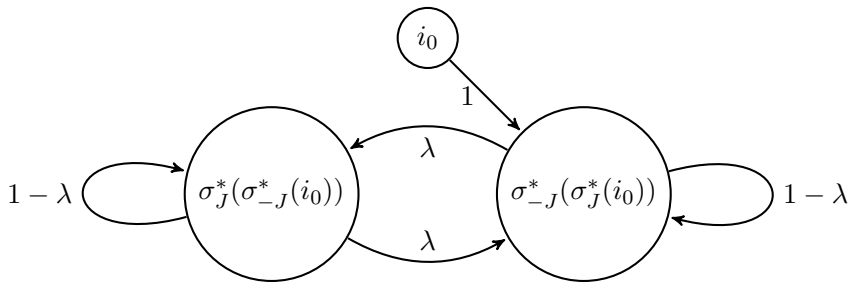
[20] The detailed proof of Theorem 3 is available on request.

**Theorem 3**

*Let $i_0 \in \mathcal{I}$ be the status-quo policy in $t = 1$. Then there exists some $\widehat{\theta} > 0$ such that for all $0 < \theta < \widehat{\theta}$, $\mathcal{G}_\theta^{i_0}$ has a unique P-MSMPE, referred to as $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$ and characterized as follows:*

*(i) If $0 \leq \Gamma^{i_0} < \Pi + \frac{c}{2}$, then $\sigma^*$ is characterized by the Markov transition diagrams given in (i)–(iii) of Theorem 1.*

*(ii) If $\Pi + \frac{c}{2} \leq \Gamma^{i_0} \leq \frac{1}{2}$, then $\sigma^*$ is characterized by*




*where the equilibrium policy choices are determined according to the best response given in Proposition 1 and $J \in \{L, R\}$ is a party such that $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$.*

Statement *(i)* of the above result states that Theorem 1 is robust to voters being farsighted if $\theta$ is low and $\Gamma^{i_0} < \Pi + \frac{c}{2}$. The proof is based on the continuity in $\theta$ of the median voter's objective function. However, Theorem 1 fails to describe the equilibrium dynamics of the extended model if $\Gamma^{i_0} \geq \Pi + \frac{c}{2}$. Let us briefly explain why we cannot use a continuity argument in this case. By Theorem 1*(iii)*, it follows that if $\Gamma^{i_0} \geq \Pi + \frac{c}{2}$ and $\theta = 0$, the policy implemented by the first-period office-holder $J$ satisfies $\left|\sigma_J^*(i_0) - \frac{1}{2}\right| = \Pi + \frac{c}{2}$. Thus, if $J$ has normal capacity, the second election is a tie and $J$ is re-elected. Now, if $\theta > 0$, this tie turns into a majority for the challenger. By sequential rationality, the voters anticipate in the first election that $J$ would lose the second election and thus $-J$ is already elected into office in $t = 1$. This corresponds to the behavior described by the diagram in Theorem 3*(ii)*.

Although for large initial policy polarization the equilibrium behavior with non-myopic voters is not the same as in the baseline model, we can show that as long as $\theta$ is sufficiently low Theorem 2 is robust to voters being farsighted:[21]

[21] The detailed proof of Theorem 4 is available on request.

**Theorem 4**

*Let $\lambda \in (0,1]$ and $\beta \in (0,1)$, and let also $i_0 \in \mathcal{I}$ be the status-quo policy in $t = 1$. Then there exists $\tilde{\theta} > 0$ such that, for all $0 < \theta < \tilde{\theta}$, statements (i) and (ii) of Theorem 2 hold in $\sigma^* = (\sigma_v^*, \sigma_L^*, \sigma_R^*)$, the unique P-MSMPE of $\mathcal{G}_\theta^{i_0}$.*

## 6.2 Non-myopic parties

We assume next that parties (but not voters) are non-myopic and investigate how the results in Theorems 1 and 2 change under these circumstances. We have both parties discounting future payoffs with a common discount factor $\psi \in (0,1)$. This parameter refers to the sophistication of parties' behavior regarding future outcomes. As in Section 6.1, $\psi$ might not coincide with the social discount factor $\beta$. Since voters are myopic, they only foresee (and care about) the policies that each of the candidates would implement in the first term after the elections. Given the status-quo policy $i_0 \in \mathcal{I}$, we use $\mathcal{G}_\psi^{i_0}$ to denote the modification of the game $\mathcal{G}^{i_0}$ where parties' discount factor is $\psi$. We need to adapt the notion of equilibrium used in our baseline model to this modified setting. To do so, we first define a particular subset of the set of Stationary Markov strategies.

**Definition 4**

*A Simple Stationary Markov Strategy profile $\sigma = (\sigma_v, \sigma_L, \sigma_R)$ is a Stationary Markov Strategy profile where for each party $K \in \{L, R\}$ the best-response function $\sigma_K : \mathcal{I} \to \mathcal{I}$ that maps the status-quo policy into the policy of the current period can be written as*

$$\sigma_K(i_{t-1}) = \min\left\{\max\left\{\mu_K - \chi_K^-, i_{t-1}\right\}, \mu_K + \chi_K^+\right\} \tag{9}$$

*for $\chi_K := (\chi_K^-, \chi_K^+) \in [0, \mu_K] \times [0, \mu_{-K}]$ such that $\chi_R^- = \chi_L^+ = \underline{\chi}$ and $\chi_R^+ = \chi_L^- = \overline{\chi}$.*

A party strategy that is part of a Simple Stationary Markov Strategy profile is called *simple*. According to (9), when parties play in accordance with a simple strategy, they are actually choosing the level of persistence in policy-making that they will apply anytime they are in power. Note that we also assume that parties' strategies are symmetric in the sense that $\chi_K^- = \chi_{-K}^+$. Since a simple strategy for party $K$ can be fully characterized by the pair $\chi_K = (\chi_K^-, \chi_K^+)$, we simplify (and slightly abuse) notation and write $\sigma_K = \chi_K$. Figure 5 is a generalization of Figure 2 and illustrates the shape of a simple strategy of a party.

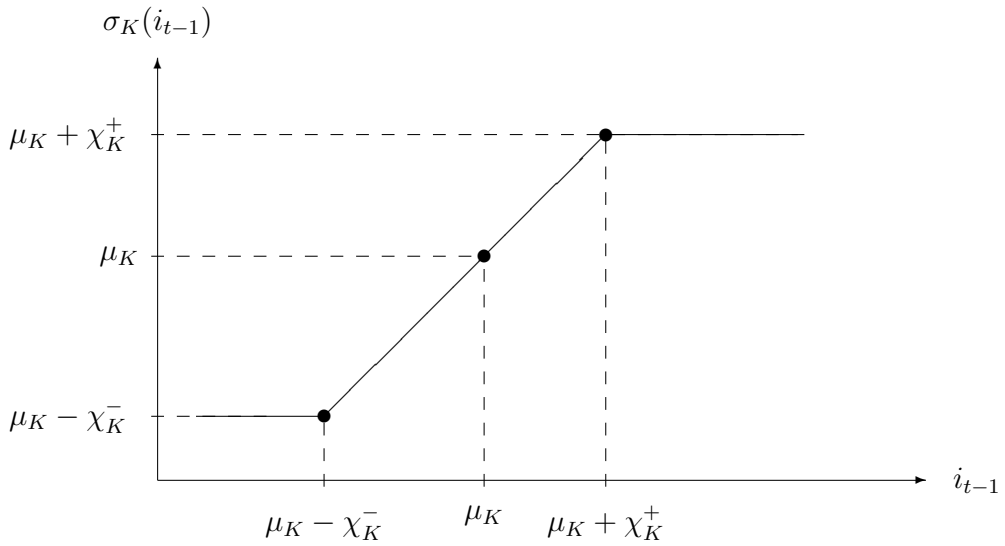


Figure 5: A Simple Strategy for Party $K$ given the status-quo policy $i_{t-1}$.

We are now in a position to define the notion of equilibrium for $\mathcal{G}_\psi^{i_0}$.

**Definition 5**

*A Voter-Myopic Stationary Markov Perfect Equilibrium (V-MSMPE) of $\mathcal{G}_\psi^{i_0}$ is a profile of Simple Stationary Markov Strategies $\sigma^* = (\sigma_v^*, \chi_L^*, \chi_R^*)$ such that for each $t \in \{1, 2, \ldots\}$, each $i_{t-1} \in \mathcal{I}$, and $k \in K$ denoting the incumbent in period $t$, we have*

$$\chi_K^*(i_{t-1}) \in \underset{i_t \in \mathcal{I}}{\operatorname{argmax}} \left\{ U_K\left(i_{t-1}, i_t, a_k\right) + \mathbb{E}_{\mathcal{S}} \left[ \sum_{t' \geq t+1} \psi^{t'-t} \cdot U_K\left(i_{t'-1}, i_{t'}, g_{t'}\right) \right] \right\}$$

*and, for all $i \in [0, 1]$,*

$$\sigma_i^*(K, i_{t-1}) = K \Leftrightarrow U_i\left(i_{t-1}, \chi_K^*(i_{t-1}), a_k\right) \geq U_i\left(i_{t-1}, \chi_{-K}^*(i_{t-1}), a_{-k}\right),$$

*where $\mathcal{P}(\mathcal{S}, \sigma^*, i_t) = (i_t, i_{t+1}, i_{t+2}, \ldots)$ (or $\mathcal{A}(\mathcal{S}, \sigma^*, i_t) = (g_{t+1}, g_{t+2}, \ldots)$) is the path of policies in $\mathcal{I}$ (or outputs in $\mathcal{B}$) that follows $i_t$, the decision of the office-holder, when $\mathcal{S} = (s_{t+1}, s_{t+2}, \ldots)$ is the realization of the stochastic process containing the capacity shocks and voters and parties decide in accordance with $\sigma^*$.*

We assume again that $A \gg 0$, so that incumbents whose capacity is low are never re-elected, and also use $E(\sigma_v, K, i_{t-1}) \in \{L, R\}$ to denote the outcome of an election when the incumbent has zero capacity, as in (7). To solve the model, i.e. to find the equilibria of the game, we first need to find the optimal level of policy persistence for parties and then characterize the optimal

behavior of voters. This enables us to formulate the following result, which is the counterpart of Theorem 1 when parties are farsighted:[22]

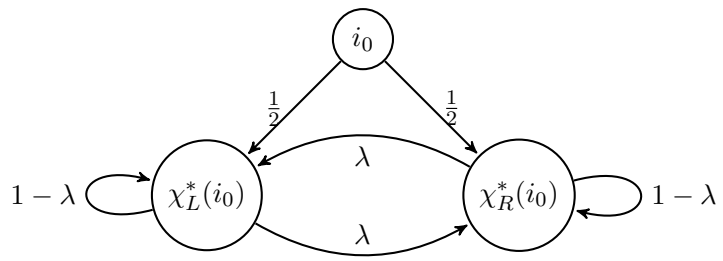
**Theorem 5**

*Let* $c \in \left[0, \frac{2\Pi}{1+\psi(2\lambda-1)}\right] \cup \left[\frac{2\Pi+1}{1-\psi}, \infty\right)$ *and let* $i_0 \in \mathcal{I}$ *be the status-quo policy in* $t = 1$*. Then* $\mathcal{G}_\psi^{i_0}$ *has a unique V-MSMPE, referred to as* $\sigma^* = (\sigma_v^*, \chi_L^*, \chi_R^*)$*. Depending on* $\Gamma^{i_0}$*,* $\sigma^*$ *is characterized by one of the following Markov transition diagrams, where*

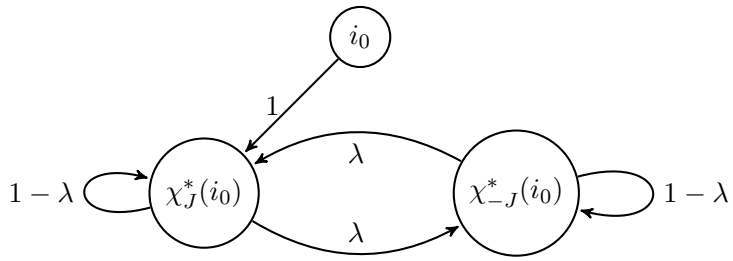
$$\chi_L^* = (\overline{\chi}^*, \underline{\chi}^*) = \left(\frac{c}{2} \cdot (1-\psi), \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right),$$
$$\chi_R^* = (\underline{\chi}^*, \overline{\chi}^*) = \left(\frac{c}{2} \cdot [1+\psi(2\lambda-1)], \frac{c}{2} \cdot (1-\psi)\right),$$

*and* $J \in \{L, R\}$ *is a party such that* $|\mu_J - i_0| \leq |\mu_{-J} - i_0|$*:*

*(i) If* $\Gamma^{i_0} = 0$*, then* $\sigma^*$ *is characterized by*

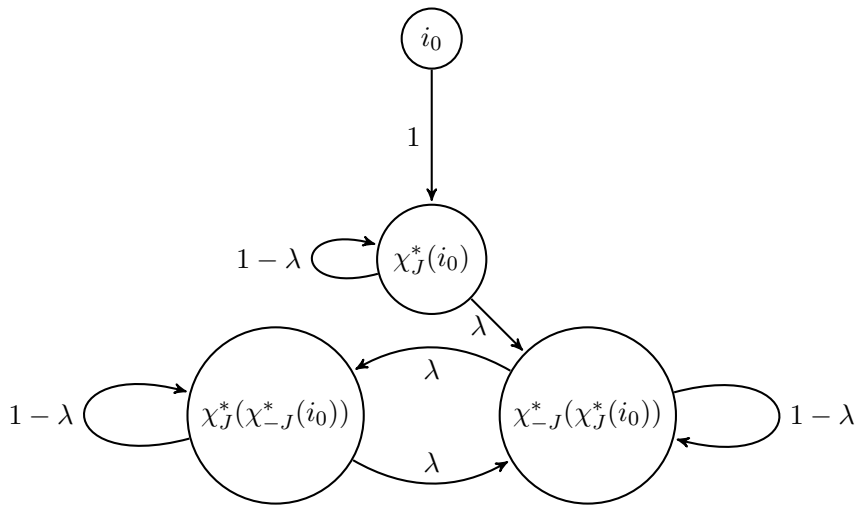



*(ii) If* $0 < \Gamma^{i_0} \leq \min\left\{\left|\Pi - \underline{\chi}^*\right|, \frac{1}{2}\right\}$*, then* $\sigma^*$ *is characterized by*




*(iii) If* $\min\left\{\left|\Pi - \underline{\chi}^*\right|, \frac{1}{2}\right\} < \Gamma^{i_0} \leq \frac{1}{2}$*, then* $\sigma^*$ *is characterized by*

[22] The detailed proof of Theorem 5 is available on request.




The above result shows that, except for $c \in \left(\frac{2\Pi}{1+\psi(2\lambda-1)}, \frac{2\Pi+1}{1-\psi}\right)$, the equilibrium dynamics of the extended model are qualitatively identical to those described in Theorem 1. The case of intermediate values of $c$, in which there exists no equilibrium, is discussed below. When $c$ is not intermediate, however, the existence (and uniqueness) of equilibria is guaranteed and the transition probabilities are the same as in the baseline model. More particularly, this means that the policy path also tends to a stochastic alternation between two policies equidistant to $\frac{1}{2}$. However, since office-holders are farsighted, the equilibrium policy choices now depend on $\psi$. If $\psi$ tends to zero, the baseline model is recovered. The reason is that

$$\lim_{\psi \to 0} \overline{\chi}^* = \lim_{\psi \to 0} \underline{\chi}^* = \frac{c}{2}.$$

Two observations concerning policy choices are worth mentioning. First, if $\Gamma^{i_0} > \left|\Pi - \underline{\chi}^*\right|$ and thus initial polarization is large, the first-period office-holder will implement an *extreme* policy during his tenure in office until he is deselected. The baseline model also features such a property. However, because $\overline{\chi}^* = \frac{c}{2} \cdot (1-\psi) < \frac{c}{2}$, such an extreme policy will be closer to the median voter's preferred policy than under the assumption that parties (and office-holders) are myopic. The intuition behind this moderation effect is the following: While a myopic first-period office-holder merely weighs $U^c(i_1, i_0)$ against $U_K^{\mathcal{I}}(i_1)$, a farsighted candidate anticipates (on behalf of his party) that accepting a larger policy shift in the first period (which will bring about a higher cost) may increase the party's expected future utility. As a consequence, non-myopic candidates will choose policies in the first period that are weakly closer to their bliss-point and hence more moderate, because the initial status-quo policy is very extreme. This moderating effect is stronger, the larger $\psi$ is.

Second, by Theorem 5, if the marginal cost of change is low, namely if $c \leq \frac{2\Pi}{1+\psi(2\lambda-1)}$, the long-term policies are now given by

$$\mu_L + \underline{\chi}^* = \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \quad \text{and} \quad \mu_R - \underline{\chi}^* = \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)].$$

If and only if $\lambda > \frac{1}{2}$ these policies are more moderate than their counterparts from Proposition 1, $\mu_L + \frac{c}{2}$ and $\mu_R - \frac{c}{2}$. On the other hand, if $c \geq \frac{2\Pi+1}{1-\psi}$, the long-term policies are more moderate than in the baseline model if and only if $\lambda < \frac{1}{2}$. Accordingly, precisely how the parties' farsightedness will affect long-term policy polarization is ambiguous and depends on the parameters $c$ and $\lambda$. In particular, the fact that parties are farsighted may not be beneficial for society in the long run.

Finally, recall that we have said that the equilibrium described in Theorem 5 does not exist for all $c \in [0, \infty)$. If $c \in \left(\frac{2\Pi}{1+\psi(2\lambda-1)}, \frac{2\Pi+1}{1-\psi}\right)$, in particular, it can be shown that the only potential equilibrium—playing according to $\chi_L^* = (\overline{\chi}^*, \underline{\chi}^*)$ and $\chi_R^* = (\underline{\chi}^*, \overline{\chi}^*)$, as described in Theorem 5—is not an equilibrium for all $i_0$ with $0 \leq \Gamma^{i_0} \leq \left|\Pi - \underline{\chi}^*\right|$. More precisely, upon observing a status-quo policy $i_{t-1}$ such that $\mu_R - \underline{\chi}^* < i_{t-1} < \mu_R - \overline{\chi}^*$, an $R$-candidate has an incentive to move away from $i_{t-1}$ and choose a policy closer to his bliss-point. Analyzing the behavior of welfare as a function of $c$ is thus not possible for all $c \in [0, \infty)$. This implies that we cannot state a formal result concerning the robustness of Theorem 2 to the parties' being non-myopic. However, it follows from continuity in $\psi$ that for positive but small $\psi$, the qualitative behavior of welfare as a function of $c \in \left[0, \frac{2\Pi}{1+\psi(2\lambda-1)}\right] \cup \left[\frac{2\Pi+1}{1-\psi}, \infty\right)$ is unchanged if political parties are concerned with outcomes in future elections.

## 6.3 Convex costs of change

In this last extension we analyze the robustness of our findings to costs of change being convex instead of linear. More precisely, we consider a variation of the game $\mathcal{G}^{i_0}$, which we denote by $\mathcal{G}_\eta^{i_0}$. We assume that

$$U^c(i_{t-1}, i_t) = -c \cdot |i_{t-1} - i_t|^\eta, \tag{10}$$

with $\eta > 1$. The assumption in (10) influences the candidates' and the voters' behavior. Due to the non-linearity of the cost-of-change term, the optimal policy choice and the optimal election decision are given in each period by an implicit function of the previous policy choice.[23] With non-linear costs of change, the best response of the office-holder in $t = 2$ exhibits full responsiveness

[23] Details of the analysis describing the agents' behavior are available on request—see also Gersbach et al. (2015).

to changes of the status-quo policy. This is different to the case where costs of change are linear. Figure 6 illustrates this for different values of the cost parameter $c$.

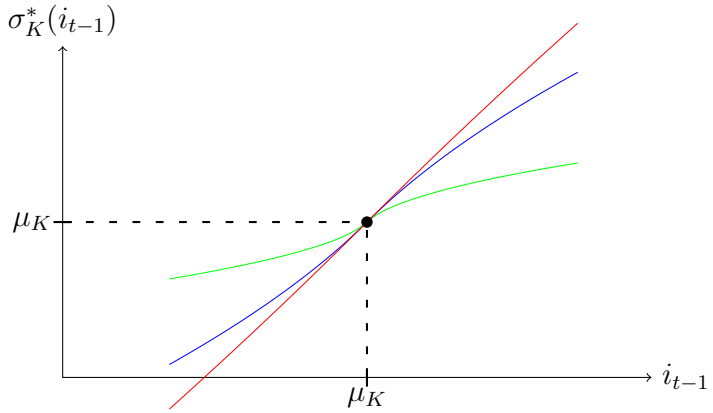



Figure 6: Best response of an office-holder with bliss point $\mu_K$ to policy $i_{t-1}$ for $c = 1/3$ (green), $c = 4/3$ (blue), and $c = 4$ (red) when costs of change are strictly convex, with $\eta = 3/2$.

When costs of change are convex, we cannot therefore analytically derive all the components needed to describe the equilibrium behavior using Markov transition diagrams. As a consequence, we cannot derive a closed-form expression of $W(c)$. However, we can numerically compute all equilibrium paths of policy choices—and thus the corresponding re-election probabilities—for any finite-time horizon $T$ and then analyze the resulting values of welfare, which we denote by $W_T(c)$.

In Figure 7, we can see that $W(c) = \lim_{T\to\infty} W_T(c)$ is approximated reasonably well by $W_T(c)|_{T=10}$ for $\beta = 0.4$.

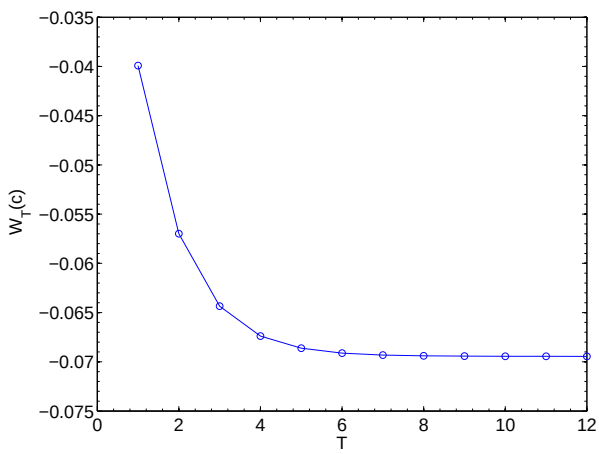
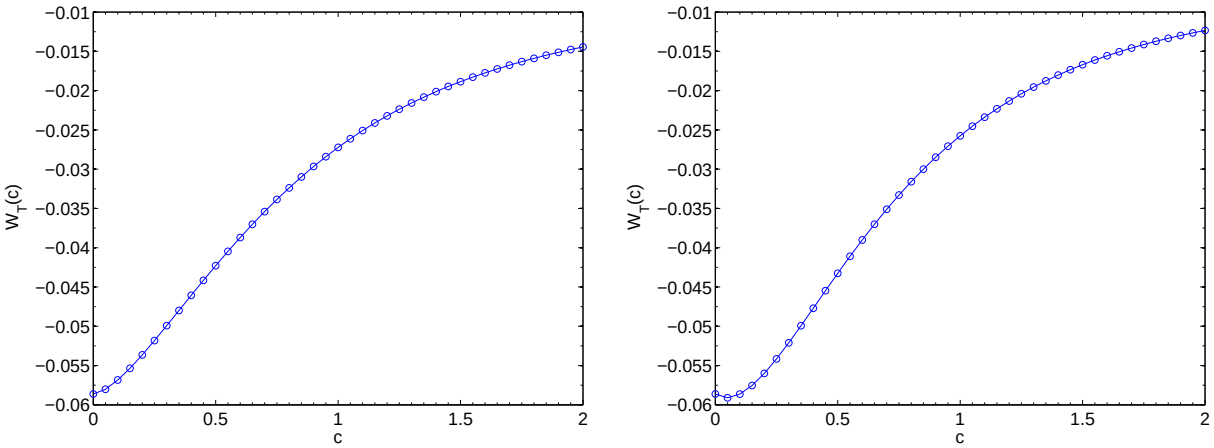



Figure 7: Simulated values of welfare as a function of the time horizon $T$, for $c = 0.3$ and $\beta = 0.4$. The other parameter values are $\Gamma^{i_0} = 0$, $\lambda = 0.2$, $\eta = 1.5$, and $\Pi = 0.3$.

To explore the qualitative behavior of $W(c)$, we thus define the time horizon as $T = 10$ and analyze the numerically computed values of $W_T(c)|_{T=10}$ for varying parameter values. Figures 8(a)–8(b) depict welfare as a function of $c$ when initial policy polarization is low.

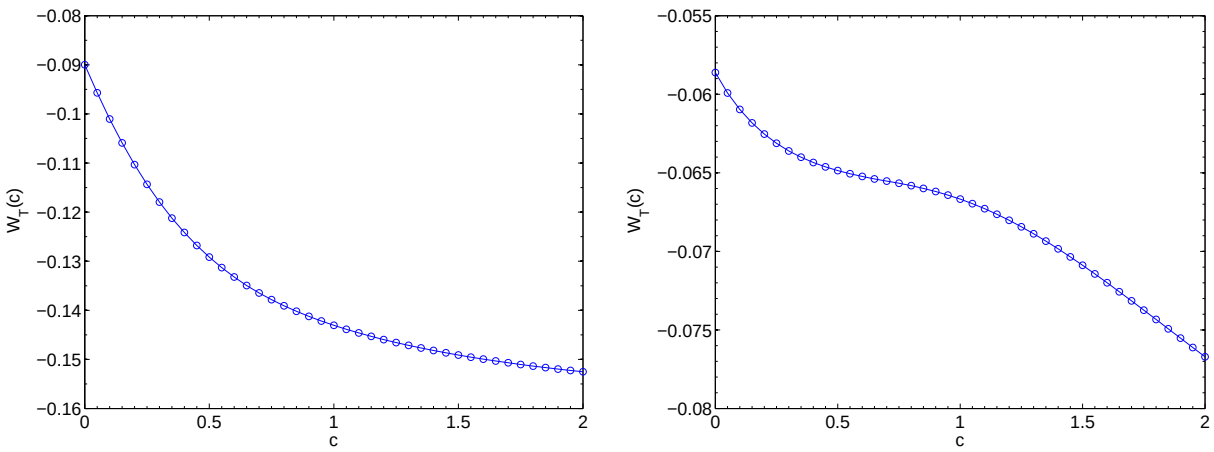



(a) $W_T(c)$ for $\lambda = 0.2$.

(b) $W_T(c)$ for $\lambda = 0.6$.

Figure 8: Welfare as a function of $c \geq 0$, for $\Gamma^{i_0} = 0.1$. The following parameter values have been used: $\Pi = 0.3$, $\beta = 0.9$, $\eta = 1.5$, $T = 10$, and $\lambda = 0.2$ (or $\lambda = 0.6$).

The corresponding graphs for a larger value of $\Gamma^{i_0}$ are given in Figures 9(a)–9(b).




(a) $W_T(c)$ for $\beta = 0.4$.

(b) $W_T(c)$ for $\beta = 0.9$.

Figure 9: Welfare as a function of $c \geq 0$, for $\Gamma^{i_0} = 0.4$. The following parameter values have been used: $\Pi = 0.3$, $\lambda = 0.2$, $\eta = 1.5$, $T = 10$, and $\beta = 0.4$ (or $\beta = 0.9$).

Figures 7–9(b) are evidence for a strong robustness of Theorem 2 with respect to convexity of costs

of change. More precisely, if $\Gamma^{i_0}$ is low, welfare is not highest for small marginal-cost-of-change levels, whereas welfare is highest for small values of $c$ if $\Gamma^{i_0}$ is large. Moreover, for low values of $\Gamma^{i_0}$, any $c > 0$ yields higher welfare than zero costs of change—given that $\lambda$ is small and $\beta$ is sufficiently large. As in Theorem 2, this is not necessarily the case if initial policy polarization is large.

# 7 Our Model Predictions: A Discussion

Over and above the theoretical insights, the model and the results enable us to formulate a series of predictions regarding actual policy-making, all of which could be empirically tested. To recall, we have shown that:

1. Incumbents have an advantage over challengers in elections.
2. In the long run, there is an alternation between policies, the distance between which is proportional to (but lower than) the difference in the parties' bliss points. Moreover, policy shifts occur only when a new party comes into office. In particular, the incumbents' policies are persistent during their tenure.
3. Intermediate levels of marginal costs of change lead to lower levels of policy polarization in the long run.
4. If political instability is high, the more shortsighted the candidates are, the higher policy polarization is.

A confirmation—or a rebuttal—of the above hypotheses requires a detailed analysis that lies beyond the scope of the present paper. We will limit ourselves to a brief discussion of each of the above four hypotheses in the light of other contributions in the literature and some empirical evidence. Doing so will provide support for our modeling assumptions and results.

First, in the last few decades, over 90% of the incumbents in the US House of Representatives have been successful when standing for re-election (Levitt and Wolfram, 1997). Erikson et al. (1993) find that US governors have comparable advantages when seeking re-election: since World War II the re-election rates of governors have been rising and reached nearly 90% in the period 2010–2013.[24] Many scholars argue that these high re-election rates are partially due to the so-called

[24]The growing re-election rates can be found under `http://governors.rutgers.edu/on-governors/us-governors/when-governors-seek-re-election` (retrieved 18 May 2015).

incumbency advantage: being in office offers politicians a range of policy and non-policy tools that are not available to their challengers. In our model, the incumbency advantage is a consequence of the possibility for incumbents to leverage on the partial commitment tool offered by costs of change. This enables incumbents to choose a policy that will guarantee their re-election unless they experience a negative (exogenous) shock.

Second, the number of policy dimensions a government can influence is large. Policy alternation can thus be defined for a bundle of policies, or rather on a policy-by-policy basis. Focusing on the highest marginal tax rate in the US as a remarkable example of a one-dimensional policy, both the Democratic and Republican administrations have been reversing their opponent's decisions ever since Reagan's tax cuts.[25] Policy alternation is featured in some recent papers such as Nunnari and Zápal (2014) as a matter of fact. Additionally, as Wiesehomeier and Benoit (2009) show for the case of many Latin-American countries, polarization in parties' ideologies tends to be more pronounced than the differences in actual policies chosen by different parties. The fact that policy polarization is lower than party polarization is also discussed in Alesina and Rosenthal (2000). Further, Budge et al. (2010) argue that parties become more partisan right after winning elections, less partisan right after losing elections, and that they do not tend to substantially change policies while they are in power (i.e., policies are persistent unless government switches from one party to the other). Budge et al. (2010) also show that, to some extent, this pattern is consistent with what can be observed in US politics. Policy persistence, in particular, is a property evidenced in abundant theoretical and empirical research—we refer to Section 2 and Forand (2014) in particular, as well as the examples and references therein.

Third, in a narrow sense the parameter $c$ corresponds to the costs imposed on all citizens per unit of policy change. However, in a broader sense costs of change can also be related to institutional factors of the political system, e.g. checks and balances. In most democracies, the larger the policy reform, the more hurdles—judiciary sanction, qualified majority, or majority in two chambers, for instance—the proponents of the change have to overcome. The potential impact of such hurdles on policy-making has been discussed in the literature: Diermeier and Myerson (1999) study the consequences of bicameralism, Shepsle and Weingast (1987) discuss the power of committees in legislatures, and Huber (1992) studies further restrictive legislative procedures. It is well-known that a strong system of checks and balances can lead to legislative gridlock, as was already a concern for Hamilton and Madison in the wake of the new Constitution for the US. Nevertheless, our results indicate that even filibuster-like instruments that are used, for instance, in the US Senate

[25] Source: Tax Foundation, see `http://taxfoundation.org/` (retrieved 18 November 2016).

to delay or block policy changes may not always have a negative impact. Indeed, in democracies with low or medium political polarization they may have a positive impact. The reason is that intermediate difficulties in changing the status quo may prevent moderate policies that appeal to the median voter from being replaced by extreme policies targeted at the constituency of one party only. These and other instruments that create hurdles in the policy-making process may nonetheless become harmful when policy polarization is high. To sum up, our paper offers a rationale for a developed system of checks and balances as a stabilizer in the policy-making process in democracy.[26]

Fourth, there seems to be widespread agreement that policy polarization has increased in recent times, especially in the US for both the House of Representatives and the Senate (see e.g. Poole and Rosenthal (2001) and Theriault (2008)) and for presidential platforms (see e.g. Budge et al. (2001)). Simultaneously, political systems all over the world seem to be increasingly unstable, partly due to a rise in electoral support for outsiders.[27] This has eroded the control of party elites and has added a great deal of uncertainty, eventually leading to outcomes as unexpected as Brexit or the election of Donald Trump. The currently observable high levels of policy polarization are potentially due to the parties' lack of farsightedness in politically unstable times or the parties' loss of control over their candidates' agenda.

# 8 Conclusion

We have developed an infinite-horizon model of electoral competition to analyze the long-term social consequences of policy changes being costly. We have found that the equilibrium policy path tends to an infinite sequence of policies that are equidistant to the median voter and more moderate than the office-holders' bliss points. The behavior of welfare as a function of $c$, the marginal cost of change, is driven by the following trade-off: On the one hand, low values of $c$ avoid a potential transition from extreme to moderate policies being overly costly. On the other hand, intermediate values of $c$ lead to more moderate long-term policies. Moreover, as long as the initial policy is close to the median and the social discount factor is sufficiently large, welfare is highest for intermediate marginal-cost-of-change levels and positive costs of change never decrease

[26]This Aristotelian flavor is not exclusive to the costs of change in our model. For instance, there is evidence indicating a positive correlation between a constitution of medium page length (long enough to be relevant, but short enough to be flexible) and GDP per capita—see e.g. Tsebelis and Nardi (2016) for OECD countries.

[27]For more details, we refer to e.g. `http://www.forbes.com/sites/bowmanmarsico/2015/09/16/political-insiders-and-outsiders-by-the-numbers/#1a69f41072fa` (retrieved 17 November 2016).

welfare. By contrast, for high levels of initial policy polarization welfare is highest for low or zero marginal costs of change.

Our analysis seems likely to open up a variety of further inquiries and applications. For instance, the impact of costs of change in systems with more than two parties or the possibility of endogenizing party platforms are possible avenues for further research. Remarkably, our results also suggest that the major effects of costs of change on long-term policy outcomes already arise when voters and office-holders look only one term ahead. In turn, from a behavioral perspective, this may provide a rationale for issues such as why voters only concentrate on the consequences of their votes for the next term. This property of voter behavior calls for further investigation.

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# Appendix

**Proof of Proposition 1**

Let $k \in K$ be the office-holder in period $t$, with $t \geq 1$. For a given $(t-1)$-th period policy choice $i_{t-1}$, $k$ chooses policy $i_t$ so that his party's instantaneous utility in period $t$, as given in (2), is maximized. Recall that candidates are myopic and do not care about re-election. So the office-holder maximizes

$$U_K(i_{t-1}, i_t, g_t) = a_{kt} - (i_t - \mu_K)^2 - c \cdot |i_{t-1} - i_t|.$$

We note that $U_K(i_{t-1}, i_t, g_t)$ is differentiable with respect to $i_t$ on $(0,1)\backslash\{i_{t-1}\}$. We distinguish two cases, depending on the relative size of $i_{t-1}$ and $\mu_K$.

<u>*Case 1:*</u> $i_{t-1} < \mu_K$

In this case,

$$\frac{dU_K(i_{t-1}, i_t, g_t)}{di_t} = \begin{cases} -2(i_t - \mu_K) + c & \text{if } 0 < i_t < i_{t-1}, \\ -2(i_t - \mu_K) - c & \text{if } i_{t-1} < i_t < 1. \end{cases}$$

Therefore, $U_K(i_{t-1}, i_t, g_t)$ is strictly increasing if and only if

$$0 < i_t < \max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\},$$

which implies that

$$\underset{i_t \in [0,1]}{\operatorname{argmax}}\, U_K(i_{t-1}, i_t, g_t) = \left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}\right\}.$$

<u>*Case 2:*</u> $i_{t-1} \geq \mu_K$

Analogous reasoning leads to

$$\underset{i_t \in [0,1]}{\operatorname{argmax}}\, U_K(i_{t-1}, i_t, g_t) = \left\{\min\left\{\mu_K + \frac{c}{2}, i_{t-1}\right\}\right\}.$$

Finally, combining *Case 1* and *Case 2* yields

$$\underset{i_t \in [0,1]}{\operatorname{argmax}}\, U_K(i_{t-1}, i_t, g_t) = \left\{\min\left\{\max\left\{\mu_K - \frac{c}{2}, i_{t-1}\right\}, \mu_K + \frac{c}{2}\right\}\right\}.$$

This completes the proof.

□

**Proof of Proposition 2**

Without loss of generality, we can assume that $i_{t-1} \in \mathcal{I} \cap \left[\frac{1}{2}, 1\right]$. The results for $i_{t-1} \in \mathcal{I} \cap \left[0, \frac{1}{2}\right)$

follow by symmetry. Let $K \in \{\emptyset, L, R\}$ denote the incumbent's party at the moment of the $t$-th election. We will now assume that $K \neq \emptyset$ and provide a detailed proof of statement *(ii)*. Statement *(i)* follows immediately by adapting the tie-breaking rule.

When deciding whether to elect $k \in R$ or $k' \in L$ in the $t$-th election, any voter $i \in [0,1]$ will compare the instantaneous utilities that he receives from the two candidates' being in office. We can assume that both candidates have capacity $a_t = 0$, because a low-capacity incumbent is never re-elected anyway. So voter $i$ strictly prefers $k \in R$ to be in office if

$$\Delta U_i(i_{t-1}) := U_i(i_{t-1}, i_{kt}, g_{kt}) - U_i(i_{t-1}, i_{k't}, g_{k't}) > 0,$$

which, by (1), is equivalent to

$$\Delta U_i(i_{t-1}) = -(i_{kt} - i)^2 - c \cdot |i_{t-1} - i_{kt}| + (i_{k't} - i)^2 + c \cdot |i_{t-1} - i_{k't}| > 0.$$

From Proposition 1, we know that in any MSMPE $(\sigma_v^*, \sigma_L^*, \sigma_R^*)$ of $\mathcal{G}^{i_0}$, the policy choices $i_{kt}$ and $i_{k't}$ are given by $\sigma_R^*(i_{t-1})$ and $\sigma_L^*(i_{t-1})$, respectively. Thus

$$\Delta U_i(i_{t-1}) = -[\sigma_R^*(i_{t-1}) - i]^2 - c \cdot |i_{t-1} - \sigma_R^*(i_{t-1})| + [\sigma_L^*(i_{t-1}) - i]^2 + c \cdot |i_{t-1} - \sigma_L^*(i_{t-1})|.$$

Since $i_{t-1} \geq \frac{1}{2}$, it follows that $J = R$ and $-J = L$, where $J \in \{L, R\}$ satisfies $|\mu_J - i_{t-1}| \leq |\mu_{-J} - i_{t-1}|$. Moreover, an incumbent is re-elected if and only if he receives a vote-share of at least $\frac{1}{2}$. Thus, if $K = J$—i.e., $k \in J = R$ is the incumbent—, the incumbent will be re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \geq 0,$$

because $\Delta U_i(i_{t-1})$ is increasing in $i$. Analogously, if $K = -J$—i.e., $k' \in -J = L$ is the incumbent—, the incumbent will be re-elected if and only if

$$\Delta U_{\frac{1}{2}}(i_{t-1}) \leq 0.$$

We now distinguish two different cases regarding the relative size of $c$ and $\Pi$ and analyze the sign of $\Delta U_{\frac{1}{2}}(i_{t-1})$ as a function of $i_{t-1}$ in each case.

<u>*Case 1:*</u> $\frac{c}{2} < \Pi$

*Case 1a:* $i_{t-1} \in \left[\frac{1}{2}, \mu_R - \frac{c}{2}\right]$
In this case, $\sigma_R^*(i_{t-1}) = \mu_R - \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$ and thus

$$\Delta U_i(i_{t-1}) = 2i(2\mu_R - 1 - c) - (2\mu_R - 1 - 2ci_{t-1}).$$

As a consequence, $\Delta U_{\frac{1}{2}}(i_{t-1}) > 0$ for $i_{t-1} \in \left(\frac{1}{2}, \mu_R - \frac{c}{2}\right]$ and $\Delta U_{\frac{1}{2}}(i_{t-1}) = 0$ for $i_{t-1} = \frac{1}{2}$. So, if $k \in R$ is the incumbent—i.e., $K = J$—, he will always be re-elected. On the other hand, if $K = -J = L$, an incumbent will only be re-elected for $i_{t-1} = \frac{1}{2}$.

*Case 1b:* $i_{t-1} \in \left(\mu_R - \frac{c}{2}, \mu_R + \frac{c}{2}\right)$
Since $\sigma_R^*(i_{t-1}) = i_{t-1}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, we obtain

$$\Delta U_{\frac{1}{2}}(i_{t-1}) = \underbrace{\left(\mu_L + \frac{c}{2} - i_{t-1}\right)}_{<0} \cdot \underbrace{\left(\mu_L - \frac{c}{2} + i_{t-1} - 1\right)}_{<0} > 0.$$

Thus, the incumbent is always re-elected if $K = J = R$, whereas the incumbent is never re-elected if $K = -J = L$.

*Case 1c:* $i_{t-1} \geq \mu_R + \frac{c}{2}$
Since $\sigma_R^*(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, it follows that $\Delta U_{\frac{1}{2}}(i_{t-1}) = 0$ for all $i_{t-1} \geq \mu_R + \frac{c}{2}$. Thus, for both $K = J = R$ and $K = -J = L$, the incumbent is always re-elected.

<u>*Case 2:*</u> $\frac{c}{2} \geq \Pi$

*Case 2a:* $i_{t-1} \in \left[\frac{1}{2}, \mu_L + \frac{c}{2}\right]$
In this case, $\sigma_R^*(i_{t-1}) = \sigma_L^*(i_{t-1}) = i_{t-1}$, so that $\Delta U_i(i_{t-1}) = 0$ for all $i \in [0, 1]$ and all $i_{t-1} \in \left[\frac{1}{2}, \mu_L + \frac{c}{2}\right]$. Thus, incumbents are always re-elected.

*Case 2b:* $i_{t-1} \in \left(\mu_L + \frac{c}{2}, \mu_R + \frac{c}{2}\right)$
Now, $\sigma_R^*(i_{t-1}) = i_{t-1}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$. Thus, we obtain the same results as in *Case 1b*.

*Case 2c:* $i_{t-1} \geq \mu_R + \frac{c}{2}$
Since $\sigma_R^*(i_{t-1}) = \mu_R + \frac{c}{2}$ and $\sigma_L^*(i_{t-1}) = \mu_L + \frac{c}{2}$, it follows that incumbents are always re-elected, as in *Case 1c*.

The combination of *Cases 1–2* yields statement *(ii)* of Proposition 2. As mentioned above, statement *(i)* can be shown by using the first-election tie-breaking rule in the computations of *Cases 1–2*. This completes the proof of Proposition 2.

□

**Proof of Theorem 2**

From Theorem 1 it follows that, for given $\Gamma^{i_0} \in \left[0, \frac{1}{2}\right]$ and $\Pi \in \left[0, \frac{1}{2}\right]$, the expression for $W(c)$ depends on whether $c \geq 0$ satisfies

$$\textit{1)}\ \left|\Pi - \frac{c}{2}\right| \geq \Gamma^{i_0} \quad \text{or} \quad \textit{2)}\ \left|\Pi - \frac{c}{2}\right| < \Gamma^{i_0}.$$

The first case combines *(i)* and *(ii)* of Theorem 1 and the second case corresponds to *(iii)* of Theorem 1. Subsequently, the following definitions will be useful:

$$\omega := \max\left\{\Pi - \frac{c}{2}, 0\right\} \quad \text{and} \quad \gamma := \left|\sigma^*_{-J}(i_0) - \frac{1}{2}\right|, \tag{11}$$

where $J \in \{L, R\}$ denotes a party that satisfies

$$|\mu_J - i_0| \leq |\mu_{-J} - i_0|.$$

We will now derive an expression for $W(c)$ in each of the above two cases.

<u>*Case 1:*</u> $\left|\Pi - \frac{c}{2}\right| \geq \Gamma^{i_0}$

Let us compute the three components of $W(c)$, as defined in (8). From the Markov transition diagrams in *(i)* and *(ii)* of Theorem 1, it follows that

$$\begin{aligned} EU^c(c) &= (1-\beta)\cdot\left(-c\left|\sigma^*_J(i_0) - i_0\right| - 2c\omega\cdot\lambda\cdot\sum_{t=2}^{\infty}\beta^{t-1}\right) \\ &= -(1-\beta)\cdot c\cdot\left|\sigma^*_J(i_0) - i_0\right| - 2c\omega\beta\cdot\lambda \end{aligned}$$

and

$$EU^{\mathcal{I}}_{\frac{1}{2}}(c) = -(1-\beta)\cdot\sum_{t=1}^{\infty}\beta^{t-1}\gamma^2 = -\gamma^2.$$

Moreover, since a low-capacity incumbent is never re-elected in equilibrium, it follows that $EU^{\mathcal{B}}(c) = 0$. So, by (8),

$$W(c) = -\gamma^2 - (1-\beta)\cdot c\cdot\left|\sigma^*_J(i_0) - i_0\right| - 2c\omega\beta\cdot\lambda. \tag{12}$$

<u>*Case 2:*</u> $\left|\Pi - \frac{c}{2}\right| < \Gamma^{i_0}$

Using the Markov transition diagram in *(iii)* of Theorem 1, we can compute $EU^c(c)$ and $EU^{\mathcal{I}}(c)$. On the one hand,

$$\begin{aligned} EU^c(c) = &-c(1-\beta)\cdot\left[\left|\sigma^*_J(i_0) - i_0\right| + \lambda\left(\beta\left|\sigma^*_J(i_0) - \sigma^*_{-J}(i_0)\right| + \lambda\sum_{t=3}^{\infty}2\omega\beta^{t-1}\right)\right] \\ &+ (1-\lambda)\cdot\beta\cdot\left[EU^c(c) + c(1-\beta)\cdot\left|\sigma^*_J(i_0) - i_0\right|\right] \end{aligned}$$

and thus

$$\begin{aligned} EU^c(c) = -\frac{c\cdot(1-\beta)}{1-(1-\lambda)\beta}\cdot\Bigg[&\left|\sigma^*_J(i_0) - i_0\right|\cdot[1-\beta(1-\lambda)] \\ &+ \lambda\cdot\beta\cdot\left(\left|\sigma^*_J(i_0) - \sigma^*_{-J}(i_0)\right| + \frac{2\omega\beta}{1-\beta}\cdot\lambda\right)\Bigg]. \end{aligned} \tag{13}$$

On the other hand,

$$EU^{\mathcal{I}}_{\frac{1}{2}}(c) = -(1-\beta)\cdot\left[\left(\sigma^*_J(i_0)-\frac{1}{2}\right)^2+\lambda\cdot\frac{\gamma^2\beta}{1-\beta}\right]+(1-\lambda)\cdot\beta\cdot EU^{\mathcal{I}}_{\frac{1}{2}}(c),$$

which yields

$$EU^{\mathcal{I}}_{\frac{1}{2}}(c) = -\frac{(1-\beta)}{1-(1-\lambda)\beta}\cdot\left[\left(\sigma^*_J(i_0)-\frac{1}{2}\right)^2+\lambda\cdot\frac{\gamma^2\beta}{1-\beta}\right]. \tag{14}$$

Since $EU^{\mathcal{B}}(c)=0$, as in *Case 1*, (13) and (14) imply that

$$\begin{aligned} W(c) = -\Bigg\{ &\left(\sigma^*_J(i_0)-\frac{1}{2}\right)^2+\lambda\cdot\frac{\gamma^2\beta}{1-\beta}+c\cdot\Bigg[|\sigma^*_J(i_0)-i_0|\cdot[1-\beta(1-\lambda)] \\ &+\lambda\cdot\beta\cdot\left(\left|\sigma^*_J(i_0)-\sigma^*_{-J}(i_0)\right|+\frac{2\omega\beta}{1-\beta}\cdot\lambda\right)\Bigg]\Bigg\}\cdot\frac{(1-\beta)}{1-(1-\lambda)\beta}. \end{aligned} \tag{15}$$

To sum up, $W(c)$ is given by (12) and (15) in *Case 1* and *Case 2*, respectively. We will now build on (12) and (15) to analyze $W(c)$ as a function of $c\geq 0$ and to complete the proof of Theorem 2.

A) Proof of *Theorem 2(i)*

Let $\Gamma^{i_0}\leq\Pi$. Then,

$$\left|\Pi-\frac{c}{2}\right|\geq\Gamma^{i_0}\Leftrightarrow\frac{c}{2}\in[0,\min\{\mu_R-i_0,i_0-\mu_L\}]\cup[\max\{\mu_R-i_0,i_0-\mu_L\},\infty).$$

Therefore, $W(c)$ is given by

$$\begin{cases}(12) & \text{if } 0\leq\frac{c}{2}\leq\min\{\mu_R-i_0,i_0-\mu_L\},\\ (15) & \text{if } \min\{\mu_R-i_0,i_0-\mu_L\}<\frac{c}{2}<\max\{\mu_R-i_0,i_0-\mu_L\},\\ (12) & \text{if } \frac{c}{2}\geq\max\{\mu_R-i_0,i_0-\mu_L\}.\end{cases}$$

Without loss of generality, we can assume that $i_0\geq\frac{1}{2}$. Then, $\sigma^*_J(i_0)=\sigma^*_R(i_0)$ and $\sigma^*_{-J}(i_0)=\sigma^*_L(i_0)$. Plugging (11) as well as the choices $\sigma^*_R(i_0)$ and $\sigma^*_L(i_0)$ (as given by Proposition 1) into (12) and (15) yields

$$W(c)=\begin{cases}W^A_1(c) & \text{if } 0\leq\frac{c}{2}\leq\mu_R-i_0,\\ W^A_2(c) & \text{if } \mu_R-i_0<\frac{c}{2}<\Pi,\\ W^A_3(c) & \text{if } \Pi\leq\frac{c}{2}<i_0-\mu_L,\\ W^A_4(c) & \text{if } \frac{c}{2}\geq i_0-\mu_L,\end{cases}$$

where

$$W^A_1(c)=-\left(\mu_R-\frac{c}{2}-\frac{1}{2}\right)^2-c\left(\mu_R-\frac{c}{2}-\frac{1}{2}\right)\cdot(1-\beta(1-2\lambda))+c(1-\beta)\left(i_0-\frac{1}{2}\right),$$

$$W_2^A(c) = -\frac{1-\beta}{1-(1-\lambda)\beta}\left[\left(i_0 - \frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)^2 \right.$$
$$\left. + \lambda\beta c\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)\cdot\frac{1-\beta(1-2\lambda)}{1-\beta} + \lambda\beta c\left(i_0 - \frac{1}{2}\right)\right],$$

$$W_3^A(c) = -\frac{1-\beta}{1-(1-\lambda)\beta}\left[\left(i_0 - \frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)^2 - \lambda\beta c\left(\mu_L + \frac{c}{2} - i_0\right)\right],$$

and

$$W_4^A(c) = -\left(i_0 - \frac{1}{2}\right)^2.$$

Now, we can show that

$$\frac{dW_1^A(c)}{dc} > 0 \quad \text{if } \lambda < \frac{1}{2}, \tag{16}$$

and

$$\frac{d^2W_1^A(c)}{dc^2} > 0 \quad \text{if } \lambda \geq \frac{1}{2}. \tag{17}$$

Similarly, we find that

$$\frac{dW_2^A(c)}{dc} > 0 \quad \text{if } \beta \geq \frac{1}{2(1-2\lambda)} \tag{18}$$

and

$$\frac{d^2W_2^A(c)}{dc^2} > 0 \quad \text{if } \beta < \frac{1}{2(1-2\lambda)}. \tag{19}$$

Furthermore,

$$\frac{d^2W_3^A(c)}{dc^2} < 0 \quad \text{if } \beta > \frac{1}{2} \tag{20}$$

and

$$\frac{dW_4^A(c)}{dc} = 0. \tag{21}$$

Finally, since

$$W(2(i_0-\mu_L),0) > W(0,0) \quad \text{and} \quad W(2(i_0-\mu_L),0) > W(2(\mu_R-i_0),0)$$

are satisfied for any $\frac{1}{2} \leq i_0 < \mu_R$, the combination of (16), (17), (18), (19), (20), and (21) completes the proof of statement *(i)*, with

$$\beta^*(\lambda) = \frac{1}{2(1-2\lambda)}.$$

Note that $\beta^*(\lambda) \in \left(\frac{1}{2}, 1\right)$ for $\lambda \in \left(0, \frac{1}{4}\right)$.

B) Proof of *Theorem 2(ii)*

Let $\Gamma^{i_0} > \Pi$. Then,

$$\left|\Pi - \frac{c}{2}\right| \geq \Gamma^{i_0} \Leftrightarrow \frac{c}{2} \in \left[\max\{\mu_R - i_0, i_0 - \mu_L\}, \infty\right).$$

Without loss of generality, we can assume that $\mu_R < i_0 \leq 1$. Then, $W(c)$ is given by

$$\begin{cases} (15) & \text{if } 0 \leq \frac{c}{2} < i_0 - \mu_L, \\ (12) & \text{if } \frac{c}{2} \geq i_0 - \mu_L. \end{cases}$$

It is useful to consider two cases.

*Case B.1:* $i_0 - \mu_R \geq \Pi$

In this case, we obtain

$$W(c) = \begin{cases} W_1^B(c) & \text{if } 0 \leq \frac{c}{2} < \Pi, \\ W_2^B(c) & \text{if } \Pi \leq \frac{c}{2} \leq i_0 - \mu_R, \\ W_3^B(c) & \text{if } i_0 - \mu_R < \frac{c}{2} < i_0 - \mu_L, \\ W_4^B(c) & \text{if } \frac{c}{2} \geq i_0 - \mu_L, \end{cases}$$

where

$$\begin{aligned} W_1^B(c) = &- \frac{(1-\beta)}{1-(1-\lambda)\beta} \Bigg\{ \left(\mu_R + \frac{c}{2} - \frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)^2 \\ &+ c\left(i_0 - \mu_R - \frac{c}{2}\right)(1-(1-\lambda)\beta) + 2\lambda\beta c\left[\mu_R - \frac{1}{2} + \frac{\lambda\beta}{1-\beta}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)\right] \Bigg\}, \end{aligned}$$

$$\begin{aligned} W_2^B(c) = -\frac{(1-\beta)}{1-(1-\lambda)\beta} \Bigg[ &\left(\mu_R + \frac{c}{2} - \frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 \\ &+ c\left(i_0 - \mu_R - \frac{c}{2}\right)(1-(1-\lambda)\beta) + 2\lambda\beta c\left(\mu_R - \frac{1}{2}\right) \Bigg], \end{aligned}$$

$$W_3^B(c) = -\frac{1-\beta}{1-(1-\lambda)\beta}\left[\left(i_0 - \frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 - \lambda\beta c\left(\mu_L + \frac{c}{2} - i_0\right)\right],$$

and

$$W_4^B(c) = -\left(i_0 - \frac{1}{2}\right)^2.$$

We can show that

$$\frac{d^2 W_1^B(c)}{dc^2} > 0 \quad \text{and} \quad \frac{d^2 W_3^B(c)}{dc^2} \geq 0 \quad \text{hold if } \beta \leq \frac{1}{2}. \tag{22}$$

Similarly,

$$\frac{dW_3^B(c)}{dc} < 0 \quad \text{if } \beta > \frac{1}{2} \quad \text{and} \quad \frac{dW_2^B(c)}{dc} < 0. \tag{23}$$

Moreover, we can show that

$$W_1^B(0) > W_1^B(2\Pi) \quad \text{if } \beta < \beta_1 \tag{24}$$

and

$$-\left(i_0-\frac{1}{2}\right)^2 = W(2(i_0-\mu_L),0) < W(0,0) = -\left(\mu_R-\frac{1}{2}\right)^2, \tag{25}$$

where

$$\beta_1 := \begin{cases} \frac{\Gamma^{i_0}\left(2\Gamma^{i_0}-\Pi\right)}{2\left(\Gamma^{i_0}-\Pi\right)\left(\Gamma^{i_0}+\Pi\right)} & \text{if } \lambda = \frac{\Gamma^{i_0}-2\Pi}{\Gamma^{i_0}} \\ \frac{(5-\lambda)\Pi-2(2-\lambda)\Gamma^{i_0}+\sqrt{\left[(5-\lambda)\Pi-2(2-\lambda)\Gamma^{i_0}\right]^2+8[2\Pi-(1-\lambda)\Gamma^{i_0}](2\Gamma^{i_0}-\Pi)}}{4\left[2\Pi-(1-\lambda)\Gamma^{i_0}\right]} & \text{otherwise.} \end{cases} \tag{26}$$

Finally, the combination of (22), (23), (24), and (25) completes the proof of statement *(ii)* in *Case B.1*, the critical $\beta$ being

$$\beta_1^{**} = \min\left\{\beta_1, \frac{1}{2}\right\}. \tag{27}$$

*Case B.2:* $i_0-\mu_R<\Pi$

Now we obtain

$$W(c) = \begin{cases} W_1^B(c) & \text{if } 0 \le \frac{c}{2} \le i_0-\mu_R, \\ W_{ii}^B(c) & \text{if } i_0-\mu_R < \frac{c}{2} < \Pi, \\ W_3^B(c) & \text{if } \Pi \le \frac{c}{2} < i_0-\mu_L, \\ W_4^B(c) & \text{if } \frac{c}{2} \ge i_0-\mu_L, \end{cases}$$

where $W_1^B(c)$, $W_3^B(c)$, and $W_4^B(c)$ are unchanged compared to *Case B.1* and

$$W_{ii}^B(c) = -\frac{1-\beta}{1-(1-\lambda)\beta}\Bigg\{\left(i_0-\frac{1}{2}\right)^2 + \frac{\lambda\beta}{1-\beta}\left(\mu_R-\frac{c}{2}-\frac{1}{2}\right)^2 + \lambda\beta c\left[i_0-\mu_L-\frac{c}{2}+2\frac{\lambda\beta}{1-\beta}\left(\mu_R-\frac{c}{2}-\frac{1}{2}\right)\right]\Bigg\}.$$

We can show that

$$\frac{d^2W_{ii}^B(c)}{dc^2} > 0 \quad \text{if } \beta \le \frac{1}{2}, \tag{28}$$

$$\lim_{c\to 2(i_0-\mu_R)^-} \frac{dW_1^B(c)}{dc} < 0 \quad \text{if } \beta < \beta_2, \tag{29}$$

and

$$W_{ii}^B(2(i_0-\mu_R)) > W_{ii}^B(2\Pi) \Leftrightarrow \beta < \frac{\Gamma^{i_0}}{4\lambda(\Gamma^{i_0}-\Pi)}, \tag{30}$$

where

$$\beta_2 := \frac{\Gamma^{i_0}-(2-\lambda)\Pi}{2\left[(4\lambda^2-\lambda+1)\Gamma^{i_0}-(6\lambda^2-3\lambda+1)\Pi\right]} + \frac{\sqrt{\left[\Gamma^{i_0}-(2-\lambda)\Pi\right]^2+4\Pi\left[(4\lambda^2-\lambda+1)\Gamma^{i_0}-(6\lambda^2-3\lambda+1)\Pi\right]}}{2\left[(4\lambda^2-\lambda+1)\Gamma^{i_0}-(6\lambda^2-3\lambda+1)\Pi\right]}. \tag{31}$$

Together with (22), (23), and (25) from *Case B.1*, the combination of (28), (29), and (30) completes the proof of statement *(ii)* in *Case B.2*, the critical $\beta$ now being

$$\beta_2^{**} = \min\left\{\beta_2, \frac{\Gamma^{i_0}}{4\lambda(\Gamma^{i_0} - \Pi)}, \frac{1}{2}\right\}. \tag{32}$$

Finally, combining *Case B.1* and *Case B.2* completes the proof of statement *(ii)*, with

$$\beta^{**}(\lambda) = \min\{\beta_1^{**}, \beta_2^{**}\} = \min\left\{\beta_1, \beta_2, \frac{\Gamma^{i_0}}{4\lambda(\Gamma^{i_0} - \Pi)}, \frac{1}{2}\right\}$$

and $\beta_1$, $\beta_2$, $\beta_1^{**}$, and $\beta_2^{**}$ defined in (26), (31), (27), and (32), respectively.

□

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