Carbon tax and OPEC's rents under a ceiling constraint

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1 Introduction

Three key lessons stand out from the Copenhagen Summit in December 2009. First, the agreement recognizes the need for the temperature rise to stay below 2 degrees Celsius, which is usually associated with a greenhouse gas concentration ceiling of 450 ppm (IPCC [2007]). This shared objective is a significant progress obtained in Copenhagen. The two other lessons are less constructive: the inability of the largest emitting countries to reach even a basic effective agreement on an international policy architecture designed to aim towards this common objective; and OPEC's hostility to any international agreement which would finally result in a sensible contraction of world oil demand.

That OPEC doesn't contemplate introducing a climate policy, and reacts negatively when oil importing countries do, is not entirely surprising. As mentioned by Wirl and Dockner [1995], an example of such strategic OPEC's reaction is the oil price increase of \$4 per barrel in the 1992's first-half, matching the first step of the EEC proposal of a hybrid energy-carbon tax[1]. The crude oil price increase allows OPEC to make climate policy in some ways useless, since it may be sufficient to trigger the demand decrease whished for by consuming countries. By doing so, OPEC captures to some extent what we can call the carbon or climate rent.

However, oil consuming countries can also behave as a coalition and adopt a carbon tax which allows them to reap some part of the oil rent[2]. A global climate agreement of oil importing countries coordinating their carbon taxation could be interpreted as a consumers' cartel, would it happen to exist[3]. The implementation of a high carbon tax

[1] The initial level of this tax was set to \$3 per barrel, and the tax had to increase by \$1 a year until 2000. Finally, it was not adopted.

[2] Besides objectives of budget financing and energy savings, the high levels of fuel taxes in the European countries can be viewed also as an attempt to capture a part of the scarcity and monopoly rents. On average, between one-third and one-half of the total price of unleaded gasoline is excise tax (38,7% in Denmark, 43,6% in France, 44,5% in United-Kingdom and 46,7% in Germany (IEA [2009]).

[3] Some oil consuming countries have already implemented carbon taxes: Finland, Denmark, Norway and Sweden since 1991, Switzerland in 2008, but these policies remain isolated. Cap and trade schemes have the same consequences but even the largest existing system, the EU-ETS, cannot be interpreted as a global agreement on climate policy.

could prompt OPEC to lower its producer price in order to limit the decrease of oil demand.

Increases in oil price between 1998 and 2008, and especially the very strong increase of summer 2008, have led in many OECD countries the idea that what the government must control is the consumer price of oil and that, for doing so, taxes on energy should be "floating", or "additional", to use two terms coined in France: they should decrease when the producer price increases, and *vice versa*. This proposal is clearly a bad one, as OECD [2006] points out, for strategic reasons: "If oil importers start to reduce taxes in order to stabilise tax-inclusive fuel prices, oil exporters will know that they can at no risk increase their resource rents by restraining their production and thus increase crude prices further. Normally such actions would trigger reductions in demand that could reduce the incomes of the oil producers, but the demand reductions will be absent if tax-inclusive user prices are kept stable by tax reductions."

What is then the result of this two-sided strategic behaviour? Does the producers' cartel capture the climate rent or does the consumers' cartel capture the scarcity and monopoly rents? Is the time path of extraction more or less conservative than without strategic behaviour? What are the consequences of strategic climate policy in terms of welfare and in terms of distribution among the two coalitions? These are the main questions we address in this paper.

They have to some extent already been addressed from a theoretical point of view by Wirl [1994], [1995], Tahvonen [1995], [1996], and more recently by Rubio and Escriche [2001] and Liski and Tahvonen [2004]. These papers solve a similar differential game between the producers' and the consumers' cartels, in a framework where global warming creates damages to consumers' welfare. They study the Markov Perfect Nash Equilibrium (MPNE) of the game, and compare it to some benchmarks.

Liski and Tahvonen [2004] "consolidate, clarify, and extend" the results obtained in these papers. They show that the optimal design of the carbon tax in the presence of two-sided interactions generally deviates from a Pigouvian tax that internalizes only the environmental damage. Moreover, considering linear strategies, they solve explicitly the MPNE and prove that when pollution is not too severe, the carbon tax shifts more rents than necessary for internalizing the environmental externality and the buyers' payoff is higher than without carbon tax. They also study the time profile of the carbon tax and the producer price: If the damage is small, the carbon tax is decreasing with time and the producer price is increasing, if the damage is intermediate, both are increasing, if the damage is large, the carbon tax is increasing and the producer price decreasing.

The aim of the present paper is to understand what happens when keeping the idea of a differential non-cooperative game between a coalition of buyers and a cartel of sellers, but substituting the framework of a ceiling on the atmospheric carbon concentration to the damage function framework. This ceiling framework was introduced in the theoretical litterature by Chakravorty, Magné and Moreaux [2006a], [2006b]. One of its justifications is institutional: international negociations about climate change have often organized the discussion not in term of damages but around concentration ceilings –typically 450 or 550 ppm– corresponding to different rises of temperature. This approach is now ingrained in the Copenhagen agreement. Another justification lies in the large uncertainties characterizing the estimation of the damages due to global warming. Following the work of Allen *et al.* [2009] and Meinshausen *et al.* [2009], it has recently been argued that the correct way of thinking about global warming is in terms of a global "CO_2 emission budget", which overtaking would trigger temperature rises above 2°C, because of all the uncertainties in the carbon cycle, the climate response to an increase in the atmospheric carbon concentration, and the natural decay. This global carbon budget is exactly equivalent to a cap on carbon atmospheric concentration, a ceiling constraint, provided that natural carbon absorption by sinks is negligible.

Whereas in general only linear-quadratic structures of the game make it possible to obtain explicit solutions, which are linear, the ceiling constraint introduces here an intrinsic non-linearity in the model. We keep the linear-quadratic structure, but are nevertheless able to obtain non-linear solutions of the MPNE.

We obtain implicit feedback rules and explicit time paths of extraction, carbon tax and producer price. We show that the producer and consumer price are monotonically increasing in time, whereas the carbon tax may be first decreasing and then increasing, and is monotonically increasing near the ceiling. Moreover, strategic interactions lead to the discontinuity of the producer price and the carbon tax at the ceiling. The carbon tax shifts more rent than necessary for an environmental motive: consumers are able to reap some part of the scarcity and monopoly rents. We then compare the MPNE and three benchmarks. The comparison with the efficient equilibrium allows us to assess to what extent the carbon tax of the MPNE departs from the Pigouvian tax of the efficient equilibrium, designed to correct the environmental problem only. The comparison with the open loop equilibrium highlights the impact of the producers' and consumers' strategic behaviours. The comparison with the cartel without carbon tax equilibrium allows us to see whether consumers may gain in terms of welfare when they adopt a common environmental policy.

The paper is organized as follows. Section 2 presents the assumptions of the model, solves the Markov-perfect Nash equilibrium and studies its properties. Section 3 studies the three benchmarks against which the properties of the MPNE will be assessed: the efficient equilibrium, the open-loop equilibrium of the game, and the cartel without carbon tax equilibrium. The equilibria are compared in Section 4. Section 5 concludes.

2 Markov-perfect Nash Equilibrium

We study the strategic interactions between a cartel of fossil fuel producers and a coalition of consumers coordinating their carbon emission taxation to fight global warming, in a differential games framework of analysis. Whereas producers set the fossil fuel price, consumers set the carbon tax in order to meet an atmospheric carbon concentration constraint.

2.1 Consumers' area

In the consumers' area, the utility of the representative consumer is derived from the use of the fossil resource. The utility function is denoted $u(x_t)$, where x is the consumed resource flow. It is assumed to be quadratic and concave:

$$u(x) = ax - \frac{b}{2}x^2, \quad a > 0, \quad b > 0. \tag{1}$$

$u'(0) = a$ is the choke price, for which demand becomes nil.

The initial resource stock is X_0, the stock still in the ground at date t is X_t. The additional (to the pre-industrial level) atmospheric carbon concentration at date t is Z_t. We assume that natural carbon absorption is nil, so that the additional atmospheric carbon concentration at date t is strictly equal to the stock yet extracted and burnt at this date: $Z_t = X_0 - X_t$. This assumption has two justifications: firstly, natural absorption by sinks (oceans, forests) is uncertain and likely to decrease while carbon concentration increases; secondly and more technically, it allows us to consider only one stock in the problem and obtain tractable solutions. Climate policy takes the form of a ceiling constraint: $X_0 - X_t \leq \overline{Z}$ where $\overline{Z}$ is the ceiling on the additional carbon stock. This may be interpreted as a specific damage function, where the damage is nil before the ceiling and infinite at the ceiling. Alternatively, $\overline{Z}$ can be seen as the global carbon budget allowing humanity to contain the rise of temperature under 2°C.

The buyers' regulator sets a unit carbon tax for the area to control the pollution accumulation due to the resource use, taking into account the demand function of the representative consumer, given by:

$$u'(x) = p + \theta, \tag{2}$$

where p is the producer price and θ the carbon tax. The regulator maximizes on the whole horizon the discounted net surplus, difference between the consumers' utility and the amount paid to producers, subject to the law of carbon accumulation in the atmosphere and the ceiling constraint. Tax revenues are reimbursed as lump-sum transfers to consumers, so they do not appear into the net surplus. The damage from pollution doesn't appear either, but instead there is a ceiling on the atmospheric carbon stock.

A Markov solution is looked for, so the buyers' regulator, when choosing the carbon tax θ, takes into account the fact that the producer price depends on the resource stock, that is $p(X)$. The buyers' regulator understands the sellers' reaction to the state variable X, that sums up the whole past of the game and in particular its own carbon tax strategy.

The buyers' regulator problem is[4]:

$$V_c^m(X_0) = \max_{\theta_t} \int_0^\infty e^{-\rho t} \left[u\left(x\left(p\left(X_t\right) + \theta_t\right)\right) - p_t x\left(p\left(X_t\right) + \theta_t\right)\right] dt$$

$$\text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x\left(p\left(X_t\right) + \theta_t\right) \\ X_0 - X_t \leq \overline{Z} \\ X_0, \ \overline{Z} \text{ given.} \end{array} \right. \tag{3}$$

Let λ_c be the shadow price of the resource and ω the Lagrange multiplier associated to the ceiling constraint. First order optimality conditions and the complementarity slackness condition read[5]:

$$\theta_t = \lambda_{ct} \tag{4}$$

$$\dot{\lambda}_{ct} = \rho\lambda_{ct} + p'(X_t)x_t - \omega_t \tag{5}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ct}X_t = 0 \tag{6}$$

$$\omega_t \geq 0, \ X_t - X_0 + \overline{Z} \geq 0, \ \omega_t(X_t - X_0 + \overline{Z}) = 0. \tag{7}$$

[4]Superscript m for Markov. We will use in what follows the superscripts o for optimum/efficient equilibrium, ol for open loop equilibrium and c for cartel without tax equilibrium.

[5]To solve the MPNE, we use the maximum principle instead of the Hamilton–Jacobi–Bellman equation that is generally used in differential games, because it yields directly strategies depending on the state variable, here X. The reason is that the value function is potentially non-differentiable at the ceiling, and that consequently discontinuities of the feedback rule at the ceiling are possible. The use of the maximum principle allows us to overcome this difficulty. Notice that the non-differentiability of the value function at the ceiling is equivalent to the discontinuity of the stock shadow price at the ceiling. We study this potential discontinuity in Appendix B. We solve the same problem using the Hamilton–Jacobi–Bellman equation in Appendix E, to convince the reader that we actually obtain the same strategies.

Let T_m be the date at which the ceiling becomes binding. Given the structure of the problem, if the initial atmospheric carbon concentration is lower than the ceiling, which we assume, the ceiling won't be binding before T_m ($\omega_t = 0 \ \forall t < T_m$), and will remain binding forever after T_m ($\omega_t > 0 \ \forall t \geq T_m$). After the ceiling is reached, the resource consumer price, sum of the producer price and the carbon tax, remains equal to $u'(0)$, the price at which demand is choked, and $x = 0$.

Before the ceiling i.e. $\forall t < T_m$, equations (4) and (5) yield the following evolution of the carbon tax:

$$\dot{\theta}_t = \rho\theta_t + p'(X_t)x_t. \tag{8}$$

(8) integrates into:

$$\theta_t = \left(e^{-\rho T_m}\theta_{T_m-}\right)e^{\rho t} - \int_t^{T_m} e^{-\rho(s-t)}p'(X_s)x_s ds. \tag{9}$$

The carbon tax is the sum of two terms.

The first term on the right-hand side of equation (9) is the "pure Pigouvian tax" of Liski and Tahvonen [2004], in this ceiling framework. It is of the Hotelling form. Absent any damage i.e. without ceiling, T_m would tend to infinity and the first term would be nil by the transversality condition (6), since X_t would tend in the long run to a strictly positive value[6]. There would then exist no environmental motive for taxation, i.e., in the terminology of Liski and Tahvonen, no Pigouvian motive. Intuition suggests that the more stringent the ceiling the smaller T_m and the higher $e^{-\rho T_m}\theta_{T_m-}$, which represents the initial carbon tax in the case without strategic interactions, that is in the pure pigouvian case.

The second term on the right-hand side of (9) is the strategic component of the carbon tax. It represents the "pure import tariff" of Liski and Tahvonen [2004] if $p'(.) < 0$, and their "pure import subsidy" if $p'(.) > 0$. In the first case, the carbon tax is positive even absent any damage. Consumers tax oil more heavily than the environmental motive would require, and are thus able to reap to some extent the scarcity and monopoly rents of producers. In the second case, consumers subsidize oil consumption to correct the monopoly distorsion.

After the ceiling i.e. $\forall t \geq T_m$, equations (4) and (5) yield the following evolution of the carbon tax:

$$\dot{\theta}_t = \rho\theta_t - \omega_t. \tag{10}$$

Using the transversality condition (6), (10) integrates into:

$$\theta_t = \int_t^{\infty} e^{-\rho(t-s)}\omega_s ds. \tag{11}$$

2.2 Producers' area

Producers face a unit extraction cost depending on the resource stock still in the ground. The smaller this stock the higher the marginal extraction cost: the last drop of oil is very costly to extract. More precisely, the unit extraction cost is $c(X_t)$, with $c(X) > 0$, $c'(X) < 0$. We use the following linear specification:

$$c(X) = c_1 - c_2 X, \quad c_1 > 0, \quad c_2 > 0. \tag{12}$$

[6] This is because of assumption (14) on the extraction cost function.

We make the assumption that initial extraction is profitable:

$$c(X_0) < u'(0) \Leftrightarrow X_0 > \frac{c_1 - a}{c_2}. \tag{13}$$

With a constant marginal extraction cost, scarcity is purely physical. Here, scarcity can be economic, in the sense that the marginal cost of extraction of the last drop of oil can be higher than the choke price. Following Liski and Tahvonen [2004], we therefore assume that economic scarcity is binding:

$$c(0) > u'(0) \Leftrightarrow c_1 > a. \tag{14}$$

Then the last drop will never be extracted; producers will stop extraction before and leave some oil in the soil. Without any environmental constraint –no ceiling in our framework, they will leave in the ground a stock X_∞ defined by:

$$c(X_\infty) = u'(0) \Leftrightarrow X_\infty = \frac{c_1 - a}{c_2}. \tag{15}$$

We assume that producers do not intend to adopt any climate policy, but are perfectly aware that consumers do.

The sellers' regulator, when choosing the producer price, maximizes on the whole horizon its discounted profits, subject to the law of evolution of the resource stock:

$$\begin{aligned} V_p^m(X_0) = \max_{p_t} \int_0^\infty e^{-\rho t} \left(p_t - c(X_t)\right) x\left(p_t + \theta\left(X_t\right)\right) dt \\ \text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x\left(p_t + \theta\left(X_t\right)\right) \\ X_0 \text{ given.} \end{array} \right. \end{aligned} \tag{16}$$

Producers are assumed to understand the reaction function of buyers to the state variable. As this one sums up the whole past, it is assumed that the sellers understand that the buyers react to the sellers' strategy relative to the producer price.

The first order conditions give the producers' price strategy and the evolution of the shadow price of the resource λ_p, together with the transversality condition:

$$p_t = c(X_t) - \frac{x\left(p_t + \theta\left(X_t\right)\right)}{x'\left(p_t + \theta\left(X_t\right)\right)} + \lambda_{pt} \tag{17}$$

$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + (c'(X_t) + \theta'(X_t))x\left(p_t + \theta\left(X_t\right)\right) \tag{18}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt}X_t = 0. \tag{19}$$

$c(X) - \frac{x(.)}{x'(.)}$ is the static component of the monopoly price. The scarcity rent must be added to this static component.

Integrating equation (18) forward before the ceiling ($t < T_m$) yields:

$$\lambda_{pt} = \left(e^{-\rho T_m}\lambda_{pT_m-}\right)e^{\rho t} - \int_t^{T_m} e^{-\rho(s-t)}(c'(X_s) + \theta'(X_s))x_s ds. \tag{20}$$

The scarcity rent exhibits a Hotelling component, and a second component expressing the fact that a marginal amount of stock extracted at a given date s affects future profits by $c'(X_s) + \theta'(X_s)$, because of increased extraction costs and the variation of the consumers tax rate, which must be reflected in the current price. While the extraction cost effect clearly reduces future profits and increases the scarcity rent and today's price, the tax effect is at this stage ambiguous.

With the specification (1) adopted for the utility function, the demand function is:

$$x(p+\theta) = \frac{a-(p+\theta)}{b}, \tag{21}$$

and (17) reads before the ceiling :

$$p = \frac{1}{2}\left[c(X) + a - \theta(X) + \lambda^p\right]. \tag{22}$$

This formulation highlights the two effects of the carbon tax on the producer price: a negative static effect through the monopoly price (when θ increases, the cartel decreases p to support demand, and *vice versa*), and a dynamic effect through the scarcity rent. The sign of this last effect is at this stage indeterminate. If it is positive, it can be interpreted as a preemptive behaviour of the cartel: when the cartel extracts one unit of oil it knows that it will increase the atmospheric carbon concentration and that consequently the consumers' coalition will increase the carbon tax, and it reacts by increasing the producer price.

Consider now what happens after the ceiling is reached. Integrating (18) forward after T_m and using the transversality condition (19) shows that the scarcity rent is nil after the ceiling, which makes sense since the resource won't be extracted anymore. Then (17) yields $p_t = c(X_0 - \overline{Z}) = \overline{p}$, $t \geq T_m$. Then, since $p_t + \theta_t = u'(0)$ $t \geq T_m$, $\theta_t = u'(0) - \overline{p} = \overline{\theta}$, $t \geq T_m$.

Notice that nothing insures neither that the scarcity rent is continuous at the ceiling, i.e. that $\lambda_{pT_m-} = 0$, nor that the producer price and the carbon tax are.

2.3 The non-linear solution

In general, the Markov Perfect Nash Equilibria of differential games are studied under the assumptions of quadratic objective functions (utility, profit) and linear cost functions. Linear solutions are then the only ones that can be computed throughout analytically, even if other solutions may exist. But here, the ceiling constraint introduces an intrinsic non-linearity in the problem. Hence it is natural that the linear solution holds only if this constraint is not binding, that is if climate policy is sufficiently lenient, so that the increase in unit extraction cost triggers the stop of extraction for economic reasons before the ceiling is reached. We make the assumption that the ceiling is sufficiently tight, so that it requires to stop extraction before the date at which it would have stopped spontaneously, and to leave more fossil fuel in the ground. The study of the linear solution, valid under the opposite assumption, is relegated to Appendix A.

The equilibrium of the game is characterized by equations (4) and (22) giving the carbon tax and the producer price depending on the shadow price of the resource respectively for the consumers and the producers, and equations (5) and (18) giving the evolution of these shadow prices.

2.3.1 Strategies before the ceiling

Considering the shadow prices λ_c and λ_p as functions of the resource stock, notice that $\dot{\lambda}_c = \lambda_c'(X)\dot{X} = -\lambda_c'(X)x$, and that the same holds for $\dot{\lambda}_p$. Then the sum of equations (8) giving the evolution of the carbon tax before the ceiling and (18) giving the evolution of the scarcity rent yields, at the equilibrium:

$$-\left(\theta'(X)+\lambda_p'(X)\right)x=\rho\left(\theta(X)+\lambda_p(X)\right)+(p'(X)+c'(X)+\theta'(X))x,$$

i.e.

$$\rho\left(\theta(X)+\lambda_p(X)\right)+(p'(X)+c'(X)+2\theta'(X)+\lambda_p'(X))x=0. \tag{23}$$

Differentiating (22) w.r.t. X and replacing in (23) yields:

$$\rho\left(\theta(X)+\lambda_p(X)\right)+\frac{3}{2}(c'(X)+\theta'(X)+\lambda_p'(X))x=0. \tag{24}$$

Wirl and Dockner [1995] suggests (in a somewhat different framework) to introduce the total marginal value of the resource stock, composed here of the sum of the unit extraction cost, the carbon tax and the scarcity rent:

$$\Phi\left(X\right)=c(X)+\theta(X)+\lambda_p(X). \tag{25}$$

Equation (24) then reads:

$$\rho\left(\Phi(X)-c(X)\right)+\frac{3}{2}\Phi'(X)x=0. \tag{26}$$

However, from (22),

$$p(X)+\theta(X)=\frac{1}{2}\left[a+\Phi(X)\right] \tag{27}$$

which can also be written:

$$a-bx=\frac{1}{2}\left[a+\Phi(X)\right]. \tag{28}$$

By elimination of x, (26) and (28) yield a non-linear differential equation[7] in X:

$$\Phi'\left(X\right)=\frac{4\rho b}{3}\left(\frac{\Phi(X)-c\left(X\right)}{\Phi(X)-a}\right). \tag{29}$$

We show in Appendix A that a linear solution to this equation exists, but that it is valid only in the case where the ceiling constraint is not binding. We want to find a solution in the case of a sufficiently severe climate policy, such that:

$$X_0-\overline{Z}>\frac{c_1-a}{c_2}=X_\infty. \tag{30}$$

This solution is not linear. It is not possible to find $\Phi(X)$ analytically, so we resort to a phase diagram (Figure 1).

On the phase diagram, $\Phi'(X)=0\Leftrightarrow\Phi(X)=c(X)$; the admissible zone is under the line $\Phi(X)=a$, since $p+\theta=\frac{a+\Phi(X)}{2}<a\Rightarrow\Phi(X)<a$; $\Phi'(X)<0$ when $\Phi(X)>c(X)$, and $\Phi'(X)>0$ when $\Phi(X)<c(X)$; finally, $\Phi'(X)\rightarrow-\infty\Leftrightarrow\Phi(X)=a$. Starting from $\Phi(X_0)>c(X_0)$, the unique stable arm is travelled along from the right to the left, towards the equilibrium $\Phi(X_0-\overline{Z})=a$. X decreases, and as $\Phi'(X)<0$, $\Phi(X)$ increases.

[7] Of course, using the Hamilton–Jacobi–Bellman equations leads to the same differential equation (see Appendix E).

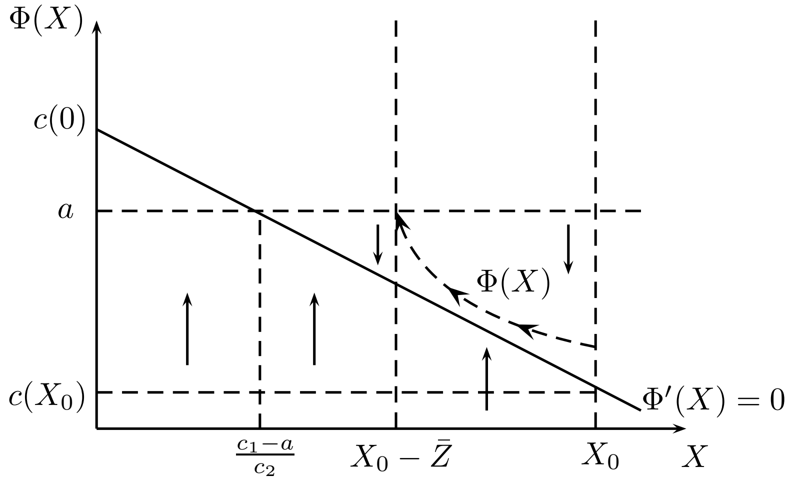

Figure 1: Phase diagram $(X, \Phi(X))$

We can now express the feedback rules for extraction, the scarcity rent, the carbon tax and the producer price. Equation (28) is equivalent to

$$x = \frac{1}{2b}\left[a - \Phi(X)\right]. \tag{31}$$

From (18),

$$\rho\lambda_p(X) + \Phi'(X)x = 0. \tag{32}$$

i.e., with (31) and (29),

$$\lambda_p(X) = \frac{1}{2\rho b}\left[\Phi(X) - a\right]\Phi'(X) = \frac{2}{3}\left[\Phi(X) - c\left(X\right)\right]. \tag{33}$$

From (24), using (31), (33) and (29),

$$\theta(X) = \frac{1}{4\rho b}\left[\Phi(X) - a\right]\Phi'(X) = \frac{1}{3}\left[\Phi(X) - c\left(X\right)\right]. \tag{34}$$

Finally, from (27), using (34),

$$p(X) = \frac{1}{6}\Phi(X) + \frac{1}{3}c(X) + \frac{a}{2}. \tag{35}$$

On the phase diagram, the carbon tax $\theta(X)$ can be read as one third of the vertical distance between the curve $\Phi(X)$ and the line $\Phi'(X) = 0$ (see equation (34)).

Notice that $\theta = \lambda_p/2$. This implies that the dynamic strategic effect of the carbon tax on the producer price is stronger than the static monopoly effect (see equation (22)), and that consequently the producers' cartel is able to preempt to some extent the carbon tax, as explained by Wirl [1995]. Now, let's consider the payoffs of the game. We show in Appendix E that the sellers' payoff, $V_p^m(X_0)$, is equal to two times the buyers' payoff, $V_c^m(X_0)$. The producers' cartel takes two-third of the payoff of the game.

2.3.2 At the ceiling

At the ceiling, x is nil, X is equal to $X_0 - \overline{Z}$, and we have shown that, $\forall t \geq T_m$:

$$p_t = \overline{p} = c\left(X_0 - \overline{Z}\right) \tag{36}$$
$$\theta_t = \overline{\theta} = a - c\left(X_0 - \overline{Z}\right) \tag{37}$$
$$\omega_t = \overline{\omega} = \rho\overline{\theta}. \tag{38}$$

The producer price at the ceiling is equal to the unit extraction cost. The level of the carbon tax is then such that demand is totally choked, since the consumer price is equal to the choke price a. The scarcity rent is nil.

2.3.3 Properties of the MPNE

Proposition 1 ***MPNE***

(i) Before the ceiling, the consumer and producer prices are monotonically increasing; the carbon tax may be first decreasing and then increasing, and is always increasing near the ceiling. The carbon tax includes an import tariff element.

(ii) The carbon tax and the producer price are not continuous at the ceiling, whereas the consumer price is. When reaching the ceiling, the carbon tax jumps upwards and the producer price jumps downwards to the marginal extraction cost.

Proof. Equation (27) shows that the consumer price is monotonically increasing in time, since $\Phi'(X) < 0$ and X is monotonically decreasing in time. Moreover, (27) and (36)–(37) show that the consumer price is continuous at the juncture, and equal to the choke price $u'(0) = a$.
Equation (34) yields:

$$\theta'(X) = \frac{1}{3}\left[\Phi'(X) - c'(X)\right].$$

As $\Phi'(X) < 0$ and $c'(X) < 0$, the sign of $\theta'(X)$ is indeterminate. However,

$$\lim_{X \to X_0 - \overline{Z}} \theta'(X) = -\infty,$$

and, by continuity, $\theta'(X) < 0$ and the carbon tax is increasing in time near the ceiling.
Equation (35) yields:

$$p'(X) = \frac{1}{6}\Phi'(X) + \frac{1}{3}c'(X) < 0.$$

Hence the producer price is increasing with time.
This proves part (i) of the proposition.
When $X \to X_0 - \overline{Z}$, (34) and (35) show that $\theta(X) \to \frac{1}{3}[a - c\left(X_0 - \overline{Z}\right)]$ and $p(X) \to \frac{1}{3}[2a + c\left(X_0 - \overline{Z}\right)]$. However at the ceiling, according to (37) and (36), $\theta_t = \overline{\theta} = a - c\left(X_0 - \overline{Z}\right)$ and $p_t = \overline{p} = c\left(X_0 - \overline{Z}\right)$. This proves part (ii) of the proposition. ■

2.3.4 Time paths before the ceiling

Though the feedback rules cannot be expressed as closed-form functions of X, it is possible to obtain the time paths of extraction, carbon tax and producer price implied by the MPNE (Figure 2).

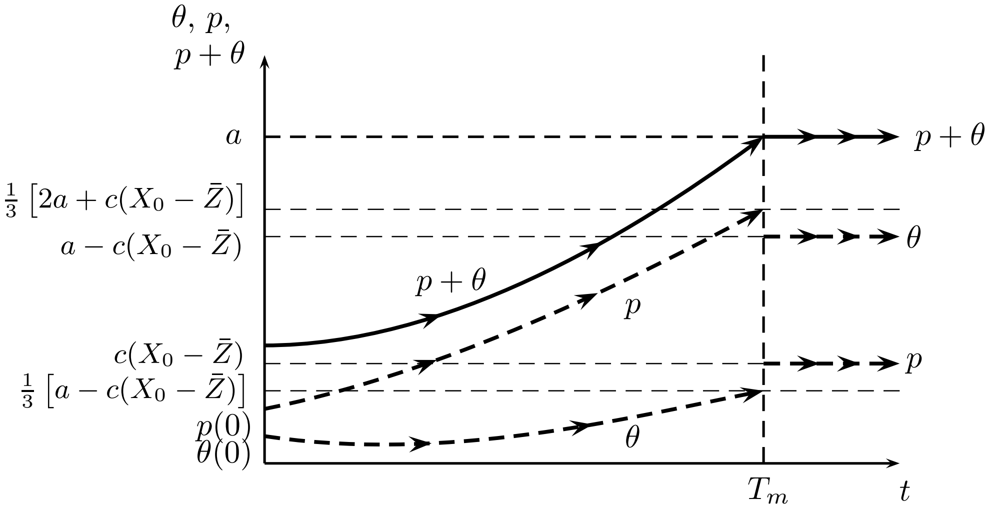

Figure 2: MPNE: time paths of the producer price, the carbon tax and the consumer price

Differentiating (31) w.r.t. time and using (29) yields:

$$\dot{x} = -\frac{2\rho}{3}\left(\frac{\Phi(X) - c(X)}{\Phi(X) - a}\right)\dot{X}. \tag{39}$$

Notice that $\dot{x} = -\ddot{X}$ and that, from (31), $\Phi(X) - a = -2bx = 2b\dot{X}$. Equation (39) then reads:

$$\ddot{X} = \frac{\rho}{3b}\left(\Phi(X) - c(X)\right) = \frac{\rho}{3b}\left(2b\dot{X} + a - c(X)\right).$$

Replacing $c(X)$ by its linear specification (12), we obtain a second order differential equation in X :

$$3b\ddot{X} - 2\rho b\dot{X} - \rho c_2 X + \rho(c_1 - a) = 0. \tag{40}$$

The solution of this differential equation is:

$$X_t = \alpha_1 e^{v_1 t} + \alpha_2 e^{v_2 t} + \frac{c_1 - a}{c_2}, \tag{41}$$

with:

$$v_1 = \frac{\rho}{3}\left(1 + \sqrt{1 + \frac{3c_2}{\rho b}}\right) > 0 \text{ and } v_2 = \frac{\rho}{3}\left(1 - \sqrt{1 + \frac{3c_2}{\rho b}}\right) < 0. \tag{42}$$

If the ceiling constraint never binds, that is if $X_0 - \overline{Z} \leq \frac{c_1 - a}{c_2}$, the v_1 root can be ruled out as X_t can neither become negative (if $\alpha_1 < 0$) nor increase (if $\alpha_1 > 0$). In such a case, the solution is the linear one we derive in Appendix A, where X converges asymptotically to X_∞. Under the opposite assumption, the two roots must be conserved and we obtain the non-linear solution.

Three equations are needed to determine implicitly the three unknown α_1, α_2, and T_m: the initial condition X_0, the condition at the ceiling $X_{T_m} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling[8], $x_{T_m} = 0$. These conditions read:

$$X_0 = \alpha_1 + \alpha_2 + \frac{c_1 - a}{c_2} \tag{43}$$

$$X_0 - \overline{Z} = \alpha_1 e^{v_1 T_m} + \alpha_2 e^{v_2 T_m} + \frac{c_1 - a}{c_2} \tag{44}$$

$$0 = v_1 \alpha_1 e^{v_1 T_m} + v_2 \alpha_2 e^{v_2 T_m}. \tag{45}$$

Equations (44) and (45) yield:

$$\alpha_1 = -\frac{v_2}{v_1 - v_2} \left(X_0 - \overline{Z} - X_\infty \right) e^{-v_1 T_m} > 0 \tag{46}$$

$$\alpha_2 = \frac{v_1}{v_1 - v_2} \left(X_0 - \overline{Z} - X_\infty \right) e^{-v_2 T_m} > 0. \tag{47}$$

Then (43) yields

$$\frac{v_1 e^{-v_2 T_m} - v_2 e^{-v_1 T_m}}{v_1 - v_2} = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}, \tag{48}$$

which gives T_m implicitly. It is an increasing function of $\overline{Z}$.

We can now obtain the time paths of the extraction, consumer price (from (2)), scarcity rent (from (32) and (39)), carbon tax and producer price:

$$x_t = -\dot{X}_t = -v_1 \alpha_1 e^{v_1 t} - v_2 \alpha_2 e^{v_2 t} \tag{49}$$

$$p_t + \theta_t = a - b x_t = a + b(v_1 \alpha_1 e^{v_1 t} + v_2 \alpha_2 e^{v_2 t}) \tag{50}$$

$$\lambda_{pt} = -\frac{2b}{\rho} \dot{x}_t = \frac{2b}{\rho} \left(v_1^2 \alpha_1 e^{v_1 t} + v_2^2 \alpha_2 e^{v_2 t} \right) \tag{51}$$

$$\theta_t = \frac{1}{2} \lambda_{pt} = \frac{b}{\rho} \left(v_1^2 \alpha_1 e^{v_1 t} + v_2^2 \alpha_2 e^{v_2 t} \right) \tag{52}$$

$$p_t = a + \frac{b}{\rho} \left[v_1 \alpha_1 \left(\rho - v_1 \right) e^{v_1 t} + v_2 \alpha_2 \left(\rho - v_2 \right) e^{v_2 t} \right]. \tag{53}$$

3 Benchmarks

Three possible benchmarks are studied, against which the properties of the MPNE will be assessed: the efficient equilibrium, the open loop equilibrium of the game, and the cartel without carbon tax equilibrium. The first one allows us to assess whether the monopoly power of the producers' cartel and the strategic behaviour of the two players lead to too much or too little extraction, compared to what is optimal. The second one allows us to assess separately the effect of the strategic behaviours on the producer price and the carbon tax. The last one is the proper benchmark, as argued by Liski and Tahvonen [2004], since it is the pre-tax situation. The comparison of the pre-tax and the MPNE outcomes for consumers shows whether consumers benefit in terms of welfare from the implementation of the carbon tax.

[8]The proof is relegated to Appendix B.

3.1 Optimum and efficient equilibrium

3.1.1 Optimum

The world central planner's problem reads[9]:

$$V^*(X_0) = \max_{x_t} \int_0^\infty e^{-\rho t}\left[u(x_t) - c(X_t)x_t\right] dt$$

$$\text{s.t.} \left| \begin{array}{l} \dot{X}_t = -x_t \\ X_0 - X_t \leq \bar{Z} \\ X_0 \text{ given.} \end{array} \right. \tag{54}$$

Denoting by ν_t the shadow price of the resource stock and ω_t the Lagrange multiplier associated to the ceiling constraint, first order optimality conditions and the complementarity slackness condition are:

$$u'(x_t) = c(X_t) + \nu_t \tag{55}$$

$$\dot{\nu}_t = \rho\nu_t + c'(X_t)x_t - \omega_t \tag{56}$$

$$\lim_{t\to\infty} e^{-\rho t}\nu_t X_t = 0 \tag{57}$$

$$\omega_t \geq 0,\ X_t - X_0 + \overline{Z} \geq 0,\ \omega_t(X_t - X_0 + \overline{Z}) = 0. \tag{58}$$

Before the ceiling, the marginal utility on the optimal path is the sum of the marginal extraction cost and of a rent ν_t, encompassing the scarcity rent and the carbon shadow value, in this simplified framework where the same stock characterizes the fossil resource stock in the ground and the atmospheric carbon concentration.

It is possible to show that extraction x is continuous at the juncture, in the same line as for the MPNE (see Appendix B). Then, from (55), the costate ν is also continuous at the juncture.

Differentiating (55) w.r.t. time and using (56) yields:

$$u''(x)\dot{x} = c'(X)\dot{X} + (\rho\nu + c'(X)x - \omega) = \rho\nu - \omega.$$

Before the ceiling, $\omega = 0$ and we get, using (55) again, the optimal extraction path:

$$u''(x)\dot{x} = \rho\left(u'(x) - c(X)\right). \tag{59}$$

With the specifications adopted for the utility and extraction cost functions, (59) reads:

$$b\ddot{X} - \rho b\dot{X} - \rho c_2 X + \rho(c_1 - a) = 0, \tag{60}$$

a linear differential equation of the second order, as in the MPNE, but with different coefficients.

The solution is:

$$X_t = \beta_1 e^{u_1 t} + \beta_2 e^{u_2 t} + \frac{c_1 - a}{c_2}, \tag{61}$$

with

$$u_1 = \frac{\rho}{2}\left(1 + \sqrt{1 + \frac{4c_2}{\rho b}}\right) > 0 \text{ and } u_2 = \frac{\rho}{2}\left(1 - \sqrt{1 + \frac{4c_2}{\rho b}}\right) < 0. \tag{62}$$

[9] Notice that it is exactly equivalent to the problem without ceiling but with an initial resource stock $X_0 - \overline{Z}$.

As in the case of the MPNE, three boundary conditions allow us to obtain the unknown β_1, β_2 and T_o : the initial condition X_0, the condition at the ceiling $X_{T_o} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling, $x_{T_o} = 0$. With the same argument as in the MPNE, it is possible to show that $\beta_1, \beta_2 > 0$.

At date T_o, the ceiling is reached, and we have, $\forall t \geq T_o$:

$$\begin{cases} X_t = X_0 - \overline{Z} \\ x_t = 0 \\ \nu_t = u'(0) - c(X_0 - \overline{Z}) \\ \omega_t = \rho\nu_t. \end{cases} \tag{63}$$

3.1.2 Efficient competitive equilibrium

The decentralization of the optimum leads to an efficient competitive equilibrium, provided that the right environmental tax –redistributed by lump-sum transfers to consumers– is implemented in the consumers' area.

The demand function of the representative consumer is given by:

$$u'(x) = p + \theta.$$

On the producer side:

$$V_p^o(X_0) = \max_{x_t} \int_0^\infty e^{-\rho t}\left[p_t - c(X_t)\right] x_t dt \tag{64}$$
$$\text{s.t.} \quad \dot{X}_t = -x_t, \quad X_0 \text{ given.}$$

Denoting by λ_p the scarcity rent, the first order conditions read:

$$p_t = c(X_t) + \lambda_{pt} \tag{65}$$
$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + c'(X_t)x_t \tag{66}$$
$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt}X_t = 0. \tag{67}$$

The equilibrium is then defined by:

$$u'(x_t) = c(X_t) + \lambda_{pt} + \theta_t. \tag{68}$$

Differentiating equation (68) w.r.t. time and using (66) yields:

$$b\ddot{X} - \rho b\dot{X} - \rho c_2 X + \rho(c_1 - a) = \dot{\theta} - \rho\theta. \tag{69}$$

Comparing (60) and (69) shows that for the equilibrium to be an optimum the carbon tax must grow at the discount rate before the ceiling.

Extraction x and the marginal extraction cost $c'(X)$ being continuous at the ceiling, (66) implies that $\dot{\lambda}_p - \rho\lambda_p$ is continuous, and yields, integrating forward and using the transversality condition (67)[10],

$$\lambda_{pt} = -\int_t^\infty e^{-\rho(s-t)}c'(X_s)x_s ds, \quad \forall t \geq 0.$$

[10]Notice that the same reasoning cannot be made in the case of the MPNE, since we are not sure that $\theta'(X)$ is continuous.

Then, as $x_s = 0$, $s \geq T_o$, $\lambda_{pT_o} = 0$. The scarcity rent is continuous at the juncture, and equal to 0. Then the carbon tax is also continuous, and it is equal to $\overline{\theta}$ given by (37). This allows us to obtain the initial level of the carbon tax: $\theta_0 = \overline{\theta} e^{-\rho T_o}$. This optimal tax is the efficient (first best) Pigouvian tax.

Equations (65) and (66) show that the producer price is monotonically increasing before the ceiling. Moreover, by (65), the producer price is continuous at the juncture and equal to $\overline{p}$ given by (36).

These results are summarized in the following proposition.

Proposition 2 ***Efficient equilibrium***

(i) Before the ceiling, the carbon tax is a pure Pigouvian tax, growing at the discount rate, and the producer price is monotonically increasing.

(ii) The carbon tax and the producer price are continuous at the ceiling.

3.2 Open loop equilibrium

In this case, the players base their strategies on time alone.

The consumers' regulator problem is similar to problem (3), but for the fact that he doesn't adopt any strategic behaviour. He takes the producer price as given. Equally, the producers' regulator problem is similar to (16), but for the fact that he takes the carbon tax as given. Strategic interactions are absent, except of course at the initial date.

The first order optimality conditions are, on the consumers' side:

$$\theta_t = \lambda_{ct} \tag{70}$$

$$\dot{\lambda}_{ct} = \rho\lambda_{ct} - \omega_t \tag{71}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ct} X_t = 0, \tag{72}$$

which shows that before the ceiling the carbon tax is a Pigouvian tax, growing at the discount rate.

On the producer side, the first order optimality conditions read:

$$p_t = \frac{1}{2}\left[c(X_t) + a + \lambda_{pt} - \theta_t\right] \tag{73}$$

$$\dot{\lambda}_{pt} = \rho\lambda_{pt} + c'(X_t)x_t \tag{74}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{pt} X_t = 0. \tag{75}$$

The equilibrium before the ceiling is characterized by:

$$u'(x) = p + \theta \Longleftrightarrow a - bx = \frac{1}{2}\left[c(X) + a + \theta + \lambda_p\right]. \tag{76}$$

Differentiating (76) w.r.t. time and using (71) and (74) yields:

$$2b\ddot{X} - 2\rho b\dot{X} - \rho c_2 X + \rho\left(c_1 - a\right) = 0, \tag{77}$$

again a linear differential equation of the second order, but with different coefficients.

The solution has the same form as in the MPNE and the efficient equilibrium:

$$X_t = \gamma_1 e^{w_1 t} + \gamma_2 e^{w_2 t} + \frac{c_1 - a}{c_2}, \tag{78}$$

with
$$w_1 = \frac{\rho}{2}\left(1+\sqrt{1+\frac{2c_2}{\rho b}}\right) > 0 \text{ and } w_2 = \frac{\rho}{2}\left(1-\sqrt{1+\frac{2c_2}{\rho b}}\right) < 0. \tag{79}$$

As in the cases of the MPNE and the efficient equilibrium, three boundary conditions allow us to obtain the unknown γ_1, γ_2 and T_{ol}, the date at which the ceiling is reached: the initial condition X_0, the condition at the ceiling $X_{T_{ol}} = X_0 - \overline{Z}$, and the fact that extraction becomes nil at the ceiling, $x_{T_{ol}} = 0$. It is possible to show that $\gamma_1, \gamma_2 > 0$.

Proposition 3 ***Open loop equilibrium***

(i) Before the ceiling, the carbon tax is a Pigouvian tax growing at the discount rate, whereas the producer price may be first increasing and then decreasing, and is decreasing when approaching the ceiling.

(ii) The carbon tax and the producer price are continuous at the ceiling.

Proof. The consumer price is increasing as:

$$p + \theta = a + b\dot{X} \Rightarrow \frac{d(p+\theta)}{dt} = b\ddot{X} > 0.$$

As the carbon tax before the ceiling is exponential, the sign of $\dot{p}$ is indeterminate. However at the juncture at date T_{ol}, using (73) and (74),

$$\dot{p}_{T_{ol}-} = \frac{\rho}{2}\left(c\left(X_0 - \overline{Z}\right) - a\right) < 0.$$

This proves part (i) of the proposition.
To prove part (ii), remark that λ_p is continuous at the juncture by the same argument as in the efficient equilibrium case. Its continuity implies that of the carbon tax (from (76)) and the producer price. ■

The discontinuity of the carbon tax and the producer price at the juncture at the MPNE is then the consequence of the strategic behaviour of the players, since this discontinuity does'nt exist at the open loop equilibrium.

3.3 Cartel equilibrium without carbon tax

The buyers' problem is very simple as it does not take into account any environmental damage:

$$u'(x) = p.$$

The sellers' regulator problem is the same as in the open loop game, but for the fact that the carbon tax is nil.

The equilibrium is characterized by equation (77), as in the open loop game; but now no constraint prevents the economy from extracting all what is economically profitable: producers leave $X_\infty = \frac{c_1 - a}{c_2}$ in the ground. In the solution of the linear differential equation (77), the positive exponential has to be ruled out as there is no ceiling. Therefore:

$$X_t = \left(X_0 - X_\infty\right) e^{w_2 t} + X_\infty \tag{80}$$
$$x_t = -w_2\left(X_t - X_\infty\right) \tag{81}$$

with w_2 given in (79). This implies the following proposition:

Proposition 4 ***Cartel without tax equilibrium***

The resource price is an increasing and concave function of time. It converges towards the choke price a when $t \to \infty$.

4 Comparison of equilibria

The comparison of the MPNE and the efficient equilibrium allows us to assess to what extent the carbon tax of the MPNE departs from the Pigouvian tax of the efficient equilibrium, designed to correct the environmental problem, and to see if the game is more or less conservative than what is optimal. The comparison of the MPNE and the open loop equilibrium highlights the impact of the producers' and consumers' strategic behaviours. Finally, the comparison of the MPNE and the cartel without carbon tax equilibrium allows us to see whether consumers may gain in terms of welfare when they adopt a common environmental policy.

Proposition 5 *(i) The MPNE is more conservative than the open loop equilibrium, in the sense that initial extraction is lower. Both are excessively conservative, compared to what is efficient. The ceiling is reached later in the MPNE than in the open loop equilibrium, and later in the open loop equilibrium than in the efficient equilibrium.*

(ii) The ranking of the payoffs for the producers' and the consumers' area respectively are: $V_p^{ol}(X_0) > V_p^m(X_0) > V_p^o(X_0) = 0$ *and* $V_c^o(X_0) > V_c^{ol}(X_0) > V_c^m(X_0)$.

(iii) The open loop equilibrium is more conservative than the cartel without tax equilibrium. The producers' area is always better off in the cartel without tax case, whereas when the ceiling is not too tight, the consumers' area gets a higher payoff in the MPNE and in the open loop equilibrium than in the cartel without tax equilibrium.

Proof. (i) We prove in Appendix C that $x_0^o > x_0^{ol} > x_0^m$ and that $T_m > T_{ol} > T_o$.
We can make use of the Bellman equations to show that moreover x_0^o is very significantly higher than x_0^{ol}. The sum of the value functions of the producers and the consumers at the open loop equilibrium are:

$$V^*\left(X_0\right) = \frac{b}{2\rho}\left(x_0^o\right)^2$$
$$V_c^{ol}\left(X_0\right) + V_p^{ol}\left(X_0\right) = \frac{3b}{2\rho}\left(x_0^{ol}\right)^2.$$

Both only depend on initial extraction on the respective paths. By definition of the optimum, the payoff of the game is lower than that of the efficient equilibrium. Consequently:

$$x_0^o > \sqrt{3}x_0^{ol}.$$

(ii) The ranking of payoffs is deduced from the ranking of initial extractions (except for the producers' payoff at the efficient equilibrium), since $V_p^m\left(X_0\right) = \frac{\left(x_0^m\right)^2}{\rho}$, $V_p^{ol}\left(X_0\right) = \frac{\left(x_0^{ol}\right)^2}{\rho}$, $V_c^o\left(X_0\right) = \frac{\left(x_0^o\right)^2}{2\rho}$, $V_c^m\left(X_0\right) = \frac{\left(x_0^m\right)^2}{2\rho}$, $V_c^{ol}\left(X_0\right) = \frac{\left(x_0^{ol}\right)^2}{2\rho}$. The producers' payoff at the efficient equilibrium is nil. Notice that we cannot deduce the ranking of initial extractions from the ranking of the dates at which the ceiling is reached and the fact that total extraction is the same in the three equilibria, because in some of the equilibria extraction can be convex-concave. It may be the cas for instance in the MPNE since θ_t can be first increasing and then decreasing and $\ddot{x}_t = -\frac{\rho}{b}\dot{\theta}_t$.
(iii) From (81), we have:

$$x_0^c = -w_2\left(X_0 - X_\infty\right),$$

and we deduce from (78):

$$x_0^{ol} = -\gamma_1 w_1 - \gamma_2 w_2 \quad \text{with } \gamma_1 + \gamma_2 = X_0 - X_\infty,\ \gamma_1, \gamma_2 > 0,\ w_1 > 0, w_2 < 0.$$

Hence

$$x_0^{ol} = -\gamma_1 (w_1 - w_2) - w_2 (X_0 - X_\infty) = -\gamma_1 (w_1 - w_2) + x_0^c < x_0^c.$$

We can compare directly the producers' payoff as the Hamilton–Jacobi–Bellman equation can be used and so:

$$V_p^c(X_0) > V_p^{ol}(X_0) > V_p^m(X_0) > V_p^o(X_0) = 0.$$

However for the consumers the Hamilton–Jacobi–Bellman equation cannot be used in the cartel without tax case, since there is no state variable in their problem, and the consumers' payoff must be computed directly:

$$\begin{aligned} V_c^c (X_0) &= \max_x \int_0^\infty e^{-\rho t} [u(x_t) - p_t x_t] \, dt \\ &= \frac{b}{2} \int_0^\infty e^{-\rho t} x_t^2 dt. \end{aligned}$$

Extraction x_t is given by (80) and (81), which show that $x_t = x_0 e^{w_2 t}$. Then

$$V_c^c (X_0) = \frac{b}{2(\rho - 2w_2)} (x_0^c)^2, \quad \text{with } \rho - 2w_2 = \rho \sqrt{1 + \frac{2c_2}{\rho b}} > 0. \tag{82}$$

It proves that:

$$V_c^c (X_0) < \frac{b}{2\rho} (x_0^c)^2.$$

Then $x_0^{ol} < x_0^c$ doesn't necessarily imply $V_c^{ol}(X_0) < V_c^c(X_0)$. It may exist cases in which the contrary prevails. See Appendix D. ■

This Proposition contains at least three strong results.

First of all, when the two players act strategically, the sellers win.

Secondly, consumers and producers are both better off in the open loop equilibrium than in the MPNE, that is absent strategic interactions. In this sense, behaving strategically is a lose-lose situation, both parties ending up being worse off.

Thirdly, when the ceiling is not too tight the consumers gain from introducing a carbon tax, with respect to the case without tax.

5 Conclusion

Studying the MPNE of a game between two coalitions of oil producing and oil consuming countries, Liski and Tahvonen [2004] show, within the damage approach, that the carbon tax is not purely Pigouvian. If the damage is not too severe, it includes an import tariff element and exceeds the present value of marginal damages, allowing oil consuming countries to reap resource rents from the cartel of oil producers, whereas for a serious damage this element is an import subsidy and the strategic tax falls short of the Pigouvian one. The optimal design of the strategic tax (import subsidy or import tariff, tax increasing or decreasing in time) depends on the value of the parameter of the linear damage function, featuring the severity of the damage. This severity also determines the temporal profile of the strategic producer price: increasing when the damage is not too severe, decreasing otherwise. In terms of payoffs, Liski and Tahvonen conjecture that the strategic tax reduces the producers' payoff and enhances the consumers' payoff whatever the severity of the damage.

We revisit this game within the ceiling approach. We obtain a monotonically increasing strategic producer price before the ceiling, and a carbon tax which may be decreasing or increasing at the beginning of the planning horizon, but is always increasing near the ceiling, and this independently on the stringency of the ceiling. Moreover, in this framework, the strategic tax includes an import tariff element whatever the stringency of the ceiling. These results challenge the robustness of the conclusions of the existing litterature.

Compared to the open loop solution, behaving strategically is a lose-lose situation, both parties ending up being worse off. But compared to the pre-tax situation (the cartel equilibrium), we prove that when the ceiling is not too tight the consumers gain from introducing a carbon tax, whereas the producers always lose. We confirm, on this subject, the conclusion of Liski and Tahvonen [2004].

The practical discussions about the introduction of a carbon tax very often concentrate on the distributive consequences of the tax within each country and between countries adopting the environmental policy and countries refusing to do so, without considering a central actor in the climate change game, namely fossil fuel producers. We have in this paper contributed to fill this gap, in a two-zones framework. It would nevertheless be very useful to distinguish between two different zones of oil consuming countries, a "Kyoto zone", setting a common carbon tax, and a "non-Kyoto zone" refusing to do so. In this three players game, the oil cartel's power should be enhanced, but the consequences of the unilateral climate policy on Kyoto and non-Kyoto countries is not trivial and deserves further research.

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Appendix

A The linear solution of the MPNE

We look for a linear solution $\Phi(X) = A + BX$ to equation (29). By identification of the terms, it is easy to see that the coefficients A and B satisfy the following equations:

$$B^2 - \frac{4\rho b}{3}B - \frac{4\rho b}{3}c_2 = 0,$$

$$A = \frac{\frac{4\rho b}{3}c_1 - Ba}{\frac{4\rho b}{3} - B}.$$

The first equation admits two real roots, one positive and one negative. As we must have $\Phi'(X) < 0$ (see the phase diagram on Figure 1), B is equal to the negative root:

$$B = \frac{2\rho b}{3}\left(1 - \sqrt{1 + \frac{3c_2}{\rho b}}\right).$$

We then have

$$A = \frac{2c_1 - \left(1 - \sqrt{1 + \frac{3c_2}{\rho b}}\right)a}{1 + \sqrt{1 + \frac{3c_2}{\rho b}}}.$$

The solution to equation (29) must satisfy $\Phi(X_0 - \overline{Z}) = a$ if climate policy is stringent that is if $X_0 - \overline{Z} > X_\infty$, and $\Phi(X_\infty) = a$ if climate policy is lenient that is if $X_0 - \overline{Z} \leq X_\infty$. However, it is easy to check that $A + BX_\infty = a$. The linear solution is then the solution of the game in the case of a lenient climate policy. In the opposite case, as Φ is linearly decreasing, $A + B(X_0 - \overline{Z}) < a$, and the linear solution cannot be the solution of the game.

B Continuity of x at the juncture at the MPNE

We use the following result of optimal control (see Seierstad and Sydsaeter [1987] pp. 318–319) applied to the consumers' problem: the costate is continuous at T_m if $\left(X_t - X_0 + \overline{Z}\right)_{t=T_m} = 0$ and if $\left.\frac{d\left(X_t - X_0 + \bar{Z}\right)}{dt}\right|_{t=T_m}$ is not continuous (sufficient condition).

The first part of the condition is true by definition of T_m, the date at which the ceiling is reached.

Assume that $\left.\frac{d\left(X_t - X_0 + \bar{Z}\right)}{dt}\right|_{t=T_m} = \dot{X}_{T_m}$ is not continuous i.e. $\dot{X}_{T_{m-}} \neq 0$.

Hence from the previous result the costate θ is continuous at the juncture. Moreover, before the juncture, $\lambda_p = 2\theta$ is also continuous, and so is the sum of the costates of the consumers' and producers' problems $\lambda_p + \theta$.

Remind that at the ceiling $\lambda_p = 0$ and $\theta = a - c(X_0 - \overline{Z})$. Consequently when $t \to T_{m-}$ we must have $\theta + \lambda_p \to a - c(X_0 - \overline{Z})$.

However before the ceiling equation (28) yields $\theta + \lambda_p = a - c(X) + 2b\dot{X}$. Consequently, when $t \to T_{m_-}$, $\theta + \lambda_p \to a - c(X_0 - \overline{Z}) + 2b\dot{X}_{T_m}$.

Hence $\dot{X}_{T_m-} = 0$: a contradiction. We conclude that $\dot{X}$ is continuous at the juncture.

C Comparison of the initial extractions, dates at which the ceiling is reached and payoffs in the three equilibria with ceiling

The structure of the solution is the same in the 3 equilibria with ceiling: MPNE, efficient equilibrium and open loop equilibrium. Equation (48) gives the date T_m at which the ceiling is reached in the case of the MPNE:

$$\frac{v_1 e^{-v_2 T_m} - v_2 e^{-v_1 T_m}}{v_1 - v_2} = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}. \tag{C1}$$

The same applies *mutatis mutandis* for the other equilibria.

Let's define

$$F_v(T) = \frac{v_1 e^{-v_2 T} - v_2 e^{-v_1 T}}{v_1 - v_2}. \tag{C2}$$

We have:

$$F_v(0) = 1, \quad F_v(\infty) = +\infty, \quad F_v'(T) > 0, \quad F_v'(0) = 0, \quad F_v''(T) > 0.$$

$F_v(T)$ is an increasing and convex function of T, with an initial value of 1. (C1) reads

$$F_v(T_m) = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty} > 1. \tag{C3}$$

Then the solution T_m exists and is unique.

With obvious notations, we have, for the efficient and open loop equilibria respectively, $F_u(T_o) = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}$ and $F_w(T_{ol}) = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}$.

It can be proved that for $\forall T > 0$, $F_u(T) > F_w(T) > F_v(T)$. Let's prove it for the efficient equilibrium and the MPNE.

Indeed, if $T \sim 0$, a second-order approximation yields $F_v(T) \sim 1 - \frac{1}{2} v_1 v_2 T^2$ and $F_u(T) \sim 1 - \frac{1}{2} u_1 u_2 T^2$, with, according to (42) and (62), $-\frac{1}{2} v_1 v_2 = \frac{\rho c_2}{6b} < -\frac{1}{2} u_1 u_2 = \frac{\rho c_2}{2b}$. If, on the contrary, $T \gg 0$, $e^{-v_1 T} \sim 0$ and $e^{-u_1 T} \sim 0$. Hence:

$$\frac{F_v(T)}{F_u(T)} \sim \frac{v_1}{u_1} \frac{u_1 - u_2}{v_1 - v_2} \frac{e^{-v_2 T}}{e^{-u_2 T}} < 1$$

since $\frac{v_1}{u_1} \frac{u_1 - u_2}{v_1 - v_2} < 1$ and $u_2 - v_2 < 0$. Thus the $F_v(T)$ graph is under the $F_u(T)$ graph $\forall T > 0$, which yields $T_m > T_o$.

We prove along the same line that $T_{ol} > T_o$ and $T_m > T_{ol}$.

Consider now initial extraction in the MPNE. We have:

$$x_0^m = -\alpha_1 v_1 - \alpha_2 v_2 \quad \text{with } \alpha_1 + \alpha_2 = X_0 - X_\infty.$$

Hence

$$x_0^m = -\alpha_1 (v_1 - v_2) - v_2 (X_0 - X_\infty) = v_2 \left(X_0 - \overline{Z} - X_\infty\right) e^{-v_1 T_m} - v_2 (X_0 - X_\infty)$$

i.e.

$$x_0^m = -v_2 \left[(X_0 - X_\infty) - \left(X_0 - \overline{Z} - X_\infty\right) e^{-v_1 T_m}\right].$$

Let's define

$$G_v(\overline{Z}) = -v_2 \left[(X_0 - X_\infty) - \left(X_0 - \overline{Z} - X_\infty\right) e^{-v_1 T_m}\right]. \tag{C4}$$

Totally differentiating (C4) yields:

$$\frac{dG_v}{d\overline{Z}} = -v_2 \left[1 + v_1 \left(X_0 - \overline{Z} - X_\infty\right) \frac{dT_m}{d\overline{Z}}\right] e^{-v_1 T_m}, \tag{C5}$$

whereas totally differentiating (C1) yields:

$$-\frac{v_1 v_2}{v_1 - v_2} \left[e^{-v_2 T_m} - e^{-v_1 T_m}\right] \frac{dT_m}{d\overline{Z}} = \frac{X_0 - X_\infty}{\left(X_0 - \overline{Z} - X_\infty\right)^2}$$

i.e.

$$-\frac{v_1 v_2}{v_1 - v_2} \left[\frac{v_1 - v_2}{v_1} \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty} + \left(\frac{v_2}{v_1} - 1\right) e^{-v_1 T_m}\right] \frac{dT_m}{d\overline{Z}} = \frac{X_0 - X_\infty}{\left(X_0 - \overline{Z} - X_\infty\right)^2}$$

i.e.

$$-v_2 \left[(X_0 - X_\infty) - \left(X_0 - \overline{Z} - X_\infty\right) e^{-v_1 T_m}\right] \frac{dT_m}{d\overline{Z}} = \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}$$

i.e.

$$\frac{dT_m}{d\overline{Z}} = \frac{1}{x_0^m} \frac{X_0 - X_\infty}{X_0 - \overline{Z} - X_\infty}. \tag{C6}$$

Hence

$$\frac{dG_v}{d\overline{Z}} = -v_2 \left[1 + v_1 \left(X_0 - X_\infty\right) \frac{1}{x_0^m}\right] e^{-v_1 T_m} > 0.$$

When $\overline{Z} \to 0$ (extremely tight ceiling), $G_v(\overline{Z}) \to 0$. When $\overline{Z} \to X_0 - X_\infty$ (non-binding ceiling), $G_v(\overline{Z}) \to -v_2 \left(X_0 - X_\infty\right)$. Moreover, $\frac{dG_v}{d\overline{Z}}\big|_{\overline{Z} \to 0} = +\infty$. Finally, it is easy to show that $G_v(\overline{Z})$ is concave.

The same applies for $G_w(\overline{Z})$ and $G_u(\overline{Z})$, the initial extractions in the open loop and efficient equilibria.

As $G_v(\overline{Z})$, $G_w(\overline{Z})$ and $G_u(\overline{Z})$ are increasing and concave functions of $\overline{Z}$, both nil and with an infinite slope at the origin, and as $G_v(X_0 - X_\infty) < G_w(X_0 - X_\infty) < G_u(X_0 - X_\infty)$ since $-v_2 < -w_2 < -u_2$, we can conclude that $x_0^m < x_0^{ol} < x_0^o \ \forall \overline{Z} > 0$.

D Comparison of the payoffs of the two games and the cartel without carbon tax equilibrium when the ceiling is not too tight

The payoff of the consumers at the MPNE is $V_c^m\left(X_0\right) = \frac{\left(x_0^m\right)^2}{2\rho}$. When the ceiling is not too tight, according to (C4) $x_0^m \sim -v_2 \left(X_0 - X_\infty\right)$. Then

$$V_c^m\left(X_0\right) \sim \frac{b}{2\rho} \left[v_2 \left(X_0 - X_\infty\right)\right]^2.$$

Now, the payoff of the consumers in the cartel without tax case is, according to (82) and (81):

$$V_c^c\left(X_0\right) = \frac{b}{2\rho} \frac{1}{\sqrt{1 + \frac{2c_2}{\rho b}}} \left[w_2 \left(X_0 - X_\infty\right)\right]^2.$$

Hence:

$$\frac{V_c^m\left(X_0\right)}{V_c^c\left(X_0\right)} \sim \sqrt{1 + \frac{2c_2}{\rho b}} \left(\frac{v_2}{w_2}\right)^2 = \frac{4}{9} \sqrt{1 + \frac{2c_2}{\rho b}} \frac{\left(1 - \sqrt{1 + \frac{3c_2}{\rho b}}\right)^2}{\left(1 - \sqrt{1 + \frac{2c_2}{\rho b}}\right)^2}.$$

Easy but tedious computations show that the right-hand side member is an increasing function of $\frac{c_2}{\rho b}$. Moreover, for $\frac{c_2}{\rho b} \sim 0$, it is equivalent to $1 + \frac{c_2}{\rho b} > 1$. Hence:

$$\frac{V_c^m(X_0)}{V_c^c(X_0)} > 1.$$

When the ceiling is not too tight, the consumer area has a greater payoff in the MPNE than in the monopoly without carbon tax.

E MPNE: dynamic programming

Producers

The Bellman equation is:

$$\begin{aligned}\rho V_p^m(X) &= \max_p \left\{ (p - c(X))\, x(p + \theta(X)) + \frac{dV_p^m(X)}{dt} \right\} \\ &= \max_p \left\{ \left(p - c(X) - V_p^{m\prime}(X)\right) x(p + \theta(X)) \right\}\end{aligned}$$

The FOC reads:

$$x(p + \theta(X)) + \left(p - c(X) - V_p^{m\prime}(X)\right) x'(p + \theta(X)) = 0.$$

Hence the producers' price strategy:

$$p = c(X) - \frac{x(p + \theta(X))}{x'(p + \theta(X))} + V_p^{m\prime}(X).$$

The maximized Bellman equation reads:

$$\rho V_p^m(X) = -\frac{x^2}{x'}.$$

Computing the derivative with respect to X_t and using the envelope theorem yields:

$$\begin{aligned}\rho V_p^{m\prime}(X) &= \left(-c'(X) - V_p^{m\prime\prime}(X)\right) x + \left(p - c(X) - V_p^{m\prime}(X)\right) x'\theta'(X) \\ &= -\left(c'(X) + \theta'(X) + V_p^{m\prime\prime}(X)\right) x.\end{aligned}$$

Hence the equation giving the evolution of the marginal value of the resource stock for the producers:

$$V_p^{m\prime\prime}(X)x = -\rho V_p^{m\prime}(X) - \left(c'(X) + \theta'(X)\right) x$$

i.e.

$$\frac{dV_p^{m\prime}(X)}{dt} = \rho V_p^{m\prime}(X) + \left(c'(X) + \theta'(X)\right) x.$$

Consumers

The Bellman equation before the ceiling is:

$$\begin{aligned}\rho V_c^m(X) &= \max_\theta \left\{ u(x) - p(X)x + \frac{dV_c^m(X)}{dt} \right\} \\ &= \max_\theta \left\{ u(x) - \left(p(X) + V_c^{m\prime}(X)\right) x \right\}\end{aligned}$$

The FOC read:

$$u'(x) = p(X) + V_c^{m\prime}(X).$$

Hence the consumers' tax strategy:

$$\theta = V_c^{m\prime}(X).$$

The maximized Bellman equation reads:

$$\rho V_c^m(X) = u(x) - u'(x)x$$

Computing the derivative with respect to X_t and using the envelope theorem yields:

$$\begin{aligned}\rho V_c^{m\prime}(X) &= u'(x)x' - \left(p(X) + V_c^{m\prime}(X)\right)x' - \left(p'(X) + V_c^{m\prime\prime}(X)\right)x \\ &= -\left(p'(X) + V_c^{m\prime\prime}(X)\right)x\end{aligned}$$

Hence the evolution of the marginal value of the resource stock for consumers:

$$V_c^{m\prime\prime}(X)x = -\rho V_c^{m\prime}(X) - p'(X)x$$

i.e.

$$\frac{dV_c^{m\prime}(X)}{dt} = \rho V_c^{m\prime}(X) + p'(X)x.$$

MPNE

The strategies before the ceiling are:

$$\begin{aligned}p &= c(X) - \frac{x(p+\theta)}{x'(p+\theta)} + V_p^{m\prime}(X) \\ \theta &= V_c^{m\prime}(X).\end{aligned}$$

and so the equilibrium is characterized by:

$$u'(x) = p + \theta = c(X) - \frac{x(p+\theta)}{x'(p+\theta)} + V_p^{m\prime}(X) + V_c^{m\prime}(X).$$

Let

$$\Phi(X) = c(X) + V_c^{m\prime}(X) + V_p^{m\prime}(X).$$

the equilibrium equation then reads:

$$u'(x) = p + \theta = \Phi(X) - \frac{x(p+\theta)}{x'(p+\theta)},$$

i.e., with the quadratic specification for the utility function:

$$a - bx = p + \theta = \Phi(X) + a - (p+\theta) \Rightarrow \begin{cases} p + \theta = \frac{1}{2}\left(\Phi(X) + a\right) \\ x = \frac{a - \frac{1}{2}(\Phi(X)+a)}{b} = \frac{1}{2b}\left(a - \Phi(X)\right). \end{cases}$$

The FOC, the maximized Bellman equations and the equations of evolution of the marginal value of the stock read:

$$
\begin{aligned}
p &= c(X) + a - (p+\theta) + V_p^{m\prime}(X) \\
\theta &= V_c^{m\prime} \\
\rho V_p^m(X) &= bx^2 = \frac{1}{4b}\left(a - \Phi(X)\right)^2 \\
\rho V_c^m(X) &= \frac{b}{2}x^2 = \frac{1}{8b}\left(a - \Phi(X)\right)^2 \\
\left(V_p^{m\prime\prime}(X) + c'(X) + \theta'(X)\right)x &= -\rho V_p^{m\prime}(X) \\
\left(V_c^{m\prime\prime}(X) + p'(X)\right)x &= -\rho V_c^{m\prime}(X)
\end{aligned}
$$

Notice that (i) $V_p^m(X) = 2V_c^m(X)$, (ii) $V_p^m(X_0)$ and $V_c^m(X_0)$ are function of the initial extraction x_0 only, and vary in the same direction with x_0.

The sum of the maximized Bellman equations yields:

$$
\rho\left(V_p^m(X) + V_c^m(X)\right) = \frac{3}{8b}\left(a - \Phi(X)\right)^2,
$$

while the sum of the equations of evolution of the marginal value of the stock reads:

$$
\left(\Phi'(X) + p'(X) + \theta'(X)\right)x = -\rho\left(\Phi(X) - c(X)\right)
$$

i.e.

$$
\left(\Phi'(X) + \frac{1}{2}\Phi'(X)\right)\frac{1}{2b}\left(a - \Phi(X)\right) = -\rho\left(\Phi(X) - c(X)\right)
$$

i.e.

$$
\Phi'(X) = \frac{4\rho b}{3}\frac{\Phi(X) - c(X)}{\Phi(X) - a}
$$