ETH zürich

Autonomous Mobile Robots

Exercise 3: Line fitting and extraction for robot localization

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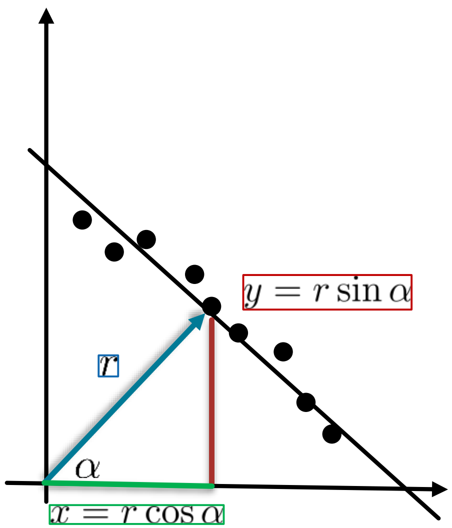
Line in polar parameters

$$x \cos \alpha + y \sin \alpha = r$$



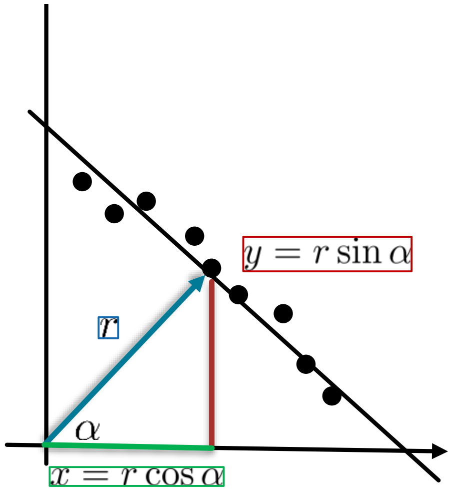
$$x^2 + y^2 = r^2 \quad (1)$$

With $x = r \cos \alpha$ and $y = r \sin \alpha$:

$$x^2 + y^2 = r^2 \quad (2)$$

$$x \cos \alpha + y \sin \alpha = r \quad (3)$$

Line fitting / Line regression



$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2$$

Solution of (r,α): $\nabla S = 0$, i.e.

$$\frac{\partial S}{\partial \alpha} = 0$$
$$\frac{\partial S}{\partial r} = 0$$

Task:

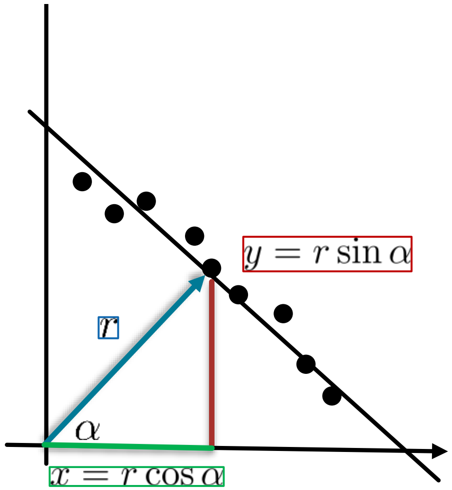
- Derive α.

You will need the following identities:

$$\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \quad (1)$$
$$\cos^2(\alpha) - \sin^2(\alpha) = 2\cos(\alpha) \quad (2)$$

Line fitting / Line regression

$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2 \quad (1)$$



$$\frac{\partial S(r,\alpha)}{\partial r} = 2\sum_i (r - x^i \cos\alpha - y^i \sin\alpha) = 0 \quad (1)$$

$$Nr - \cos\alpha \sum_i x^i - \sin\alpha \sum_i y^i = 0 \quad (2)$$

$$r = \cos\alpha \frac{\sum_i x^i}{N} + \sin\alpha \frac{\sum_i y^i}{N} \quad (3)$$

$$= x_c \cos\alpha + y_c \sin\alpha \quad (4)$$

That is, the line passes through the centroid (for a squared cost function).

Line fitting / Line regression

$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2 \quad (1)$$

$$\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \quad (2)$$

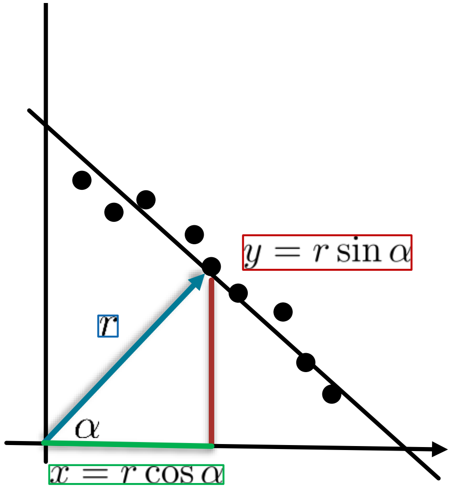
$$\cos^2(\alpha) - \sin^2(\alpha) = 2\cos(\alpha) \quad (3)$$

$$r = x_c \cos\alpha + y_c \sin\alpha \quad (4)$$

$$\frac{\partial S(r,\alpha)}{\partial \alpha} = \frac{\partial}{\partial \alpha} \sum_i (x_c \cos\alpha + y_c \sin\alpha - x^i \cos\alpha - y^i \sin\alpha)^2 \quad (1)$$

$$= \frac{\partial}{\partial \alpha} \sum_i (\cos\alpha(x_c - x^i) + \sin\alpha(y_c - y^i))^2 \quad (2)$$

$$= 2\sum_i (\tilde{x}\cos\alpha + \tilde{y}\sin\alpha)(-\tilde{x}\sin\alpha + \tilde{y}\cos\alpha) \quad (3)$$

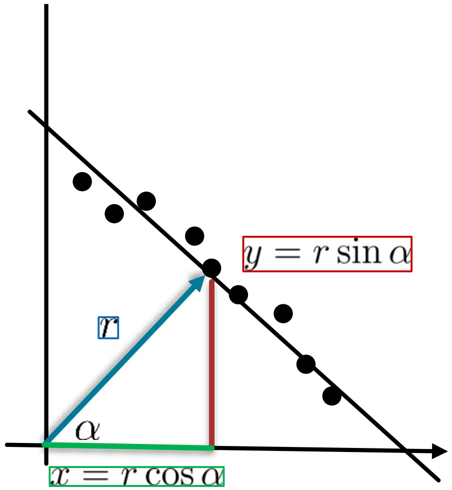


Line fitting / Line regression

$$\frac{\partial S(r,\alpha)}{\partial \alpha} = \frac{\partial}{\partial \alpha}\sum_i (x_c \cos\alpha + y_c \sin\alpha - x^i \cos\alpha - y^i \sin\alpha)^2 \quad (1)$$

$$= \frac{\partial}{\partial \alpha}\sum_i (\cos\alpha(x_c - x^i) + \sin\alpha(y_c - y^i))^2 \quad (2)$$

$$= 2\sum_i (\tilde{x}\cos\alpha + \tilde{y}\sin\alpha)(-\tilde{x}\sin\alpha + \tilde{y}\cos\alpha) \quad (3)$$



$$-\cos\alpha\sin\alpha\sum\tilde{x}^2 + \cos^2\alpha\sum\tilde{x}\tilde{y} - sin^2\alpha\sum\tilde{x}\tilde{y} + \sin\alpha\cos\alpha\sum\tilde{y}^2 = 0$$

$$\sin\alpha\cos\alpha\sum(\tilde{y}^2 - \tilde{x}^2) + (\cos^2\alpha - \sin^2\alpha)\sum\tilde{x}\tilde{y} = 0$$

$$\sin(2\alpha)\sum(\tilde{y}^2 - \tilde{x}^2) + 2\cos(2\alpha)\sum\tilde{x}\tilde{y} = 0$$

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = -\frac{2\sum\tilde{x}\tilde{y}}{\sum(\tilde{y}^2 - \tilde{x}^2)}$$

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{-2\sum\tilde{x}\tilde{y}}{\sum(\tilde{y}^2 - \tilde{x}^2)}\right)$$