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L. Bretschger and A. Vinogradova

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# Preservation of Agricultural Soils with Endogenous Stochastic Degradation

Lucas Bretschger and Alexandra Vinogradova*

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**Abstract**

Soils are often subject to environmental shocks which are caused by negative externalities linked to overexploitation. We present a stochastic model of a dynamic agricultural economy where natural disasters are sizeable, multiple, and random. Expansion of agricultural activities raises effective soil units (an index of quality and quantity) but contributes to an aggregate loss of soil-protective ecosystem services, which increases the extent of soil degradation at the time of a shock. We provide closed-form analytical solutions and show that optimal development is characterized by a constant growth rate of effective soil units and crop consumption until an environmental shock arrives causing both variables to jump downwards. Optimal policy consists of spending a constant fraction of output on soil preservation. This fraction is an increasing function of the shocks arrival rate, degradation intensity of agricultural practices, and the damage intensity of environmental impact. Implications for the optimal propensity to save are also discussed. An extension of the model provides a solution for the optimal preservation policy when both the hazard rate and damages are endogenous.

JEL Classification: Q18, Q54, O13, O44

**Key Words**: Soil conservation, stochastic degradation, agriculture, environment, uncertainty, natural disasters.

*Both Center of Economic Research, CER-ETH, Zurich, Switzerland. Tel: +41-44-632-21-92, email: lbretschger@ethz.ch, avinogradova@ethz.ch

# 1 Introduction

## 1.1 Externalities and Risk

Risk is an inherent element of all agricultural activities. The initial development of agriculture itself was a response to the immense risks of relying on hunting and gathering for food (Hardaker et al. 2004). The impact of risk has not disappeared, of course, but it has changed its character and has shifted to different areas. It is still inevitable, because agrarian markets are interlinked with aggregate risks in the economy and, in particular, because ecological systems and weather conditions are subject to unpredictable perturbations. While farmers often understand how to respond to risks on an individual basis, the agrarian sector as a whole may suffer from suboptimal development when adverse shocks are induced by negative externalities of aggregate market activities.

Important negative external effects arise from the agricultural sector itself, creating potentially harmful changes in the ecological systems supporting agriculture (Lichtenberg 2004). For example, deforestation, land use changes, or irrigation may cause soil erosion and nutrient depletion. The use of pesticides, animal wastes, and soil siltation may contaminate surface and ground waters. Salinization of rivers may damage crop production in downstream areas, while irrigation and land clearing may lead to land loss to selenium and salt drawn up from subsoils.

There is a widespread concern that the clearing of Asia's upland mountain forests exacerbated the damages from floods in Bangladesh, Cambodia, China, India, Thailand, Vietnam and elsewhere in the Asian lowlands. Transboundary floods that affected India and Pakistan in 2014 resulted in losses of at least US$ 18 billion; the largest damage was the river basin flood in India causing 1281 fatalities and a loss of US$ 16 billion. While not all damages can be attributed to fading protection, forests can reduce the volume of water arriving in nearby streams and rivers, or at least, spread its arrival over a greater time span.

The long-run development of an agriculture-dependent economy is determined by its decisions to invest in productive inputs as well as in preservation measures. Investment in agricultural expansion may have positive internal effects but may entail negative external effects. Specifically, the enlargement and improvement of soils are aimed at raising productivity and profits for a single farmer but may, at the same time, harm ecosystem services protecting agricultural land making it vulnerable to degradation and natural disasters which are not easily predictable. To correct for this dynamic market failure, soil-protection measures have the potential to improve the overall welfare in the economy. This type of an environmental problem in agriculture warrants a thorough investigation from a theoretical perspective.

## 1.2 Model and Findings

In the present paper we develop a dynamic model of an agricultural economy, where agricultural practices generate harmful externalities which result in adverse shocks to the stock of effective soil. An effective unit of soil is measured by the quantity of arable land and also by its quality characteristics, which are determined by the biological, chemical, and physical attributes (Smith et al. 2000). The magnitude of damages at the time of a shock are linked to the harmful production activities and can therefore be controlled by the planner through an appropriate choice of preservation measures and investment in accumulation of the productive input. Examples of such shocks include major floods, droughts and landslides due to increased exposure of agricultural soils when protective forests are cut. This is an important issue especially in mountainous or in tropical regions where the lack of protection may cause substantial losses of land and yield. Given that random shocks constitute a central part of agricultural production, they need to be taken into account within an appropriate modeling framework, which we propose in the present paper.

Our model delivers closed-from analytical solutions for the optimal growth rate of crop consumption, the gross saving propensity, and the optimal soil preservation policy. The optimal development in the economy is characterized by the consumption rate and the effective-soil stock which grow at the same constant rate until a shock arrives causing a downward jump in both variables. The size of the jump is endogenously determined and depends on the shock probability, protection efficiency, damage intensity and the intertemporal substitution elasticity. Our main findings can be summarized as follows. First, we confirm that the damage-related parameters, such as the shock arrival rate, the extent of harmful agricultural practices, soil exposure intensity, and damage intensity, have the expected negative effect on the optimal crop-consumption growth rate. Agricultural productivity naturally fosters the optimal growth rate, while soil protection efficiency may have either a positive or a negative effect, which we explain in detail. Second, we show that the optimal preservation policy consists of devoting a constant fraction of output to soil protection. This fraction is an increasing function of the shock arrival rate, agricultural technology, proportion of harmful byproducts, exposure component, and damage intensity. The role of each parameter is therefore identified precisely. This is a desirable characteristic, especially when it comes to formulating policy prescriptions. Third, we show that the optimal propensity to save and invest crucially depends on the elasticity of intertemporal consumption substitution and therefore widely adopted logarithmic preferences, favored for their simplicity and tractability, may deliver misleading policy conclusions. Fourth, as long as agricultural labor force increases the marginal productivity of land, it also raises the optimal growth rate of crop consumption and the optimal fraction of output devoted to soil protection. Finally, in an extension of the model, we show how the optimal growth rate and preservation policy change when not only the damages but also the arrival rate of shocks are endogenous. There we assume that the larger the extent of harmful agricultural practices the higher are the chances of a negative shock to occur.

Overall, the existence of negative externalities calls for corrective policies, which adequately reflect the uncertain nature of environmental shocks and accommodate important parameters and conditions such as protection efficiency and population growth. Our results are relevant for the formulation of optimal soil protection measures in agrarian societies, e.g. in less developed countries, or in the agrarian sector in developed economies. They also help us understand from today's perspective the implementation of protective policies in past periods of now developed world. Given the close link to the food security challenge the considered risks belong to an important class of decision problems in agriculture. To the best of our knowledge the present paper is the first to analyze this question in a tractable theoretical model.

## 1.3 Contribution to the Literature

The crucial role of the "nearly forgotten resource" – soil – for food security and human health has recently been highlighted in Wall and Six (2015) who argue that agricultural practices have "increased soil erosion to rates much greater than those of soil formation." Many important contributions in the literature deal with the complex interplay between agriculture and ecosystem services. Dalea and Polasky (2007) analyze the environmental impacts of agricultural practices on a wide range of ecological services such as water nutrient cycling, soil retention, quality, pollination, carbon sequestration, and biodiversity conservation. They also show that ecosystem services have a positive impact on agricultural productivity. Heal and Small (2002) study agriculture as a producer and consumer of ecosystem services and stress that the quantity and quality of ecosystem services depend on the joint actions of many dispersed resource users. Pfaff (1996) confirms empirically that greater soil quality in the Brazilian Amazon is associated with more deforestation at the level of individual farms, which then diminishes protection of soils at the aggregate level. We build on these fundamental relationships in our framework by explicitly modeling the decrease of protective ecoservices as an externality of agricultural activity.

Ehui et al. (1990) study a two-sector model of agriculture and forestry to show the dynamic interactions between deforestation and agricultural productivity. They derive optimal rules when forest clearing raises growth of agricultural output but at the same time damages long-run productivity. Grepperud (1997) extends the previous studies by analyzing time-limited effects of optimal soil conservation measures and long-lasting effects on the soil base. Cacho (2001) analyzes the impact of agroforestry on arable land prone to degradation in the presence of forest externalities. It is concluded that appropriate forestry is able to reduce land degradation and to contribute substantially to a sustainable use of soil. In the study of the tropical forest ecosystem in Bangladesh, Islam and Weil (2000) argue that "human population pressures upon land resources have increased the need to assess impacts of land use change on soil quality." Our paper departs from the deterministic framework used in these papers, introducing random environmental shocks which have been classified as "production uncertainty" and qualified as a "quintessential feature of agricultural production" (Moschini and Hennessy 2001, p. 90).

Most studies of dynamic decisions under risk in agricultural economics are empirical. An early exception is Hertzler (1991), who provides an overview of the mathematical tools and various exemplary applications to the field. A paper related to our study is Shively (2001) which also considers a dynamic stochastic model of soil preservation but does not provide analytical solutions to the theoretical problem. The focus of Shively's analysis is on the role of farm size and liquidity constraints for the decision of subsistence-oriented households to make a one-time investment in protective installations reducing the risk of soil erosion. Thus, the dynamics of the investment process are not considered. His main conclusion is that public policy should aim at enhancing saving and insurance mechanisms for small farms which are most likely to face liquidity and subsistence constraints.

Soil degradation can be addressed by different types of policies. An interesting example from history is the very strict Forest Act of 1876 in Switzerland, which was adopted as a political response to the massive soil degradation caused by recurring floods after the clearing of mountain forests. More recently, the use of international aid for land protection has emerged as a research topic. In a theoretical two-sector model, Ollivier (2012) studies the dilemma between economic growth and deforestation and the long term impacts of international transfers aimed at preserving tropical forests. She concludes that, for low transfer schemes, the agricultural output increases with the transfer even though less land is under cultivation. The impact of land heterogeneity and incomplete information on optimal transfers to agriculture is analyzed in Chiroleu-Assouline and Roussel (2014). Using the example of carbon sequestration in agricultural soils these authors show how optimal contracts can be designed leading to truthful revelation of information by agricultural firms. Labrière et al. (2015) empirically estimate the impact of contour planting, no-till farming and use of vegetative buffer strips on the reduction of soil erosion and find enormous potentials. They conclude that the government or natural resource managers can help decrease soil losses on a large scale. Finally, it has been stressed in the literature that corrective measures need not be implemented by public policies only. Studying alternatives to governmental policies, Lopez (2002) deals with the endogenous evolution of rural environmental institutions which is particularly relevant for poor tropical areas where the agricultural and natural resource base is fragile. Our model refers to a planner internalizing the external effects in the economy. This is a generic approach to policy, so that we do not have to specify whether the political actors are public or private and, similarly, domestic or foreign.

The remainder of the paper is organized as follows. Section 2 develops our baseline framework. In Section 3, we present the main results with respect to the optimal growth rate, soil protection, and saving propensity. Section 4 extends the model to account for the endogenous arrival rate. Finally, Section 5 concludes.

# 2 The Framework

## 2.1 The Model

We consider an agrarian society or an agrarian sector which produces output (e.g. crop) using effective soil/land units, denoted by $S_t$, and labor, denoted by $L$, as inputs. Labor is supplied inelastically. The agricultural yield, denoted by $Y_t$, is assumed to follow a Cobb-Douglas structure: $Y_t = A_t S_t^a L^b$, where $A_t$ represents the level of total factor productivity at time $t$. We shall assume that productivity in agricultural sector is augmented through "learning-by-doing" process, which we model as $A_t = \xi S_t^{1-\gamma} L^{\gamma}$, $\gamma \in [0, 1]$, exhibiting constant returns to scale. The parameter $\xi$ stands for a given level of agricultural technology. Let us assume that $\gamma = a$ and we can then rewrite the production function as

$$Y_t = \xi S_t L^{\beta}, \quad \beta = \gamma + b \in (0, 1). \tag{1}$$

The stock of effective soil units, $S_t$, is measured as an index of soil quantity and quality (e.g. infiltration rate, content of organic matter, structure, nutrient content and soil depth) at time $t$. Production process results in byproducts which are potentially harmful for effective soil and lead to deterioration of soil-protecting ecosystem services. We assume that there exists a relationship between output quantity and soil degradation. Specifically, let us postulate that a unit of output is accompanied by $\eta$ units of harmful practices which cause damage to the ecosystem (Clarke 1992). Exploitation of ecosystems leads to their weakening or even exhaustion, which in turn reduces their capacity to protect and preserve either the quantity and/or the quality of agricultural land, making the soil vulnerable to degradation shocks. One may think, for example, of deforestation to increase availability of arable land as a "harmful practice" adopted by farmers to increase their land plots. Deforestation, especially in mountainous regions, causes landslides and exposes crops to winds which may reduce soil quality or even partly destroy crop fields. We assume that arrival of a shock/disaster (e.g. landslide, drought, strong wind, etc.) follows a Poisson process with a constant intensity $\lambda$. When a shock occurs, an endogenously-determined amount $\Delta_t \in [0, S_t]$ of the existing effective soil units is degraded. The economy has a possibility to reduce soil degradation by adopting preservation/protection measures (e.g. contour hedgerows, antierosion ditches, grass strips, radical terraces). We assume that a share $\theta_t$ of output is spent on protection, resulting in protection expenditure $E_t = \theta_t Y_t$. The remaining share $(1 - \theta_t)Y_t$ is split between current crop consumption, $C_t$, and investment in accumulation of soil ecosystem services (either an increase in quantity or quality through use of organic fertilizers and trace nutrients, prevention of soil erosion). Total ecosystem protection, $\Omega(E_t)$, is an increasing function of the protection expenditure: $\Omega'(E_t) > 0$. Let us assume that the protection function exhibits constant returns to scale (e.g. a doubling of expenditure on protective measures will double the protection services) with the parameter $\omega$

denoting the effectiveness or efficiency of protection measures. Then the net damages to the ecosystem, denoted by $D_t$, can be expressed as the difference between the impact of harmful practices and protection services:

$$D_t = \eta Y_t - \omega E_t. \tag{2}$$

It is clear from the landslide example that the size of the landslide and thus the deterioration impact are directly and positively related to the magnitude of deforestation. We thus assume that degradation of effective soil units at the time of a shock is proportional to the damages to the ecosystem. We shall also assume that there is a possibility of soil degradation occurring due to natural disasters, that is the magnitude of degradation is then independent of the man-made harmful practices and is equal to a fraction $\sigma \in [0, 1)$ of the total stock $S_t$. The total degradation impact may be written as:

$$\Delta(D_t, S_t) = \delta D_t + \sigma S_t, \tag{3}$$

where $\delta$ measures the degradation intensity of man-made damages and $\sigma$ measures the exposure to natural degradation even in the absence of any man-made activities.

The agrarian economy's objective is to maximize the expected discounted value of utility over an infinite planning horizon with respect to consumption, $C_t$, and the share of output devoted to protection, $\theta_t$, subject to the stochastic process which describes the evolution of effective soil units over time. The utility function takes a standard CRRA form, $U(C_t) = \frac{C_t^{1-\varepsilon}-1}{1-\varepsilon}$, where $1/\varepsilon$ is the intertemporal substitution elasticity. Specifically, the planner's program is

$$\max_{C_t, \theta_t} \quad \mathbb{E}_0 \left\{ \int_0^\infty \frac{C_t^{1-\varepsilon} - 1}{1 - \varepsilon} e^{-\rho t} dt \right\} \tag{4}$$

s.t.

$$dS_t = [(1 - \theta_t) Y_t(S_t, L) - C_t]dt - (\delta D_t + \sigma S_t) dq_t, \tag{5}$$

$$D_t = \eta Y_t(S_t, L) - \omega \theta_t Y_t(S_t, L), \tag{6}$$

$$Y_t = \xi S_t L^\beta, \tag{7}$$

where $\mathbb{E}_0$ is the expectations operator, $dq_t$ is an increment of the Poisson process with a constant arrival rate $\lambda$ and $\rho$ is the constant rate of time preference. We also require that the soil stock and consumption are non-negative and $\theta_t \in (0,1)$.

## 2.2 Solving the Model

Denoting by $V(S)$ the value function associated with the optimization problem described in (4) - (7), the Hamilton-Jacobi-Bellman (HJB) equation may be written as

$$\rho V(S) = \max \left\{ U(C) + V'(S)[(1-\theta)Y(S,L) - C] + \lambda \left[ V(\tilde{S}) - V(S) \right] \right\}, \tag{8}$$

where $V(\tilde{S})$ is the value function after the occurrence of a shock which depends on the new effective soil stock $\tilde{S} = S - \Delta(D,S)$. Time subscripts are omitted when there is no ambiguity. The first-order conditions consist of

$$C \quad : \quad U'(C) - V'(S) = 0, \tag{9}$$

$$\theta \quad : \quad -V'(S)Y + \lambda\delta\omega V'(\tilde{S})Y = 0, \tag{10}$$

$$\begin{aligned} S \quad : \quad & \rho V'(S) = V''(S)[(1-\theta)Y - C] + V'(S)\xi L^{\beta}(1-\theta) + \\ & + \lambda \left\{ V'(\tilde{S}) \left[ 1 - \sigma - \delta(\eta - \omega\theta)\xi L^{\beta} \right] - V'(S) \right\}. \end{aligned} \tag{11}$$

The optimality conditions are complemented by the transversality condition for $S$, the non-negativity constraints on $C$, $S$, and the requirement $\theta \in [0,1)$.

Using Itô's Lemma and Eq. (11) we compute the differential of $V'(S)$ and, using (9), we obtain an explicit solution for the law of motion of the crop consumption rate:

$$\frac{dC}{C} = \frac{1}{\varepsilon} \left\{ \xi L^{\beta} \left( 1 - \frac{\eta}{\omega} \right) + \frac{1-\sigma}{\omega\delta} - \rho - \lambda \right\} dt + \left( \frac{\tilde{C}}{C} - 1 \right) dq, \tag{12}$$

where the consumption rate following a shock, $\tilde{C}$, is a constant fraction $\mu$ of the pre-jump rate:

$$\tilde{C} = \mu C, \quad \mu \equiv (\lambda\omega\delta)^{\frac{1}{\varepsilon}} \in [0,1) \tag{13}$$

It follows that the last term on the RHS is negative and it represents the downward jump in consumption every time a shock occurs.

The first term on the RHS of (12) represents what we label the "trend" consumption growth rate. Specifically, while a shock has not arrived, crop consumption grows at the constant rate, defined as

$$g \equiv \frac{1}{\varepsilon}\left\{\xi L^{\beta}\left(1-\frac{\eta}{\omega}\right)+\frac{1-\sigma}{\omega\delta}-\rho-\lambda\right\}. \tag{14}$$

When a shock occurs, consumption jumps down to the new level, $\widetilde{C}$, and then continues to grow at the rate $g$ until the next shock.

It can be shown that the value function of the problem, satisfying the HJB equation and certain limiting conditions (see, e.g., Sennewald and Wälde 2006), is of the form

$$V(S)=\frac{\psi^{-\varepsilon}S^{1-\varepsilon}-\frac{1}{\rho}}{1-\varepsilon}, \tag{15}$$

where $\psi$ is a function of the parameters of the model:

$$\psi \equiv \frac{1}{\varepsilon}\left\{\rho-(1-\varepsilon)\left[\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta}+\xi L^{\beta}\left(1-\frac{\eta}{\omega}\right)\right]+\lambda\left[1-(\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\}.$$

**Proposition 1:** *The solution of the maximization problem given by (4) - (7) is characterized by the following:*
*(i) optimal consumption is proportional to the effective soil stock;*
*(ii) optimal protection expenditure is a constant fraction of output;*
*(iii) consumption, effective soil stock, and protection services increase at the same constant rate, given by (14), between two subsequent shocks.*

**Proof:** The result in (i) follows immediately from (9) and (15), so that:

$$C^{*}=\psi S. \tag{16}$$

Statement (ii) follows from (10) and (15); by combining the two expressions we find that the optimal share of output devoted to protection services is given by:

$$\theta^* = \frac{\eta}{\omega} - \frac{1 - \sigma - (\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^\beta \omega\delta}. \tag{17}$$

The non-negativity constraint on $D$ requires that $\theta^* \leqslant \frac{\eta}{\omega}$ (see (6)). At the same time, $\theta^*$ must be non-negative, so that both conditions lead to the inequality $0 \leqslant \frac{\eta}{\omega} - \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^\beta \omega\delta} \leqslant \frac{\eta}{\omega}$. After some rearrangements, we obtain

$$1 - \sigma - \xi L^\beta \eta\delta \leqslant (\lambda\omega\delta)^{\frac{1}{\varepsilon}} \leqslant 1 - \sigma, \tag{18}$$

which is the necessary restriction on the parameters of the model to ensure the existence of an interior solution.

To prove (iii), note that the stochastic time path of the soil stock can be found analytically by substituting the optimal controls (16) and (17) in (5) and solving the resulting stochastic differential equation

$$dS_t = [(1-\theta^*)\xi L^\beta - \psi]S_t dt - \left[\delta(\eta - \omega\theta^*)\xi L^\beta + \sigma\right] S_t dq_t.$$

The solution is given by

$$S_t = S_0 e^{[(1-\theta^*)\xi L^\beta - \psi]t + \ln\left[1-\sigma-\delta(\eta-\omega\theta^*)\xi L^\beta\right] q_t}.$$

We can verify that the term in the exponent involving the logarithm is well-defined since the argument of the logarithm is unambiguously positive and is equal to (using (17))

$$1 - \sigma - \delta(\eta - \omega\theta^*)\xi L^\beta = (\lambda\omega\delta)^{\frac{1}{\varepsilon}} > 0.$$

Substituting the solution for $\theta^*$ in $[(1-\theta^*)\xi L^\beta - \psi]$, we obtain the following stochastic path of the effective soil stock

$$S_t = S_0 e^{gt + \frac{1}{\varepsilon}\ln(\lambda\omega\delta) q_t}, \tag{19}$$

where the term $q_t$ in the exponent is responsible for the discontinuous downward jump at the time of a shock. The jump is downward since $\ln(\lambda\omega\delta)$, which multiplies $q_t$, is negative. When $q_t = 0$, $S_t = S_0 e^{gt}$, i.e. the effective soil stock improves at the constant rate $g$, so that consumption and effective soil stock grow at the same rate as long as a shock has not arrived, in line with (16). Expenditure on protective measures, equal to a fraction $\theta^*$ of output, evolves over time according to

$$E_t = \theta^* \xi S_t L^\beta = \frac{1}{\omega}\left[\eta\xi L^\beta \delta - (1-\sigma) + (\lambda\omega\delta)^{\frac{1}{\varepsilon}}\right] S_0 e^{gt + \frac{1}{\varepsilon}\ln(\lambda\omega\delta)q_t}$$

showing that it grows at the trend rate $g$ while $q_t = 0$. ■

# 3 Analysis of the Solution

In this Section we provide a characterization of the optimal consumption growth and the optimal protection policy. In particular, we are interested in the effects of the key parameters such as exposure and damage intensities, protection efficiency, shock arrival probability, levels of labor force and technology.

## 3.1 Consumption Growth

The trend growth rate of crop consumption is given by $g$, which may be written as

$$g = \frac{1}{\varepsilon}\left\{\xi L^\beta \left(1 - \frac{\eta}{\omega}\right) - \rho + \lambda\left(\frac{1}{\lambda\omega\delta} - 1 - \frac{\sigma}{\lambda\omega\delta}\right)\right\}. \tag{20}$$

The expression reveals that the consumption rate is increasing over time if the effective discount rate, which includes not only the pure rate of time preference $\rho$ but also the Poisson intensity $\lambda$, is relatively low, formally $g > 0 \Leftrightarrow \xi L^\beta\left(1 - \frac{\eta}{\omega}\right) + \frac{1-\sigma}{\omega\delta} > \rho + \lambda$.

The expression resembles the Keynes-Ramsey formula which is widely known in standard growth and macro models. The Keynes-Ramsey growth rate is typically equal to the difference between the real interest rate (usually the marginal productivity of physical capital) and the rate of pure time preference, multiplied by the elasticity of intertemporal consumption substitution (EICS). We note that in Eq. (20) the economy's implicit real interest rate is given by the first term inside the parentheses. It does not only include the marginal productivity of soil input ($\xi L^{\beta}$) but also the effect of harmful agricultural practices adjusted by the protection efficiency, i.e, the term $\eta/\omega$. It follows that soil deterioration has an unambiguously negative growth effect. This adverse effect may be reduced by either increasing the protection efficiency, $\omega$, or decreasing the proportion of harmful agricultural practices, $\eta$.

The last term in Eq. (20), multiplying $\lambda$, represents the effect of uncertainty, which includes the exposure and the jump components. The former is represented by the term $\frac{\sigma}{\lambda\omega\delta}$ and captures the slow-down effect due to the damage exposure of the proportion $\sigma$ of the total stock. On the other hand, the jump component, represented by the term $\frac{1}{\lambda\omega\delta} - 1$, contributes to a *faster* consumption growth as compared to the standard Keynes-Ramsey formula. The tern $\frac{1}{\lambda\omega\delta}$ is equal to the ratio of marginal utilities of post- to pre-jump consumption and therefore it is larger than unity (see also Eqs. (13) and (18)) implying that the term $\frac{1}{\lambda\omega\delta} - 1$ is positive. The optimal stochastic consumption path is therefore tilted counterclockwise, as compared to the consumption path in a deterministic Keynes-Ramsey model. The economy starts with a relatively low consumption rate at the beginning of the planning horizon, which implies the presence of the precautionary-saving motive, including saving for financing of protective measures. The result is analogous to what has been found in the literature on precautionary savings under uncertainty.[1] The peculiarity of the current setting is that the gross savings are endogenously split between two purposes: accumulation of effective soil units and soil protection measures. It is clear that expansion of protective measures directly reduces soil degradation, while accumulation of effective units has a double-sided effect. On the one hand, a larger stock of effective soil units implies more output and thus more harmful byproducts. On the other hand, having a larger stock of effective soil creates an "emergency buffer" for the rainy days - when a disaster strikes. In Section 3.3 we

[1] See, e.g., Wälde (1999), Toche (2001), Steger (2005).

discuss in more detail the economy's optimal saving rate.

The responses of the crop-consumption growth rate to changes in the fundamental parameters of the model are summarized in Proposition 2. It is important to distinguish between the effect of the expected frequency of disasters and the effect of the overall uncertainty. The former takes into account only the arrival rate $\lambda$. The latter includes both the arrival rate and the damage caused by a shock, as reflected in the last term in Eq. (20).

**Proposition 2:** *The optimal growth rate of crop consumption is:*
*(i) a decreasing function of the shock arrival rate ($\lambda$), proportion of harmful byproducts ($\eta$), exposure intensity ($\sigma$), and damage intensity ($\delta$);*
*(ii) an increasing function of the agricultural productivity ($\xi$) and labor force ($L$);*
*(iii) either an increasing or a decreasing function of soil protection efficiency ($\omega$), depending on the parameter constellation.*

**Proof:** Follows from comparative statics (Eq. (20)):

$$\begin{aligned}
(i):&\quad \frac{\partial g}{\partial \lambda} = -\frac{1}{\varepsilon} < 0, \quad \frac{\partial g}{\partial \eta} = -\frac{\xi L^{\beta}}{\varepsilon \omega} < 0, \quad \frac{\partial g}{\partial \sigma} = -\frac{1}{\varepsilon \omega \delta} < 0, \quad \frac{\partial g}{\partial \delta} = -\frac{1-\sigma}{\varepsilon \omega \delta^2} < 0,\\
(ii):&\quad \frac{\partial g}{\partial \xi} = \frac{1}{\varepsilon} L^{\beta}\left(1 - \frac{\eta}{\omega}\right) > 0, \quad \frac{\partial g}{\partial L} = \frac{1}{\varepsilon}\beta \xi L^{\beta}\left(1 - \frac{\eta}{\omega}\right) > 0,\\
(iii):&\quad \frac{\partial g}{\partial \omega} = \frac{1}{\varepsilon \omega^2}\left(\xi L^{\beta}\eta - \frac{1-\sigma}{\delta}\right) \gtrless 0. \quad \blacksquare
\end{aligned}$$

The effect of the arrival rate ($\lambda$) on the optimal growth rate is directly proportional to the negative of the elasticity of intertemporal consumption substitution. Although there is not a general consensus on the magnitude of this elasticity, the empirically plausible range of values lies between 1 and 3. This suggests that if the frequency of disasters were to rise in the future due to a weakened ecosystem, the agrarian economy may experience an important growth slowdown. The intuition behind the effects of $\eta$ and damage intensity $\delta$ is rather straightforward and has already been discussed.

An interesting result concerns the effect of the protection efficiency $\omega$, which is in general ambiguous. On the one hand, a higher $\omega$ directly improves protection efficiency and contributes to soil preservation thus enhancing the growth rate through the first term in Eq. (20). On the other hand, a higher $\omega$ also means a lesser degradation of effective soil units and a smaller jump in consumption rate at the time of a disaster (the jump-smoothing effect). The ratio of post- to pre-shock marginal utilities of consumption is reduced (see Eq. (13)) and this contributes to a growth slowdown through the last term in Eq. (20). The overall effect of $\omega$ on $g$ is positive when the marginal productivity of effective land units is relatively high (i.e. the level of technology ($\xi$) and/or agricultural labor force ($L$) is relatively high), or the proportion of harmful byproducts ($\eta$) is relatively large or the degradation intensity ($\delta$) is relatively high. This suggests that economies with a relatively high damage intensity of production and with a higher degree of vulnerability to shocks, such as numerous agriculture-dependent developing economies, may enjoy substantial gains in terms of their growth rates by adopting (more) efficient protective measures. At the same time, economies with a relatively higher total factor productivity (such as advanced economies) may also experience an improvement in the growth rate of their agrarian sectors by enhancing their soil-protection technologies.

## 3.2 Optimal Soil Protection

How much of the current resources to devote to soil preservation is an important policy question, especially in economies heavily relying on agricultural production. We have shown in the previous section that it is optimal to allocate a specific constant fraction of output to protection activities. The solution for the optimal output share $\theta^*$ is reproduced from Eq. (17) for convenience

$$\theta^* = \frac{\eta}{\omega} - \frac{1 - \sigma - (\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^\beta \omega\delta}. \tag{21}$$

If $\theta^*$ were simply equal $\eta/\omega$, then all the man-made damage ($D_t$) would be eliminated (see Eq. (6)). Given that $\theta^* < \eta/\omega$, it is not optimal for the economy to eliminate all the harmful byproducts. If we ignore the exposure component of damages for the moment by setting $\sigma = 0$, we see that this policy is optimal only if the intertemporal substitution elasticity is zero (or coefficient of relative risk aversion is infinite). For finite $\varepsilon$, the optimal protection share falls short of 100% due to the "jump" effect. In fact, by bringing all the terms in (21) to the common denominator, we see that the optimal $\theta^*$ depends on the difference between the marginal degradation caused by an extra unit of accumulated soil ($\xi L^{\beta}\eta\delta$) and the magnitude of the jump ($1-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}$) in the effective soil stock (and also consumption) when a shock occurs. The jump effect works to reduce $\theta^*$. The presence of the exposure component ($\sigma$) works in the opposite direction to increase $\theta^*$. The following proposition summarizes the effects of the fundamental parameters of the model on the optimal protection share.

**Proposition 3:** *The optimal fraction of output devoted to soil protection is:*
*(i) an increasing function of the event arrival rate ($\lambda$), agricultural technology ($\xi$), labor force ($L$), proportion of harmful byproducts ($\eta$), exposure component ($\sigma$), and damage intensity ($\delta$),*
*(ii) either a decreasing or an increasing function of protection efficiency ($\omega$), depending on the parameter constellation.*

**Proof:** The results can be obtained from the following comparative statics, using Eq. (21)

$$(i): \quad \frac{\partial\theta^*}{\partial\lambda} = \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}-1}}{\xi L^{\beta}\varepsilon} > 0, \quad \frac{\partial\theta^*}{\partial\xi} = \frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi^2 L^{\beta}\omega\delta} > 0, \quad \frac{\partial\theta^*}{\partial L} = \beta\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^{\beta+1}\omega\delta} > 0,$$

$$\frac{\partial\theta^*}{\partial\eta} = \frac{1}{\omega} > 0, \quad \frac{\partial\theta^*}{\partial\sigma} = \frac{1}{\xi L^{\beta}\omega\delta} > 0, \quad \frac{\partial\theta^*}{\partial\delta} = \frac{1+\frac{1-\varepsilon}{\varepsilon}(\lambda\omega\delta)^{\frac{1}{\varepsilon}}-\sigma}{\xi L^{\beta}\omega\delta^2} > 0,$$

$$(ii): \quad \frac{\partial\theta^*}{\partial\omega} = \frac{1-\xi L^{\beta}\eta\delta+\frac{1-\varepsilon}{\varepsilon}(\lambda\omega\delta)^{\frac{1}{\varepsilon}}-\sigma}{\xi L^{\beta}\omega^2\delta} \gtrless 0. \quad \blacksquare$$

The intuition behind the results in (i) is rather straightforward. A higher frequency of disasters ($\lambda$) requires more preservation measures in order to better protect the land from degradation in the event of an adverse shock. If governments happen to misperceive the true arrival rate $\lambda$, the preservation policy would be sub-optimal. Specifically, if the perceived $\lambda$ is lower than the true one, there is too little preservation. This might happen due to a regime switch from a low to a high shock frequency while the general expectations, if based on past experience, lag behind.

The total factor productivity ($\xi$) and the agricultural labor force ($L$) raise output and thus act in the same direction as $\eta$, damage intensity ($\delta$) and exposure component ($\sigma$).

The statement in (ii) warrants some further comments. The reason for the ambiguous sign in $\partial\theta^*/\partial\omega$ is that there are three effects which operate in different directions. They can be analyzed by examining the expression in (21). First, there is a direct effect of $\omega$ on the optimal protection share, operating through the first term on the RHS of (21): Better protective technology requires a smaller expenditure on soil preservation, all else equal. Second, a better protection efficiency has a positive effect on the economy's growth rate (provided $\xi L^\beta \eta\delta > 1 - \sigma$), which in turn calls for a larger protection expenditure to compensate for an increase in soil exploitation. If $\xi L^\beta \eta\delta < 1 - \sigma$, the reverse is true. Finally, protection efficiency also affects the size of the downward jump in the consumption rate and in the soil stock when an adverse shock occurs (the last term in (21)). The direction of this latter effect depends on the intertemporal substitution elasticity, $1/\varepsilon$. When it is relatively high, above unity, the effect of $\omega$ on the downward jump is positive. Conversely, if EICS is below unity, the effect is negative. Overall, the first (direct) effect contributes to a decrease in the share of output devoted to soil protection; the second (growth) effect contributes to an increase or a decrease in protection; while the third (jump) effect can also be either positive or negative, depending on the intertemporal substitution elasticity.

**Corollary 1:** *If the intertemporal substitution elasticity is above (below) unity,*
*(i) the optimal protection share is convex (concave) in the arrival rate;*
*(ii) the response of the protection share to a change in the arrival rate is more (less) pronounced when protection technology is more (less) efficient and when damage intensity is relatively large (small).*

**Proof:** Follows directly from

$$(i) \quad : \quad \frac{\partial^2 \theta^*}{\partial \lambda^2} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\lambda^{\frac{1}{\varepsilon}-2} (\omega\delta)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1,$$

$$(ii) \quad : \quad \frac{\partial^2 \theta^*}{\partial \lambda \partial \omega} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\omega^{\frac{1}{\varepsilon}-2} (\delta\lambda)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1,$$

$$\frac{\partial^2 \theta^*}{\partial \lambda \partial \delta} = \left(\frac{1}{\varepsilon} - 1\right) \frac{\delta^{\frac{1}{\varepsilon}-2} (\omega\lambda)^{\frac{1}{\varepsilon}-1}}{\xi L^\beta \varepsilon} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1. \quad \blacksquare$$

The Corollary implies that, when the frequency of disasters is already relatively high, a further increase in the frequency should be associated with a more (less) than proportional increase in protection measures if the intertemporal substitution elasticity is greater (smaller) than unity.

**Corollary 2:** *If the intertemporal substitution elasticity is*
*(i) below 3, then the optimal protection share is concave in the damage intensity;*
*(ii) below 2, then the response of the protection share to a change in the damage intensity is less pronounced when protection technology is more efficient. (These conditions are sufficient but not necessary.)*

**Proof:** Follows directly from

$$(i) \quad : \quad \frac{\partial^2 \theta^*}{\partial \delta^2} = \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}} (1 - 3\varepsilon) + 2\varepsilon^2 \left[ (\lambda\omega\delta)^{\frac{1}{\varepsilon}} - (1 - \sigma) \right]}{\xi L^\beta \omega \delta^3} \gtrless 0,$$

$$(ii) \quad : \quad \frac{\partial^2 \theta^*}{\partial \delta \partial \omega} = \left(\frac{1 - \varepsilon}{\varepsilon}\right)^2 \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\xi L^\beta \omega^2 \delta^2} - \frac{1 - \sigma}{\xi L^\beta \omega^2 \delta^2} \gtrless 0. \quad \blacksquare$$

These results formally support the argument that it is optimal to increase soil protection when the magnitude of damages or the expected frequency of disasters increase. Our model predicts that the optimal increase in the protection share should be more (less) than proportional to an increase in the frequency of events if the intertemporal substitution elasticity is relatively high (low). The intuition here is straightforward. A higher elasticity of intertemporal consumption substitution implies that the economy is easily willing to forgo current consumption in exchange for more consumption in the future and thus an increase in the current protection expenditure is less burdensome. In the limiting case $\varepsilon = 1$ (logarithmic utility), $\theta^*$ is linear in $\lambda$ and monotone-increasing and concave in the damage intensity, $\frac{\partial^2\theta^*}{\partial\delta^2} = -\frac{2(1-\sigma)}{\xi L^\beta \omega\delta^3} < 0$. It can be either monotone-decreasing and convex or monotone-increasing and concave in protection efficiency, depending on whether $\xi L^\beta \eta\delta \gtrless 1-\sigma$.

## 3.3 Propensity to Save

In addition to choosing the optimal soil preservation policy, the economy must also decide on another crucial variable, which is how much to invest in expansion of effective soil units (improvement in soil quantity or quality). The economy's gross savings are therefore endogenously split between soil preservation and soil augmentation. The ratio of gross savings to output represents the economy's propensity to save (PTS), which we denote by $s$. We are particularly interested, from the macroeconomic perspective, in how the possibility of adverse shocks impacts on $s$. Moreover, knowing how shocks affect $\theta^*$ and $s$ allows us to deduce their impact on investment in soil stock accumulation. Using Eq. (16), we may express $s$ as

$$s = 1 - \frac{\psi}{\xi L^\beta} = \frac{1}{\xi L^\beta \varepsilon}\left\{\xi L^\beta - \rho + (1-\varepsilon)\frac{1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}} - \xi L^\beta\eta\delta}{\omega\delta} - \lambda\left[1-(\lambda\omega\delta)^{\frac{1-\varepsilon}{\varepsilon}}\right]\right\}. \tag{22}$$

Note that when log-utility is assumed ($\varepsilon \to 1$), the expression simplifies to $1 - \frac{\rho}{\xi L^\beta}$ and thus excludes the risk-related parameters altogether. This simplified preference structure implies that occurrence of random disasters would only cause a reallocation between investment in effective soil accumulation and preservation, but not between consumption and gross savings. When $\varepsilon$ is different from unity, the effects of the key parameters characterizing negative agricultural shocks on $s$ are summarized in

**Proposition 4:** *If the EICS is above (below) unity, the optimal propensity to save is*

*(i) a decreasing (increasing) function of the disaster arrival rate ($\lambda$) and proportion of*

*harmful agricultural practices ($\eta$);*

*(ii) an increasing (decreasing) function of the damage intensity ($\delta$) and protection efficiency ($\omega$).*

**Proof:** Follows directly from:

$$
\begin{aligned}
(i) \quad : \quad & \frac{\partial s}{\partial \lambda} = \frac{1}{\xi L^{\beta} \varepsilon} \left\{ (\lambda\omega\delta)^{\frac{1}{\varepsilon}-1} - 1 \right\} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \lessgtr 1, \\
& \frac{\partial s}{\partial \eta} = -\frac{1-\varepsilon}{\varepsilon\omega} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \lessgtr 1, \\
(ii) \quad : \quad & \frac{\partial s}{\partial \delta} = \frac{(1-\varepsilon)}{\xi L^{\beta} \varepsilon} \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}}}{\omega\delta^{2}} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1, \\
& \frac{\partial s}{\partial \omega} = \frac{(1-\varepsilon)}{\xi L^{\beta} \varepsilon} \frac{(\lambda\omega\delta)^{\frac{1}{\varepsilon}} + \xi L^{\beta}\eta\delta}{\omega^{2}\delta} \gtrless 0 \Leftrightarrow \frac{1}{\varepsilon} \gtrless 1.
\end{aligned}
$$

The value of the intertemporal substitution elasticity appears to be crucial for resolving the ambiguity in the effects on the saving propensity. For instance, an increase in $\lambda$ causes an unambiguous increase in the share of output devoted to soil preservation ($\theta^*$) but it leads to a decline in $s$ if $1/\varepsilon > 1$ and to an increase in $s$ if $1/\varepsilon < 1$. It follows that, when the elasticity is relatively high, the optimal response of the economy to an increase in disaster frequency is to increase both its soil preservation expenditure and current consumption - at the expense of investment in soil expansion. By contrast, when the elasticity is relatively low, an increase in the preservation share is accompanied by a *reduction* in both consumption and soil accumulation ($\frac{\partial s}{\partial \lambda} < \frac{\partial \theta^*}{\partial \lambda}$, see the exact expression for $\frac{\partial \theta^*}{\partial \lambda}$ in the proof of Proposition 2). The intuition behind these results lies in the understanding of the intertemporal consumption smoothing. If EICS is relatively low, this means that the economy is less willing to reallocate consumption possibilities over time. An increase in $\lambda$ implies a higher chance of having a lower income in the future and thus a lower consumption. A low EICS dictates a preference for a smoother time-profile of consumption and thus it is optimal to lower the current consumption in response to an increase in $\lambda$ as a lower consumption is also anticipated in the future. Therefore $s$ increases, while current consumption and investment in soil accumulation fall. If EICS is relatively high, this means that the economy is more easily willing to reallocate consumption over time, so that the intertemporal smoothing is less important as compared to the overall lifetime consumption possibilities. As $\lambda$ increases, indicating a more likely decline in consumption in the future, the economy's optimal response is to increase soil preservation measures and current consumption as well. This is akin to the "precautionary consumption" phenomenon (Müller-Fürstenberger and Schumacher 2015). A similar reasoning can be applied to analyze the effects of an increase in harmful agricultural practices ($\eta$), damage intensity ($\delta$) and protection efficiency ($\omega$).

We note that when $\varepsilon$ approaches unity, the derived impact of all the parameters - and most importantly those characterizing adverse events, $\lambda$, $\eta$, and $\delta$ - are at the lower bound of the empirically plausible impact range.

Finally, population size in general has an ambiguous effect on $s$:

$$\frac{\partial s}{\partial L} = \frac{\beta}{\varepsilon \xi L^{\beta+1}} \left\{ \rho + \lambda + \frac{\varepsilon[1-\sigma-(\lambda\omega\delta)^{\frac{1}{\varepsilon}}]-(1-\sigma)}{\omega\delta} \right\} \gtrless 0.$$

On the one hand, an increase in labor raises the productivity of each soil unit which results in a higher optimal crop consumption growth, see Eq. (20). On the other hand, we see in Eq. (21) that more labor increases the marginal damage as a side-product of soil expansion, calling for a higher output share to be used for soil protection. This reflects the dual impact of population pressure prominently appearing in the literature: it offers a potential for raising aggregate output but imposes a higher challenge for policy making, requiring tougher soil protection in the optimum. With logarithmic preferences, however, the effect is unambiguously positive: $\frac{\partial s}{\partial L} = \frac{\beta\rho}{\xi L^{\beta+1}} > 0$.

# 4 Endogenous Arrivals

In this section we relax one of the assumptions of our baseline model, namely the exogeneity of the shock arrival rate. Harmful agricultural practices, such as deforestation to increase arable land, for example, may not only relate to the extent of the damage to the soil in the event of, say, a landslide but they may also cause an increase in the frequency of landslides. In this augmented model, soil preservation measures have a double-sided protective role since they reduce the damage on impact but also the probability of shock occurrence.

Let us assume that the arrival rate is an increasing function of the damages to the ecosystem per effective soil unit, $\lambda_t = \lambda(v_t)$, $v_t \equiv D_t/S_t$ and $\lambda'(v_t) > 0$. The HJB equation reads (omitting the time subscripts):

$$\rho V(S) = \max\left\{U(C) + V'(S)[(1-\theta)Y(S,L) - C] + \lambda(v)[V(\widetilde{S}) - V(S)]\right\}$$

and the optimality conditions consist of:

$$C \ : \ U'(C) - V'(S) = 0, \tag{23}$$

$$\theta \ : \ -V'(S)Y(S,L) + \frac{d\lambda(v)}{d\theta}[V(\widetilde{S}) - V(S)] + \lambda(v)V'(\widetilde{S})\xi S L^{\beta}\omega\delta = 0, \tag{24}$$

$$\begin{aligned} S \ : \ & \rho V'(S) = V''(S)[(1-\theta)Y(S,L) - C] + V'(1-\theta)\xi L^{\beta} + \frac{d\lambda(v)}{dS}[V(\widetilde{S}) - V(S)] + \\ & +\lambda(v)\left[V'(\widetilde{S})[1-\sigma-\delta(\eta-\theta\omega)\xi L^{\beta}] - V'(S)\right]. \end{aligned} \tag{25}$$

Given that the ecosystem damages are proportional to agricultural output, we have $v_t = (\eta - \omega\theta_t)\xi L^{\beta}$, so that we can write $\lambda_t = \lambda(\theta_t)$ with $d\lambda_t/d\theta_t = -\omega\xi L^{\beta}\lambda'(v_t) < 0$. It can be shown that the value function has the same form as in the benchmark model, $V(S) = \frac{\bar{\psi}^{-\varepsilon}S^{1-\varepsilon} - 1/\rho}{1-\varepsilon}$, where $\bar{\psi}$ is a function of the parameters of the model. Condition (24) then simplifies to

$$-V'(S) - \omega\lambda'(v)\frac{[V(\widetilde{S}) - V(S)]}{S} + \lambda(v)V'(\widetilde{S})\omega\delta = 0$$

or

$$\lambda(v)\delta(1-\sigma-v\delta)^{-\varepsilon} - \lambda'(v)\frac{(1-\sigma-v\delta)^{1-\varepsilon} - 1}{1-\varepsilon} = \frac{1}{\omega},$$

which contains an implicit solution for $v$, and thus for the optimal constant abatement share, which we denote $\theta^v$. The solution for $\theta^v$ depends on the choice of the functional form of $\lambda(v)$. For example, if $\lambda$ is linear in $v$ with the proportionality constant $a$ and the utility is logarithmic, then $\theta^v$ can be found in closed form:

$$\theta^v = \frac{\eta}{\omega} - \frac{(1-\sigma)\left(1 - \frac{1}{W(e^{1+1/a\omega}(1-\delta))}\right)}{\xi L^\beta \omega \delta},$$

where $W(.)$ is the LambertW function. Comparing $\theta^v$ with $\theta^*$ of the benchmark model, we find that

$$\theta^v \geqslant \theta^* \Leftrightarrow a \geqslant \frac{1}{\omega\left(\frac{1-\sigma}{\lambda\omega\delta} - 1 - \ln \lambda\omega\delta\right)} \equiv \bar{a}.$$

That is, the share of output devoted to soil preservation in a scenario with endogenous hazard rate exceeds the one in the benchmark model if and only if the marginal impact of damage-to-soil ratio on the hazard rate exceeds a specific threshold, $\bar{a}$. This threshold is an increasing function of the damage intensity ($\delta$), exposure intensity ($\sigma$), and arrival rate of the benchmark model ($\lambda$). For a specific parameter constellation, $\bar{a}$ may be negative and thus $\theta^v$ is always larger than $\theta^*$. The restriction on parameters in this case is such, however, that this outcome is relatively unlikely. More precisely, the requirement $\frac{1-\sigma}{\lambda\omega\delta} - 1 - \ln \lambda\omega\delta < 0$ is met only when $\lambda\omega\delta \in (1-\sigma, 1)$, i.e., very close to unity.

The parameter of the value function, $\bar{\psi}$, which also represents the proportion of crop consumption in total soil stock, is equal to:

$$\bar{\psi} = \frac{1}{\varepsilon}\Big\{\rho - (1-\varepsilon)(1-\theta^v)\xi L^\beta - \lambda\big(v(\theta^v)\big)\left[(1-\sigma-v(\theta^v)\delta)^{1-\varepsilon} - 1\right]\Big\}.$$

The optimal trend consumption growth rate, which we label $g^v$, can be found by applying the Itô's Lemma on $V'(S)$ and combining with condition (25) (following the same steps as in the benchmark model):

$$g^v = \frac{1}{\varepsilon}\Big\{(1-\theta^v)\xi L^\beta - \rho + \lambda\big(v(\theta^v)\big)\left[(1-\sigma-v(\theta^v)\delta)^{1-\varepsilon} - 1\right]\Big\}. \tag{26}$$

It can be shown that $g^v$ is smaller than $g$ if $\theta^v > \theta^*$ and the marginal impact of $v$ on the hazard rate is sufficiently small. To see this, substitute $\theta^*$ for $\theta^v$ in Eq. (26) and compare the result with $g$ of Eq. (20). If $a$ is sufficiently small (but still above $\bar{a}$ defined above) and $\varepsilon > 1$ (which we take to be the relevant range), then $g^v|_{\theta^v=\theta^*} < g$. Also note that $g^v$ is a decreasing function of $\theta^v$, so that if $\theta^v > \theta^*$, then $g^v < g^v|_{\theta^v=\theta^*}$, which implies that $g^v < g$ as well. The exact condition on the magnitude of $a$ reads $\bar{a} \leqslant a < \frac{\lambda\delta}{1-\sigma-(\lambda\omega\delta)^{1/\varepsilon}}$.

# 5 Conclusions

The present paper considers an agrarian economy which produces output employing two essential inputs, agricultural soil and labor. Production process, accompanied by harmful agricultural practices, weakens the ecosystem and diminishes its protective services. The agricultural soils become vulnerable to random environmental shocks which lead to soil degradation. In the benchmark model we assume that shocks arrive at a constant Poisson rate. Subsequently, we relax this assumption and endogenize the hazard rate as well. Such a scenario is observed in numerous developing countries with large agricultural sectors. In an attempt to raise yields and profits, farmers clear forests to gain arable land but at the same time they lose the protective services of forests and make their land exposed to landslides, floods, droughts, winds and similar calamities. To ensure sustained yields, it thus becomes necessary to adopt soil preservation measures (e.g. installation of contour hedgerows to prevent landslides). Since the extent of soil degradation depends positively on the magnitude of harmful agricultural activities and negatively on the protection efforts, an optimal soil expansion and preservation policy can be designed to maximize the economy's expected lifetime welfare. In the present article we provide a clear-cut closed-form solution to this dynamic stochastic problem.

The optimal development of the economy is characterized by the soil stock and consumption rate which grow at the same constant rate until a disaster occurs causing a downward jump in both variables. The percentage reduction in consumption, i.e. the size of the jump, is constant and depends on the arrival rate, the damage intensity, the protection efficiency and the intertemporal substitution elasticity (EICS). The optimal soil preservation strategy consists of devoting a constant fraction of output to protection measures. This fraction is an increasing function of the Poisson arrival rate, the damage intensity, the level of agricultural technology and agricultural labor force. It may be either increasing or decreasing in the protection efficiency due to three counteracting forces, the direct effect, the jump effect and the growth effect. The EICS appears to play a crucial role in determining how the economy's propensity to save responds to changes in the key parameters, including those characterizing adverse shocks. For a relatively high value of EICS (above unity), we find that an increase in disaster frequency leads to a decline in the saving propensity, implying that soil conservation measures and current consumption increase at the expense of soil stock expansion. An increase in the damage intensity of shocks leads to an increase in both protection measures and saving propensity. Consequently, an increase in the disaster frequency and in the damage intensity, while both having a positive impact on preservation measures, have diverging effects on the propensity to save and thus on how consumption possibilities are spread over time. When we allow for the arrival rate to depend on economic activity and to increase with the magnitude of harmful agricultural practices, we find that the optimal fraction of output devoted to preservation measures may exceed the one derived in the benchmark model.

Soil conservation has recently reemerged as an important issue in policy debate, especially in developing economies where a significant share of population still relies on agriculture for subsistence. The present article provides sound theoretical foundations for the analysis of optimal growth in agrarian economies and for the formulation of policy prescriptions with respect to soil expansion and preservation nexus.

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