Profitable contract menus in competitive insurance markets with adverse selection*

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VERY PRELIMINARY

Abstract

Can profit-making contract menus exist in competitive insurance markets under adverse selection when the single-crossing property does not hold? We show that the answer to this question is related to the equilibrium non-existence problem in the standard Rothschild-Stiglitz framework where single-crossing holds: In a framework where cross-subsidizing contracts can be sustained in equilibrium, then profit-making contract menus cannot exist. If however equilibrium contracts in the standard framework are always individually non-loss-making, then profit making contract menus can occur when single-crossing does not hold.

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Keywords: Adverse selection; insurance market; failure of single crossing

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1 Introduction

Several authors (Smart 2000; Villeneuve 2003; Wambach 2000; Sonnenholzner and Wambach 2009) have shown in different contexts that in insurance markets with adverse selection where the single crossing property (SCP) does not hold, profit-making allocations can be sustained in equilibrium. The single crossing property may be violated if customers are additionally heterogeneous and privately informed with respect to a second dimension other than risk that affects the willingness to pay for insurance. The profit-making contracts are then such that the indifference curves of a high risk and a low risk person are tangential to each other such that cream skimming, i.e. offering a contract which only attracts the low risk type, is not possible.

Recently, Snow (2009) has argued that the result of positive profits is an artefact of the assumption that in existing models insurers are assumed to only offer a single contract each. With a menu of contracts, it is always possible to construct two separating contracts which both types of risks prefer and which are jointly profit-making, thus making the initial contracts unstable. However, these contracts themselves can also not constitute an equilibrium, as a deviating insurer would rather than offering the cross-subsidizing contract menu prefer to offer the profit-making contract only. Thus the whole problem boils down to the question on what happens if the initial contracts do not constitute an equlibrium. Snow (2009) proceeds in arguing that by appealing to the approaches of Wilson (1977) or Hellwig (1987), in equilibrium firms will offer cross-subsidizing, jointly zero profit-making contracts.

While the observation by Snow that a pair of cross-subsidizing contracts might be better for both types of risks is not contentious, in this article we show that his conclusion about the possibility of equilibria with jointly profit-making contracts is not correct. By linking the analysis to the discussion on existence of equilibrium in the standard Rothschild and Stiglitz (1976) model, we show that depending on the specification of insurance market interaction, profit-making contract menus can exist in equilibrium, even if firms offer contract menus.

The basic model we analyze follows Rothschild and Stiglitz (1976) in the timing of

the game where at the first stage, insurers offer a menu of contracts, and in the second stage customers choose a contract. In addition to heterogeneity of risk types, customers furthermore differ in privately known degree of risk aversion. In this game, an equilibrium (non-)existence problem occurs that is similar to the one in the standard Rothschild-Stiglitz framework where in the original model if the proportion of high risks in the population is large, no equilibrium in pure strategies exists.
There are several ways on how to reconcile existence of equilibrium in the standard Rothschild-Stiglitz framework, which we will discuss below. The important distinction between these approaches is that, depending on the specification of the game, either the (second-best efficient) Wilson-Miyazaki-Spence (WMS) contracts constitute the equilibrium contracts, or the (inefficient) Rothschild-Stiglitz (RS) contracts. Applying these concepts to the present case where the SCP does not hold, a similar differentiation applies: If the game specification is such that in the standard framework the WMS contracts will prevail, then profit-making contract menus will not occur in equilibrium. If however the game is such that RS contracts would result in the standard framework, then profit-making contract menus can exist in equilibrium.

We proceed as follows. In the next section we provide an overview of the situations where the single crossing property is violated. We will discuss one application in more detail, showing how with single contract offers profit-making contracts can occur, while with offers of menu of contracts this equilibrium becomes unstable. In section 3 we discuss recent attempts to approach the equilibrium non-existence problem in the standard Rothschild-Stiglitz framework. We then proceed to show that by adapting these to the case where SCP is violated, either equilibria with jointly profit-making contracts or cross-subsidizing contracts can occur depending on the game specification, even if firms offer contract menus. The results are summarized in the conclusion.

2 Adverse selection models without the single-crossing property

In the classic RS model with two risk types, the high risk indifference curve is always flatter than the low risk indifference curve, as the high risk is always willing to pay more for one more unit of further insurance compared to the low risk. This property is known as the single-crossing property.

However, while this property seems plausible, several applications argue that this does not necessarily have to be the case. In de Meza and Webb (2001), agents exert effort to prevent a loss. It can be shown that heterogeneity with respect to risk attitude then gives rise to so-called advantageous selection, i.e. more risk averse agents exert more effort and thus have a lower risk of accident, but they have a higher willingness to pay for insurance due to their higher degree of risk aversion. In a similar vein, Sonnenholzner and Wambach (2009) take as given that agents have different rates of time discounting. Thus those who value the future more, provide more effort to prevent a loss occurring, while they are also willing to pay more for insurance.

While these models combine effort provision with heterogeneous preferences, the case we are going to discuss here in more detail, follows Smart (2000), Villeneuve (2003) and Wambach (2000) who consider multidimensional adverse selection. These authors assume that agents do not only differ with respect to risk, but additionally differ in their privately known degree of risk aversion. Consider the specific case where there is a high risk type with low risk aversion, e.g. a wealthy high risk type, and a low risk type with high risk aversion, e.g. a poor low risk type. In Figure 1 we show the indifference curves of the two types. The curved line through H is the indifference curve of the high-risk type, going through the point of full insurance at her fair premium. The other curved line going through $\hat{L}$ is the indifference curve of the low risk type. As this type is more risk averse, her indifference curve is at some points flatter than that of the high risk type. Thus we can have a situation as in Figure 1, where both indifference curves are tangential to each other.

Before going further, let us first specify the model more formally. There is a continuum of individuals of mass 1. Each individual faces two possible states of nature:

in state 1, no loss occurs and the endowment is w_{01}, in state 2 a loss occurs and the endowment is w_{02} with $w_{01} > w_{02} > 0$. Individuals have a twice continuously differentiable strictly concave von Neumann Morgenstern utility function $v(w, \theta)$, where θ is a parameter for risk aversion. Insurance is provided by firms in the set $F := \{1, ..., f, ...r\}$. Individuals differ with respect to risk and risk preferences. To be precise, there are two types: type hh has accident probability π_h and risk aversion parameter θ_h whereas type ll has accident probability π_l and risk aversion parameter θ_l where $0 < \pi_l < \pi_h < 1$ and $-\frac{v''(w,\theta_l)}{v'(w,\theta_l)} > -\frac{v''(w,\theta_h)}{v'(w,\theta_h)}$. Thus, hh is the high risk but high risk tolerant type, and ll the low risk but more risk averse type. Firms are risk neutral and do not know, ex ante, any individual's type.[1]

In this setup, the single-crossing property might fail if the difference in the degree of risk aversion is sufficiently large and/or the difference in accident probabilites is sufficiently small. In the remainder of the paper, we will focus on this case where the single-crossing property is violated.

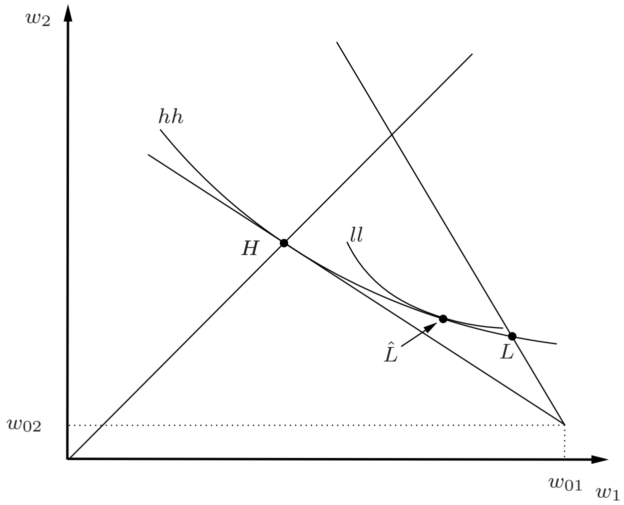

Figure 1: Failure of single crossing property

Single contract offers

We will now discuss the case when insurers are restricted to offer a single contract each.[2] For latter use, we describe the timing of the game formally:

[1]Note that, for a given contract, only the private information on the risk type dimension is payoff-relevant for insurers.

[2]See also Smart (2000), Wambach (2000), Villeneuve (2003).

Stage 0: The type of each individual is chosen by nature. Each individual has a chance of λ, $0 < \lambda < 1$ to be a hh type, and of $(1 - \lambda)$ to be a ll-type.

Stage 1: Each insurer $f \in F$ offers a single contract $\omega^f = (P^f, I^f) \in \Omega = \{(P, I)|I \leq w_{01} - w_{02}\}$ that specifies a premium P^f and an indemnity I^f.[3]

Stage 2: Individuals choose a contract or remain uninsured.

The contracts H and L in Figure 1 display the standard RS contracts: Full insurance for the high risk type at her fair premium, and partial insurance for the low risk type at her fair premium, such that the high risk type is indifferent between the contracts. These standard RS contracts (H and L) do not constitute an equilibrium, as L is not the best possible contract for the low risk type which satisfies the incentive compatibility constraint and which makes no loss. Now, since each insurer only offers a single contract, if the share of high risks is sufficiently high such that a profitable pooling contract does not exist, the contracts underlying H and $\hat{L}$ constitute an equilibrium, i.e. some insurers offer contract H and some offer contract $\hat{L}$. High risks buy contract H and low risks buy contract $\hat{L}$. Interestingly, contract $\hat{L}$ is profit-making. This is an equilibrium because any deviation from $\hat{L}$ does not only attract low risks but also high risks as the indifference curves are tangential to each other. There is no scope to cream-skim, i.e. to attract the low risks only.
For the purpose of this article, we will henceforth call H and $\hat{L}$ the 'RS-type contracts'.

Menu of contracts
Snow (2009) shows that the above result of an equilibrium in which some firms make a strictly positive profit no longer holds if, in the basic game, insurers are allowed to offer a menu of contracts rather than a single contract each. So compared with the situation above, stage 1 changes to:

[3] $I > w_{01} - w_{02}$ is ruled out for moral hazard considerations.

Stage 1': Each firm $f \in F$ offers a finite set of contracts $\Omega^f = \left\{\omega_1^f, \omega_2^f, ..., \omega_k^f\right\}$ where a contract $\omega_l^f = (P_l^f, I_l^f) \in \Omega$ specifies a premium P_l^f and an indemnity I_l^f.

Now consider the contract pair H', L' in Figure 2. These contracts are chosen such that high risks prefer contract H', low risks prefer contract L', and as these contracts are close to H and $\hat{L}$ they are jointly profit making. While H' makes a loss, the profit obtained from the low risks with L' is more than enough to compensate the losses. Thus an insurer would offer H' and L' (if all the others offer either H or $\hat{L}$), attract all consumers and make a profit which is larger than offering $\hat{L}$ only. Thus, the profit making contract $\hat{L}$ cannot be tendered in equlibrium if a menu of contracts is considered.
However, as already noted by Snow, H' and L' cannot constitute an equilibrium either. If every insurer offers H' and L', a single insurer might only offer L', i.e. the profit making contract, leaving the loss making contract for the others. If sufficiently many insurers opt for this, then those who are still offering H' would make a loss overall, which cannot be an equilibrium outcome.

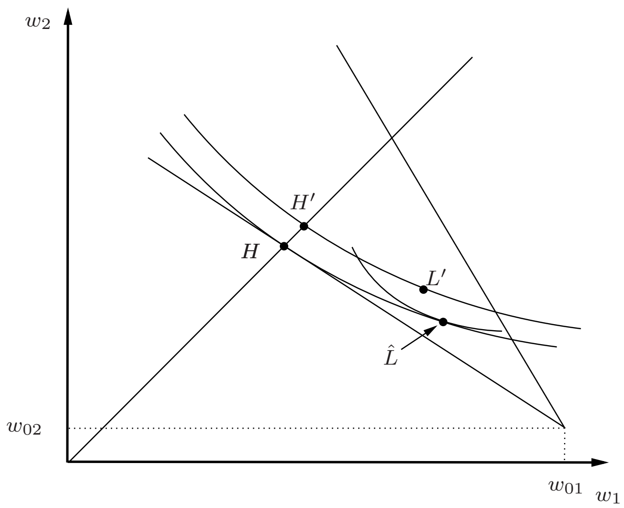

Figure 2: Profitable deviation with contract menus

So to decide whether profit making contracts can exist or not, one has to go deeper into the issue of how the equilibrium will look like. Snow refers in this context to the approaches by Wilson (1977) and Hellwig (1987). As Hellwig (1987) considers only single contract offers, Snow argues that adapting Hellwig to contract menus or applying the Wilson anticipatory equilibrium concept yields an equilibrium with the cross-subsidizing, jointly zero profit-making WMS contracts. However, Wilson (1977) or Hellwig (1987) are not the only models to provide solutions to the Rothschild-Stiglitz equilibrium non-existence puzzle. There are further approaches in the literature (that allow insurers to offer contract menus.) We will discuss these approaches in the next section before we analyze the equilibrium in this model where the single crossing property is violated.

3 The equilibrium non-existence debate

The literature tackling the equilibrium non-existence problem in the Rothschild-Stiglitz framework is large. Research has addressed the non-existence problem by considering mixed strategies (Dasgupta and Maskin, 1986), introducing equilibrium concepts that differ from Nash-equilibrium (Wilson 1977; Riley 1979), extending the dynamic structure of the game (Jaynes 1978; Hellwig 1987; Engers and Fernandez 1987; Asheim and Nilssen 1996; Mimra and Wambach 2011) or modifying assumptions about insurer or contract characteristics (Inderst and Wambach 2001; Faynzilberg 2006; Picard 2009; Mimra and Wambach 2010).[4]

In terms of the resulting equilibrium allocation, these different approaches can be broadly distinguished into models that yield the (inefficient) RS allocation (e.g. Riley 1979; Engers and Fernandez 1987; Inderst and Wambach 2001), even if they are not equilibrium contracts in the RS framework, and models that yield the second-best efficient Wilson-Miyazaki-Spence allocation (e.g. Asheim and Nilssen 1996, Picard 2009, Mimra and Wambach 2010, 2011).[5] To provide an intuition for this result, we

[4]There are yet methodically different strands in the literature. Ania, Tröger, and Wambach (2002) take an evolutionary game theory approach, Guerrieri, Shimer, and Wright (2010) consider a competitive search model; other directions use cooperative concepts (Lacker and Weinberg, 1999) or a general-equilibrium framework (see e.g. Dubey and Geanakoplos (2002) or Bisin and Gottardi (2006)).

[5]The RS allocation is inefficient in those cases where the RS contracts do not constitute an equilibrium in the original model. In case these contracts are equilibrium contracts, i.e. if the number of high risks is sufficently large, the RS contracts are second-best efficient.

will focus on two contributions which lead to the two different allocations.

In Mimra and Wambach (2010), which is based on Faynzilberg (2006), insurers choose capital ex ante. Thus, depending on the level of capital, insolvency might occur. Mimra and Wambach (2010) show that an equilibrium always exists in which insurers choose zero upfront capital in the first stage and offer the WMS contracts in the second stage. The WMS allocation can be sustained for the following reason: Consider a cream-skimming deviation, where a deviant insurer attracts all low risk types. If high risks stay at their non-deviant insurer, this insurer will go insolvent as he only serves high risks and holds zero upfront capital (recall that in a cross-subsidizing WMS allocation the high risk contract is loss making). This leads to a deterioration of high risk type expected utility. Thus the high risk type may prefer to choose the cream skimming contract as well, which renders the deviation unprofitable.[6]

In contrast, in Inderst and Wambach (2001) insurers are capacity constrained. This constraint, which might result from limited capacity to deal with applications, limited capital which requires firms not to take on too many contracts, etc., implies that if too many insured come to the firm, there will be rationing and customers might have to search for an alternative insurer which is costly. In this setup, the RS contracts are equilibrium contracts even if they do not constitute an equilibrium in the original RS setup for the following reasoning: Suppose every insurer apart from one (the deviator) offers the RS contracts. The deviator offers a menu of contracts which is better for both types and profit-making, as long as all insurees go to this insurer, i.e. if the loss making high types are cross-subsidized by the profit-making low types. However as this deviator is capacity constrained, a customer at this deviator risks being rationed and will thus incur search costs. So insurees compare the risk of being rationed with the gain in expected utility from applying for the contract at the deviator. Now since for any deviating offer the gain in expected utility of a high risk

[6] Further models that yield the WMS allocation are Picard (2009) and Asheim and Nilssen (1996). Picard (2009) endogenizes the contract structure: insurers can offer contracts in which customers share the profits or losses of the insurer, as e.g. in a mutual. This sustains the WMS allocation as a cream-skimming deviation imposes losses on the high risk type WMS contract such that high risks as well prefer the deviating contract. In Asheim and Nilssen (1996), insurers can renegotiate contracts with their own customers on a nondiscriminatory basis. The WMS allocation is the unique equilibrium allocation as a cream-skimming contract, lying outside the efficiency region, will be renegotiated to an efficient contract that yields high risks higher utility than their WMS contract.

type is always larger than that of low risk types since the high risk type indifference curve is flatter, a continuation equilibrium exists in which only high-risks turn up at the deviator. This makes deviation unattractive.[7]

Thus, even if there might be a contract menu that would be preferred to the RS contracts in the standard framework where single-crossing holds, there are models of competitive insurance markets in which the RS contracts are equilibrium contracts. We will now proceed to show that the underlying logic carries over to the case where the single-crossing property does not hold, with the consequence that RS-type equilibria with positive profits exist even though firms can offer contract menus. Put broadly, we will show that the RS versus WMS dichotomy carries over to case of violation of the single-crossing property.

4 Equilibrium allocations in adverse selection insurance markets without the single-crossing property

To demonstrate the link between the possibility of profit-making allocations in competitive markets with adverse selection and the equilibrium non-existence problem in the standard Rothschild-Stiglitz model, we will apply the two models discussed above in turn, where one yields the WMS allocation in the single-crossing framework, and one yields the RS allocation.

[7]Further models that yield the RS allocation are Engers and Fernandez (1987), Guerrieri et al. (2010) and Ania et al. (2002). Engers and Fernandez (1987) model Riley's reactive equilibrium by allowing insurers to add contracts repeatedly in a second stage. This sustains the RS allocation as cream-skimming on deviations renders any deviation unprofitable. Guerrieri, Shimer, and Wright (2010) combine adverse selection with competitive search theory. An equilibrium with the RS allocation exists as with bilateral matching, a pooling offer to attract a cross-section of types does not attract the good types. The good types are discouraged from searching since their outside option, trying to obtain a separating contract, is more attractive. Note that the economics behind this is similar to the above discussed capacity constraints. Ania et al. (2002) model dynamics in insurance markets using evolutionary game theory. RS contracts are the long run outcome of the evolutionary game if insurers experiment only locally since the RS contracts cannot be destabilized by small changes in contracts.

4.1 WMS allocation

Consider the framework of Mimra and Wambach (2010), but assume that individuals are of types hh or ll and the single crossing property is violated. The timing and specification of the game is the following:

Stage 0: The type of each individual is chosen by nature. Each individual has a chance of λ, $0 < \lambda < 1$ to be a hh type, and of $(1-\lambda)$ to be a ll-type.

Stage 1: Each firm $f \in F$ decides on the level of its upfront capital K^f. There are nonnegative opportunity costs to holding capital.[8]

Stage 2: Each firm $f \in F$ offers a finite set of contracts $\Omega^f = \left\{\omega_1^f, \omega_2^f, ..., \omega_k^f\right\}$ where a contract $\omega_l^f = (P_l^f, I_l^f) \in \Omega$ specifies a premium P_l^f and an indemnity I_l^f.

Stage 3: Individuals choose their insurance contract.

Stage 4: Losses are realized.

Stage 5: Insurance firms pay out indemnities. If total claims C^f at firm f are less than total assets A^f, the insurance firm fully settles claims. If total claims C^f exceed total assets A^f, then the insurance firm pays out the assets and defaults on the remaining claims. The ex ante known insolvency rules specify proportional payout, i.e. a customer at firm f who has bought contract ω_g^f and has realized a loss receives a fraction $\beta^f = A^f/C^f$ of her indemnity claim I_g^f.

In analogy to the standard case where individuals only differ with respect to risk, we define the WMS contracts as the contract menu that maximizes utility of the low-risk type ll subject to incentive compatibility and an overall nonnegative profit constraint.

With crossing indifference curves at the ll-type WMS contract, the problem of equilibrium existence is exactly the same as in the framework where single crossing holds, that is cream-skimming needs to be prevented.[9] The logic in Mimra and Wambach (2010) then applies.

[8]Equilibrium existence does not depend on whether opportunity costs of capital are zero or positive.

[9]Observe that the WMS contracts are such that at the ll-type WMS contract, there is no point of tangency of indifference curves as otherwise a nonnegative-profit contract menu preferred by both types exists.

Proposition 1. *If insurers choose capital ex-ante, an equilibrium that yields the cross-subsidizing, jointly zero profit-making WMS allocation always exists.*

Proof. The proof proceeds along the same lines as the proof of Proposition 3 in Mimra and Wambach (2010) and is therefore omitted. □

We illustrate the result in Figure 3, which displays the equilibrium contracts ω_{hh}^{WMS} and ω_{ll}^{WMS}. Suppose a deviating insurer offers a cream-skimming contract ω_{CS}. If only low risks choose this contract, hh-type utility deteriorates if they choose their WMS contract at an insurer offering WMS and holding zero upfront capital as the insurer goes insolvent. With zero ex ante capital, the allocation for the high risks shifts downwards as shown in Figure 3. Then, however, the cream-skimming contract is attractive to hh-types as well, which in turn makes it unprofitable for the deviator. In equilibrium insurers hold zero capital, as putting in more capital only increases incentives of other insurers to cream skim low risks from an insurer.

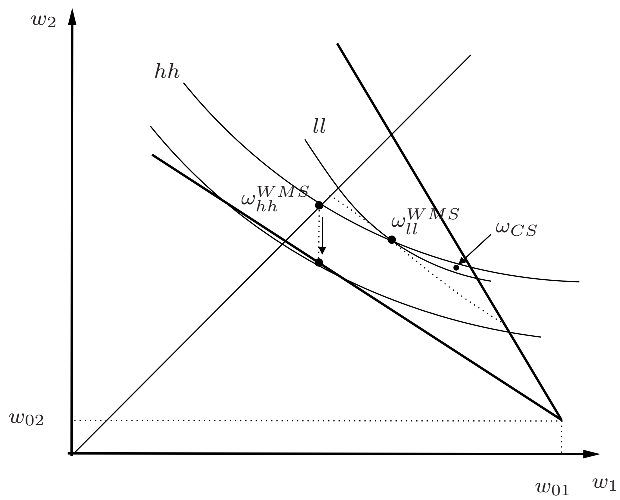

Figure 3: WMS equilibrium contracts and deterioration of hh-type utility at WMS insurer under cream-skimming when insurers hold no upfront capital

4.2 Profit-making RS-type allocation

We will now consider the case in which under single crossing the RS contracts are equilibrium contracts. For this purpose, we consider the framework of Inderst and

Wambach (2001), but assume that individuals are of types hh or ll and the single crossing property is violated.

The specification and timing of the game is the following: Firm $f \in F$ has a fixed (integer) capacity of $k^f > 0$. There are $N = \{1, ..., N\}$ individuals. It holds that $k^f < N$ for all $f \in F$ and $\sum_{f \in F'} \geq N$ for all sets $F' = F \setminus \{f\}$ with $f \in F$, i.e. no firm can serve the whole market, but all but one firm together are sufficient to serve the whole market. The timing is the following:

Stage 0: The type of each individual is chosen by nature. Each individual has a chance of λ, $0 < \lambda < 1$ to be a hh type, and of $(1 - \lambda)$ to be a ll-type.

Stage 1: Each firm $f \in F$ offers a finite set of contracts $\Omega^f = \left\{\omega_1^f, \omega_2^f, ..., \omega_k^f\right\}$ where a contract $\omega_l^f = (P_l^f, I_l^f) \in \Omega$ specifies a premium P_l^f and an indemnity I_l^f.

Stage 2: Individuals choose an insurer f and a contract ω_l^f or remain uninsured. If the number of customers choosing insurer f, denoted by n^f, does not exceed k^f, each customer obtains her desired contract. If n^f is larger than k^f, the insurer has to apply a rationing scheme. Rationing occurs randomly over all applicants. If some customers do not obtain a contract, Stage 3 follows.

Stage 3: The rationed customer can either choose to remain uninsured or she can visit any other firm f' which still has free capacity available and pick a contract. Search for a new insurer is costly, search costs measured in utility units are $u > 0$. If a customer does not obtain a contract at the next insurer, Stage 3 is repeated. If all customers are either served or have exited the market, the game ends.

Let $v_t^E(\omega)$ denote the expected utility of type t from contract ω. For notational convenience, let $v_{hh}^H = v_{hh}^E(H)$ and $v_{ll}^{\hat{L}} = v_{ll}^E(\hat{L})$.

For capacity constraints to be relevant, search costs u should neither be too high nor too low and the capacity of a single firm compared to the market sufficiently low. To describe the latter, let $k^M = \max_{f \in F} k_f$ be the maximum capacity of a single firm in the market and let ρ^M denote the rationing probability of an individual that chooses a contract at the firm with the maximum capacity expecting that all high risks but no low risk turns up at that firm. Furthermore, let v_{hh}^P denote the expected utility of a hh-type under his preferred contract with the low-risk type fair premium.

Assumption 1. $v_{ll}^{\hat{L}} - u > v_t^E(0,0)$ and $v_{hh}^H - u > v_t^E(0,0)$. Furthermore, $(1-\rho^M)v_{hh}^P + \rho^M(v_{hh}^{\text{RS-type}} - u) < v_{hh}^H$.

We now claim, that in this framework, profit-making contracts can occur in equilibrium when single crossing is violated even if firms offer contract menus.

Proposition 2. *If insurers are capacity constrained and Assumption 1 holds, an equilibrium with the jointly profit-making RS-type contracts exists.*

Proof. See Appendix. □

The logic of the proof is illustrated by Figure 4 below.

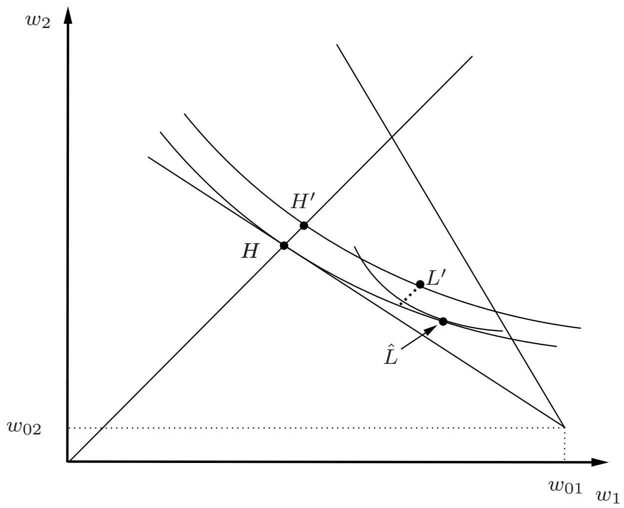

Figure 4: Gain in expected utility from deviating contract menu

Figure 4 shows the RS-type equilibrium contracts H and $\hat{L}$ and one possible deviating contract menu H' and L'. For this deviating contract menu and indeed for any possible deviating contract menu, the gain in expected utility for low risk types is never higher than the gain in expected utility for high risk types. This can be seen as follows: Consider any contract which makes the low risk better off. The willingness to pay for such a contract can be seen by a shift from this contract along the 45 degree line to the initial indifference curve. In Figure 4, this is indicated by the dotted line. As the high risk type must weakly prefer her (deviating) high risk type contract H' compared to L' , the willingness to pay of the high risk for H' is larger than or equal to her willingness to pay for L'. But, as can be seen from Figure 4, this willingness to pay is larger than the willingness to pay of the low risk type.[10] Now since low risks never gain more than high risks, a continuation equilibrium exists in which only high risks turn up at the deviating insurer: If sufficiently many high risks go to the insurer, there will be rationing, and thus the low risks would prefer to stay with their original insurer. However, if only high risks turn up, the deviator would make a loss, which makes deviation unattractive in the first place. Thus, the profit-making RS-type contracts constitute an equilibrium.

5 Conclusion

This paper shows that the debate on whether profit-making contract menus can occur in a competitive market with adverse selection is related to the equilibrium non-existence problem in the original Rothschild-Stiglitz model.

If the Wilson-Miyazaki-Spence concept holds in the standard model, i.e. insurers offer cross-subsidizing contracts in equilibrium in those cases where the standard Rothschild-Stiglitz model does not have an equilibrium in pure strategies, then also in a model where the single-crossing property does not hold, profit-making contract menus will not occur. In equilibrium there will be cross subsidization with zero profits for the insurer.

If however concepts hold where in the standard Rothschild-Stiglitz model insurers offer the Rothschild-Stiglitz contracts always, then in the case where the single-crossing property is violated, profit-making contract menus can occur in equilibrium.

Thus the question of whether profit-making contracts can occur or not is related to a topic which goes beyond the application considered here: As the Wilson-Miyazaki-Spence contracts are second-best efficient, while the Rothschild-Stiglitz contracts are

[10]Even the best self-selecting deviation for the ll-types, i.e. a deviation where the ll-type contract moves along the 45 degree line from $\hat{L}$, does not yield a *higher* willingness to pay of low risks than of high risks, such that the gain in expected utility for low risks is never *higher* than the gain in expected utility of high risks.

(Pareto) inefficient, only the latter call for government intervention as a consequence of adverse selection, at least as long as a Pareto standard is used for welfare considerations. Thus if adverse selection is a cause for governmental intervention, then profit-making contract menus might be considered feasible.

Appendix

Proof of Proposition 2.

The proof follows proof of Proposition 1 in Inderst and Wambach (2001).

We claim that one possible equilibrium strategy for firms is to offer both RS-type contracts each. To show this, we need to discuss the equilibrium strategy of the customers. Let h be a search history. A strategy of a customer is a map from history h into its action space, consisting out of the choice to visit some firm and pick a particular contract or to exit the market. Consider the case where all firms have offered the two RS-type contracts. After any search history h, let $m(h)$ be an ordered set of sellers who still have unused capacity $k(h)$. Let $n(h)$ be an ordered set of buyers who have not yet found contracts. Define $f(i,h)$ as the smallest index $f \in \{1,2,...,m(h)\}$ satisfying $\sum_{f'=1}^{f} k_{f'}(h) \geq i$. The equilibrium strategy of customer $i \in \{1,2,...,n(h)\}$ is to choose firm $f(i,h)$ and to pick the RS-type contract according to her type. By doing so the customers always solve their coordination problem and in equilibrium, they are all served in Stage 2. No customer has an incentive to deviate, as all are served with the best possible contract on offer and no rationing occurs. All firms make positive profit with these contracts.

To support the resulting allocation as an equilibrium, it remains to specify strategies if a single firm deviates to a different menu of contracts in order to show that there is no profitable deviation, i.e. no deviation in which a firm can earn a higher profit than under the above specification.

Consider now the case where all firms offer the two RS contracts apart from firm $\bar{f}$ whose offer if given by the contract menu $\Omega^{\bar{f}} = \left\{\bar{\omega}_1^{\bar{f}}, \bar{\omega}_2^{\bar{f}}, ..., \bar{\omega}_k^{\bar{f}}\right\}$. Rationing, if it occurs, is the same for all customers of a single firm. Therefore it suffices to concentrate on the two contracts which are optimal for each type. Let $\bar{\omega}_{hh}$ be the contract out of $\Omega^{\bar{f}}$ which the high-risk types prefers, and define $\bar{\omega}_{ll}$ for the low-risk type analogously. Both contracts may well be the same.

We only consider the interesting case, where $v_{ll}^{E}(\bar{\omega}_{ll}) \geq v_{ll}^{\hat{L}}$ and $\bar{P}_{ll} - \pi_l \bar{I}_{ll} > 0$.[11] The deviating offer is supposed to attract some low-risk types. Due to incentive compatibility and since high-risk and low-risk type indifference curves are at a point of tangency at $\hat{L}$, it follows that $v_{hh}^{E}(\bar{\omega}_{hh}) \geq v_{hh}^{H}$. We claim that in this case there exists a (continuation) equilibrium where only high-risk types turn up at the deviator. We construct the continuation equilibrium such that all customers visit $\bar{f}$ in Stage 2 with a probability ϕ if and only if they are of the high-risk type. With probability $1 - \phi$ a high-risk type turns to a firm where she obtains her RS-type contract with probability 1. Low-risk types choose a firm other than $\bar{f}$ with probability 1 in order to buy their RS-type contract. Formally, the equilibrium strategies are as follows: After any search history h, let $m'(h)$ be an ordered set of non-deviating sellers who still have unused capacity $k_f(h)$. Let n(h) be an ordered set of buyers who have not yet found contracts. Define $f'(i,h)$ as the smallest index f satisfying $\sum_{f'=1}^{f} k_{f'}(h) \geq i$. In Stage 2, the strategy of customer i is to go to firm $f'(i,h)$ and choose her RS-type contract, if she is of the low-risk type. If she is of the high-risk type, her strategy is to go to firm $f'(i,h)$ with probability $1-\phi$ and to firm $\bar{f}$ with probability ϕ and choose the appropriate contracts. For any further stage, and after any search history, if firm $\bar{f}$ has no free capacity, customer i will go to firm $f'(i,h)$ and choose the RS-type contract according to her type. If firm $\bar{f}$ still has free capacity, take any continuation equilibrium from there on, which we know does exist.

We will now show that the above mentioned ϕ is uniquely determined. For a given ϕ calculate the expected rationing probability $\rho(\phi)$ for an individual who contemplates choosing firm $\bar{f}$

$$\rho(\phi) = 1 - \sum_{m=0}^{N-1} \binom{N-1}{m} (\phi\gamma)^m (1-\phi\gamma)^{N-1-m} \min\left\{1, k_{\bar{f}}/m+1\right\}$$

Note that $\rho(\phi)$ is strictly increasing in ϕ. Define by $v_t^D(\bar{\omega}_t, \rho) = (1-\rho)v_t(\bar{\omega}_t) + \rho(v_t^{RS-type} - u)$ the expected utility of a type $t \in \{hh, ll\}$ who chooses the deviating

[11] For the other cases, see Inderst and Wambach (2001)

firm, and, if rationed, buys the RS-type contract in the next round. There exists a unique $0 < \phi < 1$ satisfying

$$v_{hh}^{D}(\bar{\omega}_{hh}, \rho(\phi)) = v_{hh}^{H} \tag{1}$$

$v_{hh}^{D}(\bar{\omega}_{hh}, \rho)$ is strictly decreasing and continuous in ρ, while $\rho(\phi)$ is strictly increasing and continuous in ϕ. Moreover, it holds that $v_{hh}^{D}(\bar{\omega}_{hh}, \rho(0)) > v_{hh}^{H}$ while $v_{hh}^{D}(\bar{\omega}_{hh}, \rho(1)) < v_{hh}^{H}$. The latter assertion follows directly from Assumption 1, by noting that $\rho(1) = \rho^{M}$, which was defined before invoking Assumption 1.

It now remains to show that, given the uniquely chosen $0 < \phi < 1$ satisfying (1), it is indeed optimal for any low-risk type not to visit $\bar{f}$. Recall first that by Assumption 1 and our specification of continuation strategies, an individual implements her RS-type contract at Stage 3 if she was rationed when visiting $\bar{f}$ at Stage 2. It therefore remains to show that $v_{ll}^{D}(\bar{\omega}_{ll}, \rho(\phi)) \leq v_{ll}^{\hat{L}}$. This holds if $v_{hh}^{E}(\bar{\omega}_{hh}) - v_{hh}^{H} \geq v_{ll}^{E}(\bar{\omega}_{ll}) - v_{ll}^{\hat{L}}$, i.e. by substituting from incentive compatibility $v_{hh}^{E}(\omega_{ll}^{\hat{L}}) = v_{hh}^{H}$ and $v_{hh}^{E}(\bar{\omega}_{hh}) \geq v_{hh}^{E}(\bar{\omega}_{ll})$, if

$$v_{ll}^{E}(\bar{\omega}_{ll}) - v_{ll}^{\hat{L}} \leq v_{hh}^{E}(\bar{\omega}_{ll}) - v_{hh}^{E}(\omega_{ll}^{\hat{L}}).$$

Since at $\hat{L}$ the high-risk type and low-risk type indifference curves are tangential to each other, for any contract ω with $v_{ll}^{E}(\omega) \geq v_{ll}^{\hat{L}}$ and $P - \pi_l I > 0$, it holds that $v_{ll}^{E}(\omega) - v_{ll}^{\hat{L}} \leq v_{hh}^{E}(\omega) - v_{hh}^{E}(\omega_{ll}^{\hat{L}})$.

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