Controlling Externalities in the Presence of Rent Seeking

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Abstract

Contests are a common method to describe the distribution of many different types of rents. Yet in many of these situations the utilisation of the prize plays an important role in determining agents' payoffs and incentives. In this paper, we investigate the incentives to expend effort for a prize that produces consumption externalities and consider alternative regulatory policies. We find relatively more global consumption externalities will increase (decrease) rent seeking when consumption externalities are negative (positive). We show how introducing Pigouvian taxation (possibly with revenue transfer) and Coasean bargaining alters equilibrium effort and payoffs. Pigouvian taxation tends to reduce both effort and payoffs whereas this is not always the case for Coasean bargaining. In the presence of sufficiently large consumption externalities, establishing Pigouvian taxation coupled with some element of lump-sum transfer may reduce costly rent seeking effort and improve the welfare of some agents compared to other approaches.

Keywords: externalities, contest, natural resources

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1 Introduction

Conventional rent-seeking frameworks allow analysis of many political and economic interactions where agents expend effort to win a prize which produces private benefits, such as litigation, political campaigns, sport events, R&D patents, conflicts and natural resource rights allocation (Congleton et al. 2008b; Konrad, 2009). However, in many cases, the consumption or utilisation of the rent plays a fundamental role in determining agents' utilities and incentives to invest in effort. In particular, consumption of the rent often produces externalities.

It has long been known how to deal with externalities, namely the implementation of Pigouvian taxation or Coasean bargaining, yet it is currently unclear how these mechanisms perform in rent seeking contests under the presence of externalities. For example, would producers lobby more to obtain a share of carbon dioxide emission rights than to obtain a share of radioactive waste rights? Do pollution taxes or tradable pollution permit markets perform better in controlling rent dissipation? It turns out, the extent of rent seeking depends on whether consumption produces positive or negative externalities and the extent of the 'globalness'. Pigouvian taxation and Coasean bargaining produce very different effects on rent seeking strategies and payoffs which depend, in part, on the level of marginal externalities. A potentially successful solution is to establish Pigouvian taxation coupled with some element of lump-sum transfer which may reduce costly rent seeking effort and improve the welfare of some agents compared to other approaches.

In conventional contests, negative externalities exist as an agent's probability of obtaining a prize declines with an increase in rivals' effort (see, for example, Hillman and Riley, (1989), Nitzan (1994), Congleton et al. (2008a) and Konrad (2009)). Yet, other externalities exist in contests, such as spillovers from patent races, damage to infrastructure due to military conflicts and so on (Congleton, 1989; Chung, 1996; Lee and Kang, 1998; Shaffer, 2006). In order to analyse externalities in contests, it has generally been assumed that the level of aggregate effort alters the size of the contestable rent. For example, one can consider labour tournaments where the increase in (productive) effort by workers results in a larger surplus for all in the organisation. Yet restricting analysis to

contests that only produce externalities as a result of an endogenously determined rent (i.e. aggregate efforts influencing the rent) may not help to explain all types of externalities present in contests. Importantly, it is possible that while a contest prize remains fixed, agents' consumption of the prize produces additional benefits (costs)—*consumption* externalities. For example, the capturing of natural resource rights (such as coal, gas, oil, fisheries and forestry), may all produce externalities *independent* of aggregate effort. Our prize is a private, excludable and rivalrous rent where the consumption of the prize produces effects in the form of both private benefits (damages) and 'global' externalities (for rivals).[1] An important distinction in our model is that the act of consumption produces *transferable* externalities (Bird, 1987).[2] This means that an agent can transfer (a portion of) externalities to rival agents in the contest by consuming more of the rent.

The work most relevant to our paper is by Shogren and Crocker (1991), who consider a contest with transferable externalities among agents.[3] They show when agents have the ability to invest in protection against environmental externalities that over-protection may occur when externalities are transferable among agents. It is possible that the notion of transferability externalities can be discussed in a much broader context. In our paper, consumption externalities are similar to transferable externalities in that, a change in the distribution of consumption will alter the levels of externalities each agent experiences. Yet there a number of subtle differences. First, our externalities are consumption-based, therefore it is as a consequence of consumption that externalities occur and not *ad hoc* externalities in which agents have to protect against. This means that the prize in our contest is a rivalrous and excludable rent. Second, consumption externalities have an additional effect based on private consumption. Therefore, an agent that consumes the

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[1] Contests have also been considered where the prize is a public good (Katz et al., 1990; Ursprung, 1990; Baik, 1993; Gradstein, 1993; Linster, 1993; Riaz et al., 1995; Baik, 2008; Epstein and Mealem, 2009). The conventional approach is to allow individuals (within a group) to independently invest in effort where the group with the largest aggregate effort wins the group-specific public good. In general, it has been found that, within the winning group, individuals with higher valuations invest more effort whereas lower valuing individuals tend to free ride.

[2] Unrelated to the context of consumption externalities, Appelbaum and Katz (1986) were the first to formally consider the effort of agents when the benefits and costs of a rent are transferred among agents.

[3] The general theme of externalities can also been considered in a number of contexts, such as contest where agents can be altruists or envious (e.g. Konrad, 2004) and multiple agents where the losers are not indifferent about who wins the rent (e.g. Linster, 1993).

rent will produce global externalities but also experience a private consumption effect. We provide further insight for the results of Shogren and Crocker (1991) and show this is a special case of our framework, where over-effort occurs as a result of relatively large global externalities compared to the private consumption effect. We then compare effort levels and payoffs in the contest in which Pigouvian and Coasean solutions are implemented.

In this paper, we allow agents to invest in effort to win a share of a rivalrous and excludable resource which is then fully utilised. Firstly, an agent's consumption of the rent may provide additional private effects. Second, agents' consumption of the rent may produce externalities for rivals. The consumption externalities effect is in addition to those experienced in conventional contests. We find that in the equilibrium of the contest, agents' optimal actions decrease in the index of relative globalness, so that an increase in relative globalness will increase (decrease) actions for negative (positive) externalities. When the regulator has the ability to introduce Pigouvian taxation or allow ex post Coasean bargaining, we find that taxation reduces effort whereas this does not always happen for Coasean bargaining. When Coasean bargaining is introduced agents with high values of the rent always improve their position (as they experience no externalities from rivals) whereas for lower value agents it is ambiguous and depends on the relative globalness of the consumption effect, the bargaining power of the agents and the asymmetry in agents' valuation of the rent. We also show that for sufficiently large marginal consumption externalities, Pigouvian taxation in which tax revenues are transferred to the agent with the lowest value may be a desirable mechanism due to Pareto improvements. In particular, efforts are maintained at conventional Pigouvian levels while the payoff for the low value agent is larger than under Coasean bargaining.

Our work thus contributes to the growing literature on contests by exploring how the degree of relative 'globalness' of consumption externalities interact with conventional contest externalities in a simple framework. This allows us to analyse the incentive to invest in effort to obtain resources that produce externalities of varying degrees of 'globalness'. Allowing externalities to be created as a result of consumption provides an additional method to consider contests. This may highlight important issues in rent seeking for

natural resources, conflict, patent races and so on, where the consumption of the prize appears to have important implications for agents' incentives to invest effort. Further, important policy implications occur due to the implementation of Pigouvian taxation or Coasean bargaining prior to the contest. These results can be directly applied to contests over natural resources. Pigouvian taxes (pollution taxes) and Coasean bargaining (tradable permit markets) are now commonly experienced in environmental regulation (Freeman and Kolstad, 2007). This paper adds to the literature on the desirability of Pigouvian taxes or Coasean bargaining for the control of resources by providing evidence of significant differences in incentives under both regulatory mechanisms.

The remainder of the paper is structured as follows: Section 2 sets out the model when consumption externalities exist. Section 3 introduces Pigouvian taxes and Coasean bargaining and shows the changes to equilibrium effort and payoffs and Section 4 has some concluding remarks.

2 The model

Consider a set of agents $\Theta = \{1, 2, ..., n\}$ that participate in a complete information contest by investing in effort s_i $\forall i \in \Theta$ in order to win a share of a rent at a sunk linear cost. The contestable rent $A \in \mathbb{R}_+$ is rivalrous, excludable and common knowledge to all agents. However, the rent may produce consumption effects. That is, the consumption of the rent may produce local private benefits as well as 'global' externalities on rival agents.

To represent agent i's share of the rent, we define a share function for agent i given by a conventional Tullock (1980) contest:[4]

$$
L_i = \begin{cases} \frac{s_i}{s_i + s_{-i}} A & \text{if } \max\{s_i, s_{-i}\} > 0 \\ \frac{A}{n} & \text{otherwise} \end{cases}
\tag{1}
$$

where $-i = \{1, \ldots, i-1, i+1, \ldots, n\}$.

In order to provide insight into the behaviour of agents in the presence of consumption

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[4]For convenience, we discuss a rent that is divisible and is shared among agents. However, our framework may allow interpretations of both divisible and indivisible prizes when agents are risk neutral.

externalities, we separate the net benefit of obtaining the rent into *attainment* benefits and *consumption* benefits (damages). For the *attainment* benefits, agent i obtains benefits from winning the share of the prize which we represent by L_i and is determined by (1). The attainment benefit is determined solely by agent i's share of the rent and is independent of the consumption of the rent. This benefit is the value placed on an agent's successful attainment (share) of the rent, for example, this could represent the value of a patent in a R&D contest, the value of natural resources won in a rent seeking game, the value of a wage in a job promotion contest, and so on.

Additional benefits (or costs) may occur due to the consumption of the rent. Firstly, agent i's utilisation of the rent may produce "global" externalities that affect rivals. For example, this could include the consumption of resource rights (i.e. pollution damage) or the utilisation of patents which produce positive externalities in terms of technological spillovers. We denote global externalities (borne by agent i) as ϕL_{-i} where L_{-i} is the share of the prize obtained by rivals and $\phi > 0$ is a parameter signalling the extent of consumption externalities, that is, the extent of 'globalness' of consumption externalities, where a larger ϕ represents more 'global' externalities (in the absolute sense). Second, when agent i consumes a portion of the rent, it may experience alterations to its own *private benefit L_i*. For example, for negative externalities, one can consider the reduction in its own benefit due to the pollution created. Throughout the paper, we refer to these effects as *private consumption benefits* and denote this by υL_i where L_i is the share won by agent i in the contest and $\upsilon > 0$ is a parameter denoting the extent of private consumption benefits. This effect is rivalrous and excludable with the capturing of the rent. Throughout we assume full utilisation of the rent by agents.[5] Therefore, agent i's aggregate influence from rent utilisation is given by the summation of global externalities and private consumption benefits affected by consumption given by:

$$E_i = \omega \left[\upsilon L_i + \phi L_{-i} \right] \tag{2}$$

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[5]Introducing a second stage where the agent has the option to only utilise a proportion of the rent that is won $\lambda_i \in [0, 1]$, so that local consumption externalities of the equilibrium rent are given by $\lambda_i \omega \upsilon L_i^*$ will result in corner solutions for costless consumption. In particular, for negative (positive) local externalities the agent decides to consume none (all) of the rent.

where ω indicates whether externalities are positive ($\omega > 0$) or negative ($\omega < 0$).

2.1 Equilibrium strategy

To demonstrate how contests with consumption externalities differ from conventional contests, one can compare equilibrium rent seeking efforts. To do this, let us denote a benchmark model where no consumption externalities exist ($E_i = 0$) and denote the benchmark rent seeking effort by s_i^B. The following proposition provides a comparison of symmetric equilibrium effort $s_i^* = s_{-i}^* = s^*$ from a contest with externalities and the benchmark model.

Proposition 1 *If $\omega\left[\upsilon - \phi\right] \gtreqless 0$ then $s^* \gtreqless s^B$*

Proof. See Appendix A. ∎

To the extent that effort is larger than the benchmark depends on whether consumption externalities are either positive ($\omega > 0$) or negative ($\omega < 0$) and the relative marginal *globalness* of this effect, that is, the absolute difference in $\upsilon - \phi$.

Consider positive consumption externalities $\omega > 0$, where global externalities may be formed due to rivals' consumption of the rent (ϕ) and private consumption benefits are also realized (υ). If $\upsilon > \phi$ then the marginal benefit from private consumption benefits is larger than the marginal benefit from global positive externalities. Therefore, an incentive exists for agent i to increase effort. It follows that even in the presence of positive consumption externalities, effort will be larger than the benchmark model. However, when $\upsilon < \phi$ the marginal benefits from local consumption externalities are smaller than those from global externalities. Here we observe the incentive to free ride, as the benefits due to rivals' consumption produce larger benefits to agent i than her own benefits from the private consumption benefits.

Similar logic applies to negative consumption externalities $\omega < 0$, in that if $\upsilon < \phi$, rivals' marginal (negative) consumption externalities are relatively large compared to the marginal damages from the private consumption benefits experienced by agent i's consumption, hence, agent i decides to increase effort in order to reduce the relative

increase in negative externalities produced by rivals (Shogren and Crocker, 1991). Finally when $\upsilon > \phi$, the marginal damage from private consumption benefits is larger than the marginal damage from the global negative externalities so agent i chooses to reduce rent seeking.

In the existing literature, Shogren and Crocker (1991) are able to show that over-effort occurs for negative transferable externalities (and a decrease in effort caused by filterable externalities). Here, Proposition 1 provides deeper insight for the results of Shogren and Crocker (1991) in that we find *relative* global negative externalities tend to produce over effort in contests. Yet we find additional cases where private consumption benefits can alter these results. Proposition 1 shows that it is the relative size of local benefits (costs) and global externalities (and not simply the existence of externalities) that determines whether an agent over (or under) invests in effort. This is important when we begin to consider the utilisation of resources. For example, the consumption of natural resources has the potential to produce both private consumption damages and global negative externalities and therefore it depends on the relative size of these externalities as to whether agents will over- (under-) invest in effort.[6]

It is clear from Proposition 1 that even in the presence of private consumption benefits (damages) and global externalities, effort may not differ from the benchmark level (however the payoffs will be different). When $\upsilon = \phi$, the marginal private benefits (costs) are equal to the marginal gain (cost) from the global positive (negative) externalities. For example, this may be applicable to contests over carbon dioxide rights. Carbon dioxide is

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[6]An equivalent analysis can also consider group contests. Additional consumption externalities arise from the consumption of the remaining members in group i. This we denote as $L_i \omega \kappa \left[[1-a] \frac{S_i - s_{ik}}{S_i} + \frac{a(n-1)}{n} \right] A$. where $a \in [0,1]$ is an exogenous parameter which describes the characteristics of the sharing rule within group i (Nitzan, 1991) and κ is a parameter denoting the level of intra-group externalities. When $a = 0$, the share to each agent is based on agent k's effort relative to all other group i members $\frac{s_{ik}}{S_i}$. When $a = 1$, the prize won by group i is shared equally among all members. We find that for $\upsilon = 0$ that $\frac{\partial S}{\partial \kappa} \gtrless 0$ if and only if $a \lessgtr 1/n$. When $a \to 1$ (and hence $a > 1/n$) the sharing rule tends in favour of equal distribution which is independent of agents' effort. Therefore, given an increase in fellow group members' consumption externalities, an incentive exists to reduce effort in order to reduce the exposure of externalities by reducing the share of the group's winning share. This is a perverse incentive in that the agent, in order to reduce exposure from fellow group members' externalities, would rather lower their team share of the prize in the group contest. This is a direct consequence of not being able to transfer externalities by investing in effort (and hence consumption) as $a \to 1$. When effort (and hence consumption) can transfer externalities, that is, when $a \to 0$ ($a < 1/n$), then an increase in group members' consumption externalities results in an incentive to increase effort in order to not only crowd out rivals, but also to crowd out fellow group members from consuming the prize.

a 'pure' transboundary pollutant where the (marginal) damages caused by the utilisation of the rights are independent of the geographical area in which they are produced ($\upsilon = \phi$). Therefore, this contest may produce effort levels similar to a contest with no consumption externalities. In these cases, both effects types perfectly counterbalance each other.

Like many rents, the effects of consumption may persist throughout time. It is easy to illustrate the augmented effect of persistent externalities.[7] To represent this, we allow consumption externalities to persist throughout (infinite) time and 'decay' at a rate η in each year (from the initial consumption) with a discount factor $\rho = \frac{1}{1+r}$, where $r > 0$ is the discount rate. Therefore, present value consumption externalities are given by: $\frac{1}{1-\eta\rho}\omega[\upsilon - \phi]$. When $\eta = 0$ consumption externalities are simply the value in the present period whereas when $\eta = 1$ we have indefinite consumption externalities (which is discounted throughout time). The varying levels of persistence can be seen by focusing on resource rights where the levels of decay can vary dramatically from a few weeks (such as methane emissions) to hundreds of years (such as radioactive material).

An increase in persistence will increase the distortion in equilibrium rent seeking effort. If $\omega[\upsilon - \phi] > 0$ then $\frac{\partial s^*}{\partial \eta} > 0$ whereas if $\omega[\upsilon - \phi] < 0$ then $\frac{\partial s^*}{\partial \eta} < 0$. When marginal consumption externalities are positive and the effect is relatively 'local' then agent i will increase rent seeking effort. Therefore, as persistence increases, this augments the marginal benefit to agent i so that rent seeking effort is increased.

2.2 Asymmetric valuations

For tractability, let us assume a contest consists of two participating agents $k = L, H$ which have asymmetric values of the rent. In particular, agents L and H have values of the rent A_L and A_H respectively, where, without loss of generality, we assume that $A_L < A_H$ so that agent H has a larger valuation of the rent than agent L.

We focus, for the rest of the paper, on negative consumption externalities and nor-

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[7]Here we assume that consumption of the prize is a 'one-off' event and in future periods only decaying consumption externalities are experienced. Therefore the strategic decisions are static. It is also possible to introduce a dynamic game in that we have repeated consumption with a stock of externalities being influenced by the historical levels of externalities and current period consumption. For dynamic and repeated contests see Cairns (1989), Leininger and Yang (1994), Wirl (1994) and Shaffer and Shogren (2008)

malise the size of externalities to $\omega = -1$ and the private consumption benefit (damage) so that $v = 0$. It is also possible to consider positive consumption externalities of varying degrees. Each agent experiences a level of negative externalities equal to the consumption of the opponent's share of the rent. Formally, agents' payoff functions are given by:

$$\Pi_L = \frac{s_L}{s_L + s_H} A_L - \frac{\phi s_H}{s_L + s_H} A_L - s_L \tag{3}$$

$$\Pi_H = \frac{s_H}{s_L + s_H} A_H - \frac{\phi s_L}{s_L + s_H} A_H - s_H \tag{4}$$

From (3) and (4), each agent obtains the rent equal to their investment in effort relative to total effort with the addition of negative externalities from rivals' consumptions.

Differentiating (3) and (4) with respect to s_L and s_H respectively and solving for optimal values of effort s_L^* and s_H^* yields:[8]

$$s_k^* = (1 + \phi) \frac{\gamma}{(1 + \gamma)^2} A_k \qquad \text{for } k = L, H \tag{5}$$

where $\gamma \equiv \frac{A_L}{A_H}$. The corresponding equilibrium payoffs for agent k is:

$$\Pi_k^* = \frac{A_k}{(1 + \gamma)} \left[\gamma - \phi - \frac{\gamma(1 + \phi)}{(1 + \gamma)} \right] \qquad \text{for } k = L, H \tag{6}$$

From (5), equilibrium effort is distorted up due to negative externalities whilst payoffs are distorted downwards compared to the case without consumption externalities. As shown in Proposition 1, as marginal negative externalities increase, equilibrium effort also increases, to crowd out the rivals and reduce negative consumption externalities.

Note that (5) and (6) can be rewritten so that $\hat{A}_L = (1 + \phi)A_L$ and similarly for agent H. It follows that for valuations $\left(\hat{A}_L, \hat{A}_H \right)$ the analysis has similarities to conventional asymmetric valuation contests (Nti, 1999). In particular, the valuation ratio is identical, in that:

$$\frac{\hat{A}_L}{\hat{A}_H} = \frac{(1 + \phi)A_L}{(1 + \phi)A_H} = \frac{A_L}{A_H} = \gamma \tag{7}$$

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[8] The second order conditions are satisfied at the optimal values of rent-seeking.

We now show how agents' rent seeking efforts and payoffs change given alternative regulatory mechanisms to control externalities.

3 Equilibrium effort and payoffs under the establishment of Pigouvian taxation and Coasean bargaining

3.1 The regulator's optimal choice of price or quantities

As before, let us assume that in the regulator's economy there are two agents with asymmetric valuations which obtain benefit from obtaining a share of the aggregate regulator's prize which produces consumption externalities. To control for externalities, the regulator has the ability to establish Pigouvian and Coasean mechanisms. In our context, externalities are transferable in that an agent's consumption can transfer externalities to the rival. As Baumol and Oates (1988) and Bird (1987) explain, to reach an efficient outcome, a unit tax should be set equal to the marginal social damage caused by transferring externalities to the rival. In this case, the Pigouvian tax is levied at $\tau^* = \phi$. Further, allowing the prize to be ex-post allocated reaches an efficient outcome. The agent with the highest willingness to pay (to possibly avoid exposure from externalities) can purchase the prize and compensate the rival in the form of price per unit. Similar to the Pigouvian tax, the amount of ex post reallocated rent allowed for an efficient outcome is equal to the total optimal damage $\frac{\phi s_L^*}{s_L^* + s_H^*} A_H^* + \frac{\phi s_H^*}{s_L^* + s_H^*} A_L^*$. As it is well known how Pigouvian taxation and Coasean bargaining are optimally chosen in a regulatory system (See, for example, Bamoul and Oates, 1988), we instead focus on the consequence for equilibrium rent seeking and payoffs given these mechanisms are chosen optimally by the regulator. In particular, it is clear that, given complete certainty in the benefits and costs of the production of externalities, the use of price (Pigouvian taxes) and quantity (Coasean bargaining) mechanisms should yield the same result (Weitzman, 1974). In our context, this means from the viewpoint of expected costs, allowing a rent to be ex post reallocated

will result in the same outcome as if a tax per unit of externality is levied. However, an open question arises with respect to whether Pigouvian taxes or Coasean bargaining is the preferred regulatory mechanism when the resources are allocated through a contest. In particular, how do Pigouvian taxation and Coasean ex post reallocation responses to consumption externalities differ in their influence on socially wasteful rent seeking effort and agents' payoffs?

3.2 Pigouvian taxation

The first approach to controlling externalities is the introduction of a tax based on consumption and hence the transfer of externalities. In this context, an agent that wins a share of the rent must also pay the unit tax applicable to the damage caused by their consumption externalities.[9] For the tax rate τ^* based on the level of consumption externalities produced by agent L and H, their payoffs become:

$$\Pi_L^P = \frac{s_L}{s_L + s_H} A_L - \frac{\phi s_H}{s_L + s_H} A_L - \frac{\tau^* s_L}{s_L + s_H} A_L - s_L \qquad (8)$$

$$\Pi_H^P = \frac{s_H}{s_L + s_H} A_H - \frac{\phi s_L}{s_L + s_H} A_H - \frac{\tau^* s_H}{s_L + s_H} A_H - s_H \qquad (9)$$

which is solved for (s_L^P, s_H^P).

From (8), agent L 's tax burden is equal to the amount of externalities produced by its consumption $\frac{\tau^* s_L}{s_L + s_H} A_L$ and similarly for agent H. As the marginal rate of externalities are assumed to be identical for both agents (the marginal increase in agents' share of the rent is equal to ϕ), one simply needs to levy a common tax rate $\tau_L^* = \tau^* = \tau_H^*$.[10]

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[9]Here, the tax is based on the share of the rent consumed by each player. Other possible taxes exist which may include taxation of realised profits or rent-seeking expenditures (Glazer and Konrad, 1999; Katz and Rosenberg, 2000; Epstein and Nitzan, 2002).

[10]It is also possible to have asymmetric tax rates based on the heterogeneous distribution of the externality. For example, this occurs in many pollution problems that have non-uniform transboundary spillover rates, such as the case for SO_2 emissions in the United States (Ellerman et al., 2000).

3.3 Coasean bargaining

An alternative solution is to allow the rent to be reallocated among the agents ex post.[11] The clearest example of such a process is the recent introduction of tradable pollution permit markets where firms through the (partial) process of rent seeking obtain an initial allocation of permits and are allowed to ex post trade (Hanley and MacKenzie, 2009).

We follow a framework similar to Dari-Mattiacci et al. (2009) but allow the market price of the rent to be determined by the bargaining power of both agents and independent of shares won in the contest. Formally, the price is determined by

$$\mu = \alpha A_L + (1 - \alpha) A_H \tag{10}$$

where α is the bargaining power of agent H and $A_L < \mu < A_H$.[12] From (10), an increase in the bargaining power of the high valuing agent (agent H) results in the market price of the rent decreasing. This determines a common value of the rent, which as we will see below, has important implications for effort strategies.

Given an ex post reallocation of the rent is possible, it is efficient to allow low value agents to sell their share of the rent to high value agents. As agent L is the lowest value agent, any share of the rent won on the contest is sold to agent H at the price determined in (10). This price is at least as big as agent L's valuation of the rent. Therefore, agent L sells all rent won in the contest to agent H and obtains the revenue $\frac{s_L}{s_L + s_H}\mu$ but experiences all consumption externalities ϕA_L. Formally, agent L's payoff function is:

$$\Pi_L^C = \frac{s_L}{s_L + s_H}\mu - \phi A_L - s_L \tag{11}$$

which is optimally solved for s_L^C.

For agent H, additional to the share of rent won in the contest $\frac{s_H}{s_L + s_H}A_H$, it has the opportunity to purchase the remaining rent at a price that is lower that its own

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[11]Ex post reallocation (resale) has been extensively discussed by Gupta and Lebrun (1999), Zheng (2002), Haile (2003), Dari-Mattiacci et al. (2009), Garratt and Tröger (2006), Hafalir and Krishna (2008) and Sui (2009).

[12]In order for tractability, we assume exogenous bargaining however there are other possibilities. Most bargain problems proceed with exogenous bargaining power, such as the Nash bargaining program.

valuation. Therefore the additional benefit obtained due to Coasean bargaining is given as $\frac{s_L}{s_L + s_H}(A_H - \mu)$. As agent H now consumes the entire rent after ex post reallocation, it no longer experiences any negative externalities from the rival. Formally, agent H 's payoff function is denoted by:

$$\Pi_H^C = \frac{s_H}{s_L + s_H} A_H + \frac{s_L}{s_L + s_H}(A_H - \mu) - s_H \tag{12}$$

which is optimally solved for s_H^C.

3.4 Comparison of Pigouvian and Coasean solutions

3.4.1 Equilibrium strategies

As shown in Appendix B, the equilibrium effort levels for both policy mechanisms are now independent of marginal consumption externalities: $s_k^P = \frac{\gamma}{(1+\gamma)^2} A_k$, $s_k^C = \frac{\mu}{4}$ for $k = L, H$. By comparing the equilibrium solutions obtained from (5)-(12), we can directly compare the effects of Pigouvian and Coasean solutions compared to both the benchmark model and the model where consumption externalities exist. The next proposition solves the equilibrium effort solutions and provides a ranking.

Proposition 2 *The ranking of agents' equilibrium efforts are, for agent L:*

$$\begin{aligned} s_L^B &= s_L^P < s_L^C \\ s_L^P &< s_L^* \end{aligned}$$

Similarly, for agent H:
$$s_H^B = s_H^P < s_H^*$$

where there exists a threshold bargaining power $\alpha^*(H) \equiv \frac{1-\gamma}{(1+\gamma)^2}$ such that

$$s_H^C \lessgtr s_H^P \text{ if } \alpha^0 \gtrless \alpha^*(H)$$

Proof. See Appendix B. ∎

Proposition 2 shows that introducing Pigouvian taxation reduces effort in the contest compared to a contest with uncontrolled consumption externalities and the effort chosen is identical to a contest with no consumption externalities $s_k^B = s_k^P$.[13] This is easily understood as the Pigouvian tax directly targets agents' external costs of their externalities. In terms of comparing Pigouvian and Coasean solutions, the intuition is as follows. Coasean bargaining allows agent H to obtain the rent by trading which softens her incentive to supply effort. We can see this further by analysing $\alpha^*(H)$, the bargaining power of agent H that equates effort from Pigouvian and Coasean policies. For $\alpha^0 > \alpha^*(H)$ a sufficiently high bargaining power of agent H will result in an effort level lower than that in the Pigouvian case, as the rent can be purchased at a lower price on the ex post market. If agent H reduces effort in the contest, this increases competition in the contest and this coupled with the fact that agent L can sell the rent on the market (for a higher value) results in agent L having larger rent seeking under Coasean bargaining.

Interesting results occur when one considers effort of Coasean bargaining compared to a contest with uncontrolled externalities (s_k^*). Allowing for an additional regulatory mechanism to control for externalities may, in some cases, actually increase the equilibrium level of socially unproductive effort. The following corollary provides a comparison of the effort of agents.

Corollary 1 *For rents that produce consumption externalities, allowing Coasean bargaining may either increase or decrease rent seeking for both agents relative to the case with uncontrolled externalities ($s_k^C \gtrless s_k^*$ for $k = L, H$).*

Proof. See Appendix B. ∎

A counter-intuitive result from Corollary 1 shows that allowing Coasean bargaining may actually increase the total socially unproductive effort. In the existing literature, Dari-Mattiacci et al. (2009) find a similar result without the inclusion of consumption externalities. To see the intuition of our result, let us directly compare the equilibrium efforts

---

[13]One can also extend this to an incomplete information setting (Malueg and Yates, 2004; Fey, 2008). Allowing types $A_i \in \{A_L, A_H\}$, each agent has a probability of drawing A_L with $1/2$ and A_H with $1/2$. Given agents' own types are private information and their rival's type is drawn from the probability distribution, we find qualitatively similar results in that $s_k^B = s_k^P < s_k^*$ for $k = L, H$. Therefore, Pigouvian taxation in the incomplete information setting continues to reduce efforts.

for Coasean bargaining and no additional policy respectively, $s_k^C = \frac{\mu}{4}$; $s_k^* = (1+\phi)\frac{\gamma}{(1+\gamma)^2}A_k$ for $k = L, H$. There are two important differences. Firstly, introducing Coasean bargaining eliminates the dependence on consumption externalities ϕ. This occurs as Coasean bargaining allows the lowest value agent to suffer all externalities due to selling all rents to the high value agent. Effort is now independent of the relative globalness of negative externalities. It follows then that $s_k^C > s_k^*$ when the relative globalness is low. This is in line with Proposition 1 as the lower relative globalness of externalities results in lower equilibrium effort (s_k^*). Second, notice that in Coasean bargaining each agent selects effort based on the market value of the rent instead of simply their own valuation. Therefore, it is possible to have $s_k^C > s_k^*$ when the market value of the rent is relatively high. This can occur for two main reasons. From (10), the market value of the rent is determined by both the bargaining power of agents (α) and their valuations. Therefore, both agents increase effort when either α is low (the low value agent has improved bargaining power) or when agent H's valuation increases (i.e. a low γ). Given a reduction in the level of α, agent H now has to pay relatively more for agent L's share of the rent, hence there is an incentive for agent H to increase effort to avoid paying the additional price. Similarly, as agent L has a better bargaining position it has an incentive to increase effort to obtain a larger share of the rent to sell to agent H. Similar logic applies for an increase in the value agent H places on the prize.

From Proposition 2, it is unclear in terms of aggregate efforts, which policies produce the highest and lowest aggregate efforts as agent L invests more effort under Coasean bargaining whereas agent H may invest more under Pigouvian taxation. Denoting aggregate efforts by F, direct comparison of aggregate efforts show that $F^P = F^B < F^*$. As expected, the Pigouvian and benchmark efforts are identical and lower effort compared to a contest that does not control externalities. However, the ranking when one considers Coasean bargaining is ambiguous. For $F^C \gtrless F^P$ then

$$\frac{\alpha(\gamma - 1) + 1}{2} - \frac{\gamma}{(1 + \gamma)} \gtrless 0 \tag{13}$$

and similarly for $F^C \gtreqless F^*$ then

$$\frac{\alpha(\gamma - 1) + 1}{2} - (1 + \phi)\frac{\gamma}{(1 + \gamma)} \gtreqless 0 \tag{14}$$

From (13), it can be seen that aggregate rent seeking effort from Pigouvian and Coasean solutions are equal $F^C = F^P$ when the sufficient condition is met: $\alpha^{*F} \equiv \frac{1}{1+\gamma}$. This shows that the value of bargaining power where $F^C = F^P$ is given by the range $[\frac{1}{2}, 1]$. As shown above, we can observe that Pigouvian taxation produces the lowest aggregate rent seeking when $\alpha^0 < \alpha^{*F}$, that is, when agent H has low bargaining power. Indeed, Pigouvian taxation always results in lower aggregate effort when agent L has the majority of the bargaining power, that is, $\alpha^0 < \frac{1}{2}$. However, when we compare aggregate efforts under Coasean bargaining to aggregate efforts in the contest with uncontrolled externalities, we see that the degree of globalness does have a role to play. From (14), an increase in the degree of globalness places downward pressure on the aggregate efforts of Coasean policies, thus making it more likely to be smaller than aggregate efforts from a contest with uncontrolled externalities. It is clear that, given $F^C < F^P$, we must also have $F^C < F^*$. However, when $F^C > F^P$, the degree of globalness will determine whether $F^C > F^*$ or $F^C < F^*$.

3.4.2 Equilibrium payoffs

Let us now turn to the comparison of equilibrium payoffs for both agents. The next corollary provides a comparison of payoffs.

Corollary 2 *The equilibrium payoffs for agent L are:*

$$
\begin{aligned}
\Pi_L^P &< \Pi_L^* < \Pi_L^B \\
\Pi_L^P &< \Pi_L^C; \Pi_L^C \gtreqless \Pi_L^B; \Pi_L^C \gtreqless \Pi_L^*
\end{aligned}
$$

and for agent H are:

$$\Pi_H^P < \Pi_H^* < \Pi_H^B < \Pi_H^C$$

Proof. See Appendix C. ∎

Corollary 2 shows that for both agents, the introduction of a Pigouvian tax produces the lowest possible payoff from the contest. In particular, it can be seen that Coasean bargaining tends to produce larger payoffs for both agents than Pigouvian taxation. Our results show that the ranking for agent H is clear: Coasean bargaining produces the largest payoffs. Agent H has the ability to purchase additional shares of the rent on the market at a price lower than agent H's value. As a consequence, agent H consumes the entire rent and experiences no negative externalities (which is borne solely by agent L). Indeed, this is the reason why Coasean bargaining for agent L may produce an ambiguous payoff ranking compared to the benchmark and uncontrolled externalities models. The payoff ranking for agent L crucially depends on the relative globalness of externalities ϕ. We find for a large ϕ, agent L will obtain a lower payoff compared to the benchmark model and when externalities are uncontrolled.

3.5 Pigouvian taxation with lump-sum transfers

Up to this point, we have not discussed what happens to the Pigouvian tax revenue. In this setting, it is possible for the regulator to redistribute tax revenues to the participating agents. To begin, notice that, in terms of agents' rent valuations, aggregate tax revenue is given by:

$$\phi A_L \leq \frac{\phi\left(s_L A_L + s_H A_H\right)}{s_L + s_H} \leq \phi A_H \tag{15}$$

Using this tax revenue, It is possible to show that Pareto improvements do exist, compared to conventional Pigouvian taxation. In particular,

Proposition 3 *A Pareto improvement occurs in a Pigouvian taxation system when tax revenues are redistributed to the agent with the lowest value. That is, $\Pi_L^{PN} \geq \Pi_L^B \geq \Pi_L^P$ and for threshold consumption externalities $\phi^T \equiv \frac{\alpha(1-\gamma)+1}{4\gamma} - (\frac{\gamma}{1+\gamma})^2$, $\Pi_L^{PN} \geq \Pi_L^C \geq \Pi_L^P$ for $\phi > \phi^T$.*

Proof. See Appendix D. ∎

Proposition 3 shows that transferring all tax revenue to the agent with the lowest value produces a Pareto improvement. As the revenue is distributed lump-sum, rent seeking efforts are still maintained at their conventional Pigouvian levels. Further, the payoff for agent L is at least as large as the payoff obtained from the benchmark model (with no consumption externalities). Notice that this occurs with the lowest amount of tax revenue distributed to agent L (ϕA_L).[14] Importantly, transferring the tax revenue to agent H, even the largest possible amount ϕA_H, cannot increase payoffs above the benchmark or Coasean solutions. One issue with Coasean bargaining is that agent L will experience all consumption externalities and as a consequence, their payoff is low. However, given Pigouvian taxation with revenue recycling and where consumptions externalities are sufficiently large ($\phi \geq \phi^T$), it follows that agent L's payoff is larger than under Coasean bargaining.

4 Conclusions

Many situations that involve expending effort for common contestable rents are affected by the consumption of the rent. Given this, it is important to understand how equilibrium effort changes when consumption externalities exists and how alternative policies to control externalities affect effort levels and payoffs. Unlike standard contest problems, the utilisation of the rent can produce local benefits (costs) as well as both positive and negative global externalities from rivals' consumption. Therefore, the purpose of this paper is to investigate incentives behind effort when the consumption of an excludable and rivalrous rent produces a consumption effect and compare the effects of Pigouvian and Coasean regulatory mechanisms.

Our simple model allows agents to rent seek over a common contestable rent which has the potential to produce consumption externalities. These consumption externalities are

---

[14]Improvements in agents' payoffs can be seen more clearly when we assume symmetric agents. In such a cases, the revenue transfer is ϕA. It is easy to show that a Pareto improvement exists between the contest with consumption externalities and a contest with revenue neutral Pigouvian taxation for *any* tax revenue transfer. Further, aggregate payoffs also increase with respect to the benchmark contest without consumption externalities given the transfer belongs to the set where for transfers $R_i > \frac{\phi A}{4}$, $R_i + R_{-i} \leq \phi A$ for transfers R_i, R_{-i}.

in addition to those experienced in conventional contests. We find that in the equilibrium of the contest, agents' optimal actions decrease in the index of relative globalness. The intuition for this result is as follows. For positive consumption externalities, agents have an incentive to decrease effort when relative globalness increases as the agent can free ride. Considering negative consumption externalities, an increase in globalness results in an incentive to increase effort in order to crowd out rivals' allocation of the rent, reduce rivals' share of consumption and hence negative externalities that they produce. Allowing additional polices to control externalities, namely Pigouvian taxation and Coasean bargaining, alters the amount invested in effort. In particular, Pigouvian taxation tend to lower the amount of resources used where this may not always happen under Coasean bargaining.

Considering the findings in this paper, it is important for governments and policymakers to understand when, and to what extent, effort plays an important role in determining the composition of the contestable rent. To this end, regulators need to carefully consider the size of the rent and the characteristics of consumption externalities before initiating regulation. In particular, for many policy considerations, Pigouvian taxation coupled with some element of lump-sum transfer may reduce costly rent seeking effort and improve the welfare of some agents compared to other approaches. Future work may focus on intertemporal aspects of consumption externalities where the utilisation of the prize and the subsequent exposure to externalities may happen in multiple time periods and where externalities persist throughout time. Further studies may focus on the informational settings within this model and introduce asymmetric information among agents.

Acknowledgements

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Appendix A

Proof of Proposition 1:

Proof. The objective function for agent i with consumption externalities is given by:

$$\max_{s_i} \Pi_i = L_i + E_i - s_i \text{ for all } i \in \Theta$$

substituting in (1) and solving, the symmetric Nash equilibrium is given by:

$$s^* = \omega \left[v - \phi \right] \frac{(n-1)}{n^2} A$$

For the conventional Tullock (1980) contest the symmetric Nash equilibrium is:

$$s^B = \frac{(n-1)}{n^2} A$$

comparison of the two rent seeking strategies yields the result. ∎

Appendix B

Proof of Proposition 2:

Proof. We start by solving the Pigouvian tax and Coasean bargaining payoff functions. For a Pigouvian tax, differentiating (8) and (9) with respect to s_L and s_H respectively, yields:

$$
\begin{aligned}
\frac{\partial \Pi_L}{\partial s_L} &= \frac{s_H}{(s_L + s_H)^2} A_L + \frac{\phi s_H}{(s_L + s_H)^2} A_L - \frac{\tau^* s_H}{(s_L + s_H)^2} A_L - 1 \\
\frac{\partial \Pi_H}{\partial s_H} &= \frac{s_L}{(s_L + s_H)^2} A_H + \frac{\phi s_L}{(s_L + s_H)^2} A_H - \frac{\tau^* s_L}{(s_L + s_H)^2} A_H - 1
\end{aligned}
$$

equating each to zero and solving, we obtain the effort strategies for agents L and H respectively:

$$
\begin{aligned}
s_L^P &= A_H \beta \left(\frac{A_L}{A_L + A_H} \right)^2 \\
s_H^P &= A_L \beta \left(\frac{A_H}{A_L + A_H} \right)^2
\end{aligned}
$$

where $\beta = (1 - \tau^* + \phi)$. The optimal Pigouvian tax holds when $\tau^* = \phi$ which reduces the equilibrium effort strategies to:

$$s_k^P = \frac{\gamma}{(1+\gamma)^2} A_k \tag{A1}$$

where $\gamma \equiv \frac{A_L}{AH} \in (0,1)$ and $k = L, H$.

Let us now consider effort in the presence of Coasean bargaining. Differentiating (11)

and (12) with respect to s_L and s_H respectively, yields

$$\frac{\frac{s_H}{(s_L + s_H)^2}\mu - 1}{\frac{s_L}{(s_L + s_H)^2}A_H - \frac{s_L}{(s_L + s_H)^2}(A_H - \mu) - 1}$$

equating to zero and rearranging gives:

$$\frac{s_H}{(s_L + s_H)^2}\mu - 1 = 0$$
$$\frac{s_L}{(s_L + s_H)^2}\mu - 1 = 0$$

solving this we obtain the equilibrium effort strategies of agents L and H with the potential for ex post reallocation:

$$s_L^C = \frac{\mu}{4} = s_H^C \tag{A2}$$

Let us start with the case for agent L. Given the equilibrium of effort for agent L in (5), (A1) and (A2), we can now compare equilibrium payoffs. It is clear from (A1) that $s_L^B = \frac{\gamma}{(1+\gamma)^2}A_L = s_L^P$. Next for $s_L^P < s_L^C$ (A1) and (A2) yield:

$$s_L^P = \frac{\gamma}{(1 + \gamma)^2}A_L < \frac{\mu}{4} = s_L^C$$

which can be transformed to:

$$\frac{\gamma^2}{(1 + \gamma)^2} - \frac{\alpha(\gamma - 1) + 1}{4} < 0$$

which always holds for $\gamma > 0$. For completeness we also can show $s_L^P < s_L^*$ where using (A1) and (5) yields:

$$s_L^P = \frac{\gamma}{(1 + \gamma)^2}A_L < (1 + \phi)\frac{\gamma}{(1 + \gamma)^2}A_L = s_L^*$$

which holds as $1 < (1 + \phi)$.

Let us turn to agent H. For $s_H^B = s_H^P$ it is clear to see that:

$$s_H^B = \frac{\gamma}{(1+\gamma)^2} A_H = s_H^P$$

To show $s_H^P < s_H^*$:

$$s_H^P = \frac{\gamma}{(1+\gamma)^2} A_H < (1+\phi)\frac{\gamma}{(1+\gamma)^2} A_H = s_H^*$$
$$1 < (1+\phi)$$

which holds given $\phi, \gamma > 0$. The case of s_H^P and s_H^C is ambiguous as

$$\frac{\gamma}{(1+\gamma)^2} - \frac{\alpha(\gamma-1)+1}{4}$$

has ambiguous sign. Setting to zero and rearranging for α yields $\alpha^*(H) \equiv \frac{1-\gamma}{(1+\gamma)^2}$ where $\alpha^*(H)$ occurs when $s_H^P = s_H^C$. Therefore, for $\alpha^0 \gtrless \alpha^*(H)$ then $s_H^C \lessgtr s_H^P$. ∎

Proof of Corollary 1:

Proof. For $s_L^C < s_L^*$ we have

$$s_L^C = \frac{p}{4} < (1+\phi)\frac{\gamma}{(1+\gamma)^2} A_L = s_L^*$$
$$\frac{\alpha(\gamma-1)+1}{4} - (1+\phi)\frac{\gamma^2}{(1+\gamma)^2}$$

which has an ambiguous sign and finally $s_H^C < s_H^*$:

$$s_H^C = \frac{p}{4} < (1+\phi)\frac{\gamma}{(1+\gamma)^2} A_H = s_H^*$$
$$\implies \frac{\alpha(\gamma-1)+1}{4} - (1+\phi)\frac{\gamma}{(1+\gamma)^2}$$

also with ambiguous sign. ∎

Appendix C

Proof of Corollary 2:

Proof. Using the equilibrium effort strategies found in the proof of Proposition 2 and substituting into the payoff functions yields the following:

$$
\begin{aligned}
\Pi_L^P &= A_L\left(\left(\frac{\gamma}{(1+\gamma)}\right)^2 - \phi\right) \\[2mm]
\Pi_H^P &= A_H\left(\left(\frac{\gamma}{(1+\gamma)}\right)^2 - \phi\right) \\[2mm]
\Pi_L^C &= \frac{\mu}{4} - \phi A_L \\[2mm]
\Pi_H^C &= A_H - \frac{3\mu}{4}
\end{aligned}
$$

Let us begin by comparing the payoffs for agent L:

$\Pi_L^P < \Pi_L^B$:

$$
\begin{aligned}
\Pi_L^P &= A_i\left(\left(\frac{\gamma}{(1+\gamma)}\right)^2 - \phi\right) < \frac{\gamma^2}{(1+\gamma)^2}A_i = \Pi_i^B \\
&\implies -A_i\phi < 0
\end{aligned}
$$

which holds.

$\Pi_L^P < \Pi_L^C$:

$$
\begin{aligned}
\Pi_L^P &= A_L\left(\left(\frac{\gamma}{(1+\gamma)}\right)^2 - \phi\right) < \frac{\mu}{4} - \phi A_L = \Pi_L^C \\
&\implies \frac{\gamma^3}{(1+\gamma)^2} - \frac{\alpha(\gamma-1)+1}{4} < 0
\end{aligned}
$$

which holds for given parameters.

$\Pi_L^P < \Pi_L^*$:

$$\Pi_L^P = A_L \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) < \frac{A_L}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right] = \Pi_L^*$$

$$\implies \left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi < \frac{\gamma}{(1+\gamma)} - \frac{\phi}{(1+\gamma)} - \frac{\gamma(1+\phi)}{(1+\gamma)^2}$$

$$\implies -\phi\gamma^2 < 0$$

which must also hold.

$\Pi_L^* < \Pi_L^B$:

$$\Pi_L^* = \frac{A_L}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right] < \frac{\gamma^2}{(1+\gamma)^2} A_L = \Pi_L^B$$

$$\implies -\phi - 2\phi\gamma < 0$$

Π_L^C and Π_L^* is ambiguous as:

$$\frac{p}{4} - \phi A_L - \frac{A_L}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right]$$

$$\implies \frac{\alpha(\gamma-1)+1}{4} < (>) \frac{\gamma^3}{(1+\gamma)^2}(1+\phi)$$

For Π_L^C and Π_L^B it is also ambiguous:

$$\frac{\alpha(\gamma-1)+1}{4} - \phi\gamma - \frac{\gamma^3}{(1+\gamma)^2}$$

$$\implies \frac{\alpha(\gamma-1)+1}{4} < (>) \frac{\gamma^3}{(1+\gamma)^2} + \phi\gamma$$

We now do similar analysis for Agent H:

$\Pi_H^P < \Pi_H^B$:

$$\Pi_H^P = A_H \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) < \frac{\gamma^2}{(1+\gamma)^2} A_H = \Pi_H^B$$

$$\implies -\phi < 0$$

which always holds.

$\Pi_H^P < \Pi_H^C$:

$$\Pi_H^P \quad = \quad A_H \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) < A_H - \frac{3\mu}{4} = \Pi_H^C$$

$$\implies \quad \left(\frac{\gamma}{(1+\gamma)} \right)^2 - (1+\phi) - \frac{\alpha(\gamma-1)+1}{4} < 0$$

$\Pi_H^P < \Pi_H^*$:

$$\Pi_H^P \quad = \quad A_H \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) < \frac{A_H}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right] = \Pi_H^*$$

$$\implies \quad \left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi < \frac{\gamma}{(1+\gamma)} - \frac{\phi}{(1+\gamma)} - \frac{\gamma(1+\phi)}{(1+\gamma)^2}$$

$$\implies \quad -\phi\gamma^2 < 0$$

Which must also hold

$\Pi_H^C > \Pi_H^*$:

$$\implies \quad A_H - \frac{3\mu}{4} > \frac{A_H}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right]$$

$$\frac{1}{4} \quad > \quad \frac{3\alpha(\gamma-1)}{4} + \frac{\gamma^2 - \phi(1+2\gamma)}{(1+\gamma)^2}$$

$\Pi_H^* < \Pi_H^B$:

$$\Pi_i^* \quad < \quad \Pi_i^B$$

$$\implies \quad \frac{A_H}{(1+\gamma)} \left[\gamma - \phi - \frac{\gamma(1+\phi)}{(1+\gamma)} \right] < \frac{\gamma^2}{(1+\gamma)^2} A_H$$

$$\implies \quad -\phi - 2\phi\gamma < 0$$

which always holds.

$\Pi_H^C > \Pi_h^B$:

$$\Pi_H^C \quad > \quad \Pi_H^B$$

$$\implies \quad 1 - \frac{\gamma^2}{(1+\gamma)^2} - \frac{3\alpha(\gamma-1)+1}{4} > 0$$

■

Appendix D

Proof of Proposition 3:

Proof. To compare $\Pi_L^{PN} \geq \Pi_k^B$ this can be rewritten as:

$$A_L \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) + R_L \geq \frac{\gamma^2}{(1+\gamma)^2} A_L$$

where R_L is the lump-sum transfer to agent L. This is reduced to:

$$-\phi A_L + R_L \geq 0$$

From expression 15 we know that $R_L \geq \phi A_L$. Therefore it holds that $-\phi A_L + R_L \geq 0$. Next, let us compare $\Pi_L^{PN} \geq \Pi_L^C$ which is:

$$A_L \left(\left(\frac{\gamma}{(1+\gamma)} \right)^2 - \phi \right) + R_L \geq \frac{\mu}{4} - \phi A_L$$

and this means that $\Pi_L^{PN} \geq \Pi_L^C$ when for $\phi > \phi^T$ where ϕ^T is given by:

$$\phi^T = \frac{\alpha(1-\gamma)+1}{4\gamma} - (\frac{\gamma}{1+\gamma})^2$$

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