Public Debt in Economies with Heterogeneous Agents*

Anmol Bhandari, David Evans, Mikhail Golosov, Thomas J. Sargent

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Abstract

We study public debt in an economy in which taxes and transfers are chosen optimally subject to heterogeneous agents' diverse capacities to pay. We assume a government that commits to policies and can enforce tax and debt payments. If the government enforces perfectly, asset inequality is determined in an optimum competitive equilibrium but the level of government debt is not. In addition, welfare increases if the government commits not to enforce private debt contracts and introduces borrowing frictions. By doing so, it reduces competition on debt markets and gathers monopoly rents from providing liquidity. Regardless of whether the government chooses to enforce private debt contracts, the level of initial government debt does not affect an optimal allocation, but the distribution of net assets does.

Key words: Distorting tax. Transfers. Government debt. Ricardian equivalence.

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If, indeed, the debt were distributed in exact proportion to the taxes to be paid so that every one should pay out in taxes as much as he received in interest, it would cease to be a burden.... if it were possible, there would be [no] need of incurring the debt. For if a man has money to loan the Government, he certainly has money to pay the Government what he owes it. Simon Newcomb (1865, p.85)

1 Introduction

To understand whether a government's debt is too high or too low requires knowing who owes what, when, to whom. That impels studying balance sheets of both creditors and debtors as well as the budget sets that appear in a coherent economic model and leads to distinguishing superficial from substantive features by tracking and properly consolidating assets and liabilities. We seek a workable description of features of government debt that affect continuation allocations and prices. For that purpose, this paper studies an economy with people who differ in their productivities and a government that administers a non-linear tax on labor earnings. Agents and the government trade one-period bonds. There is no capital. The economy starts with an exogenously given distribution of debt across agents and the government. Taxes are restricted by agents' abilities to pay. Public policies are chosen at time 0 i.e., the government *commits*.

The structure of budget constraints implies that the cross-section distribution of initial net assets, not gross assets, affects the set of feasible allocations that can be implemented in competitive equilibria. An increase in initial government debt that is shared equally among all agents leaves the distribution of net assets unchanged and therefore also leaves an equilibrium allocation unaltered. This outcome embodies ideas proclaimed by Simon Newcomb (1865) in the quotation above. The logic applies to structures with and without physical capital, with complete or incomplete asset markets, and with more general tax structures.

The role of government debt crucially depends on how well private debt contracts are enforced. If both tax and debt obligations are enforced perfectly, then agents' borrowing is restricted only by their abilities to repay their debts and an optimal level of government debt is indeterminate. In this case, any sequence of government

debts is optimal and a version of Ricardian equivalence holds despite the fact that taxes distort private agents' decisions. The dynamics of asset inequality, however, share some of the qualitative features of the dynamics of debt in representative agent models in which transfers are ruled out.

We also show that welfare increases if the government commits not to enforce private debt contracts so that agents can borrow only up to an exogenous, ad hoc debt limit. In this case, government debt provides an additional instrument to affect equilibrium allocations. The gains come from monopoly power on the asset market that the government acquire by restricting the ability of private agents to provide liquidity. What matters for this result is not the size of the debt limit per se, but the inability of agents to use anticipated transfers to relax their current borrowing constraints. An optimal level of government debt is determined by a trade-off between obtaining monopoly rents and distorting agents' intertemporal marginal rates of substitution.

There is a sizable literature in macro about debt and Ricardian equivalence, going back at least to Barro (1974). It is well understood that in representative agent economies the role of debt hinges on whether lump sum taxes are allowed. But in the context of those models, there is no inherent economic reason to rule them out. Proportional labor taxes, often assumed in such models, are counterfactual since transfers are a large part of modern tax systems (see, e.g., Figure 1). In our settings, agents are heterogeneous, taxes are restricted by agents' ability to pay, and the government chooses taxes to maximize a weighted average of lifetime utilities of agents.

Werning (2007) obtained counterparts to our results about net versus gross asset positions in a complete markets economy with heterogeneous agents, an affine tax structure, and transfers that are unrestricted in sign. Because he allowed unrestricted taxation of initial assets, the initial distribution of assets played no role. Our lemma 2 and its corollaries extend Werning's results by showing that all distributions of gross assets among private agents and the government that imply the same net asset positions lead to the same equilibrium allocation, a conclusion that holds for market structures beyond complete markets. Werning (2007) characterized optimal allocations and distortions in complete market economies, while Bhandari et al. (2015) and Werning (2012) investigate how precautionary savings motives that

incomplete markets impart both to private agents and to a benevolent government affect optimal allocations.[1]

Our results on desirability of weak enforcement of private debt contracts build on insights in Yared (2012, 2013), who showed that it may not be optimal to undo agents' borrowing frictions even though the government has ability to do so. Bassetto (1999) studied the role of taxation and debt limits in the economies with heterogeneity in which transfers are ruled out.

The rest of our paper is organized as follows. In Section 2, we lay out a baseline environment in which taxes are restricted to be affine functions of labor income and agents are heterogeneous in labor earnings but do not face idiosyncratic uncertainty. In Section 3, we study an economy in which agents' borrowing is restricted only by their ability to pay. In Section 4, we study an economy in which agents face more stringent borrowing constraints. We show that our results extend to idiosyncratic shocks in Section 5 and to richer taxes constrained only by informational frictions in Section 6.

2 Environment

Time is discrete and infinite. There is a government and I types of agents each of mass n_i for $i \in \{1, 2, \ldots, I\}$ with $\sum_{i=1}^{I} n_i = 1$. Preferences of an agent of type i over stochastic processes for consumption $\{c_{i,t}\}_t$ and labor supply $\{l_{i,t}\}_t$ are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right), \tag{1}$$

where $\mathbb{E}_t$ is a mathematical expectations operator conditioned on time t information and β a discount factor. We assume that U^i is increasing and concave in $(c, -l)$. The labor supply of agent i lies in a set $\left[0, \bar{L}_i\right]$. We allow $\bar{L}_i$ to be infinite.

Uncertainty is summarized by a shock s_t governed by an irreducible Markov process that takes values in a finite set S. We let $s^t = (s_0, ..., s_t)$ denote a history of shocks having joint probability density $\pi_t(s^t)$. We use boldface letters $\boldsymbol{x}$ to denote a sequence $\{x_t\left(s^t\right)\}_{t \geq 0, s^t}$. We write $s^{t'} \in s^{t''}$ for $t'' > t'$ if the first t' elements of

[1] Other recent pertinent papers include Azzimonti et al. (2008a,b) and Correia (2010). These authors study optimal policy in economies in which agents are heterogeneous in skills and initial assets.

$s^{t''}$ constitute $s^{t'}$. When it does not cause confusion, we use x_t to denote a random variable with a time t for all s^t. Finally, we define a set of infinite histories $\mathbb{S}^\infty$ such that $s^\infty \in \mathbb{S}^\infty$ satisfies $\pi_t(s^t) > 0$ for all $s^t \in s^\infty$.

Shock s_t affects government expenditures $g_t(s_t)$ and productivity of each individual $\{\theta_{i,t}(s_t)\}_i$. An agent of type i who supplies l_i units of labor produces $y_i \equiv \theta_i(s_t) l_i$ units of output. Feasible allocations satisfy

$$\sum_{i=1}^{I} n_i c_{i,t} + g_t = \sum_{i=1}^{I} n_i \theta_{i,t} l_{i,t}. \tag{2}$$

Agents and the government issue and trade riskless one period discount bonds. At date t, history s^t the price is denoted by $q_t(s^t)$. Let the cumulation of past prices at t, s^t be $Q_t(s^t) \equiv \prod_{k \leq t, s^k \in s^t} q_k(s^k)$. We denote asset holdings of agents and the government in period t by $\{b_{i,t}\}_i$ and B_t, respectively. We use a convention that negative values denote net indebtedness of the agent or the government. Households and the government begin with assets $\{b_{i,-1}\}_{i=1}^I$ and B_{-1}, respectively. Asset holdings satisfy market clearing conditions

$$\sum_{i=1}^{I} n_i b_{i,t} + B_t = 0 \text{ for all } t \geq -1. \tag{3}$$

In each period, the government imposes a tax on labor earnings $\mathcal{T}_t(y_t)$. To be comparable to the literature, we assume throughout most of this section that $\mathcal{T}_t$ are affine functions

$$\mathcal{T}_t(y_t) = -T_t + \tau_t y_t. \tag{4}$$

Such affine tax functions approximate actual tax and transfer programs pretty well; see Figure 1 from Heathcote et al. (2016).[2]

As will be indicated from our proofs, our results directly extend to more general non-linear income tax schedules $\mathcal{T}_t(y_t)$ and to even richer tax systems. We discuss these later.

The government budget constraint with affine taxes is

$$g_t + q_t B_t = \tau_t \sum_{i=1}^{I} n_i \theta_{i,t} l_{i,t} + B_{t-1} - T_t. \tag{5}$$

[2]We have truncated the plot at 162K USD, which is the 95th income (pre-tax) percentile, to emphasize the role of transfers specially for poor agents.

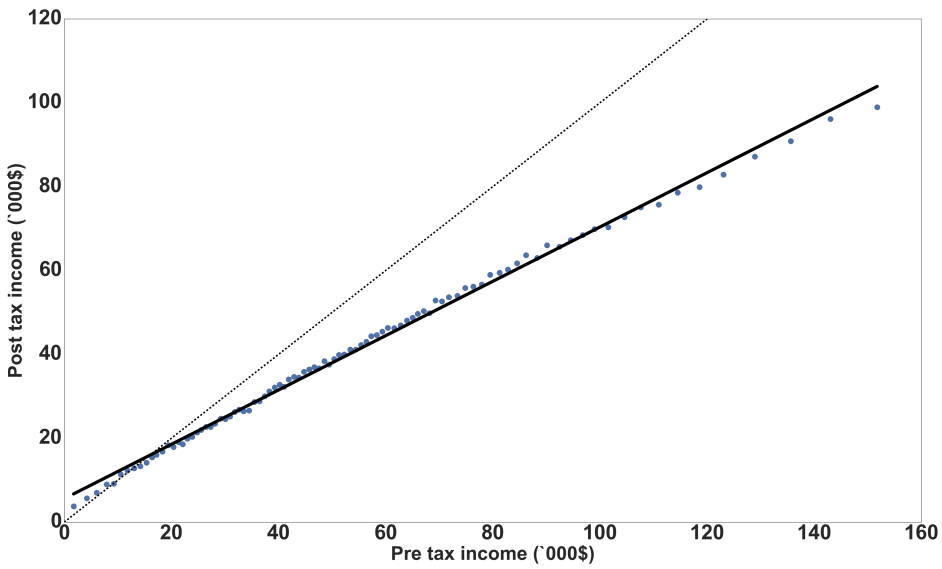

Figure 1: Representation of U.S. tax and transfer system from the PSID 2000-06 and TAXSIM. Source: Heathcote et al. (2016). The dotted line is the 45 degrees line and the solid line is a linear fit.

A government's preferences over stochastic process for consumption and work are ordered by

$$\mathbb{E}_0 \sum_{i=1}^{I} n_i \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t}\right) \tag{6}$$

where $\omega_i \geq 0$, $\sum_{i=1}^{I} \omega_i = 1$ is a set of Pareto weights.

A type i agent's budget constraint at $t \geq 0$ is

$$c_{i,t} + q_t b_{i,t} = \left(1 - \tau_t\right) \theta_{i,t} l_{i,t} + b_{i,t-1} + T_t. \tag{7}$$

In competitive equilibrium, agent i maximizes utility (1) by choosing sequences $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ that satisfy budget constraints (7). Without further restrictions on debt holdings, this problem is ill-posed because it allows agents to achieve infinite utility by running Ponzi schemes. To rule out explosive debt paths, we require sequences $\boldsymbol{b}_i$ to be bounded from below. Later we consider more stringent constraints on private debt.

In the spirit of Lucas and Stokey (1983), we study government policies $(\boldsymbol{\tau}, \boldsymbol{T}, \boldsymbol{B})$ that maximize welfare criterion (6) in a competitive equilibrium, given an initial distribution of assets $\left(\{b_{i,-1}\}_i, B_{-1}\right)$. We are interested in two questions

- How does the level of the initial government debt B_{-1} affect welfare in the optimal equilibrium?
- What determines properties of an optimal path of government debt $\boldsymbol{B}$?

The first of these questions allows us to think about legacy costs of past debt. We want to understand the implications of our neoclassical model for such costs. The answer to the second question also allow us to shed light on what determines an optimal level of debt (if it exists) and how quickly the government should converge to it.

The assumption that agents are heterogeneous affects our answers. In a representative agent economy it is well understood that the answers to these questions depend on whether lump-sum taxes are available (see Barro (1974)). If agents are identical, there is little reason to think that lump sum taxes are infeasible. Much of the macro literature rules out such taxes by implicitly alluding to unmodeled heterogeneity and the presence of a subset of poor agents who cannot afford to pay such taxes. By modeling such poor agents explicitly, we can study the optimal tax policy without relying on ad hoc restrictions on transfers. Instead, since our transfers $\boldsymbol{T}$ are anonymous, the budget constraints of the poorest agents endogenously restrict their sign and magnitude.

Our answers depend in part on borrowing constraints. We interpret these constraints as arising from the ability or inclination of a government to punish agents who default on their obligations. As a benchmark, we start with the loosest borrowing limits: these allow agents to borrow amounts that are feasible for them to repay in all future states. These limits are rationalized by the government being willing and able to impose the harshest punishments on agents who ever default. We then discuss stricter limits on private borrowing.

3 Optimal debt under natural debt limits

We start with the situation in which consumers face the loosest possible borrowing constraints. We call these the natural debt limits and begin with some standard definitions

Definition 1. An *allocation* is a sequence $\{\boldsymbol{c}_i, \boldsymbol{l}_i\}_i$. An *asset profile* is a sequence

$(\{\boldsymbol{b}_i\}_i, \boldsymbol{B})$. A *price process* is a sequence $\boldsymbol{q}$. A *tax policy* is a sequence $(\boldsymbol{\tau}, \boldsymbol{T})$.

Definition 2. A *competitive equilibrium with natural debt limits* given initial assets $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ is a $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ such that (i) $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ maximize (1) subject to (7) and $\boldsymbol{b}_i$ is bounded below for all i; (ii) constraints (2), (3), and (5) are satisfied.

Definition 3. An *optimal* competitive equilibrium with natural debt limits given initial asset $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ is a competitive equilibrium with natural debt limits that maximizes(6).

A short discussion of our terminology is in order. The natural debt limit terminology was popularized by Aiyagari (1994). He considered an economy with finite after tax endowment and utility defined over non-negative consumption. When the equilibrium interest rate $\frac{1}{q_t}$ is strictly greater than one (this condition can be relaxed to require that $\boldsymbol{Q}$ is a strictly positive and summable sequence[3]) Aiyagari required that an agent's debt does not exceed the present value of his maximum after-tax income in the worst shock sequence.[4] Formally, the maximum income of agent i in state s^t is $Y_{i,t}(s^t) \equiv \max\left\{(1-\tau_t(s^t))\,\theta_{i,t}(s^t)\,\bar{L}_i, 0\right\} + T_t(s^t)$ and the present value of his maximum income in the worst shock sequence is

$$D_t\left(\boldsymbol{Y_i}; s^t\right) \equiv \inf_{s^\infty \in \mathbb{S}^\infty : s^t \in s^\infty} \sum_{k>t, s^k \in s^\infty} \frac{Q_{k-1}\left(s^{k-1}\right)}{Q_t\left(s^t\right)} Y_{i,k}(s^k). \tag{8}$$

The natural debt limit requires that if an agents i's consumption is bounded below and $\boldsymbol{Q}$ is a summable sequence, then agent i's assets are constrained by

$$b_{i,t}(s^t) \geq -D_t\left(\boldsymbol{Y_i}; s^t\right) \text{ for all } t, s^t. \tag{9}$$

The following lemma indicates that our definition of competitive equilibrium simply extends Aiyagari's notion of borrowing constraints to situations in which his definition of a natural debt limit is ill-posed.

Lemma 1. *Suppose that U^i is defined only for $c \geq 0$, $\boldsymbol{Y}_i$ is bounded above and bounded below away from zero, and $\boldsymbol{Q}$ is a strictly positive summable sequence. Then $\boldsymbol{b}_i$ satisfies the natural debt limit if and only if $\boldsymbol{b}_i$ is bounded below.*

[3]That is, $\sum_t Q_t(s^t)$ exists for all s^∞.

[4]When the gross interest rate is less than one so that the present value of income is infinite he imposed an explicit lower bound on debt.

Proof. Since $\boldsymbol{Y}_i$ is bounded above and $\boldsymbol{Q}$ is a strictly positive summable sequence, the sequence $D_t(\boldsymbol{Y_i})$ is bounded above and therefore (9) implies that $\boldsymbol{b}_i$ is bounded below.

Suppose that $\boldsymbol{b}_i$ is bounded below but does not satisfy the natural debt limit at some history s^t. In particular, for some $\epsilon > 0$ suppose that

$$b_{i,t}(s^t) = -D_t\left(Y_i; s^t\right) - \epsilon. \tag{10}$$

Use agent i's budget constraint and impose non-negativity of consumption to get

$$q_{t+1}(s^{t+1})b_{i,t+1}(s^{t+1}) \quad \leq \quad Y_{i,t+1}(s^{t+1}) - D_t\left(Y_i; s^t\right) - \epsilon.$$

Using the definition of $D_t(Y_i; s^t)$, there exists $\underline{s}_{t+1}$ that occurs with strictly positive probability such that

$$b_{i,t+1}(s^t, \underline{s}_{t+1}) \leq -D_{t+1}\left(Y_i; s^t, \underline{s}_{t+1}\right) - \frac{\epsilon}{q_{t+1}(s^t, \underline{s}_{t+1})}.$$

Repeating this process, we get

$$b_{i,t+N}(s^t, \underline{s}_{t+1} \ldots \underline{s}_{t+N}) \leq -D_{t+1}\left(Y_i; s^t, \underline{s}_{t+1} \ldots \underline{s}_{t+N}\right) - \frac{\epsilon}{\Pi_{j=0}^{N} q_{t+1+j}(s^t, \underline{s}_{t+1} \ldots \underline{s}_{t+1+j})}.$$

Since $\boldsymbol{Q}$ is summable, $\lim_N \Pi_{j=0}^{N} q_{t+1+j}(s^t, \underline{s}_{t+1} \ldots \underline{s}_{t+1+j}) = 0$ and hence for any constant B there exists an $\mathcal{N}(B)$ sufficiently large that

$$b_{i,t+N}(s^t, \underline{s}_{t+1} \ldots \underline{s}_{t+N}) \leq -B - \epsilon \qquad \forall N \geq \mathcal{N}(B),$$

and thus we obtain a contradiction. □

Our definition of an optimal competitive equilibrium allows the government to optimize over taxes and transfers $(\boldsymbol{\tau}, \boldsymbol{T})$. Since competitive equilibrium is defined only over taxes that all consumers can afford to pay (i.e., for which each consumer's budget set is nonempty), this definition endogenously imposes restrictions on feasible tax policies.

We start with an important result.

Lemma 2. *Given* $\left(\{b_{i,-1}\}_i, B_{-1}\right)$, *let* $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ *be a competitive equilibrium with natural debt limits. For any bounded sequences* $\left\{\hat{\boldsymbol{b}}_i\right\}_i$ *and* $\left\{\hat{b}_{i,-1}\right\}_i$ *that satisfy*

$$\hat{b}_{i,t} - \hat{b}_{I,t} = b_{i,t} - b_{I,t} \text{ for all } t \geq -1, i \in [1, 2, \ldots, I-1]$$

there exist sequences $\left(\hat{\boldsymbol{T}}, \hat{\boldsymbol{B}}\right)$ *such that* $\left(\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$ *is a competitive equilibrium with natural debt limits given* $\left(\left\{\hat{b}_{i,-1}\right\}_i, \hat{B}_{-1}\right)$.

Proof. For any bounded $\left\{\hat{\boldsymbol{b}}_i\right\}_i$ let $\Delta_t \equiv \hat{b}_{I,t} - b_{I,t}$ for all $t \geq -1$. Define, for all $t \geq -1$,

$$\hat{T}_t = T_t + q_t \Delta_t - \Delta_{t-1}, \quad \hat{B}_t = B_t + \Delta_t. \tag{11}$$

The sequence $\left(\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$ satisfies (2), (3), and (5), so it remains to show that $\left(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right)$ is the optimal choice given $\left(\boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$. Observe that $\left(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right)$ satisfies budget constraint

$$\begin{aligned}
c_{i,t} &= (1-\tau_t)\,\theta_{i,t} l_{i,t} + b_{i,t-1} - q_t b_{i,t} + T_t \\
&= (1-\tau_t)\,\theta_{i,t} l_{i,t} + (b_{i,t-1} - b_{I,t-1}) - q_t\,(b_{i,t} - b_{I,t}) + T_t + b_{I,t-1} - q_t b_{I,t} \\
&= (1-\tau_t)\,\theta_{i,t} l_{i,t} + \left(\hat{b}_{i,t-1} - \hat{b}_{I,t-1}\right) - q_t\left(\hat{b}_{i,t} - \hat{b}_{I,t}\right) + T_t + b_{I,t-1} - q_t b_{I,t} \\
&= (1-\tau_t)\,\theta_{i,t} l_{i,t} + \hat{b}_{i,t-1} - q_t \hat{b}_{i,t} + \hat{T}_t.
\end{aligned}$$

Suppose that $\left(\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right)$ is not an optimal choice for consumer i, in the sense that there exists some other sequence $(\boldsymbol{c}_i', \boldsymbol{l}_i', \boldsymbol{b}_i')$ that give consumer i higher utility given $\left(\boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$. The sequence $(\boldsymbol{c}_i', \boldsymbol{l}_i', \boldsymbol{b}_i' - \boldsymbol{\Delta})$ satisfies (7) and (9) given $(\boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ and gives strictly higher utility than $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$. Therefore, $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ cannot be a part of a competitive equilibrium $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$, a contradiction. □

We summarize the answers to two questions posed in Section 2 by means of two propositions that follow from Lemma 2.

Proposition 1. *For any pair* B_{-1}', B_{-1}'', *there are asset profiles* $\left\{b_{i,-1}'\right\}_i$ *and* $\left\{b_{i,-1}''\right\}_i$ *such that an optimum equilibrium allocations with natural debt limit starting from* $\left(\left\{b_{i,-1}'\right\}_i, B_{-1}'\right)$ *and from* $\left(\left\{b_{i,-1}''\right\}_i, B_{-1}''\right)$ *are the same. These asset profiles satisfy*

$$b_{i,-1}' - b_{I,-1}' = b_{i,-1}'' - b_{I,-1}'' \text{ for all } i.$$

Proposition 1 asserts that it is not total government debt but how its ownership is distributed that matters for equilibrium allocations. To understand why, suppose that we increase the initial level of government debt from 0 to some arbitrary level B_{-1}'. If transfers $\boldsymbol{T}$ were to be held fixed, the government would want to increase

taxes $\boldsymbol{\tau}$ to collect a present value of revenues sufficient to repay B'_{-1}. Since deadweight losses are convex in the tax rate, higher levels of debt would then impose disproportionately larger distortions, which makes higher levels of debt particularly bad. But consider how this conclusion would change if we were to allow the government to adjust transfers. To find optimal transfers, we need to know how holdings of government debt B'_{-1} are distributed. Suppose that agents hold equal amounts of the new debt. In this case, each unit of debt repayment achieves the same redistribution as one unit of transfers. Since the original level of transfers at zero government debt is optimal, the best policy for the government with debt B'_{-1} is to reduce transfers by exactly the amount of the increase in per capita debt. As a result, both the distorting taxes $\boldsymbol{\tau}$ and allocations remain unchanged. This example illustrates ideas expressed by Simon Newcomb (1865, p. 85) in the quotation with which we began this paper.

This logic is sensitive to the assumption that holdings of additional government debt are equal across agents. Suppose instead that the government debt is owned disproportionately by high-earnings households meaning that inequality is higher in economies with higher government debt; the optimal fiscal response would typically call for an increase in both tax rates $\boldsymbol{\tau}$ and transfers $\boldsymbol{T}$. The conclusion would be the opposite if government debt were to be disproportionately owned by low-earnings households.[5]

Proposition 1 cautions against comparing debt burdens across countries based purely on aggregate quantities like debt to GDP ratios. Assuming that governments generally want to redistribute from high-earning to low-earning households, public debt that is held widely by private agents or government agencies typically will be less distorting than public debt held by agents in the right tail of the earning distribution or by foreign investors. Similarly, our result cautions against lumping both explicit debt and implicit promises (such as Social Security obligations) into one headline number without adjusting for heterogeneity across holdings of various types of debts.

Another implication of Lemma 2 is that the path of government debt in the optimal competitive equilibrium with natural debt limits is indeterminate.

Proposition 2. *(**Ricardian equivalence**) Suppose that an optimal equilibrium with*

[5]It is straightforward to extend our analysis to an open economy with foreign holdings of domestic debt. The more government debt is owned by the foreigners, the higher are the distortions the government will need to impose.

a natural debt limit given $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ *exists. Then any bounded* $\boldsymbol{B}$ *is part of an optimal competitive equilibrium.*

Although Proposition 2 asserts that the level of government debt is indeterminate, there are close parallels between optimal allocations in our economy and representative agent Ramsey models of debt. To illustrate this, assume that $I = 2$ and that both U^is are differentiable and satisfy Inada conditions. Then using standard arguments, one can show that $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ is a competitive equilibrium if and only if it satisfies

$$c_{2,t} + \frac{U_{l2,t} l_{2,t}}{U_{c2,t}} - \left(c_{1,t} + \frac{U_{l1,t} l_{1,t}}{U_{c1,t}}\right) + q_t \left(b_{2,t} - b_{1,t}\right) = b_{2,t-1} - b_{1,t-1}, \tag{12a}$$

$$\frac{U_{l1,t}}{\theta_1 U_{c1,t}} = \frac{U_{l2,t}}{\theta_2 U_{c2,t}}, \tag{12b}$$

$$c_{1,t} + c_{2,t} + g_t \leq \theta_1 l_{1,t} + \theta_2 l_{2,t}, \tag{12c}$$

$$\frac{U_{ci,t}}{\mathbb{E}_t U_{ci,t+1}} = \frac{\beta}{q_t}. \tag{12d}$$

An inspection of these equations indicates that we can define a net asset position $\mathfrak{b}_t \equiv b_{2,t} - b_{1,t}$ and represent its dynamics recursively using standard techniques with $\mathfrak{b}_t$ being a state variable. By normalizing $b_{2,t} = 0$ we can interpret $\mathfrak{b}_t$ to be government debt. This structure bears a close resemblance to some representative agent Ramsey models (e.g. Aiyagari et al. (2002); Farhi (2010)), although it might have substantially different qualitative and quantitative implications, some of which we investigate in Bhandari et al. (2016 fothcoming) and Bhandari et al. (2015).

Lemma 2 and its implication in the form of Propositions 1 and 2 hold in more general environments too. For example, we could allow agents to trade all conceivable Arrow securities and still show that equilibrium allocations depend only on agents' net asset positions. Similarly, our results hold in economies with capital, or with arbitrary non-linear income tax schedule $\mathcal{T}_t(y_t)$.

4 Imperfect debt enforcement and ad hoc borrowing constraints

The analysis of the previous section closely follows the Ramsey tradition of answering normative questions. At the outset we specify sequences of instruments available to

the government ($\boldsymbol{\tau}$, $\boldsymbol{T}$ and $\boldsymbol{B}$ in our case) and assume that the government commits to those sequences in period -1. Optimizing over a set of competitive equilibria associated with those sequences implicitly assumes that the government has the ability to pick the equilibrium with the highest welfare from that set. That is, the government has a *technology* that allows it perfectly to implement an equilibrium allocation associated with its policies.

To elaborate the implementation issue, consider a situation in which agents decide to make alternative choices that render some budget constraints violated, for example, by some agents not working enough to be able to meet their tax liabilities. An implicit enforcement technology assumption would require the government to impose punishments sufficiently harsh to prevent agents from pursuing such policies "off-equilibrium". If consumption is bounded by 0 and $\lim_{c\to 0} U^i(c,l) = -\infty$ for all i, l, it is sufficient to specify that the government commits to seizing all of an agent's labor and asset income in a period in which he cannot pay its prescribed taxes. But if the utility function is bounded from below additional non-pecuniary punishments may be needed.

The same assumption of perfect enforcement would extend to repayment of private debts – agents never fail to repay their debts in equilibrium presumably because the punishments from default are sufficiently severe from failing to do so. Thus, the equilibrium definition in Section 3 indirectly requires not only that the government has the ability to enforce payments, but also that it exercises this ability to enforce both tax and debt payments.

In this section, we stay within the boundaries of a conventional Ramsey analysis and focus on whether it is desirable for the government to enforce both tax and debt obligations and whether it can improve welfare by committing to enforce some type of payments and not others. Formally, we capture this decision in the simplest form by assuming that agents can borrow up to an ad hoc debt limit

$$b_{i,t} \geq -\underline{b} \tag{13}$$

for some exogenously given $\underline{b} \geq 0$. We interpret these constraints as arising from imperfect government debt enforcement: the government imposes an arbitrary high punishment on agents if they default on any debt less than $\underline{b}$ and no punishment for any default on debt over $\underline{b}$; the case $\underline{b} = 0$ is interpreted as the government's

refusing to enforce any private debt contracts. The natural debt limit considered in the previous section is a limit that arises when agents are punished for any debt default.[6] Note that we maintain the assumption that the government enforces tax liability perfectly: thus, we study whether it is optimal to enforce taxes and debt contracts differentially.[7]

Definition 4. A *competitive equilibrium with an ad hoc debt limit* given initial assets $\left(\{b_{i,-1}\}_i, B_{-1}\right)$ is a $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ such that (i) $(\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i)$ maximize (1) subject to (7) and (13) for all i; (ii) constraints (2), (3), and (5) are satisfied.

To understand what determines the path of debt, we first show that, in general, it is optimal for the government *not* to enforce private contracts. Restricting private borrowing allows more flexibility to the government in managing its own debt service costs. In fact, the optimal path of debt is pinned down by these considerations in contrast to alternative accounts that emphasize that a government should issue debt to *increase* liquidity because there is a lack of other means of savings. We begin with our main proposition for this section.[8]

Proposition 3. *If there are tax policies that support any allocation* $(\boldsymbol{c}, \boldsymbol{l})$ *as a competitive equilibrium allocation with a natural debt limit, then there are tax policies that support* $(\boldsymbol{c}, \boldsymbol{l})$ *as a competitive equilibrium allocation with an ad hoc debt limit with any* $\underline{b}$*. If* $(\boldsymbol{c}, \boldsymbol{l})$ *can be supported as a competitive equilibrium allocation with an ad hoc debt limit* $\underline{b}'$*, it can also be supported as a competitive equilibrium allocation with ad hoc debt limit* $\underline{b}''$ *for any* $\underline{b}''$*.*

Proof. Let $\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i$ be a competitive equilibrium allocation and debt with a natural debt limit. Let $\Delta_t \equiv \max_i \{\underline{b} - b_{i,t}\}$. Define $\hat{b}_{i,t} \equiv b_{i,t} + \Delta_t$ for all t. By Lemma 2, $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is also a competitive equilibrium allocation with natural debt limits.

[6] We believe that another fruitful way to study the role of debt is to drop the full commitment assumption and explicitly specify strategies for all histories for agents and the government as was done by Bassetto (2002) in a closely related context of monetary economics and the fiscal theory of price level. We hope to pursue this line of work.

[7] Bryant and Wallace (1984) describe how a government can use borrowing constraints as part of a welfare-improving policy to finance exogenous government expenditures. Sargent and Smith (1987) describe Modigliani-Miller theorems for government finance in a collection of economies in which borrowing constraints on classes of agents produce the rate of return discrepancies that Bryant and Wallace manipulate.

[8] Our proposition builds on Yared (2012, 2013), who showed that the planner may find it optimal not to undo agents' borrowing constraints even when that is feasible.

Moreover, by construction $\hat{b}_{i,t} - \underline{b} = b_{i,t} + \Delta_t - \underline{b} \geq 0$. Therefore, $\hat{\boldsymbol{b}}_i$ satisfies (13). Since agents' budget sets are smaller in the economy with ad hoc debt limits and $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ lies in this smaller budget set, then $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is also an optimal choice for agents in the economy with exogenous borrowing constraints $\underline{b}$. Since all market clearing conditions are satisfied, $\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i$ is a competitive equilibrium allocation and asset profile.

For the proof of the second part, let $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ be a competitive equilibrium with debt limit $\underline{b}'$. Define $\Delta_t \equiv \underline{b}' - \underline{b}''$, and construct $\left(\hat{T}, \hat{B}\right)$ as in (11), $\hat{b}_{i,t} = b_{i,t} + \Delta_t$ for all i, t. Then we can show that $\left(\left\{\boldsymbol{c}_i, \boldsymbol{l}_i, \hat{\boldsymbol{b}}_i\right\}_i, \hat{\boldsymbol{B}}, \boldsymbol{q}, \boldsymbol{\tau}, \hat{\boldsymbol{T}}\right)$ is a competitive equilibrium with limit $\underline{b}''$ by using arguments from Lemma 2. □

A remarkable implication of Proposition 3 is not only that the government finds it optimal to treat transfers and debt differently, but that the weakest possible enforcement of private debt contracts is optimal. Without loss of generality, we can assume that agents cannot borrow.

Corollary 1. *Welfare in optimum equilibrium with ad hoc debt limits is higher than welfare in the optimum equilibrium with natural debt limits and does not depend on the value of the debt limit* $\underline{b}$.

A crucial difference between ad hoc debt limits studied in this section and natural debt limits in the previous section is how they depend on the tax policy. While the lower bound on debt is endogenous and policy-dependent in the Section 3 discussion of natural debt limits, it is exogenous in this section where we have ad hoc debt limits. Policy invariance of the debt limit here implies that changing the timing of transfers can change the set of agents who are up against their borrowing limits. This gives the government the ability to increase welfare.

The previous discussion also highlights a critical assumption underlying the results of this section: asymmetric enforcement of taxes and private debt. If the government allowed agents to use transfers as collateral for private borrowing, postponing transfers into the future would relax agents' borrowing constraints and undo a government's ability to gain from pushing people against their borrowing constraints. Asymmetry between enforcement of debt obligations and tax obligations is common in practice. For example, in the U.S. it is illegal to use future social security payments

as collateral and it is typically easier to discharge unsecured debt than tax liabilities through bankruptcy.

Various authors have studied Ramsey policies in economies with ad hoc constraints (13) and pointed out that Ricardian equivalence fails and consequently that the optimal debt is determined.[9] Thus, in the context of the results of the previous section, our Proposition 2 would generally not hold when agents are subject to the ad hoc constraint (13). In the following example we investigate the sources of welfare gains that came from from limiting agents' opportunities to borrow.

Example 1. Suppose that there are two types of agents with equal mass. Agent 1 cannot work and has preferences $c_{1,t}$. Agent 2 has preferences $u\left(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma}\right)$ with $\gamma > 0$. Agent 2's productivity satisfies $\theta_{2,t} = 1$ if t is even and $\theta_{2,t} = 0$ if t is odd. There are no government expenditures. The government puts Pareto weight 1 on agent 1's utility. All agents start with no initial assets.

Consider first the optimum equilibrium when debt enforcement is perfect so that agents face a natural debt limit. In this case, agent 1's preferences imply that the only feasible equilibrium interest rate in this economy satisfies $q_t = \beta$ for all t. The government's objective function makes it want to maximize the present value of tax revenues, evaluated at this interest rate. Given the assumption about agent 2's preferences, the optimal tax rate is $\tau_t = \bar{\tau}$ for all t, where $\bar{\tau}$ is the top of the Laffer curve tax rate.[10] This implies that welfare with natural debt limits is $\sum_t \beta^t T_t = \frac{1}{2}\frac{\bar{Z}}{1-\beta^2}$. The timing of transfers is indeterminate by Lemma 2. Without loss of generality the optimum can be attained by setting $b_{1,t} = 0$ for all t, and $(\boldsymbol{B}, \boldsymbol{T})$ that jointly solve the following equations for all t

$$T_{2t+1} = B_{2t}, \quad T_{2t} + \beta B_{2t} = \bar{Z}, \quad B_{2t+1} = 0 \tag{14}$$

and

$$u'\left((1-\bar{\tau})\,\bar{l} + 2\beta B_{2t} - \frac{1}{1+\gamma}\bar{l}^{1+\gamma}\right) = u'\left(T_{2t+1} - 2B_{2t}\right).$$

[9]For instance, see Woodford (1990); Aiyagari and McGrattan (1998); Azzimonti et al. (2014). Some commentators observed that this construction implicitly requires that it is easier to extract a dollar from an agent in taxes than in debt service. Our analysis indicates that it is an optimal choice for the government to choose arrangements that produce this outcome even if the same technology is available for enforcing both types of payments.

[10]It is easy to verify that they satisfy $\bar{\tau} = \frac{\gamma}{1+\gamma}$, $\bar{l} = \left(\frac{1}{1+\gamma}\right)^{1/\gamma}$, $\bar{Z} = \gamma\left(\frac{1}{1+\gamma}\right)^{1+1/\gamma}$.

Agent 2's budget constraint and (2) imply that $B_{2t} < 0$ for all t. Thus, the government issues debt in even periods that agent 2 uses to smooth marginal utility intertemporally. The government repays this debt in odd periods by levying (negative) lump sum transfers. We denote that optimum equilibrium with natural debt limits using these transfer and debt sequence as $(\{\boldsymbol{c}_i^{nat}, \boldsymbol{l}_i^{nat}, \boldsymbol{b}_i^{nat}\}_i, \boldsymbol{B}^{nat}, \boldsymbol{q}^{nat}, \boldsymbol{\tau}^{nat}, \boldsymbol{T}^{nat})$.

Now consider the economy in which private debt constraints are not enforced, so that agent's debt must satisfy

$$b_{i,t} \geq 0 \text{ for all } i, t. \tag{15}$$

Observe that $(\{\boldsymbol{c}_i^{nat}, \boldsymbol{l}_i^{nat}, \boldsymbol{b}_i^{nat}\}_i, \boldsymbol{B}^{nat}, \boldsymbol{q}^{nat}, \boldsymbol{\tau}^{nat}, \boldsymbol{T}^{nat})$ still satisfy the agents' budget constraints with this additional debt limits so that it is also an equilibrium in the economy without private borrowing. We now construct a welfare improving equilibrium.

One can show that $(\{\boldsymbol{c}_i, \boldsymbol{l}_i, \boldsymbol{b}_i\}_i, \boldsymbol{B}, \boldsymbol{q}, \boldsymbol{\tau}, \boldsymbol{T})$ is part of an equilibrium with ad hoc limits (15) if and only if budget constraints (7) holds for both agents (with $\theta_{1,t} = 0$ for all t), feasibility (2) and (3) and borrowing constraints (15) are satisfied, and the following equations hold:

$$l_{2,t}^{\gamma} = (1 - \tau_t)\, \theta_{2,t}, \tag{16a}$$

$$q_t u'\left(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma}\right) \geq \beta u'\left(c_{2,t+1} - \frac{1}{1+\gamma} l_{2,t+1}^{1+\gamma}\right), \tag{16b}$$

$$\left[q_t u'\left(c_{2,t} - \frac{1}{1+\gamma} l_{2,t}^{1+\gamma}\right) - \beta u'\left(c_{2,t+1} - \frac{1}{1+\gamma} l_{2,t+1}^{1+\gamma}\right)\right] b_{2,t} = 0 \tag{16c}$$

$$q_t \geq \beta, \tag{16d}$$

$$[q_t - \beta]\, b_{1,t} = 0. \tag{16e}$$

Equation (16a) is the optimality condition for labor of agent 2; equations (16b)-(16e) are optimality conditions for savings that hold with inequality only if the agent's assets are zero, and with equality otherwise.

A key observation about these conditions is that there equilibrium q_t that are higher than the discount factor β when assets of agent 1 are at zero. We show in the appendix that for any $\varrho \geq \beta$ one can construct an equilibrium in which $\tau_t = \bar{\tau}$ and $b_{1,t} = 0$ for all t and an(inverse of) interest rate sequence $\boldsymbol{q}(\varrho) = (\varrho, \beta, \varrho, \beta, ...)$. This

equilibrium is supported by transfers and debt sequence $(\boldsymbol{T}(\varrho), \boldsymbol{B}(\varrho))$ that satisfies

$$T_{2t+1}(\varrho) = B_{2t}(\varrho), \quad T_{2t}(\varrho) + \varrho B_{2t}(\varrho) = \bar{Z}, \quad B_{2t+1}(\varrho) = 0, \tag{17}$$

which generalizes (14). Differentiate to obtain

$$\begin{aligned} \frac{\partial}{\partial \varrho} T_{2t+1}(\varrho) &= \frac{\partial}{\partial \varrho} B_{2t}(\varrho) \\ \frac{\partial}{\partial \varrho} T_{2t}(\varrho) + \varrho \frac{\partial}{\partial \varrho} B_{2t}(\varrho) + B_{2t}(\varrho) &= 0 \end{aligned}$$

Since welfare is simply $\sum_t \beta^t T_t(\varrho)$, it follows that for ϱ close to β lowering equilibrium interest rates (increasing ϱ) improves welfare:

$$\frac{\partial}{\partial \varrho} \sum_t \beta^t T_t(\varrho) \bigg|_{\varrho=\beta} = -\sum_t \beta^{2t+1} B_{2t}^{nat} > 0.$$

Since $\varrho = \beta$ corresponds to welfare in the optimum equilibrium with natural debt limits, this also proves that welfare with ad hoc limits is strictly higher.

Example 1 illustrates the key driving force behind the determination of the optimal quantity of debt. If the government issues debt in equilibrium, it is generally better off if interest payments on that debt are lower. In the economy with natural debt limits, equilibrium interest rates are determined implicitly by competition between the government and agent 1 to supply savings ("liquidity") to agent 2. Even though in that equilibrium agent 1 does not supply liquidity, he would, by issuing private risk-less debt, as soon as the interest rate drops below the inverse β^{-1} of his rate of time preference. When private debt contracts are unenforceable, agent 1 cannot issue riskless debt and the government becomes a monopolist supplier of liquidity to agent 2. By using its monopoly power, it can extract additional surplus from agent 2 by issuing debt at a lower interest rate.

The results in Woodford (1990), Aiyagari and McGrattan (1998) are often interpreted as justifying a role for government debt to increase liquidity of savings instruments. Our discussion suggests that the government should *decrease* the aggregate supply of liquidity by limited enforcement of private debt contracts and use the additional market power thereby acquired in providing liquidity to extract monopoly rents. This force runs counter to a common opinion that the role of public debt is to increase liquidity of agents who are borrowing constrained.

While the optimal continuation level of government debt is often determined in equilibrium, the initial level of government debt is irrelevant for welfare in the same sense as in Proposition 1.

Proposition 4. *Proposition 1 holds in the economy with ad hoc debt limits. If* $\boldsymbol{B}'$ *is the optimal path of debt given* $\left(\left\{b'_{i,-1}\right\}_i, B'_{-1}\right)$*, then* $\boldsymbol{B}'$ *is also the optimal path of debt given* $\left(\left\{b''_{i,-1}\right\}_i, B''_{-1}\right)$ *if* $b'_{i,-1} - b''_{i,-1}$ *is independent of* i*.*

Proof. Suppose $(\boldsymbol{\tau}', \boldsymbol{T}')$ are the optimal taxes in the economy with initial assets $\left(\left\{b'_{i,-1}\right\}_i, B'_{-1}\right)$. Define sequence $\boldsymbol{T}''$ as $T''_0 = T'_0 + b'_{I,-1} - b''_{I,-1}$ and $T''_t = T'_t$ for all $t > 0$. Following the same steps as in the proof of Lemma 2 we can verify that $(\boldsymbol{\tau}', \boldsymbol{T}'')$ are optimal taxes in the economy with initial assets $\left(\left\{b''_{i,-1}\right\}_i, B''_{-1}\right)$. □

To understand why the initial level of government debt is welfare-irrelevant, note that the welfare gains in the example 1 are obtained from the government's ability to influence prices of future debt. The value of legacy debt with which the government enters period 0 was set in the past and is not affected by future policies. Thus, the initial debt level does not play a role different from that in Section 3. Note that Proposition 4 shows that not only welfare but also the optimal debt path is independent of the level of initial government debt B_{-1} (though they generally do depend how the initial assets are distributed across agents). Thus, transitions to an optimal debt level take exactly one period, independently of the size of the initial debt.

5 Idiosyncratic risk

It is relatively straightforward to allow for idiosyncratic income risk.[11] There are I groups of agents each with a measure n_i of ex-ante identical individuals. We maintain the aggregate shocks $s_t \in S$ and introduce idiosyncratic shocks that are drawn from $\tilde{s}_i \in \tilde{S}_i$ for all $i \in I$ where S and $\left\{\tilde{S}_i\right\}_{i \in I}$ are finite sets. We use $S^t, \tilde{S}_i^t$ to denote t period Cartesian products and $S^\infty, \tilde{S}_i^\infty$ as the space of infinite sequences of aggregate and idiosyncratic shocks for each group. We use π_t to denote the probability measure over Borel sets of S^t and π as the extension to the Borel sets of S^∞. In an analogous

[11] See for instance Bewley (1986), Aiyagari (1994), Huggett (1993) or Krusell and Smith (1998).

fashion, we define $\mu_{i,t}$ and μ_i to be probability measures over Borel sets of $\tilde{S}_i^t$ and $\tilde{S}_i^\infty$, respectively, for each $i \in I$.

We impose that $\pi_t(s^t) > 0$ for all $s^t \in S^t$ and $\mu_{i,t}(\tilde{s}_i^t) > 0$ for $\tilde{s}_i^t \in \tilde{S}_i^t$ for all t. Next, we assume that conditional on the history of aggregate shocks s^t, the idiosyncratic shocks $\tilde{s}_i^t$ are distributed independently and identically across members of group i. We assume a Law of Large numbers: for any Borel subset set B of $\tilde{S}_i^t$, the measure $\mu_{i,t}(B)$ denotes the fraction of type i agents that have histories $\tilde{s}_i^t \in B$.

The productivity of an agent in group i is now described by functions $\theta_i\left(s, \tilde{s}_i\right)$ while, as before, government expenditures are $g(s)$. Time t realizations after history $(s^t, \tilde{s}_i{}^t)$ are denoted $\theta_{i,t}(s^t, \tilde{s}_i^t)$ and $g_t(s^t)$. Bond prices, tax policy, and government debt policy are measurable with respect to aggregate shocks s^t and individual consumption, leisure, and asset choices depend on $(s^t, \tilde{s}_i{}^t)$.

Individual budget constraints remain as in equation (7). We modify the resource constraint to reflect that there is a continuum of agents: for any history s^t of aggregate shocks,

$$\sum_i n_i \int_{\tilde{s}_i^t \in \tilde{S}_i^t} c_{i,t}(s^t, \tilde{s}_i^t) d\mu_{i,t}(\tilde{s}_i^t) + g_t(s^t) = \sum_i n_i \int_{\tilde{s}_i^t \in \tilde{S}_i^t} \theta_{i,t}(s^t, \tilde{s}_i^t) l_{i,t}(s^t, \tilde{s}_i^t) d\mu_{i,t}(\tilde{s}_i^t)$$

$$\sum_i n_i \int_{\tilde{s}_i^t \in \tilde{S}_i^t} b_{i,t}(s^t, \tilde{s}_i^t) d\mu_{i,t}(\tilde{s}_i^t) + B_t(s^t) = 0$$

Initial conditions consist of $\left(\{b_{i,-1}, \tilde{s}_{i,-1}\}_{i \in I}, s_{-1}\right)$. The natural debt limit in equation (8) is constructed by taking the infimum over joint histories $s^\infty, \tilde{s}_i^\infty \in S^\infty \times \tilde{S}_i^\infty$.[12] The definitions of competitive equilibrium and optimal competitive equilibrium (with either natural debt limits or ad hoc debt limits) are unchanged.

The arguments in Lemma 2 prevail here too. Any perturbation Δ_t such that $\hat{B}_t = B_t + \Delta_t$ and individual assets $\hat{b}_{it} = b_{i,t} - \Delta_t$ preserve pairwise differences in assets; then adjusting transfers as in equation (11) keeps all budget sets unaltered. This implies that Lemma 2 and also Proposition 1 - 4 continue to hold when we allow for idiosyncratic income risk.

[12] More formally we would have

$$D_t\left(\boldsymbol{Y_i}; s^t, \tilde{s}_i^t\right) \equiv \inf_{s^\infty, \tilde{s}_i^\infty \in S^\infty \times \tilde{S}_i^\infty s^t, \tilde{s}_i{}^t \in s^\infty \times \tilde{s}_i^\infty} \sum_{k > t, s^t, \tilde{s}_i{}^t \in s^\infty \times \tilde{s}_i^\infty} \frac{Q\left(s^{k-1}\right)}{Q\left(s^t\right)} Y_{i,k}(s^k, \tilde{s}_i^k).$$

6 Informationally-constrained optimal taxes

The analysis of the previous sections followed the Ramsey tradition when we a priori restricted attention to taxes of a particular form, such as affine taxes (4). An alternative approach is to start with explicit information constraints on the government and to derive implications for government policy. This approach originated in the work of Mirrlees (1971) and was introduced to macro by Golosov et al. (2003) and Werning (2007). In this section, we investigate the role of debt and taxes when government actions are restricted by informational frictions only.

An *informationally-constrained optimum* is a sequence $\{\mathbf{c}_i, \mathbf{l}_i\}_i$ that maximizes (6) subject to feasibility (2) and the constraints that specify information about agents that is available to the government. *Informationally-constrained taxes* are tax functions that use observable variables as their arguments; *optimal informationally-constrained taxes* implement an informationally-constrained optimum as a competitive equilibrium.

A standard assumption since Mirrlees (1971) is that the government does not observe individual's labor supply $l_{i,t}$ or productivity $\theta_{i,t}$ but that it does observe labor earnings $y_{i,t}$. We maintain this assumption throughout this section. The role of public and private debt depends critically on whether an individuals' assets and consumption are observable. If agents' assets are observable, public or private debt plays no interesting role: *any* sequence $(\boldsymbol{B}, \{\boldsymbol{b}_i\}_i)$ that satisfies feasibility (3) can be supported in an optimal competitive equilibrium. The reason is simple. Let $\left\{\mathbf{c}_i^{ob}, \mathbf{l}_i^{ob}\right\}_i$ be an informationally-contained optimum with observable assets and let $\boldsymbol{y}_i^{ob}$ be defined by $y_{i,t}^{ob} \equiv \theta_{i,t} l_{i,t}^{ob}$. The government can implement $\left\{\mathbf{c}_i^{ob}, \mathbf{l}_i^{ob}\right\}_i$ by offering agents a menu of I tax schedules of the form $\{\mathcal{T}_t(y_t, b_{t-1}, b_{-1}, i)\}_t$ and letting agents permanently self-select into one of them in period $t = -1$. One constrained optimum tax schedule simply sets

$$\mathcal{T}_0(y_0, b_{-1}, i) = \theta_{i,0} l_{i,0}^{ob} - c_{i,0}^{ob} - b_{-1}$$
$$\mathcal{T}_t(y_t, b_{t-1}, b_{-1}, i) = \theta_{i,t} l_{i,t}^{ob} - c_{i,t}^{ob}$$

so long as $y_t = y_{i,t}^{ob}$, $b_{-1} = b_{-1,i}$ and $b_{t-1} = 0$ and $\mathcal{T}_t(y_{i,t}, b_{i,t-1}, b_{-1}, i) = \infty$ (or any sufficiently high number) for any other (y_t, b_{t-1}). This tax sequence de facto shuts down all assets markets by penalizing agents for choosing anything other than $b_{i,t} = 0$

and restricting agents' budget sets to sequences $\left\{\mathbf{c}_i^{ob}, \mathbf{l}_i^{ob}\right\}_i$. Incentive compatibility then ensures that an optimum is implemented. This is not a unique way to implement it. Instead of shutting down asset markets entirely, one can choose any sequence $\{\boldsymbol{b}_i\}_i$ and re-define taxes appropriately to ensure that in an optimal equilibrium agent i chooses $\boldsymbol{b}_i$.[13]

When agents' assets are unobservable, the problem becomes more interesting. We assume that interactions in asset markets are anonymous and that agents and the government can issue and buy debt, but that it is impossible for the government to ascertain an individual agent's asset holdings. This also requires that individual consumption is not observable.

The informationally-constrained optimum can be characterized by invoking the Revelation principle and setting up a mechanism design problem.[14] We now define an informationally-constrained optimal allocation with unobservable assets associated with a mechanism design problem that determines labor income $\{\mathbf{y}_i\}_i$ and payments $\{\mathbf{x}_i\}_i$ as well as a debt sequence $\mathbf{B}$. A reporting strategy is a function $r : I \to I$. A mechanism $\{\mathbf{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ is feasible if there exists an allocation $\{\mathbf{c}_i, \mathbf{l}_i\}_i$, asset choices $\{\mathbf{b}_i\}_i$, a reporting strategy r and bond prices $\mathbf{q}$ such that each agent i chooses $\{\mathbf{c}_i, \mathbf{l}_i\}_i$, $\mathbf{b}_i$, $r(i)$ to maximize (1) subject to the budget constraint

$$c_{i,t} + q_t b_{i,t} = x_{r(i),t} + b_{i,t-1}, \tag{18}$$

with $b_{i,t}$ satisfying either natural or ad hoc debt limits. Prices $\mathbf{q}$ are such that debt market clearing (3) and feasibility

$$\sum_i n_i c_{i,t} = \sum_i n_i y_{r(i),t} \tag{19}$$

are satisfied. A feasible mechanism $\{\mathbf{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ is incentive compatible if the associated reporting strategy $r(i) = i$. An informationally constrained optimum is an incentive compatible mechanism $\{\mathbf{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ such that the associated allocation $\{\mathbf{c}_i, \mathbf{l}_i\}_i$ maximizes (6).

The ability of agents to trade assets anonymously lowers welfare, which implies that the government would find it optimal to minimize enforcement of private debt

[13] It is easy to smooth the constructed tax function to implement the optimum with continuous or differentiable tax functions (see, for example, Kocherlakota (2005), Werning (2009), Grochulski and Kocherlakota (2010)). The conclusion that neither public nor private assets are pinned down continues to hold in such implementations.

[14] See Golosov and Tsyvinski (2007) for details.

contracts. Given that, we first analyze this economy when private borrowing is subject to the ad hoc limit (13). Then we discuss how our conclusions would change if debt enforcement on private markets is perfect.

Consider any incentive compatible mechanism $\{\mathbf{x}_i, \mathbf{y}_i\}_i$ and $\mathbf{B}$ and let $\{\mathbf{b}_i, \mathbf{c}_i\}_i$ be the associated optimal asset and consumption choices and let $\mathbf{q}$ be bond prices. A necessary condition for incentive compatibility is

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, \frac{y_{i,t}}{\theta_{i,t}} \right) \geq \mathbb{E}_{-1} \left[U^i \left(c_{j,0} + b_{i,-1} - b_{j,-1}, \frac{y_{j,0}}{\theta_{i,0}} \right) + \sum_{t=1}^{\infty} \beta^t U^i \left(c_{j,t}, \frac{y_{j,t}}{\theta_{i,t}} \right) \right], \tag{20}$$

for all pairs i, j. The left side is the utility of agent i when he receives allocation $\{\mathbf{x}_i, \mathbf{y}_i\}$. This should be at least as high as utility from claiming a bundle $\left(\mathbf{x}_j, \mathbf{y}_j\right)$ and choosing asset profile $\boldsymbol{b}_j$ on the anonymous market at the same prices $\mathbf{q}$. The payoff from that choice is the right side of constraint (20). In principle, agent i can further increase his utility from bundle $\left(\mathbf{x}_j, \mathbf{y}_j\right)$ if he chooses some other asset profile $\boldsymbol{b}'$, but as we show below, if trading is subject to ad hoc debt limits, an optimally chosen debt sequence $\boldsymbol{B}$ prevents such retrading.

Let $\left\{\mathbf{c}_i^{adhoc}, \mathbf{y}_i^{adhoc}\right\}_i$ be a maximizer of the objective function (6) subject to feasibility (3) and the incentive constraint (20). Let $B_t = \underline{b}$ for all t and choose any $\boldsymbol{q}$ that satisfies

$$\begin{aligned} q_t &\geq \beta \frac{\mathbb{E}_t U_c^i \left(c_{j,t+1}^{adhoc}, \frac{y_{j,t+1}^{adhoc}}{\theta_{i,t+1}} \right)}{U_c^i \left(c_{j,t}^{adhoc}, \frac{y_{j,t}^{adhoc}}{\theta_{i,t}} \right)} \text{ for } t > 0, \text{ all } i, j, \\ q_0 &\geq \beta \frac{\mathbb{E}_0 U_c^i \left(c_{j,1}^{adhoc}, \frac{y_{j,1}^{adhoc}}{\theta_{i,1}} \right)}{U_c^i \left(c_{j,0}^{adhoc} + b_{i,-1} - b_{j,-1}, \frac{y_{j,0}^{adhoc}}{\theta_{i,0}} \right)} \text{ for all } i, j. \end{aligned} \tag{21}$$

Choose sequence $\left\{\mathbf{x}_i^{adhoc}\right\}_i$ such that

$$c_{i,t}^{adhoc} - q_t \underline{b} = x_{i,t}^{adhoc} + \mathring{b}_{i,t},$$

where $\mathring{b}_{i,t} = -\underline{b}$ for $t > 0$ and $\mathring{b}_{i,t} = b_{i,-1}$ for $t = 0$.

For an agent of i type who claims sequence $\boldsymbol{x}_j^{adhoc}$ and faces debt prices $\boldsymbol{q}$, it is optimal to borrow up to the maximum debt limit $\underline{b}$ and therefore obtain the after-tax consumption allocation $c_{j,0}^{adhoc} + b_{i,-1} - b_{j,-1}$ and $\left\{c_{j,t}^{adhoc}\right\}_{t>0}$. Constraint (20) ensures

that the optimal report is $r(i) = i$ for all i, verifying that $\left\{\mathbf{c}_i^{adhoc}, \mathbf{y}_i^{adhoc}\right\}_i$ is indeed an informationally-constrained optimum. This optimum can be implemented by a sequence of tax functions of the form $\{\mathcal{T}_t(y_t, i)\}_{t,t}$.

Observe that the government debt $\boldsymbol{B}$ plays the same role here as it did in Section 4. When agents face ad hoc borrowing constraints the government can affect interest rates by choosing the level of its debt. As in Section 4, the government exploits monopoly power on asset markets and lowers interest rates. The size of the borrowing constraint $\underline{b}$ is irrelevant for welfare because the government covers its interest expenses by adjusting the stream of tax payments to agents without affecting final allocations.

Like our discussion in Section 4, it is crucial for this result that private debt contracts are enforced imperfectly. If agents can trade on anonymous markets subject only to the natural debt limit, the government loses its ability to influence interest rates through $\boldsymbol{B}$, so the Ricardian equivalence result of Proposition 2 reemerges. Since equation (21) would hold with equality in equilibrium with natural debt limits, welfare would be lower.

The role of the initial debt level and initial asset inequality also mirrors that described in Propositions 1 and 4. The absolute level of government debt B_{-1} per se does not affect welfare in the constrained optimum, but asset inequality does, as can be seen from the right hand side of (20).

It is straightforward to generalize these results to economies with idiosyncratic shocks, richer asset markets, and capital.[15] We summarize our analysis in the following proposition.

Proposition 5. *The initial level of debt B_{-1} does not affect welfare with optimal informationally-constrained taxes, but the level of initial asset inequality $b_{i,-1} - b_{I,-1}$ generally does. Necessary conditions for the optimal path of debt $\mathbf{B}$ to be determinate are anonymous asset trades and ad hoc borrowing constraints. Welfare is higher in the economy with ad hoc debt limits than in the economy with natural debt limits.*

[15] Under mild technical assumptions (e.g. assumption 1 in Kocherlakota (2005)) one can implement informationally-constrained optimal allocations using tax schemes that depend on past histories of individual incomes. In a related paper, Bassetto and Kocherlakota (2004) show that allowing tax functions that are sufficiently flexible in terms of history dependence makes the path of debt irrelevant.

7 Concluding remarks

The spring of 2013 witnessed a lively debate in newspapers and economic magazines about the accuracy and meaning of empirical correlations between output growth rates and ratios of government debt to GDP inferred from data sets assembled by Reinhart and Rogoff (2010). From the perspectives of this paper and of Werning (2007), those correlations and some of the contributions to those debates are difficult to interpret because our models tell us that total government debt *per se* does not impinge on allocations, government transfers, or tax rates. A principal message of this paper is that without exogenous restrictions on transfers, the level of government debt doesn't matter. What matters is how ownership of government debt is distributed. Depending on society's attitudes toward unequal distributions of consumption and work, the cross-section distribution of government debt across assets can matter very much. To interpret those Reinhart-Rogoff facts country-by-country, we would want to know much more about how the distributions of net assets across people have varied across countries and how they have interacted with risks to interest rates and to the underlying sources of unequal productivities across people. An optimal path of debt is determined when agents' abilities to borrow are restricted because that allows prospective public debt issues to affect interest rates. However, this path is independent of the current debt level.

We restricted our analysis to economies in which the government commits to future policies. We believe that a promising direction for research is to explore the role of debt in economies in which a government cannot commit. As our discussion in Section 4 suggests, imperfect commitment may impose additional restrictions on transfers and debt that are feasible in equilibrium. We leave this extension to future work.

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A Example 1

We will show that there is a competitive equilibrium associated with the price sequence $(\varrho, \beta, \varrho, \beta, \ldots)$ with consumption allocation for agent 2 given by

$$c_{2,t} = (\beta/\varrho)^{-1/\sigma}\left((\beta/\varrho)^{-1/\sigma} + \varrho\right)^{-1}\left(\frac{\gamma}{1+\gamma}\bar{l}^{1+\gamma} + \frac{1}{2}\bar{\tau}\bar{l}\right) + \frac{1}{1+\gamma}\bar{l}^{1+\gamma}$$

and

$$c_{2,t+1} = \left((\beta/\varrho)^{-1/\sigma} + \varrho\right)^{-1}\left(\frac{\gamma}{1+\gamma}\bar{l}^{1+\gamma} + \frac{1}{2}\bar{\tau}\bar{l}\right)$$

where t is even. The consumption allocation for agent 1 is then pinned down by the aggregate resource constraint

$$c_{1,t} = \frac{1}{2}\bar{l} - c_{2,t}$$

and

$$c_{1,t+1} = -c_{2,t+1}$$

for even t. In order for agent 1 to hold zero assets, the government's transfer policy must be $T_t = c_{1,t}$ for all t.

It is clear that agent 1's budget constraint and Euler equation is satisfied for all t and thus agent 1 is optimizing for the given interest rate sequence. With a little algebra, it is possible to show that the asset positions

$$b_{2,t} = \bar{l}^{1+\gamma} + T_t - c_{2,t}$$

and

$$b_{2,t+1} = 0,$$

for t even, satisfy agent 2's budget constraint. Moreover,

$$\left(c_{2,t} - \frac{1}{1+\gamma}\bar{l}^{1+\gamma}\right)^{-\sigma} = \frac{\beta c_{2,t+1}^{-\sigma}}{\varrho}$$

and

$$c_{2,t+1}^{-\sigma} > \left(c_{2,t+2} - \frac{1}{1+\gamma}\bar{l}^{1+\gamma}\right)^{-\sigma}$$

for even t. Therefore the agent 2's Euler equation is satisfied for all t with the agent being borrowing constrained in odd periods. We conclude that since, under the above allocation, both agents are optimizing [16] and feasibility is imposed by construction there exists a competitive equilibrium for the interest rate sequence $(\varrho, \beta, \varrho, \beta, \ldots)$.

[16] Optimal labor choice for agent 2 is imposed by $l_t = \bar{l}$ in even periods and 0 otherwise.