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Abstract

This paper investigates competition among financial intermediaries in a finite-trader version of the Diamond and Dybvig (1983) economy under no aggregate uncertainty. The economy is populated by self-interested financial intermediaries that compete strategically over deposit contracts offered to consumers. Both exclusive and nonexclusive competition perspective are considered, in both cases multiple equilibria arise if banks do not have an initial endowment. When financial intermediaries have a sufficient level of endowment, regardless the competition perspective adopted, the first best allocation is the unique equilibrium allocation.

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Keywords: financial intermediation, deposit contracts.

1 Introduction

The Diamond and Dybvig (1983) model has become a reference model in the literature on financial intermediation. The model offers a rationale for the existence of financial intermediaries into the economic system: the intermediaries are able to provide liquidity services to consumers in the presence of an otherwise uninsurable event[1]. At the same time it raises a link between the use of deposit contracts, one of the instruments an intermediary can rely on to provide liquidity insurance, and the fragility of the financial system.

The model, then, has been adopted to study the allocation of liquidity risk in interbank markets (see, e.g., Bhattacharya and Gale, 1987, and Allen and Gale, 1997), and as a reference model to study the fragility of financial systems (see Allen and Gale, 2009).

Still a satisfactory characterization of the competition among financial intermediaries in economies

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[1]A financial intermediary does not play any role in an Arrow-Debreu economy, i.e. an economy where markets are perfect and complete. See, e.g., Freixas and Rochet (2008, Chapter 2) on this subject.

à la Diamond-Dybvig is missing. The present work pursues such an objective considering financial intermediaries that compete through menus of nonexclusive deposit contracts. When contracts are nonexclusive a consumer can simultaneously deal with several intermediaries. The nonexclusive competition perspective offers novel results, with respect to the exclusive competition scenario, in terms of allocations that can be sustained in equilibrium in economies subject to asymmetric information. Bizer and DeMarzo (1992) has been one of the first paper to analyse contractual externalities in financial markets subject to moral hazard, when contracts are nonexclusive. The paper shows that the presence of moral hazard can generate contractual externalities between existing and new loans, also in case debt is fully prioritised. Indeed new loans can change the borrower's optimal action, and thus can affect the expected return of existing debts. Attar et al. (2011) consider a seller who has private information about the quality of the good he sells, the good is perfectly divisible. The buyers compete by offering menus of non-exclusive contracts. The equilibrium features are different from those that arise under exclusive competition: an equilibrium exists under mild conditions, moreover when an equilibrium exists the seller trades his whole endowment or does not trade at all[2]. Ales and Maziero (2009) adopt a nonexclusive perspective in a private value economy. The authors consider a dynamic Mirrleesian economy, where consumers can be affected by idiosyncratic labour productivity shocks, the shock is a consumer's private information. In the economy financial intermediaries compete, by providing insurance services, through nonexclusive contracts. They show that in equilibrium consumers cannot acquire any insurance, while under exclusivity they can do so. Hence there is a complete breakdown of the insurance market.

Since the presence of asymmetric information is a key element in Diamond and Dybvig (1983), it is interesting to investigate whether or not competition among financial intermediaries can lead to a non efficient provision of liquidity services, and also if the way in which competition is defined affects the set of allocations that can be sustained in equilibrium. Papers that have addressed issues close to the one we pursue are: Adao and Temzelides (1998) and Farhi et al. (2009).

The purpose of Adao and Temzelides (1998) is: first to explicitly model the deposit decision of consumers; and second to investigate in which bank to deposit when there are two identical banks that offer different demand deposit contracts. With respect to the first issue, the authors consider the presence of a unique bank in the economy, this bank offers a deposit contract through which the

[2] In the same economy, competition through menus of exclusive contracts leads to inexistence issues while when an equilibrium exists the seller trades different quantities according to the quality of his good, in line with the results obtained in Rothschild and Stiglitz (1976).

first best allocation in the economy can be achieved. The consumers can accept or reject the offer, in the last case they invest directly in the available assets. With respect to the second issue, the authors consider an economy in which there are two financial intermediaries that offer two given menus of deposit contracts. Both menus of contracts consist of a deposit and of two possible profiles of withdrawals among which the consumers can choose at $t = 1$, after they have acquired their type. One bank offers a menu of deposit contracts through which the first best allocation can be achieved, while the other bank offers a menu of deposit contracts through which can be achieved an allocation that is strictly dominated by the first best one. The authors show that in both cases there are equilibria in which the consumers do not get the first best payoffs. Consider the second issue, suppose all consumers but one deposit in the bank that does not offer the first best allocation, then the best response of the last consumer is to deposit in the same bank, since a bank can afford liquidity provision only by pooling consumers with different liquidity needs. Section 3.2 discusses in more details the results achieved by these authors.

More recently Farhi et al. (2009) considered an economy in which competing intermediaries offer liquidity services in presence of a secondary market in which consumers can engage in hidden trades as in Jacklin (1987). In this economy consumers can, at the same time, deal with an intermediary and privately trade with other consumers in a competitive market, this can be another way to introduce nonexclusivity[3]. The authors show that competitive equilibria are inefficient[4], but to restore efficiency is sufficient to introduce a wedge between the interest rate implicit in the optimal allocations and the economy's marginal rate of transformation. They also characterize a simple way to implement the optimum: a liquidity floor that imposes to financial intermediaries to invest a given ratio of their liabilities in the short-term asset[5].

Before to present the model it is worth to highlight the main differences between the Diamond

[3]Farhi et al. (2009, Section 1: Introduction) give the following interpretation about contracts and hidden side trades the consumer can engage in: "This second friction (*hidden side trades*) can be interpreted as the case where contracts with financial intermediaries cannot be made exclusive. Arguably, both observability of certain financial market transactions and nonexclusivity become more relevant with the increasing sophistication of financial markets. Agents can and do engage in a variety of financial market transactions and routinely deal with several different intermediaries".

[4]If an intermediary offers a positive insurance against liquidity shocks then consumers, or other intermediaries, can exploit arbitrage opportunities as long as the interest rate in the secondary market and the gross return of the productive technology are different. But only offers with no positive insurance provision ensure the absence of such arbitrage opportunities. The inefficiency result due to arbitrage opportunities was already stated in Hellwig (1994) and Allen and Gale (2004) facing similar problems.

[5]In this way the interest rate in the secondary market is kept sufficiently low. Indeed a higher interest rate makes profitable for (impatient) consumers deviations consisting in early withdrawals within a deposit scheme and trades in the secondary market.

and Dybvig economy presented here and the classical screening model presented in Rothschild and Stiglitz (1976). The economy we rely on is with private values, since the consumers' private information is not an argument of the intermediaries' objective functions. To understand why it is so, let us consider an economy in which there is a unique intermediary who offers deposit contracts, what is relevant for the intermediary is the amount and the timing of the withdrawal made by a consumer and not the consumer's type per se. A second difference is that a consumer's participation decision with an intermediary is made ex-ante, i.e. before the consumer receives the information about his type. Moreover we consider financial intermediaries that compete through menus of nonesclusive contracts.

The paper is organized as follows: the reference economy is described in Section 2, Section 3 states and studies the competition game among financial intermediaries under both exclusive and nonexclusive perspective. Section 4 concludes. Appendix A presents the main features of the first best, the autarkic, and the second best allocation.

2 The Model

We built on the economy presented in Diamond and Dybvig (1983). The economy lasts three periods: $t = 0, 1, 2$. There is a unique good that serves at the same time as consumption good and as investment good. Two technologies are available in the economy, and all economic agents have access to them:

- the *productive technology*, or long-term asset, is available only at time $t = 0$, per unit invested guarantees a gross return equal to $\hat{R}$ at time $t = 2$, while if the production is interrupted at $t = 1$ the gross return per unit invested is equal to L. In particular, we assume the liquidation value of the productive technology is equal to one, $L = 1$. This makes the production side of the economy identical to the one presented in Diamond and Dybvig (1983)[6];
- the *storage technology*, or short-term asset, and per unit invested in a given period guarantees a gross return equal to 1 in the next period.

[6]Indeed in their model, a unique asset is present, the asset is available in each $t = 0, 1$. If the investment takes place at t, then the asset guarantees a gross return equal to 1 if liquidated in the period $t + 1$, while guarantees a gross return equal to $R > 1$ if liquidated in $t + 2$.

Given the assumption on L, the productive technology weakly dominates the storage technology in each investment decision[7].

The economy is populated by two types of economic agents: consumers and financial intermediaries (Henceforth we will also use the term insurance providers). Each consumer has an endowment equal to e at $t = 0$, while he does not receive any endowment at $t = 1$ and $t = 2$. Consumers have a utility function that is defined over their consumption at time $t = 1$ and $t = 2$. Moreover they can be of two types, and uncertainty about consumers' type is resolved only at $t = 1$.

In the economy there are also financial intermediaries, who compete providing liquidity services to consumers. They can offer, at $t = 0$, menus of deposit contracts. In particular, the economy is populated by a finite number, $N \in \mathbb{N}$, of financial intermediaries. They have a system of preferences that is linear in their wealth at time $t = 2$. If the wealth at the end of each period is denoted with γ_i for $i = 0, 1, 2$, then the objective function of an intermediary is given by[8]:

$$v_i(\gamma_0, \gamma_1, \gamma_2) = \gamma_2 \quad \text{with} \quad i \in \{1, 2, \ldots, N\}$$

Consumers' Types and Aggregate Uncertainty

We rely on an economy with only two consumers: consumer a and consumer b. As anticipated a consumer at time $t = 1$ can be of two types: θ_I or θ_P. We will also refer to a θ_I type as an impatient consumer and to a θ_P type as a patient consumer. The type determines the utility function of the consumer. The system of preferences imposes that an impatient consumer only values consumption at $t = 1$ while a patient consumer is indifferent between consuming in either periods:

$$u(c_1, c_2; \theta) = \begin{cases} u(c_1) & \text{if} \quad \theta = \theta_I \\ u(c_1 + c_2) & \text{if} \quad \theta = \theta_P \end{cases} \tag{1}$$

As standard in the literature, we will refer to a baseline utility function $u : \mathbb{R}_+ \to \mathbb{R}$ that is twice continuously differentiable, increasing, strictly concave, and that satisfies Inada conditions:

$$\lim_{c \to 0} u'(c) = \infty \quad \text{and} \quad \lim_{c \to \infty} u'(c) = 0 \tag{2}$$

[7]The structure of the production side, although redundant, offers the possibility to generalize our analysis in further research.

[8]Given the possibility to rely on the storage technology, financial intermediaries can transfer resources without incurring in any cost. Therefore a system of linear preferences that accounts for potential consumption in $t = 0$ and $t = 1$ will not change our analysis.

moreover the relative risk aversion coefficient has to be greater than one everywhere for both types:

$$-\frac{c\,u''(c)}{u'(c)} > 1 \quad \forall c > 0 \tag{3}$$

This last assumption guarantees that the per type profile of consumption associated to the first best allocation is smoother than the profile the consumer can achieve in autarky[9].

Notice that the assumed system of preferences satisfies an extreme version of single crossing condition, the indifference curves of an impatient consumer are vertical in the (c_1, c_2) space, while those of a patient consumer are linear and decreasing in the same space (see Figure 6). Consider the marginal rate of substitution between consumption at time $t = 1$ and consumption at time $t = 2$:

$$\tau_i = \frac{\dfrac{\partial u\left(c_1, c_2; \theta_i\right)}{\partial c_1}}{\dfrac{\partial u\left(c_1, c_2; \theta_i\right)}{\partial c_2}} \quad \text{for} \quad i = \text{I,P} \quad \text{and} \quad \forall\left(c_1, c_2\right) \in R_+^2$$

Then:

$$\tau_P = -\frac{u'\left(c_1 + c_2\right)}{u'\left(c_1 + c_2\right)} = -1$$

We can conclude that to renounce at a marginal unit of consumption good in $t = 1$, an impatient consumer needs an infinite compensation in terms of consumption good in $t = 2$; whereas a patient consumer, to renounce at a marginal unit of consumption good in $t = 1$, needs a compensation equal to an additional unit of consumption good in $t = 2$. In this sense an impatient consumer values more consumption in $t = 1$ than a patient consumer does.

The uncertainty about consumers' type is modelled assuming that at $t = 1$ two possible states of nature can realize. Both states are equiprobable, meaning that each state realizes with probability $1/2$. In state 1, consumer a is impatient and consumer b is patient. In state 2, consumer a is patient while consumer b is impatient. The uncertainty structure is summarized in Table 1. Hence in our economy there is *no aggregate uncertainty* since for sure one consumer will be impatient and the other will be patient.

The economy incorporates the main ingredients of the Diamond and Dybvig (1983) model, but since in each state of nature there is only one patient consumer, the coordination problems among

[9] On the role played by such assumption see von Thadden (1999, Section 2.2). In particular the author assumes that consumers have a constant intertemporal relative risk aversion, $u'(c) = c^{-a}$ with $a > 0$, and shows how the features of first best allocation changes with a. When $a > 1$ the optimal consumption profile is smoother than the autarkic one, while when $a < 1$ the opposite is true.

Table 1. States of Nature and Consumers' types

State of Nature	Consumers' type	
	Consumer a	Consumer b
1	θ_I	θ_P
2	θ_P	θ_I

impatient consumers that trigger bank runs do not arise in the present setting.

2.1 The Set of Incentive Feasible Allocations

In this section we define the set of feasible incentive compatible allocations, henceforth incentive feasible allocations. Appendix A supports the analysis presented in this section by characterizing: the first best allocation, the autarkic allocation and the second best allocation.

We consider an economy in which there is a unique insurance provider. We proceed presenting the indifference curves of the consumers and the iso-profit curves of the insurance provider.

The marginal rate of substitution of a representative consumer is:

$$\text{MRS}_c = -\frac{u'\left(c_1(\theta_I)\right)}{u'\left(c_2(\theta_P)\right)}$$

Let us consider the expected profit of an insurance provider that offers a profile of consumption $(c_1(\theta_I),\, c_2(\theta_P))$ and gets acceptance by both consumers, raising an amount of resources equal to $2Z$ from them. Since the insurance provider, in this case, does not face any aggregate uncertainty, then, if the consumption profile is incentive compatible, the profit of the insurance provider is:

$$\Pi\left(2Z, (c_1(\theta_I), c_2(\theta_P))\right) = \hat{R}\left[2Z - c_1(\theta_I)\right] - c_2(\theta_P)$$

Hence the marginal rate of substitution for an insurance provider is:

$$\text{MRS}_p = -\hat{R}$$

Along the 45 degree line, that guarantees a degenerate profile of consumption, the iso-profit curves of the insurance provider are steeper than the indifference curves of the consumers, in particular:

$$\left|\text{MRS}_p\right| = \hat{R} > 1 = \left|\text{MRS}_c\right|$$

Thus, starting from a certain profile of consumption, if we move toward north-west along the same indifference curve of the consumer, the lender increases his profits while the consumer receives the same expected utility, hence there are gains from further trades. Figure 1 presents the main features of the indifference curves and iso-profit curves.

The set of incentive feasible allocations can be defined considering the following constraints:

- the individual rationality constraint of the consumers,

$$\frac{1}{2}\Big[u(c_1(\theta_I)) + u(c_2(\theta_P))\Big] \geq \frac{1}{2}\left[u\left(e\right) + u\left(\hat{R}e\right)\right] = U_0 \tag{IRC}$$

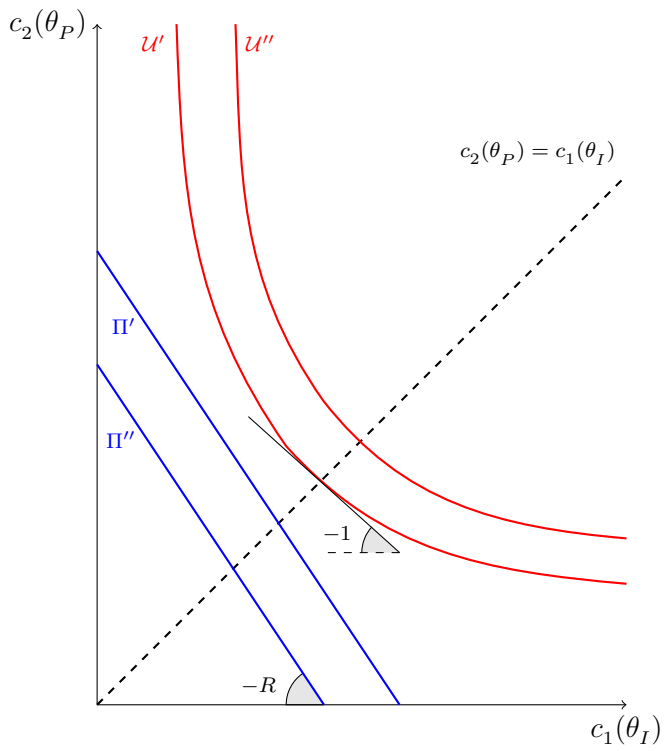
- the aggregate feasibility constraint of the economy,

$$c_1(\theta_I) + \frac{1}{\hat{R}}c_2(\theta_P) \leq 2e \tag{AFC}$$

- and the incentive compatibility constraint of the patient consumer,

$$c_2(\theta_P) \geq c_1(\theta_I) \tag{ICC}$$

Figure 1. Indifference and Iso-Profit Curves



Note: In red two indifference curves of the consumer, in blue two iso-profit curves of the insurance provider.

Notice that if an allocation satisfies the participation constraint of the consumers but it is not incentive compatible then cannot be afforded by the insurance provider. Indeed if it satisfies the consumer's participation constraint then $c_1(\theta_I) > e$, and since it is not incentive compatible any consumer will pretend to be impatient. In this case the bank will "default" since he has no sufficient resources to satisfy the aggregate withdrawals demand: $2c_1(\theta_I) > 2e$. The set of incentive feasible allocations, then, can be defined as:

$$\mathcal{F} = \Big\{ (c_1(\theta_I), c_2(\theta_P)) : \text{(IRC), (AFC) and (ICC) are satisfied.} \Big\}$$

2.1.1 Monopolistic Allocation

The monopolistic insurance provider does not face any aggregate uncertainty, hence his problem can be stated as:

$$\max_{c_1(\theta_I), c_2(\theta_P)} \hat{R}\left[2e - c_1(\theta_I)\right] - c_2(\theta_P) \tag{MAP}$$

subject to:

$$c_2(\theta_P) \leq \hat{R}\left[2e - c_1(\theta_I)\right] \tag{4}$$

$$\frac{1}{2}\Big[u\left(c_1(\theta_I)\right) + u\left(c_2(\theta_P)\right)\Big] \geq \frac{1}{2}\left[u\left(\hat{R}e\right) + u\left(e\right)\right] = U_0 \tag{5}$$

$$c_1(\theta_I) \geq c_1(\theta_P) \tag{6}$$

$$c_2(\theta_P) \geq c_1(\theta_I) \tag{7}$$

To approach the problem stated above we rely on a relaxed problem that only considers the rationality constraint of the representative consumer and then we show that at the solution of the relaxed problem both the incentive constraints are satisfied:

$$\max_{c_1(\theta_I), c_2(\theta_P)} \hat{R}\left[e - c_1(\theta_I)\right] - c_2(\theta_P)$$

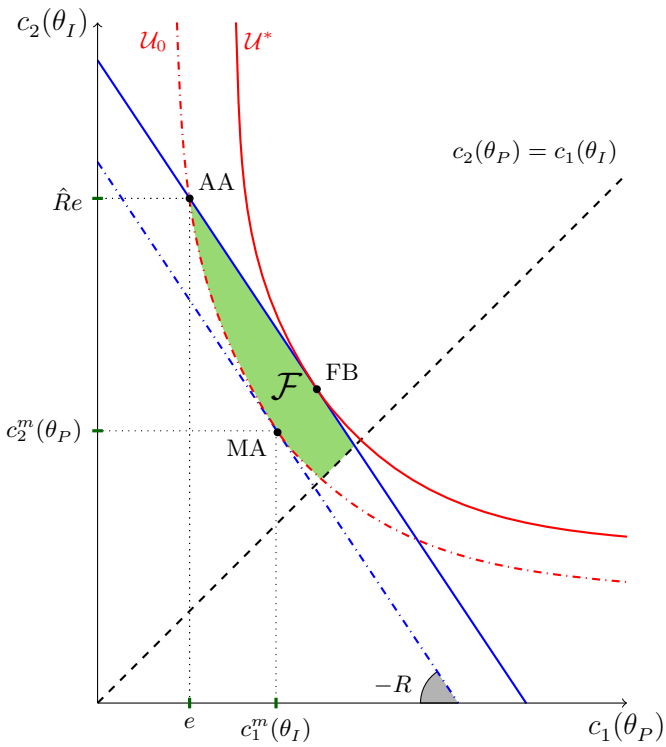
$$\text{s.t.} \quad u\left(c_1(\theta_I)\right) + u\left(c_2(\theta_P)\right) = u\left(e\right) + u\left(\hat{R}e\right)$$

The FONCs of the problem are:

$$\hat{R} - u'(c_1(\theta_I)) = 0$$

$$1 - u'(c_2(\theta_P)) = 0$$

Figure 2. Set of Incentive Feasible Allocations and Monopolistic Allocation



Note: In green, the set of incentive feasible allocations, $\mathcal{F}$. The monopolistic allocation is denoted by MA, the allocation characterized by the tangency of the iso-profit curve (in blue dashed) of the monopolist and the participation constraint of the consumer (curve U_0 in red).

Hence, the solution to the relaxed problem, $(c_1^m(\theta_I), c_2^m(\theta_P))$, has to be a solution of the following system of equations:

$$u(c_1^m(\theta_I)) + u(c_2^m(\theta_L)) = u(e) + u\left(\hat{R}e\right) \tag{8}$$

$$u'(c_1^m(\theta_I)) = \hat{R}\, u'(c_2^m(\theta_P)) \tag{9}$$

From (9), since $\hat{R} > 1$, $u'(c)$ is decreasing, and given the assumption stated in equation (3), we have:

$$\hat{R}e > c_2^m(\theta_P) > c_1^m(\theta_I) > e \tag{10}$$

Hence $(c_1^m(\theta_I), c_2^m(\theta_P))$ satisfies both the consumers' incentive constraints, and is therefore the unique solution to the considered problem. In the characterization of the monopolistic allocation it is important that whenever the allocation proposed by the monopolist is in the $\mathcal{F}$ set, then both consumers accept the proposed allocation[10].

[10] To exploit the relevance of this assumption see Section 3.2.

Figure 2 summarises the results achieved in this section.

3 Competition Game

In this section we present the structure of the competition game, we consider both exclusive and nonexclusive competition. Results presented in Sections 3.3.2 and 3.3.3 crucially depend on Assumption 1 stated in Section 3.3.

3.1 The Extensive Form Game

The interaction between the economic agents follows the extensive form presented below:

- At time $t = 0$:

  (0.1) Simultaneously each insurance provider offers a menu of deposit contracts. $\mathcal{M}^i$ will represent the offer made by insurance provider i, with $i = 1, \ldots, N$. While $\mathcal{M} = \Pi_{i=1}^{N} \mathcal{M}^i$ denotes the product set of all offered menus.

  The offer of an insurance provider consists of: $\mathcal{M}^i \subset \{(z, W)\colon z \in [0, e],\ W \subset \mathbb{R}^2_+\}$. If a consumer accepts a contract (z, W) in the menu, he has to deposit in bank i the amount z at time $t = 0$; while W is the set of withdrawal profiles among which, at time $t = 1$, he can choose. A withdrawal profile, $w = (w_1, w_2) \in W$, consists of a pair of repayments made by the bank to the consumer respectively at time $t = 1$ (w_1), and at $t = 2$ (w_2).

  To include in our setting the decision of a consumer not to deal with any insurance provider we assume that, for each i, $\mathcal{M}^i$ includes at least the degenerate contract $d_0 = (0, (0, 0))$[11]. Henceforth the word contract will be mainly used to refer to a non null contract.

  (0.2) Consumers observe the offers made by all insurance providers, and simultaneously accept one deposit contract from each offered menu.

  At the same time each consumer has to deposit the amount prescribed by each signed contract. If a consumer picks contract d_0 from each menu, then he has to deposit nothing and has to receive nothing from each bank.

  (0.3) Each bank observes each consumer's acceptance choice in his own menu. Then banks simultaneously implement their portfolio decisions. Consumers, if they have resources left, define their private portfolio simultaneously.

[11] In this way, to state the difference between exclusive and nonexclusive competition it is sufficient to impose that under exclusivity at most one accepted contract can be different from d_0, while under nonexclusivity this is no longer the case, any element in the set of accepted contracts can be different from d_0.

- At time $t = 1$:
  - (1.1) Each consumer privately knows his own type.
  - (1.2) After having known his type each consumer picks a consumption profile from each accepted deposit contract.
  - (1.3) Banks make their repayments at $t = 1$.

    Each consumer receives the gross return at $t = 1$ of his own portfolio and makes his consumption choice at $t = 1$ and his investment decision in the short-term asset from $t = 1$ to $t = 2$.
- At time $t = 2$:
  - (2.1) Investments in the short-term asset (from $t = 1$ to $t = 2$) and in the long-term asset realize. Banks make their repayments at $t = 2$.

    Each consumer makes his time $t = 2$ consumption decision.

As solution concept we will refer to perfect Bayesian equilibrium, and we will focus on equilibrium in pure strategies.

3.2 Coordination between Consumers and Competition between Intermediaries

Let us focus on an economy in which competition is exclusive, so that each consumer can accept a contract from at most one insurance provider. With respect to the usual competition game à la Bertrand in which two identical liquidity providers compete offering menus of deposit contracts, in our setting the provider that offers a better allocation to consumers could be unable to attract them.

In this section we will take as given the offers of the intermediaries and study the deposit and withdrawal game between consumers. To simplify assume that $N = 2$ and suppose that insurance providers 1 and 2 offer the following menus:

$$\mathcal{M}^1 = \left\{ d_0; \left\{ e, \{(c_1^*(\theta_I), 0), (0, c_2^*(\theta_P))\} \right\} \right\}$$
$$\mathcal{M}^2 = \left\{ d_0; \left\{ e, \{(c_1^m(\theta_I), 0), (0, c_2^m(\theta_P))\} \right\} \right\}$$

Hence insurance provider 1 offers a menu to sustain the first best allocation, whereas insurance provider 2 offers a menu to sustain the monopolistic allocation. Denote with d_0 the strategy not to deal with any insurance provider. In case a consumer decides to deal with an insurance provider, a strategy should prescribe which provider to deal with, and a withdrawal time according to the type he receives

in $t = 1$. A strategy for a consumer can be denoted with a triple, (j_1, j_2, j_3) with $j_k = 1, 2$ for $k = 1, 2, 3$; where the first component denotes the insurer the consumer deals with, the second component prescribes when to withdraw in case the consumer is impatient, and the last component prescribes when to withdraw when the consumer is patient. Then, the set of possible strategies for a consumer consists of 9 elements: $\{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2),d_0\}$.

Table 2 characterizes, in a bi-matrix, the payoffs associated to each possible profile of strategies of the game. To determine the payoffs we assume that, whenever the bank has not enough resources to face the aggregate withdrawals, the bank has to be liquidated and the obtained resources have to be equally distributed to the depositors[12]. Consider for instance the profile $(112, 112)$: both consumers deal with insurance provider 1, and both of them withdraw at $t = 1$ if impatient or at $t = 2$ if patients. Then if state of nature 1 realizes, consumer a is impatient while consumer b is patient, hence consumer a withdraws at $t = 1$ and consumer b withdraws at $t = 2$. The associated payoffs are: $u\left(c_1^a(\theta_I)\right)$ and $u\left(c_2^*(\theta_P)\right)$. If state of nature 2 realizes, consumer a is patient while consumer b is impatient, hence consumer a withdraws at $t = 2$ and consumer b withdraws at $t = 1$. The associated payoffs are: $u\left(c_2^*(\theta_P)\right)$ and $u\left(c_1^*(\theta_I)\right)$. Hence the expected utility, for both consumers, is $\frac{1}{2}\left[u\left(c_1^a(\theta_I)\right) + u\left(c_2^*(\theta_P)\right)\right]$.

The coordination game has three Bayes Nash Equilibria: $(112, 112)$, $(212, 212)$ and (d_0, d_0)[13]. This fact has relevant implications for our competitive game, since in equilibrium, also in this very simple example in which there is an insurance provider that offers a menu through which consumers can achieve the first best allocation, the consumers can choose not to deal with any insurance provider or to accept the menu through which the monopolistic allocation is achieved. These results are raised in Adao and Temzelides (1998). They consider an economy populated by a finite number of consumers and show that taken as given the offers of two insurance providers there could be multiple equilibria in the subgame that follows from these offers, in some of them liquidity providers can achieve strictly positive profits (In the coordination game above, this is the case when both consumers choose to deal with liquidity provider 2, that offers the monopolistic allocation). Moreover relying on forward induction they reach the following result (Adao and Temzelides, 1998, Proposition 9):

[12]Given the offers we considered if consumers deposit in different banks, then when the impatient consumer tries to withdraw he can at most be able to get the deposit he made in $t = 0$.

[13]In our example a run will never occur, indeed in any state of nature there is only one patient consumer, therefore if the allocation provided by a insurer is incentive feasible there are always sufficient resources to guarantee the withdrawal designed for the patient consumer.

Table 2. Coordination Game among Consumers

a \ b	$(1,1,1)$	$(1,1,2)$	$(1,2,1)$	$(1,2,2)$	$(2,1,1)$	$(2,1,2)$	$(2,2,1)$	$(2,2,2)$	d_0
$(1,1,1)$	$u(e), u(e)$	$\mathcal{U}_1^*, \mathcal{U}_2^*$	$\mathcal{U}_1^*, \frac{1}{2}u(e)$	$u(c_1^*), \frac{1}{2}u(c_2^*)$	$u(e), u(e)$	$u(e), \mathcal{U}_2^m$	$u(e), \frac{1}{2}u(e)$	$u(e), \frac{1}{2}u(c_2^m)$	$u(e), \underline{\mathcal{U}_0}$
$(1,1,2)$	$\mathcal{U}_2^*, \mathcal{U}_1^*$	$\underline{\mathcal{U}^*}, \underline{\mathcal{U}^*}$	$\mathcal{U}_2^*, \frac{1}{2}u(e)$	$\underline{\mathcal{U}^*}, \frac{1}{2}u(c_2^*),$	$\mathcal{U}_2^*, \frac{1}{2}u(e)$	$\mathcal{U}_2^*, \frac{1}{2}\mathcal{U}_2^m$	$\mathcal{U}_2^*, \frac{1}{2}u(e)$	$\mathcal{U}_2^*, \frac{1}{2}u(c_2^m)$	$u(e), \mathcal{U}_0$
$(1,2,1)$	$\frac{1}{2}u(e), \mathcal{U}_1^*$	$\frac{1}{2}u(e), \mathcal{U}_2^*$	$\frac{1}{2}u(c_1^*), \frac{1}{2}u(c_1^*)$	$\frac{1}{2}u(c_1^*), \frac{1}{2}u(c_2^*)$	$\frac{1}{2}u(e), u(e)$	$\frac{1}{2}u(e), \mathcal{U}_2^m$	$\frac{1}{2}u(e), u(e)$	$\frac{1}{2}u(e), \frac{1}{2}u(c_2^m)$	$\frac{1}{2}u(e), \underline{\mathcal{U}_0}$
$(1,2,2)$	$\frac{1}{2}u(c_2^*), \frac{1}{2}u(c_1^*)$	$\frac{1}{2}u(c_2^*), \underline{\mathcal{U}^*}$	$\frac{1}{2}u(c_2^*), \frac{1}{2}u(c_1^*)$	$\frac{1}{2}u(c_2^*), \frac{1}{2}u(c_2^*)$	$\frac{1}{2}u(c_2^*), u(e)$	$\frac{1}{2}u(c_2^*), \mathcal{U}_2^m$	$\frac{1}{2}u(c_2^*), \frac{1}{2}u(e)$	$\frac{1}{2}u(c_2^*), \frac{1}{2}u(c_2^m)$	$\frac{1}{2}u(c_2^*), \mathcal{U}_0$
$(2,1,1)$	$u(e), u(e)$	$u(e), \mathcal{U}_2^*$	$u(e), \frac{1}{2}u(e)$	$u(e), \frac{1}{2}u(c_2^*)$	$u(e), u(e)$	$\mathcal{U}_1^m, \mathcal{U}_2^m$	$\mathcal{U}_1^m, \frac{1}{2}u(e)$	$u(c_1^m), u(c_1^m)$	$u(e), \underline{\mathcal{U}_0}$
$(2,1,2)$	$\mathcal{U}_2^m, u(e)$	$\mathcal{U}_2^m, \mathcal{U}_2^*$	$\mathcal{U}_2^m, \frac{1}{2}u(e)$	$\mathcal{U}_2^m, \frac{1}{2}u(c_2^*)$	$\mathcal{U}_2^m, \mathcal{U}_1^m$	$\underline{\mathcal{U}^m}, \underline{\mathcal{U}^m}$	$\mathcal{U}_2^m, \frac{1}{2}u(e)$	$\underline{\mathcal{U}^m}, \frac{1}{2}u(c_2^m)$	$\mathcal{U}_2^m, \underline{\mathcal{U}_0}$
$(2,2,1)$	$\frac{1}{2}u(e), u(e)$	$\frac{1}{2}u(e), \mathcal{U}_2^*$	$\frac{1}{2}u(e), \frac{1}{2}u(e)$	$\frac{1}{2}u(e), \frac{1}{2}u(c_2^*)$	$\frac{1}{2}u(e), \mathcal{U}_1^m$	$\frac{1}{2}u(e), \mathcal{U}_2^m$	$\frac{1}{2}u(c_1^m), \frac{1}{2}u(c_1^m)$	$\frac{1}{2}u(c_1^m), \frac{1}{2}u(c_2^m)$	$\frac{1}{2}u(e), \underline{\mathcal{U}_0}$
$(2,2,2)$	$\frac{1}{2}u(c_2^m), u(e)$	$\frac{1}{2}u(c_2^m), \mathcal{U}_2^*$	$\frac{1}{2}u(c_2^m), \frac{1}{2}u(e)$	$\frac{1}{2}u(c_2^m), \frac{1}{2}u(c_2^*)$	$\frac{1}{2}u(c_2^m), \frac{1}{2}u(c_1^m)$	$\frac{1}{2}u(c_2^m), \underline{\mathcal{U}^m}$	$\frac{1}{2}u(c_2^m), \frac{1}{2}u(c_1^m)$	$\frac{1}{2}u(c_2^m), \frac{1}{2}u(c_2^m)$	$\frac{1}{2}u(c_2^m), \underline{\mathcal{U}_0}$
d_0	$\underline{\mathcal{U}_0}, u(e)$	$U_0, \mathcal{U}_2^*$	$\underline{\mathcal{U}_0}, \frac{1}{2}u(e)$	$U_0, \frac{1}{2}u(c_2^*)$	$\underline{\mathcal{U}_0}, u(e)$	$\underline{\mathcal{U}_0}, \mathcal{U}_2^m$	$\underline{\mathcal{U}_0}, \frac{1}{2}u(e)$	$\underline{\mathcal{U}_0}, \frac{1}{2}u(c_2^m)$	$\underline{\mathcal{U}_0}, \underline{\mathcal{U}_0}$

a The row player is consumer a, the column player is consumer b. The Coordination Game has three Bayes Nash Equilibria: $(112, 112)$, $(212, 212)$ and (d_0, d_0). Where: $c_1^* = c_1^*(\theta_I)$; $c_2^* = c_2^*(\theta_P)$; $c_1^m = c_1^m(\theta_I)$; $c_2^m = c_2^m(\theta_P)$; $\mathcal{U}_1^* = \frac{1}{2}[u(e) + u(c_1^*(\theta_I))]$; $\mathcal{U}_2^* = \frac{1}{2}[u(e) + u(c_2^*(\theta_P))]$; $\mathcal{U}_1^m = \frac{1}{2}[u(e) + u(c_1^m(\theta_I))]$; $\mathcal{U}_2^m = \frac{1}{2}[u(e) + u(c_2^m(\theta_P))]$; $\mathcal{U}^* = \frac{1}{2}[u(c_1^*(\theta_I)) + u(c_2^*(\theta_P))]$; $\mathcal{U}_0 = \frac{1}{2}\left[u(e) + u\left(\hat{R}e\right)\right]$.

Remark 1. In the considered coordination game, when consumers sequentially make their acceptance decisions, if consumers are restricted to pure strategies, the only forward induction sequential equilibrium is the one that supports the first best allocation.

From this section should be clear that the beliefs of a consumer about the behaviours of other consumers are the driving force to sustain, for instance, the monopolistic allocation in the considered coordination game. Indeed suppose consumer a believes that consumer b will deposit in bank 2 and well behave then for consumer a deposit in bank 2 and well behave is a best response, and the same is true for consumer b. In this way the consumers can coordinate towards the bank (or autarky also in presence of a unique bank that offers the first best allocation) that offers an allocation that is strictly dominated by the first best allocation. The same rationale applies to the extensive form game we presented in Section 3.1. Suppose both consumers believe that the other consumer will deal with intermediary 1 regardless the profile of offers they receive, then it is optimal for both consumers to deal with intermediary 1, moreover intermediary 1 given the systems of beliefs and strategies of the consumers can offer the monopolistic allocation and his offer will be accepted by the consumers.

3.3 Financial Intermediaries with Endowment

In this section we show that once the intermediaries have a sufficient level of initial resources a unique allocation can be sustained in equilibrium: the first best allocation.

We assume that each liquidity provider has an endowment equal to $\bar{f} - e$, where $\bar{f}$ is the highest flat profile of consumption that is feasible[14]:

$$\bar{f} = \frac{2\hat{R}e}{1+\hat{R}} < 2e \tag{11}$$

If we reconsider the coordination game presented in the Section 3.2, under the assumption made above, each liquidity provider can afford the withdrawals prescribed by the first best allocation, also if he attracts only one consumer (at $t = 1$, if an insurance provider is able to attract at least one consumer he will have a total asset greater than $c_1^*(\theta_I)$). Hence both consumers will have a strictly dominant strategy: to deposit in bank 1, and to withdraw at $t = 1$ if impatient or at $t = 2$ if patient. In this way, with certainty, a consumer obtains the expected utility associated to the first best allocation, regardless of the strategy the other consumer chooses.

[14]From the feasibility constraint we get $\bar{f} = \hat{R}(2e - \bar{f}) \Leftrightarrow \bar{f}(1 + \hat{R}) = 2\hat{R}e$.

The rest of this section is based on the following assumption:

Assumption 1. Each liquidity provider has an endowment, at $t = 0$, equal to $\bar{f}$. Where $\bar{f}$ is defined by equation (11).

Under this assumption, all insurance providers can afford any profile of consumption that is in the set of incentive feasible allocations, but if an intermediary attracts only one consumer he makes a strictly negative expected profit any time the withdrawal made available at $t = 1$ is greater than e. Indeed in this last case the provider has to liquidate the investment made with his own endowment. While if he attracts both consumers he can afford any allocation in the set of incentive feasible allocations without making losses.

3.3.1 An Analysis of the Consumer's Choices

Given our setting, it is useful to analyse the structure of the consumers' decisions. Again for simplicity let us restrict to the case $N = 2$. Consider a consumer that accepts two deposit contracts, one from each liquidity provider. Let us denote the two contracts with $d^1 = \{z^1, W^1\}$ and $d^2 = \{z^2, W^2\}$, where W^1 and W^2 are both subsets of $\mathbb{R}^2_+$. Let us consider now the optimal choice of the consumer at $t = 1$, when he has to choose which elements of W^1 and W^2 to pick. Since the financial intermediary, by attracting at least one consumer, can afford any allocation in the set of incentive feasible allocations, then the consumer's choice will only depend on his own type but not anymore on the choice made by the other consumer. If he is impatient, the highest level of utility the consumer can achieve is:

$$u\left(e - z_1 - z_2 + \tilde{w}_1^1 + \tilde{w}_1^2\right)$$

where: $\tilde{w}_1^i = \max\{w_1^i\}$, with $(w_1^i, w_2^i) \in W^i$, for $i = 1, 2$. That is, he will pick, from each set of consumption profiles, the one that guarantees the highest consumption in $t = 1$. Moreover the consumer will liquidate his private portfolio getting: $e - z_1 - z_2$.

In case the consumer is patient, he has to choose the contracts $w^i = (w_1^i, w_2^i) \in W^i$, for $i = 1, 2$, that maximizes:

$$u\left(\hat{R}\left(e - z_1 - z_2\right) + \sum_{i=1}^{2} w_i^1 + \sum_{i=1}^{2} w_i^2\right)$$

To each possible deposit contract $d^i = (z^i, W^i)$ we can assign the two following values, $(\tilde{w}^i(d^i;\theta_I)$, $\tilde{w}^i(d^i;\theta_P))$:

$$\begin{cases} \tilde{w}^i(d^i;\theta_I) = \max\{w_1^i\} & \text{for} \quad (w_1^i, w_2^i) = w^i \in W^i \\ \tilde{w}^i(d^i;\theta_P) = \max\{w_1^i + w_2^i\} & \text{for} \quad (w_1^i, w_2^i) = w^i \in W^i \end{cases}$$

Therefore, at $t = 0$, given the menu offered by the two insurance providers $(\mathcal{M}^1, \mathcal{M}^2)$ a consumer will choose the contracts $\tilde{d}^1 = \left(\tilde{z}^1, \tilde{W}^1\right) \in \mathcal{M}^1$, $\tilde{d}^2 = \left(\tilde{z}^2, \tilde{W}^2\right) \in \mathcal{M}^2$ in order to maximize:

$$u\Big(e - z^1(d^1) - z^2(d^2) + \tilde{w}^1(d^1;\theta_I) + \tilde{w}^2(d^2;\theta_I)\Big) + \\ + u\left(\hat{R}\left(e - z^1(d^1) - z^2(d^2)\right) + \tilde{w}^1(d^1;\theta_P) + \tilde{w}^2(d^2;\theta_P)\right)$$

3.3.2 The Exclusive Competition Game

Let us now characterize the equilibrium allocations that can arise in the competition game. Remember that when competition is exclusive each consumer can accept at most one no null contract.

Proposition 1. The unique equilibrium allocation of the exclusive competition game is the first best allocation.

Proof of Proposition 1.

At first we present the profile of strategies through which the first best allocation can be sustained in equilibrium, then we show that there are no other allocations that can be sustained in equilibrium. We start achieving the result for $N = 2$ and then address the general case of $N > 2$.

Profile of strategies to sustain the first best allocation.

Consider the following profile of strategies:

- Two insurance providers (provider 1 and 2) offer a menu that consists of a unique deposit contract, the one that guarantees the first best allocation:

$$\mathcal{M}^i = \Big\{d_0; \Big\{z = e; W = \{(c_1^*(\theta_I), 0), (0, c_2^*(\theta_P))\}\Big\}\Big\} \quad \text{for} \quad i = 1, 2.$$

- The other insurance providers offer $M^i = \{d_0\}$, with $i = 3, \ldots, N$;
- Both consumers accept the offer from the same insurance provider: provider 1 or 2. Moreover both consumers withdraw at $t = 1$ if impatient and at $t = 2$ if patient.

No insurance provider has a strictly profitable deviation, because there will be always at least one

provider that offers the allocation associated to the highest ex-ante utility a consumer can achieve in our setting, i.e. the first best allocation, regardless of the choices made by the other consumer.

Uniqueness of the equilibrium allocation when $N = 2$.

It is left to prove that the first best allocation is the unique allocation that can be sustained in equilibrium. To achieve this result we will consider two possible cases:

(i) The consumers accept contracts from different insurance providers;

(ii) The consumers accept contracts from the same insurance provider, the two accepted contracts can be different.

Consider case (i). Any insurance provider can afford, without making losses, at most a consumption profile that belongs to the set of feasible allocations of the autarkic consumer. Therefore the unique allocation, in this set, that satisfies the participation constraint of the consumer is the autarkic allocation. Hence both accepted contracts have to consist in the autarkic allocation.

In this case any insurance provider has a profitable deviation, that consists in adding to his menu the following contract:

$$\tilde{C} = \{z = e, W = \{(c_1^*(\theta_I) - \varepsilon, 0), (0, c_2^*(\theta_P))\}\}$$

notice that for ε small enough the above contract can lye in the set of incentive feasible allocations and that the expected utility can be higher than the one achieved choosing the autarkic allocation, therefore for ε small enough both consumers will find optimal to accept this contract. Furthermore since both consumers will accept the new contract proposed by the deviant provider, then the deviant will be able to sustain the above allocation (there is no aggregate uncertainty in this case and the feasibility constraint of the deviant will be equal to the feasibility constraint of the social planner) making strictly positive profits. Therefore under case (i) any allocation different from the first best allocation cannot be sustained in equilibrium.

Consider now case (ii), suppose that both consumers accept a contract from insurance provider j, then insurance provider $k \neq j$ has a strictly profitable deviation. Again, provider k can add to his menu the following contract:

$$\tilde{C}^k = \{z = e, W = \{(c_1^*(\theta_I) - \varepsilon, 0), (0, c_2^*(\theta_P))\}\}$$

For ε small enough, both consumers will sign the above contract, and also in this case the deviant

can make strictly positive profits.

Uniqueness when $N > 2$.

Since we have only two consumers, only two insurance providers will get acceptance and there will be always an inactive provider who can exploit a profitable deviation in line with the one depicted in case (ii).

Competition game with free entry.

Under free entry there is always an inactive provider: the entrant. Hence there is always an insurance provider that can exploit a profitable deviation in line with the one depicted in case (ii). ■

3.3.3 The Nonexclusive Competition Game

In the nonexclusive competition game, the consumers can accept contracts from different insurance providers.

A first result that can be stated when competition is nonexclusive, is that in equilibrium an allocation different from the first best allocation can be sustained only if all insurance providers make strictly positive profits (this implies that all insurance providers have to be active, i.e. they have to get at least one contract accepted by a consumer).

Lemma 1. If an allocation, different from the first best allocation, is sustained in equilibrium then all insurance providers have to make strictly positive profits.

Proof of Lemma 1.

Suppose that an allocation different from the first best is an equilibrium allocation. Assume that all insurance providers are active[15], but one does not make strictly positive profits in the supposed equilibrium, then he can add to the menu offered the following contract:

$$\tilde{C} = \left\{ z = e, W = \{(c_1^*(\theta_I) - \varepsilon, 0), (0, c_2^*(\theta_P))\} \right\}$$

For ε small enough the deviant provider will be able to attract both consumers, moreover since the accepted contracts asks for the entire endowment of the consumer, we can disregard the possibility that a consumer deals with other insurance providers, i.e. the above contract will be the unique accepted contract by each consumer. ■

[15] If there is an inactive provider then he makes zero profits by definition and the results stated in Lemma is achieved.

Now we are ready to characterize the equilibrium allocation of the nonexclusive competition game.

Proposition 2. The unique equilibrium allocation of the nonexclusive game is the first best allocation.

Proof of Proposition 2.

The structure of the proof follows the Proof of Proposition 1.

Profile of strategies to sustain the fist best allocation.

The profile of strategies to sustain the first best allocation is the one proposed in the proof of Proposition 1, namely:

- Two insurance providers (provider 1 and 2) offer a menu that consists of a unique deposit contract, the one that guarantees the first best allocation:

$$\mathcal{M}^i = \left\{ d_0; \left\{ z = e; W = \{(c_1^* (\theta_I), 0), (0, c_2^* (\theta_P))\} \right\} \right\} \quad \text{for} \quad i = 1, 2.$$

- The other insurance providers offer $M^i = \{d_0\}$, with $i = 3, \ldots, N$;
- Both consumers accept the offer from the same insurance provider: provider 1 or 2. Moreover both consumers withdraw at $t = 1$ if impatient and at $t = 2$ if patient.

No insurance provider has a strictly profitable deviation, because there will be always at least one provider that offers the allocation associated to the highest ex-ante utility a consumer can achieve in our setting, i.e. the first best allocation, regardless of the choices made by the other consumer.

Uniqueness of the equilibrium allocation when $N = 2$.

To address the uniqueness issue we will distinguish among two main cases.

First we will consider the case in which each consumer accepts one contract, but from different insurance providers. In this case we are under case (i) of the Proof of Proposition 1. The profitable deviation considered there applies also in this case.

It is left to prove that our result holds in the case at least one consumer accepts contracts from different insurance providers. The argument applied below builds on Section 3.3.1. The two consumers have to choose one contract from each menu offered. Suppose in equilibrium the two deposit contracts chosen by consumer a are $d^1(a) = (z^1(a), W^1(a)) \in \mathcal{M}^1$ and $d^2(a) = (z^2(a), W^2(a)) \in \mathcal{M}^2$; similarly for consumer b, the two contracts chosen are $d^1(b) = (z^1(b), W^1(b)) \in \mathcal{M}^1$ and $d^2(b) = (z^2(b), W^2(b)) \in \mathcal{M}^2$. According to the analysis of the optimal choice of the consumer, we

will have that the associated ex-ante profiles of consumption are:

$$\left(\tilde{c}_1\left(a;\theta_I\right),\tilde{c}_2\left(a;\theta_P\right)\right)=\left(e-\sum_{i=1}^{2}z^i\left(a\right)+\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(a\right);\theta_I\right),\hat{R}\left(e-\sum_{i=1}^{2}z^i\left(a\right)\right)+\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(a\right);\theta_P\right)\right)$$
$$\left(\tilde{c}_1\left(b;\theta_I\right),\tilde{c}_2\left(b;\theta_P\right)\right)=\left(e-\sum_{i=1}^{2}z^i\left(b\right)+\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(a\right);\theta_I\right),\hat{R}\left(e-\sum_{i=1}^{2}z^i\left(b\right)\right)+\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(a\right);\theta_P\right)\right)$$

where with the letters $\tilde{c}_l\left(j;\theta_i\right)$ for $j=a,b$, $i=I,P$ and $l=1,2$, we denote to the best allocation consumer j can achieve when he is of type θ_i at time l when he accepts contracts $d^1\left(a\right)$ and $d^2\left(a\right)$ in $t=0$. Notice that consumers have to be indifferent between the two supposed equilibrium allocations:

$$\frac{1}{2}\Big[u\left(\tilde{c}_1\left(a;\theta_I\right)\right)+u\left(\tilde{c}_2\left(a;\theta_P\right)\right)\Big]=\frac{1}{2}\Big[u\left(\tilde{c}_1\left(b;\theta_I\right)\right)+u\left(\tilde{c}_2\left(b;\theta_P\right)\right)\Big]$$

To be an equilibrium profile of consumption, the associated expected profits made by both insurance providers, according to Lemma 1, have to be strictly positive:

$$\Pi^i=z^i\left(a\right)+z^i\left(b\right)-\frac{1}{2}\Big[\tilde{w}^i\left(d^i\left(a\right);\theta_I\right)+\tilde{w}^i\left(d^i\left(b\right);\theta_P\right)\Big]-\frac{1}{2}\Big[\tilde{w}^i\left(d^i\left(a\right);\theta_P\right)+\tilde{w}^i\left(d^i\left(b\right);\theta_I\right)\Big]>0$$

for $i=1,2$. Then $\Pi=\Pi^1+\Pi^2>0$. Let us consider the ex-ante profile of consumption, between the two chosen in equilibrium by the consumers, that gives the highest expected profits in aggregate to the providers:

$$\max_{k=a,b}\left\{\Pi\left(k\right)=\frac{1}{2}\left[\hat{R}\left(\sum_{i=1}^{2}z^i\left(k\right)-\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(k\right);\theta_I\right)\right)+\hat{R}\sum_{i=1}^{2}z^i\left(k\right)-\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(k\right);\theta_P\right)\right]\right\}\Leftrightarrow$$
$$\max_{k=a,b}\left\{\Pi\left(k\right)=\frac{1}{2}\left[\hat{R}\left(2\sum_{i=1}^{2}z^i\left(k\right)\right)-\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(k\right);\theta_I\right)\right]-\frac{1}{2}\sum_{i=1}^{2}\tilde{w}^i\left(d^i\left(k\right);\theta_P\right)\right\}$$

Let us denote with $\hat{k}$ the consumer that chooses the most profitable allocation in aggregate, for $\hat{k}\in\{a,b\}$, since this is the most profitable allocation this means that the associated expected profits are at least equal to $\frac{1}{2}\Pi$.

At this stage of the analysis, one of the insurance provider is able to exploit a strictly profitable deviation adding the following contract to his menu:

$$C'=\left\{e,\left\{\left(\tilde{c}_1\left(\hat{k};\theta_I\right)+\varepsilon,0\right),\left(0,\tilde{c}_2\left(\hat{k};\theta_P\right)\right)\right\}\right\}$$

indeed, for ε small enough the aggregate profits of the deviant are approximately close to Π or even

greater, since the deviant is able to attract both consumers. Notice that having designed a deviation that asks the entire endowment to both consumers, we do not have to explore the possibility that at the deviation stage a consumer accepts contracts from other insurance providers. The profit that the deviant is able to achieve is:

$$\begin{aligned}
\Pi^d &= \hat{R}\left[2\left(e-\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right)-\tilde{c}_1\left(\hat{k};\theta_I\right)-\varepsilon+2\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right]-\tilde{c}_2\left(\hat{k};\theta_P\right)\\
&= \hat{R}\left[2\left(e-\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right)-\left(e-\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right)-\sum_{i=1}^{2}\tilde{w}^i\left(\hat{k};\theta_I\right)-\varepsilon+2\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right]-\tilde{c}_2\left(\hat{k};\theta_P\right)\\
&= \hat{R}\left[e-\sum_{i=1}^{2} z^i\left(\hat{k}\right)-\sum_{i=1}^{2}\tilde{w}^i\left(\hat{k};\theta_I\right)-\varepsilon+2\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right]-\left[\hat{R}\left(e-\sum_{i=1}^{2} z^i\left(\hat{k}\right)\right)+\sum_{i=1}^{2}\tilde{w}^i\left(\hat{k};\theta_P\right)\right]\\
&= \hat{R}\left[2\sum_{i=1}^{2} z^i\left(\hat{k}\right)-\sum_{i=1}^{2}\tilde{w}^i\left(\hat{k};\theta_I\right)\right]-\sum_{i=1}^{2}\tilde{w}^i\left(\hat{k};\theta_P\right)-\hat{R}\varepsilon=2\Pi\left(\hat{k}\right)-\hat{R}\varepsilon
\end{aligned}$$

For ε small enough this profit can be made arbitrarily close to Π[16], and since $\Pi > \Pi^i$, for $i = 1, 2$, the deviation is profitable.

Uniqueness when $N > 2$.

The argument adopted above, for the case in which in the economy are present only two insurance providers applies also in the case $N > 2$. Indeed we have constructed a profitable deviation that guarantees, for each aggregate profits associated to a candidate equilibrium allocation different from the first best allocation, profits arbitrarily close to the aggregate ones. Since in equilibrium all insurance providers have to make strictly positive profits, for any candidate equilibrium the intermediary with the lowest level of expected profits achieves at most a profit equal to $\frac{\Pi}{N}$. Hence the deviation depicted above can guarantee a level of profits arbitrarily close to Π, therefore it follows that also in this case the deviation is strictly profitable.

Competition game with free entry.

Consider as a candidate equilibrium allocation an allocation different from the first best, in a competitive game with free entry the entrant is inactive by definition. Then he can offer a menu that consists of:

$$\tilde{C} = \left(z = e, W = \left(\left(c_1^*(\theta_I) - \varepsilon, 0\right), \left(0, c_2^*(\theta_P)\right)\right)\right)$$

As usual, for ε small enough, this will be a strictly profitable deviation. ■

[16] Actually since $\Pi(\hat{k}) \geq \frac{1}{2}\Pi$ then the profit can be also greater than Π.

4 Conclusion

We considered an economy à la Diamond and Dybvig (1983) with a finite number of financial intermediaries and two consumers. Financial intermediaries are self-interested, i.e. they have to maximize their own profits, and compete by offering liquidity services to consumers in an economy with no aggregate uncertainty.

Given the coordination problems between consumers, we showed that multiple allocations can be sustained in equilibrium under both exclusive and nonexclusive competition. By imposing that financial intermediaries have a sufficient level of endowment we achieve that the first best allocation arises as the unique equilibrium allocation of the competition game, since in this case the standard undercutting mechanism à la Bertrand is restored.

Given that the endowment of the consumers is fixed and known, there is a simple way to impose exclusivity when intermediaries compete through menus of deposit contracts: an intermediary can ask for the entire endowment of the consumer.

In the Diamond and Dybvig (1983) model the presence of informational asymmetries do not play an active role in the characterization of the set of incentive feasible allocations, the first and second best allocation coincide, we aim to extend the analysis to economies in which informational asymmetries have a relevant role in that respect.

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A Appendix

A.1 First Best Allocation

In this section we characterize the first best allocation: the allocation that can be achieved by a social planner that only cares about the expected utility of the representative consumer, under the assumption that the social planner can define consumption profiles that are contingent on the consumer's type. The consumer's expected utility is defined by:

$$U((c_1(\theta_I), c_2(\theta_I)), (c_1(\theta_P), c_2(\theta_P)); \theta) = \frac{1}{2}\left[u\Big(c_1(\theta_I), c_2(\theta_I); \theta_I\Big) + u\Big(c_1(\theta_P), c_2(\theta_P); \theta_P\Big)\right]$$

Remember that the productive technology weakly dominates the storage technology, hence without loss of generality we can focus on the case in which the social planner invests all the resources collected in the productive technology. Since there is no aggregate uncertainty, the planner's problem can then be stated as follows:

$$\max_{x, \{c_1(\theta_i), c_2(\theta_i)\}_{i=I,P}} u\Big(c_1(\theta_I)\Big) + u\Big(c_1(\theta_P) + c_2(\theta_P)\Big) \tag{FBP}$$

subject to:

$$0 \leq x \leq 2e \tag{12}$$

$$0 \leq c_1(\theta_I) + c_1(\theta_P) \leq x \tag{13}$$

$$c_2(\theta_P) \leq \hat{R}\left[x - c_1(\theta_I) - c_1(\theta_P)\right] \tag{14}$$

where x represents the investment in the productive technology.

First, notice that it is never optimal to have $c_1(\theta_P) > 0$. Indeed the social planner can avoid the early liquidation of the investment associated to $c_1(\theta_P)$ and increase the utility of the patient consumer: $u\left(c_2(\theta_P) + \hat{R}c_1(\theta_P)\right) > u\left(c_1(\theta_P) + c_2(\theta_P)\right)$. For similar reasons $c_2(\theta_I) = 0$. Moreover constraint (14) is binding. Suppose not, the social planner can increase $c_2(\theta_P)$ and this will not affect any other constraint but increases the value of the objective function. Also constraint (12) is binding, meaning that $x = 2e$. Suppose not, the social planner can increase x and this will relax constraint (14) without affecting any other constraint. Furthermore, thanks to Inada conditions, constraint (13) is slack at the optimum. Hence the social planner's problem reduces to the maximization of the objective function subject to the following feasibility constraint:

$$c_2(\theta_P) = \hat{R}\left[2e - c_1(\theta_I)\right] \tag{15}$$

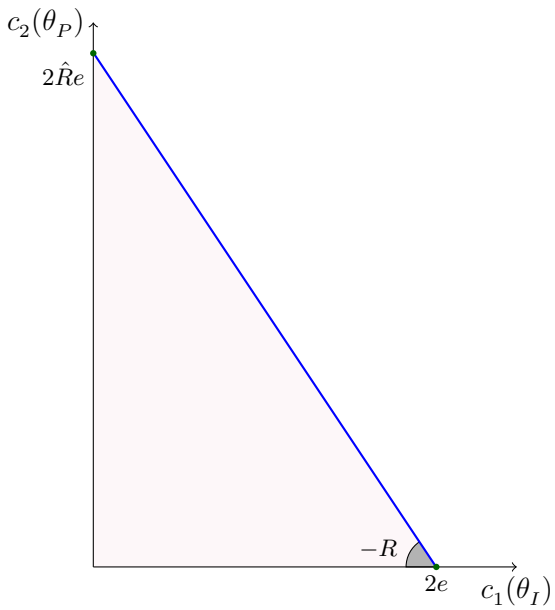
At this stage we can fully characterize the feasibility constraint in the Cartesian space $(c_1(\theta_I), c_2(\theta_P))$. This is done in Figure 3.

Substituting the feasibility constraint into the objective function we can characterize the FONC of the social planner's problem:

$$u'\left(c_1(\theta_I)\right) = \hat{R}\, u'\left(c_2(\theta_P)\right) \tag{16}$$

From (16), since $\hat{R} > 1$, $u'(c)$ is decreasing, and given the assumption stated in equation (3), we

Figure 3. Feasible Allocations



Note: The area in purple below the feasibility constraint represents the set of feasible consumption profiles for the social planner.

have:

$$\hat{R}e > c_2^*(\theta_P) > c_1^*(\theta_I) > e \tag{17}$$

The last inequality is due to the fact that $cu'(c)$ is decreasing in c, indeed from condition (3) follows:

$$-\frac{c \cdot u''(c)}{u'(c)} > 1 \Rightarrow u'(c) + c \cdot u''(c) < 0 \Rightarrow \frac{\partial (c \cdot u'(c))}{\partial c} < 0$$

From this, we can state that:

$$e \cdot u'(e) > \hat{R}e \cdot u'\left(\hat{R}e\right)$$

So that $c_1^*(\theta_I) > e$. Given this result, from the feasibility constraint (see equation (15)) is easy to show that $\hat{R}e > c_2^*(\theta_P)$.

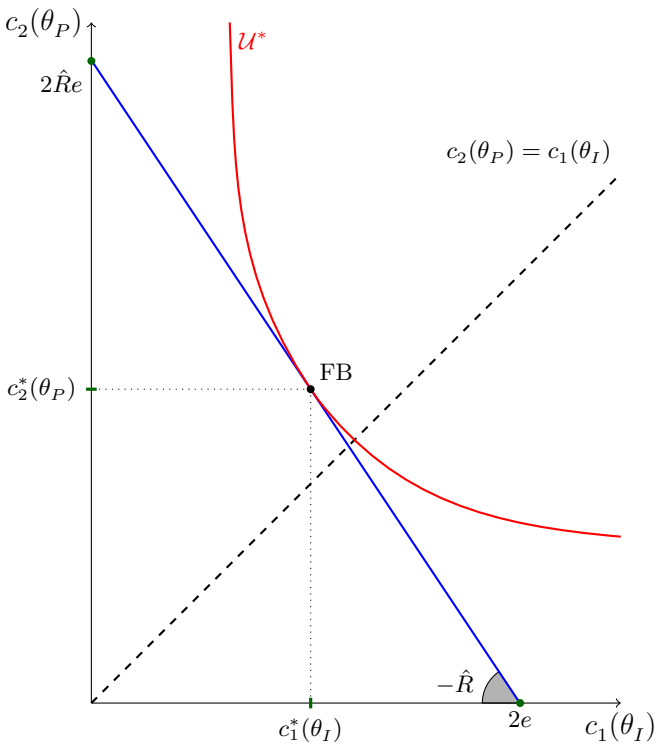
Figure 4 depicts the features of the optimal allocation in the $(c_1(\theta_I), c_2(\theta_P))$ space. The optimal allocation $(c_1^*(\theta_I), c_2^*(\theta_P))$ is above the 45 degree line. Rearranging equation (16) we obtain that at the optimum the ex-ante indifference curve of the representative consumer has to be tangent to the feasibility constraint:

$$\frac{u'(c_1^*(\theta_I))}{u'(c_2^*(\theta_P))} = \hat{R} \tag{18}$$

A.2 Autarkic Allocation

Autarky refers to an economy where there are no markets in which consumers can trade and no intermediaries that can provide insurance to consumers. Notice that in autarky, if a consumer is patient, it is never optimal for him to consume at $t = 1$, since postponing consumption at $t = 2$

Figure 4. First Best Allocation



Note: FB denotes the first best allocation. Such an allocation can be provided by the social planner investing all the resources in the productive technology and liquidating $c_1^*(\theta_I)$ at $t = 1$.

he can increase his utility. Hence the autarkic allocation can be obtained as the solution of the following problem:

$$\max_{x, c_1(\theta_I), c_2(\theta_P)} u\left(c_1(\theta_I)\right) + u\left(c_2(\theta_P)\right) \tag{AAP}$$

subject to:

$$x \leq e \tag{19}$$

$$c_1(\theta_I) \leq x \tag{20}$$

$$c_2(\theta_P) \leq \hat{R}x \tag{21}$$

All the constraints are binding at the optimum, so the autarkic allocation consists of:

$$c_1^a\left(\theta_I\right) = e \quad \text{and} \quad c_2^a\left(\theta_P\right) = \hat{R}e$$

The set of feasible allocations for an autarkic consumer is:

$$\mathcal{F}_a = \left\{(c_1(\theta_I), c_2(\theta_P)) : \; c_1(\theta_I) \leq e \text{ and } c_2(\theta_P) \leq \hat{R}e\right\}$$

We can also compare this feasibility constraint with the one of the social planner, the set of autarkic feasible allocations is a strict subset of the set of feasible allocations for the social planner. Moreover

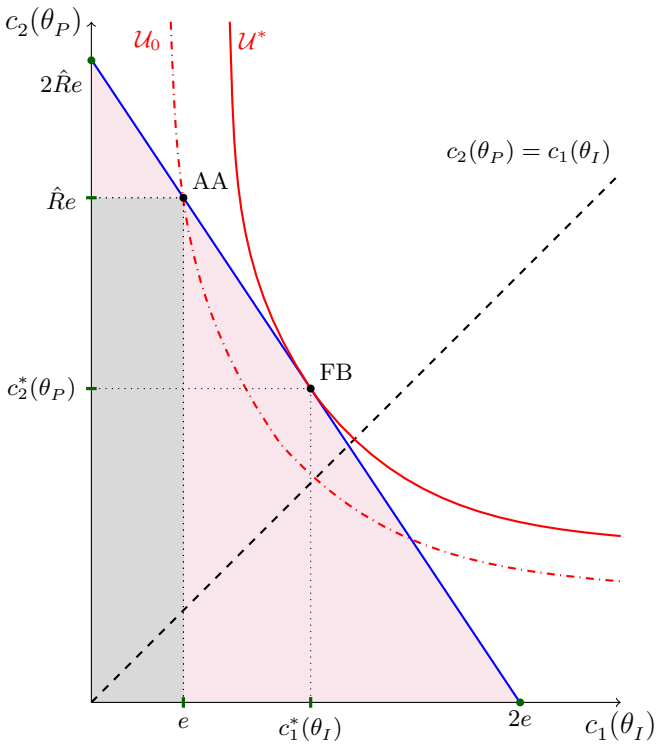
the autarkic allocation is on the feasibility constraint of the social planner:

$$\hat{R}\left[2e - c_1^a\left(\theta_I\right)\right] = c_2^a\left(\theta_P\right) \Leftrightarrow \hat{R}\left[2e - e\right] = \hat{R}e$$

Notice that the autarkic allocation can never be the first best allocation, since $e < c_1^*(\theta_I)$.

The social planner, collecting all the resources in the economy, is able to reduce the difference in the ex-ante consumption profile of the representative consumer, moving along the feasibility constraint from the autarkic allocation toward the first best allocation. Figure 5 depicts the features of the autarkic optimal allocation. From the figure we can observe that the first best exhibits a smoother per type consumption profile than the one a consumer can achieve in autarky. Moreover the investment needed in $t = 0$ to afford the first best consumption of an impatient consumer is greater than his endowment, hence there is cross-subsidisation across types.

Figure 5. Autarkic Allocation



Note: The grey area represents the set of feasible allocations for an autarkic consumer. The Autarkic Allocation (AA) is on the social planner feasibility constraint but on the left of the First Best Allocation (FB).

A.3 Second Best Allocation

In this section we characterize the features of the second best allocation, i.e. we consider the problem of a social planner that has to maximise the expected utility of a representative consumer in the case the consumers' type is private information. The problem can be stated as:

$$\max_{x, c_1(\theta_I), c_2(\theta_P)} u\Big(c_1(\theta_I)\Big) + u\Big(c_1(\theta_P) + c_2(\theta_P)\Big) \tag{SBP}$$

subject to:

$$0 \leq x \leq 2e \tag{22}$$

$$0 \leq c_1(\theta_I) + c_1(\theta_P) \leq x \tag{23}$$

$$0 \leq c_2(\theta_P) \leq \hat{R}\left[x - c_1(\theta_I)\right] \tag{24}$$

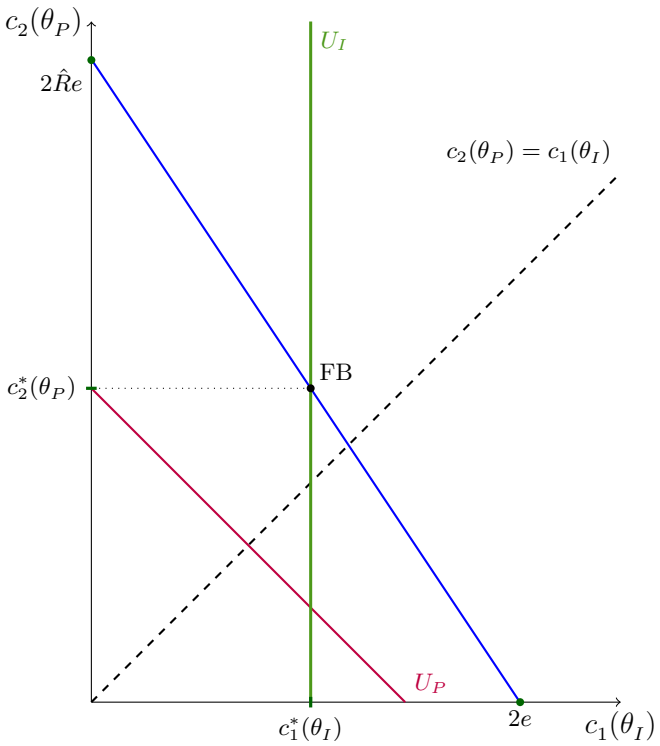
$$u\left(c_1(\theta_I)\right) \geq u\left(c_1(\theta_P)\right) \tag{25}$$

$$u\left(c_2(\theta_P) + c_1(\theta_P)\right) \geq u\left(c_2(\theta_I) + c_1(\theta_I)\right) \tag{26}$$

where constraints (25) and (26) are the incentive compatibility constraints of the impatient and patient consumer respectively.

It is easy to check that the first best allocation satisfies the two incentive compatibility constraints, this follows immediately from condition (17). Figure 6 reconsiders Figure 4, showing that no consumer's type has an incentive to take the allocation designed for the other type.

Figure 6. First Best Allocation and Incentive Compatibility Constraints



Note: At the First Best Allocation (FB), none of the incentive constraints is violated. In green the indifference curve of the impatient consumer while in violet the one of the patient consumer associated to the first best allocation.

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