A Traffic Equilibrium Model with Paid and Unpaid Parking

Thomas F. Rutherford and Renger van Nieuwkoop
Center for Energy Policy and Economics
Swiss Federal Institute of Technology

Motivation

One has only to observe a flustered driver, desperately trying to park his car when there is no parking to be found, in order to see that we do not always act in accordance with the rationality principle.

Karl Popper

Downtown traffic congestion is a problem in most big cities around the world. Cruising for parking can play a big role in the amount of congestion. According to Shoup (2011) almost 30% of the cars on downtown streets are cruising for parking. Cruising can cause additional congestion, waste of fuel, air pollution and waste of time. Governments try to reduce the negative effects of cruising by technical measures or by policy measures like road tolls or parking fees. These policy instruments can be evaluated using a special kind of parking model.

Model Description

We use a traffic flow model in which agents interact on a road network and search for a parking slot. Households can differ both with respect to their origin and valuation of their time. Intersections (nodes in the network) have paid and curb (unpaid) parking facilities with fixed capacity. Parking slots at paid locations are allocated through fees, which equilibrate supply and demand. Search intensity influences the random distribution of curbside parking slots to agents choosing to look. Decisions about where to park reflect trade-offs between time (driving, search and walking) and the money (parking fees or road tolls). The model is formulated as a mathematical program with complementarity constraints, with, at the upper level, a nonlinear programming problem, minimizing total user cost and, at the lower level, the user equilibrium problem formulated as a nonlinear complementarity problem. The model is used to find the optimal design of a toll or parking fee taking into account the individual optimization of travel time. The model can be solved using readily available non-linear solvers. There is no need for heuristic algorithms and complete enumeration of all possible paths in the network.

Mathematical Description

We **minimize the total time costs** on the network:

$$OBJ^C = \sum_{h,i} v_h \left[w_i Z_{hi} + \sum_{a(j,i)} \tau_{ji} (X_{hji} + Y_{hji})(\phi + S_{hi} + \pi_{hi} w_i) Y_{hji} \right]$$

where v_h is the value of the time for household h, w the expected time spent walking to the CBD, Z agents entering paid parking, τ_{ij} the time for traveling from node i to j, X agents deciding to drive to the next node without searching, Y agents searching, π the probability of finding a parking place, and S, the search intensity.

The **conservation of flow condition** states that the difference between the agents traveling to a node and the agents leaving this node should be equal to the number of agents with this node as destination:

$$\sum_{a(j,i)} X_{hji} - \sum_{a(i,j)} X_{hij} + Z_{hj} + \sum_{a(j,i)} Y_{hji} - \sum_{a(i,j)} [1 - \pi_{hj}] Y_{hij} - d_{hj} = 0 \perp T_{hj}$$

where T_{hj} is the expected time in continuation should no parking place be found.

Arbitrage conditions: either drive to the adjacent node without searching for curbside parking:

$\tau_{ij} + T_{hj} + \frac{t_{ij}}{v_h} \geq T_{hi} \quad \perp \quad X_{hij} \geq 0$ or pay the fee and take a paid parking spot:

$$w_i + \frac{P_i}{v_h} \geq T_{hi} \quad \perp \quad Z_{hi} \geq 0$$

or drive to and search for curbside parking at adjacent node:

$$\tau_{ij} + \frac{t_{ij}}{v_h} + S_{hj} + \phi + \pi_{hj} \left[w_j + \frac{f_j}{v_h} \right] + (1 - \pi_{hj}) T_{hj} \geq T_{hi} \quad \perp \quad Y_{hij} \geq 0$$

Supply equals demand for paid parking:

$$(1 - \theta_j) \bar{K}_j = \sum_h Z_{hj} \perp \quad p_j \geq 0$$

where θ_i is the fraction of curbside parking and $\bar{K}_j$ is the total parking available.

The **probability of finding a parking place** is $\pi_{hj} = 1 - \mu_j (S_{hj} + \phi)^{-\gamma}$ where μ_j is the shadow price multiplier on curbside parking and ϕ the minimal searching time.

Supply equals demand for curbside parking:

$$\theta_j \bar{K}_j = \sum_h \left[\pi_{hj} \sum_{i \in a(i,j)} Y_{hij} \right] \quad \perp \mu_j$$

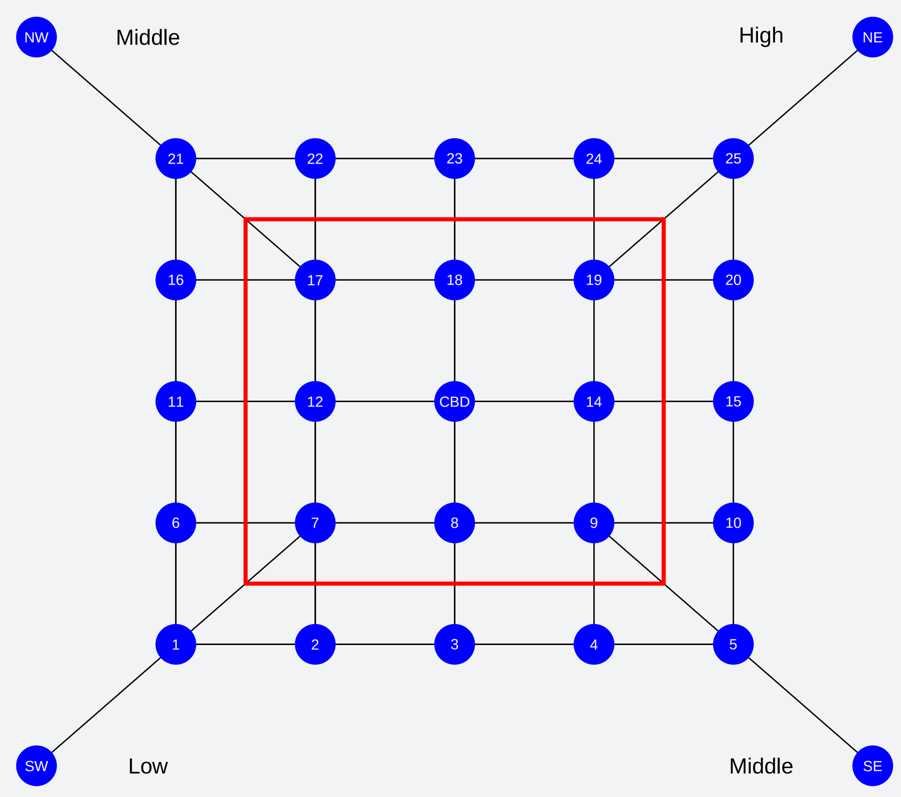
$$1 - \left(T_{h,j} - w_j - \frac{f_j}{v_h} \right) \gamma \mu_j (S_{h,j} + \phi)^{-1-\gamma} > 0 \quad \perp \quad S_{hj}$$

Aggregate arc flow and the **travel time** on the arcs are given by:

$F_{a(i,j)} = \sum_h (X_{h,i,j} + Y_{h,i,j})$ and $\tau_a = \alpha_a (1 + \beta_a F_a^4)$

where α_a is the uncongested travel time and β the congestion coefficient.

Numerical Example



Preliminary Results

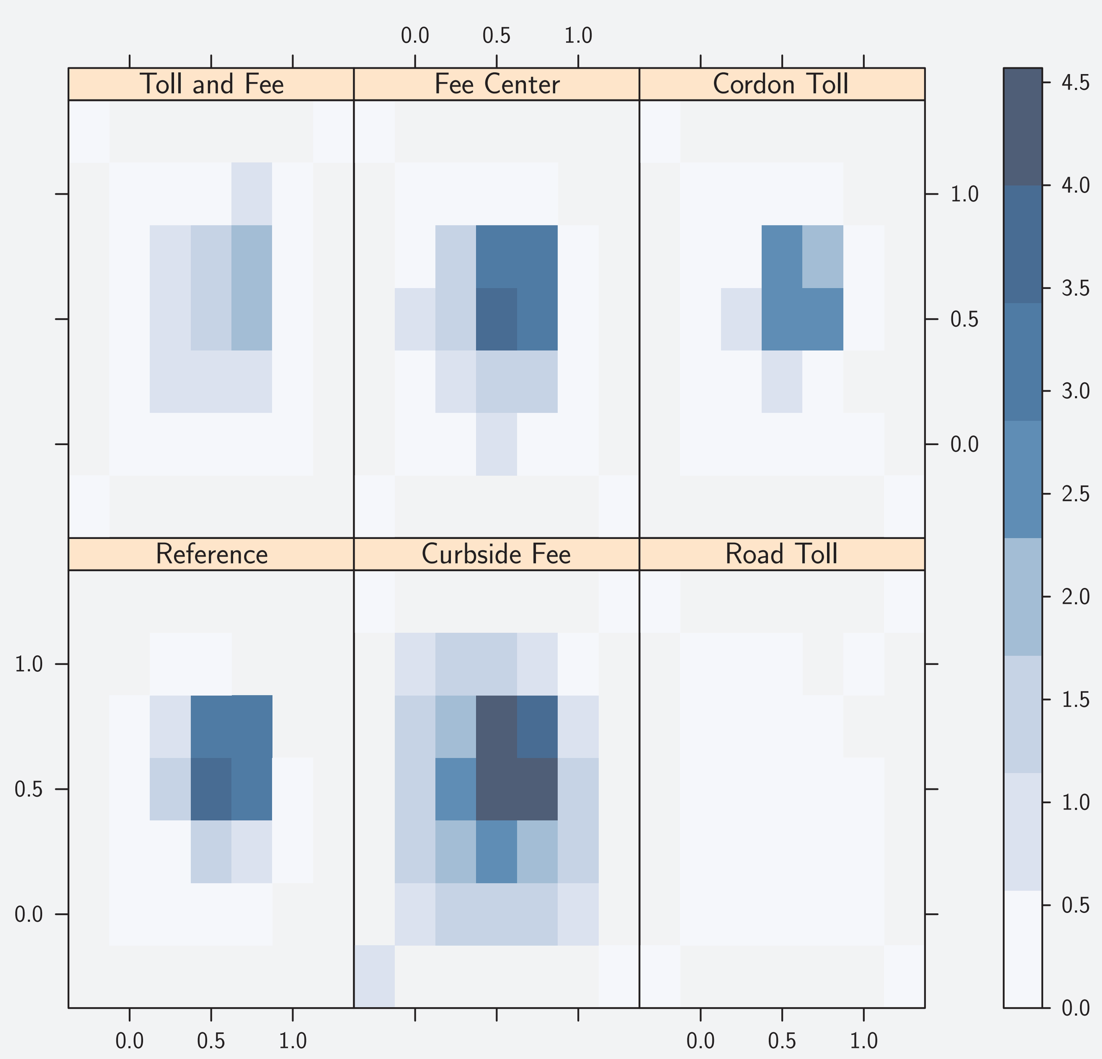
Table: Changes in congestion (%) and fraction paid parking (% points) for homogeneous agents.

Parameter	Curbside Fee	Road Toll	Toll and Fee	Fee Center	Cordon Toll
Objective	-11.17	-11.22	-11.22	-6.69	-2.04
Total Driving Time	-13.43	-13.52	-13.50	-15.56	2.46
Curbside Parking	-0.00	-0.00	-0.00	10.40	-0.00
Time Driving	-13.43	-13.52	-13.52	-15.57	2.46
Time Walking	6.66	6.66	6.66	17.19	2.28
Time Searching	-45.43	-45.43	-45.43	-40.06	-42.80

Table: Changes in congestion (%) and fraction paid parking (% points) for heterogeneous agents.

Parameter	Curbside Fee	Road Toll	Toll and Fee	Fee Center	Cordon Toll
Objective	-13.66	-17.27	-17.30	-4.52	-1.01
Total Driving Time	-18.70	-23.96	-23.71	-12.45	-6.17
Curbside Parking	0.53	-0.80	-0.80	2.51	0.34
Time Driving	-18.71	-24.01	-23.73	-12.46	-6.18
Time Walking	12.51	11.96	11.96	14.41	5.10
Time Searching	-46.85	-46.85	-46.85	-25.40	-23.11

Figure: Parkhouse Price for Heterogeneous Households



Conclusions

- With homogeneous agents, introducing a curbside fee, a road toll or a combination have the same impact.
- With heterogeneous agents the combination of curbside fee and road toll is the most efficient policy.
- A cordon toll is less efficient than any other policy.