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HARNESSING WINDFALL REVENUES IN DEVELOPING ECONOMIES:

Sovereign wealth funds and optimal tradeoffs between citizen dividends, public infrastructure and debt reduction

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Abstract

A windfall of foreign aid or natural resource revenue faces government with choices of how to manage public borrowing, public asset accumulation, and the distribution of funds to households (across time and household types), particularly when the windfall is both anticipated and temporary. These choices are acute if some households do not have access to credit markets and are unable to smooth consumption, and if the country as a whole is not a price-taker in international capital markets – both reasonable descriptions of many developing countries experiencing resource (or aid) booms. We analyse the optimal policy actions for countries in this position and show that the usual permanent income hypothesis prescription of engineering a permanent increase in consumption financed by borrowing ahead of the windfall and then accumulating a Sovereign Wealth Fund (SWF) is not optimal. Heavily indebted countries with a small windfall should both increase current consumption and accumulate capital to accelerate their development. Only if the windfall is large relative to initial debt is it optimal to build a SWF. We study the intricate dynamic trade-offs faced when using the windfall to pay off debt and possibly accumulate a SWF, build public infrastructure and hand out citizen dividends. Finally, we show that a more sophisticated range of instruments (e.g., an asset holding subsidy) makes the trade-offs easier.

Keywords: windfall public revenues, risk premium on foreign debt, public infrastructure, private investment, credit constraints, expropriation, optimal fiscal policy, debt management, Sovereign Wealth Fund, asset holding subsidy, developing economies

JEL codes: E60, F34, F35, F43, H21, H63, O11, Q33

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1. Introduction

A temporary windfall of foreign aid or natural resource revenues poses interesting policy challenges. Should the revenues be used for government investment in public infrastructure to stimulate economic activity? Should the government use the windfall income to reduce government debt and thereby lower interest rates and boost private sector investment? Should the extra income be used to provide more education, health care and other public goods to improve the quality of life or transferred directly to citizens through tax cuts? Alternatively, revenues could be used to transform exhaustible resource assets into interest-earning foreign assets by setting up a Sovereign Wealth Fund for future generations. This is a bewildering array of policy options and the most appropriate option depends on what stage of development the economy is and what kind of constraints the economy faces.

The conventional consumption smoothing and debt management guidelines based on the permanent income hypothesis are familiar from the tax smoothing literature (Barro, 1979) or the optimal use of the current account (e.g., Sachs, 1981). These arguments underlie much of the advice for the setting up of a Sovereign Wealth Fund (SWF) proffered by the International Monetary Fund (e.g., Davis, et al., 2002; Barnett and Ossowski, 2003; Segura, 2006; Leigh and Olters, 2006; Olters, 2007; Basdevant, 2008). Although the main insights of this advice are sound, they ignore essential problems of developing economies struggling to grow in the face of various market failures, and may therefore be only of limited relevance.

Our objective is to provide a rigorous analysis of how to address these policy choices in relatively poor countries with less than perfect access to capital markets. We focus on welfare-maximizing government choices between three options: using the windfall for private (or public) consumption; spending on public assets that raise income and the marginal productivity of private investment; and altering the country's foreign asset/ debt position. While we focus on responses to windfalls, our analysis of these choices is of more general interest for policy formulation, particularly in developing economies.

Central to our analysis are two market imperfections that we think are important aspects of many economies. The first is that working households find it hard to borrow against future wage income so Ricardian debt neutrality is unlikely to hold. To capture this we suppose that there is a class of credit-constrained households who have no access to capital markets, living entirely from current wage income and government transfers. There is also a class of capitalists ('rentiers') who do not face these constraints, and make intertemporal consumption and saving decisions. The presence of credit constrained households means that there is a role for government to smooth consumption by varying taxes paid by or subsidies handed out to these households. This may involve building a sovereign wealth fund (SWF) to pay a continuing stream of citizen dividends.

The second imperfection is that the country is not necessarily a price taker in international credit markets. A country with a relatively low level of foreign debt relative to its earning potential has a small risk of default and can borrow on international markets at the world interest rate. But beyond a certain level of foreign indebtedness, it faces an upward-sloping supply schedule of foreign debt. An indebted country then faces an interest premium, the size of which depends on the stock of foreign debt (private and public combined). The premium might be a consequence of the perceived likelihood of default, although we do not model this explicitly. The consequence of this premium is that indebted economies will have high domestic interest rates, low capital-labour ratios and little investment in public infrastructure, and thus low wages and low per capita income. Our primary focus will be on such economies. In the absence of a foreign exchange windfall they are on a trajectory of capital accumulation and rising consumption, and we examine how the windfall can be optimally used to alter this trajectory.

To focus on the main public finance issues at hand we abstract from many important elements of the problem. We use a single sector model in which there are no problems in absorbing expenditure, either from an appreciation of the real exchange rate and its adverse impact on the traded sector (the Dutch disease, Corden and Neary, 1982; van Wijnbergen, 1984; Sachs and Warner, 1997) or from supply bottlenecks in particular domestic sectors such as construction. We also abstract from uncertainty and from political economy concerns.

The outline of the paper is as follows. Section 2 first sets up the benchmark for a country with a relatively low level of foreign debt whose citizens are unable to smooth consumption but whose government can do it for them. In this benchmark the home interest rate is pegged to the world interest rate. Application of the permanent income hypothesis then shows that it is best to use temporary windfall revenue to have an immediate and permanent boost to citizen dividends and private consumption, this paid for by borrowing ahead of the windfall and then by the interest from a SWF accumulated during the windfall.

Section 3 analyses what one should do with an anticipated windfall in heavily indebted economies. Consumption smoothing is no longer optimal. Instead, an immediate increase in consumption is followed by rapid debt reduction in order to get the interest rate down. With small windfalls, the economy's growth path is accelerated, but no SWF is built up. Only if windfalls are large relative to initial debt will it be optimal to build an SWF and associated permanent increase in consumption. In both these cases, consumption is relatively more skewed towards current (typically poorer) generations than is the case with the permanent income hypothesis benchmark. Section 4 extends our analysis to allow for domestic production with foreign-owned capital and domestic labour. Section 5 puts forward our more general model of a developing economy containing private and public investment and a class of domestic capitalists with access to credit markets. The government does not only want to help its credit-constrained citizens to smooth consumption but also uses part of the windfall revenue for public infrastructure to boost economic activity. In addition, we allow for a wider range of government tax instruments. After deriving and characterising optimal fiscal policy for this economy, we present some simulations to highlight the various tradeoffs in this more general model. Section 6 discusses some extensions to allow for Dutch disease effects, habit persistence, irreversible investment in public infrastructure, partisan politics and optimal depletion of natural resources. Section 7 concludes and summarises our guidelines for how to cope with windfall revenue in a developing economy and contrasts them with the advice given by the International Monetary Fund and many others.

2. Benchmark: the permanent income hypothesis

We first consider a small open economy with relatively small foreign indebtedness (or with net foreign assets) that can borrow or lend at the world interest rate. This economy has exogenous and constant non-oil output Y. Consumers receive a lump sum transfer or citizen dividend T from the government so their consumption is given by $Q = Y + T$. The government is the only agent in the economy that has access to the international capital market, so foreign debt F corresponds to public debt. It chooses transfers T and public consumption G to maximise utility of its citizens, $\int_0^\infty \left(\frac{Q^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} \right) \exp(-\rho t) \mathrm{d}t$, where $\psi \geq 0$ is the weight given to public consumption, σ is the elasticity of intertemporal substitution and the rate of time preference is denoted by ρ. Maximisation is subject to the budget constraint $\dot{F} = r^* F + G + T - N = r^* F + G + Q - Y - N$, with fixed initial debt F_0 and exogenous world interest rate r^*, assumed to equal the rate of time preference, $\rho = r^*$. N stands for the flow of windfall revenue from the sale of oil or foreign aid, all of which accrues to government. This flow of revenue is announced (A) at date $t = 0$; it starts to flow at some date at or beyond $t = 0$ which we refer to as the date of extraction (E); it may cease flowing, or be depleted (D), at some later date.

The conditions for optimal government policy are familiar. The intertemporal efficiency condition states that consumption of the government and its citizens are smoothed over time, $\dot{G} = \dot{Q} = 0$, and that lump-sum transfers adjust to achieve this. The intratemporal efficiency condition requires that public and private consumption move up and down together, $G = \psi^\sigma Q$. Combining these conditions with the present-value budget constraint gives

$$(1)\quad \begin{aligned} &Q=\left(\frac{1}{1+\psi^{\sigma}}\right)(N^P+Y-r^*F), \quad G=\left(\frac{\psi^{\sigma}}{1+\psi^{\sigma}}\right)(N^P+Y-r^*F),\\ &T=\left(\frac{1}{1+\psi^{\sigma}}\right)(N^P-\psi^{\sigma}Y-r^*F), \quad T+G=N^P-r^*F \text{ and } \dot{F}=N^P-N. \end{aligned}$$

where $N^P(t)$ is the *permanent* level of resource revenue at each date, defined as the amortised value of the stream of future revenues, $N^P(t)\equiv r^*\int_t^{\infty}\exp\left(-r^*(z-t)\right)N(z)\mathrm{d}z$. Debt accumulation/decumulation equal to the difference between the permanent level and current flow of resource wealth, $\dot{F}=N^P-N$, ensures that N^P-r^*F is held constant, as can be seen by differentiating the definition of permanent resource revenue to give $\dot{N}^P=r^*(N^P-N)$, and hence $\dot{N}^P-r^*\dot{F}=0$.

Figure 1: Use of Sovereign Wealth Fund to manage temporary windfall revenue according to the permanent income hypothesis

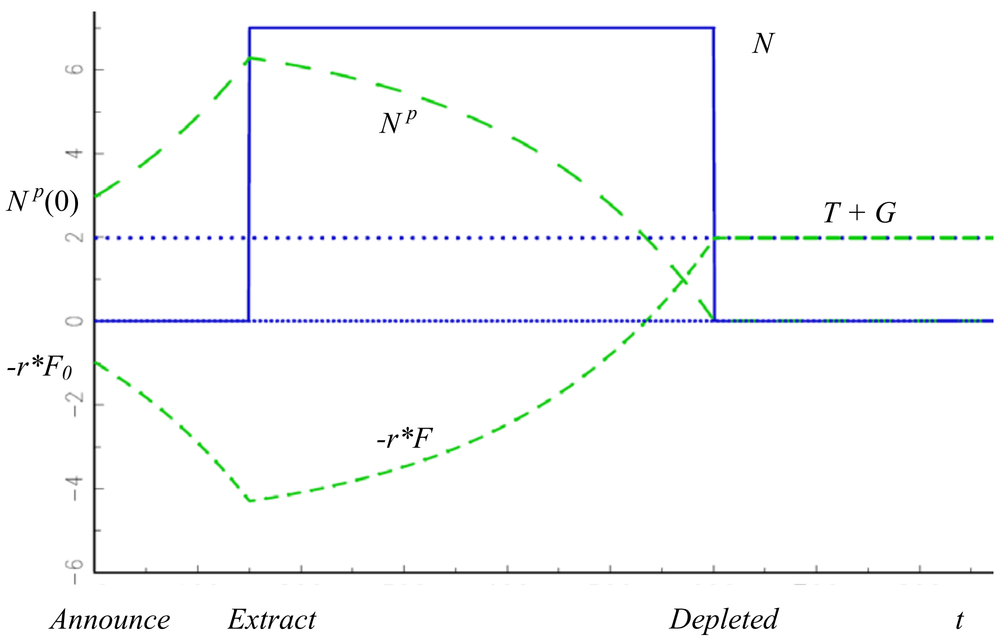


Figure 1 summarises the responses derived in (1) to an anticipated temporary windfall with revenue flow given by the step function N. The vertical axis measures income flows, so an economy with initial debt F_0 (for illustration, $F_0 > 0$) has an associated income flow (negative debt service) of $-r^*F_0$. The windfall is discovered at date $t = 0$, and permanent resource revenue is given by curve N^P. This rises as the date of extraction comes closer, and falls once extraction begins, going to zero when the windfall ends. There is a permanent rise in public spending equal to the permanent value on discovery of the temporary bonanza. Prior to the discovery $T + G = r^* F_0$ and following the discovery this jumps to $T + G = r^* F_0 + N^P(0)$, as illustrated by the horizontal line $T + G$. Ahead of the windfall revenue, the country borrows abroad, with the path of debt illustrated by - r^*F, this mirroring the shape of N^P since $T + G = N^P - r^* F$. Debt rises until the windfall revenue comes in, at which date the country starts paying off debt and eventually builds up assets abroad sufficient to sustain the permanent increase in $T + G$. The temporary stream of windfall revenue is thereby transformed into a permanent stream of income from abroad. The assets accumulated abroad correspond to a Sovereign Wealth Fund (SWF) of the type used by Norway. The revenue from the Fund provides for a constant level of citizen dividends and public consumption each period. The government thus steps in to smooth consumption of its citizens even though they do not have access to the international capital market themselves.

This yields the following guidelines for coping with windfall revenue:

- the country borrows and runs a current-account deficit while it anticipates windfall revenue in the future ($N^P > N$), but pays off foreign debt and then builds up a buffer (e.g. a Sovereign Wealth Fund) and thus runs a current-account surplus during the temporary windfall revenue ($N > N^P$);
- total spending (public consumption plus citizen dividends) goes up immediately upon the news of the windfall one-for-one with the permanent level of windfall revenue;
- the amount of foreign assets that are built up at the end of the windfall generate just sufficient revenue to finance the permanent increase in public spending;
- the share of total spending devoted to citizen dividends decreases with the weight given to public consumption;
- government transfers ensure that private consumption is completely smoothed over time and increases with permanent windfall revenue and income and decreases with interest on outstanding debt.

3. Developing economies: departure from the permanent income hypothesis

The benchmark of using debt to smooth consumption may be applicable for countries able to borrow or lend unlimited amounts at a given world interest rate. Most developing economies have high levels of foreign debt relative to their earning potential and have to struggle with relatively high domestic interest rates r. We capture this with a supply schedule of foreign debt, where for low values of foreign indebtedness the home interest rate is pegged to the world interest rate and for high levels of indebtedness (greater than $\bar{F}$) the home interest rate rises above the world interest rate:

(2) $$r = r^* \text{ for } F \le \bar{F} \text{ and } r = r^* + \Pi(F) > r^* \text{ for } F > \bar{F},$$

$$\text{with } \Pi(\bar{F}) = \Pi'(\bar{F}) = 0 \text{ and } \Pi'(F) > 0,\ F > \bar{F}.$$

Figure 2: The cost of foreign borrowing:

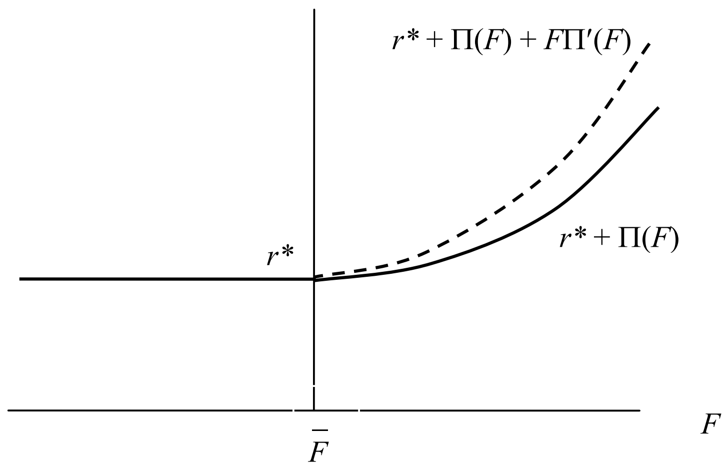


Figure 2 portrays this supply schedule. One can interpret $\Pi(F)$ as an international risk premium on foreign debt to capture the risk of default, but we do not model that. In the macroeconomics literature (e.g. Turnovsky, 1997, section 2.6; Schmitt-Grohé and Uribe, 2003), it is common to close small open economy models by specifying a supply schedule of foreign debt which slopes upwards for all F. Although this is analytically convenient, it has the unattractive feature of implying a unique steady-state value $F(\infty) = \bar{F}$ which is independent of windfall revenue. This is in contrast to the permanent income hypothesis under which, as we saw in section 2, countries chose their steady-state value of F by, for example, building a SWF. To capture both the interest premium and endogeneity of the steady-state value of F, we suppose that economies above some level of indebtedness face a premium, $\Pi(F) > 0$ and $\Pi'(F) > 0$, while below some level of debt countries are price takers at r^*, so that $\Pi(F) = 0$ and $\Pi'(F) = 0$. We assume that $\bar{F} \ge 0$ so $F\Pi'(F) \ge 0$ for all $F \ge \bar{F}$, as illustrated in figure 2.

As in section 2, the government maximises utility of its citizens subject to its budget constraint $\dot{F} = [r^* + \Pi(F)]F + T + G - N, F(0) = F_0$. The intratemporal efficiency condition $G = \psi^\sigma Q$ can be used to write the current-account dynamics as

(3) $$\dot{F} = [r^* + \Pi(F)]F + (1+\psi^\sigma)Q - Y - N, \quad F(0) = F_0.$$

Perfect consumption smoothing is no longer optimal, since the marginal cost of borrowing is not equal to the pure time preference rate. The intertemporal efficiency condition (the first-order condition for the optimal consumption path) is

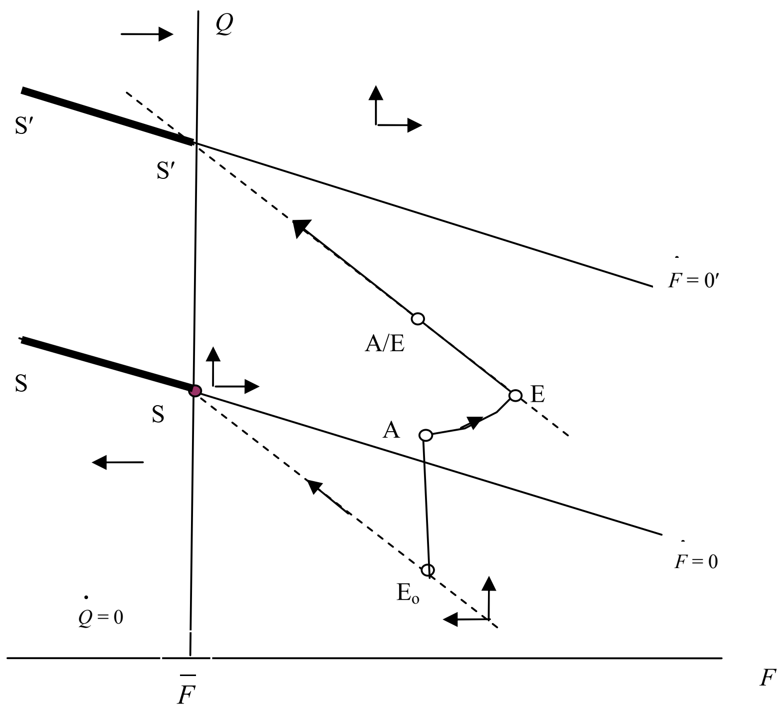
(4) $$\dot{Q} = \sigma Q[r^* + \Pi(F) + F\Pi'(F) - \rho].$$

This says that consumption is low – and therefore rising – if the marginal cost of foreign borrowing (or marginal return to accumulating foreign assets) exceeds the rate of pure time preference. The marginal cost of foreign borrowing now includes the risk premium $\Pi(F)$ and the value of any change in the premium, $F\Pi'(F)$. Even though $r^* = \rho$, an indebted country faces a higher world interest rate (and marginal cost of foreign funds) so has an incentive to postpone consumption and save. Only when F is reduced to $\overline{F}$ does consumption become stationary.

Figure 3.1 portrays the phase-plane diagram corresponding to (3)-(4). Looking first at the lower part of the figure, the $\dot{F} = 0$ locus slopes downwards; above it consumption is high and foreign debt increases and below it foreign debt declines over time. $\dot{Q} = 0$ is not a line, but the set of all values of $F \leq \overline{F}$. Countries with substantial foreign debt $F > \overline{F}$ face high domestic interest rates and prefer to save; hence their consumption tends to increase over time. Conversely, countries with $F \leq \overline{F}$ have interest rates equal to the world interest rate and (by assumption) their rate of time preference, so have constant consumption. This system has a set of stationary points, given by the line S-S. For an economy which finds itself with $F \leq \overline{F}$, this line segment is unstable, so Q must jump to S-S; this is precisely the permanent income hypothesis. For an economy with $F > \overline{F}$ there is a unique saddlepath, illustrated by the dashed line.

Consider therefore an indebted economy without windfall revenue which starts out at point E_0 on figure 3.1. This economy faces relatively high interest rates and is gradually catching up to its final steady state at point S. Along the saddlepath (dashed line), the economy saves a lot and thus consumption grows relatively fast. As it pays off its foreign debt, the domestic interest rate falls so that the propensity to save and the growth rate fall. In the long run the economy has paid off its foreign debt ($F = \overline{F}$), the domestic interest rate has fallen to the world interest rate, and private and public consumption have risen to their steady-state values.

Figure 3.1: Consumption and investment response to a permanent windfall



3.1. A permanent windfall

The effects of a permanent and constant flow of foreign revenue (oil or foreign aid) are shown in figure 3.1. Once the flow starts being extracted the $\dot{F} = 0$ locus shifts up, moving the steady state from S-S to S′-S′ with dynamics based around this new steady state. If extraction occurs from the date of announcement, then consumption immediately jumps, Q going from E_0 to point A/E; the economy then converges along the new saddlepath towards the new steady state S′. If the economy was in steady state initially, it would jump instantaneously to its new steady state with higher consumption.

What if the permanent increase in foreign windfall revenue is anticipated some time ahead of extraction? At announcement, consumption immediately increases to point A. During the interval between announcement and extraction, the government borrows abroad and thus pushes up foreign debt and interest rates at home as illustrated by the movement up and to the right. Once extraction commences the economy must be on the new saddlepath at point E. The government stops borrowing and starts to pay off the foreign debt. As the economy moves along the saddlepath to S′ the interest rate gradually falls to the world interest rate, economic growth tapers off and consumption rises to the new steady state value. Anticipated oil or foreign aid revenues thus imply that a debt-ridden country adds to its debt before the revenue comes on stream, and this finances higher consumption. Debt is eventually paid off but – as a direct consequence of the resource windfall being permanent and constant -- the economy does not accumulate a SWF.

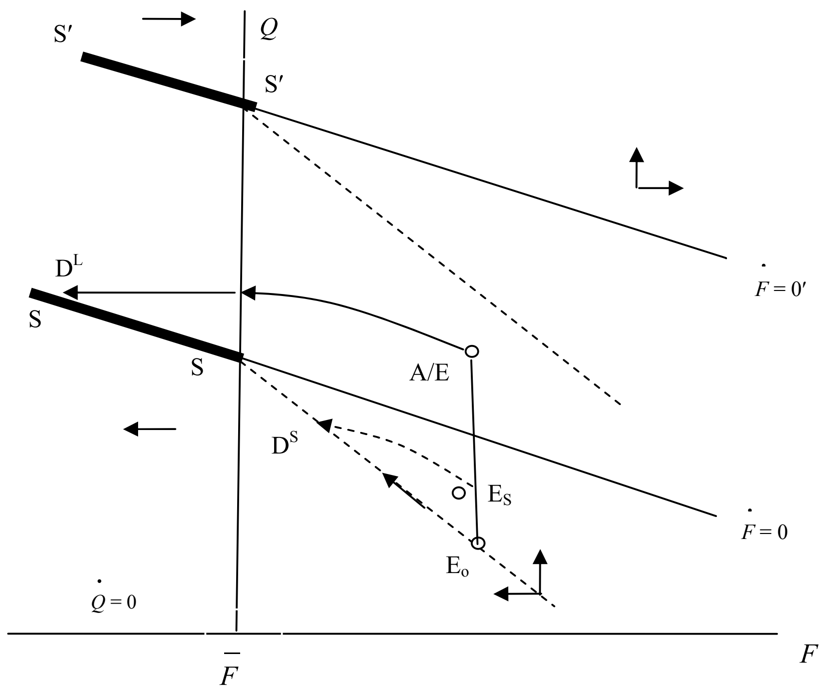
3.2. A temporary windfall

The more interesting case is that in which the revenue flows from the windfall are known to be temporary, addressed in figure 3.2. We assume that revenues follow a step function (as in figure 1), so the system is under the influence of the stationary at S′- S′ only during the period during which the resource is extracted. For simplicity in construction of this figure, we assume that extraction commences at the date of announcement.

The analysis now depends critically on whether the windfall is 'small' or 'large'. When the windfall is small the economy jumps from E_0 to E_S as the government immediately raises transfers to its citizens and boosts private and public consumption. Under the dynamics associated with S′ the government pays off debt relatively fast since the jump in consumption is less than the flow of revenue. The marginal cost of capital remains above the rate of time preference so consumption continues to rise. At the date when the resource is depleted, the economy has to hit the original saddlepath (point D^S) and it then continues up this path to the steady state S. The windfall raises consumption at all dates, but does not raise the long run level of consumption to which the economy asymptotically converges. In contrast to the permanent income hypothesis there is no SWF, since the economy's initial low level of consumption means that it is optimal to use the windfall proportionately more on the current generation.

A larger windfall is associated with a larger initial jump in consumption, and with the establishment of a SWF to finance permanently higher consumption. This is illustrated in figure 3.2 by the jump to point A/E. As with the small-windfall case, during the period of extraction consumption is higher and debt is being repaid, but now it is optimal to repay all debt (reach $\overline{F}$) before the resource is depleted. Consumption therefore increases until this point is reached and the marginal cost of capital reaches the rate of time preference. Consumption reaches its permanent value and foreign assets continue to be built up to the level sufficient to sustain this consumption once resource revenues cease. At the date the resource runs out (point D^L) the economy becomes stationary. The economy has to reach the original stationary, S–S, at the date the resource is depleted and the windfall ceases, this determining the size of the jump to A/E. The size of the long-run SWF (i.e., $-F(\infty)$) is now endogenously determined. For example, a higher initial debt or a smaller and less protracted windfall will reduce the size of the terminal SWF. The boundary between the 'large' and 'small' windfall is when points D^S and D^L coincide at foreign debt level $\bar{F}$.

Figure 3.2: Temporary windfall and permanent spending



3.3. Summing up

The main lesson from this section is that with an interest premium on foreign borrowing, perfect smoothing of public and private consumption is no longer optimal. Consumption will jump up at the date of announcement, but not the whole way to its steady state value. More resources will be devoted to bringing down the domestic interest rate. A SWF will be built up only if the resource windfall is temporary and large enough to move the economy out of the regime in which it faces a premium on its foreign debt.

Notice that we have not allowed the announcement of the windfall to have any direct effect on the 'creditworthiness' of the economy. Such a direct effect would shift the $\Pi(F)$ function; for example, the risk premium might be specified as $\pi = \Pi(F - \varsigma N^P / r^*)$ with the coefficient $0 < \varsigma < 1$ indicating the extent to which foreign lenders are prepared to take the present discounted value of future oil revenues into account. With this specification, upon discovery of the oil or news of the planned impulse to foreign aid, credit worthiness improves and the home interest rate falls. Consequently, the country borrows more ahead of the windfall and thus there will be a bigger boom in private and public investment. After the windfall has occurred, credit worthiness rapidly falls and domestic interest rates rise unless the economy has accumulated sufficient assets to prevent this happening. There is a danger of over-borrowing (e.g., Mansoorian, 1991; Manzano and Rigobon, 2001). We do not discuss this any further here, and would argue that such a direct effect is not rational. Along an optimal path the government will spend all the revenue, so any improvement in credit worthiness is illusory.

4. Allowing for domestic production with foreign-owned capital

To draw out the implications of this analysis more fully we now endogenise non-resource output, by allowing for physical capital in production, while retaining for the moment the assumption that there are no private domestic asset holders. Non-resource domestic income is now given by the Cobb-Douglas production function $Y = K^{\alpha} L^{1-\alpha}$, $0 < \alpha < 1$, where K denotes capital and L employment of labour, normalised to unity. Profit maximisation requires that the marginal product of capital, net of depreciation δ_K, equals the interest rate, so that $\alpha K^{\alpha-1} - \delta_K = r^* + \Pi(F) = r$. Notice that a reduction in F that reduces the interest rate will now lead to an increase in capital employed in the economy and hence an increase in non-resource income, so K is a non-decreasing function of F.

In the absence of private domestic asset holders, all of K is foreign owned. We define net national (non-resource) wealth as the physical capital stock net of foreign claims, $B \equiv K - F$.[1] The relationship between F and K implicitly defines $K = \mathrm{K}(B)$ according to $\alpha K^{\alpha-1} - \delta_K = r^* + \Pi(K - B)$ with $\mathrm{K}'(B) \equiv \partial \mathrm{K} / \partial B = K\Pi' / \{K\Pi' + (1-\alpha)(r+\delta_K)\}$, this gradient having positive value for $K(B) - B > \overline{F}$ and zero otherwise. Dependence of K on B also gives (from the production function and profit-maximising conditions) wage rate $W = W(B)$ and interest rate $r(B)$.

Current account dynamics now include private investment in physical capital, I_K, so the budget constraint (3) becomes

$$(3') \qquad \dot{F} = rF + (1 + \psi^{\sigma})Q + I_K - Y - N$$

[1] An equivalent way to see this is to note that, with no private asset holders, $-B$ is public debt; foreigners own the capital stock and public debt, so $F = K + (-B)$.

Using $\dot{B} = \dot{K} - \dot{F}$, the capital accumulation equation, $\dot{K} = I_K - \delta_K$ and that factor payments exhaust income, $Y = (r + \delta_K)K + W$, this can be expressed as

$$(3'') \qquad \dot{B} = r(B)B - (1 + \psi^{\sigma})Q + W(B) + N.$$

i.e. the change in national wealth is national income minus consumption expenditures. Maximising social welfare $\int_0^\infty \left(\frac{Q^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1 - 1/\sigma} \right) \exp(-\rho t) \mathrm{d}t$ subject to this budget constraint and initial value $B(0) = B_0$ yields the dynamic efficiency condition[2]

$$(4') \qquad \dot{Q} = \sigma Q[\mathrm{r}(B) + \mathrm{W}'(B) + B\mathrm{r}'(B) - \rho] \text{ for } B < K(B) - \overline{F} \text{ and } \dot{Q} = 0 \text{ otherwise.}$$

It is easy to establish that $\mathrm{W}'(B) + B\mathrm{r}'(B) = [1 - B / \mathrm{K}(B)](1 - \alpha)(r + \delta_K)\mathrm{K}'(B)$.

The dynamic system in B and Q is qualitatively similar to that in F and Q illustrated in figures 3.1 and 3.2, and we therefore choose to illustrate optimal paths by simulation of an example presented in the panels of figure 4.1. Time is on the horizontal axis, and scaling of the vertical axes is achieved by having the long-run stationary value of income equal to unity. Parameter values are given in appendix 1; we assume that $\overline{F} = 0$ and that the time dimension is scaled such that the horizontal axis can be (loosely) interpreted as years. The solid lines give the path of an economy which starts out with national wealth B_0, set at half its long-run value and which experiences no shocks. The economy has positive initial foreign debt and converges smoothly to its stationary value with accumulation of assets, decumulation of foreign debt, falling interest rates, and rising income and consumption.

The effect of a temporary anticipated *'small'* windfall is given by the dashed lines. Initial asset values are as in the base case, but at year zero there is an announcement of a flow of resource revenue between years 16 and 40. The flow is equal to 5% of long-run stationary non-oil income. At the date of announcement it is optimal to increase consumption, this financed by foreign borrowing so F rises to a level above what it otherwise would have been. Higher foreign debt translates into higher r, lower K and lower non-oil income, Y. Once the revenue flow comes on stream, F is paid off more rapidly than was the case absent the windfall, with the associated rapid fall in r and increase in K and Y. Each of these variables crosses its non-windfall path during the period of revenue flow. At the date when the windfall revenue ceases (t = 40, at the kink) the economy reverts to its previous path, but earlier than would otherwise have been the case. Thus, the dashed lines converge to the same value as the solid ones, but are shifted to the left by some 30 time periods. The top right hand panel gives the composition of the economy's assets. The lower dashed line gives the evolution of $B = K - F$, while the upper line adds to this the present value of remaining resource stock.

In summary, optimal use of the windfall involves increased consumption from the date at which the resource is discovered, and faster asset accumulation (debt decumulation) from the date windfall revenue flows. This brings forward the economy's development path, but does not lead to the formation of a SWF.

Figure 4.1: A small temporary anticipated windfall

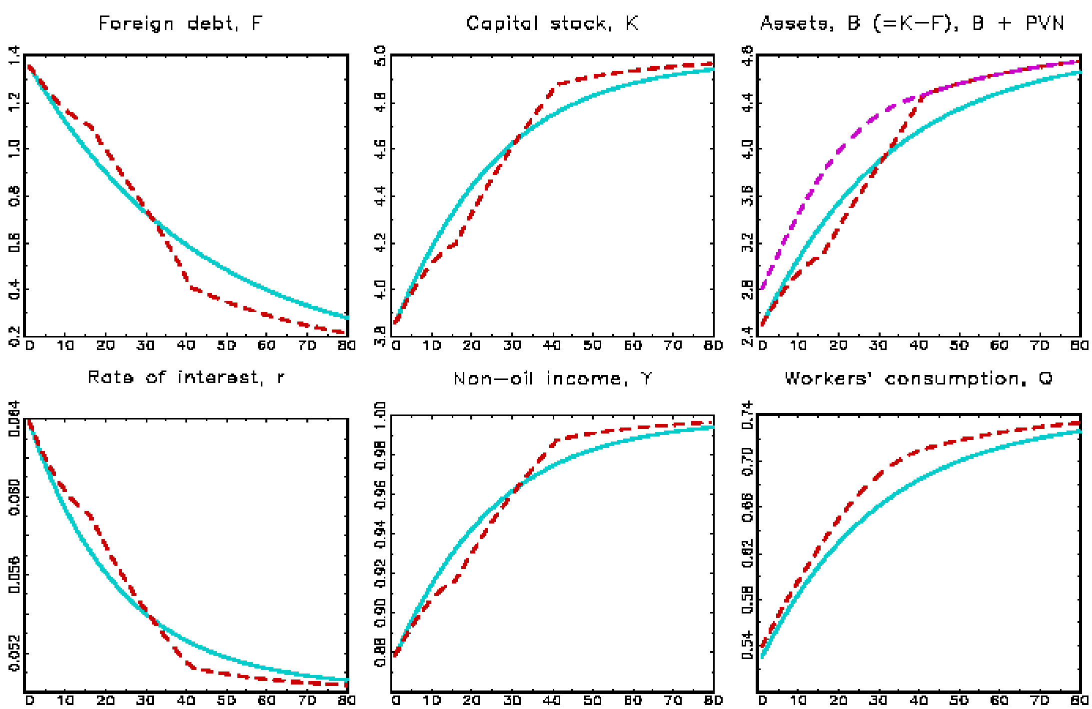
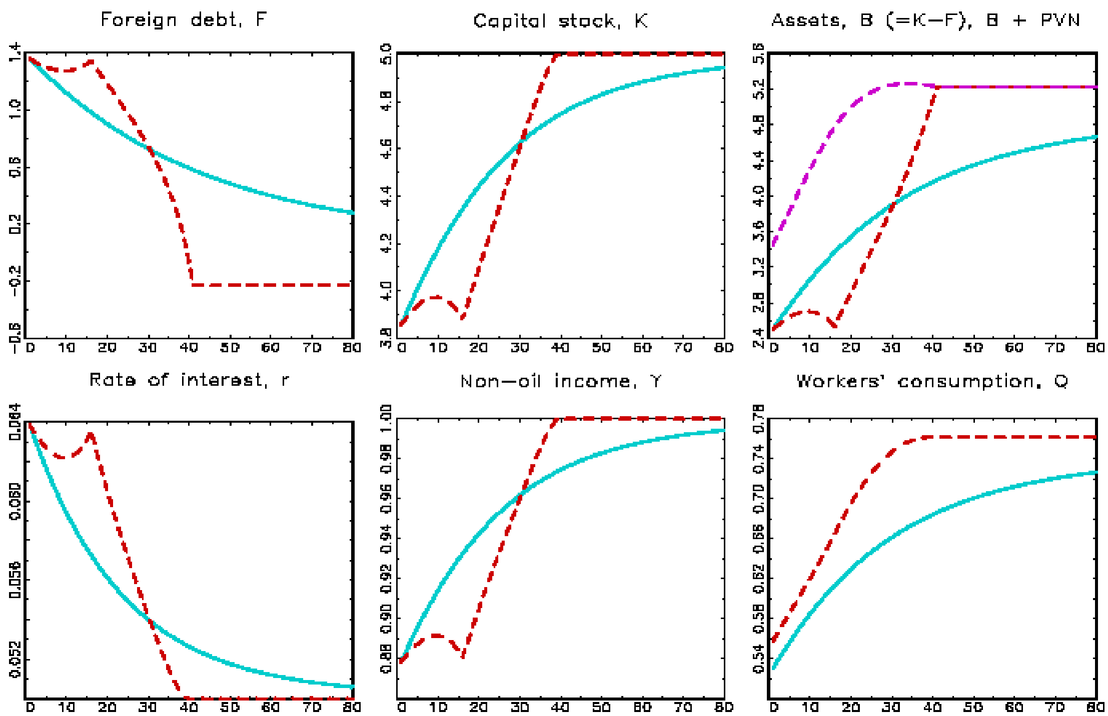


Figure 4.2 repeats the analysis for a *'large'* windfall. The solid lines are as before, and the dashed lines represent the trajectory with a revenue flow equal to 16.7% of long-run GDP between years 16 and 40. Effects are now quantitatively larger and (as in figure 3.2) large enough to provide the economy with sufficient resources to create a SWF which will sustain permanently higher consumption after the windfall has ceased. Thus, F reaches $\overline{F}$ and $r = r^*$ around date $t = 37$; capital stock and income have attained their long-run values and further revenues lead to $F < 0$ and the build-up of a SWF, so that consumption is permanently higher. Optimal use of this 'large' windfall has achieved three things: an immediate increase in

[2] Formal derivation of this is undertaken in the more general setting of section 5.

consumption; faster capital accumulation and growth of income and consumption growth; and permanently higher consumption.

Figure 4.2: A large temporary anticipated windfall



5. Private asset holders and public infrastructure

We now move to a richer description of the economy, in which we allow some private domestic agents to access capital markets and take optimal forwards looking decisions. For simplicity, we suppose that these 'capitalists' receive no labour income, while workers remain credit constrained. In addition, we suppose that government invests in public infrastructure, has at its disposal various tax/ transfer instruments and (as before) has access to the international capital market. The resulting model captures some essential features of a developing economy with windfall revenue:

- many credit-constrained workers and few capitalists with access to capital markets;
- private investment in capital and public investment in infrastructure;
- an upward-sloping supply schedule of foreign debt; and
- a government which faces a trade-off between spending the windfall on transfers to workers and capitalists, public infrastructure and/or public consumption while managing debt and its impact on interest rates, investment and economic activity.

Most small open economy models with incomplete asset markets have steady states that depend on initial conditions and furthermore have equilibrium dynamics with a random walk component. To ensure stationarity and a unique steady state, macroeconomists often postulate an upward-sloping supply schedule of foreign debt over the whole range (i.e. $\overline{F} = -\infty$).[3] Our assumption that the supply schedule of foreign debt is upward sloping only for $F > \overline{F}$ seems realistic and implies that it may pay to set up a SWF as already discussed in sections 3 and 4. We abstract from capital adjustment costs, so volatility of investment in response to variations in the risk premium may be somewhat too large.

5.1. Workers and capitalists

Workers supply one unit of labour and are credit constrained. They consume their wage income and any transfers received from the government, $Q = W + T$. Capitalists have access to domestic capital markets and are thus able to smooth their consumption C. They face the budget constraint $\dot{A} = (r - \tau_A)A - C + T_A$, where A denotes capitalists' holdings of financial assets, r the rate of return on these assets, τ_A an asset holding tax and T_A lump-sum transfers. The optimality condition for capitalists states that their growth in consumption must be proportional to the gap between the interest rate (net of the asset holding tax) and their rate of pure time preference ρ:

$$\dot{C} / C = \sigma(r - \tau_A - \rho) . \tag{5}$$

We assume that if the asset holding tax is used, then its proceeds are handed back to capitalists in a lump sum manner. In the absence of any other lump-sum transfers, their budget constraint is therefore $\dot{A} = rA - C$, although the tax still has a distortionary effect on consumption choices, as in (5). Capitalists can either save by buying government bonds A_B or purchasing equity in domestic firms A_E, so their asset holdings are $A = A_B + A_E$.

Since capitalists are Ricardian consumers, the timing of lump-sum transfers is immaterial and only their present value matters. Any lump-sum transfers that the government might make to capitalists over and above the restitution of revenues from the asset holding tax can thus be represented by a transfer at time zero. Denoting such transfers by T^*, capitalists' flow and present-value budget constraint can be written as

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[3] Alternatives are to have an endogenous discount rate, convex portfolio adjustment costs or asset markets with a complete menu of state-contingent claims (Schmitt-Grohé and Uribe, 2003), but we do not explore these as a debt-elastic risk premium seems relevant for developing economies. The

(6) $$\dot{A} = rA - C,\ A(0) = A_0 + T^* \text{ or } \int_0^\infty \exp\left(-\int_0^t r(\mathrm{s})\mathrm{ds}\right)C(\mathrm{t})\mathrm{dt} = A_0 + T^*.$$

Substitution of (5) into (6) yields the following consumption function for capitalists:[4]

(7) $$C(0) = \mathrm{P}\left(A_0 + T^*\right) \text{ where } \mathrm{P} \equiv \left(\int_0^\infty \exp\left[-\sigma\rho\mathrm{t} - (1-\sigma)\int_0^t r(\mathrm{s})\mathrm{ds}\right]\mathrm{dt}\right).$$

5.2. Domestic production

Income is taxed at a uniform rate τ. Production Y uses capital ('equipment') K, labour L and now also public infrastructure S. Infrastructure consists not only of seaports, airports, roads and railroads, but also of education, health or any other public investment that boosts the productivity of private production. We assume a Cobb-Douglas production function with constant returns to scale with respect to K and L, i.e. $Y = K^{\alpha} L^{1-\alpha} S^{\gamma}$ where $0 < \gamma < 1-\alpha < 1$. We exclude larger values of γ to rule out endogenous ever-increasing growth. Firms maximise after-tax profits, so the after-tax marginal products of capital and labour equal the user cost of capital (consisting of the domestic interest rate r plus depreciation charge δ_K) and the wage rate, respectively. Setting the labour force equal to unity, the equilibrium level of the capital stock follows from $\alpha(1-\tau) K^{\alpha-1} S^{\gamma} - \delta_K = r$ where r is the domestic interest rate. This, in turn, equals the world interest rate r^* plus the premium determined by the level of the country's foreign debt, according to the schedule (2).

The total capital stock of the economy is $K + S$. Domestic capitalists have assets A, and B now denotes *public* net wealth. The remaining part of the capital stock is owned by foreigners, so foreign debt is $F = K + S - A - B$. Notice that here we assume that private and public liabilities – equity and government bonds – are perfect substitutes. Foreign liabilities thus defined correspond to the excess of public debt over private bond holdings plus net import of capital. The equilibrium level of the capital stock K is thus implicitly defined by

(8) $$\alpha(1-\tau)K^{\alpha-1}S^{\gamma} - \delta_K = r = r^* + \Pi(K + S - A - B),$$

which gives rise to the following relationships:

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alternative of portfolio adjustment costs when asset holdings are different from some long-run level gives qualitatively similar conclusions.

[4] If $\sigma = 1$, capitalists' consumption equals $C = \rho A$. In general, the capitalists' propensity to consume out wealth P depends on the interest rate. If $r(\mathrm{t}) = r$ is a constant for all $t \geq 0$, then we have $\mathrm{P} = \sigma\rho +$

(9) $$K = K\left(\overset{+}{S}, A \overset{+}{+} B, \overset{-}{\tau}\right), \text{ and hence } r = r\left(\overset{+}{S}, A \overset{-}{+} B, \overset{-}{\tau}\right),$$

$$W = W\left(\overset{+}{S}, A \overset{+}{+} B, \overset{-}{\tau}\right) \text{ and } Y = Y\left(\overset{+}{S}, A \overset{+}{+} B, \overset{-}{\tau}\right).$$

Equations (9) are of course a generalisation of the dependence of K, r, W and Y on B which we used in section 4. Appendix 2 gives explicit expressions for the partial derivatives of the relationships in (9). The responses are intuitive. A higher stock of assets owned by capitalists, A, or the government, B, corresponds to lower foreign liabilities and thus pushes down the risk premium and the domestic interest rate. Consequently, capital, wage income and output increase. A higher level of public infrastructure or a lower tax rate boost the after-tax marginal productivity of capital and labour, hence increase the demand for capital and boost output. As a result, the domestic interest rate and wage rate rise. For given $A + B$, a higher public infrastructure also increases foreign liabilities and pushes up the domestic rate of interest. This has offsetting effects on capital use and the wage rate. Other parameters of the model shift these functions; for example, a higher world interest rate pushes up the domestic interest rate and depresses the use of capital by firms and thus domestic production.

5.3. The government budget and dynamics of foreign debt

The government receives tax income τY and windfall revenue N. It spends on public consumption G, infrastructure I_S and transfers to workers T. Public net wealth is B, so government debt is $D = S - B$, i.e. public net wealth, B, is the stock of public assets minus debt. The government borrows and issues debt D at rate of interest r. The government budget constraint is thus given by:

(10) $$\dot{D} = rD + G + I_S + T - \tau Y - N .$$

Noting that accumulation of public infrastructure is given by $\dot{S} = I_S - \delta_S S$ where $\delta_S \geq 0$ is the depreciation rate of public infrastructure, and that $T \equiv Q - W$, the flow and present-value government budget constraints can be written as:

(11) $$\dot{B} = rB + N + \tau Y - G - (r + \delta_S)S - Q + W, \quad B(0) = B_0 - T^* \text{ and}$$

$(1-\sigma)r$. If the intertemporal substitution effect is weaker than the income effect (i.e., $\sigma < 1$), it follows that a higher interest rate boosts the capitalists' marginal propensity to consume out of wealth.

$$\int_0^\infty \exp\left[-\int_0^t r(s)ds\right]\left[G(t)+(r(t)+\delta_S)S(t)+T(t)\right]dt + T^* \le B_0 + \int_0^\infty \exp\left[-\int_0^t r(s)ds\right]\left[\tau(t)Y(t)+N(t)\right]dt.$$

Equation (11) is the natural generalisation of (3′′).

It is important to be precise about initial conditions. We generally take A_0 and B_0 as fixed, although in appendix 1 we briefly discuss the implications of lump-sum transfers between government and capitalists, T^*, in which case it is just the sum $A_0 + B_0$ which is fixed (see budget constraints (6) and (11)). The evolution of the budget constraints of the capitalists and of government are given by differential equations (6) and (11) and these can be combined using $F = K + S - A - B$, to give the dynamics of foreign debt,

$$\text{(12)} \qquad \dot{F} = rF + C + Q + G + I_S + I_K - Y - N, \quad F(0) = S(0) + K(0) - A_0 - B_0 .$$

The non-oil trade deficit (i.e., the excess of public and private spending over production) minus the sum of oil income and interest income on foreign assets increases indebtedness of the nation.[5] The no-Ponzi condition implies that the present discounted value of net exports of goods and services minus oil exports must cover initial foreign liabilities. These liabilities jump on impact if the government borrows for infrastructure or firms import capital.

5.4. Optimal government policy.

The social welfare function is

$$\text{(13)} \qquad U = \int_0^\infty \exp(-\rho t)\left[\frac{Q^{1-1/\sigma} + \theta C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma}\right]dt ,$$

where $\theta \ge 0$ is the relative weight given to utility of capitalists.[6]

The present-value Pontryagin function for this optimal control problem is

$$\text{(14)} \qquad \begin{aligned} H \equiv &\left[Q^{1-1/\sigma} + \theta C^{1-1/\sigma} + \psi G^{1-1/\sigma}\right]\sigma/(\sigma-1) + \eta\left[r(S, A+B, \tau)A - C\right] + \\ &\mu\left[(B-S)r(S, A+B, \tau) + N + W(S, A+B, \tau) + \tau Y(S, A+B, \tau) - Q - G - \delta_S S\right] + \\ &\lambda\sigma C\left[r(S, A+B, \tau) - \tau_A - \rho\right] \end{aligned}$$

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[5] By Walras' law this condition follows from the other equilibrium conditions and budget constraints.
[6] If $M < 1$ is the number of capitalists, a utilitarian social welfare function implies $\theta = M^{1/\sigma}$, but θ could be higher or lower if the power of capitalists in the government is more or less than that of workers.

where η, μ and λ are the present-value co-state variables corresponding to A, B and C, respectively, and the dynamics of these variables are given by (6), (11) and (5), with (9). The government chooses public consumption G, workers' consumption Q (via transfers T, where $Q = W + T$), infrastructure S, the asset holding tax τ_A, and possibly also the income tax τ to maximise, and η, μ and λ to minimise (14). Application of the Maximum Principle gives the following static efficiency conditions:

(15a) $$\psi G^{-1/\sigma} = Q^{-1/\sigma} = \mu,$$

(15b) $$\frac{\partial H}{\partial S} = \left[\mathrm{W}_S + (B-S)\mathrm{r}_S + \tau \mathrm{Y}_S - r - \delta_S \right]\mu + \mathrm{r}_S A\eta + \sigma C \mathrm{r}_S \lambda = 0.$$

(15c) $$\frac{\partial H}{\partial \tau_A} = -\sigma\lambda C = 0 \text{ and}$$

(15d) $$\frac{\partial H}{\partial \tau} = \left[\mathrm{W}_\tau + (B-S)\mathrm{r}_\tau + Y + \tau \mathrm{Y}_\tau \right]\mu + A\mathrm{r}_\tau \eta + \sigma C \mathrm{r}_\tau \lambda = 0,$$

We thus see from (15a) that the marginal utility of public and workers' consumption should equal the marginal value of an extra unit of public infrastructure or one less unit of government debt. We call μ the marginal (utility) cost of public funds. Optimal consumption by workers and public consumption are decreasing functions of the marginal cost of public funds. Equations (15b), (15c) and (15d) correspond to the first-order conditions for public infrastructure, the asset holding tax and the income tax rate, respectively.

The Maximum Principle also yields the following dynamic efficiency conditions:

(16a) $$\rho\eta - \dot{\eta} = \frac{\partial H}{\partial A} = \left[\mathrm{W}_A + (B-S)\mathrm{r}_A + \tau \mathrm{Y}_A \right]\mu + \left(r + \mathrm{r}_A A \right)\eta + \sigma C \mathrm{r}_A \lambda$$

(16b) $$\rho\mu - \dot{\mu} = \frac{\partial H}{\partial B} = \left[\mathrm{r} + \mathrm{W}_A + (B-S)\mathrm{r}_A + \tau \mathrm{Y}_A \right]\mu + \mathrm{r}_A A\eta + \sigma C \mathrm{r}_A \lambda$$

(16c) $$\rho\lambda - \dot{\lambda} = \frac{\partial H}{\partial C} = \theta C^{-1/\sigma} - \eta + \lambda\sigma(r - \tau_A - \rho),$$

together with the transversality conditions

$$\lim_{t\to\infty} \exp(-rt)\eta(t)A(t) = 0, \ \lim_{t\to\infty} \exp(-rt)\mu(t)B(t) = 0 \text{ and } \lim_{t\to\infty} \exp(-rt)\lambda(t)C(t) = 0.$$

We now illustrate and discuss this solution for alternative sets of government instruments

5.5. Case: Public infrastructure, transfer payments, and optimal asset holding subsidy

The first case we analyse is that where the government is optimising public infrastructure, *S*, transfer payments to workers and hence their consumption, *Q*, and also setting an asset holding tax τ_A (recalling that this is rebated to capitalists in a lump-sum manner, so has an incentive but not an income effect). The first-order condition (15c) implies $\lambda(t) = 0$, $\forall t \geq 0$ and hence, from (16c), $\theta C^{-1/\sigma} = \eta$. Using this and equation (15a), the optimal time paths of consumption follow from (16a) and (16b) as:

(17a) $$\dot{C}/(\sigma C) = r - \rho + r_A A + \{W_A + (B-S)r_A\}\left(C/Q\right)^{1/\sigma}/\theta,$$

(17b) $$\dot{Q}/(\sigma Q) = r - \rho + W_A + r_A\left\{B - S + A\theta\left(Q/C\right)^{1/\sigma}\right\}.$$

The asset accumulation equations are as before, (6) and (11), which we reproduce here as:

(17c) $$\dot{A} = rA - C,$$

(17d) $$\dot{B} = r(B-S) + N + W - Q - G - \delta_S S.$$

The first-order condition for the optimal choice of public infrastructure is,

(17e) $$W_S = r + \delta_S - r_S[B - S + A\theta\left(Q/C\right)^{1/\sigma}].$$

Interpretation of this system is as follows. First, the consumption of both workers and capitalists, *Q* and *C*, can jump at the discovery date, while (with no initial lump-sum transfer to capitalists) both private and public asset levels, *A* and *B,* are historically determined.

Second, the long-run equilibrium involves $F = \overline{F}$, $\Pi = \Pi' = 0$ and $r = r^* = \rho$. $\Pi = 0$ implies that $r_A = r_S = W_A = 0$ and $W_S = w\gamma/(S(1 - \alpha))$ (see appendix 2). Stationary values of *S, K* and hence factor prices and income are therefore independent of consumption and asset levels and straightforward to calculate as functions of $r^* = \rho$ only.

Third, the term $\theta\left(Q/C\right)^{1/\sigma}$ that occurs in the equations above captures income distributional concerns. *If* government used an initial lump-sum transfer to capitalists, T^*, then optimality would require $\mu = \eta$ and hence $\theta C^{-1/\sigma} = Q^{-1/\sigma}$ at all dates.[7] Absent this control, the term $\theta(Q/C)^{1/\sigma}$ may differ from unity, being greater than unity if capitalists' assets and hence consumption are relatively low.

Fourth, the capitalists' consumption path given in (17a) is decentralised by the asset holding tax, τ_A. Comparing (17a) with (5), the optimal value of this tax is

$$\tau_A = -\{W_A + (A + B - S)r_A\} + \left\{1 - \theta\left(Q/C\right)^{1/\sigma}\right\}\left[W_A + (B - S)r_A\right]. \tag{18}$$

The first term in curly brackets is greater than zero if $F > \bar{F}$. The intuition is that saving reduces the interest rate paid by the economy on foreign debt and foreign-owned assets, but this is not internalised by individual capitalists. There should therefore be an asset holding subsidy, which reduces capitalists' consumption (and raises its rate of growth). This argument is tempered by income distributional concerns (the remaining terms in (18)), if marginal utilities of consumption for workers are capitalists are not aligned.

Finally, we illustrate the solution by means of a simulation example. Figure 5 is analogous to figure 4.1, illustrating the dynamic adjustment path to a *'small'* temporary anticipated flow of windfall revenue. As before, solid lines illustrate the case without the windfall, and dashed lines with. Initial values of A and B are set such that the economy's assets are just half their steady state value, and the government does not use T^* to equalise marginal utilities of consumption. Windfall revenues are 10% of long run income accruing between years 16 and 40.

The effect of a temporary windfall is given by the dashed lines, and can be thought of in three phases. At the date of announcement, the government increases borrowing, and public sector debt rises during the period between announcement and commencement of extraction. The primary reason is that it is optimal to raise workers' consumption through a sharp reduction in taxation, although there is also a small initial increase in public sector investment. Since most of government borrowing goes to finance consumption, its net asset position, B, deteriorates. Some of this is financed by foreign debt, F, being higher than it otherwise would have been. As a consequence, interest rates are higher and private and public capital accumulation lower, making wages and income lower than they otherwise would have been. In short, the optimal response is to borrow to finance consumption, even though this depresses investment and non-oil income.

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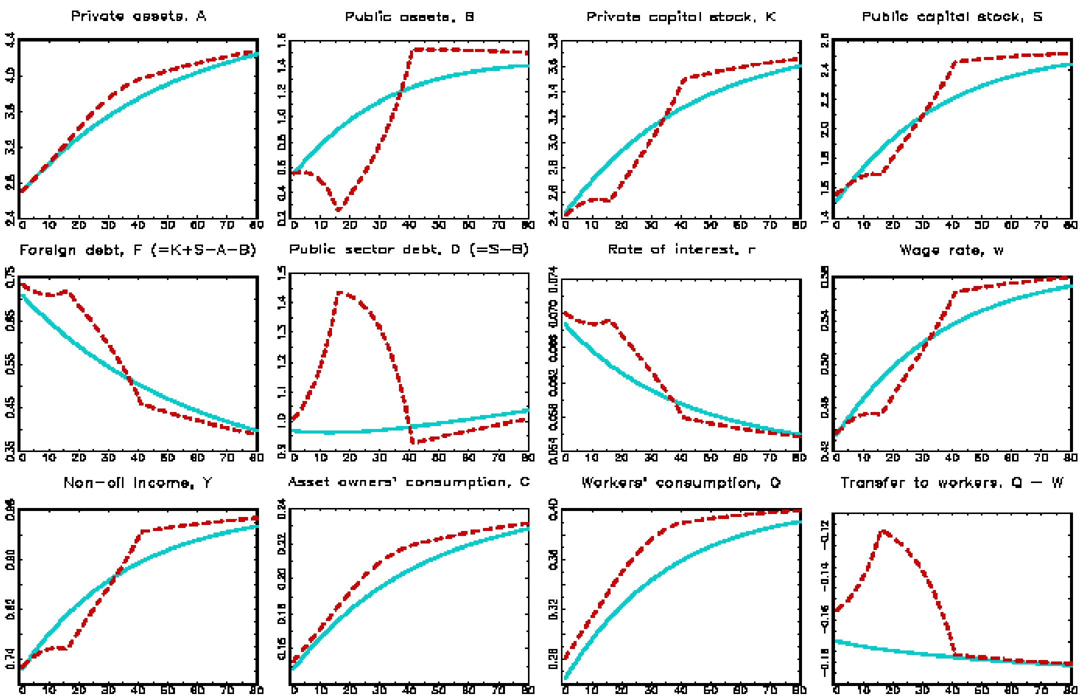
[7] By differentiating (14) with respect to dA, dB with $dA + dB = 0$. From (7), the initial capital transfer to capitalists is given by $T^* = (\theta^{\sigma} / \mathrm{P})Q(0) - A_0$, which increases directly with the optimal initial level of capitalists' consumption and decreases one-for-one with the initial capitalists' asset holdings.

The second phase is the period during which revenue is being received. Debt, both domestic and foreign, is paid off rapidly and as a consequence the interest rate falls. The government also increases infrastructure spending rapidly and this, combined with the falling interest rate, leads to a rapid increase in the private capital stock, income and wages. Notice that rising wages mean that consumption rises smoothly even though the transfer to workers falls sharply, this financing the debt reduction and infrastructure spending. The third phase is when resource revenues come to an end. In this 'small' case no SWF is built up (as in figure 4.1) and the economy resumes its asymptotic convergence to steady state, but with the growth trajectory brought forwards.

Figure 5: A small temporary anticipated windfall in general model with public infrastructure and private asset accumulation



When the windfall is 'large' the first two phases are qualitatively similar, but foreign debt attains level $\overline{F}$ during the period of revenue flow. At this point the domestic interest rate becomes equal to the world rate and the rate of time preference. From this date on consumption and the non-oil production side of the economy are stationary. Production variables (capital stocks, non-oil income and wages) and the consumption and assets of the capitalists have reached their steady state values. These are the same values as are attained asymptotically without the resource revenue, but the windfall allows them to be attained much earlier. The consumption of workers is however higher than its non-oil steady state

value. The amount by which it is higher is exactly equal to the interest payments that can be sustained in perpetuity from the sovereign wealth fund that is about to be established.
Once resource revenue flow stop, the economy continues in this stationary state in perpetuity.

5.6. Case: Public infrastructure and transfers without an asset holding subsidy

The optimal capital asset holding subsidy in figure 5 is initially quite small (about –0.029) and then converges to zero in the long run. It does not seem to affect the qualitative nature of our simulations very much. However, an asset holding subsidy to capitalists will not be very popular in many developing countries. Since such a time-varying use of the capital asset holding subsidy/tax is seldom seen in practice, it may be more realistic to suppose that the asset holding subsidy is not available (i.e., $\tau_A = 0$). It is then not generally the case that $\lambda = 0$. In fact, the steady-state value of λ is only zero if the long-run income tax rate is zero and the long-run level of foreign liabilities is zero as in steady state $\overline{F} + \left(\frac{\tau}{1-\tau}\right)K = \sigma C Q^{1/\sigma}\lambda$. If the economy builds up a SWF in the long run, long-run λ is thus negative. Hence, deviations of λ from zero may occur during the transient phase in which case the government has an incentive to re-optimise. Government policy is then time inconsistent. Inability to directly control the rate of change of capitalists' consumption also means that changes in C induced by other policy instruments are of non-zero value.

5.7. Smoothing with no risk premium on foreign debt

The case of no risk premium (i.e., $r = r^*$) seems relevant for countries without substantial foreign debt such as Norway. Flight capital as a policy choice (e.g., formation of a SWF) has been discussed before in a different context (Collier, Hoeffler and Pattillo, 2001; Collier and Gunning, 2005). When we exclude a risk premium on foreign debt in our most general model discussed in this section 5, it is optimal to have no distortionary income tax and a constant national income share of public infrastructure services (appendix 3). The optimal public infrastructure, capital stock, economic activity and wage are independent of the political strength of capitalists and demand factors, increase with the weight of public infrastructure in private production, and decline with the world interest rate and the depreciation rates of infrastructure and equipment. They do not depend on asset holdings or windfall revenue.

The government schedules transfers to workers to fully smooth their consumption. These transfers and workers' consumption are high if the political strength of capitalists θ is low. It is also optimal to smooth public consumption. Public consumption is high if the political priority to public consumption is high and to capitalists is low. The initial transfer to capitalists is high if permanent oil revenues are high, initial government debt or capitalists' assets are low, the political strength of capitalists is high and the weight of public consumption is low.

As in section 2, the government financial surplus must equal *temporary* revenues from abroad; total public spending is determined by *permanent* windfall revenues. The government thus runs a surplus during the windfall and borrows in anticipation of the windfall. Borrowing finances the initial increases in public infrastructure and capital transfers to capitalists. Long-run increases in spending are financed by the interest earned on the foreign assets accumulated in the SWF after the windfall revenue ceases.

With no risk premium, the international capital market is used to fully smooth consumption of credit-rationed workers and spending on public consumption and public infrastructure. The permanent increase in private consumption and public spending is financed by borrowing in the short run and by interest derived from the SWF in the long run. These policy prescriptions differ sharply from the ones highlighted in sections 6.1 and 6.2, which are more relevant for heavily indebted, developing economies.

5.8. Summing up

Other policy combinations have been explored (including those with an initial capital transfer to capitalists), but are not presented here. Simulations indicate a high degree of qualitative – and quantitative – similarity between the various cases with an endogenous risk premium on foreign debt. The main findings therefore appear to be robust. Namely that:

- Public and private consumption should increase ahead of resource revenue flow; this to a lesser extent than is the case under the permanent income hypothesis, but nevertheless leading to an increase in domestic interest rates and consequent reduction in the growth of the capital stock and income.
- During the period of revenue flow there is rapid debt reduction, private investment and public infrastructure investment, this causing wages to rise and making the optimal consumption trajectory consistent with much lower transfers (or higher taxes) on workers.
- This investment brings forward the asymptotic growth trajectory followed by the economy once resource revenues have terminated and – only if the windfall is large relative to the state of development of the economy – may also be associated with a SWF and permanently higher consumption level.

We thus see that windfall revenues are used to bring down foreign debt in heavily indebted countries and then set up a sovereign wealth fund**,** and boost private and public consumption and also private and public investments. The international capital market is used to smooth consumption by funding the long-run increase in consumption out of interest on sovereign funds, but smoothing is not complete as foreign indebtedness first has to be reduced to lower domestic interest rates and boost economic activity.

6. Extensions

6.1. Dutch disease effects

So far, we assumed one single traded consumption good, which is also produced at home. In the Dutch disease literature it is common to allow for a traded and a non-traded consumption good. An oil windfall then typically leads to an appreciation of the real exchange rate – a rise in the relative price of non-traded goods – and thus to a decline of the traded sector and an expansion of the non-traded sector (e.g., Corden and Neary, 1982). It is interesting to investigate how this would affect our insights on what to do with a temporary windfall in oil revenues in a relatively poor, credit-constrained economy.[8] To allow for Dutch disease effects, it helps to postulate a fixed specific factor in the non-traded sector for otherwise interest rates, wage rates, capital intensities and the real exchange rate are all pinned down by the world interest rate and the supply side of the economy (e.g., Heijdra and van der Ploeg, 2008).[9] Fiscal policy can then be called upon to soften the blow to the traded sector and the consequent faltering of the engine of growth. The policy challenge for the government is not only to step in and help credit-constrained workers to smooth consumption, invest in public infrastructure and manage debt and a SWF appropriately, but also to subsidise the traded sector to avoid loss of learning by doing and other growth externalities.

6.2. Habit persistence, irreversible public infrastructure and partisan politics

Section 5 captures many of the issues of what to do with windfall oil revenues in credit-constrained economies. However, the government may be myopic for political reasons or due to competing fractions leading to a common-pool problem (cf., the voracity effect discussed in Tornell and Lane, 1999) in which case it is reasonable to suppose that the rate of pure time preference of the government exceeds the world interest rate, $\rho > r^*$. In that case, the marginal cost of public funds rises over time and the government brings forwards public consumption and spending on public infrastructure, but postpones taxation. Furthermore, many of problems to do with partisan politics are not addressed in our framework. For example, an incumbent government, who is worried about being removed from office by a political rival with preference for a different type of public goods, typically issues too much debt and spends too much on its own preferred type of consumption goods (Alesina and

[8] In Heijdra and van der Ploeg (2008) these questions are pursued in a three-sector, specific-factor economy with overlapping generations, but without credit constrained consumers. This way there is also no Ricardian debt neutrality.

[9] An alternative is to introduce intrasectoral adjustment costs for transferring capital between the traded and non-traded sectors.(Morshed and Turnovsky, 2004).

Tabellini, 1990). Of course, the political distortions may be exacerbated if the incumbent uses the windfall oil revenues to opportunistically pacify the electorate with special subsidies and transfers in an attempt to stay in office. The experience of Chavez in Venezuela indicates that such inefficient populist policies may be sustained for as long as the windfall oil revenues last. In practice, politically political parties often have bigger partisan disagreement about the type of public investment projects. One party may prefer to invest in schools and hospitals while another party in dams and infrastructure. Also, one party may wish to invest in one district and another party in another district. Public investments are much more irreversible than spending on public consumption goods, because costs of scrapping or adjusting public investment projects are often substantial. In that case, the political distortions not only lead to excessive public debt but also to excessive levels of investment by political parties in their own pet projects (Beetsma and van der Ploeg, 2007). Again, these political distortions are likely to be exacerbated in economies with large windfall oil revenues. On the other hand, in such political contexts, governments may find it more attractive to invest in public infrastructure rather than in a SWF because the former is more difficult to be raided by future political rivals than foreign assets held in a SWF (e.g., Collier and Gunning, 2005). Another complication is highlighted by the IMF (e.g., Leigh and Olters, 2006; Olters, 2007). Countries flush with natural resource revenues are likely to become addicted to excessively high levels of public spending (viz. habit persistence) and may find it difficult to wean them of it again once oil revenues dry up.

6.3. Optimal depletion of natural resources

So far, we have analysed how a developing country should adjust its fiscal policy and debt management in response to an exogenous, anticipated, temporary windfall. An important extension is to allow for endogenous depletion of natural resources. The normal guideline for this is the Hotelling rule (Hotelling, 1931) which states that a country should be indifferent between, on the one hand, extracting the oil, selling it and getting a market return on the proceeds, and, on the other hand, keeping the oil in the ground and enjoying the capital gains. In other words, the extraction rate is determined by the condition that the rate of change in the Hotelling rents (i.e., marginal revenue minus marginal extraction costs) should equal the market rate of interest. Such a rule can allow for monopoly power on the word market of oil. But even with a given trend in the world price of oil, increasing marginal extraction costs ensure that the depletion path is well determined. The key challenge is to find out how the Hotelling rule needs to be modified in a second-best world with several distortions, where the government faces a variety of tough public finance dilemmas of the type discussed in section 5. Does it still make sense to save and have a current-account surplus matching the Hotelling rents in a distorted economy? How is the recommendation of a Sovereign Wealth Fund implied by the permanent income hypothesis affected by endogenous depletion of oil?

6.4. Stochastic volatility of future windfall revenue

In practice, countries face stochastic shocks to windfall revenue. This may arise from unpredictability of proven oil reserves or foreign aid. They could also arise from stochastic volatility of future oil prices. There may also be uncertainty about future extraction costs. It is then prudent for a government to accumulate a precautionary buffer to be prepared for adverse future shocks. This implies that the government deviates from smoothing consumption (even when the home and world interest rates are pegged to each other) as the country initially saves more for the buffer. Expected consumption will thus grow over time if adverse shocks do not materialise.

7. Concluding remarks

The traditional advice for economies experiencing an anticipated and temporary windfall in natural resource or aid revenues is based on the permanent income hypothesis. Ahead of the windfall, the country should borrow on the international market to finance a permanent increase in public consumption and a permanent increase in transfers or cut in taxation in order to boost private consumption. During the windfall, the debt is repaid and subsequently assets are accumulated in a sovereign wealth fund. After the windfall, the interest on the SWF pays for the permanent increase in public spending and private consumption.

These lessons should be modified for those developing economies whose citizens are unable to smooth consumption themselves and which are struggling with high domestic interest rates due to the premium they have to pay on large levels of foreign debt. In that case, the traditional advice based on the permanent income hypothesis is no longer valid. If the windfall is relatively small and foreign indebtedness is substantial, the government uses the windfall to bring down foreign debt and interest rates more quickly and there are no funds to build a SWF. As a result, it is optimal for consumption to gradually rise ahead, during and after the windfall rather than to be smoothed towards the steady it was converging to before the windfall. With a large windfall and relatively low levels of foreign indebtedness, consumption overshoots and it does pay to build up a SWF. In that case, the interest on the SWF permits bigger long-run increase in consumption than would have been possible without the windfall.

More generally, a government of a developing economy faces tough tradeoffs of whether to use the windfall for debt reduction, boosting private or public consumption, and investing in public infrastructure. We find that ahead of a small windfall, interest rates are higher and private and public accumulation lower than they otherwise would have been.

During the windfall the government pays of debt so that interest rates fall and the public and private capital stocks rise to levels greater than they have would been without the windfall. After the windfall, there is no SWF and the economy converges to the equilibrium it was converging to before the windfall. Although a small windfall does not raise steady-state levels of consumption, consumption does rise at all other times. With a large windfall, the main difference is that it is optimal to build up a SWF and therefore it is possible to have long-run levels of private and public consumption higher than would have been possible in the absence of a windfall. A substantial SWF arises if the windfall is large and lasts for a long time and if foreign indebtedness is not too large. The optimal time-path of citizen dividends is hump-shaped as the boost to non-oil income, stimulated by the fall in interest rates, helps to smooth consumption after the windfall has ceased.

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Appendix 1: Multiple shooting algorithm and parameters

The parameters for figures 4.1, 4.2 and 5 have been set to the following values: $\alpha = 0.4$, $\rho = r^* = 0.04$, $\sigma = 0.8$, $\psi = 0$, $\delta_K = 0.05$, $\Pi = 0.15\rho(F_+)^2$, $Y = K^\alpha\ [(\rho+\delta_K)/\alpha]^\alpha$ where the constant part scales long-run output to unity. Other parameters are described in the text. The parameters for figure r are $\alpha = 0.4$, $\gamma = 0.25$, $\rho = r^* = 0.05$, $\sigma = 0.75$, $\psi = 0$, $\delta_K = \delta_S = 0.05$, $\Pi = 0.75\rho(F_+)^2$, θ= 0.58, $Y = K^{\alpha}\ (\rho+\delta_K)^{\alpha+\gamma}\ \alpha^{-\alpha}\gamma^{-\gamma}$. Other parameters as described in the text.

We describe the algorithm for the case discussed in section 5.6 (figure 5), where the asset holding tax is set optimally and there is no initial capital transfer to capitalists ($T^* = 0$). In that case, the state-space system is given by (17a)-(17d) for the state variables $\{C, Q, A, B\}$. The saddlepath condition requires two eigenvalues with a negative real part and two with a positive real part. The system can be solved given initial conditions $A(0) = A_0$ and $B(0) = B_0$. The initial values $C(0)$ and $Q(0)$ are free to jump and are found with the aid of a multiple shooting algorithm:

1. Calculate steady-state values $C(\infty)$and $Q(\infty)$ and choose a terminal horizon t*.
2. Guess initial values for the non-predetermined variables $C(0)$ and $Q(0)$.
3. Use these initial values together with $A(0)=A_0$ and $B(0)= B_0$ to simulate the system of differential equations (17a)-(17d) from time zero to the terminal horizon, whereby at each point of time the instantaneous solutions for W, r, K, and Y conditional on the state variables $\{C, Q, A, B\}$ are found from (9) while S follows from (17e).
4. If distance between $\{C(\infty), Q(\infty)\}$ and $\{C(t^*), Q(t^*)\}$ is above some tolerance value, use a Newton-Raphson iteration to improved initial guess and go back to step 3.

The initial value of t* is chosen to be small, and then increased in steps (going back to 3 but using as starting guesses the values computed for the previous value of t*. This multiple shooting algorithm has been implemented with the computer package GAUSS. Simulations reported in the paper have $t^* = 150$.

With the asset holding tax not chosen optimally as in section 5.7, λ is no longer zero at all points of time and we thus have to solve the system (5), (6), (11) and (16a)-(16b) for the state variables $\{A, B, C, \eta, \mu, \lambda\}$. The condition for saddlepath stability requires two eigenvalues with a negative real part and four with a positive real part. With the initial transfer to capitalists T^* not set optimally, the initial conditions are $A(0) = A_0$ and $B(0) = B_0$. And one needs to shoot to find $C(0)$, $\eta(0)$, $\mu(0)$ and $\lambda(0)$. With T^* set optimally as in appendix 3, $\lambda(0) = 0$. In that case, one needs to shoot to find $C(0)$, $\eta(0)$, $\mu(0)$ and T^* with the initial conditions $A(0) = A_0 + T^*$ and $B(0) = B_0 - T^*$ and $\lambda(0) = 0$.

Appendix 2: Comparative statics of the production sector

The instantaneous value of K is found by equating the marginal product of capital, net of depreciation, to the domestic interest rate. If technology is Cobb-Douglas, then $Y = L^{1-\alpha} K^{\alpha} S^{\gamma}$ and (with $L = 1$) K is implicitly defined by:

$$r = (1-\tau)\alpha K^{\alpha-1}S^{\gamma} - \delta_K = \rho + \pi(K+S-A-B).$$

We express this relationship as $K(S, A+B, \tau)$ and the associated equilibrium values of the domestic interest rate and wage rate as $r(S, A+B, \tau)$ and $w(S, A+B, \tau)$, respectively. We note the following comparative static responses:

$$\frac{dK}{dA} = \left[\frac{\pi' K}{(1-\alpha)(r+\delta_K)+\pi' K}\right],\ \frac{dK}{dS} = \left[\frac{-\pi' K + \gamma(r+\delta_K)K/S}{(1-\alpha)(r+\delta_K)+\pi' K}\right],\ \frac{dK}{d\tau} = \left[\frac{-K(r+\delta_K)/(1-\tau)}{(1-\alpha)(r+\delta_K)+\pi' K}\right]$$

$$r_A = \left[\frac{-\pi'(1-\alpha)(r+\delta_K)}{(1-\alpha)(r+\delta_K)+\pi' K}\right],\ r_S = \frac{\pi'(r+\delta_K)(1-\alpha+\gamma K/S)}{(1-\alpha)(r+\delta_K)+\pi' K},\ r_\tau = \left[\frac{-\pi' K(r+\delta_K)/(1-\tau)}{(1-\alpha)(r+\delta_K)+\pi' K}\right]$$

$$w_A = \left[\frac{\pi'\alpha w}{(1-\alpha)(r+\delta_K)+\pi' K}\right],\ w_S = \frac{w}{S}\left[\frac{\pi'(\gamma K-\alpha S)+\gamma(r+\delta_K)}{(1-\alpha)(r+\delta_K)+\pi' K}\right] \text{ and}$$

$$w_\tau = \frac{-w}{(1-\tau)}\left[\frac{\pi' K + r + \delta_K}{(1-\alpha)(r+\delta_K)+\pi' K}\right].$$

Since $Y = w/(1 - \alpha)$, corresponding expressions for income follow directly.

Appendix 3: No international risk premium

We use $W_\tau + Y = 0$ to rewrite the optimality conditions (15b) and (15d) as:

$$\tau\text{=0 and } \mathrm{W}_{\mathrm{S}}^{*} = r^* + \delta_S \text{ or } \frac{S}{Y} = \frac{\gamma}{r^*+\delta_K}.$$

The optimal public infrastructure, capital stock, economic activity and wage are given by:

$$S = \frac{\gamma Y}{r^*+\delta_S} = \left[\left(\frac{\gamma}{r^*+\delta_S}\right)^{1-\alpha}\left(\frac{\alpha}{r^*+\delta_K}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\gamma}},\ K = \left(\frac{\alpha S^{\gamma}}{r^*+\delta_K}\right)^{\frac{1}{1-\alpha}},\ Y = \left(\frac{\alpha}{r^*+\delta_K}\right)^{\frac{\alpha}{1-\alpha}} S^{\frac{\gamma}{1-\alpha}} \text{ and}$$

$W = (1-\alpha)Y$. Since the wage and the interest rate do not depend on A, (16b) implies that

$\dot{\mu} = 0$ and thus (15a) indicates that it is optimal to completely smooth consumption by workers and government. From (1) it is also optimal to smooth capitalists' consumption, so

$$\dot{C} = \dot{Q} = \dot{G} = 0 \text{ and thus } C = \theta^{\sigma} Q = \left(\frac{\theta}{\psi}\right)^{\sigma} G = r^*(A_0 + T^*),$$

where $C = \theta^{\sigma} Q$ will definitely hold if the asset holding tax is set optimally. Transfers to workers follow from $T = Q - W$. With no international risk premium, optimal government policy is time consistent with $r=r^*$ even without an optimal asset holding tax or subsidy. The government financial surplus equals:

$$\dot{F} = \dot{D} = N^P - N \text{ and } r^*(T^* - B_0) + G + (r^* + \delta_S)S + T = N^P.$$

Upon substitution of G , S and $T = Q - W$, the last can be solved for the initial government transfer of assets to capitalists:

$$T^* = \mathrm{T}^*\left(\overset{+}{N_P}, \overset{-}{D_0}, \overset{-}{A_0}, r^*, \overset{+}{\theta}, \overset{-}{\gamma}\right).$$