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Hierarchical Growth: Basic and Applied Research*

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Abstract

We develop a model that incorporates salient features of growth in modern economies. We combine the expanding-variety growth model through horizontal innovations with a hierarchy of basic and applied research. The former extends the knowledge base, while the latter commercializes it. Two-way spillovers reinforce the productivity of research in each sector. We establish the existence of balanced growth paths. Along such paths the stock of ideas and the stock of commercialized blueprints for intermediate goods grow with the same rate. Basic research is a necessary and sufficient condition for economic growth. We show that there can be two different facets of growth in the economy. First, growth may be entirely shaped by investments in basic research if applied research operates at the knowledge frontier. Second, long-run growth may be shaped by both basic and applied research and growth can be further stimulated by research subsidies. We illustrate different types of growth processes by examples and polar cases when only upward or downward spillovers between basic and applied research are present.

Keywords: Basic research, applied research, knowledge base, commercialization, hierarchical economic growth

JEL Classification: H41, O31, O41

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"It is likely that the bulk of the economic benefits of university research come from inventions in the private sector that build upon the scientific and engineering base created by university research, rather than from commercial inventions generated directly by universities." [Henderson *et al.* (1998), p. 126]

1 Introduction

The innovation processes in both industrialized and industrializing countries are typically characterized by a hierarchy of basic and applied research. Basic research is mainly publicly funded and extends the knowledge base of an economy by generating novel ideas, theories, and prototypes that are usually of no immediate commercial use. Applied research, on the other hand, is primarily carried out by private firms, which commercialize the output of basic research by transforming it into blueprints for new products. Moreover, basic and applied research reinforce each other throughout the entire process of knowledge base extension and commercialization. For example, applied research benefits from basic research because the latter provides knowledge and methods that support the problem-solving processes at research-active private firms, whereas basic research benefits from applied research as the latter identifies unresolved problems and discovers new challenges for basic research. That is, the innovation process is characterized by two-way spillovers between basic and applied research.

This paper develops an endogenous growth model that captures the above hierarchy and interdependency of the innovation process. We analyze a closed economy with a final and an intermediate goods sector, a basic and an applied research sector, a continuum of infinitely lived households, and a government. Technological progress is the result of a hierarchical innovation process modelled along the lines sketched above: basic research is financed by the government and extends the knowledge base of the economy, whereas applied research is carried out by private firms that transform basic knowledge into blueprints for new intermediate goods. In addition, two-way spillovers between basic and applied research reinforce the productivity in each research sector. In this setup, the government can influence growth and welfare of the economy by choosing the size of the basic research sector and by granting subsidies to researchers. Both of these government activities are financed by taxes on labor income.

Our model stands in the tradition of the expanding-variety framework of growth through horizontal innovations initiated by Romer (1990).[1] The crucial features of our model are the hierarchy in the innovation process and the two-way spillovers between basic and applied research as described above. While Romer and others effectively assumed an exogenous,

[1] See Gancia and Zilibotti (2005) for a recent review of the development, extensions, and applications of the expanding-variety framework.

non-exhaustible pool of knowledge available that can be exploited by applied researchers to invent blueprints for new intermediate goods, we endogenize this knowledge pool as the output of a basic research sector. In other words, we assume that the productivity of applied research is constrained by the "knowledge frontier" of the economy which, in turn, can only be pushed outwards by basic research. In particular, growth is about to cease in the long-run unless the knowledge base of the economy is permanently expanded by basic research.

We establish the existence of balanced growth paths. Along such paths the stock of ideas and the stock of commercialized blueprints for intermediate goods grow with the same rate. Basic research is a necessary and sufficient condition for economic growth. We show that there can be two different facets of growth in the economy. First, growth may be entirely shaped by investments in basic research if applied research operates at the knowledge frontier. Second, long-run growth may be shaped by both basic and applied research and growth can be further stimulated by research subsidies. We illustrate different types of growth processes by examples and polar cases when only upward or downward spillovers are present. In the former case, a higher share of labor employed in basic research translates into higher growth rates and applied research operates at the knowledge frontier. In the latter case, growth is declining if basic research exceeds a certain threshold and ceases entirely if basic research is increased further as undertaking applied research becomes unprofitable.

So far, only few attempts have been made to analyze the impact of publicly-funded basic research in a dynamic setup. Among the first were Shell's (1966, 1967) contributions, which highlight the necessity of allocating economic resources to the process of invention. Thereby, Shell's concept of technical knowledge, which he treats as a public good financed solely by means of output taxes, corresponds closely to our notion of basic research. Similarly, Grossman and Helpman (1991) address the positive impact on growth of basic research financed by means of income taxes *vis-à-vis* the negative effect caused by the associated tax distortions. In order to evaluate the impact of various research policies, Morales (2004) considers an endogenous growth model of vertical innovation incorporating both basic and applied research performed by both private firms and the government. While these and other contributions shed light on the positive impact of publicly-funded basic research on growth, they do neither consider the hierarchy of basic and applied research nor the two-way spillovers between the two types of research in the process of innovation. Our formulation of two-way spillovers between basic and applied research follows the description of Park (1998).

The paper is organized as follows. In the following section we present empirical support for our central assumptions regarding the hierarchy and the presence of two-way spillovers in the innovation process. Section 3 sets up the formal model. In Section 4 we define and characterize the competitive equilibrium corresponding to a fixed policy scheme of the government. We identify the determinants of economic growth. In Section 5 we focus on the polar cases of upward and downward spillovers between basic and applied research. Section

7 concludes. All proofs are collected in an appendix.

2 Empirical motivation

In this section we motivate our analysis by several empirical observations. More specifically, we document the fact that basic research is primarily government-funded whereas applied research is typically carried out by private firms, and we demonstrate the hierarchy between basic and applied research as well as the existence of two-way spillovers.

2.1 Shares of basic and applied research in GDP and modes of financing

Basic and applied research expenditures constitute significant shares of GDP in most industrialized and industrializing countries. As Table 1 shows, in 2006 the average ratio of total R&D expenditures to GDP in a sample of countries with comparable data was 2.01 percent with the lowest ratio in Argentina (0.50) and the highest in Israel (4.65). Moreover, the average R&D expenditure to GDP ratio increased from 1.77 percent in 2000 to 2.01 percent in 2006. On average, basic research expenditures made up 23.07 percent of total R&D expenditures in 2006 with the lowest share in China (5.19) and the highest in the Slovak Republic (45.10). The average share of basic research expenditures in total R&D expenditures increased from 20.17 percent in 2000 to 23.07 percent in 2006.

While basic research is mainly financed through public funds, applied research is funded and performed mainly by the private sector as illustrated by Table 2. This table shows the sources of funding for basic and applied research in 2006 broken down into government and higher education institutions, whose research is mainly financed by public funds[2], business enterprises, as well as private non-profit institutions. Table 2 shows that on average 73.76 percent of basic research expenditures were financed and carried out by government and higher education institutions, while business enterprises and private non-profit institutions accounted for 22.36 and 3.89 percent, respectively. Almost the exact opposite pattern holds for the financing scheme of applied research. While government and higher education institutions on average financed and performed 24.64 percent of applied research, business enterprises and private non-profit institutions accounted for 73.73 and 1.63 percent, respectively. Taking these figures regarding the respective financing shares of basic and applied research into account, we settle on the following use of notation. Basic research corresponds to university or academic research and science, while applied research is associated with industry research.

[2] It should be noted here that in some countries income from patenting and licensing of university inventions became an additional source of financing for universities (see, for example, Colyvas *et al.*, 2002, for the case of the US, particularly after the passage of the Bayh-Dole Act in 1980).

Table 1: R&D Expenditures (Source: OECD, *Main Science and Technology Indicators*)

	Gross Domestic Expenditures on R&D as a Percentage of GDP		Basic Research Expenditures as a Percentage of Total R&D Expenditures		Applied Research Expenditures as a Percentage of Total R&D Expenditures[a]	
	2000	2006	2000	2006	2000	2006
Argentina	0.44	0.50	27.75	28.07	72.25	71.93
Australia	1.51	1.78[b]	25.81	23.17[b]	74.19	76.83[b]
China	0.90	1.42	5.22	5.19	94.78	94.81
Czech Republic	1.21	1.54	23.33	29.32	76.67	70.68
France	2.15	2.11	23.60	23.75[b]	76.40	76.25[b]
Hungary	0.78	1.00	27.19	32.96[b]	72.81	67.04[b]
Ireland	1.12	1.32	—	23.85	—	76.15
Israel	4.45	4.65	16.99	16.14	83.01	83.86
Japan	3.04	3.39	12.38	11.98[b]	87.62	88.02[b]
Korea	2.39	3.23	12.61	15.15	87.39	84.85
Portugal	0.76	0.83	25.05	25.35[b]	74.95	74.65[b]
Singapore	1.88	2.31	11.75	18.83[b]	88.25	81.17[b]
Slovak Republic	0.65	0.49	25.61	45.10[b]	74.39	54.90[b]
Switzerland	2.53	2.90[b]	27.96	28.70[b]	72.04	71.30[b]
United States	2.74	2.62	17.17[c]	18.52	82.83[c]	81.48
Average	1.77	2.01	20.17	23.07	79.83	76.93

[a]The OECD categorizes R&D into "basic research", "applied research", "experimental development" and "not elsewhere classified". We summarize the last three items under "applied research" as particularly the OECD's definition of "experimental development"(see, e.g., OECD, 2002) corresponds closely to our definition of applied research.

[b]Data from 2004

[c]Data from 2001

In the following, we use these terms synonymously.

2.2 Basic research extends the knowledge base for applied research

In most cases, the output of basic research is "embryonic" in nature, which means that it is without immediate commercial use and requires refinement through applied research before it is ready for commercialization. There are numerous prominent examples of breakthrough inventions by basic research across various disciplines that have been taken up and commercialized by applied research. The discovery of X-rays in the area of physics and life sciences, the discovery of penicillin in the field of medicine, the invention of the method

Table 2: Financing Shares of Basic and Applied Research by Sector in 2006 (Source: OECD, *Main Science and Technology Indicators*)

	Basic Research Financing Shares (% of Total Basic Research Expenditures)			Applied Research Financing Shares (% of Total Applied Research Expenditures)[a]		
	Government and Higher Education	Business Enterprise	Private Non-Profit	Government and Higher Education	Business Enterprise	Private Non-Profit
Argentina	94.61	2.60	2.81	56.42	41.26	2.32
Australia	83.75[b]	9.94[b]	6.31[b]	30.42[b]	67.46[b]	2.12[b]
China	91.28	8.72	0.00	25.51	74.49	0.00
Czech Republic	77.68	22.19	0.15	15.06	84.43	0.52
France	83.11[c]	14.38[c]	2.51[c]	21.55[c]	77.57[c]	0.89[c]
Hungary	95.54[c]	4.46[c]	0.00[c]	39.59[c]	60.41[c]	0.00[c]
Ireland	63.64	36.36	0.00	23.55	76.44	0.00
Israel	70.48	23.95	5.58	9.18	87.59	3.22
Japan	56.98[c]	40.12[c]	2.90[c]	16.88[c]	81.40[c]	1.72[c]
Korea	38.42	60.76	0.82	18.49	80.20	1.30
Portugal	66.94[c]	14.30[c]	18.77[c]	44.28[c]	46.65[c]	9.06[c]
Singapore	61.49[c]	38.51[c]	0.00[c]	26.66[c]	73.34[c]	0.00[c]
Slovak Republic	84.73[c]	15.27[c]	0.00[c]	19.69[c]	80.14[c]	0.17[c]
Switzerland	64.09[b]	29.52[b]	6.38[b]	7.82[b]	91.54[b]	0.64[b]
United States	73.65	14.26	12.09	14.45	83.09	2.46
Average	73.76	22.36	3.89	24.64	73.73	1.63

[a] As in Table 1 we summarize the OECD's notions of "applied research", "experimental development" and "not elsewhere classified" under our notion of "applied research".

[b] Data from 2004

[c] Data from 2005

of RNA interference for genetics, the derivation of the lifting line theory for aeronautics, or the invention of the method of nuclear fission for nuclear physics, to name only a few.[3] They were all invented through basic research at universities or other public research institutions, were subsequently further developed and commercialized, and had finally tremendous impact in particular industries.

The "embryonic" nature of basic research inventions and their crucial importance for giving rise to new technologies across various industries has been widely documented. Zucker *et al.* (1998) and Owen-Smith *et al.* (2002), for example, suggest that basic research inventions were fundamental to the rise and growth of biotechnology industries. Furthermore, Ander-

[3] Table 3 in the Appendix contains further examples.

son (1997) and Hirschel *et al.* (2004) highlight the importance of breakthrough theories and findings made through basic research for subsequent developments in aeronautics industries. National Academy of Engineering (2003) report that basic research has made fundamental contributions to network systems and communications, medical devices and equipment, aerospace, transportation, distribution, and logistics services, as well as to financial services industries.

Jensen and Thursby (2001) find that even the vast majority of licensed university inventions were "embryonic" in nature with more than 75 percent being merely a proof of concept without any particular commercial use in mind. At the time of licensing, only 12 percent of all university inventions were ready for commercialization while for only 8 percent their manufacturing feasibility was known. These observations lead the authors to conclude that the largest part of all licensed university inventions required further development and refinement.

A further crucial property of basic research concerns the duration between its origin in the scientific community and its impact on industrial productivity. In this regard, Adams (1990) finds that the expansion of academic knowledge exerts a positive, but lagged impact on technological change and productivity growth. By applying various measures of science within a growth-accounting framework, his findings suggest that the impact of new academic knowledge on industrial productivity does not take place instantaneously, but is rather associated with time-lags of about 20 years stemming from the time necessary to search for and adopt useful scientific knowledge in industry.

These findings suggest a rather strict hierarchy between basic and applied research. The rise and growth of new industries, as well as the associated invention and development of new products initially requires substantial basic research efforts to provide the knowledge base without a direct commercial use *per se*, upon which applied research and commercialization in industry can take place. Apart from giving rise to new industries, basic research also induces the invention and commercialization of new products in existing industries.

2.3 Two-way spillovers between basic and applied research

A crucial property of the innovation system is the existence of two-way spillovers between basic and applied research. Nelson (1993) points out that, while science often preceded the rise of new technologies, new fields of technology also often induced the rise of new fields of science. That is, throughout the entire process of product invention, development, and commercialization, basic and applied research are interdependent and mutually benefit from and reinforce each other through various channels and by various means. Examples for this interplay between basic and applied research are given, for example, by von Hippel (1988), who describes the initiating role of basic research among others in the invention of nuclear magnetic resonance and the electromagnetic lens. The subsequent interaction between basic and applied research was crucial in the further development and refinement of the nuclear magnetic resonance spectrometer and the transmission electron microscope, respectively. That is, the interdependence of basic and applied research and their mutual intensification is fundamental to the invention, development, and commercialization of new products and technologies.

2.3.1 Impact of basic on applied research

Basic research impacts on applied research, for example, through the channels of open science, such as publications, scientific reports, conferences and public meetings (Cohen *et al.*, 2002), through "embodied knowledge transfer" associated with scientists moving from basic to applied research (Zellner, 2003), collaborative and contracted research ventures as well as informal interaction between basic and applied researchers (Cohen *et al.*, 2002), joint industry-university research centers (Adams *et al.*, 2001), academic consulting (Perkmann and Walsh, 2008), the patenting and licensing of university inventions (Colyvas *et al.* 2002), or through the creation of new firms as start-ups and spin-offs from universities (Bania *et al.*, 1993).

According to Martin *et al.* (1996) and Salter and Martin (2001) publicly funded basic research impacts on applied research by generating new knowledge and information as discussed in Subsection 2.2, by training and providing skilled graduates, by developing new scientific instrumentation and methodologies, by establishing networks for knowledge diffusion, by enhancing problem-solving capacities, and by creating new firms.[4] Furthermore, the involvement of academic inventors in the subsequent commercialization of their inventions through applied research allows to employ their tacit knowledge and thereby facilitates commercialization (Zucker *et al.*, 1998, Jensen and Thursby, 2001).

Various studies have shown the positive impact of academic research on applied research. Nelson (1986), Jaffe (1989), Adams (1990), Acs *et al.* (1992), or Mansfield (1991, 1992, 1995, 1998) find a significant and positive impact of academic research on innovative activity across various industries. Furthermore, Acs *et al.* (1994) find that it is particularly small firms that benefit relatively strongly from university research. In addition, Funk (2002) suggests that basic research generates particularly strong international spillovers.

2.3.2 Impact of applied on basic research

Basic research benefits from applied research, for example, through allowing scientists to access data, instrumentation, and research material as well as to discover unresolved problems

[4] Similarly, Mowery and Sampat (2005) suggest that universities provide, for example, new scientific and technological knowledge, scientific equipment and instrumentation, prototypes for new products and processes, networks for knowledge diffusion, as well as skilled graduates and faculty.

and open challenges when performing academic consulting to industry (Mansfield, 1995, Perkmann and Walsh, 2008). Applied research might also provide basic researchers with knowledge about novel research techniques and methodologies. Furthermore, Adams *et al.* (2001) suggest that joint industry-university research centers could impact positively on basic research if the associated provision of additional faculty more than compensates for the diversion of faculty resources away from basic and towards applied research. In addition, Agrawal and Henderson (2002) suggest that basic research might also benefit from applied research through increased patenting activities of university faculty.

3 Model formulation

We consider a continuous-time model of an economy that lasts from $t = 0$ to $t = +\infty$. The economy has one production sector for a final consumption good and one production sector in which a range of differentiated intermediate goods are produced. In addition, there exist two R&D sectors. Basic research, which is funded exclusively by the government, generates ideas, theories, and prototypes and thereby extends the economy's knowledge base. Applied research, on the other hand, is carried out by private researchers who commercialize the output of basic research by transforming it into blueprints for new intermediate goods. These blueprints are protected by everlasting patents so that the intermediate goods sector operates under monopolistic competition. In order to simplify the exposition, we shall henceforth refer to the output of basic and applied research as ideas and blueprints, respectively.

3.1 Households

The economy is populated by a continuum of measure $L > 0$ of identical infinitely-lived households. There is no population growth, that is, L is a constant. The representative household derives utility from consumption according to the utility functional

$$\int_0^{+\infty} e^{-\rho t} \ln[c(t)]dt, \tag{1}$$

where $c(t)$ denotes per-capita consumption in period t. The parameter $\rho > 0$ is the common time-preference rate of the households. Each household is endowed with one unit of homogeneous labor per unit of time, which can be used for applied research, for basic research, in intermediate goods production, or in final good production. Since households are utility maximizers, they choose at each instant t that form of employment that yields the highest remuneration taking into account taxes and subsidies. We refer to this remuneration as the net real wage and denote it by $\bar{w}(t)$.

Households can use their income for consumption or for saving. They can save by holding shares in dividend paying firms. Due to no-arbitrage conditions, all these assets have the same real rate of return (dividends plus capital gains), which we denote by $r(t)$. If we define $a(t)$ as the real wealth owned by the representative household at time t, we obtain the flow budget constraint[5]

$$\dot{a}(t) + c(t) = a(t)r(t) + \bar{w}(t). \tag{2}$$

The representative household maximizes its utility given in (1) subject to the flow budget constraint (2) and the no Ponzi-game condition

$$\lim_{t\to+\infty} a(t)e^{-\int_0^t r(s)ds} \geq 0. \tag{3}$$

A necessary and sufficient condition for an optimal consumption path is that (3) holds as an equality (transversality condition) and that the Euler equation

$$\dot{c}(t)/c(t) = r(t) - \rho \tag{4}$$

is satisfied.

3.2 Final output

A single homogeneous final good is produced from labor and differentiated intermediate goods. The set of intermediate goods available at time t is the interval $[0, A(t)]$. The production function for final output is

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^{A(t)} x_i(t)^\alpha di, \tag{5}$$

where $Y(t)$, $L_Y(t)$, and $x_i(t)$ denote the rate of final output in period t and the corresponding input rates of labor and intermediate good i, respectively. The number $\alpha \in (0,1)$ is an exogenously given technological parameter.

Firms in the final good sector take the measure of available intermediate goods, $A(t)$, the real wage, $w(t)$, and the prices of intermediate goods, $p_i(t)$, as given and maximize their profit rates.[6] The necessary and sufficient first-order conditions for this profit maximization problem are

$$w(t) = (1-\alpha)Y(t)/L_Y(t) \tag{6}$$

and

$$p_i(t) = \alpha[L_Y(t)/x_i(t)]^{1-\alpha}. \tag{7}$$

[5]Here and in what follows, a dot above a variable denotes the derivative of that variable with respect to time, e.g., $\dot{a}(t) \equiv da(t)/dt$.

[6]Note the difference between the real wage $w(t)$ and the net real wage $\bar{w}(t)$. The latter differs from the former because of taxes as will be explained in Subsection 3.7.

3.3 Intermediate goods

All intermediate goods are produced by the same linear technology using labor as its only input. We assume that one unit of labor is required to produce one unit of intermediate good. The rights for the production of intermediate good i are protected by a permanent patent. The firm holding that patent is therefore a monopolist and maximizes its profit rate subject to the technological constraint and the inverse demand function given in (7). Formally, in every period t, firm i chooses $x_i(t) \geq 0$ so as to maximize

$$p_i(t)x_i(t) - w(t)x_i(t) = \alpha L_Y(t)^{1-\alpha}x_i(t)^\alpha - w(t)x_i(t). \tag{8}$$

The necessary and sufficient first-order condition for profit maximization yields

$$x_i(t) = x(t) := \left[\alpha^2/w(t)\right]^{1/(1-\alpha)} L_Y(t) \tag{9}$$

and

$$p_i(t) = p(t) := w(t)/\alpha. \tag{10}$$

All intermediate goods are sold for the same price which involves a constant markup on production costs.

Substituting (9) into (8) one finds that firm i's profit rate is given by

$$\pi(t) = (1-\alpha)\left[\alpha^{(1+\alpha)}/w(t)^\alpha\right]^{1/(1-\alpha)} L_Y(t). \tag{11}$$

The present value as of time t of the profit flow for firm i over the interval $[t, +\infty)$ is

$$V(t) = \int_t^{+\infty} e^{-\int_t^s r(s')ds'}\pi(s)ds. \tag{12}$$

$V(t)$ is the value of any intermediate goods producing firm or, equivalently, its share price at time t. Finally, we note that the total amount of labor used for the production of intermediate goods is given by

$$L_X(t) = \int_0^{A(t)} x_i(t)di = x(t)A(t) = \left[\alpha^2/w(t)\right]^{1/(1-\alpha)} A(t)L_Y(t). \tag{13}$$

3.4 Basic research

We assume that new intermediate goods are developed in a two-step procedure. In the first step, basic research generates ideas and thereby extends the economy's knowledge base, whereas in a second step these ideas are turned into blueprints for new intermediate goods through applied research. We assume that every idea can be turned into a blueprint for a single intermediate good. Thus, there is a one-to-one relationship between ideas and potential blueprints.[7] In this subsection we formulate the model for basic research, that is, we describe how new ideas are generated and how thereby the knowledge frontier of the economy evolves.

Let us denote by $B(t)$ the measure of ideas that have been generated through basic research by time t. We assume that the productivity of the basic researchers at time t depends in the form of an external effect both on the previous output from basic research, $B(t)$, and on the measure of blueprints, $A(t)$. More specifically, we assume that this productivity is a linear homogeneous function of these two variables.[8] For simplicity, we assume a Cobb-Douglas specification $\gamma_B B(t)^{1-\mu_B} A(t)^{\mu_B}$ with $\gamma_B > 0$ and $\mu_B \in (0,1)$ being exogenously given parameters. Denoting by $L_B(t)$ the total amount of labor devoted to basic research at time t, it follows that

$$\dot{B}(t) = \gamma_B B(t)^{1-\mu_B} A(t)^{\mu_B} L_B(t). \tag{14}$$

The presence of $A(t)$ in the productivity function (14) and $\mu_B \in (0,1)$ imply positive spillovers from applied to basic research. As noted above, conducting applied research allows for discovering unresolved research problems, disclosing potentially new areas of science, and applying novel instrumentation and methodologies which, in turn, impacts positively on the productivity of basic researchers. The greater is μ_B the stronger are these spillovers.

According to our formulation, basic research is undirected. That is, a basic researcher tries to invent some new idea, but not a specific one. This means that the relevant input for basic research is the total labor force devoted to it, $L_B(t)$.

3.5 Applied research

Applied researchers commercialize the ideas that have been generated through basic research by transforming them into blueprints for new varieties of intermediate goods. In contrast to basic research, this is a directed research activity in the sense that every applied researcher focuses on a single idea that has not yet been transformed into a blueprint. We shall denote by $L_A(t)$ the total amount of labor used for applied research at time t.

To formalize applied research we have to distinguish two cases[9]. First, suppose that $A(t) <$

[7] A more elaborate model would allow for a more complicated relationship between basic and applied research. For example, one could assume that several different ideas need to be combined in order to get one blueprint, or that one can use the same idea for several blueprints.

[8] Linear homogeneity implies a strong scale effect and allows the policy parameters $L_B(t)$ and $\sigma(t)$ (introduced below) to have growth effects.

[9] These cases correspond closely to the findings of Adams (1990) on the presence of significant time-lags regarding the impact of scientific knowledge on industrial productivity, which could be interpreted in two ways: Either the commercialization of basic research ideas through applied research takes time so that there is always a non-empty set of non-commercialized ideas available at time t, or it takes time to entirely develop new ideas in basic research, which are then ready for instantaneous commercialization.

$B(t)$ holds at time t. Since there is a set of measure $B(t)-A(t)$ of non-commercialized ideas available at time t, it follows that $L_A(t)/[B(t)-A(t)]$ applied researchers work on any one of them, provided that research effort is evenly distributed across all non-commercialized ideas.[10] Let $z_j(t)\Delta$ be the probability that one such idea $j \in (A(t), B(t)]$ is turned into a blueprint during the time interval $[t, t+\Delta)$, where $\Delta > 0$ is assumed to be small. Using a modelling approach analogous to that from the previous subsection, it follows that

$$z_j(t) = \gamma_A A(t)^{1-\mu_A} B(t)^{\mu_A} \frac{L_A(t)}{B(t)-A(t)},$$

where $\gamma_A > 0$ and $\mu_A \in (0,1)$ are exogenously given parameters. The total rate at which new blueprints are created at time t, $\dot{A}(t)$, can be computed as the integral of $z_j(t)$ with respect to $j \in (A(t), B(t)]$. This yields

$$\dot{A}(t) = \gamma_A A(t)^{1-\mu_A} B(t)^{\mu_A} L_A(t).$$

Second, suppose that $A(t) = B(t)$ at time t. In that case it follows that the rate at which blueprints are invented cannot exceed the rate at which new ideas are generated. Using the expression for $\dot{A}(t)$ from above, this implies that

$$\dot{A}(t) = \min\left\{\gamma_A A(t)^{1-\mu_A} B(t)^{\mu_A} L_A(t), \dot{B}(t)\right\}.$$

Substituting for $\dot{B}(t)$ from (14) and using $A(t) = B(t)$ (which we have presently assumed to hold), it follows that

$$\dot{A}(t) = \min\left\{\gamma_A L_A(t), \gamma_B L_B(t)\right\} A(t).$$

This reflects the fact that, in the case where no non-commercialized ideas are available, basic research and applied research are perfectly complementary inputs for the creation of new blueprints.

We can therefore summarize the discussion of the present subsection by the formula

$$\dot{A}(t) = \begin{cases} \gamma_A A(t)^{1-\mu_A} B(t)^{\mu_A} L_A(t) & \text{if } A(t) < B(t), \\ \min\left\{\gamma_A L_A(t), \gamma_B L_B(t)\right\} A(t) & \text{if } A(t) = B(t). \end{cases} \tag{15}$$

Whereas basic research was assumed to be a government-funded activity, applied research is conducted by private individuals provided that they face appropriate incentives. These incentives derive from the fact that blueprints can be sold to potential intermediate goods producers, who are willing to pay for a blueprint any amount up to the present value of all profits generated through the infinite life of the patent on the right to produce their particular

[10]The assumption of symmetry is made for convenience and could be justified by coordination among applied researchers to focus on all non-commercialized ideas with the same research intensity.

intermediate good. The price at which a new patent can be sold at time t is therefore given by $V(t)$ as defined in equation (12). The probability to earn this price for any given applied researcher in the time interval $[t, t+\Delta)$ is approximately equal to $\dot{A}(t)\Delta/L_A(t)$. Combining this with equation (15) we see that the expected rate of return to one unit of time spent at instant t on applied research is equal to

$$w_A(t) = \begin{cases} \gamma_A A(t)^{1-\mu_A} B(t)^{\mu_A} V(t) & \text{if } A(t) < B(t), \\ \min\{\gamma_A, \gamma_B[L_B(t)/L_A(t)]\} A(t)V(t) & \text{if } A(t) = B(t). \end{cases} \tag{16}$$

3.6 Government

The government uses two policy instruments expressed by two time paths. First, it employs $L_B(t)$ researchers at time t in basic research. Second, research activities are subsidized at the rate $\sigma(t)$. We denote the corresponding policy scheme by $P = \{L_B(t), \sigma(t)\}$.

Employing $L_B(t)$ units of labor in the basic research sector causes a net cost of $\bar{w}(t)L_B(t)$ to the government, where $\bar{w}(t)$ is the net real wage introduced in Subsection 3.1. This number already takes into account labor tax received from and research subsidies paid to basic researchers. The research subsidies paid to applied researchers amount to $\sigma(t)w_A(t)L_A(t)$. The sum of these two numbers is total government expenditure which is financed through the taxation of labor income at the rate $\tau(t)$.[11] We assume that the government is required to have a balanced budget at all times, which implies that the tax rate $\tau(t)$ is uniquely determined by the budget constraint

$$\bar{w}(t)L_B(t) + \sigma(t)w_A(t)L_A(t) = \tau(t)w_A(t)L_A(t) + \tau(t)w(t)[L_X(t) + L_Y(t)]. \tag{17}$$

The policy scheme P is feasible if $L_B(t) \in [0, L)$, $\sigma(t) \geq 0$, and $\tau(t) \in [0, 1)$ hold for all t.

3.7 Market clearing

Having described the behavior of all agents in the economy, let us now turn to market clearing. Market clearing on intermediate goods markets has already been taken into account by substituting the inverse demand functions into the profit maximization problems of intermediate goods producers. This leaves us with the markets for labor, assets, and final output.[12]

The labor market clearing condition is

$$L_A(t) + L_B(t) + L_X(t) + L_Y(t) = L. \tag{18}$$

[11] As labor supply is inelastic, this is equivalent to imposing a lump-sum tax.

[12] According to Walras' law, one of the market clearing conditions is redundant.

Moreover, there is also a no-arbitrage condition regarding the different possible uses of labor: all those uses that are actually applied must earn the same net real wage $\bar{w}(t)$. Since per-capita consumption must be positive for all t in every equilibrium, the same must be true for output. This, in turn, implies that both $L_X(t)$ and $L_Y(t)$ must be strictly positive at all times. The no-arbitrage condition is therefore

$$\bar{w}(t) = [1 - \tau(t)]w(t) \geq [1 - \tau(t) + \sigma(t)]w_A(t) \text{ with equality if } L_A(t) > 0. \tag{19}$$

Asset market clearing requires that aggregate wealth of the household sector at time t is equal to the total value of all intermediate goods producing firms, i.e.,

$$a(t) = A(t)V(t)/L \tag{20}$$

with $V(t)$ given in (12). The market for final output is in equilibrium if

$$c(t) = Y(t)/L, \tag{21}$$

because the only demand for final output is the consumption demand of the households.

4 Equilibrium

We are now ready to define an equilibrium of the model. For that purpose and for the remainder of the paper we assume that the initial values for blueprints and ideas, A_0 and B_0, respectively, are given and satisfy $0 < A_0 \leq B_0$.

Definition 1
An equilibrium associated with the initial values A_0 and B_0 and the policy scheme $P = \{L_B(t), \sigma(t)\}$ is a set of time paths $\mathcal{E} = \{Y(t)$, $x(t)$, $A(t)$, $B(t)$, $L_A(t)$, $L_X(t)$, $L_Y(t)$, $c(t)$, $r(t)$, $\bar{w}(t)$, $w(t)$, $w_A(t)$, $a(t)$, $\pi(t)$, $V(t)$, $\tau(t)\}$ such that
(i) the optimality and market clearing conditions (2), (4)-(6), (9), (11)-(21) hold for all t;
(ii) the boundary conditions $A(0) = A_0$, $B(0) = B_0$ are satisfied and (3) holds as an equality;
(iii) consumption is strictly positive at all times, i.e., $c(t) > 0$ for all t;
(iv) the tax rate is feasible, i.e., $0 \leq \tau(t) < 1$ holds for all t.

The reason why we include the requirement that consumption is strictly positive at all times is that we want to rule out the trivial equilibrium in which neither final output nor any intermediate goods are produced and, hence, consumption, profits, income, and wealth are zero at all times. Note that the condition $c(t) > 0$ implies immediately that $Y(t) > 0$ (because of (21)), which in turn implies $A(t) > 0$, $x(t) > 0$, and $L_Y(t) > 0$ (because of (5)). Hence, also $w(t) > 0$, $L_X(t) > 0$, $\pi(t) > 0$, and $V(t) > 0$ must hold in equilibrium (because of equations (6), (13), (11), and (12), respectively).

A similar argument can be made for our assumption that equilibrium requires the tax rate $\tau(t)$ to be strictly less than 1. Allowing $\tau(t)$ to become equal to 1 opens the door for some trivial and uninteresting equilibria. Ruling out these equilibria by assumption therefore greatly simplifies the presentation of the more substantial findings of our study.

4.1 The role of basic research

By assumption, basic research is a necessary condition for long-run growth in our model. As a matter of fact, the assumption that every idea can be turned into a single blueprint only, that is, $A(t) \leq B(t)$, implies that, whenever the government stops basic research forever, both $B(t)$ and $A(t)$ must remain bounded forever. This, however, means that technological progress dies out and long-run growth of output cannot be sustained.

A more interesting finding is described in the following lemma which deals with the converse of the above observation. More specifically, if government-financed basic research is bounded away from 0, private research will lead to an unbounded set of blueprints as long as the government's basic research activities do not asymptotically crowd out the entire labor force employed in production.

Lemma 1
Suppose that there exists $\varepsilon > 0$ *and* $T \geq 0$ *such that* $\varepsilon \leq L_B(t) \leq L - \varepsilon$ *for all* $t \geq T$. *Then it holds in every equilibrium that* $\lim_{t\to+\infty} A(t) = +\infty$.

The intuition for this result is that positive basic research efforts of the government imply an ever growing number of ideas, $B(t)$, which affects $w_A(t)$ positively due to spillovers from basic to applied research. This provides strong incentives for private agents to engage in applied research and the set of blueprints must therefore also grow indefinitely.

More specifically, it is easy to see that the first assumption of the lemma, namely that the measure of government-financed basic research is bounded away from zero, implies that the knowledge pool of the economy, $B(t)$, must become infinitely large. With an ever increasing amount of ideas $B(t)$, however, the positive effect of spillovers from basic research on the efficiency of applied researchers, which is reflected by the assumption $\mu_A > 0$, has at least one of the following consequences according to equation (16): Either applied research has to follow or catch up to the knowledge frontier and both basic and applied research grow without bounds, or $A(t)$ remains bounded while either the gross rate of return to applied research, $w_A(t)$, has to become arbitrarily large or the share price $V(t)$ has to approach zero. In the first case we are done, because it follows that $A(t)$ and $B(t)$ grow without bounds and approach infinity together. In the second case, where $A(t)$ is assumed to be bounded, $w_A(t)$ becomes arbitrarily large due to an ever growing number of ideas $B(t)$. However, unbounded growth of the remuneration of any form of labor requires unbounded growth of final output which is only possible if the set of blueprints and thus of new intermediate goods grows indefinitely. Hence, it follows, that $A(t)$ also needs to be unbounded, which is a contradiction. The remaining case of $A(t)$ assumed to be bounded and $V(t)$ approaching zero can be shown to occur only if final output and, hence, also the labor inputs $L_X(t)$ and $L_Y(t)$ approach zero. Together with (18) and the second assumption of the lemma, namely that basic research employment $L_B(t)$ is bounded away from L, this would imply that $L_A(t)$ remains bounded away from zero, which, again, leads to unbounded growth of $A(t)$, which is again a contradiction.

4.2 Balanced growth paths

In this subsection, we focus on time-invariant policy schemes $P = \{L_B, \sigma\}$, where both $L_B \in [0, L)$ and $\sigma \geq 0$ are constants. We examine the existence of balanced growth paths (BGPs), that is, equilibria along which all endogenous variables grow at constant rates.

Along a BGP equilibrium the labor shares $L_A(t)$, $L_X(t)$, $L_Y(t)$ as well as the tax rate $\tau(t)$ must be constant over time. We will therefore omit the time argument from these functions. Analogously, along a BGP equilibrium, the growth rate of per-capita consumption is constant. From the Euler equation (4) it follows therefore that the real interest rate is also constant, and we can simply write r instead of $r(t)$. Finally, we shall denote for any strictly positive and differentiable function v by $g_v(t)$ its growth rate at time t defined by $g_v(t) \equiv \dot{v}(t)/v(t)$.

Lemma 2
Along every BGP equilibrium $A(t)$ and $B(t)$ must grow at the same rate, that is, the equation $g_A(t) = g_B(t)$ holds for all $t \geq 0$.

The above result is a consequence of the existence of spillovers between the two research sectors as reflected by the assumptions $\mu_A > 0$ and $\mu_B > 0$. Research activities in one sector shape research activities in the other sector, and *vice versa*.

Let us henceforth denote the common growth rate of $A(t)$ and $B(t)$ by g. For the following analysis it will be convenient to define the functions $D(g)$, $F(g, L_B, \sigma)$, and $H(L_B, \sigma)$ by

$$\begin{aligned} D(g) &\equiv (1 - \alpha + \alpha^2)\rho + g, \\ F(g, L_B, \sigma) &\equiv \alpha(1 - \alpha)\gamma_A[D(g)(L - L_B) + (1 - \alpha + \alpha^2)(g + \rho)\sigma L]/D(g)^2, \\ H(L_B, \sigma) &\equiv F(\gamma_B L_B, L_B, \sigma). \end{aligned}$$

We have the following technical lemma.

Lemma 3
(a) The function $D(g)$ is continuous, strictly positive, and strictly increasing for all $g \in [0, +\infty)$.

(b) The function $F(g, L_B, \sigma)$ is continuous, strictly positive, strictly decreasing with respect to $g \in [0, +\infty)$ and $L_B \in [0, L)$, and strictly increasing with respect to $\sigma \in [0, +\infty)$.
(c) The function $H(L_B, \sigma)$ is continuous, strictly positive, strictly decreasing with respect to $L_B \in [0, L)$, and strictly increasing with respect to $\sigma \in [0, +\infty)$.

From the properties of $F(g, L_B, \sigma)$ and $H(L_B, \sigma)$ stated in Lemma 3 the following results are obvious. First, whenever $L_B \in [0, L)$ and $\sigma \in [0, +\infty)$, then there exists a unique number $\bar{g} > 0$ such that[13]

$$F(\bar{g}, L_B, \sigma) = [\bar{g}/(\gamma_B L_B)]^{\mu_A/\mu_B}. \tag{22}$$

Second, the equation $H(L_B, \sigma) = 1$ has a unique solution on the interval $L_B \in [0, L)$ if and only if $H(0, \sigma) \geq 1 > H(L, \sigma)$. In that case let us denote this solution by $\tilde{L}_B$. We define

$$\bar{L}_B = \begin{cases} 0 & \text{if } H(0,\sigma) < 1, \\ \tilde{L}_B & \text{if } H(0,\sigma) \geq 1 > H(L,\sigma), \\ L & \text{if } H(L,\sigma) \geq 1. \end{cases}$$

We are now ready to state the main result of this section. It presents both a necessary and a sufficient condition for a BGP equilibrium with growth rate g to exist.

Theorem 1
(a) If g is the common growth rate of the variables $A(t)$ and $B(t)$ in a BGP equilibrium corresponding to the constant policy scheme $P = \{L_B, \sigma\}$, then it follows that

$$g = \begin{cases} \gamma_B L_B & \text{if } L_B \leq \bar{L}_B, \\ \bar{g} & \text{if } L_B > \bar{L}_B, \end{cases} \tag{23}$$

and

$$A(t)/B(t) = \begin{cases} 1 & \text{if } 0 < L_B \leq \bar{L}_B, \\ [\bar{g}/(\gamma_B L_B)]^{1/\mu_B} < 1 & \text{if } L_B > \bar{L}_B. \end{cases} \tag{24}$$

Moreover, final output and consumption grow at the common rate $g_Y = g_c = (1-\alpha)g$ and the interest rate is given by $r = (1-\alpha)g + \rho$.
(b) Suppose that g satisfies (23) and that

$$D(g)(L - L_B) > \alpha(1-\alpha)g\sigma L. \tag{25}$$

Then there exists a BGP equilibrium corresponding to the policy scheme $P = \{L_B, \sigma\}$ along which the variables $A(t)$ and $B(t)$ grow at the rate g.

[13] This is obvious because the left-hand side of (22) is strictly positive and decreases continuously as $\bar{g}$ moves from 0 to $+\infty$, whereas the right-hand side increases continuously from 0 to $+\infty$.

4.3 Discussion

In this subsection we provide a detailed discussion of Theorem 1.

The impact of L_B

Part (a) of the theorem states conditions which the economic growth rate g and the ratio between blueprints and ideas $A(t)/B(t)$ must necessarily satisfy in equilibrium. These conditions (23)-(24) depend crucially on the threshold level $\bar{L}_B \in [0, L]$.

Let us first consider the case in which the measure of basic researchers L_B employed by the government does not exceed $\bar{L}_B$. In this situation we can see that applied researchers instantaneously commercialize any available basic research idea by turning it into a commercial blueprint. This is reflected by the result that $A(t)/B(t) = 1$ holds permanently. The reason is that, in this case, too little basic research is performed to permanently push the knowledge frontier of the economy ahead of applied research and, hence, applied researchers can completely exhaust the pool of ideas. According to (23), this implies that the overall growth rate g is only determined by the size of the publicly funded basic research sector, L_B, as well as by the productivity in the basic research sector, γ_B. Both of these variables have a strictly positive impact. In other words, in case of $L_B \leq \bar{L}_B$, basic research is the sole engine of long-run growth.

As mentioned above, in the case where $L_B \leq \bar{L}_B$, applied research always operates at the knowledge frontier. This implies in particular that applied research and, hence, overall economic growth has to cease unless positive basic research efforts permanently increase the knowledge pool of the economy. This also explains why research subsidies σ do not have any effect on the growth rate g whenever $L_B \leq \bar{L}_B$.[14] As applied research already operates at the knowledge frontier, granting subsidies in order to stimulate further applied research would be ineffective.

In contrast, if L_B exceeds the threshold value $\bar{L}_B$, basic research pushes the knowledge frontier fast enough such that, at each instant in time, a pool of non-commercialized ideas is available that has not yet been turned into blueprints through applied research. According to (24), the ratio of blueprints to ideas, $A(t)/B(t)$, is then strictly smaller than 1 showing that the available pool of non-commercialized ideas is never exhausted. In fact, the measure of non-commercialized ideas grows without bounds. The growth rate $g = \bar{g}$ is given by (22) and can be shown to lie between $\gamma_A L_A$ and $\gamma_B L_B$. Thus, in the case of $L_B > \bar{L}_B$, long-run growth is shaped by both basic and applied research together.

The intuitive reason for this result is that, while applied research stimulates growth by commercializing basic research, the potential of generating growth is upward bounded by the rate at which new ideas are created, i.e., by how fast the knowledge frontier of the economy

[14]Note, however, that the threshold level $\bar{L}_B$ itself depends on the size of σ.

is pushed ahead through basic research and how this impacts on the speed of commercialization.

We would like to emphasize that, while g is strictly increasing with respect to $L_B \in [0, \bar{L}_B]$, the relation between $g = \bar{g}$ and L_B can become more complicated for $L_B > \bar{L}_B$. This is shown by the following example.

EXAMPLE: Suppose that

$$\sigma = 0 \text{ and } L > \frac{(1-\alpha+\alpha^2)\rho}{\alpha(1-\alpha)\gamma_A}. \tag{26}$$

Simple algebra shows that these assumptions imply

$$\bar{L}_B = \frac{\alpha(1-\alpha)\gamma_A L - (1-\alpha+\alpha^2)\rho}{\alpha(1-\alpha)\gamma_A + \gamma_B} > 0. \tag{27}$$

Now assume in addition that $\mu_A = \mu_B$. In that case we can solve equation (22) to obtain

$$\bar{g} = -\frac{(1-\alpha+\alpha^2)\rho}{2} + \sqrt{\frac{(1-\alpha+\alpha^2)^2\rho^2}{4} + \gamma_A\gamma_B\alpha(1-\alpha)L_B(L-L_B)}.$$

Combining this result with (23) we see that the equilibrium growth rate increases linearly with L_B on the interval $[0, \bar{L}_B]$, whereas it is described by a unimodal function of L_B on the interval $[\bar{L}_B, L)$. It follows in particular that g is decreasing with respect to L_B on $[\max\{\bar{L}_B, L/2\}, L)$, whereas it is increasing on $[\bar{L}_B, L/2]$ provided this interval is non-empty. The latter case illustrates that the growth rate may depend positively on the public basic research input L_B even if the pool of non-commercialized ideas remains permanently non-empty. □

The example illustrates that an increase of L_B has two opposite effects on applied research. First, the associated increase of $B(t)$ triggered by a high amount of basic research affects applied research positively through knowledge spillover. Second, an increase of L_B also has a negative influence on applied research because it ties up labor that otherwise could be used for other purposes. As long as the positive spillover effect dominates, an increase of L_B induces higher growth, and both g_B and g_A are increasing with respect to L_B. As soon as L_B exceeds a critical value, however, the negative effect of reducing labor supply for applied research and production dominates and g_A becomes decreasing with respect to L_B. Any decline of g_A is associated with a corresponding decrease of g_B as spillovers to basic research diminish. That is, an increase of basic research is not only detrimental to overall growth, but also to the generation of new ideas.

The impact of research subsidies

We have already noted that research subsidies have no impact on growth for $L_B \leq \bar{L}_B$ because in that situation applied research operates at the knowledge frontier. For $L_B > \bar{L}_B$, however, equations (22) and (23) imply that the level of research subsidies, σ, has a strictly positive impact on growth. The reason is quite obvious as increasing the research subsidies lowers the relative cost of performing applied research and thus induces more labor to be employed in applied research. Furthermore, the threshold value $\bar{L}_B$ itself is an increasing function of the research subsidy σ. The reason is that granting higher research subsidies increases the incentives to work in the applied research sector, thereby shifting the labor supply towards the applied research sector, and consequently accelerates growth of $A(t)$. Therefore, for any given growth rate of $B(t)$, a higher research subsidy makes it more likely that applied research catches up to the knowledge frontier of the economy and the long-run ratio between blueprints and ideas, $A(t)/B(t)$, increases. In other words, with a higher subsidization of research, the speed of commercialization of ideas accelerates and the knowledge frontier more likely becomes a binding constraint for applied research.

Feasibility and boundary conditions

Condition (25) in part (b) of Theorem 1 ensures that the BGP equilibrium is feasible in the sense that it fulfils the requirement $\tau \in [0, 1)$; see part (iv) of Definition 1. It is obvious that τ will be equal to zero if and only if both $L_B = 0$ and $\sigma = 0$, because the only expenditures of the government are its outlays on basic research and subsidies. On the other hand, it is possible that the employment of a large number of basic researchers and/or the payment of high research subsidies would require the government to set $\tau \geq 1$, which is not feasible. To rule out this possibility, (25) imposes a joint restriction on the policy parameters L_B and σ that ensures that both government activities can be financed by a labor tax at rate $\tau < 1$.

We finally note that the threshold value $\bar{L}_B$ itself could be equal to 0. In such circumstances all positive levels of basic research input lead to a situation where the measure of blueprints is strictly smaller than the measure of ideas ($A(t) < B(t)$). Whether $\bar{L}_B = 0$ or not depends on whether $H(0, \sigma) \leq 1$ or not. From the definition of $H(L_B, \sigma)$ we can see in particular that $\bar{L}_B = 0$ holds if and only if

$$\frac{\alpha(1-\alpha)\gamma_A L(1+\sigma)}{(1-\alpha+\alpha^2)\rho} \leq 1. \tag{28}$$

5 One-directional spillovers

So far we have assumed that spillovers are present both from basic to applied research and *vice versa*. Formally, this is reflected by the assumptions $\mu_A > 0$ and $\mu_B > 0$, which we have explicitly used in the proofs of Lemmas 1 and 2 and implicitly in the proof of Theorem 1. In this section we discuss the two special cases in which non-negligible spillovers occur only in one of these directions, that is, we analyze what happens in the limits when either μ_A or μ_B approaches 0.

To this end note that none of the functions $D(g)$, $F(g, L_B, \sigma)$, and $H(L_B, \sigma)$ depends on the parameters μ_A and μ_B. This implies in particular that the threshold value $\bar{L}_B$ is independent of these parameters. However, as is clear from equation (22), the relative size of the spillover parameters μ_A and μ_B is a crucial determinant of $\bar{g}$.

5.1 Only upward spillovers

Let us start with the case where $\mu_A > 0$ is fixed and μ_B approaches 0. In this limit, the effect of the measure of blueprints on the productivity of basic researchers becomes negligible, whereas the measure of existing ideas has a strictly positive influence on the productivity of applied researchers. Hence, there is upward spillover from basic to applied research, but only negligible downward spillover from applied to basic research. As μ_B converges to 0, the right-hand side of equation (22) approaches 0 whenever $\bar{g} < \gamma_B L_B$ and it approaches $+\infty$ if $\bar{g} > \gamma_B L_B$. This proves immediately that $\bar{g}$, the unique solution of equation (22), converges to the value $\gamma_B L_B$ as μ_B approaches 0. Using this result together with condition (23) we can conclude that in the case where $\mu_B \approx 0$, the equilibrium growth rate satisfies $g \approx \gamma_B L_B$ for all settings of L_B and σ. We formally state this finding as the first result in the following lemma.

Lemma 4
In the limit, when μ_B approaches zero, the equilibrium growth rate is given by $g = \gamma_B L_B$. Furthermore, it holds that

$$\lim_{\mu_B \to 0} A(t)/B(t) = \begin{cases} 1 & \text{if } 0 < L_B \leq \bar{L}_B, \\ H(L_B, \sigma)^{1/\mu_A} < 1 & \text{if } L_B > \bar{L}_B. \end{cases}$$

This implies that, in the absence of significant downward spillovers from applied to basic research, long-run growth of the economy only depends on the amount of basic research conducted by the government, L_B, as well as on the productivity parameter γ_B. In particular, it follows that the research subsidy σ has no growth effect when only upward spillovers are present. In the case where $L_B \leq \bar{L}_B$, this finding is obvious and just repeats what we have already established for the model with two-way spillovers in section 4: if applied research operates at the knowledge frontier, overall growth is restricted by the size and productivity of the basic research sector and cannot be further increased by research subsidies. Surprisingly, however, the same result holds here also when $L_B > \bar{L}_B$, in which case the set of non-commercialized ideas is never exhausted. To understand why, first note that (14) and μ_B arbitrarily small together imply that the evolution of the knowledge frontier depends solely on the current location of this frontier, on the productivity parameter γ_B, and on the amount of labor employed in basic research: the growth rate is $g_B = \gamma_B L_B$. By subsidizing researchers at a higher rate σ, the government can make the gap between the set of commercialized blueprints and basic research ideas smaller, but, since the measure of blueprints does not feed back onto the productivity of basic researchers, this leaves the growth rate g_B unaffected. Thus, in the case of one-directional upward spillovers, long-run growth can only be stimulated through basic research.

5.2 Only downward spillovers

We now turn to the case of negligible upward spillovers, i.e., the case where $\mu_B > 0$ is given and μ_A approaches 0. In this limiting case the right-hand side of equation (22) converges to 1 whenever $\bar{g} > 0$. This implies that in the limit it must hold that

$$\bar{g} = \begin{cases} \bar{\bar{g}} & \text{if } F(0, L_B, \sigma) > 1, \\ 0 & \text{if } F(0, L_B, \sigma) \leq 1, \end{cases} \tag{29}$$

where $\bar{\bar{g}}$ is uniquely determined by the condition $F(g, L_B, \sigma) = 1$. Combining this result with (23) we obtain the following lemma in which $\bar{\bar{L}}_B$ is defined by

$$\bar{\bar{L}}_B = (1+\sigma)L - \frac{(1-\alpha+\alpha^2)\rho}{\alpha(1-\alpha)\gamma_A}.$$

Lemma 5
(a) The condition $\bar{\bar{L}}_B \leq 0$ holds if and only if $\bar{L}_B = 0$. Moreover, whenever $\bar{L}_B > 0$ then it follows that $\bar{L}_B < \bar{\bar{L}}_B$.
(b) Suppose that $\mu_B > 0$ is fixed. In the limit as μ_A approaches 0, the equilibrium growth rate is given by

$$g = \begin{cases} \gamma_B L_B & \text{if } L_B \leq \bar{L}_B, \\ \bar{\bar{g}} & \text{if } \bar{L}_B < L_B \leq \bar{\bar{L}}_B, \\ 0 & \text{if } L_B > \bar{\bar{L}}_B. \end{cases}$$

Lemma 5 indicates that there are three different facets of growth. First, if the actual amount of basic researchers L_B does not exceed the threshold $\bar{L}_B$, basic research alone shapes long-run growth. In this facet the research subsidy σ has no growth effects, because applied researchers already operate at the knowledge frontier. Second, if L_B exceeds the threshold $\bar{L}_B$, but remains beneath $\bar{\bar{L}}_B$, then growth is shaped both by basic and applied research. From the definition of $\bar{\bar{g}}$ one can see that $\bar{\bar{g}}$ is increasing with respect to σ and decreasing with respect to L_B. It follows therefore from Lemma 5 that the growth rate g is maximal if $L_B = \bar{L}_B$. Any expansion of the basic research sector beyond the threshold value $\bar{L}_B$ would be detrimental to growth. That is, along the interval $[\bar{L}_B, \bar{\bar{L}}_B]$ growth decreases until it is about to cease entirely in case L_B approaches $\bar{\bar{L}}_B$. Third, if L_B exceeds $\bar{\bar{L}}_B$ the equilibrium growth is equal to zero. These results are further illustrated by the following example.

EXAMPLE: As in the previous example assume that (26) holds. In addition to (27), this implies that

$$\begin{aligned}\bar{\bar{L}}_B &= L - \frac{(1-\alpha+\alpha^2)\rho}{\alpha(1-\alpha)\gamma_A} > \bar{L}_B,\\ \bar{\bar{g}} &= \alpha(1-\alpha)\gamma_A(L-L_B) - (1-\alpha+\alpha^2)\rho.\end{aligned}$$

Using these expressions in Lemma 5 one sees that the equilibrium growth rate g in the case of negligible upward spillovers is a continuous and piecewise linear function of L_B that increases on the interval $[0, \bar{L}_B]$, decreases on $[\bar{L}_B, \bar{\bar{L}}_B]$, and is constant and equal to 0 on $[\bar{\bar{L}}_B, L)$. □

5.3 Summary

The results for one-directional spillovers discussed above provide valuable information about the hierarchy and interdependence of basic and applied research and about their joint effect on growth. More specifically, we have seen that in the absence of significant downward spillovers from applied to basic research more publicly funded basic research always translates into faster growth. On the other hand, in the absence of significant upward spillovers from basic to applied research, growth depends positively on the public basic research input only as long as applied researchers find it optimal to completely exhaust the available knowledge base of the economy at each point in time. The general case covered by Theorem 1 can be considered as a mixture of the two special cases discussed in the present section.

6 An illustration

6.1 Replication of the US growth pattern

In this section we provide an illustrative example of above model. We replicate the US innovation pattern and growth rate.

Parameter Choices

For this purpose, we consider the following parameter values. For the elasticity of final output with respect to intermediate goods, α, we assume that new intermediates enter final goods production as new varieties of capital goods. To obtain a plausible value for α within our endogenous growth framework, we borrow from the existing empirical growth literature, where the share of capital in final goods production has been subject to various empirical estimates of the neoclassical growth model. Thereby, the quantitative extent of α crucially depends on the underlying concept of capital that is taken into account. More specifically, the conventional share of physical capital is $\alpha = 1/3$ (see, for example, Jones, 2002), which may even be as low as 0.1 for an open economy (Barro *et al.*, 1995). However, when assuming that the underlying concept of capital also includes human capital, the conventional capital share appears too narrow and one should rather focus on a broader concept of capital (see, for example, Barro and Sala-i-Martin, 1992, and Mankiw *et al.*, 1992). Thus, assuming that human capital is embodied entirely in new varieties of intermediate capital goods would imply a value of up to $\alpha = 0.8$ (Barro and Sala-i-Martin, 1992). As this last case appears to correspond closest to our model, we set $\alpha = 0.8$. Furthermore, we follow Barro and Sala-i-Martin (1992) and set the time-preference rate to $\rho = 0.05$. Finally, we set $L = 1$.

Data on basic and applied research employment are usually harder to obtain. According to National Science Board (2008), scientists and engineers made up about 4.2 percent of the total US workforce in 2006, while figures from 2003 suggest that out of these about 59 percent were employed by industry, while the remaining 41 percent were mainly affiliated with government and university institutions. This suggests that $L_B = 0.017$. In addition, we assume an equal intensity of spillovers from basic to applied research and *vice versa*, and set our spillover parameters to $\mu_A = \mu_B = 0.5$, while our research productivity parameters γ_A and γ_B are set to $\gamma_A = \gamma_B = 5.3$. Finally, we set research subsidies σ equal to zero and assume an initial amount of basic research ideas, $B(0)$, equal to 10.

Simulations

Given these parameter settings, our simulations yield that $g = 0.09$. According to Theorem 1 this implies that the growth rate of per capita GDP is $g_Y = g(1-\alpha) = 0.018$. This is equal to the average long-run growth of per capita GDP in the US per year which has been found to be about 1.8 percent (see for example, Jones, 2002, or Barro and Sala-i-Martin, 2004).

Furthermore, the share of applied researchers in the total labor force is $L_A = 0.107$. While the growth rate corresponds exactly to the average annual, long-run US growth rate stated above, the share of applied research employment L_A at first glance appears quite high as compared to the suggested value of $L_A = 0.025$ by the data of the National Science Board (2008). However, the official data about research employment are significantly downward-biased as they merely focus on scientists and engineers and also require those to have a college degree, whereby a considerable amount of researchers is not captured by these data (Jones, 2002). In that regard, a share of applied research as high as $L_A = 0.107$ appears to be not implausible. Our simulations also suggest that the US innovation and growth system operates at the knowledge frontier, that is $A(t)/B(t) = 1$. Therefore, growth is only shaped by basic research L_B as well as the research productivity parameter γ_B, while the remaining parameters have at most an impact on applied research employment L_A and on output. Finally, we note that employing $L_B = 0.017$ basic researchers, while granting no research subsidies σ corresponds to a feasible policy scheme in the sense that condition (25) of Theorem 1 is fulfilled

6.2 Policy changes

It is useful to evaluate how policy changes would impact on growth in the simulated economy. In particular, suppose we vary $L_B \in (0, 0.2]$ and $\gamma_B = (0, 10]$ in the above system. Figure 1 shows the impact of variations in the amount of basic research L_B while holding fixed all other parameters of the model. As shown in the upper graph, along the knowledge frontier the growth rate g increases linearly with the amount of basic research L_B. A continuous increase of basic research eventually results in L_B exceeding the threshold amount $\bar{L}_B$. From there on, basic research continues to have a positive, albeit diminishing effect on long-run growth. For ever larger values of L_B the growth rate would start to decline until it reaches zero when all labor would be employed in basic research. The amount of applied research L_A first increases with basic research L_B as the associated increase in the speed of creating novel ideas also increases incentives for applied research *via* spillovers. This increase of L_A reverses, however, as soon as increasing levels of basic research can only be achieved by withdrawing labor from applied research. As can be seen in the lower graph, low levels of basic research allow the economy to operate at the knowledge frontier. As soon as L_B achieves the threshold value $\bar{L}_B$, however, further increases of basic research trigger an increasing distance to the knowledge frontier. We note that the ratio of blueprints to ideas, $A(t)/B(t)$, converges to zero when L_B would converge to 1.

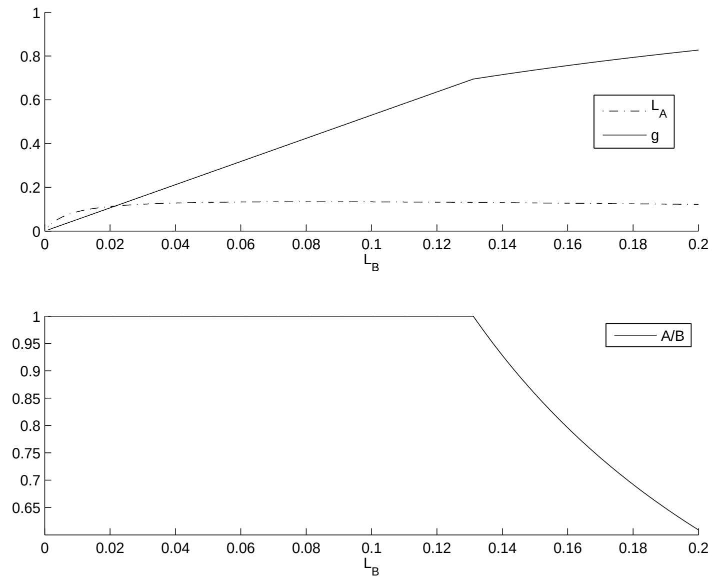

Figure 1: The impact of basic research L_B

Figure 2 shows the effect of varying the research productivity parameter γ_B while holding all other model parameters fixed. As the upper graph shows, γ_B has a strictly positive impact on growth. A higher γ_B also exerts a positive, but decreasing impact on applied research, L_A, as higher productivity of basic research, which leads to a faster growing number of ideas, also increases incentives to engage in applied research. Furthermore, as the threshold value $\bar{L}_B$ is a function of γ_B, a further continuous increase of γ_B at some stage induces basic research to be productive enough such that the number of ideas will evolve faster than the number of blueprints, and applied research falls short of basic research. That is, for sufficiently high γ_B, applied research would no longer operate at the knowledge frontier of the economy.

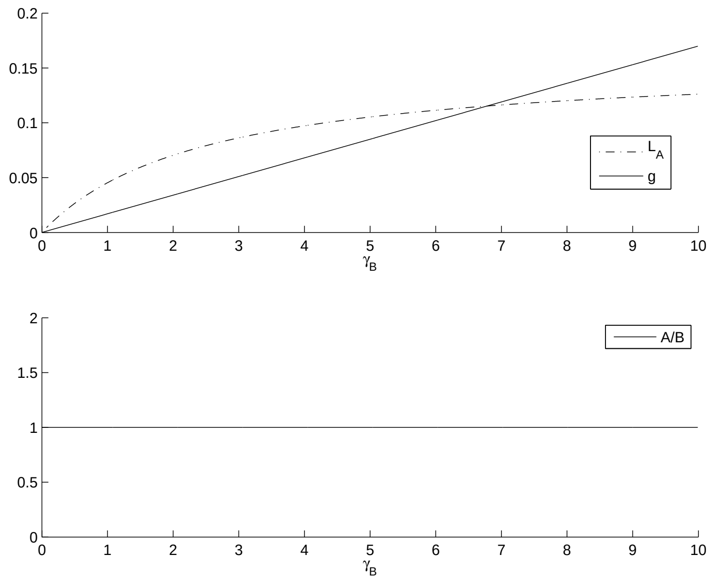

Figure 2: The impact of basic research productivity γ_B

7 Conclusion

We have developed a simple model that combines the hierarchy of basic and applied research as well as their interdependence with an expanding variety growth framework. Numerous issues deserve further scrutiny. In particular, a normative analysis of our model could be used to develop guidelines for socially optimal policies regarding the amount of basic research and research subsidies.

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Appendix

Table 3: Basic Research Ideas, Theories and Prototypes (*preliminary version*)

Invention	(Main) Inventor(s)	Time of Invention	Institutional Affiliation	(Potential) Commercial Applications	Source
X-Rays	Wilhelm Conrad Roentgen	1895	University of Wuerzburg	Radiology, X-ray equipment	Assmus (1995), Gelijns and Rosenberg (1999)
Boundary-Layer Theory	Ludwig Prandtl	1904	Institute of Technology Hanover	Airplane manufacturing, aerodynamics, fluid dynamics	Hirschel *et al.* (2004), Anderson (2005)
Insulin	Frederick Banting and Charles Best	1921	University of Toronto	Treatment of diabetes	Simoni *et al.* (2002e)
Penicillin	Alexander Fleming	1929	St. Mary's Hospital Medical School, London	Antibiotics	Lightman (2005)
Idea to Create an Extracorporeal Blood Circuit	John Gibbon	1931	Massachusetts General Hospital	Heart-lung machines	Gibbon (1978)
Nuclear Magnetic Resonance (NMR)	Isidor Rabi, Edward Purcell, Felix Bloch	1938, 1946 (Existence of NMR in Solids (Purcell) and Liquids (Bloch))	Columbia University (Rabi), Harvard University (Purcell) and Stanford University (Bloch)	Nuclear magnetic resonance spectrometer	von Hippel (1988), Gelijns and Rosenberg (1999)
Nuclear Fission (Splitting of Nuclear Atoms)	Otto Hahn, Fritz Strassmann, Lise Meitner and Otto Frisch	1939	Kaiser Wilhelm Institute for Chemistry, Berlin (Hahn and Strassmann), Academy of Sciences, Stockholm (Meitner), and University of Copenhagen (Frisch)	Nuclear energy	Lightman (2005)
Use of Acoustic Energy for Diagnostic Purposes	John Lynn, Raymund Zwemer, Arthur Chick, and August Miller	1942	Columbia University and Piezo-Electric Research Laboratories, North Bergen, New Jersey	Focused ultrasound therapies	Lynn *et al.* (1942), National Academy of Engineering (2003)
Theory of Maser	Charles Hard Townes	1951	Columbia University	Laser technologies and equipment	Hora (2000)
Flexible Gastrointestinal Endoscopy	Abraham van Heel, Harold Hopkins, and Narinder Kapany	1954	Delft University of Technology (van Heel) and Imperial College of Science and Technology, London (Hopkins and Kapany)	Flexible fiber-optic endoscopes	van Heel (1954), Hopkins and Kapany (1954), Gelijns and Rosenberg (1999)
First Implantable Cardiac Pacemaker	Rune Elmqvist and Åke Senning	1958	Karolinska Institute, Stockholm	Cardiac pacemakers	Elmqvist *et al.* (1963), Bunch and Day (2008)
Low-Frictional Hip Arthroplasty	John Charnley	1960	Manchester Royal Infirmary	Hip surgery and replacement, artificial hip joints	Donald (2007)
Packet Switching Theory	Leonard Kleinrock	1961	MIT	Computer networking	Leiner *et al.* (1997)

Table 3: Basic Research Ideas, Theories and Prototypes (*preliminary version*) (*ctd.*)

Invention	(Main) Inventor(s)	Time of Invention	Institutional Affiliation	(Potential) Commercial Applications	Source
Discovery of the Australia Antigen / Hepatitis B	Baruch Blumberg	1963	National Institutes of Health	Detection of and vaccination against Hepatitis B	Blumberg (1997)
Computer Mouse	Douglas Engelbart	1965	Stanford University	Computer mice, grapical user interfaces	Barnes (1997), Myers (1998)
Landau-de Gennes Phenomenological Theory of the Nematic Phase	Pierre-Gilles de Gennes	1971	Simon Fraser University / University of Paris, Orsay	Liquid-crystal technology	Sluckin (2000), Sluckin *et al.* (2004), Ajdari (2007), Brochard-Wyart (2007)
First Whole-Body Computerized Tomography Scanner	Robert Ledley	1973	Georgetown University	Computerized tomography scanners	Sittig *et al.* (2006)
Recombinant DNA (Gene Splicing, Genetic Engineering)	Paul Berg, Stanley Cohen and Herbert Boyer	1972-1973	Stanford University (Berg and Cohen) and University of California at San Francisco (Boyer)	Analysis of evolution and transmission of diseases (e.g., cancer), development of drugs (e.g., human insulin), agricultural applications (e.g., transgenic crops)	Cohen *et al.* (1973), Zucker *et al.* (2002), Lightman (2005)
Nuclear Magnetic Resonance Imaging	Paul Lauterbur and Peter Mansfield	1973	State University of New York at Stony Brook (Lauterbur) and University of Nottingham (Mansfield)	Nuclear magnetic resonance imaging	Lauterbur (1973), Mansfield and Grannell (1973), Leach (2004)
Human Papillomaviruses Cause Cervical Cancer	Harald zur Hausen	1976	University of Erlangen-Nuremberg	Vaccinations against cervical cancer	zur Hausen (1976, 2002)
Coronary Angioplasty	Andreas Grüntzig	1977	University of Zurich	Treatment of cardiovascular diseases	King (1996)
Method of DNA Sequencing	Allan Maxam, Walter Gilbert, and Frederick Sanger	1977	Harvard University (Maxam and Gilbert) and Medical Research Council Laboratory of Molecular Biology, Cambridge (Sanger)	Molecular biology, genetic research, identification of genetic diseases, determination of unknown DNA sequences	Maxam and Gilbert (1977), Sanger *et al.* (1977), Ahmadian *et al.* (2006)
Tissue Engineering	Joseph Vacanti and Robert Langer	1988	Children's Hospital Boston (Vacanti) and MIT (Langer)	Creation of replacement organs and body parts	National Academy of Engineering (2003), Vacanti (2006)
Theory on Flapping Insect Flight	Charles Ellington	1996	Cambridge University	Micro air vehicles	Ellington *et al.* (1996), Toon (2001)
Method of RNA Interference	Andrew Fire and Craig Mello	1998	Carnegie Institution of Washington (Fire) and University of Massachusetts Cancer Center (Mello)	Potential application in the treatment of, for example, cancer, genetic and viral diseases	Fire *el al.* (1998), Aagaard and Rossi (2007)
Superhard Material Properties of Wurtzite BN and Lonsdaleite	Zicheng Pan, Hong Sun, Yi Zhang and Changfeng Chen	2009	Shanghai Jiao Tong University (Pan and Sun) and University of Nevada (Zhang and Chen)	Design and manufacturing of superhard materials	Pan *et al.* (2009)

Useful relationships

In this subsection we collect several relationships that must hold in every equilibrium.

From (5) and (9) it follows that

$$Y(t) = \left[\alpha^2/w(t)\right]^{\alpha/(1-\alpha)} A(t)L_Y(t). \tag{30}$$

Substituting (30) into (6) we get

$$w(t) = \alpha^{2\alpha}\left[(1-\alpha)A(t)\right]^{1-\alpha}. \tag{31}$$

Substituting (31) back into (30) we obtain

$$Y(t) = \alpha^{2\alpha}(1-\alpha)^{-\alpha}A(t)^{1-\alpha}L_Y(t). \tag{32}$$

Combining (11) and (31) yields

$$\pi(t) = (1-\alpha)^{1-\alpha}\alpha^{1+2\alpha}A(t)^{-\alpha}L_Y(t), \tag{33}$$

From the Euler equation (4) one obtains that $\int_t^s r(s')ds' = \ln[c(s)/c(t)] + \rho(s-t)$. Combining this with (21) it follows that $\int_t^s r(s')ds' = \ln[Y(s)/Y(t)] + \rho(s-t)$. Substituting the latter identity into (12) and using (32) and (33) we get

$$V(t) = Y(t)\int_t^{+\infty} e^{-\rho(s-t)}\pi(s)/Y(s)ds = \alpha(1-\alpha)Y(t)\int_t^{+\infty} e^{-\rho(s-t)}A(s)^{-1}ds. \tag{34}$$

Combining (13) and (31) yields

$$L_X(t) = [\alpha^2/(1-\alpha)]L_Y(t). \tag{35}$$

Finally, by assuming $L_A(t) > 0$ and combining (17)-(19), we obtain

$$\sigma(t)L_A(t) = [1-\tau(t)+\sigma(t)][\tau(t)L - L_B(t)]. \tag{36}$$

Equation (36) represents the government's budget constraint in terms of taxes, subsidies, and the labor shares in the two research sectors in the case where applied research actually takes place.

Proof of Lemma 1

First note that both $A(t)$ and $B(t)$ are non-decreasing functions of t. Together with (14) this implies that $\dot{B}(t) \geq \gamma_B B_0^{1-\mu_B}A_0^{\mu_B}\varepsilon > 0$ holds for all $t \geq T$ which, in turn, leads to $\lim_{t\to+\infty} B(t) = +\infty$.

Now suppose that $A(t)$ remains bounded. We will show that this leads to a contradiction. As a matter of fact, since $B(t)$ diverges to $+\infty$, the boundedness of $A(t)$ implies that there exists $T_1 \geq 0$ such that $A(t) < B(t)$ holds for all $t \geq T_1$. Boundedness of $A(t)$ together with (31) implies also that $w(t)$ remains bounded. Using (19) and $\sigma(t) \geq 0$ it follows that

$$w_A(t) \leq \frac{[1-\tau(t)]w(t)}{1-\tau(t)+\sigma(t)} \leq w(t)$$

and we can therefore conclude that $w_A(t)$ remains bounded as well. Because $A(t) < B(t)$ for all $t \geq T_1$ and because $B(t)$ diverges to $+\infty$ as t approaches $+\infty$, equation (16) implies that, for $w_A(t)$ to remain bounded, $V(t)$ must converge to 0 as t approaches $+\infty$. Finally, since $A(t)$ remains bounded there exists $\tilde{A}$ such that $A(t) \leq \tilde{A}$ for all t and, hence, equation (34) implies that $V(t) \geq \alpha(1-\alpha)Y(t)/(\rho\tilde{A})$. Obviously, this shows that $\lim_{t\to+\infty} V(t) = 0$ can only hold if $\lim_{t\to+\infty} Y(t) = 0$. Because of (32) and (35) this, in turn, requires that $\lim_{t\to+\infty} L_X(t) = \lim_{t\to+\infty} L_Y(t) = 0$ as well. Combining this last result with our assumption that $L_B(t) \leq L - \varepsilon$ it follows from the labor market clearing condition (18) that there exists $T_2 \geq 0$ such that $L_A(t) \geq \varepsilon/2 > 0$ for all $t \geq T_2$. It is easily seen from (15) that this implies $\lim_{t\to+\infty} A(t) = +\infty$, which is a contradiction our assumption of boundedness of $A(t)$.

Proof of Lemma 2

By definition of a BGP equilibrium both $A(t)$ and $B(t)$ must be exponential functions of t. Moreover, because $A(t) \leq B(t)$ must hold for all t, we have either $A(t) = B(t)$ for all $t \geq 0$ or $A(t) < B(t)$ for all $t > 0$. In the former case, it is obvious that $g_A(t) = g_B(t)$ for all $t \geq 0$. In the latter case, equations (14)-(15) imply that $g_A(t) = \gamma_A L_A[A(t)/B(t)]^{-\mu_A}$ and $g_B(t) = \gamma_B L_B[A(t)/B(t)]^{\mu_B}$. Differentiating with respect to t it follows that

$$\dot{g}_A(t) = \mu_A g_A(t)[g_B(t) - g_A(t)],$$
$$\dot{g}_B(t) = \mu_B g_B(t)[g_A(t) - g_B(t)].$$

Along a balanced growth path it holds that $\dot{g}_A(t) = \dot{g}_B(t) = 0$ so that the above two equations imply $\mu_A g_A(t)[g_B(t) - g_A(t)] = \mu_B g_B(t)[g_A(t) - g_B(t)] = 0$. Because both μ_A and μ_B are strictly positive, this can only hold if $g_A(t) = g_B(t)$.

Proof of Lemma 3

Part (a) is obvious from the definition of $D(g)$. Continuity, positivity, and monotonicity of $F(g, L_B, \sigma)$ with respect to L_B and σ are also obvious. To verify the monotonicity of $F(g, L_B, \sigma)$ with respect to g just note that $F(g, L_B, \sigma) = F_1(g)[L - L_B + F_2(g, \sigma)]$ with $F_1(g) = \alpha(1-\alpha)\gamma_A/D(g)$ and $F_2(g, \sigma) = (1-\alpha+\alpha^2)(g+\rho)\sigma L/D(g)$. It is easy to see that both $F_1(g)$ and $F_2(g, \sigma)$ are strictly positive and strictly decreasing functions of g which proves that $F(g, L_B, \sigma)$ is also strictly decreasing with respect to g. Finally, part (c) follows immediately from the definition of $H(L_B, \sigma)$ and from part (b).

Proof of Theorem 1

(a) If $L_B = 0$, then (24) holds trivially and it follows that $L_B \leq \bar{L}_B$. Moreover, from (14) we obtain $g = g_B(t) = 0$ so that condition (23) is also satisfied. From now onwards, we shall therefore assume that L_B is strictly positive. Together with Lemma 1 this implies that $L_A > 0$ holds as well.[15]

As $A(t)$ and $B(t)$ grow at the common rate g, we must have $A(t) = A_0 e^{gt}$ and $B(t) = B_0 e^{gt}$ where $A_0 \leq B_0$. If $A_0 = B_0$ we have $A(t) = B(t)$ for all t and it follows from equations (14) and (15) that

$$g = \gamma_B L_B \leq \gamma_A L_A. \tag{37}$$

The BGP property implies also that $A(s) = A(t)e^{g(s-t)}$. Substituting this into (34) one obtains

$$V(t) = \frac{\alpha(1-\alpha)Y(t)}{(\rho+g)A(t)}. \tag{38}$$

Next we claim that

$$w_A(t) = \frac{\alpha(1-\alpha)gY(t)}{(\rho+g)L_A}. \tag{39}$$

To prove this claim, we distinguish the two cases $A_0 = B_0$ and $A_0 < B_0$. In the first case, equation (39) follows from using (37) and (38) in (16). In the second case, we use equations (14) and (15) to obtain

$$g = \gamma_A L_A (A_0/B_0)^{-\mu_A} = \gamma_B L_B (A_0/B_0)^{\mu_B}. \tag{40}$$

Combining (16) with (38) it follows that

$$w_A(t) = \frac{\gamma_A \alpha(1-\alpha)(A_0/B_0)^{-\mu_A} Y(t)}{\rho+g}.$$

Using the first equation in (40), it is easy to see that the above equation coincides with (39).

Substituting (32) into (39) one can see that

$$w_A(t) = \frac{\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha} g A(t)^{1-\alpha} L_Y}{(\rho+g)L_A}.$$

Substituting this together with (31) into (19) we obtain after simplifications

$$\alpha g(1-\tau+\sigma)L_Y = (1-\tau)(g+\rho)L_A. \tag{41}$$

Equations (18), (35), (36), and (41) form a system of four equations in the variables L_A, L_X, L_Y, and τ. It is straightforward to verify that the only solution of this system that satisfies

[15] Constancy of L_X and L_Y together with $\lim_{t\to+\infty} L_X(t) = \lim_{t\to+\infty} L_Y(t) = 0$ would imply that $L_X = L_Y = 0$. This would imply $Y(t) = c(t) = 0$, which is ruled out by Definition 1. Hence, Lemma 1 implies that $\lim_{t\to+\infty} A(t) = +\infty$, which is only possible if $L_A > 0$.

the conditions $\tau < 1$, $L_X > 0$, and $L_Y > 0$ is given by

$$\begin{aligned} L_A &= \alpha(1-\alpha)g[D(g)(L-L_B) + (1-\alpha+\alpha^2)(g+\rho)\sigma L]/D(g)^2, \\ L_X &= \alpha^2(g+\rho)[D(g)(L-L_B) - \alpha(1-\alpha)g\sigma L]/D(g)^2, \\ L_Y &= (1-\alpha)(g+\rho)[D(g)(L-L_B) - \alpha(1-\alpha)g\sigma L]/D(g)^2, \\ \tau &= [D(g)L_B/L + \alpha(1-\alpha)g\sigma]/D(g). \end{aligned} \tag{42}$$

Now we consider again the two cases $A_0 = B_0$ and $A_0 < B_0$ separately. In the former case, condition (37) implies $g = \gamma_B L_B$ and $g \leq \gamma_A L_A$. Using the expression for L_A from (42) we can rewrite the inequality $g \leq \gamma_A L_A$ as $F(g, L_B, \sigma) \geq 1$. Because of $g = \gamma_B L_B$ this is equivalent to $H(L_B, \sigma) \geq 1$. Due to the monotonicity of H and the definition of $\bar{L}_B$ this implies that $L_B \leq \bar{L}_B$ and the proof of the first line in (23) is complete.

In the case where $A_0 < B_0$, we can solve the two equations in (40) to express g and A_0/B_0 in terms of L_A and L_B. This yields in particular

$$g = [(\gamma_A L_A)^{\mu_B}(\gamma_B L_B)^{\mu_A}]^{1/(\mu_A+\mu_B)}.$$

Substituting the expression for L_A from (42) into this equation shows after simple algebra that $F(g, L_B, \sigma) = [g/(\gamma_B L_B)]^{\mu_A/\mu_B}$, which implies that $g = \bar{g}$. Furthermore, because of $A_0 < B_0$, $\mu_A > 0$, and $\mu_B > 0$, it follows from (40) that $\gamma_A L_A < g < \gamma_B L_B$. Substituting the expression for L_A from (42) into the first of these inequalities yields $F(g, L_B, \sigma) < 1$. Because $F(g, L_B, \sigma)$ is strictly decreasing with respect to g and because $g < \gamma_B L_B$ this implies $H(L_B, \sigma) < 1$ or, equivalently, $L_B > \bar{L}_B$. This completes the proof of the second line in (23). The fact that equation (24) holds is obvious in

the case $A_0 = B_0$ and follows from (40) in the case $A_0 < B_0$. To complete the proof of part (a) we just note that (21) and (32) imply that $g_Y(t) = g_c(t) = (1-\alpha)g$. Using this in (4), we see that $r = (1-\alpha)g + \rho$.

(b) In order to prove this part of the theorem, we use the equilibrium conditions to construct the equilibrium. Since most of the details of this construction are trivial, we focus on the less obvious steps. First of all, let $g \in [0, +\infty)$ and $L_B \in [0, L)$ be arbitrarily given and consider the numbers L_A, L_X, L_Y, and τ defined by (42). It is straightforward to see that the conditions $L_A \in [0, L)$, $L_X \in [0, L)$, $L_Y \in [0, L)$, and $\tau \in [0, 1)$ hold if and only if (25) is satisfied.

Now we use the values defined by (42) to compute the remaining endogenous variables from equilibrium conditions like (31)-(34). Having done that, we can check whether all remaining equilibrium conditions stated in Definition 1 are also satisfied. This is indeed the case if g satisfies (23). In particular, it follows from (20) and (38) that $g_a(t) = g_Y(t) = (1-\alpha)g$, which shows that (3) holds as an equality if and only if $r > (1-\alpha)g$. The latter, however, is true because of $r = (1-\alpha)g + \rho$.

Proof of Lemma 4

We have already shown in the main text that $\lim_{\mu_B \to 0} = \gamma_B L_B$. To prove the second statement, we observe that it follows from (22) and (24) that $A(t)/B(t) = F(\bar{g}, L_B, \sigma)^{1/\mu_A}$ whenever $L_B > \bar{L}_B$. Now consider the limit as μ_B approaches 0. As has been shown above, it holds that $\bar{g}$ approaches $\gamma_B L_B$ and, hence, it follows that $A(t)/B(t)$ approaches $F(\gamma_B L_B, L_B, \sigma)^{1/\mu_A} = H(L_B, \sigma)^{1/\mu_A}$ which, by the very definition of $\bar{L}_B$ and the monotonicity of $H(L_B, \sigma)$, is strictly smaller than 1 whenever $L_B > \bar{L}_B$.

Proof of Lemma 5

(a) Using (28) and the definition of $\bar{\bar{L}}_B$ it is easy to see that $\bar{L}_B = 0$ is equivalent to $\bar{\bar{L}}_B \leq 0$. Now suppose that $\bar{L}_B > 0$, that is, $H(0, \sigma) > 1$. In this case it follows from the definitions of $\bar{L}_B$ and $\bar{\bar{L}}_B$ that we have

$$F(\gamma_B \bar{L}_B, \bar{L}_B, \sigma) = H(\bar{L}_B, \sigma) \geq 1 = F(0, \bar{\bar{L}}_B, \sigma).$$

Because of the strict monotonicity of $F(g, L_B, \sigma)$ with respect to g and L_B and because of $0 < \gamma_B \bar{L}_B$ it follows from the above inequality that $\bar{L}_B < \bar{\bar{L}}_B$. This completes the proof of part (a).

(b) Because of the definitions of $F(g, L_B, \sigma)$ and $\bar{\bar{L}}_B$ we can write (29) as

$$\bar{g} = \begin{cases} \bar{\bar{g}} & \text{if } L_B < \bar{\bar{L}}_B, \\ 0 & \text{if } L_B \geq \bar{\bar{L}}_B. \end{cases}$$

Combining this with (23) and the results stated in part (a) of this lemma we obtain part (b).

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