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# Vote-Share Contracts and Learning-by-Doing\*

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## Abstract

We examine the interaction between vote-share contracts and learning-by-doing. Candidates for a political office are allowed to offer vote-share thresholds. The elected politician has to achieve at least this threshold value in his reelection result to remain in office for a second term. We assume there are learning-by-doing effects for incumbents and show that competition leads to vote-share contracts implementing the socially optimal threshold, which is above one-half. Vote-share contracts improve the average ability level of a reelected politician and increase effort in the first term of an incumbent. On the other hand, vote-share contracts reduce the probability that learning-by-doing takes place. However, the overall effect of vote-share contracts is welfare-enhancing, even under the assumption of learning-by-doing.

Keywords: elections, political contracts, vote-share thresholds, learning-by-doing effects, incumbency advantage.

JEL Classification: D7; D82; H4

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# 1 Introduction

There exists a large empirical evidence for above-average success of incumbents in reelection results.<sup>1</sup> Such incumbency advantages will be socially detrimental if they lead to the reelection of a politician with ability below average. On the other hand, there also exists an incumbency advantage beneficial for society: The learning-by-doing effects while being in office may result in higher ability and/or higher effort of the incumbent in the next period.

In this paper, we propose that candidates who compete for office should be allowed to offer vote-share contracts that diminish the incumbency advantage. Such a contract contains a vote-share threshold which may be above one-half or equal to one-half.<sup>2</sup> In the next reelection, the elected politician has to achieve this threshold value at least to stay in office for a further term. Thus, by increasing the reelection hurdle vote-share contracts are an instrument to eliminate welfare-reducing incumbency advantages. We show that this positive aspect of vote-share contracts is not outweighed by the negative aspect brought up by the diminution of socially beneficial learning-by-doing effects.

A higher reelection hurdle than one-half might have several effects. The incumbent might exert higher effort in order to improve his reelection chances. On the other hand, a high vote-share threshold might result in a lower effort choice of an incumbent if his reelection chances get too small. Furthermore, vote-share contracts might reduce the reelection probability of the incumbent. This might be beneficial if the reelection chances of an incumbent with an ability level below average get smaller. On the other hand, a lower reelection probability would be socially detrimental if a high vote-share threshold causes the deselection of an incumbent with ability above average. Finally, a lower reelection probability caused by vote-share contracts means that socially beneficial learning-by-doing effects will occur less often.

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<sup>1</sup>See e.g. Ansolabehere and Snyder (2002), who document advantages for state executives and legislators in the United States. Moreover, it is shown in their paper that incumbency advantages have become more influential over time.

<sup>2</sup>Offering a vote-share contract with a vote-share threshold of exactly one-half would be equivalent to the usually applied system of majority voting.

We use a simple two-period model in order to show that competition for vote-share contracts induces the candidates to offer voluntarily vote-share thresholds above one-half. If learning-by-doing effects are rather small, the introduction of vote-share contracts increases overall effort of the incumbent in our model. If learning-by-doing effects are rather large, introducing vote-share contracts increases the expected ability of the office-holder in the second period. Total welfare is increasing for all sizes of learning-by-doing effects if vote-share contracts are introduced.

The remainder of the paper is organized as follows: In the next section we present the related literature. We introduce our basic model in section 3. In section 4 we derive the results in the benchmark case with elections only. We analyze the results in a scenario with vote-share contracts and elections in section 5. In section 6 we examine the effect of vote-share contracts on public welfare. Section 7 contains several extensions of our basic model. Finally, section 8 concludes. The Appendix contains the proofs.

## 2 Related Literature

The paper most closely related to this one is Gersbach (2007), where the concept of vote-share contracts is proposed to alleviate negative aspects of incumbency advantage. In contrast to our paper, incumbency advantages have solely negative effects in Gersbach (2007). We show in our paper that the welfare-enhancing effect of vote-share contracts still holds when incumbency advantages also have positive aspects.

Vote-share contracts are one particular type of political contracts, which are verifiable election promises associated with remunerations or sanctions, depending on whether these promises are kept or not. Political contracts have to be certified by an independent body as they are subordinated to the rules of liberal democracy, i.e. political contracts have to be consistent with the fundamental values of democracy. There is some recent literature on political contracts (see e.g. Gersbach (2004a), Gersbach (2005), Gersbach and Müller (2006), and Gersbach and Liessem (2008)). Note that vote-share contracts feature one great advantage, compared to other types of political

contracts: They allow to tackle multi-task problems as success or failure of the incumbent depends not only on the tasks recorded in contract, but on overall performance which is crucial for reaching the vote-share threshold.

Vote-share contracts may also be seen as a special form of flexible majority rule. This concept is introduced in Gersbach (2004b). Under a flexible majority rule, the required majority depends on the voting issue. However, the majority threshold under a flexible majority rule is set by an institution not involved in the voting process, whereas in the case of vote-share contracts, the threshold value is proposed by the politicians running for office. Nevertheless, vote-share contracts are positioned in the interface of political contracts and flexible majority rules.

Finally, there exists a large literature on above-average success of office-holders in reelection results.<sup>3</sup> Several reasons are proposed in the literature for the existence of incumbency advantages.

First, we look at the case where candidates are *ex ante* homogeneous (i.e. before one of them is in office) and differ only with regard to their being incumbent or new candidate. Many advantages of incumbents may accrue from having already been in office: Campaigns are less expensive for incumbents, e.g. due to greater media coverage and face-recognition effects (Ansolabehere, Snyder and Stewart (2000) and Prior (2006)), and in addition incumbents may have better access to campaign funds (Gerber (1988)). Next, vote decisions may be influenced by the endorsement from respected elites, to which office-holders have easier access (Grossman and Helpman (1999)). Furthermore, incumbents may increase their reelection chances by providing constituency services (Cain, Ferejohn and Fiorina (1987)) and socially costly actions (Rogoff and Sibert (1988), Alesina and Cukierman (1990), Hess and Orphanides (1995, 2001), and Cukierman and Tommasi (1998)). Finally, the incumbent may have increased his value for society by improving his skills during his time in office. This process of enlarging

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<sup>3</sup>See e.g. Ashworth and Bueno de Mesquita (2008) or Gordon and Landa (2009) for a recent discussion of the literature.

capabilities is explained by learning-by-doing effects (Arrow (1962)).<sup>4</sup> Similarly, incumbents may have a larger value for the inhabitants of their district, due to the principle of seniority, which means that the agenda-setting power of politicians increases with experience (McKelvey and Riezman (1992)). Thus, the incumbent will tend to be more successful in reaping benefits for his district and – as rational voters anticipate this effect – he will have higher reelection chances. Note that while learning-by-doing effects for politicians are assumed to be beneficial for the whole society, the effects of seniority will normally be beneficial only for a specific part of the population, while they will tend to be detrimental for society as a whole.<sup>5</sup>

Second, there may be further forms of incumbency advantage when politicians are *ex ante* heterogeneous, which means that they differ beyond the dimension of their being incumbents or new candidates. There may be an *ex ante* quality difference and incumbents may have higher quality due to electoral selection (Samuelson (1984), Londregan and Romer (1993), Banks and Sundaram (1998), Zaller(1998), Ashworth (2005), Diermeier, Keane and Merlo (2005), and Ashworth and Bueno de Mesquita (2008)), or politicians may differ with respect to their ideological position. Then there might exist a pro-incumbent district partisan bias, as the ideological characteristics of a district will be relatively stable and hence, the incumbent may have an advantage against challengers with a different ideological position (Gelman and King (1990)).

Third, even for candidates being equal in their expectation value, there may exist an incumbency advantage, as risk-averse voters may prefer the incumbent to an unknown challenger (Bernhardt and Ingberman (1985), Anderson and Glomm (1992)).

Finally, there may be a further reason for the higher reelection probability of incumbents, which stems from strategic entry into the election game: Challengers may be deterred from running against incumbents with high quality (Cox and Katz (1996),

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<sup>4</sup>Arrow does not explicitly consider learning-by-doing effects of politicians, but analyzes the economic implications of learning-by-doing in general. However, his view that “Learning is the product of experience (...) and therefore only takes place during activity” is the same perspective of learning-by-doing than the one we have in mind in this paper.

<sup>5</sup>Under this perspective, the seniority argument seems to be related to the incumbency advantage accruing from providing constituency services and socially costly actions.

Stone, Maisel and Maestas (2004), and Gordon, Huber and Landa (2007)), while incumbents facing a likely defeat may retire strategically (Jacobson and Kernell (1983)).

In this paper, we will only model two forms of incumbency advantage, namely socially costly actions and socially beneficial learning-by-doing effects. Nevertheless, as our model includes positive and negative aspects of incumbency advantages this way of proceeding allows to analyze the welfare effects of introducing vote-share contracts.

### 3 The Model

The model draws on Gersbach (2007). We use a similar notation to allow an easy comparison of the results. There are two periods, denoted by  $t = 1, 2$ .

#### 3.1 Agents and Incumbency Advantage

We assume that there is a continuum of voters indexed by  $i \in [0, 1]$ . Voters elect one politician at the beginning of each period. There are two candidates, denoted by  $k$  or  $k' \in [L, R]$ , where  $L$  ( $R$ ) is a left (right) wing politician. At both election dates, the same two candidates compete for office. The victorious candidate of the first election may have a twofold incumbency advantage at the second election date:

- Due to learning-by-doing effects, the incumbent is of higher competence, which is a socially beneficial kind of incumbency advantage.
- An incumbent  $k$  may shift some part of the output to period 2, which will only be realized if he is still in office in  $t = 2$ . This type of incumbency advantage is socially detrimental for two reasons in our model frame. If there is a new officeholder  $k'$  in  $t = 2$ , then the shifted amount of output is totally lost. If  $k$  is still in office in  $t = 2$ , we assume that some part of the shifted output will get lost.

## 3.2 Policies and Utilities

The elected politician has to decide on three policies.

- Ideological Policy:  $I$

The incumbent decides in both periods on a one-dimensional ideological policy  $I \in [0, 1]$ . Voters are ordered according to their ideal points such that  $i$  is the ideal point of voter  $i$  who is affected by  $I$  with disutility  $-(i_{kt} - i)^2$ , where  $i_{kt}$  is the platform chosen by policy-maker  $k$  in period  $t$ .

- Public Project:  $P$

The incumbent  $k$  undertakes a public project in both periods. We use  $g_t$  to denote the amount of project provided in period  $t$ . We assume that all voters are homogeneous concerning the utility from the project and that  $g_t$  is given as

$$g_t = \gamma(e_{kt} + a_k), \gamma > 0, \quad (1)$$

where  $a_k$  denotes the ability of  $k$ , which is a random variable distributed uniformly on  $[-A, A]$  with  $A > 0$ , while  $e_{kt}$  stands for the effort exerted by  $k$  in period  $t$ . The incumbent incurs costs of  $C(e_{kt})$  from exerting effort.  $C(e_{kt}) = c_h e_{kt}^2$  if he is in his first term in office and  $C(e_{kt}) = c_l e_{kt}^2$  during the second term. We assume that  $c_h > c_l > 0$ , i.e. there are *learning-by-doing* effects.<sup>6</sup> As the amount of public project depends on the effort and thus on the effort costs, there may be different policy outcomes, depending on whether the incumbent is in his first or his second term in office. We will denote the amount of public project provided in the first term by  $g_t(c_h)$  and the amount provided in the second term by  $g_t(c_l)$ .

- Output-shift Policy:  $O$

The incumbent can shift the realization of a fixed amount  $\Delta > 0$  of the output from period  $t = 1$  to period  $t = 2$  if  $\gamma(e_{k1} + a_k) > \Delta$ . If the incumbent shifts  $\Delta$

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<sup>6</sup>Alternatively, one could model learning-by-doing effects by modifying the ability parameter  $a_k$ , such that  $a_k$  increases when a politician enters his second term in office. Using this model framework instead of our approach using a high/low effort-cost parameter would not change the results.

to the next period, then the realized output in period  $t = 1$  will be reduced to  $\gamma(e_{k1} + a_k) - \Delta$ . If the politician is still in office in  $t = 2$ , output  $f\Delta$  ( $0 < f < 1$ ) is realized in  $t = 2$  due to the output shift, while in the case of a new office-holder in  $t = 2$ , the shifted output is totally lost. We use  $\epsilon_k$  to denote the output-shift decision of candidate  $k$  with  $\epsilon_k = 1$  if policy-maker  $k$  shifts output in period 1, and  $\epsilon_k = 0$  otherwise. Policy option  $O$  represents a policy that requires relation-specific investments and enables the incumbent to determine the time of output realization. Examples from the executive branch are international treaties or foreign policy that require policy-specific human capital that may be lost in the case of a government change. The output-shift option is a simple means to generate a socially detrimental aspect of incumbency advantage.

To simplify the analysis, we assume that voters and politicians have a discount factor equal to 1.<sup>7</sup> We use  $V_i(\cdot, \cdot)$  to denote the lifetime utility of voter  $i$  depending on who is in office in  $t = 1$  and  $t = 2$ . For example,  $V_i(L, R)$  denotes lifetime utility of  $i$ , given that  $L$  holds office in  $t = 1$ , while  $R$  is incumbent in  $t = 2$ . There are four cases:

- $V_i(R, R) = g_1(c_h) - \epsilon_R\Delta - (i_{R1} - i)^2 + g_2(c_l) + \epsilon_R f\Delta - (i_{R2} - i)^2$ ,
- $V_i(R, L) = g_1(c_h) - \epsilon_R\Delta - (i_{R1} - i)^2 + g_2(c_h) - (i_{L2} - i)^2$ ,
- $V_i(L, R) = g_1(c_h) - \epsilon_L\Delta - (i_{L1} - i)^2 + g_2(c_h) - (i_{R2} - i)^2$ ,
- $V_i(L, L) = g_1(c_h) - \epsilon_L\Delta - (i_{L1} - i)^2 + g_2(c_l) + \epsilon_L f\Delta - (i_{L2} - i)^2$ .

The candidates derive utility from two sources:

- Benefits from policies

Politicians derive the same benefits from policies  $I$  and  $P$  as ordinary voters. Candidate  $R$ 's most preferred point with regard to policy  $I$  is denoted by  $\mu_R$  with  $\mu_R > \frac{1}{2}$ . For ease of exposition, we assume that the candidates' ideal points

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<sup>7</sup>The extension to a discount factor smaller than 1 is straightforward.

are symmetrically distributed around the median voter's ideal point which is located at one-half.<sup>8</sup> Thus, candidate  $L$ 's ideal point  $\mu_L$  is given by

$$\mu_L = 1 - \mu_R. \quad (2)$$

- Office-holding

The incumbent derives private benefits  $b$  from holding office, including his wage, but also non-monetary benefits like prestige or the satisfaction of being in power.

We use  $V_k(\cdot, \cdot)$  to denote politician  $k$ 's lifetime utility depending on who is in office in  $t = 1$  and  $t = 2$ . We look at politician  $R$ , for example, and have to distinguish four cases again:

- $V_R(R, R) = b - (i_{R1} - \mu_R)^2 - c_h e_{R1}^2 + g_1(c_h) - \epsilon_R \Delta + b - (i_{R2} - \mu_R)^2 - c_l e_{R2}^2 + g_2(c_l) + \epsilon_R f \Delta,$
- $V_R(R, L) = b - (i_{R1} - \mu_R)^2 - c_h e_{R1}^2 + g_1(c_h) - \epsilon_R \Delta - (i_{L2} - \mu_R)^2 + g_2(c_h),$
- $V_R(L, R) = -(i_{L1} - \mu_R)^2 + g_1(c_h) - \epsilon_L \Delta + b - (i_{R2} - \mu_R)^2 - c_h e_{R2}^2 + g_2(c_h),$
- $V_R(L, L) = -(i_{L1} - \mu_R)^2 + g_1(c_h) - \epsilon_L \Delta - (i_{L2} - \mu_R)^2 + g_2(c_l) + \epsilon_L f \Delta.$

### 3.3 Assumptions and Equilibrium Concept

Candidates are assumed not to be able to commit to a particular policy platform during the election campaign. The incumbent will learn his own ability  $a_k$  after he has exerted  $e_{kt}$ . Voters observe output  $g_t$ , but as ability is assumed to remain private information of the incumbent, voters are not able to distinguish how much of  $g_t$  is due to effort and how much due to ability.<sup>9</sup> Since output  $g_t$  is not contractible, it cannot be used

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<sup>8</sup>Note that there is no incumbency advantage due to ideological positions in our paper, although candidates and voters differ in their ideological opinion. However, as both candidates are symmetric to the median voter with regard to their ideological position, there is no pro-incumbent district partisan bias in our model

<sup>9</sup>This assumption is in line with Alesina and Tabellini (2007). However, note that although voters are not able to *observe* the composition between effort and ability, they may nevertheless be able to *infer* it in equilibrium.

to generate performance-related wages for politicians. The incumbent is assumed to observe  $a_k$  before he has to make his decision about shifting output. Thus, he can make  $\epsilon_k$  dependent on  $a_k$ . We assume that voters observe the outcome of policies  $I$  and  $O$  and that they vote sincerely, i.e. they vote for the candidate who generates a higher expected utility.<sup>10</sup> Furthermore, we assume  $\frac{\gamma(c_h - c_l)}{2c_h c_l} + \frac{f\Delta}{\gamma} < A$  to ensure that reelection probability in equilibrium is smaller than 1. Finally, we make the assumption that  $b$  is so large that candidates will always prefer to be in office. We are looking for perfect Bayesian Nash equilibria of the game.

### 3.4 The Overall Game

We summarize the overall game in the following figure:

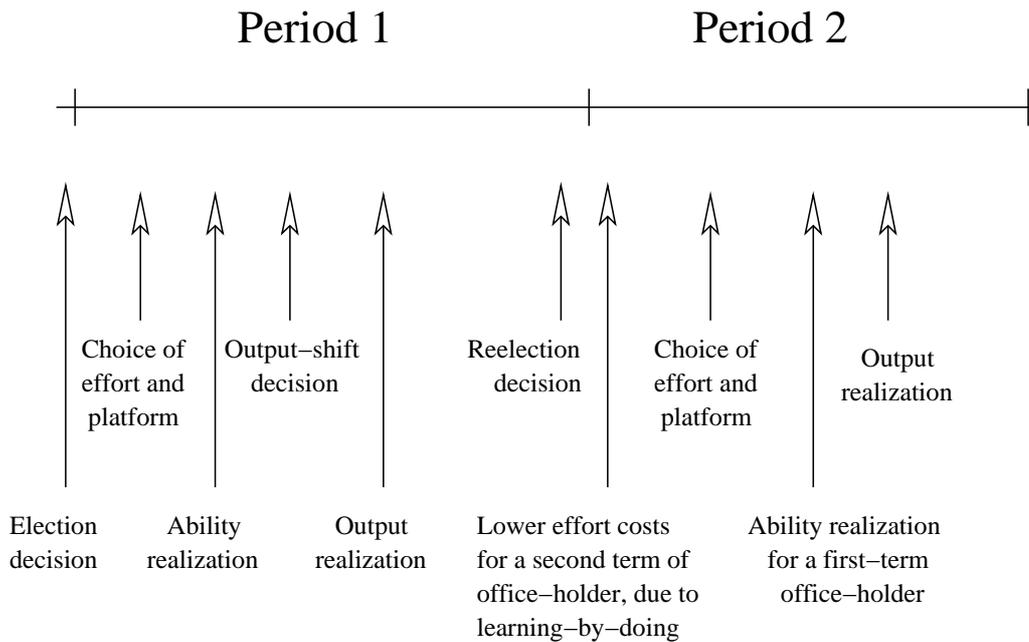


Figure 1: Time-line with elections alone

<sup>10</sup>Although the individual voter has no influence on the election outcome in the case of a continuum of voters, it is optimal for the electorate to vote sincerely, as this is the only sensible strategy for rational voters in a two-party system (see e.g. Austen-Smith (1989)).

## 4 Elections Alone

In this section, we examine the standard case without vote-share contracts and restate the results of Gersbach (2007) in the context of our model frame. We assume that the candidate with more votes is elected. If both candidates obtain the same share of votes, the probability of each candidate winning is  $\frac{1}{2}$  in  $t = 1$ , while we assume that the incumbent is reelected in the case of a tie in period two.

### 4.1 The Second Period

As candidates cannot commit to policy platforms, a policy-maker will choose his most preferred platform in  $t = 2$ . The amount of public project in  $t = 2$  depends on whether the policy-maker is in his first term (where he does not know his ability  $a_k$  and where he has a high effort-cost parameter  $c_h$ ), or in his second term (where he knows his ability  $a_k$  and where he has a low cost parameter  $c_l$ ). In the Appendix, we show:

#### Proposition 1

*Suppose, e.g., that candidate R is elected as office-holder for period 2. Then*

(i) *he will choose*

$$\alpha) \quad i_{R2}^* = \mu_R \quad \text{and} \quad e_{R2}^* = \frac{\gamma}{2c_l} \quad \text{if } R \text{ has been in office in the first period;}$$

$$\beta) \quad i_{R2}^* = \mu_R \quad \text{and} \quad e_{R2}^* = \frac{\gamma}{2c_h} \quad \text{if } R \text{ has not been in office in the first period;}$$

(ii) *the expected utility of R at the beginning of period 2 is given by*

$$\alpha) \quad V_{R2}^*(R, R) = b + \frac{\gamma^2}{4c_l} + \gamma a_R + \epsilon_R f \Delta,$$

$$\beta) \quad V_{R2}^*(L, R) = b + \frac{\gamma^2}{4c_h}.$$

### 4.2 The First Period

Both candidates win with probability  $\frac{1}{2}$  in the first election, as the median voter is indifferent between them. Without loss of generality, we will assume throughout the

remaining part of the paper that  $R$  is elected in the first election. We obtain the following Fact, which holds in every equilibrium with pure strategies:

**Fact 1**

*Suppose that candidate  $R$  is elected as office-holder for period 1. Then*

- (i) *he will choose  $i_{R1} = \mu_R$  for policy  $I$ ;*
- (ii) *voters will perfectly infer his ability  $a_R$  at the end of period 1.*

Politician  $R$  cannot gain more votes in the second election by choosing  $i_{R1} \neq \mu_R$ , as voters know that he will choose his ideal point in period 2 anyway. Part (ii) follows from the informational structure of the game. As the incumbent does not observe his ability before he exerts effort, in any pure strategy equilibrium, exactly one level of effort will be chosen, and expected by the voters. Any deviation of  $g_t$  from the expected effort times  $\gamma$  will be interpreted correctly as variation in ability, since  $a_R = \frac{g_t - \gamma \hat{e}_1}{\gamma}$ , where  $\hat{e}_1$  denotes the public's expectations about the incumbent's effort level in  $t = 1$ .

Now we want to derive the optimal behavior of the officeholder concerning  $P$  and  $O$  in  $t = 1$ . First, there may occur three cases regarding  $a_R$ :

- (i) Candidate  $R$ 's ability may be so high that he will be reelected even without output-shift policy. Then he will choose  $\epsilon_R = 0$  and will be reelected. We use  $p^0(e_{R1}, \hat{e}_1)$  to denote the probability the officeholder assigns to this eventuality.
- (ii) The incumbent may have an intermediate level of ability where he will be reelected only if he shifts output ( $\epsilon_R = 1$ ). As  $b$  is sufficiently high, the officeholder will choose the socially detrimental option  $\epsilon_R = 1$ , which implies reelection. We use  $p^1(e_{R1}, \hat{e}_1)$  to denote the incumbent's estimate of the probability of this case.
- (iii) If  $R$ 's ability is so low that he will never be reelected, irrespective of  $\epsilon_R$ , then he will choose  $\epsilon_R = 0$ . The probability of this case is  $1 - p^0(e_{R1}, \hat{e}_1) - p^1(e_{R1}, \hat{e}_1)$ .

Second, we introduce  $\tilde{a}_R(e_{R1}, \hat{e}_1)$  as candidate  $R$ 's expected level of ability, conditional on the fact that he is reelected, and show in the Appendix:

**Fact 2**

$$p^0(e_{R1}, \hat{e}_1) = \frac{1}{2} + \frac{1}{2A} \left( e_{R1} - \hat{e}_1 + \frac{\gamma(c_h - c_l)}{2c_h c_l} \right), \quad (3)$$

$$p^1(e_{R1}, \hat{e}_1) = \frac{f\Delta}{2A\gamma}, \quad (4)$$

$$\tilde{a}_R(e_{R1}, \hat{e}_1) = \frac{A + \hat{e}_1 - e_{R1}}{2} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l}. \quad (5)$$

Note that the probability of  $R$  being reelected, i.e.  $p^0(e_{R1}, \hat{e}_1) + p^1(e_{R1}, \hat{e}_1)$ , increases in  $e_{R1}$ , as for a given expectation  $\hat{e}_1$ , the incumbent can improve the public's estimate of his ability by exerting more effort. A more favorable evaluation of his ability increases his reelection chances. However, the expected level of  $R$ 's ability, contingent on the fact of his reelection, decreases with  $e_{R1}$  as an increase in  $e_{R1}$  implies that  $R$  will be reelected even for lower levels of ability. Thus,  $\tilde{a}_R(e_{R1}, \hat{e}_1)$  is lowered.

Finally, the incumbent's optimization problem can be stated in the following way<sup>11</sup>:

$$\begin{aligned} \max_{e_{R1} \geq 0} & \left\{ b + \gamma e_{R1} - c_h e_{R1}^2 - p^1(e_{R1}, \hat{e}_1) \Delta (1 - f) \right. \\ & + (p^0(e_{R1}, \hat{e}_1) + p^1(e_{R1}, \hat{e}_1)) \left( b + \gamma \left( \frac{\gamma}{2c_l} + \tilde{a}_R(e_{R1}, \hat{e}_1) \right) - \frac{\gamma^2}{4c_l} \right) \\ & \left. + (1 - p^0(e_{R1}, \hat{e}_1) - p^1(e_{R1}, \hat{e}_1)) \left( \frac{\gamma^2}{2c_h} - (\mu_R - \mu_L)^2 \right) \right\} \end{aligned} \quad (6)$$

In the Appendix, we show:

**Proposition 2**

(i)  $R$  chooses  $e_{R1}^* = \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 \right] \right\}$ .

(ii)  $R$  chooses  $\epsilon_R = 0$  and is reelected with probability

$$p^0(e_{R1}^*, e_{R1}^*) = \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l}. \quad (7)$$

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<sup>11</sup>Note that  $R$  is reelected with probability  $p^0(e_{R1}, \hat{e}_1) + p^1(e_{R1}, \hat{e}_1)$ , while  $L$  becomes the new incumbent with probability  $1 - p^0(e_{R1}, \hat{e}_1) - p^1(e_{R1}, \hat{e}_1)$ . With probability  $p^1(e_{R1}, \hat{e}_1)$ , net losses  $\Delta(1 - f)$  occur due to output-shift policies.

(iii)  $R$  chooses  $\epsilon_R = 1$  and is reelected with probability

$$p^1(e_{R1}^*, e_{R1}^*) = \frac{f\Delta}{2A\gamma}. \quad (8)$$

(iv) The average ability level of a reelected candidate is given by

$$\tilde{a}_R(e_{R1}^*, e_{R1}^*) = \frac{A}{2} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l}. \quad (9)$$

In part (i) we observe how equilibrium effort  $e_{R1}^*$  depends on the parameters. A deselected politician has utility losses due to the distance from the ideological policy of his opponent to his own ideal point and due to the fact that he has no private benefits from holding office in  $t = 2$ . Thus, the larger  $(\mu_R - \mu_L)$  and  $b$ , the higher the effort the politician is willing to invest. The higher  $A$ , the lower the marginal gain in reelection chances when  $R$  increases effort marginally. Hence, greater uncertainty regarding quality will reduce effort. The higher  $c_h$ , the lower  $e_{R1}^*$ , as exerting more effort in period 1 gets more costly for the politician. The higher  $c_l$ , the lower the learning-by-doing effects. This decreases the incumbency advantage and thus, the effort exerted in the first period is higher. The impact of  $\gamma$  is more subtle. On the one hand, higher  $\gamma$  increases the marginal value of higher effort today and the value of office tomorrow, which both raise  $e_{R1}^*$ . On the other hand, higher  $\gamma$  results in lower effort, as the utility of the opponent being in office in period 2 and the losses due to an incumbent with lower ability than average being reelected are increasing. Part (ii) of the Proposition reflects the incumbency advantage due to learning-by-doing effects. The probability of the incumbent to get reelected is larger than one-half, even without shifting output. Part (iii) reflects the additional incumbency advantage due to the output-shift policy. In the last step, we want to analyze the inefficiencies of the equilibrium results in the first period. In the Appendix, we show:

**Fact 3**

*From the voters' point of view, candidate  $R$  should be reelected if and only if*

$$a_R \geq \frac{\gamma(c_l - c_h)}{2c_h c_l}. \quad (10)$$

Thus, the socially optimal average ability level of a reelected candidate would be

$$a_R^* = \frac{A}{2} - \frac{\gamma(c_h - c_l)}{4c_h c_l}. \quad (11)$$

Thus, Proposition 2 reveals two types of inefficiency. First, with probability  $p^1(e_{R1}^*, e_{R1}^*)$ , incumbents with an ability level below average shift output to ensure reelection. Second, the average ability level of a reelected politician  $\tilde{a}_R(e_{R1}^*, e_{R1}^*)$  from equation (9) is lower than the socially optimal average ability level from equation (11). In the following, we will see that vote-share contracts can alleviate the second type of inefficiency and can additionally increase the effort of the incumbent in period 1. However, the probability of the socially wasteful output-shift does not change if vote-share contracts are introduced.

## 5 Results with Vote-Share Contracts

In this section, we examine the combination of democratic elections and vote-share contracts, and restate the results of Gersbach (2007) in the context of our model frame.

### 5.1 Vote-Share Thresholds as Political Contracts

Each candidate  $k$  is allowed to offer a vote-share contract, which occurs by stipulating a vote-share threshold  $s_k$  with  $\frac{1}{2} \leq s_k \leq 1$ . If politician  $k$  takes over office in  $t = 1$ , he must win a share of votes at least equal to  $s_k$  at the next election date to remain in office. Otherwise, the challenger  $k'$  will take office. Hence, the incumbent faces a self-imposed vote-share threshold in the election at the end of period 1. Throughout the section, we assume that  $2\mu_R - 1 < A\gamma$ , which ensures interior solutions.<sup>12</sup> To give a short summary, we display the two additions in the extended game, in comparison to our basic model, in the following figure:

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<sup>12</sup>Note that for  $2\mu_R - 1 > A\gamma$ , the incumbent may renounce exerting high effort, as his reelection chances are too low if vote share thresholds are high.

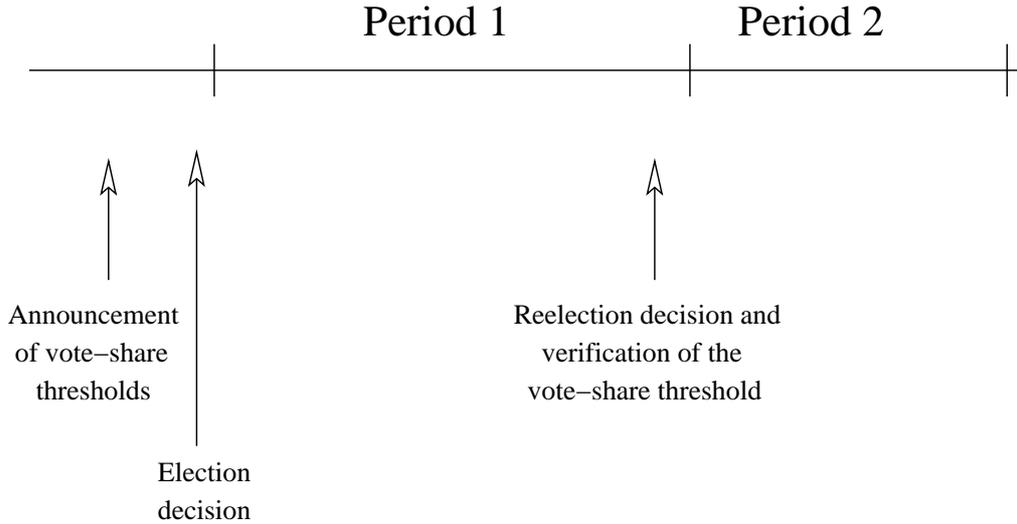


Figure 2: Changes in the time-line with elections and vote-share contracts

The first change is the additional step where both candidates are allowed to offer vote-share contracts before the first election decision takes place. Secondly, the result of the reelection decision is used to check whether the office-holder has been successful in reaching his vote-share threshold. The rest of the time-line is the same as in Figure 1 and is omitted here for the sake of clarity.

## 5.2 The Second and First Period

We denote the results in the scenario with vote-share contracts by the superscript  $V$  and assume without loss of generalization that  $R$  is elected in  $t = 1$  with a vote-share threshold  $s_R \geq \frac{1}{2}$ . In the second period,  $R$  will choose  $i_{R2}^{*V} = \mu_R$  and  $e_{R2}^{*V} = \frac{\gamma}{2c_l}$  if he is still in office, while a new office-holder  $L$  will choose  $i_{L2}^{*V} = \mu_L$  and  $e_{L2}^{*V} = \frac{\gamma}{2c_h}$ . Thus, the results will remain the same as in Proposition 1. In the Appendix, we show that equations (3), (4), and (5) have to be modified to:

### Fact 4

$$p^{0V}(e_{R1}^V, \hat{e}_1^V) = \frac{1}{2} + \frac{1}{2A} \left( e_{R1}^V - \hat{e}_1^V + \frac{\gamma(c_h - c_l)}{2c_h c_l} - \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) \right), \quad (12)$$

$$p^{1V}(e_{R1}^V, \hat{e}_1^V) = \frac{f\Delta}{2A\gamma}, \quad (13)$$

$$\tilde{a}_R^V(e_{R1}^V, \hat{e}_1^V) = \frac{A + \hat{e}_1^V - e_{R1}^V}{2} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l} + \frac{(2\mu_R - 1)(2s_R - 1)}{2\gamma}. \quad (14)$$

These equations coincide with equations (3), (4) and (5) for  $s_R = \frac{1}{2}$ . The optimal choice of  $e_{R1}^V$  is obtained by solving the optimization problem (6), where  $p^0(e_{R1}, \hat{e}_1)$ ,  $p^1(e_{R1}, \hat{e}_1)$  and  $\tilde{a}_R(e_{R1}, \hat{e}_1)$  are replaced by  $p^{0V}(e_{R1}^V, \hat{e}_1^V)$ ,  $p^{1V}(e_{R1}^V, \hat{e}_1^V)$  and  $\tilde{a}_R^V(e_{R1}^V, \hat{e}_1^V)$  from equations (12), (13), and (14), respectively. In the Appendix, we show:

**Proposition 3**

(i)  $e_{R1}^{*V} = \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 + (2\mu_R - 1)(2s_R - 1) \right] \right\}$

(ii)  $R$  chooses  $\epsilon_R = 0$  and is reelected with probability

$$p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) = \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} - \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma}. \quad (15)$$

(iii)  $R$  chooses  $\epsilon_R = 1$  and is reelected with probability

$$p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) = \frac{f\Delta}{2A\gamma}. \quad (16)$$

(iv) The average ability level of a reelected candidate is given by

$$\tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V}) = \frac{A}{2} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l} + \frac{(2\mu_R - 1)(2s_R - 1)}{2\gamma}. \quad (17)$$

From part (i), we observe that for  $s_R > \frac{1}{2}$ , the equilibrium effort level is higher, compared to elections only. The intuition is that the marginal gain from higher effort is increasing with a higher vote-share threshold. From part (iii), we learn that the probability for output-shift to occur is not reduced by the introduction of vote-share contracts. This is due to the fact that incumbents who learn that they have an ability level where they will reach their reelection hurdle if and only if they shift output will still choose the output-shift option. This intermediate ability range where the output-shift option is chosen will be shifted compared to the scenario without vote-share contracts. However, due to our assumption that ability is a uniformly distributed random variable, the probability for output-shift to occur will not change. Finally, note that the average

ability level of a reelected candidate, given by  $\tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V})$ , is increasing in  $s_R$ . Thus, larger vote shares increase the average ability of reelected incumbents.

### 5.3 Competition for Vote-Share Contracts

Now, we consider the initial stage when both candidates compete for office with vote-share contracts. The ex ante optimal vote-share threshold from the perspective of the median voter is denoted by  $s^*$  and is the solution of the following problem:

$$\max_{\frac{1}{2} \leq s_R \leq 1} \left\{ \gamma e_{R1}^{*V} + \left( p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) + p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right) \left( \gamma \tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V}) + \gamma e_{R2}^{*V} \right) + \left( 1 - p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) - p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right) \left( \gamma a_L^V + \gamma e_{L2}^{*V} \right) \right\}, \quad (18)$$

where  $e_{L2}^{*V}$  denotes the second-term equilibrium effort of the left-wing candidate if he is new in office in  $t = 2$ . We obtain

**Fact 5**

$$s^* = \min \left\{ \frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)} + \frac{\gamma^2}{4c_h(2\mu_R - 1)}; 1 \right\}. \quad (19)$$

The Fact is proven in the Appendix. Note that the value of  $s^*$  is decreasing in  $c_h$ . A larger value of  $c_h$  means, *ceteris paribus*, that the learning-by-doing effects are larger. Thus, voters are more interested in having the same politician in office during both periods and therefore the optimal vote-share, from the perspective of the median voter, is lower. However, even for  $c_h \rightarrow \infty$ , the value of  $s^*$  is larger than  $\frac{1}{2}$ , which means that even for extremely large positive aspects of incumbency advantage, a vote-share hurdle higher than one-half is socially optimal. In the Appendix, we show the following Proposition:

**Proposition 4**

(i) Both candidates  $R$  and  $L$  offer  $s^*$ . The probability of winning the first election is one-half for each candidate.

(ii)  $s^* > \frac{1}{2}$

(iii)  $s^*$  is the ex ante optimal vote-share.

From part (i) of the Proposition, we see that both candidates will offer exactly  $s^*$ .<sup>13</sup> Part (ii) of the Proposition is a first evidence for the fact that the introduction of vote-share contracts is at least not welfare-reducing from the perspective of the median voter, as otherwise, he would choose  $s^* = \frac{1}{2}$ . We will discuss the welfare effects of vote-share contracts in detail later. Part (iii) of the Proposition shows that the optimal vote-share threshold, from the perspective of the median voter, is also socially optimal.

Finally, we use  $\tilde{s}$  to denote the vote-share threshold which ensures that the incumbent will be reelected if and only if his ability is equal to or greater than  $\frac{\gamma(c_l - c_h)}{2c_h c_l}$ . Remember that this is the socially optimal lower ability boarder from equation (10) in Fact 3. By inserting  $a_R = \frac{\gamma(c_l - c_h)}{2c_h c_l}$  into condition (28), we obtain

$$\tilde{s} = \min \left\{ \frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)}; 1 \right\} \quad (20)$$

This results immediately in the following Corollary:

**Corollary 1**

$$s^* \geq \tilde{s}$$

Thus, the optimal vote-share choice from the perspective of the median voter is larger than the vote-share threshold, which ensures that no politician with ability level below the socially optimal lower boarder has a chance of getting reelected. The reason for this result is as follows: On the one hand, under the higher vote-share threshold  $s^*$ , politicians will be deselected even if their ability is slightly above the socially optimal lower boarder. On the other hand, this negative effect is outweighed by the positive effect of the larger threshold value  $s^*$  on effort. The interaction of these contrarian forces determines the optimal value  $s^*$ .

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<sup>13</sup>Note that in contrast to Gersbach and Müller (2006), there will occur no *overpromising* here, as  $s^*$  is the unique optimal point for voters and social welfare is decreasing for higher values than  $s^*$ . Thus, overpromising by offering a threshold above  $s^*$  is not profitable for a candidate, as this would result in the certain election of his opponent.

## 6 Welfare Effects

In this section, we analyze the effect of introducing vote-share contracts on public welfare in detail. We compare welfare in a scenario with and without vote-share contracts. It is intuitively clear that welfare does not change with a threshold  $s_R = \frac{1}{2}$ , since a scenario with a vote-share threshold of one-half is equivalent to the scenario with elections alone. The introduction of vote-share contracts larger than one-half has three effects, as shown in Proposition 3: The effort choice in period 1 increases, the expected ability of a reelected politician increases, while the reelection probability decreases. In the following, we examine how these three effects influence expected effort over both periods, expected ability of the office-holder in the second period and overall welfare.

### 6.1 Effects on Expected Effort

We start by analyzing the effect of introducing vote-share contracts on the expected effort over both periods. We assume that  $R$  is elected in period 1 and use  $E[e_2^*]$  to denote the expected effort of the office-holder in the second period, i.e. the effort of a reelected incumbent, weighted with his probability of being in office, plus the effort of a new office-holder, weighted with his probability of being in office. We define

$$E[e^*] := E[e_{R1}^*] + E[e_2^*] \quad (21)$$

as expected overall effort and obtain:

$$E[e^*] = e_{R1}^* + \frac{\gamma}{2c_l} \left( p^0(e_{R1}^*, e_{R1}^*) + p^1(e_{R1}^*, e_{R1}^*) \right) + \frac{\gamma}{2c_h} \left( 1 - p^0(e_{R1}^*, e_{R1}^*) - p^1(e_{R1}^*, e_{R1}^*) \right).$$

Analogously, we define  $E[e^{*V}]$  as expected effort over both periods in the scenario with vote-share contracts and obtain:

$$E[e^{*V}] = e_{R1}^{*V} + \frac{\gamma}{2c_l} \left( p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) + p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right) + \frac{\gamma}{2c_h} \left( 1 - p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) - p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right).$$

In the Appendix, we show:

**Proposition 5**

(i) *The effect of introducing vote-share contracts on expected overall effort is given by*

$$E[e^{*V}] - E[e^*] = \frac{(2\mu_R - 1)(2s_R - 1)}{4Ac_h c_l} (2c_l - c_h). \quad (22)$$

(ii) *For  $c_h < 2c_l$ , the introduction of vote-share contracts with a threshold value larger than one-half increases the expected effort over both periods.*

(iii) *For  $c_h > 2c_l$  the introduction of vote-share contracts with a threshold value larger than one-half decreases the expected effort over both periods.*

Hence, we learn from Proposition 5 that it depends on the relationship of  $c_h$  and  $c_l$  whether the effect of vote-share contracts on the expected effort over both periods is positive or negative.<sup>14</sup> The intuition for this result is as follows: An increasing spread between  $c_h$  and  $c_l$  is bad for the effect of vote-share contracts on expected overall effort, as the effect of the lower reelection probability under vote-share contracts is weighted more under larger learning-by-doing effects. Furthermore, the increase in the first period effort, under vote-share threshold, is decreasing in  $c_h$ , i.e. a higher value of  $c_h$  reduces the positive effect of vote-share contracts on the effort in period 1.

From equation (22), we see that a vote-share threshold of  $\frac{1}{2}$  has no effect on expected effort and that the (positive or negative) effect of vote-share contracts on expected effort increases with  $s_R$ , i.e. a higher threshold increases the absolute value  $|E[e_R^{*V}] - E[e_R^*]|$ .

**6.2 Effects on Expected Ability in Period 2**

In this subsection, we analyze the effect of vote-share contracts on the expected ability of the office-holder in period 2, given that  $R$  chooses  $e_{R1}^*$  in  $t = 1$ . On the one hand, there is a positive effect of vote-share contracts, as the ability of reelected candidates is raised. On the other hand, there is a negative effect, as the reelection probability

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<sup>14</sup>Note that the result from Proposition 5 would have to be modified under the assumption of a discount factor smaller than 1. Then the introduction of vote-share contracts would have a better effect on expected effort over both periods, as the lower probability of  $\frac{\gamma}{2c_l}$  being the second period effort would obtain a minor weight.

gets smaller. Thus, the probability that a new office-holder with an expected ability of zero comes into office increases. We define the expected ability of the incumbent in period 2, given that  $R$  chooses  $e_{R1}^*$  in  $t = 1$  as:

$$E[a_R(e_{R1}^*, e_{R1}^*)] := E\left[\left(p^0(e_{R1}^*, e_{R1}^*) + p^1(e_{R1}^*, e_{R1}^*)\right) \cdot \tilde{a}_R(e_{R1}^*, e_{R1}^*) + \left(1 - p^0(e_{R1}^*, e_{R1}^*) - p^1(e_{R1}^*, e_{R1}^*)\right) \cdot a_L\right] \quad (23)$$

In an analogical way, we define  $E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V})]$  in the scenario with vote-share contracts as the expected ability of the incumbent in the second period given that  $R$  chooses  $e_{R1}^{*V}$  in the first period. In the Appendix, we show:

**Fact 6**

*The effect of introducing vote-share contracts on the expected ability of the incumbent in the second period, given that the elected politician chooses  $e_{R1}^{*V}$  in the first period, i.e.  $E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V})] - E[a_R(e_{R1}^*, e_{R1}^*)]$ , is given by the following term:*

$$\frac{(2\mu_R - 1)(2s_R - 1)}{4A\gamma^2} \left( \frac{\gamma^2(c_h - c_l)}{c_h c_l} + 2f\Delta - (2\mu_R - 1)(2s_R - 1) \right). \quad (24)$$

Thus, a larger spread  $c_h - c_l$  is positive for the effect of vote-share contracts on expected ability of the incumbent in period 2. There are two intuitive reasons for this result:

- The reelection probability of the incumbent increases if either  $c_l$  decreases for given  $c_h$  or if  $c_h$  increases for given  $c_l$ . Thus, the reelection probability increases with the spread between  $c_h$  and  $c_l$ . The positive effect of vote-share contracts on expected ability, via higher ability of reelected candidates, has relatively more weight if the reelection probability is higher. Hence, a larger spread between  $c_h$  and  $c_l$  increases the expected ability in period 2.
- The ability of a reelected candidate decreases if either  $c_l$  decreases for given  $c_h$  or if  $c_h$  increases for given  $c_l$ . Thus, the ability of a reelected candidate decreases with the spread between  $c_h$  and  $c_l$ . The negative effect of vote-share contracts on expected ability, via lower reelection probability, has relatively less weight if the ability of a reelected candidate is lower. Hence, a larger spread between  $c_h$  and  $c_l$  increases the expected ability in period 2.

From equation (24), we see directly that vote-share contracts with threshold  $s_R = \frac{1}{2}$  have no effect on the expected ability. In the next step, we analyze the effect of vote-share contracts with threshold  $s^*$  on the expected ability of the incumbent in period 2. In the Appendix, we show:

**Proposition 6**

- (i) *For  $c_h > \frac{3}{2}c_l$ , the introduction of vote-share contracts with threshold  $s^*$  certainly increases the expected ability of the incumbent in the second period.*
- (ii) *For  $c_h < \frac{3}{2}c_l$ , the introduction of vote-share contracts with threshold  $s^*$  may decrease the expected ability of the incumbent in the second period.*

Thus, if vote-share contracts with threshold  $s^*$  are applied, the expected ability of the incumbent in  $t = 2$  is certainly higher than under elections alone if  $c_h > \frac{3}{2}c_l$ , i.e., if learning-by-doing effects are not too small. For  $c_l < c_h < \frac{3}{2}c_l$ , it depends on the other parameter values whether vote-share contracts with threshold  $s^*$  have positive or negative effects on the expected ability of the office-holder in period 2. For  $f\Delta$  sufficiently large, the effect of vote-share thresholds on expected ability will certainly be positive. The reason is that a large value of  $f\Delta$  means that the ability of a reelected candidate is smaller than zero and then, the lower reelection probability of the incumbent is socially beneficial, as a new candidate has an expected ability of zero.

### 6.3 Total Welfare Effects

In this subsection, we examine the total welfare effects of vote-share contracts in detail. During this analysis, we will show that vote-share contracts lead to higher welfare than elections alone, a result that could already be seen in Proposition 4. In the following, we summarize all effects of vote-share contracts on overall welfare.

- Effects on welfare via effort:
  - Vote-share contracts induce a higher effort choice in period 1.

- Vote-share contracts reduce the expected effort in period 2, as the probability decreases that the incumbent is reelected. Hence, the probability of  $\frac{\gamma}{2c_l}$  being the second period effort decreases when vote-share contracts are applied.
- Effects on welfare via ability:
  - Vote-share contracts increase the average ability of reelected incumbents.<sup>15</sup>
  - Vote-share contracts reduce the reelection probability of the incumbent, which means that the probability increases that a new incumbent with an expected ability of zero comes into office. This may be positive or negative for society depending on whether the expected ability of the first period office-holder is smaller or larger than zero.

Remember that we showed in Proposition 5 and Proposition 6 that it may depend on the relationship of  $c_h$  and  $c_l$  whether introducing vote-share contracts increases or decreases expected effort and expected ability in period 2. In the following Theorem, we show that the overall effect of introducing vote-share contracts is welfare-enhancing, independent of the relationship of  $c_h$  and  $c_l$ :

**Theorem 1**

- (i) *Welfare under vote-share contracts with a vote-share threshold  $s^*$  is higher than under elections alone.*
- (ii) *The welfare-enhancing effect of vote-share contracts is increasing in  $f$ ,  $\Delta$  and  $\gamma$ , it is decreasing in  $c_h$  and is independent of  $c_l$ .*

Theorem 1 is our main result and is proven in the Appendix. The consequence of the first part is that vote-share contracts lead to higher welfare than standard elections alone. The second part shows how the welfare-enhancing effect of vote-share contracts

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<sup>15</sup>Thus, vote-share contracts alleviate one inefficiency that occurred in the scenario with elections alone. Note, however, that vote-share contracts cannot avoid all inefficiencies, as politicians, once in office, will still shift output across time, which is socially wasteful.

depends on some of the parameters. The intuition for the effect of  $f$  and  $\Delta$  is as follows: The average ability level of a reelected candidate is decreasing in  $f$  and  $\Delta$ . As vote-share contracts decrease the probability of the incumbent to get reelected, the positive effect of introducing vote-share contracts on total welfare is increasing in  $f$  and  $\Delta$ .<sup>16</sup> The reasons for the other dependences in part (ii) are more subtle, as there are many channels by which  $\gamma$ ,  $c_h$  and  $c_l$  get effective on total welfare. These channels work in opposing directions and, in the case of  $c_l$ , just outweigh each other.<sup>17</sup> Details of these subtle dependences are omitted here. Nevertheless, we want to point at the fact that vote-share contracts get, *ceteris paribus*, more effective if learning-by-doing effects are relatively small. This result is intuitive, as the aim of vote-share contracts is to alleviate negative aspects of incumbency advantage. However, thereby, positive aspects of incumbency advantage, i.e. learning-by-doing effects, are also reduced. If the positive aspects are rather small, then vote-share contracts cause less damage in reducing these positive effects and get more effective. Vote-share contracts would be most effective if there was no learning-by-doing at all, i.e. for  $c_h = c_l$ .

As we showed in the previous two subsections, introducing vote-share contracts increases expected overall effort if  $c_h < 2c_l$ , while introducing vote-share contracts may decrease expected ability of the office-holder in period 2 if  $c_h < \frac{3}{2}c_l$ . Hence, there may be a trade-off between the effect on expected overall effort and the effect on expected ability of the office-holder in period 2. However, total welfare effects of introducing vote-share contracts are always positive, independently of  $c_h$  and  $c_l$ . The following four cases may occur, which all yield a positive effect on overall welfare:

- (i) For  $c_l < c_h \leq \frac{3}{2}c_l$ , introducing vote-share contracts increases expected overall effort, while it may decrease expected ability of the office-holder in period 2.

However, if there is an ability-decreasing effect, then it is dominated by the

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<sup>16</sup>This means that vote-share contracts become more effective if the socially detrimental output-shift policies get more harmful. Thus, vote-share contracts mitigate the negative consequences of output-shift policies, although they are not able to decrease the probability that output-shift policies occur.

<sup>17</sup>One reason for the fact that the efficiency of vote-share contracts depends on  $c_h$ , but not on  $c_l$ , might be that  $c_l$  is not contained in the first period of the optimization problem.

effort-increasing effect.

- (ii) For  $\frac{3}{2}c_l < c_h < 2c_l$ , introducing vote-share contracts increases both the expected overall effort and the expected ability of the office-holder in period 2.
- (iii) For  $c_h = 2c_l$ , introducing vote-share contracts has no influence on expected overall effort, while it increases expected ability of the office-holder in period 2.
- (iv) For  $c_h > 2c_l$ , introducing vote-share contracts decreases expected overall effort, while it increases expected ability of the office-holder in period 2. However, the ability-increasing effect dominates the effort-decreasing effect.

## 7 Extensions

In our basic model, we have described the working of vote-share contracts in a simple setup. In the following, we sketch two fruitful extensions that could be pursued to address the robustness of our result, i.e. that using vote-share contracts is welfare-enhancing.

### 7.1 No Output-shift Policy

First, one variant of our model is to assume that the socially wasteful output-shift policy is not available for the incumbent, i.e. to set  $\Delta$  equal to zero. This assumption enables us to analyze potential risks of using vote-share thresholds in the case where the incumbency advantage may only have positive effects on welfare. As one can see from Fact 5 and Theorem 1, under absence of output-shift policy,  $s^*$  will be lower and the welfare-increasing effect of vote-share contracts on welfare will be smaller. However, even for  $\Delta = 0$ , welfare under vote-share contracts with vote-share threshold  $s^*$  will be higher than under elections alone. The intuition for this result is that vote-share contracts will still be welfare-enhancing, as they result in higher expected effort and/or in higher expected ability of the office-holder in the second period.

## 7.2 Asymmetric Competition and Larger Time Horizon

A useful extension of the model is to consider two candidates who are *ex ante* non-symmetric. There are several possibilities to introduce *ex ante* asymmetry, e.g. by differing effort-cost parameters in the first term of a politician or by abandoning the assumption of symmetric ideal points concerning the ideological policy. If, for example,  $\mu_R$  is located closer to the ideal point of the median voter than  $\mu_L$ , i.e.  $\mu_L < 1 - \mu_R$ , then candidate  $R$  will have an *ex ante* advantage over his opponent  $L$ . Analyzing the consequences of such an asymmetric competition for office in  $t = 1$  on the effort choice, on the incumbency advantage in  $t = 2$ , and on the welfare-effects of vote-share contracts promises to be a fruitful extension of our model.

Moreover, a model with two periods and two candidates being already asymmetric before the first period starts may also be interpreted as the last two periods in a game with a longer time-horizon. The ex-ante asymmetry would then have been initiated by the incumbency advantage in previous periods of this repeated game. Such a model with a larger time horizon, where candidates for public office compete in each term on the basis of vote-share contracts, might be an interesting extension. First, this would make the model much more applicable for real-world situations, whereas our basic model with just two periods covers, in principle, only the case of two-period term limits, as common in the U.S. presidential elections. Second, we assume that increasing the time horizon of the model would reinforce the positive result of our basic model. For a detailed solution of a multi-period model, it would be necessary to specify the assumptions about learning-by-doing effects, i.e. to make precise assumptions whether they occur in each term, only sometimes, or even only once. However, we may state the following, even without precise assumptions about the multi-period model: As the incumbent will work hard to get reelected, the effort choice under vote-share contracts will be higher with vote-share contracts in each period, except from the last. The lower expected effort in the last period will be weighted less if there are more than two periods. Thus, we conjecture that the welfare-improving effect of vote-share thresholds

will even be higher in dynamic versions of our model.

## 8 Conclusion

In a simple model frame, we have proposed to use vote-share contracts as an instrument for restraining incumbency advantage. Vote-share contracts imply higher effort and/or higher ability of incumbents, and therefore improve the efficiency of political systems. However, the practical implementation of vote-share contracts might induce further consequences, which might reduce or even invert the positive effect of vote-share contracts. Nevertheless, we have shown that under the assumptions of our framework, it is optimal for societies to restrain the incumbency advantage of their office-holders, even if there exists a socially beneficial aspect of incumbency advantage.

One may wonder why politicians do not already use other methods today to restrain their incumbency advantage. If a politician can increase his chances to win the first election by cutting his future incumbency advantage, then one should expect politicians to make use of this mechanism in practice. A politician might, for example, announce he will spend less money for his reelection campaign in order to reduce his incumbency advantage. At first glance, this would have the same effect as a vote-share threshold above one-half. The problem is, however, that such announcements are no credible commitments, as they are not enforceable by voters. Thus, such announcements are completely worthless. The only way to avoid that announcements are just cheap talk is to embed them into the framework of enforceable political contracts. The most suitable kind of political contract to reduce welfare losses which arise from incumbency advantages are vote-share thresholds. Thus, we believe that exploring the potential of vote-share contracts as a new institution may be a fruitful path for liberal democracies.

# Appendix

## Proof of Proposition 1

Part (i) is obvious. Next, the problem of politician  $R$  in his second term is given by

$$\max_{e_{R2}} \{\gamma(e_{R2} + a_R) - c_l e_{R2}^2\},$$

which yields  $e_{R2} = \frac{\gamma}{2c_l}$ . For politician  $R$  in his first term, the problem is given by

$$\max_{e_{R2}} \{\mathbb{E}[\gamma(e_{R2} + a_R)] - c_h e_{R2}^2\}.$$

The solution is  $e_{R2} = \frac{\gamma}{2c_h}$ . This yields the second part. Finally, note for part (iii) that the expected utility from the public project for a second-term office holder  $R$  is given by  $\gamma \left( \frac{\gamma}{2c_l} + a_R \right) - c_l \left( \frac{\gamma}{2c_l} \right)^2 = \frac{\gamma^2}{4c_l} + \gamma a_R$ , while the corresponding utility for an office holder  $R$  in his first term is  $\gamma \left( \frac{\gamma}{2c_h} \right) - c_h \left( \frac{\gamma}{2c_h} \right)^2 = \frac{\gamma^2}{4c_h}$ . ■

## Proof of Fact 2

It is optimal for the median voter  $i = \frac{1}{2}$  to reelect  $R$  if this implies that his utility in  $t = 2$  is higher. Formally, this is given by

$$\gamma \left( \frac{\gamma}{2c_l} + (a_R + e_{R1} - \hat{e}_1) \right) + \epsilon_R f \Delta - (\mu_R - \frac{1}{2})^2 \geq \gamma \frac{\gamma}{2c_h} - (\mu_L - \frac{1}{2})^2,$$

where we have applied that upon observing  $g_1$ , the median voter expects the ability level of  $R$  to be  $\frac{g_1}{\gamma} - \hat{e}_1 = a_R + e_{R1} - \hat{e}_1$ . Using our assumption  $\mu_L = 1 - \mu_R$ , we obtain

$$a_R \geq -\epsilon_R \frac{f \Delta}{\gamma} - e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l}. \quad (25)$$

Condition (25) states that  $R$  is reelected if his ability level is equal or above the critical level  $-\epsilon_R \frac{f \Delta}{\gamma} - e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l}$ . Now let us look at  $R$ 's decision about  $\epsilon_R$ :

- For  $a_R \geq -e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l}$ ,  $R$  is reelected even with  $\epsilon_R = 0$ . As  $a_R$  is uniformly distributed on  $[-A; A]$ , the probability for  $a_R > -e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l}$  is given by  $p^0(e_{R1}, \hat{e}_1) = \frac{1}{2} + \frac{1}{2A} (e_{R1} - \hat{e}_1 - \frac{\gamma(c_l - c_h)}{2c_h c_l})$ .

- If  $-e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l} > a_R \geq -e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l} - \frac{f\Delta}{\gamma}$ , then it is optimal to choose  $\epsilon_R = 1$ , which prevents the officeholder from being dismissed. The probability of  $a_R$  being within this interval is given by  $\frac{f\Delta}{2A\gamma}$ .
- For  $a_R < -e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l} - \frac{f\Delta}{\gamma}$ , the ability of  $R$  is too low for him to become reelected and he will choose  $\epsilon_R = 0$  to avoid losses from output-shift policies.

Finally, we obtain the expected ability level of  $R$ , conditional on the fact that he is reelected, as the arithmetical average of  $-e_{R1} + \hat{e}_1 + \frac{\gamma(c_l - c_h)}{2c_h c_l} - \frac{f\Delta}{\gamma}$  and  $A$ , which is given by  $\tilde{a}_R(e_{R1}, \hat{e}_1) = \frac{A + \hat{e}_1 - e_{R1} + \frac{\gamma(c_l - c_h)}{2c_h c_l} - \frac{f\Delta}{\gamma}}{2}$ .

■

### Proof of Proposition 2

Together with equations (3), (4), and (5), the maximization problem (6) yields the following first-order condition:

$$\begin{aligned} \gamma - 2c_h e_{R1} + \frac{1}{2A} \left( b + \frac{\gamma^2}{4c_l} + \frac{\gamma \left( A - \frac{f\Delta}{\gamma} + \hat{e}_1 - e_{R1} + \frac{\gamma(c_l - c_h)}{2c_h c_l} \right)}{2} \right) \\ - \frac{\gamma}{2} \left( \frac{e_{R1} - \hat{e}_1 - \frac{\gamma(c_l - c_h)}{2c_h c_l}}{2A} + \frac{1}{2} + \frac{f\Delta}{2A\gamma} \right) - \frac{1}{2A} \left( \frac{\gamma^2}{2c_h} - (\mu_R - \mu_L)^2 \right) = 0. \end{aligned}$$

In equilibrium,  $\hat{e}_1 = e_{R1}$  will hold, so the equilibrium effort is given by

$$e_{R1}^* = \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 \right] \right\}.$$

We obtain part (ii) - (iv) by using the fact that  $\hat{e}_1 = e_{R1}$  will hold in equilibrium.

■

### Proof of Fact 3

Under a socially optimal reelection rule, voters should reelect  $R$  if the expected utility from the public project in period 2 is not smaller when  $R$  remains in office than under  $L$  as new office-holder. This gives the following necessary condition:

$$\gamma \left( \frac{\gamma}{2c_l} + a_R \right) \geq \gamma \left( \frac{\gamma}{2c_h} + a_L \right). \quad (26)$$

The expected ability  $a_L$  is zero. Thus,  $R$  should be reelected if and only if

$$a_R \geq \frac{\gamma(c_l - c_h)}{2c_h c_l}. \quad (27)$$

The average of  $A$  and  $\frac{\gamma(c_l - c_h)}{2c_h c_l}$  gives  $a_R^* = \frac{A}{2} - \frac{\gamma(c_h - c_l)}{4c_h c_l}$  as the optimal average ability level of a reelected candidate. ■

### Proof of Fact 4

The derivation of (12), (13), and (14) is similar to the derivation of (3), (4), and (5). With  $s_R > \frac{1}{2}$ ,  $R$  is reelected only if all voters  $i \geq 1 - s_R$  prefer him, as he needs at least  $s_R$  votes. This gives the condition

$$\gamma(e_{R2}^{*V} + (a_R + e_{R1}^V - \hat{e}_1^V)) + \epsilon_R f \Delta - (\mu_R - (1 - s_R))^2 \geq \gamma e_{L2}^{*V} - (\mu_L - (1 - s_R))^2.$$

Using  $\mu_L = 1 - \mu_R$ , one obtains

$$a_R \geq -\epsilon_R \frac{f \Delta}{\gamma} - e_{R1}^V + \hat{e}_1^V + \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) + \frac{\gamma(c_l - c_h)}{2c_h c_l}. \quad (28)$$

The right-hand side of this inequality gives the minimum ability  $R$  must have to be reelected. This minimum ability is increasing in  $s_R$ . With condition (28), it is straightforward to show that (3), (4), and (5) generalize to (12), (13), and (14). ■

### Proof of Proposition 3

The problem of the incumbent is the same as in Proposition 2, except that we have to use equations (12), (13), and (14) instead of (3), (4), and (5). The first-order condition of the maximization problem (6) is given by

$$\begin{aligned} & \gamma - 2c_h e_{R1} + \frac{1}{2A} \left( b + \frac{\gamma^2}{4c_l} + \frac{\gamma A + (2\mu_R - 1)(2s_R - 1) - f \Delta + \gamma \hat{e}_1 - \gamma e_{R1} - \frac{\gamma^2(c_h - c_l)}{2c_h c_l}}{2} \right) \\ & - \frac{\gamma}{2} \left( \frac{1}{2} + \frac{1}{2A} [e_{R1} - \hat{e}_1 - \frac{1}{\gamma}(2\mu_R - 1)(2s_R - 1) + \frac{f \Delta}{\gamma} + \frac{\gamma(c_h - c_l)}{2c_h c_l}] \right) \\ & - \frac{1}{2A} \left( \frac{\gamma^2}{2c_h} - (\mu_R - \mu_L)^2 \right) = 0. \end{aligned}$$

In equilibrium,  $\hat{e}_1 = e_{R1}$  must hold. Hence, the equilibrium effort  $e_{R1}^{*V}$  is given as

$$e_{R1}^{*V} = \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} + (2\mu_R - 1)(2s_R - 1) - f\Delta + (\mu_R - \mu_L)^2 \right] \right\}.$$

■

### Proof of Fact 5

We insert equations (15), (16), and (17) into the maximization problem (18), and use the fact that  $a_L^V = 0$ . This yields the following first-order condition:

$$\begin{aligned} \frac{(2\mu_R - 1)\gamma}{2Ac_h} - \frac{(2\mu_R - 1)}{A\gamma} \left( \frac{A\gamma + (2\mu_R - 1)(2s_R - 1) - f\Delta}{2} - \frac{\gamma^2(c_h - c_l)}{4c_h c_l} + \frac{\gamma^2(c_h - c_l)}{2c_h c_l} \right) \\ + (2\mu_R - 1) \left( \frac{1}{2} - \frac{(2\mu_R - 1)(2s_R - 1) - f\Delta}{2A\gamma} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} \right) = 0. \end{aligned}$$

Solving for  $s_R$  yields  $s^* = \frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)} + \frac{\gamma^2}{4c_h(2\mu_R - 1)}$ . Finally, the second derivative with respect to  $s_R$  is negative, which proves that  $s^*$  maximizes equation (18).

■

### Proof of Proposition 4

The reelection chances of an incumbent with offer  $s^*$  exceed  $\frac{f\Delta}{2A\gamma}$ , as  $p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) > 0$  according to our assumption  $2\mu_R - 1 < A\gamma$ . The incumbent will exert effort high enough to sustain his reelection chances, as  $b$  is sufficiently large. Deviation from  $s^*$  to a higher or a lower vote-share threshold yields the election of the opponent, as the median voter prefers the offer  $s^*$ . Hence, deviation is not profitable. Uniqueness of  $s^*$  follows in the same way. If  $k$  chooses  $s_k \neq s^*$ , then  $k'$  certainly wins the election by choosing  $s_{k'} = s^*$ . Part (ii) is obvious. For part (iii), we observe that any other vote-share threshold reduces the expected utility from public projects, as citizens are homogeneous regarding  $P$ . Furthermore, due to equation (2), aggregate utility from the ideological project does not depend on which candidate is elected. Thus, the optimal vote-share threshold, from the perspective of the median voter, is ex ante optimal.

■

### Proof of Proposition 5

By inserting the values from equation (7) and (8), we obtain

$$\begin{aligned}
E[e_R^*] &= \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 \right] \right\} \\
&\quad + \frac{\gamma}{2c_l} \left( \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{f\Delta}{2A\gamma} \right) \\
&\quad + \frac{\gamma}{2c_h} \left( 1 - \frac{1}{2} - \frac{\gamma(c_h - c_l)}{4Ac_h c_l} - \frac{f\Delta}{2A\gamma} \right). \tag{29}
\end{aligned}$$

By inserting the values from equation (15) and (16), we obtain

$$\begin{aligned}
E[e_R^{*V}] &= \frac{1}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 + (2\mu_R - 1)(2s_R - 1) \right] \right\} \\
&\quad + \frac{\gamma}{2c_l} \left( \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} - \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma} + \frac{f\Delta}{2A\gamma} \right) \\
&\quad + \frac{\gamma}{2c_h} \left( 1 - \frac{1}{2} - \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma} - \frac{f\Delta}{2A\gamma} \right). \tag{30}
\end{aligned}$$

Subtracting equation (29) from equation (30) yields, after some straightforward algebra

$$E[e_R^{*V}] - E[e_R^*] = \frac{(2\mu_R - 1)(2s_R - 1)}{4Ac_h c_l} (2c_l - c_h).$$

This gives part (i). Part (ii) and part (iii) follow directly from equation 22. ■

### Proof of Fact 6

We use the fact that the expected ability of a new left-wing office-holder in period 2 is equal to zero and insert the values from (7), (8) and (9) into equation (23). After some straightforward transformations, we obtain:

$$\begin{aligned}
E[a_R(e_{R1}^*, e_{R1}^*)] &= \frac{A}{4} - A \left( \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{f\Delta}{2A\gamma} \right)^2 \\
&= \frac{A}{4} - \frac{f^2 \Delta^2}{4A\gamma^2} - \frac{\gamma^2 (c_h - c_l)^2}{16Ac_h^2 c_l^2} - \frac{f\Delta(c_h - c_l)}{4Ac_h c_l} \tag{31}
\end{aligned}$$

Then we insert the values from (15), (16) and (17) into  $E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V})]$  and obtain

$$\begin{aligned}
E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V})] &= \frac{A}{4} - A \left( \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{f\Delta}{2A\gamma} - \frac{(2\mu_R - 1)(2s_R - 1)}{2A\gamma} \right)^2 \\
&= \frac{A}{4} - \frac{f^2 \Delta^2}{4A\gamma^2} - \frac{\gamma^2(c_h - c_l)^2}{16Ac_h^2 c_l^2} - \frac{f\Delta(c_h - c_l)}{4Ac_h c_l} + \frac{(2\mu_R - 1)(2s_R - 1)f\Delta}{2A\gamma^2} \\
&\quad + \frac{(2\mu_R - 1)(2s_R - 1)(c_h - c_l)}{4Ac_h c_l} - \frac{(2\mu_R - 1)^2(2s_R - 1)^2}{4A\gamma^2} \tag{32}
\end{aligned}$$

as the expected ability of the incumbent in the second period with vote-share contracts, given that the elected politician chooses  $e_{R1}^{*V}$  in the first period. From equation (31) and (32), we immediately obtain the following result:

$$\begin{aligned}
&E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V})] - E[a_R(e_{R1}^*, e_{R1}^*)] \\
&= \frac{(2\mu_R - 1)(2s_R - 1)}{4A\gamma^2} \left( \frac{\gamma^2(c_h - c_l)}{c_h c_l} + 2f\Delta - (2\mu_R - 1)(2s_R - 1) \right).
\end{aligned}$$

■

### Proof of Proposition 6

We insert  $s_R^*$  from Fact 5 into equation (24). Hereby, we have to distinguish two separate cases: The case where  $\frac{1}{2} < s_R^* < 1$  and the case where  $s_R^* = 1$ .

First, in the case where  $s_R^* < 1$ , we insert  $\frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)} + \frac{\gamma^2}{4c_h(2\mu_R - 1)}$  for  $s_R^*$  into equation (24) and obtain:

$$E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V}), s^*] - E[a_R(e_{R1}^*, e_{R1}^*)] = \left( \frac{f\Delta}{4A\gamma^2} + \frac{1}{8Ac_h} \right) \cdot \left( \frac{\gamma^2(2c_h - 3c_l)}{2c_h c_l} + f\Delta \right) \tag{33}$$

Second, in the case where  $s_R^* = 1$ , we insert  $s_R^* = 1$  into equation (24) and obtain:

$$E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V}), s^*] - E[a_R(e_{R1}^*, e_{R1}^*)] = \frac{2\mu_R - 1}{4A\gamma^2} \left( \frac{\gamma^2(c_h - c_l)}{c_h c_l} + 2f\Delta - (2\mu_R - 1) \right) \tag{34}$$

Furthermore, note that we know from Fact 5 that in the case  $s_R^* = 1$  the following inequality has to hold:

$$(2\mu_R - 1) \leq f\Delta + \frac{\gamma^2}{2c_h} \tag{35}$$

If we use (35) to replace the last term of equation (34), we obtain

$$E[a_R^V(e_{R1}^{*V}, e_{R1}^{*V}), s^*] - E[a_R(e_{R1}^*, e_{R1}^*)] \geq \frac{2\mu_R - 1}{4A\gamma^2} \cdot \left( \frac{\gamma^2(2c_h - 3c_l)}{2c_h c_l} + f\Delta \right) \quad (36)$$

The results of equation (33) and (36) together imply Proposition 6. ■

### Proof of Theorem 1

Part (i): Suppose, without loss of generality, that candidate  $R$  is elected in the first election. Applying vote-share contracts will be welfare-enhancing if and only if the welfare, under vote-share contracts and with a vote-share threshold  $s_R^*$  minus the welfare in the case under elections alone, is positive. This gives the following condition:

$$\begin{aligned} & \left\{ \gamma e_{R1}^{*V} + \left( p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) + p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right) \cdot \left( \gamma \tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V}) + \gamma e_{R2}^{*V} \right) \right. \\ & \quad \left. + \left( 1 - p^{0V}(e_{R1}^{*V}, e_{R1}^{*V}) - p^{1V}(e_{R1}^{*V}, e_{R1}^{*V}) \right) \cdot \left( \gamma a_L^V + \gamma e_{L2}^{*V} \right) \right\} \\ & - \left\{ \gamma e_{R1}^* + \left( p^0(e_{R1}^*, e_{R1}^*) + p^1(e_{R1}^*, e_{R1}^*) \right) \cdot \left( \gamma \tilde{a}_R(e_{R1}^*, e_{R1}^*) + \gamma e_{R2}^* \right) \right. \\ & \quad \left. + \left( 1 - p^0(e_{R1}^*, e_{R1}^*) - p^1(e_{R1}^*, e_{R1}^*) \right) \cdot \left( \gamma a_L + \gamma e_{L2}^* \right) \right\} > 0 \end{aligned}$$

By inserting  $e_{R2}^{*V} = e_{R2}^* = \frac{\gamma}{2c_l}$ ,  $e_{L2}^{*V} = e_{L2}^* = \frac{\gamma}{2c_h}$ ,  $a_L^V = a_L = 0$  and the values for  $e_{R1}^*$ ,  $p^0(e_{R1}^*, e_{R1}^*)$ ,  $p^1(e_{R1}^*, e_{R1}^*)$ ,  $\tilde{a}_R(e_{R1}^*, e_{R1}^*)$ ,  $e_{R1}^{*V}$ ,  $p^{0V}(e_{R1}^{*V}, e_{R1}^{*V})$ ,  $p^{1V}(e_{R1}^{*V}, e_{R1}^{*V})$  and  $\tilde{a}_R^V(e_{R1}^{*V}, e_{R1}^{*V})$  from Propositions 2 and 3, we obtain the following expression:

$$\begin{aligned} & \frac{\gamma}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} + (2\mu_R - 1)(2s_R^* - 1) - f\Delta + (\mu_R - \mu_L)^2 \right] \right\} \\ & \quad + \left[ \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} - \frac{(2\mu_R - 1)(2s_R^* - 1)}{2A\gamma} + \frac{f\Delta}{2A\gamma} \right] \\ & \quad \cdot \left[ \gamma \left( \frac{A}{2} + \frac{(2\mu_R - 1)(2s_R - 1)}{2\gamma} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l} \right) + \frac{\gamma^2}{2c_l} \right] \\ & \quad + \left[ 1 - \frac{1}{2} - \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{(2\mu_R - 1)(2s_R^* - 1)}{2A\gamma} - \frac{f\Delta}{2A\gamma} \right] \cdot \frac{\gamma^2}{2c_h} \\ & \quad - \frac{\gamma}{2c_h} \left\{ \gamma + \frac{1}{2A} \left[ b - \frac{\gamma^2}{4c_l} - f\Delta + (\mu_R - \mu_L)^2 \right] \right\} \\ & - \left[ \frac{1}{2} + \frac{\gamma(c_h - c_l)}{4Ac_h c_l} + \frac{f\Delta}{2A\gamma} \right] \cdot \left[ \gamma \left( \frac{A}{2} - \frac{f\Delta}{2\gamma} - \frac{\gamma(c_h - c_l)}{4c_h c_l} \right) + \frac{\gamma^2}{2c_l} \right] \\ & \quad - \left[ 1 - \frac{1}{2} - \frac{\gamma(c_h - c_l)}{4Ac_h c_l} - \frac{f\Delta}{2A\gamma} \right] \cdot \frac{\gamma^2}{2c_h} > 0 \end{aligned}$$

After some tedious calculation<sup>18</sup>, we obtain the following expression:

$$\frac{\gamma^2}{c_h} + 2f\Delta - (2\mu_R - 1)(2s_R^* - 1) > 0 \quad (37)$$

In the next step, we insert  $s_R^*$  from Fact 5 into condition (37), and show that condition (37) is always fulfilled, which proves the welfare-enhancing effect of vote-share contracts with vote-share threshold  $s_R^*$ . We have to analyze two separate cases: First, the case where  $\frac{1}{2} < s_R^* < 1$ , and second, the case where  $s_R^* = 1$ .

If  $s_R^* < 1$ , we insert  $\frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)} + \frac{\gamma^2}{4c_h(2\mu_R - 1)}$  for  $s_R^*$  in condition (37) and obtain

$$\frac{\gamma^2}{2c_h} + f\Delta > 0. \quad (38)$$

This condition is always fulfilled.

In the case where  $s_R^* = 1$ , we insert  $s_R^* = 1$  in condition (37) and obtain

$$\frac{\gamma^2}{c_h} + 2f\Delta - (2\mu_R - 1) > 0. \quad (39)$$

Furthermore, we use the fact that for  $s_R^* = 1$ , the following inequality will be fulfilled:

$$\frac{1}{2} + \frac{f\Delta}{2(2\mu_R - 1)} + \frac{\gamma^2}{4c_h(2\mu_R - 1)} \geq 1 \quad (40)$$

Inequality (40) can be transformed to

$$2f\Delta + \frac{\gamma^2}{c_h} \geq 2(2\mu_R - 1) \quad (41)$$

and, by inserting this inequality into condition (39), we immediately see that condition (39) is always fulfilled.

Part(ii): The second part of the Theorem follows directly from condition (38), where the term  $f\Delta + \frac{\gamma^2}{2c_h}$  represents the gains that accrue from the introduction of vote-share contracts. ■

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<sup>18</sup>In our calculations, we use the fact that  $s_R^* > \frac{1}{2}$ , which allows us to divide by  $(2s_R^* - 1)$ .

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