Lecture #2: Split Hopkinson Bar Systems

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Uniaxial Compression Testing

- Useful experiment to characterize the flow behavior of materials at large strains

![Diagram of uniaxial compression testing setup]
Uniaxial Compression Testing

- Axial strain definitions

\[ \varepsilon_{\text{eng}} = \frac{l}{L} - 1 \] (engineering or nominal strain)

\[ \varepsilon = \ln[1 + \varepsilon_{\text{eng}}] \] (true or logarithmic strain)

- Axial stress definitions

\[ \sigma_{\text{eng}} = \frac{F}{A_0} \] (engineering or nominal stress)

\[ \sigma = \frac{F}{A} = \sigma_{\text{eng}} [1 + \varepsilon_{\text{eng}}] \] (true stress)

Only valid for incompressible materials

\[ \sigma > 0 \] (tension)
\[ \sigma < 0 \] (compression)
Uniaxial Compression Testing

\[ F_c = -F \]

Force-displacement curve

Engineering stress-strain curve

True stress-strain curve

Applied force \( F \) [kN] vs. Change in length \( l - L_0 \) [mm]

Engineering stress [MPa] vs. Engineering strain [-]

True stress [MPa] vs. Logarithmic strain [-]
Uniaxial Compression Testing

- Numerical Simulation (→ also topic of computer lab #2)

Source: https://www.youtube.com/watch?feature=player_detailpage&v=OzzWc-SfF-w
Uniaxial Compression Testing

- Experimental challenges
  - Plastic buckling
    - H/D too large
  - Shear buckling
    - Poor alignment
    - Anisotropy
  - Barreling
    - Friction too high
    - H/D too small
  - Optimal
    - Excellent lubrication
    - H/D ~ 1.5

Recommended size for metals testing:

\[
\text{H} = 15 \text{mm} \\
\text{D} = 10 \text{mm}
\]
Strain rate in a compression experiment

![Diagram of a compression experiment showing undeformed and deformed states with strain and displacement](image)

- Nominal strain rate
  \[ \varepsilon_{eng} = \frac{l}{L} - 1 \quad \Rightarrow \quad \dot{\varepsilon}_{eng} = \frac{\dot{l}}{L} = \frac{\dot{u}}{L} \]

- True strain rate
  \[ \varepsilon = \ln[1 + \varepsilon_{eng}] \quad \Rightarrow \quad \dot{\varepsilon} = \frac{\dot{\varepsilon}_{eng}}{1 + \varepsilon_{eng}} \]

- Eq. str. strain rate
  \[ \dot{\varepsilon} = \frac{|\dot{\varepsilon}_{eng}|}{1 + \varepsilon_{eng}} \]

- \( u > 0 \) (extension)
- \( u < 0 \) (shortening)
Principle of Quasi-static Equilibrium

Compression of a cylindrical specimen

Quasi-static equilibrium

\[ F_{\text{in}}(t) \approx F_{\text{out}}(t) \]
Strain at the end of the experiment:
Average strain rate (over time):

$$T = \frac{\varepsilon_{\text{max}}}{\dot{\varepsilon}_{\text{av}}}$$

Duration of experiment

Note: $T$ is independent of specimen dimensions!

Example:

$$\varepsilon_{\text{max}} = 0.15$$
$$\dot{\varepsilon}_{\text{av}} = 500 / s$$

$$T = \frac{0.15}{500} = 0.0003 \, s = 0.3 \, ms = 300 \mu s$$
Time scale #2

Wave propagation speed: \[ c = \sqrt{\frac{E}{\rho}} \]

Specimen length: \( L \)

\[ \Delta t = \frac{L}{c} \]

Note: \( \Delta t \) does depend on specimen dimensions!

Example:

\[ c \approx 5000 m/s \]
\[ L = 10 mm \]

\[ \Delta t = \frac{10}{5 \times 10^6} = 2 \times 10^{-6} s = 2 \mu s \]
Principle of Quasi-static Equilibrium

Long time scale:

Duration of experiment

\[ T = \frac{\varepsilon_{\text{max}}}{\dot{\varepsilon}_{\text{av}}} \]

Short time scale:

Wave travel time

\[ \Delta t = \frac{L}{c} \]

Condition for quasi-static equilibrium:
(when testing an elasto-plastic material)

\[ F_{\text{in}}(t) \approx F_{\text{out}}(t) \iff \Delta t \ll T \]
Principle of Quasi-static Equilibrium

Condition of quasi-static equilibrium

\[ \Delta t \ll T \quad \iff \quad \frac{L}{c} \ll \frac{\dot{\varepsilon}_{\text{max}}}{\dot{\varepsilon}_{\text{av}}} \]

Example for challenging experiments (w/ regards to quasi-static equilibrium)

- Brittle materials, e.g. ceramics \( \varepsilon_{\text{max}} \sim 0.01 \)
- Soft materials, e.g. polymers \( c \sim 1000 m/s \)
- Materials of coarse microstructure, e.g. metallic foams \( L \gg 1 mm \)
Dynamic testing of foams

Example for challenging experiments (w/ regards to quasi-static equilibrium)

Zhao et al. (1997)
LIMITATION OF UNIVERSAL TESTING MACHINES

- Vibration issues

\[ \Delta t_2 \approx 200 \mu s \]
\[ \Delta t_1 \approx 20 \mu s \]
\[ \Delta t = 2 \mu s \]

Condition of validity

\[ T_{\text{exp}} = \frac{\varepsilon_{\text{max}}}{\dot{\varepsilon}_{\text{av}}} >> \max[\Delta t_i] \]

breaks often down at around 10/s
LIMITATION OF UNIVERSAL TESTING MACHINES

Ringing of conventional load cell

![Diagram of MTS Frame, Load Cell, Alignment Frame, Compressive Grip, Specimen, Upper Quartz Transducer, Aluminum Disks, Lower Quartz Transducer, MTS Actuator, MTS Frame.]

![Graph showing force and displacement over time with lines for force from load cell, force from upper force transducer, force from lower force transducer, and displacement from LVDT.]
SEPARATION OF TIME SCALES

Characteristic time scale of testing system $\Delta t_{sys}$ versus Duration of experiment $T_{exp} = \frac{\varepsilon_{\text{max}}}{\dot{\varepsilon}_{av}}$

Universal testing machines $\Delta t_{sys} \ll T_{exp}$

SHPB $\Delta t_{sys} \geq T_{exp}$

Universal testing machines

\[ \dot{\varepsilon} \left[ s^{-1} \right] \]
Wave Equation

- Derived wave equation for bars under the assumption of uniaxial stress

\[ c^2 \left( \frac{\partial^2 u}{\partial x^2} \right) - \frac{\partial^2 u}{\partial t^2} = 0 \]

- Longitudinal elastic wave velocity: \( c = \sqrt{\frac{E}{\rho}} \)

- The particle velocity: \( v[x, t] = \dot{u}[x, t] = \frac{\partial u}{\partial t} \)
General Solution

- General solution of wave equation

\[ \frac{1}{c^2} \left( \frac{\partial^2 u}{\partial x^2} \right) - \frac{\partial^2 u}{\partial t^2} = 0 \]

- displacement:

\[ u[x, t] = f[x + ct] + g[x - ct] \]

- strain:

\[ \varepsilon[x, t] = u'[x, t] = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial x} = \varepsilon'[x, t] + \varepsilon''[x, t] \]

- particle velocity:

\[ v[x, t] = \dot{u}[x, t] = c \frac{\partial f}{\partial x} - c \frac{\partial g}{\partial x} = c(\varepsilon'[x, t] - \varepsilon''[x, t]) \]
Elastic Wave Velocities

- Particle velocity depends on applied loading, while the wave velocity is an intrinsic material property

<table>
<thead>
<tr>
<th>Material</th>
<th>Density [g/cm³]</th>
<th>Young’s modulus [GPa]</th>
<th>Wave velocity [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td></td>
<td></td>
<td>340</td>
</tr>
<tr>
<td>Steel</td>
<td>7.8</td>
<td>210</td>
<td>5188</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>70</td>
<td>5091</td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.7</td>
<td>44</td>
<td>5087</td>
</tr>
<tr>
<td>Tungsten</td>
<td>19.3</td>
<td>400</td>
<td>4552</td>
</tr>
<tr>
<td>Lead</td>
<td>10.2</td>
<td>14</td>
<td>1171</td>
</tr>
<tr>
<td>PMMA</td>
<td>1.2</td>
<td>3</td>
<td>1581</td>
</tr>
<tr>
<td>Concrete</td>
<td>3</td>
<td>30</td>
<td>3162</td>
</tr>
</tbody>
</table>

All values are rough estimates and may vary depending on the exact material composition and environmental conditions.
Hopkinson Bar Experiment

Typical system characteristics:

- Striker bar length: \( L_s = 1000 \text{ mm} \)
- Input bar length: \( L_i = 3000 \text{ mm} \)
- Output bar length: \( L_o = 3000 \text{ mm} \)
- Bar diameter: \( D = 20 \text{ mm} \)

Specimen characteristics (for simplified theoretical analysis):

- Ideal plastic, constant force
Hopkinson Bar Experiment

rightward

leftward

total

151-0735: Dynamic behavior of materials and structures
Hopkinson Bar Experiment

rightward

leftward

total
Hopkinson Bar Experiment

rightward

leftward

total
Hopkinson Bar Experiment

rightward

leftward

total

151-0735: Dynamic behavior of materials and structures
Hopkinson Bar Experiment

rightward

leftward

total

151-0735: Dynamic behavior of materials and structures
Hopkinson Bar Experiment

rightward

leftward

total
Hopkinson Bar Experiment

rightward

leftward

total

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Hopkinson Bar Experiment
Hopkinson Bar Experiment
Hopkinson Bar Experiment

- Rightward
- Leftward
- Total

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rightward

leftward

total

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Hopkinson Bar Experiment

rightward

leftward

total

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HOPKINSON BAR TECHNIQUE

Measuring force with a slender bar

\[ \varepsilon_A(t) = \varepsilon_R(x_A,t) + \varepsilon_L(x_A,t) \]

\[ \varepsilon_A(t) = \frac{2(L-x_A)}{c} \]

\[ \varepsilon_R(x_A,t) \]

\[ \varepsilon_L(x_A,t) \]

\[ F(t) \]

\[ x_A \]

\[ L \]
HOPKINSON BAR TECHNIQUE

Measuring force with a slender bar

Force vs. Time

Duration of valid measurement

$F(t)$

$\varepsilon_A(t)$

$\varepsilon_R(x,t)$

$\varepsilon_B(t)$

$\varepsilon_L(x,t)$

$x_A$

$x_B$
HOPKINSON BAR TECHNIQUE

Direct impact experiment:

One force measurement (output bar only)

\[ u(t) \rightarrow \text{specimen} \rightarrow \text{output bar} \]

\[ \varepsilon_{out}(t) \]

\[ v_0 \rightarrow \text{specimen} \rightarrow \text{output bar} \]

\[ \varepsilon_{out}(t) \]

\[ L_{out} \]

Strategy: Experiment ends before leftward traveling wave in output bar reaches strain gage

Length of output bar

\[ L_{out} > \frac{T}{2c_{out}} \]

duration of the experiment

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HOPKINSON BAR TECHNIQUE

Split Hopkinson Pressure Bar (SHPB) experiment

Two force measurements

Two force measurements

\[ \varepsilon_L(t) \]

\[ \varepsilon_R(t) \]

\[ \varepsilon(0,t) \]

\[ \varepsilon(L_{in}/2,t) \]

\[ \varepsilon(L_{in},t) \]

\[ L_{st} \]

\[ L_{in} \]

\[ v_0 \]

\[ \text{striker} \]

\[ \text{input bar} \]

\[ \text{specimen} \]

\[ \text{output bar} \]
HOPKINSON BAR TECHNIQUE

Criterion to avoid wave superposition at strain gage position

\[ 3L_{in} / 2c > L_{in} / 2c + 2L_{st} / c \]

\[ L_{in} > 2L_{st} \]
SPLIT HOPKINSON PRESSURE BAR (SHPB) SYSTEM

Kolsky (1949)

Input force measurement:

\[ F_{in} = EA(\varepsilon_{re} + \varepsilon_{inc}) \]

\[ \varepsilon_{inc}(t) = \varepsilon_{R}^{in}(t - L_{in} / 2c) \]

\[ \varepsilon_{re}(t) = \varepsilon_{L}(t + L_{in} / 2c) \]

Strain

Time

\( \varepsilon_{L}^{in}(t) \)

\( \varepsilon_{R}^{in}(t) \)

wave "transport"

\( v_{0} \rightarrow \)

striker

input bar

specimen

output bar

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SPLIT HOPKINSON PRESSURE BAR (SHPB) SYSTEM

Calculate forces and velocities at specimen boundaries

Input bar/specimen interface:

\[ \varepsilon_R^{in}(t) \rightarrow \varepsilon_{inc}(t) \rightarrow F_{in} = EA(\varepsilon_{re} + \varepsilon_{inc}) \]
\[ \varepsilon_L^{in}(t) \rightarrow \varepsilon_{re}(t) \rightarrow v_{in} = c(\varepsilon_{re} - \varepsilon_{inc}) \]

Output bar/specimen interface:

\[ \varepsilon_R^{out}(t) \rightarrow \varepsilon_{tra}(t) \rightarrow F_{out} = EA \varepsilon_{tra} \]
\[ v_{out} = -c \varepsilon_{tra} \]

Specimen specific post-processing

Verify quasi-static equilibrium
Coupling with other measurements (e.g. high speed photography)
Calculate stress, strain and strain rate
DESIGN OF A STEEL SHPB SYSTEM

Duration of the experiment: 

\[ T = \frac{\varepsilon_{\text{max}}}{\dot{\varepsilon}_{\text{av}}} = \frac{0.5}{1000} = 500\,\mu s \]

Minimum output bar length: 

\[ L_{\text{out}} = Tc / 2 = 0.0005\,s \times 5000\,m / s / 2 = 1.25m \]

Minimum striker bar length: 

\[ L_{\text{st}} = Tc / 2 = 1.25m \]

Minimum input bar length: 

\[ L_{\text{in}} = 2L_{\text{st}} = 2.5m \]

The bar diameters need to be chosen in accordance with the forces required to deform the specimen (force associated with incident wave should be much higher than the specimen resistance)
EXAMPLE EXPERIMENT

![Graphs showing strain as a function of time, and engineering stress as a function of engineering strain.]

- **Graph 1**: Strain as a function of time for the incident bar.
- **Graph 2**: Strain as a function of time for the transmitter bar.
- **Graph 3**: Engineering stress vs. engineering strain for different strain rates.

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**Diagram:**

- **Striker**
- **Input Bar**
- **Specimen**
- **Output Bar**

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Kolsky bar system

Requirements: • Striker, input and output bar made from the same bar stock (i.e. same material, same diameter)
  • Length of input and output bars identical
  • Striker bar length less than half the input bar length
  • Strain gages positioned at the center of the input and output bars
Kolsky bar formulas

• Stress in specimen:

\[ \sigma_s(t) = \frac{EA}{A_s} \varepsilon_{tra}(t) \]

• Strain rate in specimen:

\[ \dot{\varepsilon}_s(t) = -\frac{2c}{l_s} \varepsilon_{re}(t) \]

Launching system

\[ \frac{L}{2} \quad \frac{L}{2} \quad \frac{L}{2} \quad \frac{L}{2} \]

- striker bar
- input bar
- specimen
- output bar
- strain gage
- strain gage
Wave Dispersion Effects

1D THEORY

\[ \frac{v}{2c} \]

time

compressive strain

EXPERIMENT

“Pochhammer-Chree” oscillations

long rise time

time

compressive strain

\[ \varepsilon(t) \] recorded by strain gage
Wave Dispersion Effects

**Simplified model:** axial compression only:

**Reality:** axial compression & radial expansion:

In reality, the wave propagation in a bar is a 3D problem and lateral inertia effects come into play due to the Poisson’s effect!

Inertia forces along the radial direction delay the radial expansion upon axial compression.

\[ D \quad \Delta x \quad (1 - \nu \varepsilon)D \]

\[ (1 + \varepsilon)\Delta x \]
Geometric Wave Dispersion

- Consider a rightward traveling sinusoidal wave train of wave length $\Lambda$ in an infinite bar of radius $a$ raveling at wave speed $c_\Lambda$

  $\varepsilon[x,t^*]$

  $D$

  $\Lambda$

- The wave propagation speed depends on wave length!
- The 1D theory only true for very long wave lengths (or very thin bars)
- High frequency waves propagate more slowly than low frequency waves

- Theoretical wave speed (1D analysis): $c = \sqrt{E/\rho}$
- Theoretical wave speed (3D analysis):

  $c_\Lambda / c$

  1.0

  0.5

  0.0

  0.5

  1.0

  $D / \Lambda$

Pochhammer-Chree 3D analysis

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Geometric Wave Dispersion

- Example: Rightward propagating wave in a steel bar

\[ \Lambda_1 = 40\text{mm} \quad D/\Lambda_1 = 0.5 \quad c_1 \approx 3.1\text{km/s} \quad f_1 \approx 78\text{kHz} \quad \Delta T_1 \approx 642\mu\text{s} \]

\[ \Lambda_2 = 200\text{mm} \quad D/\Lambda_2 = 0.1 \quad c_2 \approx 4.9\text{km/s} \quad f_2 \approx 25\text{kHz} \quad \Delta T_2 \approx 406\mu\text{s} \]
Geometric Wave Dispersion

- Example

\[ \varepsilon(t) \] recorded by strain gage

Chen & Song (2010)
Modern Hopkinson Bar Systems

Features:
• Striker and input typically made from the same bar stock (i.e. same material, same diameter)
• Small diameter output bar for accurate force measurement
• Similar length of all bars
• Output bar strain gages positioned near specimen end
• Wave propagation modeled with dispersion
• Strains are measured directly on specimen surface using Digital Image Correlation (DIC)
ADVANCED TOPICS related to SHPB technique

• Accurate wave transport taking geometric wave dispersion into account
• Use of visco-elastic bars (slower wave propagation than in metallic bars, more sensitive for soft materials)
• Torsion and tension Hopkinson bar systems
• Lateral inertia at the specimen level
• Friction at the bar/specimen interfaces
• Dynamic testing of materials (where quasi-static equilibrium cannot be achieved)
• Pulse shaping
• Intermediate strain rate testing
• Infrared temperature measurements
• Experiments to characterize brittle fracture
• Multi-axial ductile fracture experiments
• Experiments under lateral confinement

… and many others.
Reading Materials for Lecture #2

