Accounting for Different Uncertainties: Implications for Climate Investments

Svenja Hector

Economics Working Paper Series

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Accounting for Different Uncertainties:

Implications for Climate Investments?*

Svenja Hector

November 7, 2013

Abstract

The paper clarifies the link between changes in risk aversion and the effect on the consumption discount rate. In a general framework that can cope with various forms of uncertainty, it is shown that the response of the consumption discount rate to a change in risk aversion depends on some fundamental properties of the considered uncertainties. The application of this general result to specific forms of uncertainty extends existing results to more general forms of risk and yields a new result on preference uncertainty.

Keywords: discount rate, risk aversion, Kreps-Porteus-Selden, Risk-Sensitive preferences, uncertain preferences, climate change

JEL codes: H43, D81, Q54

Correspondence:
ETH Zurich
Department of Management, Technology and Economics
Chair of Integrative Risk Management and Economics
Zurichbergstrasse 18
8032 Zurich, Switzerland
Email: shector@ethz.ch

---

1   Introduction

Assessing optimal measures to tackle climate change is a prominent example for the evaluation of long-term investments. The consumption discount rate plays a crucial role in this context, since long time horizons amplify the discount rate's significance in determining an investment's desirability. Seemingly small differences in the discount rate have a major impact on suggested climate policy, as exemplified by the debate surrounding the Stern Review on the Economics of Climate Change and William Nordhaus' integrated assessment model DICE. Stern (2007), whose consumption discount rate of 1.4% is rather small, calls for 'strong and early action' to tackle climate change. Nordhaus (2008) matches his discount rate to a comparatively high interest rate of 4.1% and consequently argues for a more conservative 'climate policy ramp'. This discordance arises from differences in the pure rate of time preference and the intertemporal elasticity of substitution, which, together with the economic growth rate, determine the consumption discount rate in a deterministic setting.

Both, the Stern Review on the Economics of Climate Change and Nordhaus' DICE model lack a comprehensive treatment of uncertainties and the evaluation thereof. Uncertainties, however, loom large in the context of climate change. The natural sciences as well as the economic side of the evaluation contribute a considerable number of uncertainties, e.g. on climate sensitivity, the damage function, economic growth, investment payoffs, future preferences, and so on. How these uncertainties affect the assessment of climate policy does not only depend on their number and magnitude, but also on policy makers' attitude towards risk. A priori it is not clear whether increases in risk aversion impact investments positively, out of a precautionary or insurance motive, or whether a more risk averse policy maker invests less for future generations to avoid putting resources at risk. The effect from a change in the decision maker's degree of risk aversion on the discount rate may thus be positive or negative.

The present paper clarifies the link between changes in risk aversion and the effect on the consumption discount rate. For this purpose I develop a general framework that can cope with very diverse forms of uncertainty. I show within this framework that the direction of the effect from a change in the decision maker's risk attitude on the discount rate depends on some fundamental properties of the uncertainty accounted for.[1] These fundamental properties are then explored within a simple two-period endowment economy for three specific types of uncertainty, namely uncertain preferences, uncertain income, and uncertainty on an investment project. I consider a decision maker with Risk-Sensitive preferences (Hansen and Sargent, 1995), which, like Epstein-Zin preferences (Epstein and Zin, 1989), are a special case of the preference representation developed by Kreps and Porteus (1978) and Selden (1978). Contrary to the standard additive expected utility framework, as e.g. employed in the Stern Review and the DICE model, Risk-Sensitive preferences (and Epstein-Zin

---

[1]I do not draw a distinction between risk and uncertainty. Both words are used in the sense of Knightian risk. Knightian uncertainty is not considered.

preferences) allow for the disentanglement of risk and time preferences and thus render it possible to study the effect of a change in risk aversion alone. I enlarge upon the reasons for employing Risk-Sensitive preferences rather than Epstein-Zin preferences in subsection 2.2.

Related research on uncertainty and risk aversion has been conducted in the consumption/savings literature long before climate change was a widely studied topic in economics. Kihlstrom and Mirman (1974) and Kimball and Weil (2009) examine the effects of changes in risk aversion in the presence of uncertainty on investment returns and labour income, respectively. Bommier et al. (2012) explore the role of risk aversion in a non-parametric approach and investigate their results in context of income as well as return uncertainty.[2] Insights from the consumption/savings literature have more recently been applied to discounting in climate economics. Gollier (2002) complements the discounting debate by accounting for the effect of uncertainty on economic growth and in addition provides indication on the role of risk aversion. Traeger (2012) studies changes in risk aversion more explicitly, accounting for uncertainty on economic growth as well as return uncertainty.[3] The role of risk aversion given uncertainty on future preferences has neither been studied in the consumption/savings nor in the environmental economics literature. Beltratti et al. (1998) and others, however, analyze the effect of increases in preference uncertainty on the preservation of a non-renewable resource.

Similar to Gollier (2002) and Traeger (2012), the present paper adds to the discounting debate through a comprehensive treatment of risk aversion. One difference to their frameworks lies in the preference representation and the assumptions imposed on the size and distribution of the uncertainties. Gollier establishes the connection between the discount rate and risk aversion for a decision maker with general Kreps-Porteus-Selden preferences in a setting with small uncertainties. Traeger assumes normally distributed uncertainty and Epstein-Zin preferences. The present paper features Risk-Sensitive preferences and imposes no assumptions on the size or distribution of the uncertainties. Another difference to the contributions of Gollier and Traeger lies in the types of uncertainty accounted for. I account for preference uncertainty in addition to uncertainty on income and uncertainty on the investment project.

The paper's general result on the link between changes in risk aversion and the discount rate is developed in section 2 after describing the setting, the preference representation and the derivation of the consumption discount rate. This general result is then applied to specific types of uncertainty in section 3, which also provides

---

[2]The effects of increases in income and investment return uncertainty on optimal savings are analyzed by, e.g., Leland, 1968, Sandmo, 1970, Selden 1979, Drèze and Modigliani, 1972, Kimball, 1990, Bleichrodt and Eeckhoudt, 2005.

[3]The role of risk aversion is also analyzed in the wider environmental economics literature. Knapp and Olson (1996) and Epaulard and Pommeret (2003) analyze the response of optimal resource management to changes in risk aversion if net return, technological progress or stock dynamics are uncertain. Ha-Duong and Treich (2004) study the role of risk aversion in context of uncertainty about the damage from a pollution stock.

interpretations and relates to existing literature. Section 4 concludes.

2 Theory

2.1 Setting

The theoretical analysis is conducted for a decision maker (DM) who is altruistic towards people living in the present ($t = 1$) and the future ($t = 2$). His utility is derived from the felicity of the present and the future generation, which is in turn derived from consumption. The DM's purpose is the evaluation of a marginal investment in a two period endowment economy. The economy is deterministic in the first period but uncertainty enters the framework in the second period.

The first generation has exogenous income y_1 and invests an amount e in a project with rate of return r. Certain first period consumption is thus $c_1 = y_1 - e$. The second generation consumes the exogenous income y_2 and the payoff $(1 + r) e$ from the investment project. The exogenous future income as well as the investment payoff may be diminished by the random factors $\tilde{\gamma}_y$ and $\tilde{\gamma}_e$, respectively. Uncertain second period consumption is therefore $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \tilde{\gamma}_e (1 + r) e$. The decision maker evaluates the desirability of increasing project investment e by the marginal amount ε, such that first and second period consumption are $c_1 = y_1 - (e + \varepsilon)$ and $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \tilde{\gamma}_e (1 + r) (e + \varepsilon)$. In addition to the two uncertainties entering the evaluation through $\tilde{\gamma}_y$ and $\tilde{\gamma}_e$, I introduce a third random variable in the next subsection, namely $\tilde{a}$, which accounts for uncertainty on future felicity. Throughout the paper it is assumed that $y_2, r, \tilde{\alpha} > 0$, $y_1 > e \geq 0$ and $0 < \tilde{\gamma}_y, \tilde{\gamma}_e \leq 1$.

The variables $\tilde{\gamma}_y$, $\tilde{\gamma}_e$ and $\tilde{\alpha}$ are discrete random variables. Formally, a discrete random variable $\tilde{x}$ is a mapping $\Omega \to \mathbb{R}$, where Ω is the set of states of the world. For simplicity, Ω is assumed to be finite. Each state $\omega = 1, 2...N \in \Omega$ realizes with a given probability l^ω, where $\sum_\omega^N l^\omega = 1$. The value of the random variable $\tilde{x}$ when state ω is realized is denoted by x^ω. The expectation of $\tilde{x}$ is defined by $E_l [\tilde{x}] = \sum_\omega^N l^\omega x^\omega$ and the covariance between two discrete random variables $\tilde{x}_1$ and $\tilde{x}_2$ derives from $cov_l [\tilde{x}_1, \tilde{x}_2] = E_l [\tilde{x}_1 \tilde{x}_2] - E_l [\tilde{x}_1] E_l [\tilde{x}_2]$.

2.2 Preferences

The standard framework to evaluate uncertain consumption streams $(c_1, \tilde{c}_2)$ is the intertemporally additive expected utility setting. This preference representation, however, presupposes that the intertemporal elasticity of substitution (IES) is the inverse of the coefficient of Arrow-Pratt relative risk aversion (RRA). An increase in risk aversion thus entails a decrease in the IES and cannot be studied individually. In order to study the effect of a change in risk aversion alone, the additive expected

utility framework must be abandoned and a more flexible preference representation is required.

A possibility to achieve such flexibility involves the preference representations developed in Kreps and Porteus (1978) and Selden (1978). Selden provides a preference representation over certain × uncertain consumption pairs that allows for the disentanglement of risk and time preferences in a two period choice problem. Kreps and Porteus axiomatize Selden's preference representation for a T-period setting. Their recursive preference representation includes Selden's specification as the two-period special case. Key to the disentanglement of risk aversion and the intertemporal elasticity of substitution in the these frameworks is the differentiation between a cardinal atemporal expected utility function $v(\cdot)$ that represents risk preferences and an ordinal intertemporal utility function $U(\cdot)$ that represents time preferences. In a two-period setting, the general representation of Kreps-Porteus-Selden preferences is

$$U(c_1, \tilde{c}_2) = u(c_1) + \beta \phi^{-1}(E_l[\phi(u(\tilde{c}_2))]) \tag{1}$$

where $\beta > 0$ is the utility discount factor. The definition $v(\cdot) = \phi(u(\cdot))$ (with $\phi' \geq 0$, $\phi'' \leq 0$, $v', u' > 0$ and $v'', u'' < 0$) facilitates the disentanglement of risk and time preferences: Increasing the concavity of $\phi(\cdot)$ enhances the DM's risk aversion through increasing the concavity of $v(\cdot)$ without affecting the curvature of $u(\cdot)$, and thus without affecting the DM's time preferences.[4] If $\phi(\cdot)$ is linear, i.e. if $v(\cdot) = u(\cdot)$, equation (1) reduces to the standard additive expected utility representation in which time and risk preferences are entangled. A decision maker is called temporally risk averse if $\phi(\cdot)$ is strictly concave, and temporally risk neutral if $\phi(\cdot)$ is linear.

In the present paper, the 'Risk-Sensitive preferences' specification $\phi(z) = -\exp(-kz)$ (Hansen and Sargent, 1995) is employed to parameterize the general Kreps-Porteus-Selden preferences as represented by equation (1). A more prevalent parameterization is the isoelastic utility specification used by Epstein and Zin (1989) ('Epstein-Zin preferences') in their extension of Kreps and Porteus (1978) to the infinite horizon setting. The Epstein-Zin parameterization is, however, not monotonic with respect to first-order stochastic dominance, as already discussed in Chew and Epstein (1990), p. 68. Kimball and Weil (2009) illustrate that this may bring about perverted results when studying the role of risk aversion. In particular they show that increasing risk aversion can induce less precautionary savings if the risk on labour income is not restricted to be small. Bommier et al. (2012) ascribe this finding to the Epstein-Zin parameterization's failure to be 'well ordered in terms of risk aversion'. The non-compliance of this specification with ordinal dominance may imply agents that prefer first-order stochastically dominated lotteries. Within the set of stationary preferences that allow for a disentanglement of the intertemporal elasticity of substitution and risk aversion, Bommier and Le Grand (2013) identify the Risk-Sensitive preference parameterization, as employed in the present paper, as the only specification that is

---

[4]For a more detailed description of the disentanglement of risk and time preferences through the preferences of Kreps and Porteus (1978) and Selden (1978) see, e.g., Traeger (2009).

well ordered in terms of risk aversion.[5]

The framework described so far only incorporates uncertainty that enters through the argument $\tilde{c}_2$ of the second generation's felicity $u(\cdot)$. Uncertainty may, however, also exist on the contribution of the future generation's felicity to the DM's intertemporal utility. Such uncertainty can be thought of as uncertainty about future preferences, i.e. uncertainty about the felicity that the future generation gains from a given consumption level or, as in the setting of Beltratti et al. (1998), from a given level of natural resources. The felicity derived from future consumption is then $\tilde{\alpha}u(\tilde{c}_2)$, where the random multiplicator $\tilde{\alpha}$ determines whether the second generation values a given level of consumption more or less than the present generation. In particular, for $u(\cdot) > 0$ and if $\alpha^\omega > 1$ ($\alpha^\omega < 1$), the second generation values a given level of consumption more (less) than the present generation. This relation is inverted if $u(\cdot) < 0$.

Parameterizing equation (1) with the Risk-Sensitive preference specification $\phi(z) = -\exp(-kz)$ and accounting for uncertainty on future preferences, as represented by $\tilde{\alpha}$, yields

$$U(c_1, \tilde{c}_2) = u(c_1) - \frac{\beta}{k} \ln\left(E_l\left[\exp\left(-k\tilde{\alpha}u(\tilde{c}_2)\right)\right]\right), \tag{2}$$

which is the preference representation upon which the subsequent analysis builds. Under this Risk-Sensitive preference specification, a decision maker is temporally risk averse if $k > 0$ ($\phi(\cdot)$ strictly concave) and temporally risk neutral if $k \to 0$ ($\phi(\cdot)$ linear). Increasing k enhances the degree of risk aversion of the DM without affecting his time preferences.

2.3 Discounting

The consumption discount rate informs the desirability of conducting the additional marginal investment ε. Formally, a DM with preferences as represented by equation (2) considers investing ε desirable if

$$\frac{\partial U(c_1, \tilde{c}_2)}{\partial \varepsilon} \geq 0,$$

that is if investing an additional amount ε in the project increases his intertemporal utility. Solving this derivative yields

$$r \geq \delta$$

where

$$\delta = \left.\frac{u'(c_1)}{\beta} \frac{E_l\left[\exp\left(-k\tilde{\alpha}u(\tilde{c}_2)\right)\right]}{E_l\left[\exp\left(-k\tilde{\alpha}u(\tilde{c}_2)\right)\left(\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)\right)\right]}\right|_{\varepsilon=0} - 1 \tag{3}$$

---

[5]Note that the Epstein-Zin preference parameterization intersects with Hansen and Sargent's Risk-Sensitive preferences for $IES = 1$ and thus for $u(\cdot) = \ln(\cdot)$. In this special case Epstein-Zin preferences are well ordered in terms of risk aversion.

with $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \tilde{\gamma}_e (1 + r) e$. A marginal investment ε is desirable if its rate of return r exceeds the consumption discount rate δ, as defined in equation (3).[6] The consumption discount rate reflects the marginal rate of substitution between an additional unit ε of consumption in the first and in the second period, evaluated at the point where the additional investment is not yet conducted. This marginal rate of substitution gives account of the assumptions made on preferences, such as the degree of the DM's risk aversion, and assumptions on the economic setting, e.g. with respect to the considered uncertainties. The consumption discount rate constitutes a convenient tool to measure changes in the desirability of the investment ε in response to a change in risk aversion.

The term $\tilde{\gamma}_e \tilde{\alpha} u' (\tilde{c}_2)$, which I refer to as the *effective marginal utility,* is an essential part of equation (3). It is the state-dependent marginal utility gained by the decision maker in the second period if investments are increased. The higher the effective marginal utility is expected to be, ceteris paribus, the more valuable is an additional investment, as reflected in a lower discount rate. The expected size of the effective marginal utility does not only depend on the distribution of the states of the world and the state-dependent effective marginal utility itself, but also on the preferences of the Risk-Sensitive decision maker. A temporally risk averse decision maker modifies the (statistical) expectation of the effective marginal utility in such a way that the probabilities of occurrence of good states of the world are undervalued, whereas the probabilities of bad states of the world are overvalued. The more risk averse the decision maker is, the more the probabilities are modified. How good or bad a state of the world is perceived to be depends on a second essential part of equation (3), the *effective utility* $\tilde{\alpha} u (\tilde{c}_2)$. I show in the next subsection that the interrelation between the effective utility and the effective marginal utility is key in determining the effect of a change in risk aversion on the consumption discount rate.

To clarify the role of (temporal) risk aversion in the determination of the discount rate δ, I rewrite equation (3) in terms of a weighted sum:

$$\delta = \frac{1}{\beta} \frac{u'(c_1)}{\sum_\omega \pi^\omega \cdot (\gamma_e^\omega \alpha^\omega u'(c_2^\omega))} - 1 \quad \text{with} \quad \pi^\omega = l^\omega \cdot \frac{\exp(-k\alpha^\omega u(c_2^\omega))}{\sum_{\omega=1}^N l^\omega \exp(-k\alpha^\omega u(c_2^\omega))}. \quad (4)$$

The weights π^ω adjust the statistical probabilities l^ω to account for temporal risk aversion. These weights are interpretable as probabilities since $0 \leq \pi^\omega \leq 1 \ \forall \ \omega$ and $\sum_\omega \pi^\omega = 1$. I call π^ω a *risk aversion adjusted probability.* The fraction in the equation for π^ω is greater than 1 in the worst state of the world ($\alpha^\omega u (c_2^\omega)$ lowest) and smaller than 1 in the best state of the world ($\alpha^\omega u (c_2^\omega)$ highest). This suggests that the risk aversion adjusted probabilities π^ω overvalue the statistical probabilities of bad states and undervalue those of good states. This adjustment is amplified as k increases, i.e. as the decision maker becomes more risk averse. Note that π^ω approaches the statistical probability l^ω for a temporally risk neutral decision

---

[6]A more detailed derivation of the discount rate δ can be found, e.g., in Gollier (2002).

maker ($k \rightarrow 0$).[7] Equation (4) is then the discount rate of a DM with additive expected utility preferences. Such a decision maker is averse towards uncertainty on the argument of second period felicity $u(\cdot)$, yet he is neutral towards uncertainty that affects $u(\cdot)$ multiplicatively, such as $\tilde{\alpha}$.

2.4 Changes in risk aversion

The effect on the decision maker's evaluation of the intertemporal trade-off in response to a change in his degree of risk aversion is reflected in the derivative of the discount rate δ (eq. 3) with respect to k:

$$
\frac{\partial \delta}{\partial k} = \frac{u'(c_1)}{\beta \cdot (E_l[\exp(-k\tilde{\alpha}u(\tilde{c}_2))(\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2))])^2} \cdot \tag{5}
$$
$$
\left(\begin{array}{c} E_l[\exp(-k\tilde{\alpha}u(\tilde{c}_2))]\, E_l[\exp(-k\tilde{\alpha}u(\tilde{c}_2))(\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2))(\tilde{\alpha}u(\tilde{c}_2))] \\ -E_l[\exp(-k\tilde{\alpha}u(\tilde{c}_2))(\tilde{\alpha}u(\tilde{c}_2))]\, E_l[\exp(-k\tilde{\alpha}u(\tilde{c}_2))(\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2))] \end{array} \right).
$$

Using the risk aversion adjusted probabilities π^ω to define the *risk aversion adjusted expectation operator* $E_\pi[g(\tilde{x})] = \sum_\omega^N \pi^\omega g(x^\omega)$ for a random variable $\tilde{x}$ and some function $g(\tilde{x})$, equation (5) can be written as

$$
\frac{\partial \delta}{\partial k} = \frac{u'(c_1)}{\beta \cdot (E_\pi[\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)])^2} \cdot \tag{6}
$$
$$
(E_\pi[(\tilde{\alpha}u(\tilde{c}_2))(\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2))] - E_\pi[\tilde{\alpha}u(\tilde{c}_2)]\, E_\pi[\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)]).
$$

In this paper, a covariance that is constructed from a risk aversion adjusted expectation operator, i.e. $cov_\pi[g_1(\tilde{x}_1), g_2(\tilde{x}_2)] = E_\pi[g_1(\tilde{x}_1)g_2(\tilde{x}_2)] - E_\pi[g_1(\tilde{x}_1)]\, E_\pi[g_2(\tilde{x}_2)]$, is called a *risk aversion adjusted covariance*. In accordance with this denotation, the second line of equation (6) is the risk aversion adjusted covariance between $\tilde{\alpha}u(\tilde{c}_2)$ and $\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$: $cov_\pi[\tilde{\alpha}u(\tilde{c}_2), \tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)]$. If the two random variables 'effective utility' and 'effective marginal utility' satisfy a specific interrelation, the sign of this risk aversion adjusted covariance is determinate. Lemma 1 below identifies the comonotonicity characteristics (definition 1) of $\tilde{\alpha}u(\tilde{c}_2)$ and $\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$ as this particular interrelation.[8] The proof of lemma 1, which is a close analogue to theorem 43 in Hardy et al. (1934), is relegated to appendix 6.1.

Definition 1 *(Strict comonotonicity and strict countercomonotonicity).*
Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations

---

[7]The notion of 'risk aversion adjusted probabilities' reminds of the concept of 'risk neutral probabilities' in asset pricing. Yet the two concepts are not equivalent. In the present paper, the 'risk aversion adjusted probabilities' π^ω differ from the statistical probabilities l^ω only if the agent is not temporally risk neutral ($k \neq 0$). If $k = 0$, however, then $\pi^\omega = l^\omega$. In asset pricing in the contrary, a decision maker with $k = 0$ may employ 'risk neutral probabilities' that are not equivalent to the respective statistical probabilities.

[8]Comonotonicity (and countercomonotonicity) is defined in various ways in the literature. The definition provided here is based on McNeil et al. (2005), pp. 199, 200.

of a single random variable $\tilde{x}$:

$$(Z_1, Z_2) = (g_1(\tilde{x}), g_2(\tilde{x})).$$

If g_1 and g_2 are strictly increasing in $\tilde{x}$, then Z_1 and Z_2 are called comonotonic. If g_1 is strictly increasing and g_2 is strictly decreasing in $\tilde{x}$, or vice versa, then Z_1 and Z_2 are called countercomonotonic.

Lemma 1 *(Risk aversion adjusted covariance inequality).*
Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a single random variable $\tilde{x}$. If Z_1 and Z_2 are strictly comonotonic, then

$$cov_\pi[Z_1, Z_2] > 0.$$

The inequality is reversed if Z_1 and Z_2 are strictly countercomonotonic.

Applying the insight of lemma 1 to equation (6) links the effect of a change in risk aversion on the consumption discount rate to the comonotonicity characteristics of the effective utility and the effective marginal utility. This relation is summarized in proposition 1.[9]

Proposition 1 *(Effect of a change in risk aversion on δ).*
If the effective utility ($\tilde{\alpha}u(\tilde{c}_2)$) and the effective marginal utility ($\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$) are comonotonic, then the discount rate increases in response to an increase in risk aversion ($\frac{\partial\delta}{\partial k} > 0$).
If the effective utility ($\tilde{\alpha}u(\tilde{c}_2)$) and the effective marginal utility ($\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$) are countercomonotonic, then the discount rate decreases in response to an increase in risk aversion ($\frac{\partial\delta}{\partial k} < 0$).

Proof. The sign of equation (6) is the same as that of $cov_\pi[\tilde{\alpha}u(\tilde{c}_2), \tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)]$ since β, $u'(c_1)$, $(E_\pi[\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)])^2 > 0$. By application of lemma 1, the sign of $cov_\pi[\tilde{\alpha}u(\tilde{c}_2), \tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)]$ follows from the comonotonicity characteristics of $\tilde{\alpha}u(\tilde{c}_2)$ and $\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$. ∎

Proposition 1 constitutes a general result that is not restricted to specific forms of uncertainty. Information on the comonotonicity characteristics of the effective utility and the effective marginal utility suffices to determine the direction of the effect of a change in risk aversion on the consumption discount rate. The comonotonicity characteristics, however, are conditional on the type of uncertainty accounted for. In the next section I explore the comonotonicity characteristics for three different types of uncertainty and provide an example involving multiple uncertainties.

---

[9]Proposition 1 is derived for the case of a finite set of states of the world. The proof of lemma 3 in Bommier and Le Grand (2013) shows, however, that lemma 1 of the present paper also holds for continuous random variables. The general result of proposition 1 is thus extendable to the case of continuous uncertainty.

3 Application to specific types of uncertainty

The examination of the comonotonicity characteristics of the effective utility and the effective marginal utility is initially conducted for individual types of uncertainty, i.e. assuming that the remaining random variables are deterministic. Linking the results of this examination to proposition 1 yields insights on the effect of a change in risk aversion on the consumption discount rate in the presence of different types of uncertainty. The effective utility corresponding to the three different types of uncertainty is specified as follows:

case 1 ($\tilde{\alpha}$ uncertain, γ_y, γ_e deterministic) : $\tilde{\alpha}u(c_2)$ with $c_2 = \gamma_y y_2 + \gamma_e(1+r)e$

case 2 ($\tilde{\gamma}_y$ uncertain, γ_e, α deterministic) : $\alpha u(\tilde{c}_2)$ with $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \gamma_e(1+r)e$

case 3 ($\tilde{\gamma}_e$ uncertain, γ_y, α deterministic) : $\alpha u(\tilde{c}_2)$ with $\tilde{c}_2 = \gamma_y y_2 + \tilde{\gamma}_e(1+r)e$.

These three cases represent uncertain future preferences (case 1), uncertainty on future income (case 2) and uncertainty on the investment project's payoff (case 3).

In order to assign the comonotonicity characteristics of the effective utility and the effective marginal utility one needs to analyze whether they are increasing or decreasing in the considered random variable. The effective utility is increasing in the random variable $\tilde{\gamma}_y$ and, if $u(\cdot) > 0$, in $\tilde{\alpha}$. For $e > 0$, it is also increasing in $\tilde{\gamma}_e$. The effective marginal utility is increasing in $\tilde{\alpha}$, decreasing in $\tilde{\gamma}_y$ and non-monotonic in $\tilde{\gamma}_e$.[10] Combining the insights on the comonotonicity characteristics, which follow from definition 1, with the general result of this paper (proposition 1) then yields proposition 2.

Proposition 2 (*Effect of a change in risk aversion on δ for specific uncertainties*). *The effect of a change in risk aversion on the discount rate is as follows:*

Case 1: *If uncertainty exists only on future preferences and if the decision maker is risk averse with respect to this uncertainty, then the discount rate increases in response to an increase in risk aversion.[11]*

Case 2: *If uncertainty exists only on future income, then the discount rate decreases in response to an increase in risk aversion.*

Case 3: *If uncertainty exists only on the investment project's payoff and if $e > 0$, then the response of the discount rate to a change in risk aversion depends on the size of the intertemporal elasticity of substitution relative to a threshold:*
If $IES > \frac{\tilde{\gamma}_e(1+r)e}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e}$ $\forall$ ω, the discount rate increases with risk aversion.
If $IES < \frac{\tilde{\gamma}_e(1+r)e}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e}$ $\forall$ ω, the discount rate decreases with risk aversion.

The following subsections discuss these results and connect to the literature.

---

[10]See appendix 6.2.

[11]Risk aversion with respect to preference uncertainty means that the decision maker prefers $E[\tilde{\alpha}]$ to $\tilde{\alpha}$. Given $k > 0$, this is the case whenever $u(\cdot) > 0$. The relation in proposition 2 (case 1) is inverted for a DM who is risk loving with respect to $\tilde{\alpha}$, see appendix 6.2, 1.

3.1 Uncertainty on preferences

At the heart of all evaluations of intertemporal trade-offs lies the attempt to secure the well-being of generations living at different times. To this end, a decision maker may want to redistribute resources between present and future generations. The default assumption that underlies such evaluations is that agents living at different times gain the same well-being from a given amount of consumption. Rapid societal changes in the past suggest, however, that such an assumption is overly simplistic, especially if the far future is considered. Whether future developments yield an increase or a decrease in the valuation of produced or natural resources is unclear. Yet the mere presence of uncertainty on future preferences is without much doubt. Solow (1992) emphasizes the relevance of this issue in the context of sustainability. The notion of sustainability, which involves the quest for a level of future well-being that is not below ours, he argues, is problematic and vague by nature as we do not know how the well-being of future generations will be determined.

The utility multiplicator $\tilde{\alpha}$ considered in this paper can be interpreted as uncertainty about future felicity from consumption. People living in the future could be more dependent on produced goods and thus attach a higher value to their consumption, or they prefer to live more simply and thus have a lower valuation. Similarly, the felicity derived from the consumption of natural resources could be higher in the future as new insights on their usability are gained, or lower as substitutes are discovered. Would a risk averse decision maker who faces such uncertainty invest more for the future generation as his risk aversion increases, to insure for the possibility of an increased valuation? Proposition 2 (case 1) suggests that he would not. Uncertainty on future preferences implies that resources allocated to the future are put at risk since their valuation is insecure. An increasingly risk averse Risk-Sensitive decision maker therefore allocates more consumption to the present generation since he is certain about their valuation.

A small group of authors has studied the effect of uncertain preferences on the optimal allocation of a nonrenewable resource. Beltratti et al. (1998) consider the effect of uncertainty on future preferences as specified in the present paper. Yet the intertemporal trade-off in their approach is evaluated by a decision maker with additive expected utility preferences. The authors find that symmetric uncertainty on future preferences, i.e. mean preserving uncertainty in the sense of Rothschild and Stiglitz, does not affect the optimal consumption/preservation stream.[12]

It follows from proposition 2 (case 1) that Beltratti et al.'s (1998) results are not robust to changes in risk aversion. A simplified two-period version of their model can be nested in the framework of the present paper, where it constitutes the special case of temporal risk neutrality ($k \to 0$). Going from a temporally risk neutral to

---

[12]More complex forms of preference uncertainty are taken into account by Ayong Le Kama & Schubert (2004), who analyze the effect of uncertainty on the preference for environmental quality relative to the preference for consumption, and Cunha-E-Sà & Costa-Duarte (2000) and Ayong Le Kama (2012), who consider endogenous uncertainty about future preferences.

a temporally risk averse DM (i.e. increasing risk aversion) then induces an increase in the discount rate, which, in the setting of Beltratti et al., implies a decrease in the preservation of a non-renewable resource. The result of Beltratti et al. is thus extended: If the decision maker is temporally risk neutral, symmetric uncertainty about future preferences does not affect the optimal preservation of a non-renewable resource. Yet if the decision maker is temporally risk averse, then uncertainty on future preferences affects the preservation of a non-renewable resource negatively.

3.2 Uncertainty on income, investment payoff, or both

The dimension of economic growth is inherently uncertain. This uncertainty about future income levels is even increased by the possibility of detrimental effects from climate change on productive capacities. A forward-looking decision maker may thus want to take measures to insure against these negative effects on future generations, such as investing in the economy's productive capacity or in research on and development of abatement and adaptation capacities. Yet the payoff to such investments may be uncertain itself, for example due to uncertainty on technological advancements that determine the effectiveness of abatement and adaptation or due to uncertainty on damages from climate change that decrease the investment project's payoff multiplicatively.

Proposition 2 (cases 2 and 3) clarifies how an increase in the DM's risk aversion affects his willingness to invest for the future in the presence of uncertainty on future income or on the payoff from the investment. If only uncertainty on future income is accounted for (case 2), then increasing the DM's risk aversion unambiguously induces higher precautionary savings, as reflected in a decreasing discount rate. The more risk averse the decision maker is, the more he overvalues the statistical probability of bad states in which the effective marginal utility in the future is relatively high. The increasingly risk averse decision maker therefore allocates more resources to the uncertain future and thus insures against the possibility of low income levels in the second period. If only uncertainty on the investment payoff is accounted for (case 3) and if $e > 0$, then the effect of an increase in risk aversion is ambiguous. The direction of the effect on the discount rate depends on the value of the DM's intertemporal elasticity of substitution relative to the state dependent value of a threshold. An increase in risk aversion induces an increase in the discount rate if the IES is higher than the value of the threshold in the best state of the world.[13] This is always the case if $IES \geq 1$. An increase in risk aversion then amplifies the DM's aversion to putting resources at risk by investing more in a project with uncertain payoff. Consequently, the DM transfers less resources to the future. If the IES is smaller than the value of the threshold in the worst state of the world, the discount rate is decreasing in

---

[13]Note that the threshold $\frac{\tilde{\gamma}_e(1+r)e}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e}$ is increasing in $\tilde{\gamma}_e$. The state-dependent threshold thus reaches its highest (lowest) level in the best (worst) state of the world, i.e. when $\tilde{\gamma}_e$ is highest (lowest). If the IES exceeds (goes below) the threshold in the best (worst) state of the world, it also exceeds (goes below) the thresholds in the other states of the world.

risk aversion. A small IES implies a relatively high desire to smooth deterministic consumption over time. A DM with a relatively low IES thus aims to maintain the future generation's certainty equivalent consumption. As the certainty equivalent consumption is decreased by an increase in risk aversion he allocates more resources to the future. If the IES lies between the threshold evaluated in the best state of the world and the threshold evaluated in the worst state of the world we cannot make a statement on the sign of $\frac{\partial \delta}{\partial k}$. Note that the result of proposition 2 (case 3) crucially depends on assuming $e > 0$. In a setting where initial project investment is nil ($e = 0$) and uncertainty exists only on the investment project's payoff, the discount rate is unaffected by changes in risk aversion.

So far different uncertainties have only been considered individually, i.e. assuming that all other variables are deterministic. If some assumptions on the relationship between the considered uncertainties are made, it is possible to use the general result of proposition 1 to analyze the role of risk aversion in the presence of multiple uncertainties. In the context of climate change we may assume that the uncertainties on diverse variables are linked to a random variable that represents future climatic conditions. The comonotonicity characteristics of the effective utility and the effective marginal utility then depend, among other conditions, on the comonotonicity characteristics of the uncertain variables with respect to the random variable 'climate conditions'. Consider, for example, that the future income as well as the payoff of the investment are uncertain due to their dependence on random climate conditions $\tilde{x}$. A high realization of the random variable $\tilde{x}$ represents an unchanged climate and a low $\tilde{x}$ stands for detrimental climate change. Countercomonotonicity of γ_y and γ_e in $\tilde{x}$ ($\gamma_y'(\tilde{x}) > 0$, $\gamma_e'(\tilde{x}) < 0$) is given if the economic income is highest in a world with an unchanged climate while the payoff from an investment project is highest under very bad climate conditions. In this case, and if economic income varies more strongly with climate conditions than the project's payoff, an increase in risk aversion unambiguously amplifies the valuation of future consumption (see appendix 6.3, 1). Yet if γ_y and γ_e are comonotonic in $\tilde{x}$ ($\gamma_y'(\tilde{x}) > 0$, $\gamma_e'(\tilde{x}) > 0$), i.e. if both the economic income as well as the payoff from the investment project are high in good states of the world, then the sign of the derivative of the discount rate with respect to risk aversion depends on the relative magnitudes of the IES, the overall variation in consumption and the variation in the project payoff (see appendix 6.3, 2).

A result that is consistent with proposition 2 (case 2) is implicit in Gollier's (2002) examination of the effect of growth uncertainty on the socially optimal discount rate, yet his result is derived under the assumption that the uncertainty on future income is small. Similarly, Traeger (2012) shows that an increase in temporal risk aversion decreases the social discount rate for mitigation policies in an isoelastic utility framework if uncertain growth is accounted for. In addition, and in accordance with my remarks on proposition 2 (case 3) regarding the case $e = 0$, Traeger shows that temporal risk aversion with respect to an uncertain marginal investment project has no independent effect on the discount rate. Related to my example on multiple uncertainties, he shows that correlated uncertainty on the investment project induces

ambiguous reactions of the discount rate in response to risk aversion changes, depending on the sign and degree of the correlation with growth uncertainty, the variances of both uncertain variables, and the IES. Uncertainty on future income as well as uncertainty on the marginal investment project are assumed to be (jointly) normally distributed in Traeger's analysis. The earlier mentioned contribution of Kimball and Weil (2009) suggests, however, that results derived for normal (or small) uncertainties in an isoelastic utility setting may not be extendable to the case of less restricted uncertainties. In particular they show that results on the role of risk aversion in an isoelastic utility setting may be perverted if large uncertainty on future income is considered. The present paper averts these problems -and thus represents an extension of Gollier's and Traeger's results- since I do not make use of the isoelastic utility specification but rather employ Risk-Sensitive preferences. No assumptions on the size or distribution of the uncertainty are therefore necessary to derive the results on the effect of changes in risk aversion. My results are consistent with Bommier et al. (2012), who point out that the effect of risk aversion changes in presence of income uncertainties is monotonic if preferences are well ordered in terms of risk aversion, and in light of Bommier and Le Grand (2013), who identify the Risk-Sensitive preference specification to be well ordered in terms of risk aversion.

4   Conclusions

The prevalent frameworks to assess optimal climate policy, like the Stern Review and Nordhaus' DICE model, assume a degree of risk aversion that is too low in view of empirical evidence.[14] Changing the degree of risk aversion in these frameworks, however, distorts the intertemporal elasticity of substitution at the same time. A number of recent contributions have tackled this issue by developing recursive versions of DICE in order to study the effects of different types of uncertainty under empirically substantiated assumptions on risk and time preferences. Ackerman et al. (2013) do so considering climate uncertainty, Cai et al. (2013) investigate the effect of uncertainty on the economic impact of climate tipping events, Jensen and Traeger (2013) are interested in long term growth uncertainty and Crost and Traeger (2011) look at damage uncertainty. The comparison of the results from these recursive DICE models to the results of the standard DICE model yields insights on the policy implications of increased risk aversion. These recursive DICE models are rather complex, however, and account for only individual uncertainties at a time. It is thus difficult to expose the role of risk aversion and to attribute the direction of effects to specific features of the uncertainty.

The framework of the present paper is a two-period endowment economy in which

---

[14]The default assumption in the DICE model is a degree of relative risk aversion of 2 and an intertemporal elasticity of substitution of 0.5. Empirical evidence from the asset pricing literature suggests, however, a degree of relative risk aversion of 5-10 and an intertemporal elasticity of substitution of 1-1.5 (Bansal and Yaron, 2004, Vissing-Jørgensen and Attanasio, 2003).

different uncertainties can be accounted for, individually as well as simultaneously. The simplicity of the approach allows for a general result on the link between changes in the decision maker's risk aversion and the consumption discount rate by which the direction of the effect is ascribed to specific characteristics of the considered uncertainties. In particular I show that the direction of the effect from a change in risk aversion on the consumption discount rate depends on the comonotonicity characteristics of the effective utility and the effective marginal utility. The comonotonicity characteristics in turn differ between the types of uncertainties. This implies that increases in risk aversion may have very diverse effects on the evaluation of an intertemporal consumption trade-off, and thus on optimal climate policy.

The application of the general result yields a new result on the effect of risk aversion changes in the presence of preference uncertainty and extends existing results with respect to the presence of income and investment payoff uncertainties. In contrast to Beltratti et al. (1998), I find that preference uncertainty does affect the discount rate positively - if a temporally risk averse rather than a temporally risk neutral decision maker is considered. This effect is amplified as the degree of risk aversion is increased. Regarding income and investment payoff uncertainty, the application of the general result confirms previous results of Gollier (2002) and Traeger (2012), yet under less restrictive assumptions on the size or distribution of uncertainty. This extension of Gollier's and Traeger's results is rendered possible by employing the Risk-Sensitive preference representation.

5 References

ACKERMAN, F., STANTON, E.A. AND R. BUENO (2013): Epstein-Zin Utility in DICE: Is Risk Aversion Irrelevant to Climate Policy? *Environmental and Resource Economics* 56, 73-84.

AYONG LE KAMA, A. (2012): *Preservation and Endogenous Uncertain Future Preferences.* CDC Climat and Université Paris-Dauphine, Paris, France, Les Cahiers de la Chaire Economie du Climat Working Paper No. 2012-04.

AYONG LE KAMA, A. AND K. SCHUBERT (2004): Growth, Environment and Uncertain Future Preferences. *Environmental and Resource Economics* 28, 31-53.

BANSAL, R. AND A. YARON (2004): Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance* 59, 1481-1509.

BELTRATTI, A., CHICHILNISKY, G. AND G.M. HEAL (1998): Uncertain Future Preferences and Conservation. In: Chichilnisky, G., Heal, G.M. and A. Vercelli (eds.): *Sustainability: Dynamics and Uncertainty.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 257-275.

BLEICHRODT, H. AND L. EECKHOUDT (2005): Saving under rank-dependent utility. *Economic Theory* 25, 505-511.

BOMMIER, A. AND F. LE GRAND (2013): *A Robust Approach to Risk Aversion: Disentangling risk aversion and elasticity of substitution without giving up preference monotonicity.* ETH Zurich and EMLyon Business School, September 2013.

BOMMIER, A., CHASSAGNON, A. AND F. LE GRAND (2012): Comparative risk aversion: A formal approach with applications to saving behavior. *Journal of Economic Theory* 147, 1614-1641.

CAI, Y., JUDD, K.L. AND T.S. LONTZEK (2013): *The social cost of stochastic and irreversible climate change.* NBER Working Paper No. 18704, January 2013.

CHEW, S.H. AND L.G. EPSTEIN (1990): Nonexpected Utility Preferences in a Temporal Framework with an Application to Consumption-Savings Behaviour. *Journal of Economic Theory* 50, 54-81.

CROST, B. AND C. TRAEGER (2011): *Risk and Aversion in the Integrated Assessment of Climate Change.* CUDARE Working Paper No. 1104R, July 2011.

CUNHA-E-SA, M. AND C. COSTA-DUARTE (2000): Endogenous Future Preferences and Conservation. *Environmental and Resource Economics* 16, 253-262.

DREZE, J.H. AND F. MODIGLIANI (1972): Consumption Decisions under Uncertainty. *Journal of Economic Theory* 5, 308-335.

EPAULARD, A. AND A. POMMERET (2003): Optimally eating a stochastic cake: a recursive utility approach. *Resource and Energy Economics* 25, 129-139.

EPSTEIN, L.G. AND S. ZIN (1989): Substitution, Risk Aversion, and the Temporal

Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57, 937-969.

GOLLIER, C. (2002): Discounting an uncertain future. *Journal of Public Economics* 85, 149-166.

HA-DUONG, M. AND N. TREICH (2004): Risk Aversion, Intergenerational Equity and Climate Change. *Environmental and Resource Economics* 28, 195-207.

HANSEN, L.P. AND T.J. SARGENT (1995): Discounted Linear Exponential Quadratic Gaussian Control. *IEEE Transactions on Automatic Control* 40, 968-971.

HARDY, G., LITTLEWOOD, J.E. AND G. POLYA (1934): *Inequalities.* Cambridge University Press, London, UK.

JENSEN, S. AND C. TRAEGER (2013): *Mitigation under Long-Term Growth Uncertainty: Growing Emissions but Outgrowing its Consequences - Sure?* Department of Agricultural & Resource Economics, UC Berkeley, August 2013.

KIHLSTROM, R.E. AND L.J. MIRMAN (1974): Risk Aversion with Many Commodities. *Journal of Economic Theory* 8, 361-388.

KIMBALL, M. AND P. WEIL (2009): Precautionary Saving and Consumption Smoothing across Time and Possibilities. *Journal of Money, Credit and Banking* 41, 245-284.

KIMBALL, M. (1990): Precautionary Saving in the Small and in the Large. *Econometrica* 58, 53-73.

KNAPP, K.C. AND L.J. OLSON (1996): Dynamic Resource Management: Intertemporal Substitution and Risk Aversion. *American Journal of Agricultural Economics* 78, 1004-1014.

KREPS, D.M. AND E.L. PORTEUS (1978): Temporal Resolution of Uncertainty and Dynamic Choice Theory. *Econometrica* 46, 185-200.

LELAND, H.E. (1968): Saving under Uncertainty: The Precautionary Demand for Saving. *Quarterly Journal of Economics* 82, 465-473.

MCNEIL, A.J., FREY, R. AND P. EMBRECHTS (2005): *Quantitative Risk Management - Concepts, Techniques and Tools.* Princeton University Press, Princeton, NJ, USA.

NORDHAUS, W.D. (2008): *A Question of Balance: Economic Modeling of Global Warming.* Yale University Press, New Haven, CT, USA.

SANDMO, A. (1970): The Effect of Uncertainty on Saving Decisions. *Review of Economic Studies* 37, 353-360.

SELDEN, L. (1979): An OCE Analysis of the Effect of Uncertainty on Saving Under Risk Preference Independence. *Review of Economic Studies* 46, 73-82.

SELDEN, L. (1978): A New Representation of Preferences over "Certain × Uncertain" Consumption Pairs: The "Ordinal Certainty Equivalent" Hypothesis. *Econometrica* 46, 1045-1060.

SOLOW, R.M. (1991): Sustainability: An Economist's Perspective. *The Eighteenth J. Seward Johnson Lecture to the Marine Policy Center*, Woods Hole Oceonographic Institution, Woods Hole, MA, USA.

STERN, N. (2007): *The Economics of Climate Change: The Stern Review.* Cambridge University Press, Cambridge, UK.

TRAEGER, C. (2012): *Why Uncertainty Matters - Discounting under Intertemporal Risk Aversion and Ambiguity.* CUDARE Working Paper No. 1092R3, April 2012.

TRAEGER, C. (2009): Recent Developments in the Intertemporal Modeling of Uncertainty. *Annual Review of Resource Economics* 1, 261-285.

VISSING-JØRGENSEN, A. AND O.P. ATTANASIO (2003): Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion. *American Economic Review* 93, Papers and Proceedings of the One Hundred Fifteenth Annual Meeting of the American Economic Association, Washington, DC, January 3-5, 2003, 383-391.

6 Appendix

6.1 Proof of lemma 1

Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a discrete random variable $\tilde{x}$. Denote these functions as $g_1(\tilde{x})$ and $g_2(\tilde{x})$ respectively. The risk aversion adjusted covariance between Z_1 and Z_2 is then

$$cov_\pi[Z_1, Z_2] = cov_\pi[g_1(\tilde{x}), g_2(\tilde{x})]$$
$$\text{where } cov_\pi[g_1(\tilde{x}), g_2(\tilde{x})] = E_\pi[g_1(\tilde{x})g_2(\tilde{x})] - E_\pi[g_1(\tilde{x})]E_\pi[g_2(\tilde{x})] \quad (7)$$
$$\text{with } E_\pi[g(\tilde{x})] = \sum_{\omega=1}^{N} \pi^\omega g(x^\omega).$$

The risk aversion adjusted probabilities π^ω are defined by equation (4). After some rearrangements equation (7) can be written as

$$cov_\pi[g_1(\tilde{x}), g_2(\tilde{x})] = \frac{1}{2}\sum_{\omega_i=1}^{N}\sum_{\omega_j=1}^{N} \pi^{\omega_i}\pi^{\omega_j}\left(g_1(x^{\omega_j}) - g_1(x^{\omega_i})\right)\left(g_2(x^{\omega_j}) - g_2(x^{\omega_i})\right).$$

By definition 1, strict comonotonicity between the random variables Z_1 and Z_2 implies that $g_1(\tilde{x})$ and $g_2(\tilde{x})$ are both strictly increasing in $\tilde{x}$, which in turn implies that

$$\left(g_1(x^{\omega_j}) - g_1(x^{\omega_i})\right)\left(g_2(x^{\omega_j}) - g_2(x^{\omega_i})\right) > 0$$

and thus

$$cov_\pi[g_1(\tilde{x}), g_2(\tilde{x})] = cov_\pi[Z_1, Z_2] > 0.$$

Equivalently, strict countercomonotonicity between the random variables Z_1 and Z_2 implies that either of the functions $g_1(\tilde{x})$ and $g_2(\tilde{x})$ is strictly increasing in $\tilde{x}$, while the other is strictly decreasing in $\tilde{x}$. This in turn implies that

$$\left(g_1(x^{\omega_j}) - g_1(x^{\omega_i})\right)\left(g_2(x^{\omega_j}) - g_2(x^{\omega_i})\right) < 0$$

and thus

$$cov_\pi[g_1(\tilde{x}), g_2(\tilde{x})] = cov_\pi[Z_1, Z_2] < 0.$$

□

6.2 Comonotonicity characteristics of $\tilde{\alpha}u(\tilde{c}_2)$ and $\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$

To determine whether $\tilde{\alpha}u(\tilde{c}_2)$ and $\tilde{\gamma}_e\tilde{\alpha}u'(\tilde{c}_2)$ are comonotonic or countercomonotonic, I specify the type of uncertainty in the future. I consider the three different types of

uncertainty introduced in the main part of the paper:

case 1 ($\tilde{\alpha}$ uncertain, γ_y, γ_e deterministic) : $\tilde{\alpha}u\left(c_2\right)$ with $c_2 = \gamma_y y_2 + \gamma_e(1+r)e$

case 2 ($\tilde{\gamma}_y$ uncertain, γ_e, α deterministic) : $\alpha u\left(\tilde{c}_2\right)$ with $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \gamma_e(1+r)e$

case 3 ($\tilde{\gamma}_e$ uncertain, γ_y, α deterministic) : $\alpha u\left(\tilde{c}_2\right)$ with $\tilde{c}_2 = \gamma_y y_2 + \tilde{\gamma}_e(1+r)e$.

The variables $\tilde{\alpha}u\left(\tilde{c}_2\right)$ and $\tilde{\gamma}_e\tilde{\alpha}u'\left(\tilde{c}_2\right)$ are comonotonic if they are both increasing (or both decreasing) in a random variable $\tilde{\alpha}$, $\tilde{\gamma}_y$, or $\tilde{\gamma}_e$. They are countercomonotonic if either of them is increasing while the other one is decreasing in the random variable. Summarizing the results from the comonotonicity examinations of 1.-3. below and combining with proposition 1 yields proposition 2 in section 3.

1. $\tilde{\alpha}u\left(c_2\right)$ with $c_2 = \gamma_y y_2 + \gamma_e(1+r)e$
   $\frac{\partial \tilde{\alpha}u(c_2)}{\partial \tilde{\alpha}} = u\left(c_2\right) > 0$ (for $u\left(\cdot\right) > 0$)
   $\frac{\partial \gamma_e \tilde{\alpha}u'(c_2)}{\partial \tilde{\alpha}} = \gamma_e u'\left(\tilde{c}_2\right) > 0$
   If $u\left(\cdot\right) < 0$, then $\tilde{\alpha}u\left(\tilde{c}_2\right)$ and $\tilde{\gamma}_e\tilde{\alpha}u'\left(\tilde{c}_2\right)$ are countercomonotonic. In this case, by proposition 1,we have $\frac{\partial \delta}{\partial k} < 0$.
   Given $k > 0$, $u\left(\cdot\right) < 0$ whenever the DM is risk loving with respect to $\tilde{\alpha}$.

2. $\alpha u\left(\tilde{c}_2\right)$ with $\tilde{c}_2 = \tilde{\gamma}_y y_2 + \gamma_e(1+r)e$
   $\frac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{\gamma}_y} = \alpha u'\left(\tilde{c}_2\right) y_2 > 0$
   $\frac{\partial \gamma_e \alpha u'(\tilde{c}_2)}{\partial \tilde{\gamma}_y} = \gamma_e \alpha u''\left(\tilde{c}_2\right) y_2 < 0$

3. $\alpha u\left(\tilde{c}_2\right)$ with $\tilde{c}_2 = \gamma_y y_2 + \tilde{\gamma}_e(1+r)e$ (assume $e > 0$)
   $\frac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{\gamma}_e} = \alpha u'\left(\tilde{c}_2\right)\left(1+r\right)e > 0$
   $\frac{\partial \tilde{\gamma}_e \alpha u'(\tilde{c}_2)}{\partial \tilde{\gamma}_e} = \alpha u'\left(\tilde{c}_2\right) + \tilde{\gamma}_e \alpha u''\left(\tilde{c}_2\right)\left(1+r\right)e$
   then $\frac{\partial \tilde{\gamma}_e \alpha u'(\tilde{c}_2)}{\partial \tilde{\gamma}_e} \gtreqless 0 \Leftrightarrow \alpha u'\left(\tilde{c}_2\right) + \tilde{\gamma}_e \alpha u''\left(\tilde{c}_2\right)\left(1+r\right)e \gtreqless 0$:
   $u'\left(\tilde{c}_2\right) + u''\left(\tilde{c}_2\right)\gamma_y y_2 + u''\left(\tilde{c}_2\right)\tilde{\gamma}_e(1+r)e \gtreqless u''\left(\tilde{c}_2\right)\gamma_y y_2$
   $u''\left(\tilde{c}_2\right)\left(\gamma_y y_2 + \tilde{\gamma}_e(1+r)e\right) \gtreqless u''\left(\tilde{c}_2\right)\gamma_y y_2 - u'\left(\tilde{c}_2\right)$
   $1 \gtreqless \frac{\gamma_y y_2}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e} - \frac{u'(\tilde{c}_2)}{u''(\tilde{c}_2)\left(\gamma_y y_2 + \tilde{\gamma}_e(1+r)e\right)} = \frac{\gamma_y y_2}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e} + IES$
   $\frac{\tilde{\gamma}_e(1+r)e}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e} \lesseqgtr IES$
   Thus $\frac{\partial \tilde{\gamma}_e \alpha u'(\tilde{c}_2)}{\partial \tilde{\gamma}_e} \gtreqless 0 \Leftrightarrow \frac{\tilde{\gamma}_e(1+r)e}{\gamma_y y_2 + \tilde{\gamma}_e(1+r)e} \lesseqgtr IES \ \forall \ \omega$.

6.3   Uncertainty on several variables

Suppose γ_y, γ_e and α are strictly monotonic functions of a discrete random variable $\tilde{x} > 0$, such that we can write $\gamma_y\left(\tilde{x}\right)$, $\gamma_e\left(\tilde{x}\right)$, $\alpha\left(\tilde{x}\right)$. Future consumption, the derivative of the effective utility with respect to $\tilde{x}$ and the derivative of the effective marginal

utility with respect to $\tilde{x}$ are then

$$\tilde{c}_2 = \gamma_y(\tilde{x})\,y_2 + \gamma_e(\tilde{x})\,(1+r)e$$

$$\frac{\partial \alpha(\tilde{x})u(\tilde{c}_2)}{\partial \tilde{x}} = u(\tilde{c}_2)\frac{\partial \alpha(\tilde{x})}{\partial \tilde{x}} + \alpha(\tilde{x})\,u'(\tilde{c}_2)\,y_2\frac{\partial \gamma_y(\tilde{x})}{\partial \tilde{x}}$$
$$\qquad\qquad + \alpha(\tilde{x})\,u'(\tilde{c}_2)\,(1+r)e\frac{\partial \gamma_e(\tilde{x})}{\partial \tilde{x}} \tag{8}$$

$$\frac{\partial \gamma_e(\tilde{x})\alpha(\tilde{x})u'(\tilde{c}_2)}{\partial \tilde{x}} = \gamma_e(\tilde{x})\,u'(\tilde{c}_2)\frac{\partial \alpha(\tilde{x})}{\partial \tilde{x}} + \alpha(\tilde{x})\,\gamma_e(\tilde{x})\,u''(\tilde{c}_2)\,y_2\frac{\partial \gamma_y(\tilde{x})}{\partial \tilde{x}}$$
$$\qquad\qquad + [\alpha(\tilde{x})\,u'(\tilde{c}_2) + \alpha(\tilde{x})\,\gamma_e(\tilde{x})\,u''(\tilde{c}_2)\,(1+r)e]\frac{\partial \gamma_e(\tilde{x})}{\partial \tilde{x}}\,. \tag{9}$$

Suppose now that $\alpha'(\tilde{x}) = 0$ and $\gamma_y'(\tilde{x})\,y_2 > |\gamma_e'(\tilde{x})\,(1+r)e| \;\forall\;\omega$.

1. If $\gamma_y'(\tilde{x}) > 0$ and $\gamma_e'(\tilde{x}) < 0$, equations (8) and (9) yield:

   $$\frac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{x}} = \alpha u'(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) > 0$$

   $$\frac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} = \gamma_e'(\tilde{x})\,\alpha u'(\tilde{c}_2) + \gamma_e(\tilde{x})\,\alpha u''(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) \gtreqless 0$$

   $$\gamma_e(\tilde{x})\,u''(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) \gtreqless -\gamma_e'(\tilde{x})\,u'(\tilde{c}_2)$$

   $$\frac{\frac{\gamma_y'(\tilde{x})y_2 + \gamma_e'(\tilde{x})(1+r)e}{\gamma_y(\tilde{x})y_2 + \gamma_e(\tilde{x})(1+r)e}}{\frac{\gamma_e'(\tilde{x})}{\gamma_e(\tilde{x})}} \gtreqless -\frac{u'(\tilde{c}_2)}{u''(\tilde{c}_2)\left(\gamma_y(\tilde{x})y_2 + \gamma_e(\tilde{x})(1+r)e\right)} = IES$$

   thus $\dfrac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} \gtreqless 0 \Leftrightarrow \dfrac{\tilde{c}_2'(\tilde{x})\big/\tilde{c}_2(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \gtreqless IES \;\forall\;\omega$.

   This implies: If $\gamma_y'(\tilde{x}) > 0$, $\gamma_e'(\tilde{x}) < 0$: $\dfrac{\partial \delta}{\partial k} < 0$ since $IES > 0 > \dfrac{\tilde{c}_2'(\tilde{x})\big/\tilde{c}_2(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \;\forall\;\omega$.

   With $e = 0$: $\dfrac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{x}} = \alpha u'(\tilde{c}_2)\,\gamma_y'(\tilde{x})\,y_2 > 0$ and

   $$\frac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} \gtreqless 0 \Leftrightarrow \frac{\gamma_y'(\tilde{x})\big/\gamma_y(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \gtreqless IES \;\forall\;\omega.$$

2. If $\gamma_y'(\tilde{x}) > 0$ and $\gamma_e'(\tilde{x}) > 0$, equations (8) and (9) yield:

   $$\frac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{x}} = \alpha u'(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) > 0$$

   $$\frac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} = \gamma_e'(\tilde{x})\,\alpha u'(\tilde{c}_2) + \gamma_e(\tilde{x})\,\alpha u''(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) \gtreqless 0$$

   $$\gamma_e(\tilde{x})\,u''(\tilde{c}_2)\left(\gamma_y'(\tilde{x})\,y_2 + \gamma_e'(\tilde{x})\,(1+r)e\right) \gtreqless -\gamma_e'(\tilde{x})\,u'(\tilde{c}_2)$$

   $$\frac{\frac{\gamma_y'(\tilde{x})y_2 + \gamma_e'(\tilde{x})(1+r)e}{\gamma_y(\tilde{x})y_2 + \gamma_e(\tilde{x})(1+r)e}}{\frac{\gamma_e'(\tilde{x})}{\gamma_e(\tilde{x})}} \lesseqgtr -\frac{u'(\tilde{c}_2)}{u''(\tilde{c}_2)\left(\gamma_y(\tilde{x})y_2 + \gamma_e(\tilde{x})(1+r)e\right)} = IES$$

   thus $\dfrac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} \gtreqless 0 \Leftrightarrow \dfrac{\tilde{c}_2'(\tilde{x})\big/\tilde{c}_2(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \lesseqgtr IES \;\forall\;\omega$.

   This implies: If $\gamma_y'(\tilde{x}) > 0$, $\gamma_e'(\tilde{x}) > 0$: $\dfrac{\partial \delta}{\partial k} < (>) 0$ if $IES < (>) \dfrac{\tilde{c}_2'(\tilde{x})\big/\tilde{c}_2(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \;\forall\;\omega$.

   With $e = 0$: $\dfrac{\partial \alpha u(\tilde{c}_2)}{\partial \tilde{x}} = \alpha u'(\tilde{c}_2)\,\gamma_y'(\tilde{x})\,y_2 > 0$ and

   $$\frac{\partial \gamma_e(\tilde{x})\alpha u'(\tilde{c}_2)}{\partial \tilde{x}} \gtreqless 0 \Leftrightarrow \frac{\gamma_y'(\tilde{x})\big/\gamma_y(\tilde{x})}{\gamma_e'(\tilde{x})/\gamma_e(\tilde{x})} \lesseqgtr IES \;\forall\;\omega.$$

21

Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research).

13/174 B. Lanz, T. F. Rutherford and J. E. Tilton
Subglobal climate agreements and energy-intensive activities: An evaluation of carbon leakage in the copper industry

13/173 L. Blazquez, F. Heimsch and M. Filippini
The Economic Crisis and Residential Electricity Consumption in Spanish Provinces: A Spatial Econometric Analysis

13/172 A. Bommier and F. Le Grand
A Robust Approach to Risk Aversion

13/171 P. J. Agrell, M. Farsi, M. Filippini and M. Koller
Unobserved heterogeneous effects in the cost efficiency analysis of electricity distribution systems

13/170 S. Andrade de Sa , C. Palmer and S. Di Falco
Dynamics of Indirect Land-Use Change: Empirical Evidence from Brazil

12/169 L. Leinert
Does the Oil Price Adjust Optimally to Oil Field Discoveries?

12/168 F. Lechthaler and L. Leinert
Moody Oil - What is Driving the Crude Oil Price?

12/167 L. Bretschger, R. Ramer and L. Zhang
Economic effects of a nuclear phase-out policy: A CGE analysis

12/166 L. Bretschger and N. Suphaphiphat
Use Less, Pay More: Can Climate Policy Address the Unfortunate Event for Being Poor?

12/165 P. S. Schmidt and T. Werner
Verified emissions and stock prices: Is there a link? - An empirical analysis of the European Emission Trading Scheme

12/164 H. Gersbach, S. Imhof and O. Tejada
Channeling the Final Say in Politics

12/163 J. Daubanes and L. Leinert
Optimum Tariffs and Exhaustible Resources: Theory and Evidence for Gasoline

12/162 C. N. Brunnschweiler, C. Jennings and I. A. MacKenzie
Rebellion against Reason? A Study of Expressive Choice and Strikes

12/161 A. Lange and A. Ziegler
Offsetting versus Mitigation Activities to Reduce CO2 Emissions: A Theoretical and Empirical Analysis for the U.S. and Germany