Mitigation of Natural and Industrial Disasters: The Combined Roles of Insurance and Land Use Regulation*

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Preliminary Version

Abstract

Natural and industrial risks present many similarities, since they are both strongly determined by the number of people and businesses and the value of their assets located in exposed areas. The difference between natural and industrial disasters is not essentially in the nature of the physical phenomena but in the legal liability rules that apply in civil cases. Implicitly, the government is the residual claimant for disasters compensation, whereas the firm is liable for industrial disasters it causes. We analyze how location choices, risk levels and insurance price are simultaneously determined at the equilibrium in a given regulation context and under a given hazard. The equilibrium depends so on the given regulation of insurance and land use; comparative statics of equilibria with respect to different regulations enable to make recommendations for natural and industrial risks management. Indeed, our model provides a new (to our knowledge) theoretical framework to assess simultaneously insurance and land use policies and characterize their necessary coordination.

Keywords: insurance, natural disasters, industrial disasters, land use regulation

JEL classification: Q51, Q54, H23, R52

1 Introduction

Natural and industrial risks present many similarities, since they are both strongly determined by the number of people and businesses and the value of their assets located in exposed areas. As described by Mileti (1999) in his book *Disasters by Design*, disasters mainly result from human

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communities design and building. For example, the torrent floods in Brazil in January 2011 caused more than 500 deaths and were so the deadliest natural disaster in Brazil history; these fatal consequences could have been avoided if the damaged cities would not have been built in a mountain area with instable soil.[1] Indeed, populations are drawn to risky locations by risk amenities, such as the river sight or employment by industrial activities. Growing urbanism in risky locations has been recently illustrated by China. According to Fei (2010), urbanization has significantly increased during the past five decades in China, especially in megacities, which are mainly located in major rivers' reaches. "[In Shenzhen Special Economic Region,] from 1980 to 2005, percentage of agricultural land uses dropped from 86.22% to 50.16%, while urban land uses increased from 0.63% to 33.47%". A similar example is offered by neighborhoods of AZF plant in Toulouse (France), which exploded in 2001. AZF website mentions that "initially [in 1924] far from the dwelling areas, the plant has been progressively bordered by the Toulouse agglomeration"; pictures of the plant and its neighborhoods in the 1930's and in 2001 illustrate the agglomeration extension.[2] Regulating activities and location choices in high risk areas is a key element of prevention from natural and industrial risks.

The difference between natural and industrial disasters as for economists is not essentially in the nature of the physical phenomena. Moreover they may be triggered by each other via a domino effect. Natural disasters can imply industrial disasters. For example, in March 2011, the biggest earthquake in Japan history triggered not only a 33ft tsunami but also nuclear accidents. Other disasters are caused by a combination of natural and industrial hazards, as recalled by toxic mud floods due to a tank failure in an aluminium plant in Hungary in October 2010. Natural and industrial hazards can indeed mutually aggravate each other. The potential increase of seismic activity due to the Three Gorges Dam in China is for example a subject of debate among scientists (see Naik and Oster (2009)). Even past industrial accidents can aggravate consequences of natural disasters. In 2010, forest fires in Russia burned areas that were polluted following the 1986 Tchernobyl accident, contaminating agricultural crops and local populations.

Obviously, natural and industrial risks differ in the legal liability rules that apply in civil cases. We consider polar cases in which either the government is the residual claimant for disasters compensation or there is a third party - henceforth called the firm - that is liable for disasters. By convenience we call these two extreme cases respectively natural risks and industrial risks. We analyze in this paper how location choices, risk exposure and insurance price are simultaneously determined at the equilibrium in a given regulation context and under a given hazard. Note that we do not consider here any risk perception bias.

The first thing we model is how insurance by pricing risk (or not) - impacts housing market and location choices. Depending on the precision of risk classification, insurance tariffs do or do not provide incentives for locating in low-risk areas. Several empirical studies based on the hedonic prices method show that housing markets respond to natural risks insurance. Indeed, households face a tradeoff between insurance expenditures and housing prices. This tradeoff, which is the heart of our theoretical model, has been empirically verified. Bin et al. (2008), MacDonald et al. (1990), Harrison et al. (2001) compare the housing price differential for flood risk with the capitalized value of flood insurance premia between the exposed and non-exposed areas. For example, MacDonald et al. (1990) interpret the fact that the sales price reduction shortly exceeds the capitalized value of flood insurance premiums as revealing the presence of small uninsurable

[1] See http://www.liberation.fr/terre/01012313830-bresil-une-catastrophe-pas-si-naturelle.
[2] See http://www.azf.fr/l-usine-azf-de-toulouse/historique-800233.html.

costs associated with flood risk. Furthermore, Skantz and Strickland (1987) show that real estate prices do not immediately decline after a flood but "when flood insurance premiums rise dramatically approximately one year after the flood, these higher rates are capitalized into home values and prices do decline". Morgan (2007) also shows that, in Santa Rosa County (Florida), "subsidized insurance premia create a [real estate] market imbalance by reducing expected flood losses and perceived risks associated with living in floodplain areas".

The situation is quite different for industrial risks, since in theory their cost is borne by the firm which has caused damages. Household insurance against industrial disasters amounts to a victim insurance, such as in France,[3] or in many other countries, households directly lodge a complaint against the firm and no such insurance system is organized. For sake of simplicity, with exposure to natural or industrial risks, we ignore delay and associated costs or uncovered losses that could be borne by households. Therefore the impact of industrial risks on housing markets should be very limited.[4]

The second thing we model is how in turn land use determines the compensation paid in the case of a disaster. Indeed, choosing a location exposed to natural or industrial risk causes a negative externality on the community or the firm, who have respectively to pay for the cost of natural or industrial risks.[5] Therefore land use determines the cost of risk and, in the case of natural risks, the price of household insurance. Note that location choices could be made after the risk source setting up. It is the most common case with natural risks; households can also set up after the firm without any due. In both cases, there is no right of "initial land use": the community or the firm are still responsible for any disaster compensation to the newcomers.[6]

The equilibrium results from the interaction between location choices and insurance pricing. It depends on the given regulation of insurance and land use. In that framework, it is useful to compare the equilibria given by different regulations. These comparative statics enable us to assess and rank different public policies and so to make recommendations for natural and industrial risks management.

Risk is indeed determined by the combination of hazard, that is the probability that a loss occurs, and vulnerability, that is the value that could be lost. One specificity of our paper is to consider risk as fully determined by vulnerability, which depends on household location choices. In other words, we assume that hazard is given and do not consider hazard reduction. In practice, the government or the firm faces a tradeoff - often overcome by a cost benefit analysis - between reducing hazard or decreasing vulnerability. In order to reduce hazard, the government could

[3]The aim of this insurance system is only to manage the basic coverage for victims by avoiding long litigation and by covering the residual risk of no responsible identification. This system was created in 2003 in reaction to AZF explosion in 2001 which killed 31 and injured more than 4 500 persons. Every insured house has to be covered against industrial disasters and the premium is included in the house insurance premium.

[4]In practice, insurance or courts are not fully efficient nor without any delay. This explains that several empirical works also based on the hedonic prices method show that perception of industrial risks can decrease property values (see Gawande and Jenkins-Smith (2001), Kiel and McClain (1995)).

[5]In practice, imperfect compensation for industrial disasters implies that a part of their cost is borne by households. This reduces the importance of the external effect exerted by high risk location choices on the firm. However, these benefits may not overcome the bad quality of compensation provided to households in the case of loss.

[6]In France for example, this liability rule in the case of industrial risks results from the Environment Code (section L 514-19), the rules of civil liability (Civil Code, section 1382 and the following ones) and legal precedents. A unique exception can be invoked: constant and chronic pollution that would induce nuisances but no material nor health damages (Code of Building and Living, section L 112-16).

build dams, levees or retention basins; the firm can change its technology, enforce stronger safety rules or limit its production. For example, as stated in Sauvage (1997), in Saint-Gaudens the industrialist "paid 11 millions of francs [M€1.7] the containment of chlorine storage in order to reduce the danger zone radius from 850 to 500 meters around the risk source" [our translation]. We focus here on vulnerability decreasing policies based on insurance or land use regulation.

We assess in our paper insurance and land use public policies dedicated to natural risks management. Insurance against natural disasters is offered or guaranteed by the government in several countries with different tariff principles. In the United States, flood insurance with actuarial premia - and subsidy for specific risks - is provided by the federal government in the framework of the National Flood Insurance Program, which was established by the Congress in 1968. In Japan, household earthquake insurance benefits from the guarantee of the government and is based on actuarial premia.[7] The French natural disasters insurance system, created in 1982, is guaranteed by the government and based on uniform premia.[8] As location choices impact the financial exposure of the community, the government can take land, prohibit new building or limit population density to reduce the risk. In the United States, the Hazard Mitigation Grant Program organizes house selling by households; for example, "across nine states in the Midwest, more than 9,000 homeowners sold their properties after the Great Flood of 1993".[9] After these extensive floods, according to Bagstad et al. (2007), "entire towns, such as Valmeyer, Illinois, decided to move from the floodplain to higher ground, breaking an ongoing cycle of flood damage and government relief spending". In France, very exposed houses can be purchased by the government via the major natural risks prevention fund. House selling by households can be voluntary, in the framework of a sale to a private party or to the government, or imposed by the government. After Storm Xynthia in France on February 28, 2010, which caused the death of 47 people, 761 households were expropriated.

With natural risks, we show that actuarial insurance pricing implements a Pareto optimum. However, uniform insurance may be chosen for technical reasons, and restrictions on building may be necessary to substitute for imperfect internalization of natural risk. We show how red zones - that is areas where building is prohibited - can be used to implement constrained optima and how they should be set, depending on the parameters. We consider then an insurance reform switching form uniform insurance to insurance based on risk classes. We determine the acceptance of this reform by a majority vote and discuss transition management.

We then assess the firm strategy to reduce industrial risks. The firm can purchase or rent land, defining in this way a red zone in the vicinity of its plants. Several cases illustrate this fact, as mentioned by Sauvage (1997). In the United States, "the company Dow Chemical paid for example in 1991 for the moving out of a 300 inhabitants village, which was located very close to one of its chemical plants in Louisiana" [our translation]. In France, in the department of Moselle, "on the Carling petrochemical platform, one operator develops a discreet policy of buying zone 1 [the most exposed area] land and houses for sale, in order to guaranty land control in its installations neighborhoods. The dwellings are destroyed by the industrialist and

[7] The reinsurance scheme combines public and private participation. Government liability amounts to Tn¥, that is almost Bn €38.

[8] The French government offers its unlimited guarantee to one reinsurer, the Caisse Centrale de Réassurance for the coverage of natural disasters. All insured houses, buildings and cars have to be covered against natural disasters. The corresponding premium is included in housing or buildings or car insurance premia. It is supposed to be independent of exposure to natural risks.

[9] See `http://www.fema.gov/news/newsrelease.fema?id=43074`.

the set of land is kept for its own use or without any determined use, guarantying a "stopper" function with the close neighborhoods" [our translation]. The local jurisdictions or the central government can encourage the firm to buy or rent land exposed to industrial risks. As illustrated by Sauvage (1997), in Waziers, at the end of the 1980's, "the local authority possessed most of the unploughed land [...]. It obtained from the industrialist that it bought not only the acquired land for its installation purposes, but also the set of land included in the future protection area in a 240 meter radius around the hydrogen storage. Besides, the industrialist committed to buy three houses east of the site in order to destroy them" [our translation].

With exposure to industrial risks, financial incentives can make households internalize the impact of their location choices on the firm. However implementing these tools can be technically infeasible. The firm can still reserve a red zone by purchasing land. We show how red zones can implement constrained optima and how they should be set, depending on the parameters and market conditions. We consider three games that differ in the balance of power between the firm and the mayor, who defends the interests of households. We compare the efficiency of three games and the size of the different established red zones. We discuss our results by characterizing how a change in the fundamental risk parameters evolutions modifies the described equilibria.

The issue that we analyze - the impact of house insurance on household prevention measures from natural and industrial risks - inevitably crosses three fields of literature: law and economics, insurance economics and urban economics. Law economics analyzes how liability rules and/or insurance affect firms incentives for prevention (see Shavell (1982), Shavell (1986), Fluet (2002), Demougin and Fluet (2007); see Sanseverino-Godfrin (1996) for a comprehensive analysis of the legal framework of insurance and government intervention to manage catastrophic risks). We simplify this aspect by confounding the firm and its insurance. The taking of land by firms has been studied, but not in the framework of risk exposure. Blume et al. (1984) and Nosal (2001) study the efficiency of paying a compensation; Miceli and Segerson (2006) and Strange (1995) analyze the problem of holdout faced by a developer, when each individual landowner knows that each of his parcels is necessary for project completion and can postpone or even hold up the overall project. We focus here on location externalities exerted by households on the party that bears the risk - that is the community or the firm - and we assess insurance as an incentive for prevention to households.

The key role of private insurance in the mitigation of natural disasters has been specifically studied by Picard (2008), but in a different framework where prevention is not determined by location. Prevention effort is here measured by location choice riskiness. Like Frame (1998) and Frame (2001), we depart from the original theoretical urban model of Fujita and Thisse (2002) and Tatano et al. (2004)'s application to natural disasters by focusing on the tradeoff between land consumption and insurance expenditures (transport costs are ignored). Frame (1998) compares the equilibria with and without insurance. Frame (2001) shows that providing insurance to households residing in hazardous areas increases land use in exposed areas and so decreases real estate pressure in safe areas. This is why it benefits also to households living in safe areas. The author shows that it can be even beneficial to subsidize insurance when it is inefficient.

Our model adds four new perspectives. Firstly, in a complex territory with a continuum of risk exposures, we show an important case where insurance mechanisms with subsidy decrease the social welfare, even if the overexposure of the population reduces real estate pressure in safe areas. Secondly, we determine how this overexposure can be corrected - at least partially - by insurance policies or land use restrictions. We not only assess insurance and land use solutions; we also reveal and characterize their necessary coordination. Indeed, our model provides a new (to our knowledge) theoretical framework to assess simultaneously insurance and land use policies. Thirdly, we introduce a new dimension that is liability. We consider the existence of third party - different from the community - that can be affected by household location choices and analyze its impact on our results. Fourthly, we discuss concrete implementations of insurance policies. Insurance techniques based on risk classification may not be applicable because of the territory complexity, for political reasons or simply because the existence of a third party that is liable for the risk. In this case, land use restrictions are necessary. We show how red zones should be defined to improve the social welfare.

Section 2 introduces notations, important notions and sets up the model. Section 3 presents the case of natural disasters that imply public responsibility. Section 4 deals with industrial disasters for which the firm is liable. Section 5 draws comparative statics with respect to risk. Section 6 concludes.

2 The model

Our starting point is a linear urban model with perfect information. We intend to represent areas with significant risk gradients, so that the different places vary with respect to rents and insurance costs. We consider different pricings for insurance against natural and industrial disasters. Notations defined here are also listed in Appendix A.1.

Space and risk. The risk source (e.g. the river bed or the plant) is located at 0. The distance x between the source and a location determines risk exposure. The loss probability at x is denoted $p(x)$ with

$$\forall x,\ \ p(x) = \rho\, f(x), \tag{1}$$

where function $f(\cdot)$ is positive, integrable and decreasing along the space line; ρ parameterizes the intensity of risk. $[0;\bar{x}]$ is the set of inhabitable locations (see Figure 1). The safest place $\bar{x}$ can be seen as a crest. The inhabited zone may go down to the risk source. We denote x^* the leftmost (and riskiest) inhabited location in equilibrium.

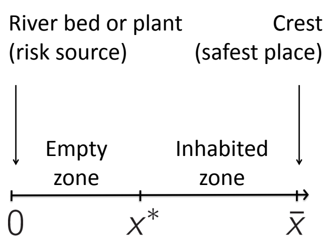


Figure 1: Space and risk

A household is a person living in a house at a location x. The house occupies a surface $s(x)$

and the density of households at location x is $n(x)$. We have

$$\int_0^{\bar{x}} n(x)dx = N, \tag{POP}$$

where N is the number of households. The available surface at every location is limited:

$$\forall x, \ n(x)\ s(x) \leq 1. \tag{SL}$$

A house located at x will be damaged only once with probability $p(x)$. The damage per house has two parts, one fixed $\lambda_F > 0$ and the other proportional to the housing surface $\lambda_S\ s$ with $\lambda_S > 0$. The damage corresponds to (re)building cost and does not depend on land value. There is no damage in empty places.

Risk is a combination of hazard and vulnerability. Here, hazard is measured by $p(\cdot)$, ρ being given. This means that all hazard reduction measures have already been taken. Vulnerability associated with building techniques (here λ_F and λ_S) is given. Endogenous vulnerability as due to location choices (household density and housing surface) is the main focus of the study.

Insurance. Insurance policies offered to cover a house of surface s located at x consist of a premium $\pi(x, s)$ and a complete reimbursement $\lambda_F + \lambda_S\ s$. Delays and associated costs are supposed to be fully compensated. Therefore risk affects location choices only through $\pi(x, s)$, which can be seen as a rent for risk. Without loss of generality, we consider premium functions with two components, one fixed and the other proportional to surface:

$$\pi(x, s) = \pi_F(x) + \pi_S(x)\ s. \tag{2}$$

Within a given community, risks are by nature highly correlated. The large number of communities on a much larger scale makes global risk tolerance high so that we can assume risk neutrality of the insurance sector. We also assume that insurers have sufficient reserves to absorb any loss so that there is no risk for households not to be fully compensated.

Insurance depends on the origin of risk. We assume that the government is the residual claimant for natural disasters compensation. This solidarity is supposed to take the form of a pure insurance system without administrative costs:

$$\int_0^{\bar{x}} \pi(x, s)n(x)dx = \int_0^{\bar{x}} p(x)\left(\lambda_F + \lambda_S\ s\right) n(x)dx. \tag{ND}$$

It could be implemented by a perfectly competitive (or perfectly controlled) private sector or by an efficient administration.

For industrial disasters, the firm is liable for the damages and we assume that it compensates completely its victims. There is no need for households to subscribe insurance:

$$\forall x, \ \forall s, \ \pi(x, s) = 0. \tag{ID}$$

We assume that households are not stakeholders of the firm.

In all scenarios, households are all insured: voluntarily when tariffs are actuarial, compulsorily when insurance is uniform (to avoid adverse selection), and by a third party for industrial risks.

Rent. The rent $r(x)$ is the price per unit of surface at x. Buying or perpetually renting a surface is equivalent; so we speak of rents henceforth. The opportunity cost of land is assumed to be null. We suppose that the collected rents are equally redistributed between them. Simply, we assume that the whole land is possessed by a fund of which households have equal shares; this structure makes sure that all reforms have a homogenous effect on households. The fund head can be seen as the mayor who acts as a proxy for households and defends their interests. All households receive

$$\bar{r} = \frac{1}{N} \int_{x^*}^{\bar{x}} r(x)s(x)n(x)dx, \tag{3}$$

which they take as given.

Households. There is a continuum of identical households of total mass N. We assume that they have no intrinsic preference for one location over another. Their utility $U(z,s)$ depends on their consumption z of the composite good (money) and on their housing surface s. The utility function is assumed to be separable

$$U(z,s) = u(z) + \alpha v(s), \ \alpha > 0. \tag{4}$$

u and v are both increasing and concave.

Households are price takers and the composite good is available everywhere at a price equal to 1. Households have an income ω and they maximize their utility under their budget constraint:

$$\max_{z,s,x} U(z,s) \tag{5}$$

$$\text{s.t. } \omega + \bar{r} \geq z + sr(x) + \pi(x,s). \tag{BC}$$

The equilibrium. As households are free to move, equilibrium is defined by the fact that no household has an incentive to change its location or its housing surface and that local and global constraints ((BC), (SL), (POP) and (ND) or (ID)) are satisfied.

Definition 1 (Equilibrium)**.** *An equilibrium is a set of functions $n(\cdot)$, $s(\cdot)$, $r(\cdot)$ and $\pi(\cdot,\cdot)$ in the considered family of insurance pricings such that the four following conditions are satisfied.*

(i) $\forall x \in [x^*; \bar{x}], \ \max_{z,s,x} U(z,s) \ \text{ s.t. } (BC)$,

(ii) (SL),

(iii) (POP),

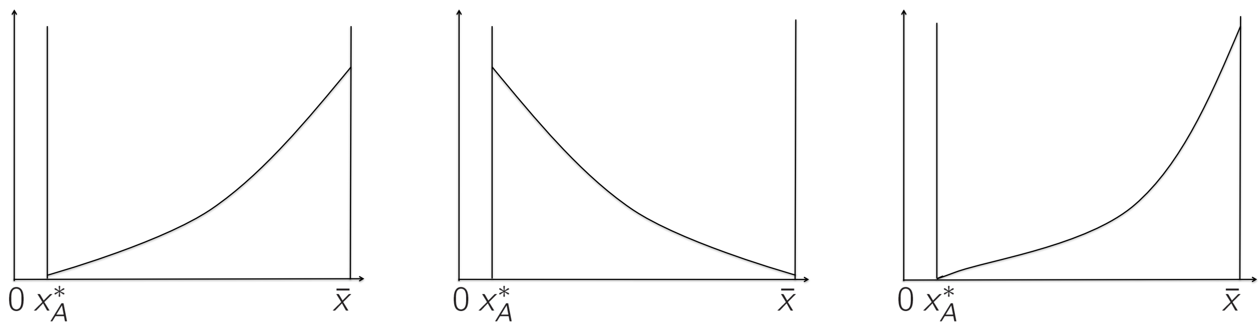
(iv) (ND) *for natural disasters or* (ID) *for industrial disasters.*

At the equilibrium, population density, housing surface and location choice, and so global and individual risk exposures are endogenous and depend on the insurance scheme. Appendix A.2 determines useful necessary conditions characterizing the equilibrium. The existence of a solution and its essential uniqueness will be proved in the scenarios we study now.

3 Natural disasters

Timing. The interaction between the insurance system and households works as follows. At stage 1, restrictions are imposed on insurance tariffs and on land use. At stage 2, households rationally anticipate their insurance cost and decide their location. At stage 3, insurers apply a pricing compliant with the given tariffs. The equilibrium of this game corresponds to Definition 1 when the liability rule is described by Equation ND.

Actuarial insurance. We consider first the actuarially fair pricing. In compensation for higher premia, rents are lower in risky areas and households can afford more space (see Figure 2). Riskiest locations may be deserted or not, depending on the parameters.



(a) Density (b) Surface per dwelling (c) Rent

Figure 2: Equilibrium under actuarial insurance

The endogenous space limit x_A^* decreases with respect to ω and can increase or decrease with respect to N. Comparative statics established by Pines and Sadka (1986) can be used here, since transportation cost is similar to the total expected cost of risk and their theorems are sufficiently general. The effect of ω is intuitive. An increase of N decreases the available space but also increases the total amount of money circulating and so rents; the net effect is ambiguous and depends on the income expansion path.

Proposition 1. *Actuarial insurance pricing implements a Pareto optimum.*

Proof. See Appendix A.3. □

Indeed, actuarial insurance makes households pay the price of risk. Therefore the cost caused by risky location choices and borne by the whole society is internalized by households. We explore now insurance inefficiency not by considering administrative costs (which Frame (2001) does). We consider instead a territory with a complex risk gradient where insurance inefficiency comes from the difficulty to insure an imperfectly known risk structure.

However, in many situations, defining the actuarial price of risk is technically infeasible. Even if information about risk exposure is public, measuring risk supposes to standardize and update these data and validate risk models, which could be quite costly for two reasons. Firstly, historical data on a few decades may not be sufficient to assess risk exposure of extreme and low probability events. Hazard maps realized by experts may not be reliable at a the dwelling scale; besides the availability or the precision of maps can be very heterogenous between areas.[10] Secondly, because of building practices or climate change, risk exposure may be evolving with respect to time.

We now consider uniform insurance. Clearly, uniform insurance generates negative externalities: if households are fully compensated and pay a uniform premium, all locations have the same value to them; by locating in exposed areas, households increase the financial exposure of the whole society. Therefore it is not clear theoretically that uniform insurance, even combined with land use restrictions, Pareto dominates the absence of insurance, whereas, as Frame (1998) shows, providing *actuarial* insurance increases the social welfare. However, absence of insurance is rarely the reference situation. In fact, as the government is the residual claimant for natural disasters, households expect financial assistance in the case of a disaster. This *charity hazard* leads us to consider provision of government assistance funded by taxes as the reference situation. This is a particular uniform insurance, taxes being almost equivalent to insurance premia.[11] We consider here a general uniform insurance combined with building restrictions. We show now that these restrictions partially substitute for imperfect internalization of risk and increase the social welfare.

Red zone under uniform pricing. All permitted locations have the same value for households and the building zone is fully and uniformly used. The government understands this and prohibits the most exposed areas, thus defining a block called *red zone* henceforth. The equilibrium on the land market implies that x^* can denote both the size of the red zone and the leftmost inhabited location (see Figure 3).

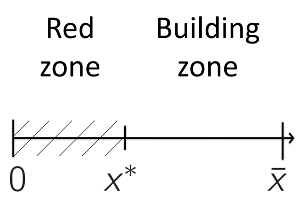


Figure 3: Red zone

We define the concept of constrained optimality in this second-best world.

[10] The maps relative to shrinking and swelling of clayey soils or to seismic activity in France are not reliable at a the dwelling scale (see `http://www.argiles.fr/` and `http://www.planseisme.fr/spip.php?article19`). Flood maps in the United States are reliable at the street scale but are not provided in all jurisdictions (see `http://www.esri.com/hazards/`). The precision of floods maps in France strongly varies between areas, depending on the implementation stage of a natural risks prevention plan (see `http://cartorisque.prim.net/`).

[11] Coate (1995) claims that the equivalence is less than perfect. Firstly, ex post assistance by government is less efficient because there is no reason to expect that people who provide assistance will choose the optimal level of assistance: assistance may rely on approximate loss assessments or discretionary decisions. Secondly, natural disasters assistance is provided by various actors (non-profit organizations, governments); the uninsured can free-ride. In this respect, the assistance providers themselves can consider that the level of assistance is not optimal. We leave these issues aside.

Definition 2 (Constrained optimality)**.** *A red zone is said to be* constrained-optimal *if it is Pareto optimal under the constraint that land use by households is uniform.*

As land use is uniform, the total expected cost of risk CR amounts to

$$CR(x) = \left(\frac{N\lambda_F}{\bar{x} - x} + \lambda_S\right) \int_x^{\bar{x}} p(t)dt. \tag{CR}$$

As the uniform premium shares equally the total expected cost of risk between households, the utility of all is equal to

$$U^*_{\text{Nat}} = u\left(\omega - \frac{CR(x^*)}{N}\right) + \alpha v\left(\frac{\bar{x} - x^*}{N}\right). \tag{6}$$

The utility increases with respect to the income ω, decreases with respect to the loss parameters λ_F, λ_S and ρ. The impact of an increase of population N on the utility is ambiguous. Indeed, increasing the number of households reduces the surface occupied by each of them but also dilutes the externality.

Proposition 2. *The constrained-optimal red zone x^*_{Nat} maximizes the utility. For an interior solution $x^*_{\text{Nat}} \in (0, \bar{x})$, x^*_{Nat} equals the marginal risk reduction (MRR) with the marginal rate of substitution (MRS) of households:*

$$-\frac{dCR(x^*_{\text{Nat}})}{dx} = \frac{\alpha v'(\frac{\bar{x} - x^*_{\text{Nat}}}{N})}{u'(\omega - \frac{CR(x^*_{\text{Nat}})}{N})}. \tag{7}$$

For corner solutions, the MRR is smaller (respectively larger) than the MRS if $x^*_{\text{Nat}} = 0$ (respectively $x^*_{\text{Nat}} = \bar{x}$).

In France, the 1995 law created natural risks prevention plans that define red zones where new building is prohibited.[12] In practice, these plans applied to one fifth of the municipalities in January 2010 (7 689 among 36 682 according to the Ministry of Ecology) and are included in the local land use plans. The red zones are defined by the central government after concertation with the mayors. Furthermore, according to the law, insurers can refuse to insure households who have built their house in prohibited areas after the implementation of the plan (Insurance Code, section L 125-6), but few houses are concerned. In the United States, there is no strict prohibition but the jurisdictions have to ensure that "structures built within the exposed areas adhere to strict floodplain management regulations" to benefit from the National Flood Insurance Program. Furthermore, flood insurance coverage is not offered to households living in high flood risk areas: "while the NFIP [National Flood Insurance Program] does not prohibit property owners from building along [fragile] coastal areas, any Federal financial assistance, including federally backed flood insurance, is prohibited".[13]

The mayors play a primary role in the implementation of these policies. We consider several jurisdictions identical to the one modeled above. Households are free to move between jurisdictions. If each jurisdiction is the residual claimant for the compensation of natural disasters that

[12] See law n^{0}95-101 of February, 2 1995 relative to the strengthening of environment protection. This law modified the law n^{0}87-565 of July, 22 1987 relative to the organisation of civil security, forest protection against fire and prevention from major risks.

[13] See http://www.floodplain.org/cmsAdmin/uploads/FEMA_Myths_about_NFIP.pdf.

occur on its ground and collects the corresponding premia, then the result of Tiebout (1956) applies: each mayor designs the optimal red zone in his jurisdiction to make it most attractive. In practice, local jurisdictions do not bear all the cost of natural risks. Mayors face a tradeoff between economic or demographic development and population protection and may not establish sufficiently large red zones. This phenomenon is named the *local government paradox* by Burby (2006), who provides several illustrations of it in the United States. Similarly, in France, certain local authorities seem to maneuver to delay natural risks prevention plans, as it became apparent at the occasion of Storm Xynthia. This is why, in addition to conditioning insurance coverage on land use regulation, the French and American Federal governments have set up public policies to link the prevention measures taken by mayors and the price of insurance purchased by households living in their jurisdictions (see Grislain-Letrémy and de Forges (2011)).

Refining zoning. Practical risk classification is rather gross and insurers use simple maps provided by government. These maps delineate location-based risk segments in which they apply the same tariff. For example, in the United States the Flood Insurance Rate Maps delimit the flood premium zones (see Hayes and Neal (2009)). In Japan, the Probabilistic Seismic Hazard Maps delineate the four earthquake premium zones (see Tsubokawa (2004)). Zoning choices shape the real estate market and insurance and have durable effect on global risk exposure. Let's compare now different zonings.

Definition 3. *A zoning is a partition of space in subintervals (zones) such that building is either prohibited or authorized on each zone; if a zone is authorized, the premium is uniform and balances the expected cost of risk over the zone.*

Uniform insurance with a red zone is a particular zoning. Actuarial insurance is another one (with an infinite number of zones).

Definition 4. *A zoning Z_2 is a refinement of the zoning Z_1 if every zone of Z_2 is a subset of a zone of Z_1 and is authorized if it belongs to an authorized zone of Z_1.*

In other words, Z_2 is a further fragmentation of Z_1 and building prohibition is (weakly) less restricted. In this sense, Z_2 is finer than Z_1.

Proposition 3. *Refining the zoning is Pareto improving.*

Proof. Consider two zonings Z_1 and Z_2, Z_2 being finer than Z_1. $\hat{p}(\cdot)$ is the unique function such that, for every zone of the partition Z_2, $\hat{p}(\cdot)$ is constant over this zone and equals the mean of $p(\cdot)$ over this zone. Given Proposition 1, Z_2 is the actuarial zoning for $\hat{p}(\cdot)$ and so to a Pareto optimum. Z_1 imposes additional constraints and thus can only lead to a Pareto inferior allocation. □

Fine zoning is costly in terms of risk assessment. The optimal fineness in the long run should be somewhere between uniform and actuarial insurance.

Transition from one zoning to another requires a long process of destructions and reconstructions. Short term costs and benefits are very likely to dominate the public debate and to determine the acceptance of the reform. In the short term, as people do not move or change their dwellings, they only see their insurance premium increase or decrease. Some zones are unchanged by the reform and their inhabitants are indifferent to the reform. In the zones that are refined, some inhabitants lose (respectively win) because their premium increases (respectively decreases). As density is uniform, comparing the number of winners and losers amounts to comparing the lengths of the corresponding zones. The government can choose these lengths to get the reform accepted.

The state may compensate the minority losers, as the American Federal state did. The Congress established the National Flood Insurance Program in 1968 and combined it with subsidies for exposed houses that were built before risk maps. This program has been more costly than foreseen: "The NFIP [...] has borrowing authority from the U.S. Department of the Treasury, and at the end of 2007, it had borrowed B$17, largely as a result of the 2004 and 2005 hurricane seasons" (see Kousky and Michel-Kerjan (2009)). An insurance reform has also been undertaken in France. The natural disasters insurance regime, created in 1982, is based on uniform premia and red zones. In the last decade, several disasters triggered debates about designing an incentive-based scheme.

4 Industrial disasters

We reinterpret the model to underline the similarities and the differences between natural and industrial risks. The potential losses due to households choices (location, surface) and the probabilities have the same structure and notation, with the difference that the firm is fully liable for all damages. If the firm were the property of the households, the issues and solutions would be similar to those presented in the previous section. To accentuate the contrast, we assume that households are not stakeholders of the firm.

We do not consider limited liability of the firm. It would certainly produce interesting effects: an excessive risk-taking by the firm and in turn less risky location choices by imperfectly protected households. We focus here on the "curse of unlimited liability": when households are fully compensated and do not pay any insurance premium they exert maximum external effect on the firm.

Location-dependent taxes. It is clearly inefficient to let households ignore the externalities they exert on the firm. Provided that the mayor receives a compensation from the firm, he could set up location-dependent taxes that implement a first-best allocation.

In France, technological risks prevention plans were created in 2003.[14] In the framework of these plans, prevention measures are required from exposed households, such as windows change, tightness works, thermal isolation of roofs.[15] Independently of their efficiency, these measures provide an additional cost to live in an exposed area, and are like location-dependent taxes. However, the coercive aspect of this policy is very limited, since there is no sanction if these measures are not implemented. Besides, though the firms consider contributing to their funding,[16] these mitigation measures are for now partially subsidized by the government via a

[14] See law n^{0}2003-699 of July, 30 2003 relative to prevention from technological and natural risks and to damages repair.

[15] These works are mandatory up to 10% of the house market value (Environment Code, section R 562-5). They typically cost €10,000 or €15,000.

[16] See debates at the national annual conference of technological risks in Douai on October, 21 2010.

tax credit, without any transfer paid by the firm for now.[17]

A finely defined limitation of population density could also work. For example, French law defines isolation polygons and areas in the very close vicinity of military pyrotechnic storages.[18] Designing the optimal location-dependent taxes or defining the optimal density requires accurate information and may be technically infeasible.

Red zones. In the absence of these sophisticated policies, available land is fully and uniformly used. The strategy of the firm consists in renting land. The firm does not need this red zone per se but only to avoid it being occupied by potential victims. Households value less the exposed areas than the firm does since they do not bear the cost of risk. Opening markets for land creates value. The fact that households are landowners make them likely to benefit from exchanges but the way competition shares this surplus between agents depends on the organization of the market.

We consider three games that differ in this respect. In all games, the firm settles in at stage 1 and the firm and the households rent land at stage 2.

Firm game. The firm holds the bargaining power: it chooses the rent per unit of land and the transfer to the community. This two-part tariff is designed so that the households rent the expected space. The firm captures the benefits of risk reduction and only compensates the households for the suffered space limitation, the no-red-zone situation being the reference.

Market game. Households and the firm are both price takers. The red zone is determined by the equilibrium on the land market.

Mayor game. The mayor holds the whole bargaining power. He sets a rent for households and another one for the firm so that the firm rents exactly the expected red zone. The mayor redistributes the benefits of risk reduction extracted from the firm, the no-red-zone situation being the reference.

In the firm game, the firm directly captures all the surplus. Indeed, utility guarantee deprives in effect households of the exchange gains. In the market game, the created value is partly recovered by households via rents. In the mayor game, the created value is entirely recuperated by households.

The three red zones are denoted x^*_{Firm}, x^*_{Market} and x^*_{Mayor}.

Proposition 4. *All the three games implement constrained-optimal red zones. For interior solutions, the red zones $x^* \in (0, \bar{x})$ are characterized by the equality between the marginal rate of transformation (MRT) of the firm and the marginal rate of substitution (MRS) of the households:*

$$-\frac{dCR(x^*)}{dx} = \frac{\alpha v'(\frac{\bar{x}-x^*}{N})}{u'(\omega + \frac{T}{N})}, \tag{8}$$

[17] In 2010, for a given dwelling, the ceiling of refunded expenditures is of €30,000 and the tax credit rate is of 30% (law n^{0}2010-1657 of December,29 2010 of finance for 2011).

[18] See law n^{0}1929-08-08 of August, 8 1929 relative to relative to urban constraints around stores and facilities used to store, handle or produce gunpowder, ammunition, fireworks or explosives.

where T is the transfer from the firm to households, that is

$$T = \begin{cases} Nc & \text{for Firm with } u(\omega + c) - u(\omega) = \alpha \left(v\left(\frac{\bar{x}}{N}\right) - v\left(\frac{\bar{x}-x^*}{N}\right)\right), & (9) \\ rx^* & \text{for Market with } r = \frac{\alpha v'(\frac{\bar{x}-x^*}{N})}{u'(\omega+\frac{rx^*}{N})}, & (10) \\ CR(0) - CR(x^*) & \text{for Mayor.} & (11) \end{cases}$$

For corner solutions, the MRT is smaller (respectively larger) than the MRS if $x^* = 0$ (respectively $x^* = \bar{x}$).

Proof. Appendix A.4 establishes the first order conditions characterizing the red zones in all games. Here follows the prof of constrained optimality of these red zones. An allocation is represented by (T, x^*).The constrained-optimal allocations are defined by the maximization of the utility of households given a minimum profit $\overline{\Pi}$:

$$\forall \overline{\Pi}, \quad \begin{cases} \max_{(T,x^*)} u\left(w + \frac{T}{N}\right) + \alpha v\left(\frac{\bar{x}-x^*}{N}\right), \\ \text{s.t. } \Pi_i - T + CR(0) - CR(x^*) \geq \overline{\Pi}. \end{cases} \tag{12}$$

The constraint is convexe in T and in x^*, since it is linear in T and in $CR(x^*)$ - with $CR(\cdot)$ monotonic and so quasi-concave. As additionally the objective is concave, the first order condition is a necessary and sufficient condition defining the constrained Pareto optima. For interior solutions $x^* \in (0, \bar{x})$,

$$-\frac{dCR(x^*)}{dx} = \frac{\alpha v'\left(\frac{\bar{x}-x^*}{N}\right)}{u'\left(w + \frac{T}{N}\right)}. \tag{13}$$

For corner solutions, the MRT is smaller (respectively larger) than the MRS if $x^* = 0$ (respectively $x^* = \bar{x}$). These conditions are satisfied by all the red zones. □

Note that we retrieve Proposition 2 (natural risk) for $T = -CR(x^*)$ in Equation 8. Formally, the households are "poorer" under natural risks than under industrial risks, since they bear the cost of natural disasters. Under exposure to industrial disasters, household wealth is the lowest in the firm game, the highest in the mayor game and intermediate in the market game. Household wealth determines their appetite for land and so the size of the red zone, as Proposition 5 says.

Proposition 5. *Assume that MRT (that is the marginal benefit of risk reduction) $-\frac{dCR(x)}{dx}$ decreases. The four red zones are ordered as follows:*

$$x^*_{\text{Nat}} \geq x^*_{\text{Firm}} \geq x^*_{\text{Market}} \geq x^*_{\text{Mayor}}. \tag{14}$$

Proof. Equation 13 can be written as

$$\frac{1}{\alpha v'\left(\frac{\bar{x}-x^*}{N}\right)}\left(-\frac{dCR(x^*)}{dx}\right) = \frac{1}{u'\left(w + \frac{T}{N}\right)}. \tag{15}$$

The MRS increases with respect to T and does not depend on x^*. The MRT does not depend on T and decreases with respect to x^*. Therefore, on the set of the constrained-optimal allocations, as the transfer T from the firm to households increases, the size of the red zone x^* decreases. In other words, the richer households are, the more expensive to "squeeze" them it is. This order is valid for interior and also corner solutions. Note that a sufficient condition to get $-\frac{dCR(x)}{dx}$ decreasing is $p(\cdot)$ convex. □

An example of land purchase decided by the firm is provided by Sauvage (1997). "The hazardous industrial plants [in Waziers, Puget-sur-Argens and Saint-Gaudens] [...] possess an important part and even the majority of the land in the concerned [protection] areas. They guaranty actually, more or less voluntarily, the land control in the protection areas that they generate" [our translation]. In Europe, following the Seveso II directive, member states of the European Union implement land use planning and red zones in the vicinity of hazardous industrial plants.[19]

Regulatory constraints could push hazardous plants towards less favored regions, at the risk of losing jobs. This threat has to be handled by developers and regulators. The tradeoff between protection of the inhabitants and economic development depends on the balance of power between governments and firms; and this balance evolves with time. After the accident of AZF in 2001, mayors were very reluctant to the maintaining of hazardous industrial activities. To preserve and promote industry in France, central and local governments decided in 2003 to subsidize industrialists directly or indirectly through the technological risks prevention plans (funding of land purchase, of the prevention measures required from the firm and from the households). The European Commission decided in 2007 that these government assistances to firms did not cause a significant competition distorsion.[20]

5 Sizing red zones

Risk is defined by the combination of hazard and vulnerability; hazard depends on the intensity of risk ρ and vulnerability depends on the demographic pressure N. Increasing ρ should be seen as a metaphor for increasing individual expected losses. We expose now the comparative statics with respect to ρ and N and the limit of the red zones as these parameters go to infinity.

As ρ (or N) increases, the cost of risk increases and therefore the value of land increases for the firm. Since the price of land increases, as renters households want less land but as landowners they are richer and want more. As policyholders, the increase of risk also impacts their tradeoff between consumption and land. In addition, the increase of N has a pure demographic effect on land demand. In the following, we compare the respective weights of these effects on the red zone by taking into account the preferences (elasticities of substitution), the risk (fixed and proportional effects) and the game played (see proof in Appendix A.5).

In all cases, the red zone equalizes the marginal rate of transformation (MRT) to the marginal rate of substitution (MRS) of the households (see Equations 7 and 8). We evaluate for all the scenarios the variations of the MRT and of the MRS with respect to ρ (then N), keeping the red zone constant to get an intuition. The comparisons are remarkably simple and they enable us to predict how the red zone should change. For example, if the MRT becomes bigger than the MRS, then the red zone should increase, as it can directly be interpreted as a bigger increase in the relative value of land for the firm than for households.

[19] The article 12 of the Seveso II directive (96/82/EC) requires from member states to consider within their land use planning policies the need of maintaining safety distances between potential accidental scenarios and the surrounding human and natural environments. Basta (2009) offers a comprehensive comparison of the national implementations of this land use planning policy between European countries.

[20] See note on April, 25 2007 from the European Commission to the French authorities relative to the government assistance N 508/2006.

Let's begin with ρ. The MRT increases in all scenarios. With natural risks, the MRS decreases (higher insurance costs reduce consumption) and in the firm game, the MRS doesn't change. Therefore the red zone increases in both cases. In the market game and the mayor game, the MRS increases. In these games, if households have a large (respectively small) elasticity of substitution,[21] then the MRS increases more (respectively less) than the MRT does and the red zone decreases (respectively increases).

Let's now examine the effect of N. Let's assume that $\lambda_F = 0$. Then an increase of N has no effect on the cost of risk and the MRT is constant in all scenarios. The evolution of the MRS depends on the evolutions of land and consumption. Land per household gets smaller in all games. Consumption increases with natural risks, as the households are more numerous to share the same risk. Therefore in that game, the MRS increases and the red zone decreases. In the other games, consumption decreases and the net impact on the MRS depends on the elasticities of substitution. If the elasticity of substitution of v' is larger (respectively smaller) than the elasticity of substitution of u', then the surface decrease has a larger (respectively smaller) impact than the consumption increase and the red zone decreases (respectively increases).

Let's assume that $\lambda_S = 0$. The MRT increases in all scenarios. As before, the evolution of the MRS depends on the evolutions of land and consumption. If the elasticity of v' is large (respectively small), then the surface effect dominates (is respectively dominated by) the other effects and the red zone decreases (respectively increases).

Empirical findings point out that the elasticities of substitution are such that $E_{v'} < E_{u'} < 1$ (see Taylor and Houthakker (2009)). In almost all cases, as the households have small elasticities of substitution, their relative value of land increases less than the one for the firm and the red zones increases with respect to risk (see Table 1).

Table 1: Comparative statics with respect to risk if $E_{v'} < E_{u'} < 1$

	ρ	N ($\lambda_F = 0$)	N ($\lambda_S = 0$)
x^*_{Nat}	$\nearrow$	$\searrow$	$\nearrow$
x^*_{Firm}	$\nearrow$	$\nearrow$	$\nearrow$
x^*_{Market}	$\nearrow$	$\nearrow$	$\nearrow$
x^*_{Mayor}	$\nearrow$	$\nearrow$	$\nearrow$

See Appendix A.5.

We now expose the limits of the red zones as these parameters go to infinity. Table 2 exposes the results in the case of a log-log utility function and a linear loss probability. As the intensity of risk increases, the red zone stretches short of a "city sanctuary" or until "squeezing" all households on the crest (see Figure 4). As population grows, the red zone narrows down to a "risk sanctuary".

Climate change is presumed to increase the intensity and the frequency of natural hazards, as recalled by the European Parliament (see Anderson (2006)) and the Intergovernmental Panel of Climate Change (see Schneider et al. (2007)). The Netherlands are in particular highly vulnerable to the sea level increase foreseen due to climate change, since about 70% of properties lie below

[21] The elasticity of u' is $-\frac{d \ln u'}{d \ln z}$; in another context, it is called the Relative Risk Aversion of u.

Table 2: Comparative statics with respect to risk in the case of a log-log utility function and a linear loss probability

	Variations w.r.t. ρ	Variations w.r.t. N	Risk sanctuary $\lim_{N\to+\infty} x^*$	City sanctuary $\lim_{\rho\to+\infty} x^*$
x^*_{Nat}	$\nearrow$	$\searrow$	$\max\left\{\bar{x} - \frac{2\alpha}{1+\alpha}\frac{\omega}{\rho\lambda_F}; 0\right\}$	$\bar{x}$
x^*_{Firm}	$\nearrow$	$\searrow$	$\max\left\{\bar{x} - \left(\frac{2\alpha\omega\bar{x}^{\alpha}}{\rho\lambda_F}\right)^{\frac{1}{1+\alpha}}; 0\right\}$	$\bar{x}$
x^*_{Market}	$\nearrow$	$\searrow$	$\max\left\{\frac{1}{1+\alpha}\bar{x} - \frac{2\alpha}{1+\alpha}\frac{\omega}{\rho\lambda_F}; 0\right\}$	$\frac{1}{1+\alpha}\bar{x}$
x^*_{Mayor}	$\nearrow$	$\searrow$ or $\nearrow$	$\max\left\{\frac{1}{1+\alpha}\bar{x} - \frac{2\alpha}{1+\alpha}\frac{\omega}{\rho\lambda_F}; 0\right\}$	$\lim_{\rho\to+\infty} x^*_{\text{Mayor}} < \bar{x}$ (†)

(†) Note that $\lim_{\rho\to+\infty} x^*_{\text{Mayor}} = \bar{x} - \frac{(1+\alpha)}{2(2+\alpha)}\frac{\lambda_F N}{\lambda_S}\left(\sqrt{1 + 4\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\frac{\lambda_S\bar{x}}{\lambda_F N}\left(\frac{\lambda_S\bar{x}}{\lambda_F N}+1\right)} - 1\right)$. If the size of the red zone increases with respect to N, the lower bound is not $\lim_{N\to+\infty} x^*_{\text{Mayor}} = \max\left\{\frac{1}{1+\alpha}\bar{x} - \frac{2\alpha}{1+\alpha}\frac{\omega}{\rho\lambda_F}; 0\right\}$ but $\lim_{N\to 0} x^*_{\text{Mayor}} = \bar{x}\left(1 - \sqrt{\frac{\alpha}{2+\alpha}}\right)$. Supplemental material details calculus and can be provided by authors on request.

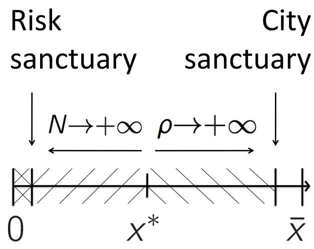


Figure 4: Risk and city sanctuaries

sea level or below river water level, according to Kok et al. (2002). This is why in 2008 the Delta Committee recommended several significant steps in water management and, among them, land purchase along the major river areas.

Climate change can indirectly increase industrial hazards as well, e.g. via natural disasters causing industrial accidents. The concentration of hazardous plants also increases the hazard. The so-called domino effect and congestions of all sorts limit the benefits of industrial concentration. Red zones are a public good to firms, and this is why an isolated industrial area is attractive to new plants and we observe numerous industrial parks in many countries. In the perspective of our analysis above, delineating a red zone amounts to fixing the degree of risk tolerated, meaning that new plants or investments should fit into the risk management plan.

Relevant examples of the impact of demographic pressure on the urbanism of risky areas are given by China for floods and by the neighborhoods of AZF plant in France for industrial disasters (see Introduction).

6 Conclusion

Our model shows that uniform insurance against natural disasters and the unlimited liability of the firm for industrial disasters provide bad incentives to households in terms on future location choices. Implementing red zones is necessary to substitute for this imperfect internalization of risk. Otherwise, population exposure will keep on increasing, leading this way to unaffordable future economic burden and major problems in terms of civil security. Our model provides indeed a general theoretical framework to characterize the necessary coordination insurance and land use policies and takes into account the key role of the mayors and the balance of power between them and the firms. Finally, our model deals with the question of future challenges. Climate change, new hazardous plants and demographic growth change the fundamental parameters of the model and we characterize how such evolutions modify the described equilibria. In this politically sensitive context, actual and future challenges of risk management require to take into account the interdependency and the substitution of insurance and land use policies.

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A Appendices

A.1 Notations

See Table 3.

A.2 Necessary conditions characterizing the equilibrium

All households are identical in terms of preferences and income and they are free to move. At the equilibrium, the utility is the same in all inhabited locations regardless of location. If not, the utility would be higher at an inhabited location x than at another inhabited location x'; then the households located in x' would move to x. U^* is the common maximum utility that can be reached considering the insurance tariff.

$$U^* = U\Big(\omega + \bar{r} - \pi_F(x) - s(x)[r(x) + \pi_S(x)], s(x)\Big). \tag{CU}$$

The land full price, denoted $p_s(x)$, is the sum of the rent for surface $r(x)$ and the rent for risk, which is the insurance premium $\pi_S(x)$.

$$p_s(x) = r(x) + \pi_S(x). \tag{16}$$

Table 3: Notations

	Fondamental parameters and functions
$[0;\bar{x}]$	Set of inhabitable locations on the space line
x^*	Riskiest inhabited location (that is size of the potential red zone)
	Households and their preferences
z	Quantity of composite good
s	Housing surface
$U(z,s)$	Utility function value for z and s consumption: $U(z,s) = u(z) + \alpha v(s)$
α	Monetary appreciation of surface
ω	Income
N	Number of households
	Risk and insurance
$p(x)$	Probability of loss at x
ρ	Intensity of risk: $p(x) = \rho f(x)$
λ_F	Marginal damage per house
λ_S	Marginal damage per unit of surface
$CR(x^*)$	Total expected cost of risk depending on x^*
	Endogenous functions
	Space
$n(x)$	Density at x
$s(x)$	Housing surface at x
$r(x)$	Rent at x
$\bar{r}$	Redistributed rent per household
	Insurance
$\pi(x,s)$	Insurance premium for a housing surface s at x
$\pi_F(x)$	Part of insurance premium corresponding to the fixed damage at x
$\pi_S(x)$	Part of insurance premium corresponding to the proportional damage at x
Π	Firm profit

Maximizing their utility under their budget constraint

$$\omega + \bar{r} = z + \pi_F(x) + s(x)[r(x) + \pi_S(x)] \tag{BC}$$

leads to

$$\frac{\partial U/\partial s}{\partial U/\partial z} = r(x) + \pi_S(x) = \frac{p_s(x)}{p_z}. \tag{CPO-EQ}$$

This is consistent, since $\omega + \bar{r} - \pi_F(x)$ appears as a net income, land full price as $p_s(x) = r(x) + \pi_S(x)$ in the budget constraint and $p_z = 1$.

The first order condition of the program with the indirect utility function V is

$$\frac{dV}{dx} = 0 = -\frac{\partial U}{\partial z}\left[\frac{ds}{dx}\Big(r(x) + \pi_S(x)\Big) + s(x)\left(\frac{dr}{dx} + \frac{\partial \pi_S}{\partial x}\right) + \frac{\partial \pi_F}{\partial x}\right] + \frac{\partial U}{\partial s}\frac{ds}{dx}.$$

Thanks to Equation CPO-EQ, we finally get

$$\frac{dV}{dx} = 0 = -\frac{\partial U}{\partial z}\left[s(x)\left(\frac{dr}{dx} + \frac{\partial \pi_S}{\partial x}\right) + \frac{\partial \pi_F}{\partial x}\right].$$

Since $\frac{\partial U}{\partial z} \neq 0$, we get

$$s(x)\left(\frac{dr}{dx} + \frac{\partial \pi_S}{\partial x}\right) + \frac{\partial \pi_F}{\partial x} = 0. \tag{TO}$$

At the equilibrium, a marginal reduction in rent is balanced by the marginal increase of insurance price. This expresses the tradeoff between land consumption and insurance expenditures.

Let us prove that the space resource constraint is binding in inhabited areas. In inhabited locations, demand for space at x is $n(x)s(x) > 0$ and supply is equal to 1. At the equilibrium, demand equals supply. We get

$$n(x)s(x) = 1. \tag{SL'}$$

In empty areas, supply is also 1 but demand is null. The price may not be determined. By convention, we fix it to 0.

Therefore an equilibrium satisfies the following necessary conditions:

$$\begin{cases} \forall x \in [0; \bar{x}] \text{ s.t. } n(x) > 0, \\ \begin{cases} U^* = U\Big(\omega + \bar{r} - \pi_F(x) - s(x)[r(x) + \pi_S(x)], s(x)\Big), & (CU) \\ \omega + \bar{r} = z + \pi_F(x) + s(x)[r(x) + \pi_S(x)], & (BC) \\ \frac{\partial U/\partial s}{\partial U/\partial z} = r(x) + \pi_S(x), & (CPO-EQ) \\ s(x)\left(\frac{dr}{dx} + \frac{\partial \pi_S}{\partial x}\right) + \frac{\partial \pi_F}{\partial x} = 0, & (TO) \\ n(x)s(x) = 1, & (SL') \end{cases} \\ (ND) \text{ } or \text{ } (ID), \\ (POP). \end{cases} \tag{17}$$

A.3 Proof of Proposition 1

Following Fujita and Thisse (2002), we prove that the equilibrium under actuarial insurance is efficient by showing that there is no feasible allocation achieving the same utility U^* and reducing the social cost. Actuarial insurance corresponds to scenario A with A used a subscript.

Let us assume that the allocation $(n(x), z(x), s(x); \hat{x} \leq x \leq \bar{x})$ is an optimum. The social cost for households to enjoy utility U^* is the sum of the total expected cost of risk and of the composite good cost, since there is no opportunity cost of land in our model. We denote $Z(s(x), U^*)$ the quantity of the composite good such that $U\Big(Z(s(x), U^*), s(x)\Big) = U^*$.

We want to solve the following program:

$$\begin{cases} \min\limits_{\hat{x}, \hat{n}(\cdot), \hat{s}(\cdot)} \int_{\hat{x}}^{\bar{x}} [p(x)\left(\lambda_F + \lambda_S \; \hat{s}(x)\right) + Z(\hat{s}(x), U^*)] n(x) dx \\ \text{s.t.} \begin{cases} (POP), \\ \forall x \in [\hat{x}; \bar{x}], \; \hat{n}(x)\hat{s}(x) = 1. \end{cases} \end{cases} \tag{18}$$

This is equivalent to this maximization program

$$\begin{cases} \max\limits_{\hat{x}, \hat{s}(\cdot)} \int_{\hat{x}}^{\bar{x}} \frac{\omega + \bar{r}_A - p(x)(\lambda_F + \lambda_S \; \hat{s}(x)) - Z(\hat{s}(x), U^*)}{\hat{s}(x)} dx \\ \text{s.t. } (POP). \end{cases} \tag{19}$$

We want to maximize the sum of amounts that can be given by households at a location x, as x describes the set of inhabited locations $[\hat{x}; \bar{x}]$. If this amount is higher than the redistributed amount in the equilibrium under actuarial insurance, $N\bar{r}_A$, then we could redistribute the increment and increase in this way the utility.

By neglecting firstly Equation POP, this problem can be solved by maximizing $[\omega+\bar{r}_A-p(x)\,(\lambda_F+\lambda_S\;\hat{s}(x))-Z(\hat{s}(x),U^*)]/\hat{s}(x)$ with respect to $\hat{s}(x)$ at each $x\geq\hat{x}$. The efficient housing surface $s(x)$ is identical to the equilibrium housing surface for each $x\geq\hat{x}$ if and only if

$$\frac{\omega+\bar{r}_A-p(x)\,(\lambda_F+\lambda_S\;s(x))-Z(s(x),U^*)}{s(x)}=\max_{z,s}\left\{\frac{\omega+\bar{r}_A-p(x)\,(\lambda_F+\lambda_S\;s)-z}{s}\text{ s.t. }U(z,s)=U^*\right\}. \tag{20}$$

We denote

$$\Psi(x,U^*)=\max_{z,s}\left\{\frac{\omega+\bar{r}_A-p(x)\,(\lambda_F+\lambda_S\;s)-z}{s}\text{ s.t. }U(z,s)=U^*\right\}. \tag{21}$$

Under actuarial insurance, $p(x)\,(\lambda_F+\lambda_S\;s)$ is equal to the exact premium paid by each household located at x. So, $\Psi(x,U^*)$ is the bid rent under actuarial insurance:

$$\Psi(x,U^*)=\max_{z,s}\left\{r_A(x,U^*)\text{ s.t. }U(z,s)=U^*\right\}. \tag{22}$$

Then, in order to maximize the object of the program, $\hat{x}$ must satisfy $\hat{x}=\max\{\tilde{x};0\}$ with $\Psi(\tilde{x},U^*)=0$, since $\Psi(\cdot,U^*)$ strictly increases. As this last equation has a unique solution $\tilde{x}_A$, we get $\hat{x}=\max\{0;\tilde{x}_A\}=x_A^*$.

Finally, we know that $(x_A^*,s_A(x))$ satisfies Equation POP. Consequently, the equilibrium under actuarial insurance is efficient. With no surprise, we find that the collected amount equals $N\bar{r}_A$:

$$\int_{x_A^*}^{\bar{x}}\frac{\omega+\bar{r}_A-p(x)\,(\lambda_F+\lambda_S\;s(x))-Z(s(x),U^*)}{s_A(x)}dx=\int_{x_A^*}^{\bar{x}}r_A(x)dx=N\bar{r}_A. \tag{23}$$

A.4 Proof of Proposition 4

We first derive the first order conditions characterizing the three red zones. Under industrial risks, the insurance premium is null. Therefore density and housing surface are constant. We denote Π_i the profit generated by the firm activity itself, that is the sale of the output reduced by the cost of production.

Firm game. In this game, we do not distinguish in the compensation c paid by the firm to each household what is relative to the rent and what is not. For the sake of simplicity, we assume that the firm pays no rent and does not contribute to the redistributed rent $\bar{r}$. The firm maximizes its profit

$$\max_{x^*_{\text{Firm}}}\Pi_{\text{Firm}}(x^*_{\text{Firm}})=\Pi_i-Nc(x^*_{\text{Firm}})-CR(x^*_{\text{Firm}}). \tag{24}$$

For an interior solution, the first order condition is

$$-\frac{dCR(x^*_{\text{Firm}})}{dx}=N\frac{dc}{dx}. \tag{25}$$

The amount c is such that household utility remains equal to the reference one. Given Equation BC, we get $\omega+c=z$ and the utility reached is so

$$U^*_{\text{Firm}}=u(\omega+c)+\alpha v\left(\frac{\bar{x}-x^*_{\text{Firm}}}{N}\right). \tag{26}$$

The reference utility is obtained by setting $x^*_{\text{Firm}}=\bar{x}$ and $c=0$ in the last equation.

$$\overline{U}=u(\omega)+\alpha v\left(\frac{\bar{x}}{N}\right). \tag{27}$$

c is so defined by

$$u(\omega + c) + \alpha v\left(\frac{\bar{x} - x^*_{\text{Firm}}}{N}\right) = u(\omega) + \alpha v\left(\frac{\bar{x}}{N}\right). \tag{28}$$

For interior solutions $x^*_{\text{Firm}} \in (0, \bar{x})$,

$$-\frac{dCR(x^*_{\text{Firm}})}{dx} = N\frac{dc}{dx} = \frac{\alpha v'(\frac{\bar{x}-x^*_{\text{Firm}}}{N})}{u'(\omega + c)}. \tag{29}$$

For corner solutions, the MRT is smaller (respectively larger) than the MRS if $x^*_{\text{Firm}} = 0$ (respectively $x^*_{\text{Firm}} = \bar{x}$).

Market game. In addition to its production activity, the firm rents a length x^*_{Market} of land at price r and bears the cost of risk. The program of the firm is

$$\max_{x^*_{\text{Market}}} \Pi(x^*_{\text{Market}}) = \Pi_i - rx^*_{\text{Market}} - CR(x^*_{\text{Market}}). \tag{30}$$

As the firm does not anticipate the impact of its decision on the market price, r does not depend on x^*. The first order condition of the program is

$$-\frac{dCR(x^*_{\text{Market}})}{dx} = r, \tag{31}$$

In this game, the firm pays the rent on the rented land and $\bar{r}$ is so collected on $[0; \bar{x}]$. Equation BC gives $z = \omega + \frac{rx^*_{\text{Market}}}{N}$. Equation CPO-EQ gives

$$\frac{\alpha v'(\frac{\bar{x}-x^*_{\text{Market}}}{N})}{u'(\omega + \frac{rx^*_{\text{Market}}}{N})} = r. \tag{32}$$

For interior solutions $x^*_{\text{Market}} \in (0, \bar{x})$,

$$-\frac{dCR(x^*_{\text{Market}})}{dx} = r = \frac{\alpha v'(\frac{\bar{x}-x^*_{\text{Market}}}{N})}{u'(\omega + \frac{rx^*_{\text{Market}}}{N})}. \tag{33}$$

For corner solutions, the MRT is smaller (respectively larger) than the MRS if $x^*_{\text{Market}} = 0$ (respectively $x^*_{\text{Market}} = \bar{x}$).

Mayor game. The mayor maximizes household utility, knowing that all benefits of renting the red zone are finally redistributed between households. The program of the mayor is so

$$\max_{x^*_{\text{Mayor}}} U^*(x^*_{\text{Mayor}}) = u\left(\omega + \frac{CR(0) - CR(x^*_{\text{Mayor}})}{N}\right) + \alpha v\left(\frac{\bar{x} - x^*_{\text{Mayor}}}{N}\right). \tag{34}$$

By similarity to the program that defines the optimal red zone under exposure to natural disasters (see Equation 6), we directly get the first order condition

$$-\frac{dCR(x^*_{\text{Mayor}})}{dx} = \frac{\alpha v'(\frac{\bar{x}-x^*_{\text{Mayor}}}{N})}{u'(\omega + \frac{CR(0)-CR(x^*_{\text{Mayor}})}{N})}, \tag{35}$$

and the optimal red zone

$$x^*_{\text{Mayor}} = x^*_{\text{Nat}}\left(\omega + \frac{CR(0)}{N}\right). \tag{36}$$

A.5 Proof of comparative statics

In each game, the red zone x^* is a function of (ρ, N) and we want here to compute the comparative statics of x^* with respect to ρ and to N. As stated by Equations 8 and 7, for interior solutions, the red zones $x^* \in (0, \bar{x})$ are characterized by the equality between the marginal rate transformation (MRT) of the firm and the marginal rate of substitution (MRS) of the households. We rewrite these equations the following way

$$MRT(x^*, \rho, N) = MRS(x^*, \rho, N) = \frac{\alpha v'(s(x^*, \rho, N))}{u'(z(x^*, \rho, N))}, \tag{37}$$

$$\text{where } MRT(x^*, \rho, N) = -\frac{dCR(x^*)}{dx}, \tag{38}$$

$$s(x^*, \rho, N) = \frac{\bar{x} - x^*}{N}, \tag{39}$$

$$z(x^*, \rho, N) = \omega + \frac{T}{N}. \tag{40}$$

We denote $MRT_1 = \frac{\partial MRT}{\partial x^*}$, $MRT_2 = \frac{\partial MRT}{\partial \rho}$ and $MRT_3 = \frac{\partial MRT}{\partial N}$ and similarly for the derivatives of s and z. T remains the transfer from the firm to households. We assume that $MRT_1 \leq 0$. Besides,

$$MRT_2 = \frac{MRT}{\rho} \leq 0, \tag{41}$$

$$MRT_3 = \frac{\lambda_F}{\bar{x} - x^*}\left(p(x^*) - \frac{P(x^*)}{\bar{x} - x^*}\right) \geq 0. \tag{42}$$

We denote $E_{u'}$ and $E_{v'}$ the elasticities of u' and v'. They are positive as u and v are concave.

$$E_{u'} = -\frac{d \ln u'}{d \ln z} = -\frac{zu''}{u'} > 0, \tag{43}$$

$$E_{v'} = -\frac{d \ln v'}{d \ln s} = -\frac{sv''}{v'} > 0. \tag{44}$$

Variations as ρ increases.

The comparative statics is given by

$$MRT_1 \frac{dx^*}{d\rho} + MRT_2 = \alpha \frac{v''}{u'} s_1 \frac{dx^*}{d\rho} - \alpha \frac{v'u''}{u'^2}\left(z_1 \frac{dx^*}{d\rho} + z_2\right), \tag{45}$$

$$\Leftrightarrow \frac{dx^*}{d\rho} \underbrace{\left(MRT_1 + \alpha \frac{v'}{u'}(E_{v'} \frac{s_1}{s} - E_{u'} \frac{z_1}{z})\right)}_{CSO} = \underbrace{-MRT_2}_{\leq 0} + \underbrace{\alpha \frac{v'}{u'} E_{u'}}_{>0} \underbrace{\frac{z_2}{z}}_{?}. \tag{46}$$

With natural disasters, in the firm game and in the mayor game, CSO is negative thanks to the second order condition of the maximization program. Thus, the sign of $\frac{dx^*}{d\rho}$ is the opposite of $-MRT_2 + \alpha \frac{v'}{u'} E_{u'} \frac{z_2}{z}$ and so depends on z_2 which varies among scenarios.

With natural risks, $z_2 = -\frac{CR(x^*)}{N\rho} < 0$ and so $\frac{dx^*_{\text{Nat}}}{d\rho} > 0$; in the firm game, $z_2 = 0$ and so $\frac{dx^*_{\text{Firm}}}{d\rho} > 0$.

In the mayor game, $z_2 = +\frac{CR(0) - CR(x^*)}{N\rho} > 0$ and the result is a priori undetermined, but assumptions relative to $E_{u'}$ enable to conclude. If $E_{u'} << 1$, then $\frac{dx^*_{\text{Mayor}}}{d\rho} > 0$; if $E_{u'} >> 1$, then $\frac{dx^*_{\text{Mayor}}}{d\rho} < 0$.

In the market game, $z_2 = 0$, but the second order condition of the maximization program only implies that $MRT_1 < 0$. Precisely, we get

$$\frac{dx^*_{\text{Market}}}{d\rho}\left(\underbrace{MRT_1}_{\leq 0} - \frac{dr}{dx}\right) = \underbrace{-MRT_2}_{\leq 0}, \tag{47}$$

$$\text{where } \frac{dr}{dx} = \underbrace{\frac{r}{N}\left(\frac{E_{v'}}{rs} + \frac{E_{u'}}{z}\right)}_{>0}\left(1 - E_{u'}\left(\omega + \frac{rx^*}{N}\right)\underbrace{\frac{\frac{rx^*}{N}}{\omega + \frac{rx^*}{N}}}_{\in]0;1[}\right)^{-1}. \tag{48}$$

If $E_{u'} \leq 1$, $\frac{dr}{dx} > 0$ and $\frac{dx^*_{\text{Market}}}{d\rho} > 0$. If $E_{u'} >> 1$ and $E_{v'} >> 1$, $\frac{dr}{dx} < 0$ and $\frac{dx^*_{\text{Market}}}{d\rho} < 0$. Table 4 summarizes the results.

Table 4: Comparative statics of x^* with respect to ρ

	$\forall u, \forall v$	Small $E_{u'}$	Large $E_{u'}$	Large $E_{u'}$, large $E_{v'}$
Nat	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
Firm	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
Market		$\nearrow$		$\searrow$
Mayor		$\nearrow$	$\searrow$	

Variations as N increases.

The comparative statics is given by

$$MRT_1\frac{dx^*}{dN} + MRT_3 = \alpha\frac{v''}{u'}\left(s_1\frac{dx^*}{dN} + s_3\right) - \alpha\frac{v'u''}{u'^2}\left(z_1\frac{dx^*}{dN} + z_3\right), \tag{49}$$

$$\Leftrightarrow \frac{dx^*}{dN}\underbrace{\left(MRT_1 + \alpha\frac{v'}{u'}(E_{v'}\frac{s_1}{s} - E_{u'}\frac{z_1}{z})\right)}_{CSO} = \underbrace{-MRT_3}_{\leq 0} + \alpha\frac{v'}{u'}(\underbrace{-E_{v'}\frac{s_3}{s}}_{>0}\underbrace{+E_{u'}\frac{z_3}{z}}_{?}). \tag{50}$$

With natural disasters, in the firm game and in the mayor game, CSO is negative. Thus, the sign of $\frac{dx^*}{dN}$ is the opposite of $-MRT_3 + \alpha\frac{v'}{u'}(-E_{v'}\frac{s_3}{s} + E_{u'}\frac{z_3}{z})$. In all games, $s_3 = -\frac{\bar{x}-x^*}{N^2} < 0$; z_3 varies among scenarios.

If λ_F is null. As MRT_3 is proportional to λ_F, $MRT_3 = 0$.

With natural risks, $z_3 = \frac{\lambda_S}{N^2}\rho\int_{x^*}^{\bar{x}} f(t)dt > 0$ and $\frac{dx^*_{\text{Nat}}}{dN} < 0$.

In the firm game, $z_3 = \frac{\partial c}{\partial N} < 0$, because $\frac{\partial c}{\partial N} = u'(w+c)^{-1}\alpha\left(-\frac{\bar{x}}{N^2}v'\left(\frac{\bar{x}}{N}\right) + \frac{\bar{x}-x^*}{N^2}v'\left(\frac{\bar{x}-x^*}{N}\right)\right)$ and $t \to tv'(t)$ is increasing, since v is concave. If $E_{u'} << E_{v'}$, then $\frac{dx^*_{\text{Firm}}}{dN} < 0$; if $E_{u'} >> E_{v'}$, then $\frac{dx^*_{\text{Firm}}}{dN} > 0$.

In the mayor game, $z_3 = -\frac{\lambda_S}{N^2}\rho\int_0^{x^*} f(t)dt < 0$. If $E_{u'} << E_{v'}$, then $\frac{dx^*_{\text{Mayor}}}{dN} < 0$; if $E_{u'} >> E_{v'}$, then $\frac{dx^*_{\text{Mayor}}}{dN} > 0$.

In the market game,

$$\frac{dx^*_{\text{Market}}}{dN}\left(\underbrace{MRT_1 - r}_{<0}\underbrace{-\frac{dr}{dx}x^*}_{?}\right) = \underbrace{-MRT_3}_{=0} + \underbrace{\frac{dr}{dN}x^*}_{?}, \tag{51}$$

where

$$\frac{dr}{dN} = \left(1 - E_{u'}(\omega + \frac{rx^*}{N})\frac{\frac{rx^*}{N}}{\omega + \frac{rx^*}{N}}\right)^{-1}\left(\underbrace{-\alpha E_{v'}v'\frac{s_3}{s}}_{>0}\underbrace{-E_{u'}(\omega + \frac{rx^*}{N})\frac{\frac{r^2x^*}{N^2}}{\omega + \frac{rx^*}{N}}}_{<0}\right). \tag{52}$$

Therefore if $E_{u'} << 1$ and $E_{u'} << E_{v'}$, $\frac{dr}{dx} > 0$ and $\frac{dr}{dN} > 0$ and so $\frac{dx^*_{\text{Market}}}{dN} < 0$. If $E_{u'} >> 1$ and $E_{u'} >> E_{v'}$, $\frac{dr}{dx} < 0$ and $\frac{dr}{dN} > 0$ and so $\frac{dx^*_{\text{Market}}}{dN} > 0$. Table 5 summarizes the results.

Table 5: Comparative statics of x^* with respect to N if $\lambda_F = 0$

	$\forall u, \forall v$	Small $E_{u'}$, large $E_{v'}$	Large $E_{u'}$, small $E_{v'}$
Nat	$\searrow$	$\searrow$	$\searrow$
Firm		$\searrow$	$\nearrow$
Market		$\searrow$	$\nearrow$
Mayor		$\searrow$	$\nearrow$

If λ_S is null. We now assume $\lambda_S = 0$. In that case $MRT_3 > 0$. With natural risks and in the mayor game, $z_3 = 0$. If $E_{v'} << 1$ then $\frac{dx^*_{\text{Nat}}}{dN} > 0$ and $\frac{dx^*_{\text{Mayor}}}{dN} > 0$; if $E_{v'} >> 1$ then $\frac{dx^*_{\text{Nat}}}{dN} < 0$ and $\frac{dx^*_{\text{Mayor}}}{dN} < 0$.

In the firm game, $z_3 < 0$. If $E_{v'} << 1$ then $\frac{dx^*_{\text{Firm}}}{dN} > 0$; if $E_{v'} >> 1$ then $\frac{dx^*_{\text{Firm}}}{dN} < 0$.

In the market game,

$$\frac{dx^*_{\text{Market}}}{dN}\left(\underbrace{MRT_1 - r}_{<0}\underbrace{-\frac{dr}{dx}x^*}_{?}\right) = \underbrace{-MRT_3}_{<0} + \underbrace{\frac{dr}{dN}x^*}_{?}, \tag{53}$$

If $E_{u'} << 1$, $\frac{dr}{dx} > 0$. Given that, if additionally $E_{v'} << 1$, $\frac{dx^*_{\text{Market}}}{dN} > 0$; if on the contrary $E_{v'} >> 1$, $\frac{dx^*_{\text{Market}}}{dN} < 0$. Table 6 summarizes the results.

Table 6: Comparative statics of x^* with respect to N if $\lambda_S = 0$

	Small $E_{v'}$	Large $E_{v'}$	Small $E_{u'}$, small $E_{v'}$	Small $E_{u'}$, large $E_{v'}$
Nat	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
Firm	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
Market			$\nearrow$	$\searrow$
Mayor	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$