Perception III:
Fundamentals of Image Processing (incl. image features)

Autonomous Mobile Robots

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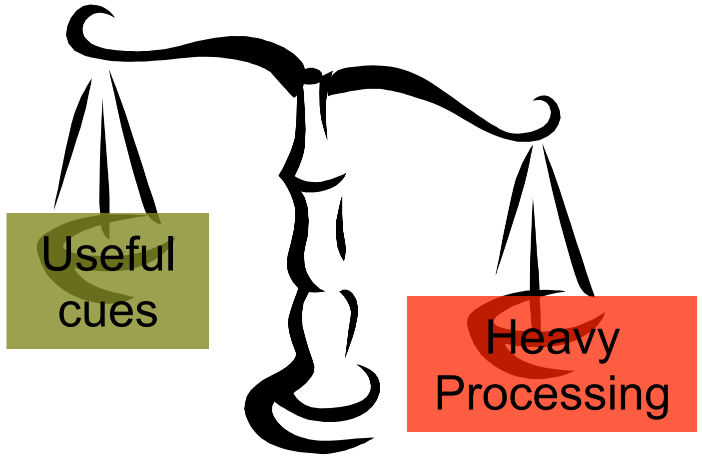
Image Intensities & Data Reduction

- Monochrome image ➪ matrix of intensity values
- Typical sizes:
  - 320 x 240 (QVGA)
  - 640 x 480 (VGA)
  - 1280 x 720 (HD)
- Intensities sampled to 256 grey levels ➪ 8 bits
- Images capture a lot of information



What is **useful**, what is **redundant**?

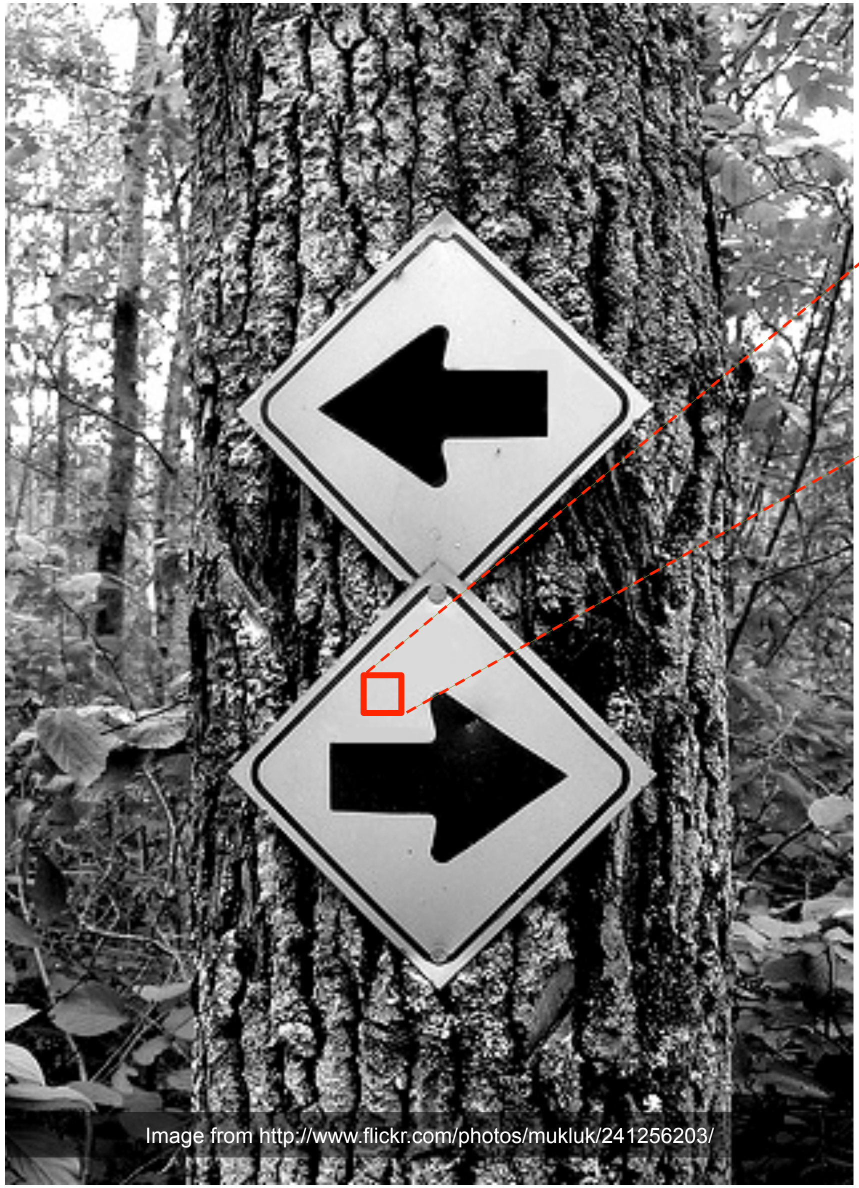
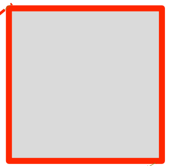


Image from http://www.flickr.com/photos/mukluk/241256203/



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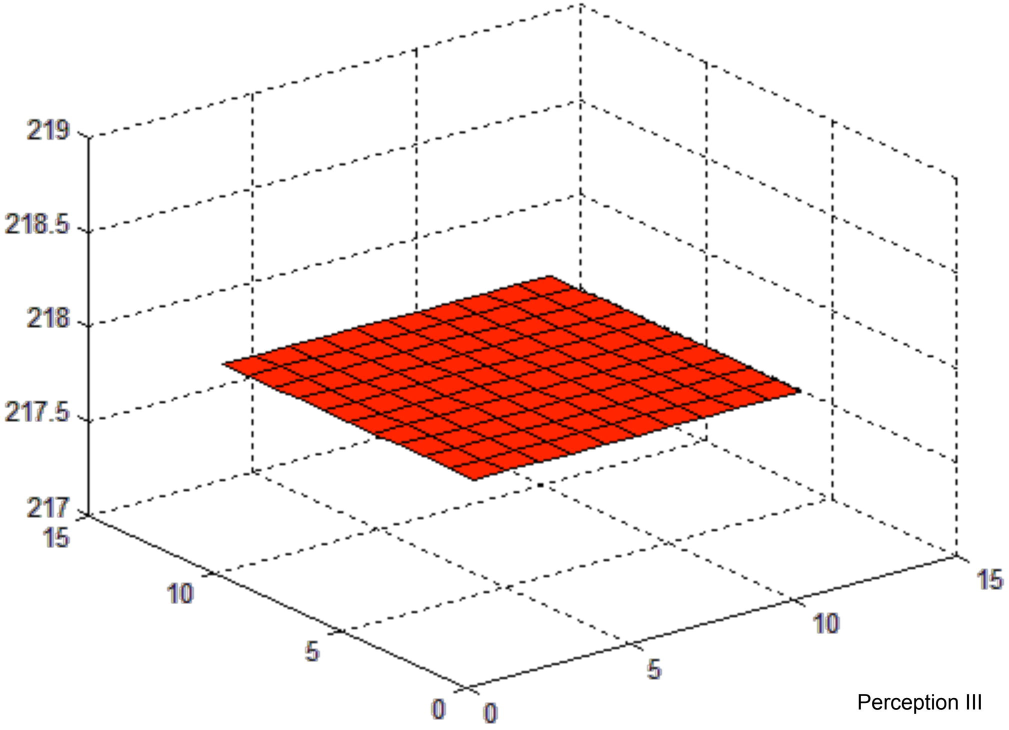

What is **useful**, what is **redundant**?

Image from http://www.flickr.com/photos/mukluk/241256203/

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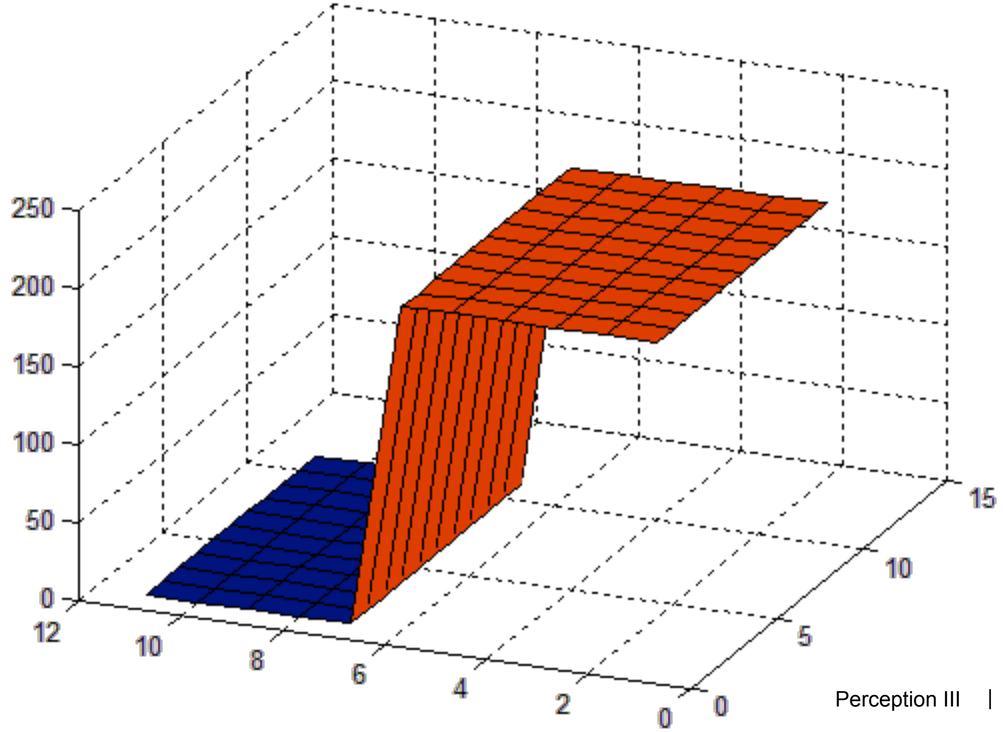

What is **useful**, what is **redundant**?

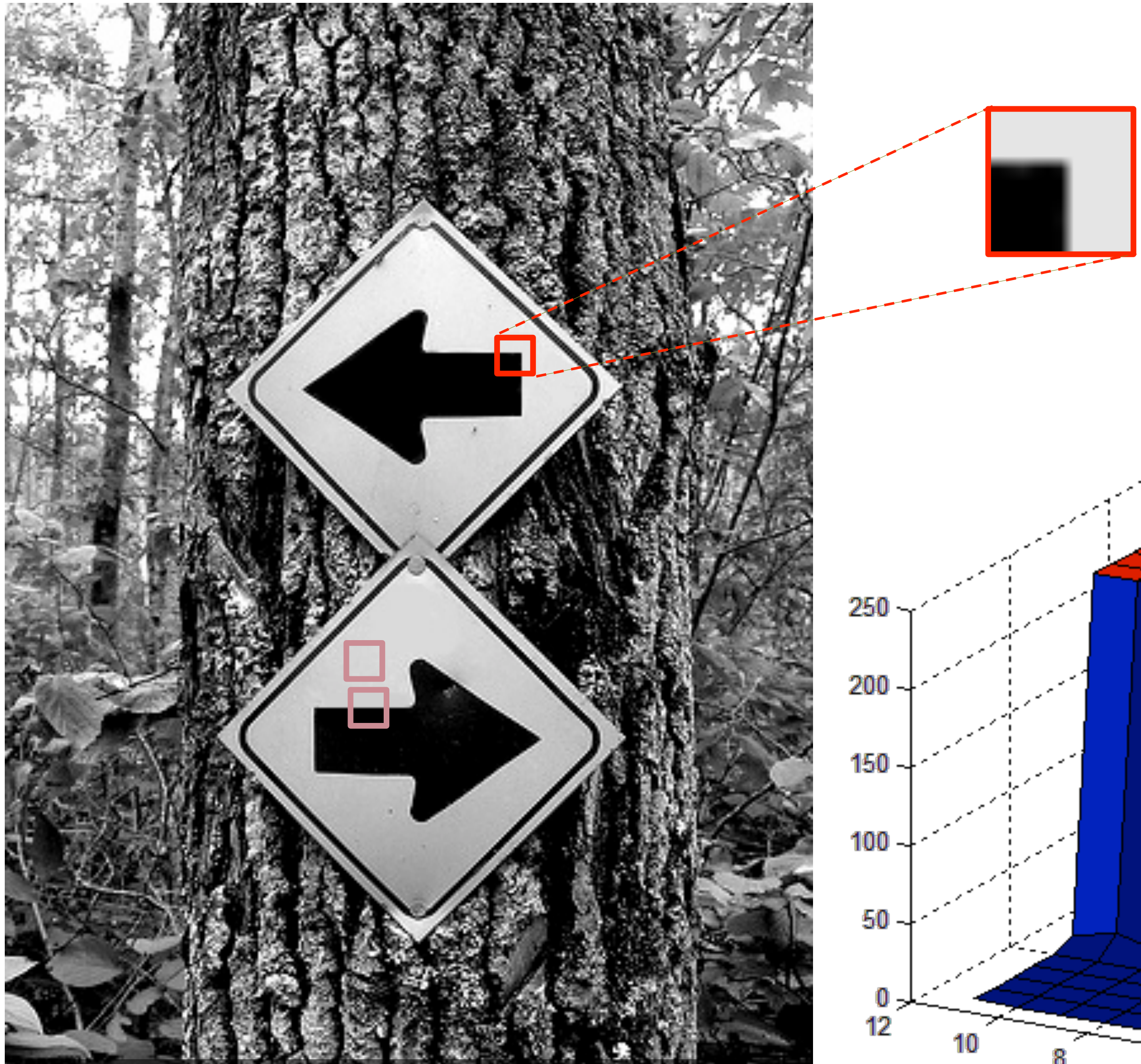

Image from http://www.flickr.com/photos/mukluk/241256203/

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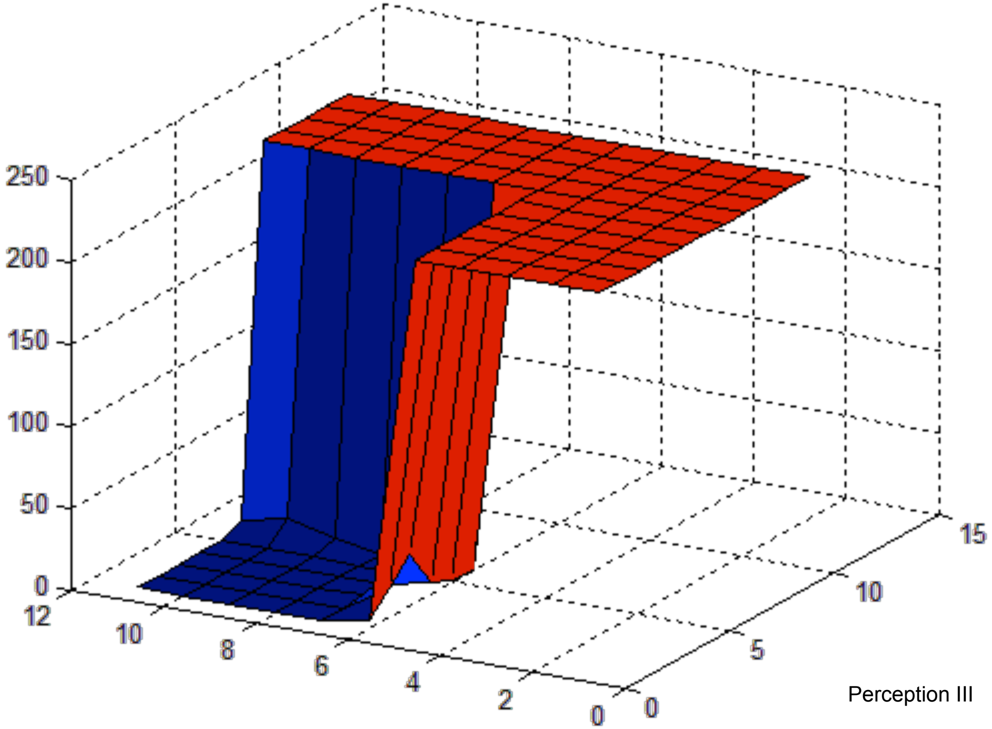


Today's Outline

- Sections **4.3 – 4.5** of the book
- Image filtering
  - Correlation
  - Convolution
- Edge / Corner extraction
- Point Features
  - Harris corners
  - SIFT features
  - + some more recent image features from the state of the art

Optional Reading:

- Harris Corner Detector: *C. Harris and M. Stephens. "A combined corner and edge detector." Alvey vision conference, 1988.* [paper]
- Shi-Tomasi features*: J. Shi and C. Tomasi. "Good features to track." IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 1994.* [paper]
- SIFT features: *D. G. Lowe. "Distinctive image features from scale-invariant keypoints."International Journal of Computer Vision (IJCV), 2004.* [paper][demo code]
- FAST corner detector: *E. Rosten, R. Porter, and T. Drummond. "Faster and better: A machine learning approach to corner detection."IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI), 2010.* [paper]
- BRIEF descriptor: *M. Calonder, V. Lepetit, C. Strecha, P. Fua. "Brief: Binary robust independent elementary features." European Conference on Computer Vision (ECCV), 2010.* [paper]
- BRISK features: *S. Leutenegger, M. Chli, and R. Y. Siegwart. "BRISK: Binary robust invariant scalable keypoints." International Conference on Computer Vision (ICCV), 2011.*[paper]
- Open source implementation of some of these methods (and others) in OpenCV.

Image filtering

- **filtering:** accept / reject certain components
- <u>example:</u> a low-pass filter allows low frequencies ⇨ blurring (smoothing) effect on an image – used to reduce image noise
- Smoothing can be achieved not only with **frequency filters**, but also with **spatial filters**.

Low-pass filtering:
retains low-frequency components (smoothing)

High-pass filtering:
retains high-frequency components (edge detection)

Image filtering | spatial filters

- S_{xy} : neighborhood of pixels around the point (x,y) in an image I
- Spatial filtering operates on S_{xy} to generate a new value for the corresponding pixel at output image J

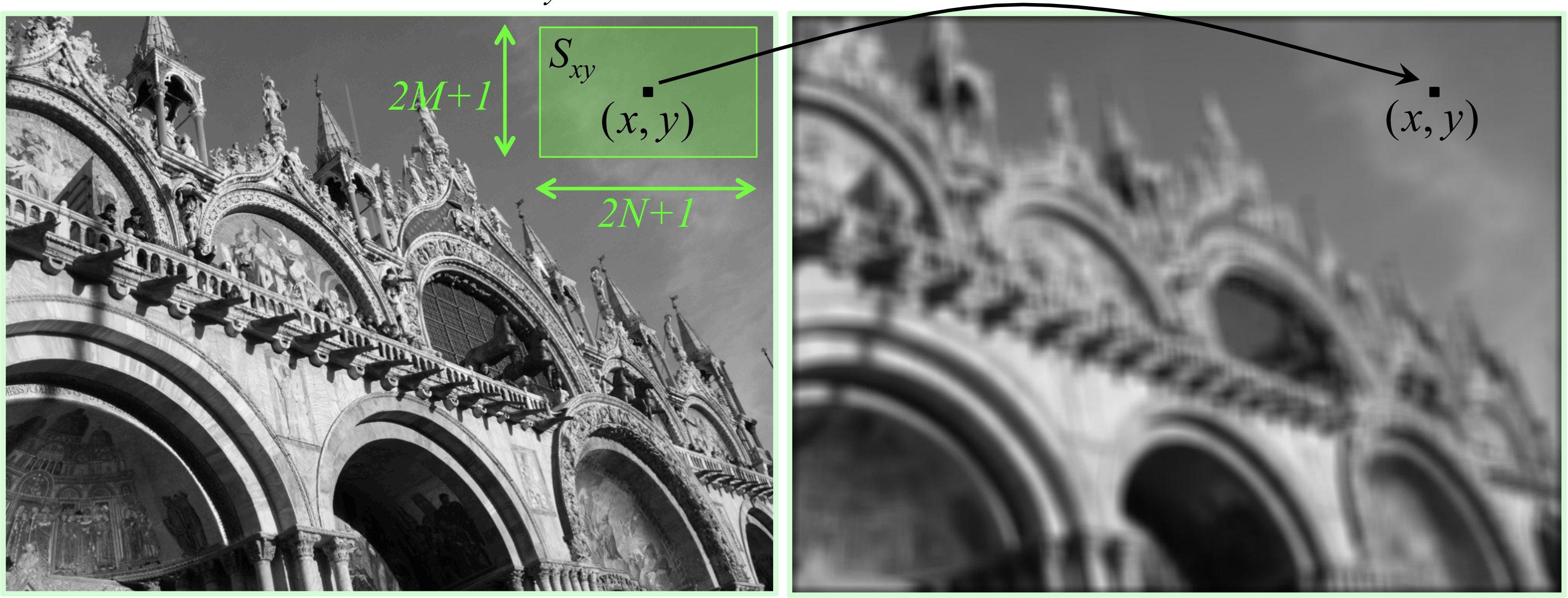


Image I Filtered Image $J = F(I)$

- For example, an averaging filter is: $J(x,y) = \frac{\sum_{(u,v)\in S_{xy}} I(u,v)}{(2M+1)(2N+1)}$

Image filtering | linear, shift-invariant filters

- **Linear:** every pixel is replaced by a linear combination of its neighbours
- **Shift-invariant:** the same operation is performed on every point on the image
- Why filter?
  - Noise reduction, image enhancement, feature extraction, …
- Basic & very useful filtering operations:
  - Correlation
  - Convolution
- Brief study of these filters in the simplest case of 1D images (i.e. a row of pixels) & their extension to 2D

Image filtering | correlation

- An averaging filter

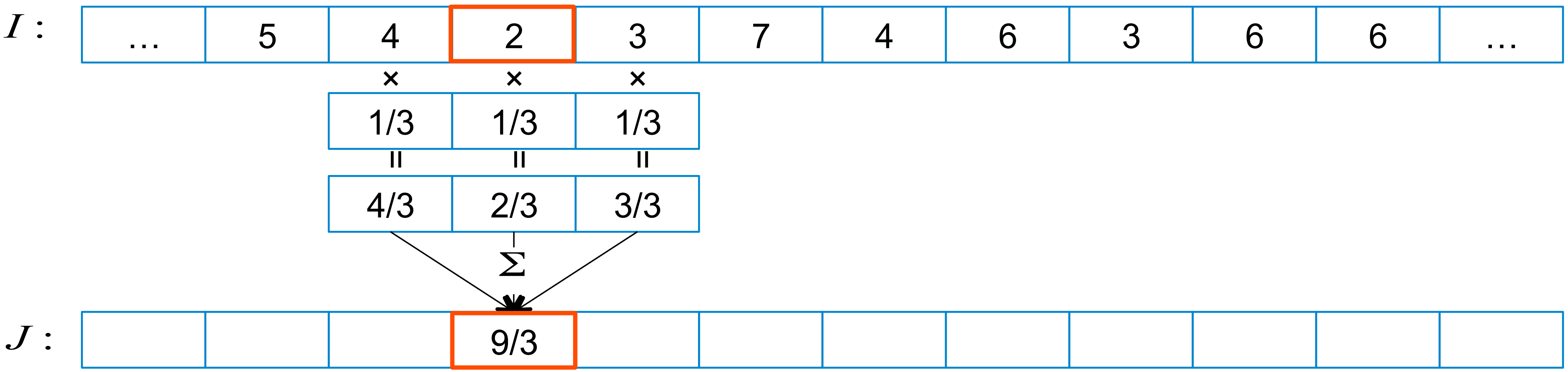
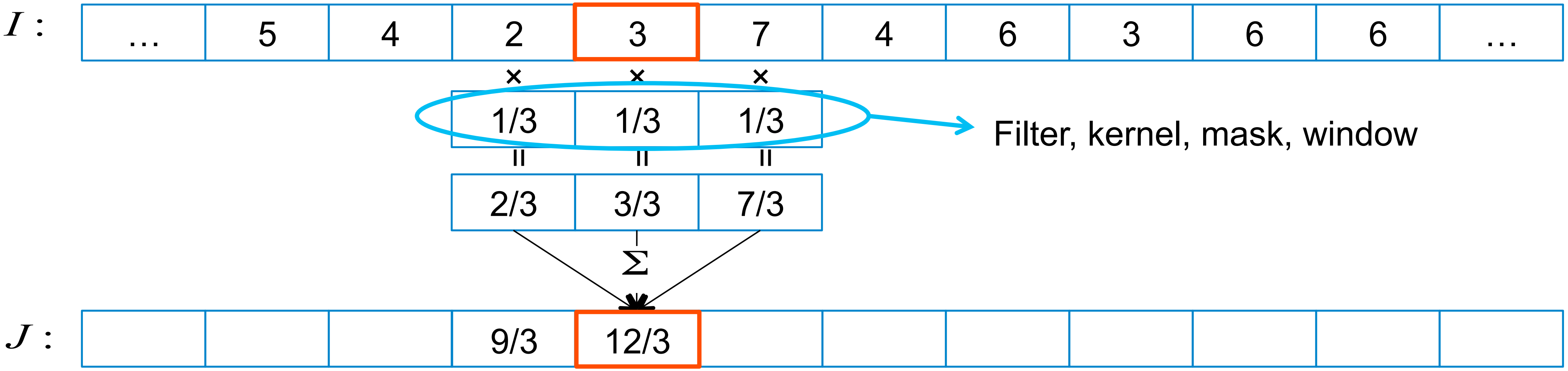


Image filtering | correlation

- An averaging filter

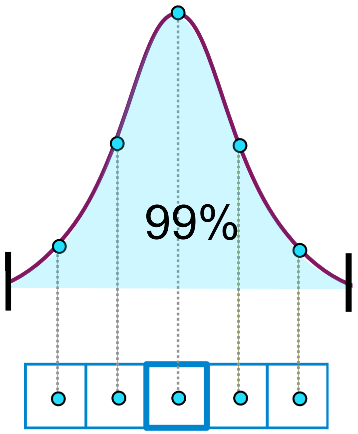


- Formally, Correlation is $J(x) = F \circ I(x) = \sum_{i \in [-N,N]} F(i) I(x+i)$

- In this smoothing example $F(i) = \begin{cases} 1/3, i \in [-1,1] \\ 0 \;\;, i \notin [-1,1] \end{cases}$

Image filtering | constructing filter from a continuous fn

- Common practice for image smoothing:
  use a Gaussian



Normalize filter so that values always add up to 1

$$g(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu = 0$

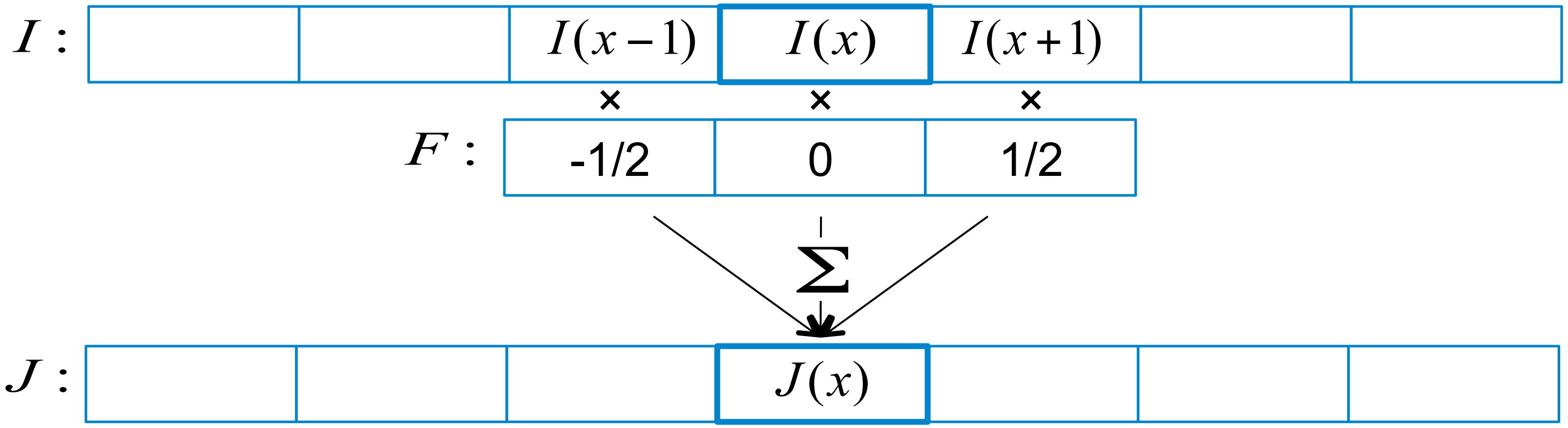
σ : controls the amount of smoothing

- Near-by pixels have a bigger influence on the averaged value rather than more distant ones

Image filtering | taking derivatives with correlation

- **Derivative** of an image:
  quantifies how quickly intensities change
  (along the direction of the derivative)

- Approximate a derivative operator:



$$J(x) = \frac{I(x+1) - I(x-1)}{2}$$

Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**

Correspondence Search | the problem

- **goal**: identify image regions / patches in the left & right images, corresponding to the same scene structure
  - Typical **similarity measures**: Normalized Cross-Correlation (NCC) , Sum of Squared Differences (SSD), Sum of Absolute Differences (SAD), …
  - **Exhaustive** image search can be computationally very expensive! *Can we search for correspondences more efficiently?*

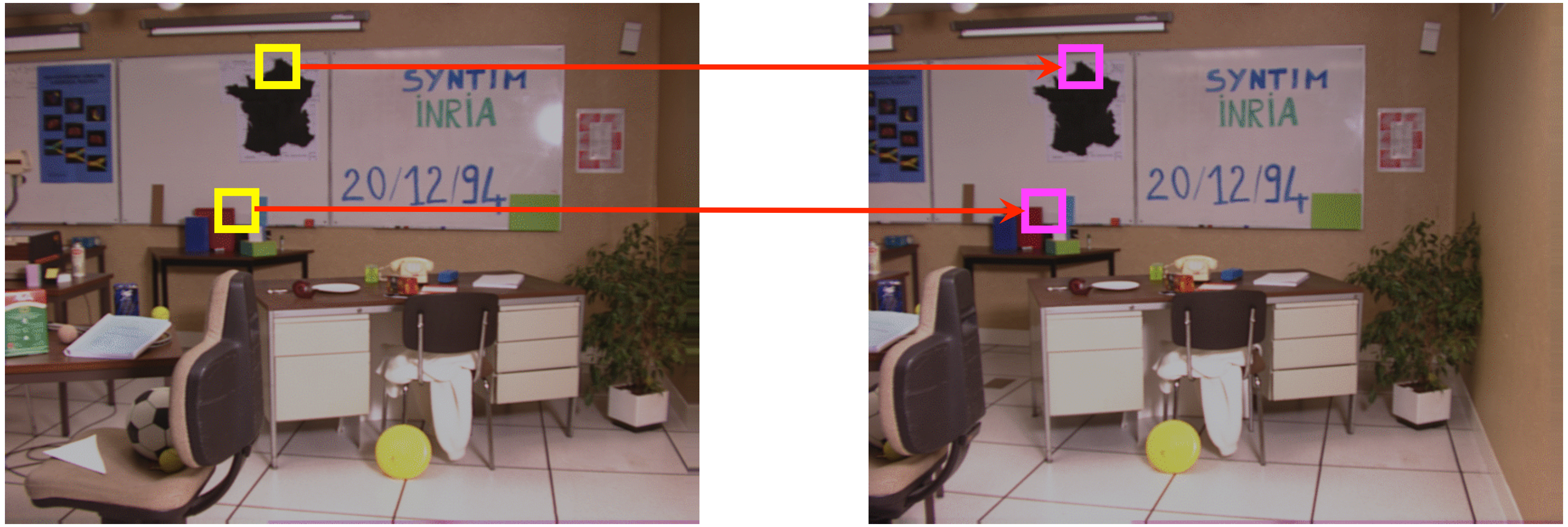


Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**
- Filter = template

3	8	3

⇨ test it against all image locations

$I:$

3	2	4	1	3	8	4	0	3	8	7	7

- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^{N} \left(F(i) - I(x+i)\right)^2$$

$J:$

26	37	21	50	54	1	50	65	59	16	42	17

Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**
- Filter = template

3	8	3

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I :

3	2	4	1	3	8	4	0	3	8	7	7

- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^{N}\left(F(i)-I(x+i)\right)^2 = \sum_{i=-N}^{i=N}\left(F(i)\right)^2 + \sum_{i=-N}^{i=N}\left(I(x+i)\right)^2 - 2\sum_{i=-N}^{i=N}\left(F(i)I(x+i)\right)$$

Correlation

J :

26	37	21	50	54	1	50	65	59	16	42	17

- Similarity measure: Correlation? – maximize

Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**

- Filter = template

3	8	3

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I :

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Correlation

J :

26	37	21	50	54	1	50	65	59	16	42	17

- Similarity measure: Correlation? – maximize

J :

30	37	41	29	51	85	56	21	48	86	101	77

Image filtering | NCC: Normalized Cross Correlation

- Find locations in an image that are similar to a **template**
- Filter = template

3	8	3

⇨ test it against all image locations

$I:$

3	2	4	1	3	8	4	0	3	8	7	7

- Correlation value is affected by the magnitude of intensities
- Similarity measure: Normalized Cross Correlation (**NCC**) – maximize

$$\frac{\sum_{i=-N}^{i=N}\left(F(i)I(x+i)\right)}{\sqrt{\sum_{i=-N}^{i=N}\left(F(i)\right)^2}\sqrt{\sum_{i=-N}^{i=N}\left(I(x+i)\right)^2}}$$

$J:$

0.919	0.759	0.988	0.628	0.655	0.994	0.691	0.464	0.620	0.860	0.876	0.859

Image filtering | ZNCC: Zero-mean NCC

- Find locations in an image that are similar to a **template**
- Filter = template

3	8	3

⇨ test it against all image locations

I :

3	2	4	1	3	8	4	0	3	8	7	7

- Correlation value is affected by the magnitude of intensities
- Similarity measure: Zero-mean Normalized Cross Correlation (**ZNCC**) – maximize

$$\frac{\sum_{i=-N}^{i=N}\left(F(i)-\mu_F\right)\left(I(x+i)-\mu_{I_x}\right)}{\sqrt{\sum_{i=-N}^{i=N}\left(F(i)-\mu_F\right)^2}\sqrt{\sum_{i=-N}^{i=N}\left(I(x+i)-\mu_{I_x}\right)^2}}$$

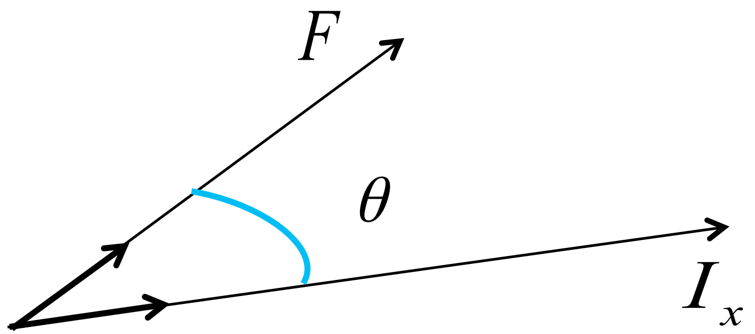
, where

$$\begin{cases} \mu_F = \dfrac{\sum_{i=-N}^{N} F(i)}{2N+1} \\ \mu_{I_x} = \dfrac{\sum_{i=-N}^{N} I(x+i)}{2N+1} \end{cases}$$

Image filtering | correlation as a dot product

- Considering the filter F and the portion of the image I_x as vectors ⇨ their correlation is:

$$\langle F, I_x \rangle = \|F\| \|I_x\| \cos\theta$$



- In **NCC** and **ZNCC** we consider the unit vectors of F and I_x , hence we measure their similarity based on the angle θ

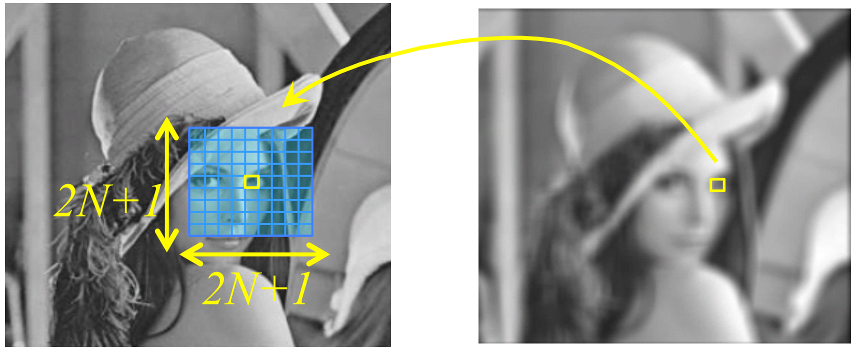
Image filtering | correlation in 2D

$$F \circ I(x,y) = \sum_{j \in [-M,M]} \sum_{i \in [-N,N]} F(i,j) I(x+i, y+j)$$

- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

This example was generated with a 21x21 mask



- If $size(F) = (2N+1)^2$ i.e. this is a square filter
  - 2D Correlation ⇨ no. multiplications per pixel $= (2N+1)^2$
    no. additions per pixel $= (2N+1)^2 - 1$

Image filtering | correlation in 2D

$$F \circ I(x,y) = \sum_{j \in [-M,M]} \sum_{i \in [-N,N]} F(i,j) I(x+i, y+j)$$

- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

"separable" filter

- If $size(F) = (2N+1)^2$ i.e. this is a square filter
  - 2D Correlation ⇨ no. multiplications per pixel $= (2N+1)^2$
    no. additions per pixel $= (2N+1)^2 - 1$
  - 2 × 1D Correlation ⇨ no. multiplications per pixel $= 2(2N+1)$
    no. additions per pixel $= 4N$

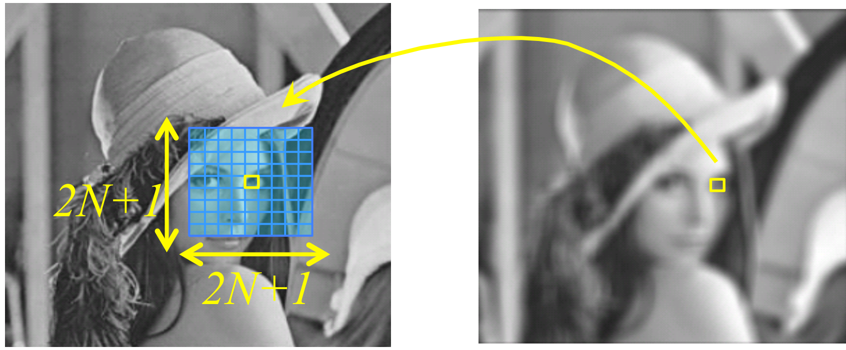


Image filtering | correlation in 2D

$$F \circ I(x,y) = \sum_{j \in [-M,M]} \sum_{i \in [-N,N]} F(i,j) I(x+i, y+j)$$

- Example:
  Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

"separable" filter

- If $size(F) = (2N+1)^2$ i.e. this is a square filter
  - 2D Correlation ⇨ no. multiplications per pixel $= (2N+1)^2$
    no. additions per pixel $= (2N+1)^2 - 1$
  - 2 × 1D Correlation ⇨ no. multiplications per pixel $= 2(2N+1)$
    no. additions per pixel $= 4N$
  - 2 × 1D Correlation ⇨ no. multiplications per pixel $= 1$
    (with const. factor) no. additions per pixel $= 4N$

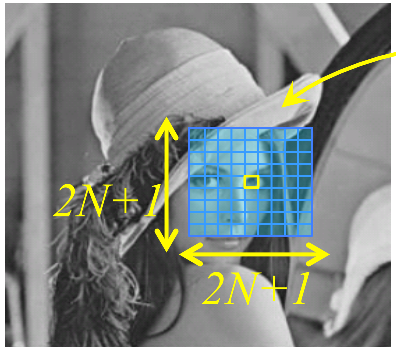


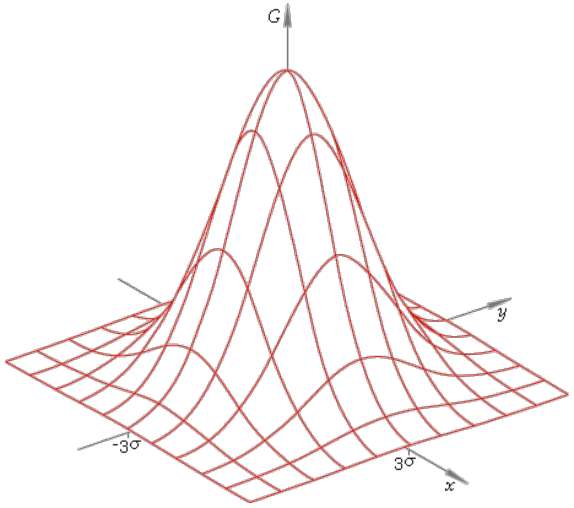
Image filtering | 2D gaussian smoothing

- A general, 2D Gaussian $G(x,y) = \frac{1}{2\pi |S|^{1/2}} e^{-\frac{1}{2}\begin{pmatrix} x \\ y \end{pmatrix} S^{-1} \begin{pmatrix} x & y \end{pmatrix}}$

- We usually want to smooth by the same amount in both x and y directions $S = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$

- So this simplifies to:

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}}_{g_\sigma(x)} \cdot \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}}}_{g_\sigma(y)}$$



- Another separable filter

Image filtering | convolution

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- Convolution is ***equivalent*** to Correlation with a flipped filter before correlating
- CONVOLUTION: $J(x) = F * I(x) = \sum_{i \in [-N,N]} F(i) I(x-i)$
- CORRELATION: $J(x) = F \circ I(x) = \sum_{i \in [-N,N]} F(i) I(x+i)$
- Likewise, in 2D we flip the filter both horizontally & vertically

$$J(x,y) = F * I(x,y) = \sum_{j \in [-M,M]} \sum_{i \in [-N,N]} F(i,j) I(x-i, y-j)$$

- Key difference between correlation and convolution is that **convolution is associative**:

$$F * (G * I) = (F * G) * I$$

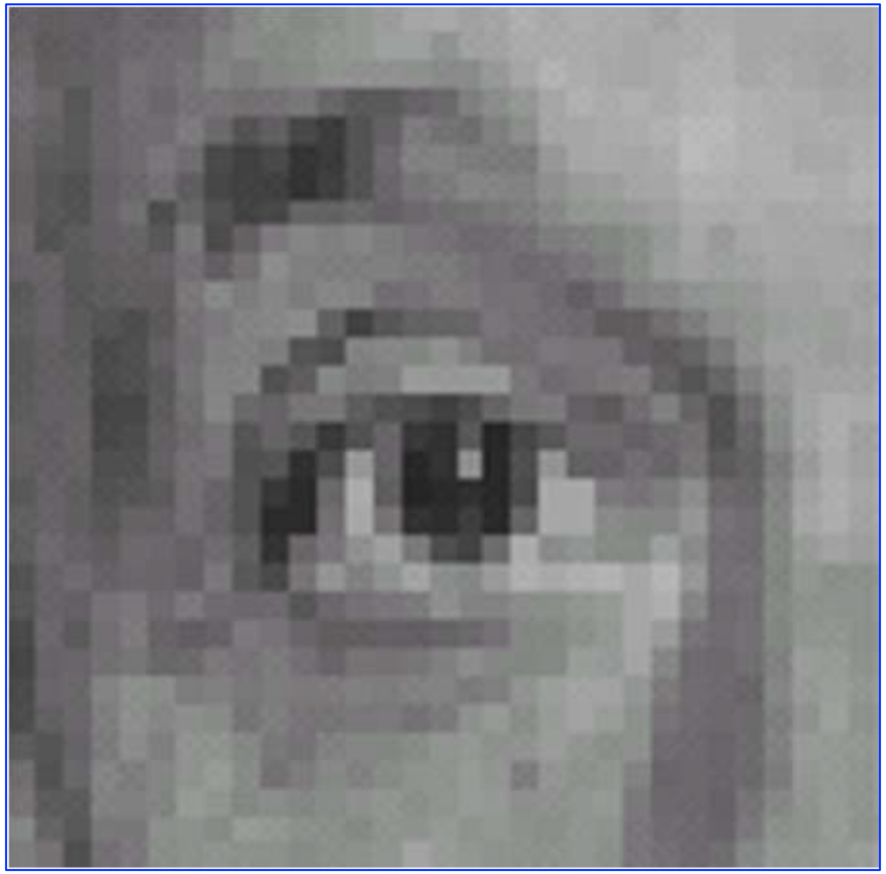
- Very useful!
- Example: smooth an image & take its derivative ⇨ convolve the Derivative filter with the Gaussian Filter & convolve the resulting filter with the Image

So if $F = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$F' = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

Then, $F * I(x) = F' \circ I(x)$

Image filtering | examples



original image

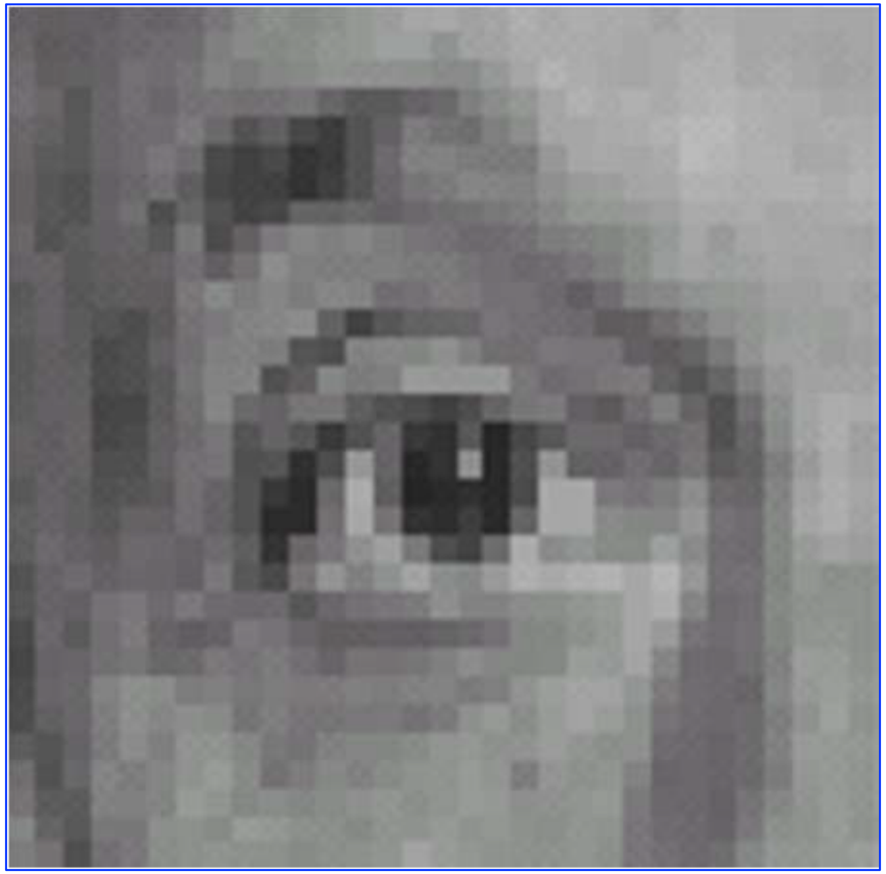
*

0	0	0
0	1	0
0	0	0

=

filtered (no change)

Image filtering | examples

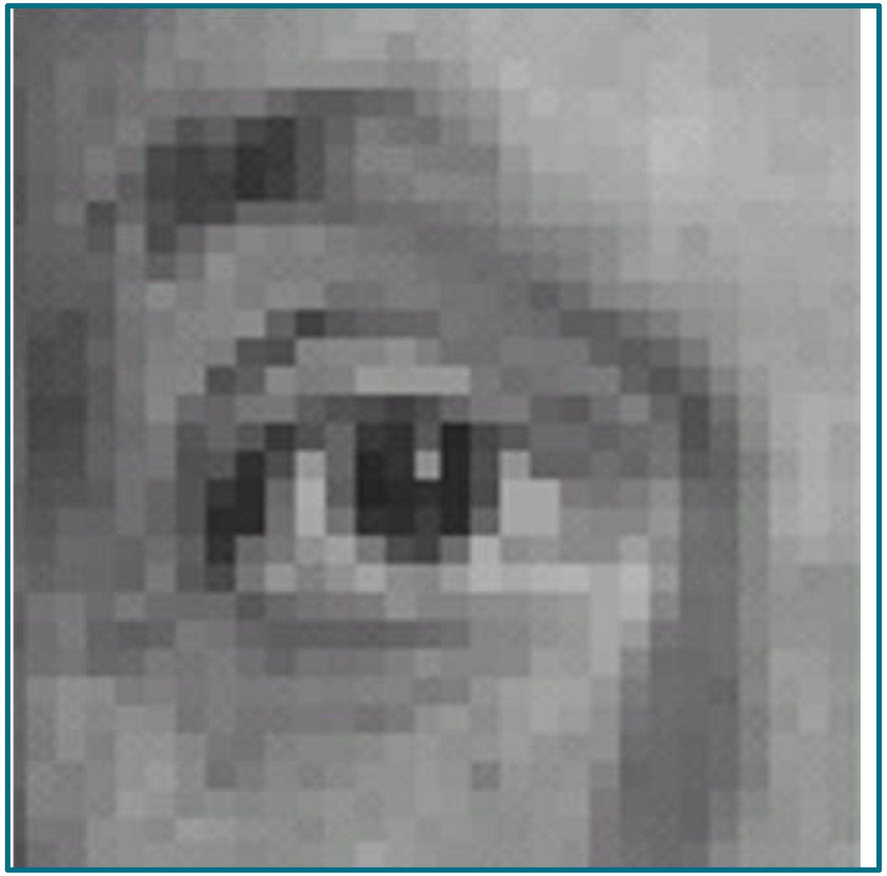


original image

*

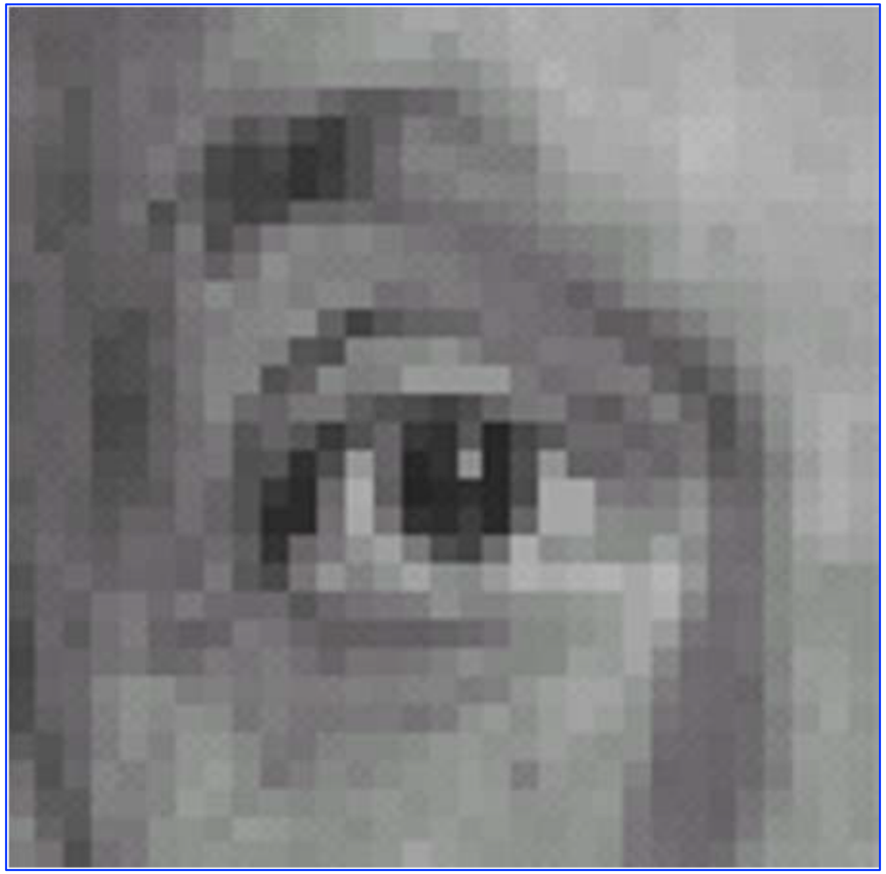
0	0	0
0	0	1
0	0	0

=



filtered (shifted left by 1 pixel)

Image filtering | examples

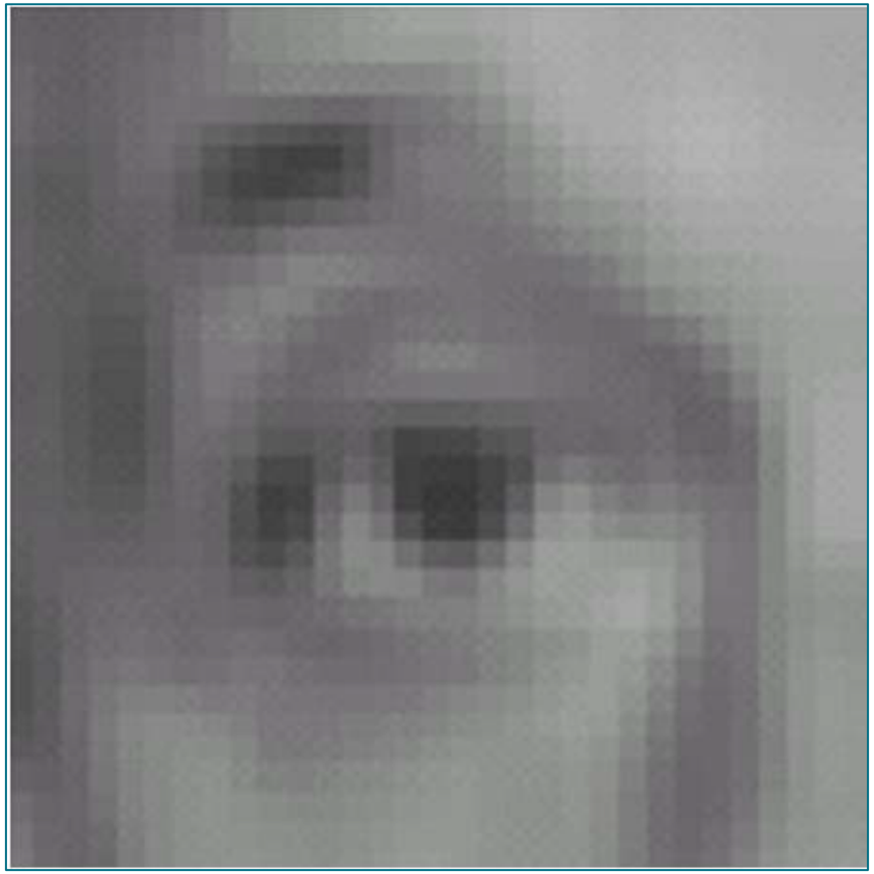


original image

*

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=



filtered (blurred with a box filter)

Image filtering | examples

- What does blurring take away?

original image

\-

smoothed (5x5)

=

detail

Image filtering | examples

- Let's add it back:

original image

+ a

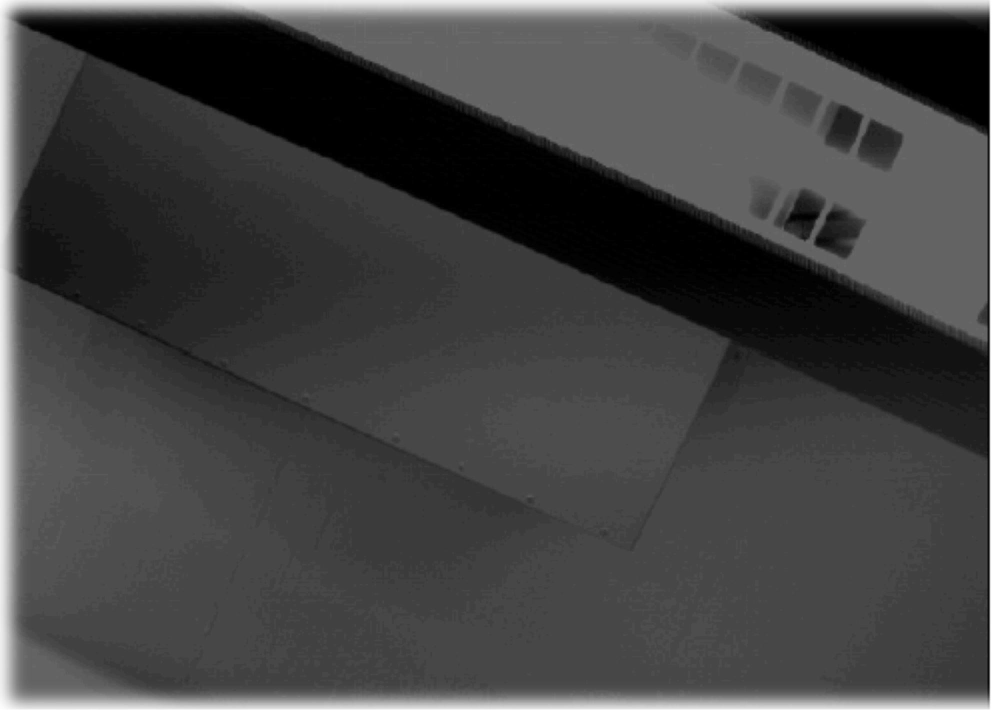
detail

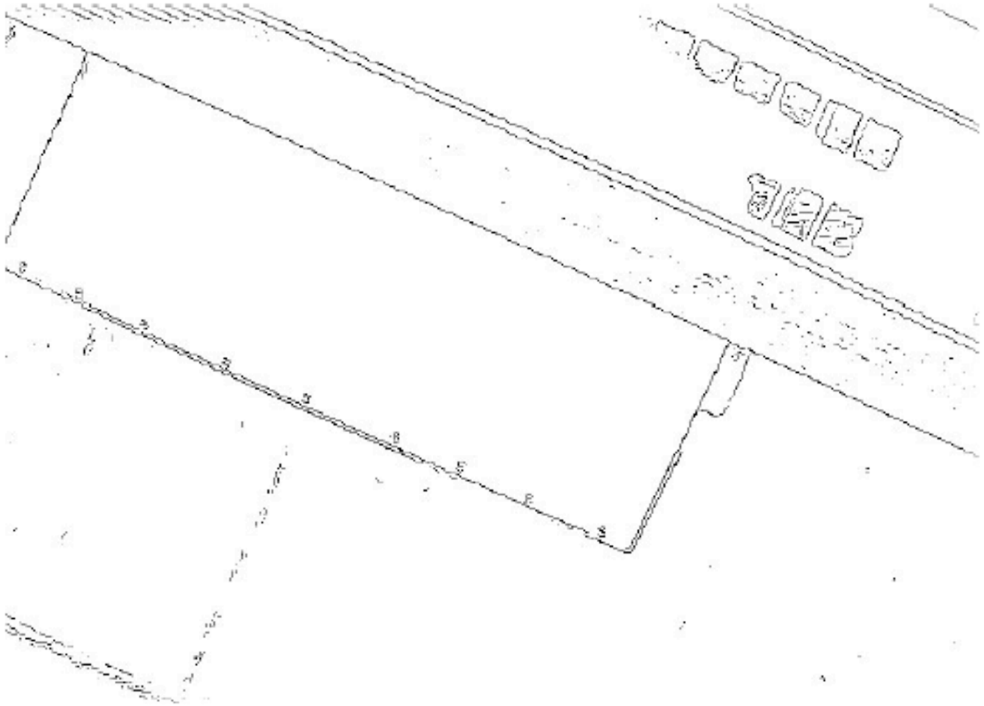
=

sharpened

Edge detection

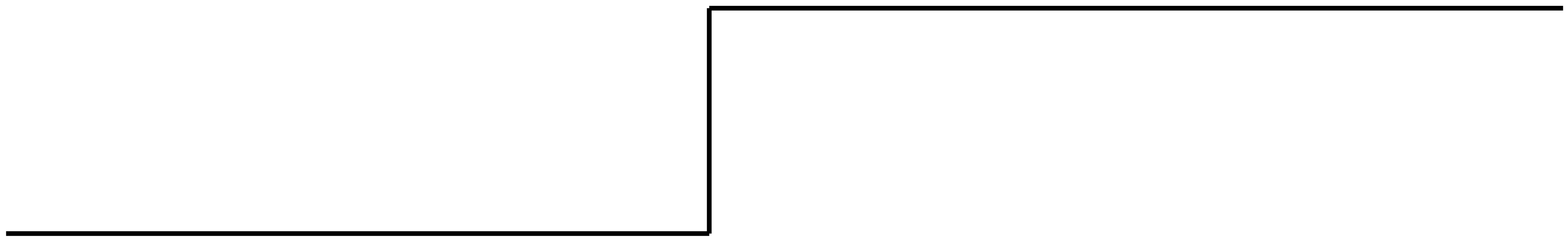
- Ultimate goal of edge detection: an idealized line drawing.
- Edge contours in the image correspond to important scene contours.





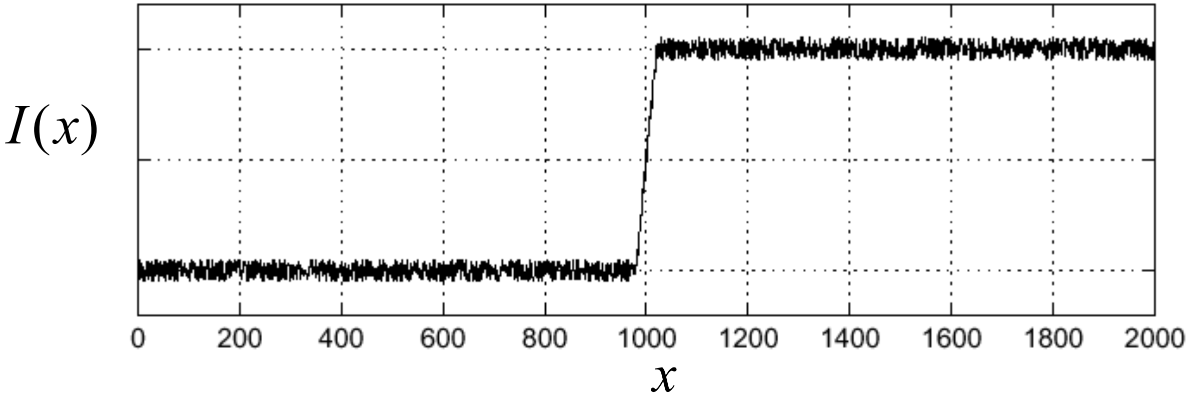
Edge detection | edge = intensity discontinuity in 1 direction

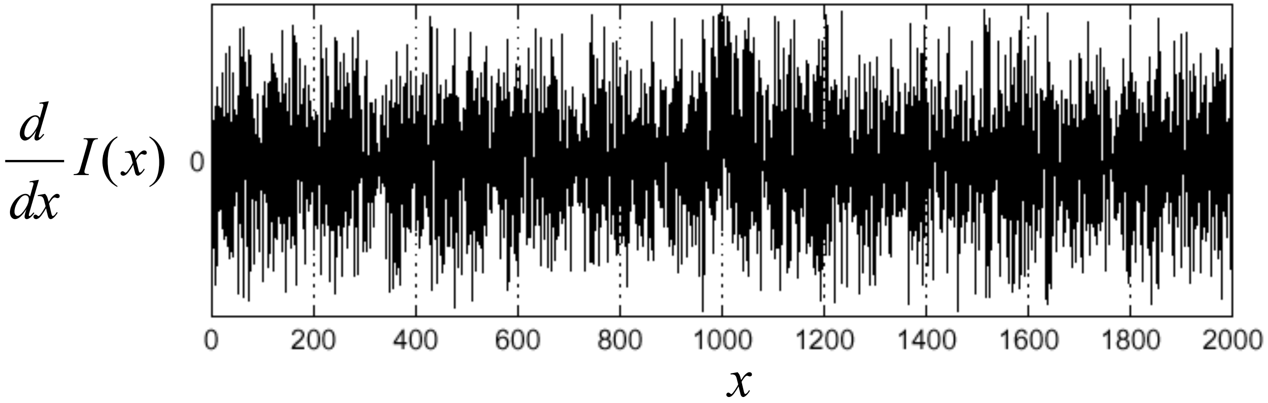
- Edges correspond to sharp changes of intensity
- **How to detect an edge?**
  - Change is measured by 1st order derivative in 1D
  - Big intensity change ⇨ magnitude of derivative is large
  - Or 2nd order derivative is zero.



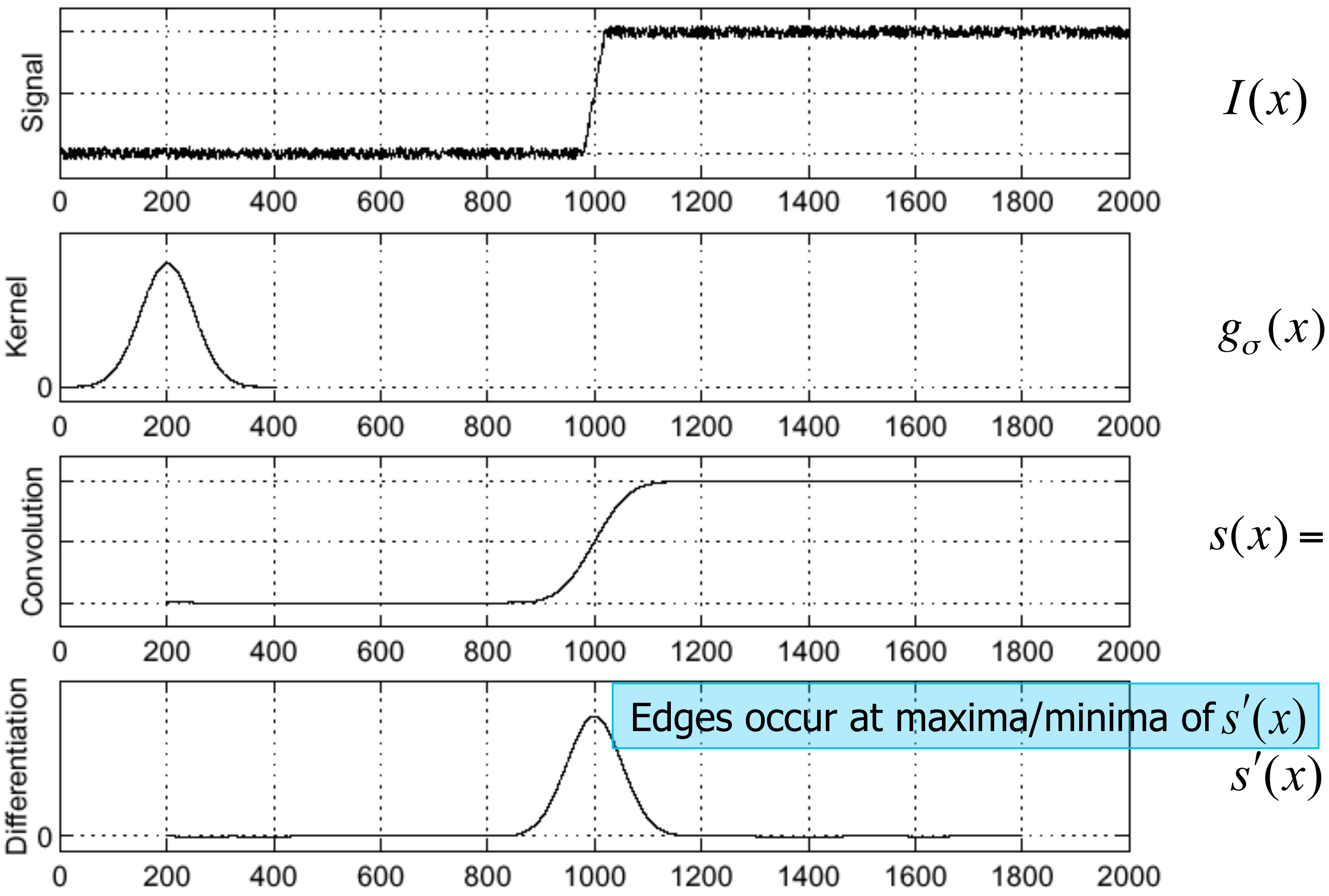
1D edge detection |

- Consider a single row or column of the image, where image intensity shows an obvious change





1D edge detection | solution: smooth first



$$I(x)$$

$$g_\sigma(x)$$

$$s(x) = I(x) * g_\sigma(x)$$

$$s'(x) = \frac{d}{dx}(s(x))$$

- Where is the edge?

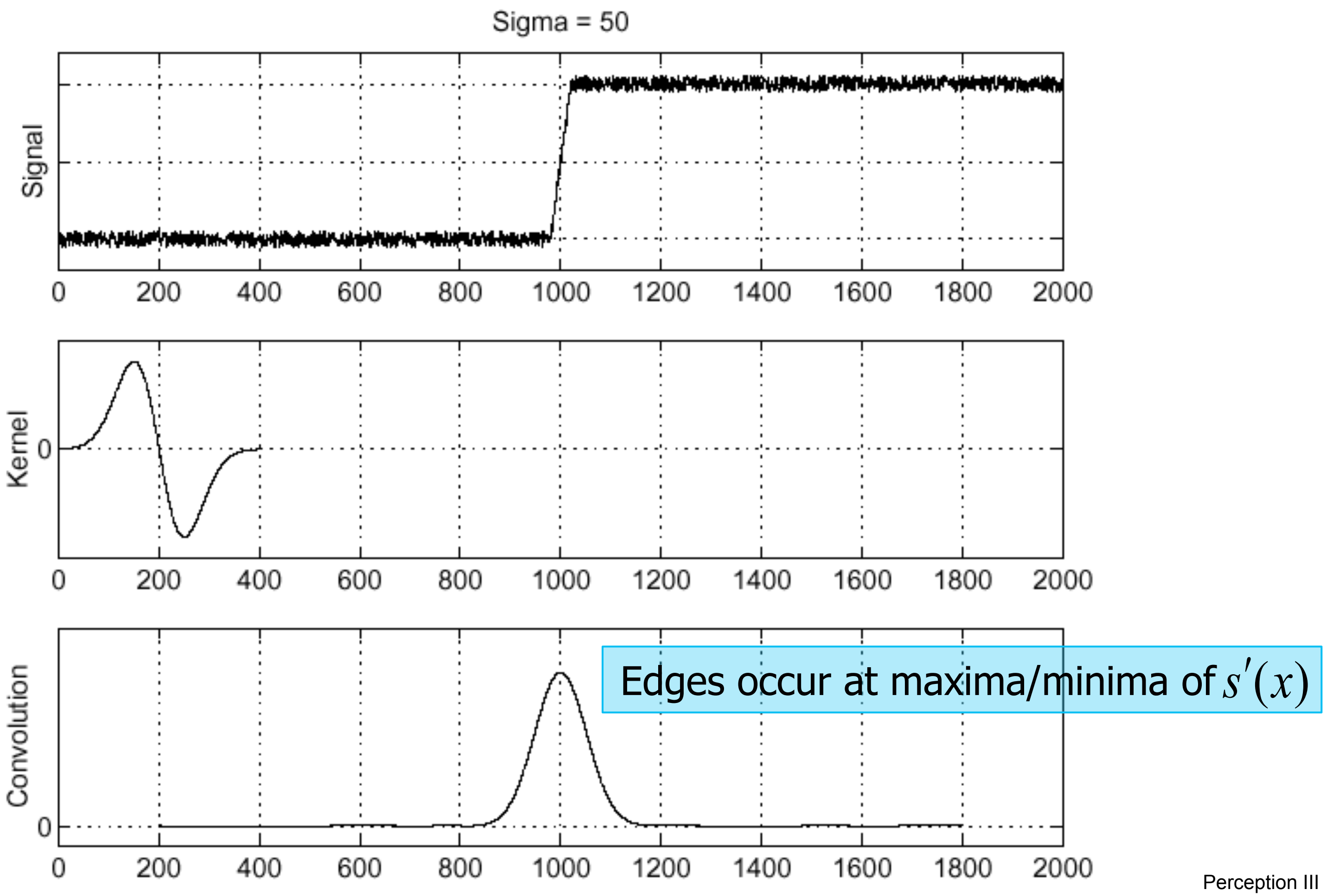
1D edge detection | derivative theorem of convolution

- $s'(x) = \frac{d}{dx}\left(g_\sigma(x) * I(x)\right) = g'_\sigma(x) * I(x)$
- This saves us one operation:

$I(x)$

$g'_\sigma(x) = \frac{d}{dx} g_\sigma(x)$

$s'(x) = g'_\sigma(x) * I(x)$



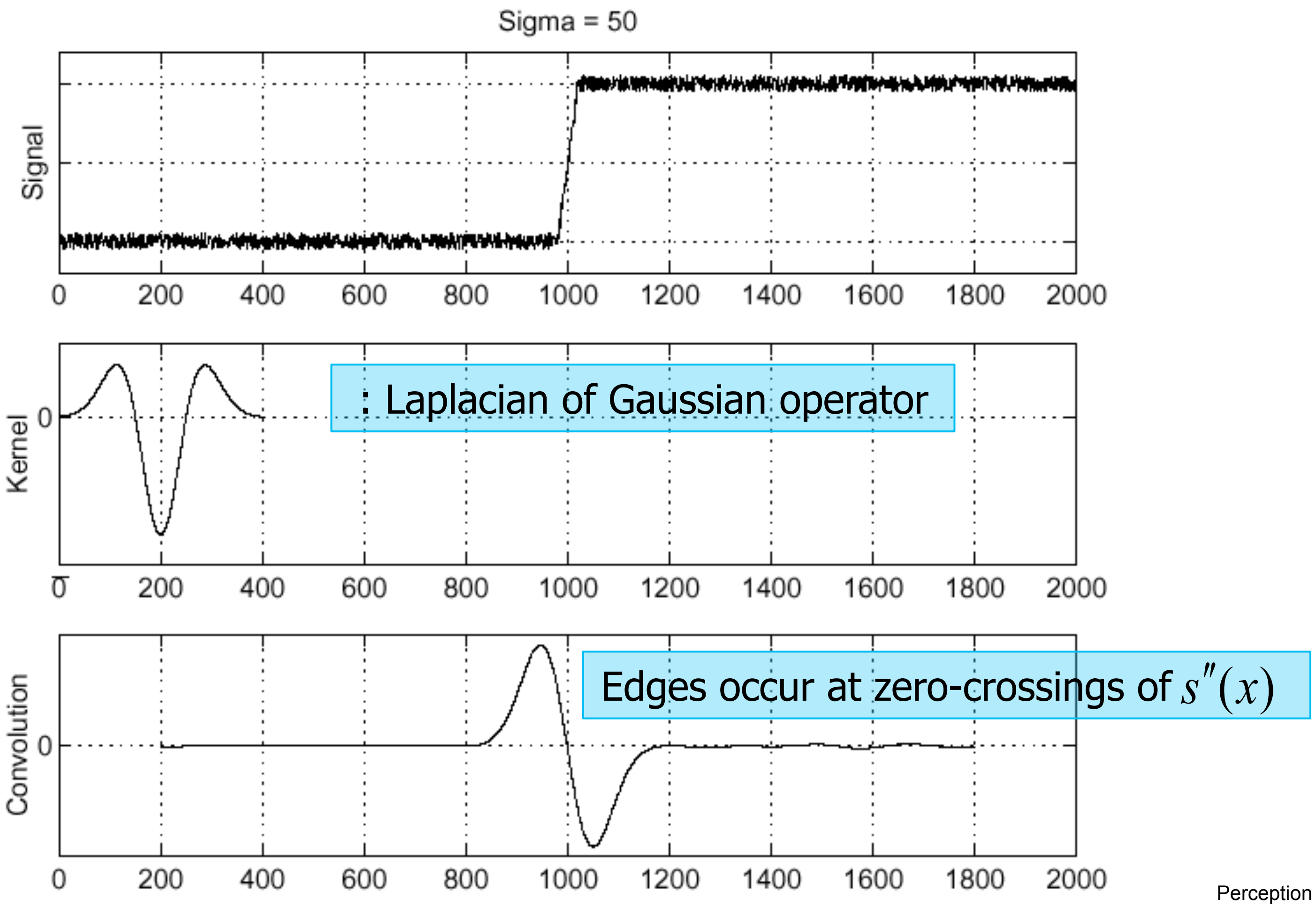
1D edge detection | zero-crossings

- Locations of Maxima/minima in $s'(x)$ are equivalent to zero-crossings in $s''(x)$

$$I(x)$$

$$g''_\sigma(x) = \frac{d^2}{dx^2} g_\sigma(x)$$

$$s''(x) = g''_\sigma(x) * I(x)$$



2D edge detection

Usually use a separable filter such that:
$G_\sigma(x,y) = g_\sigma(x) g_\sigma(y)$

- Find gradient of smoothed image in both directions

$$\nabla S = \nabla(G_\sigma * I) = \begin{bmatrix} \dfrac{\partial(G_\sigma * I)}{\partial x} \\ \dfrac{\partial(G_\sigma * I)}{\partial y} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial G_\sigma}{\partial x} * I \\ \dfrac{\partial G_\sigma}{\partial y} * I \end{bmatrix} = \begin{bmatrix} g'_\sigma(x) g_\sigma(y) * I \\ g_\sigma(x) g'_\sigma(y) * I \end{bmatrix}$$

- Discard pixels with $|\nabla S|$ (i.e. edge strength) below a certain threshold
- **Non-maxima suppression**: identify local maxima of $|\nabla S|$
  ⇨ detected edges

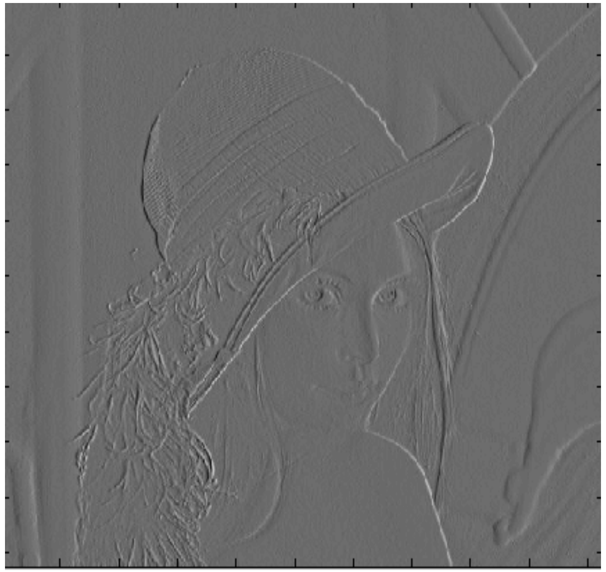
2D edge detection | example

I : original image (Lena)

2D edge detection | example using the Canny edge detector

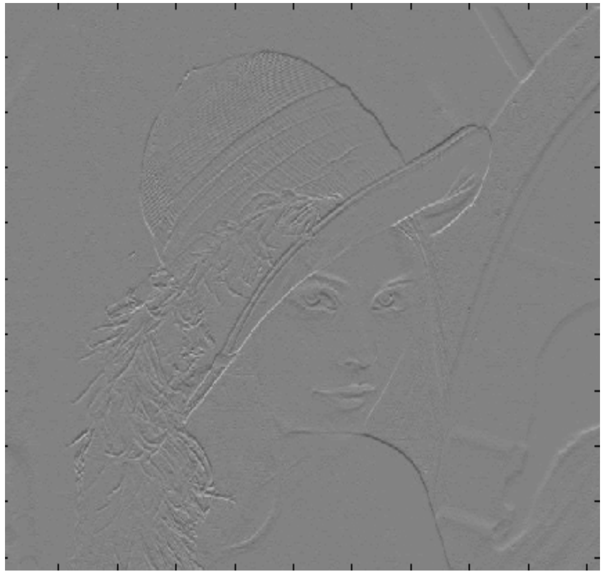
$$|\nabla S| = \sqrt{S_x^2 + S_y^2}$$

$|\nabla S|$: Edge strength



$$S_x = \frac{\partial (G_\sigma * I)}{\partial x}$$

$$\nabla S = \nabla (G_\sigma * I)$$



$$S_y = \frac{\partial (G_\sigma * I)}{\partial y}$$

2D edge detection | example using the Canny edge detector

Thresholding $|\nabla S|$

2D edge detection | example using the Canny edge detector

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Thinning: non-maximal suppression
⇨ **edge image**

2D edge detection | partial derivatives of an image

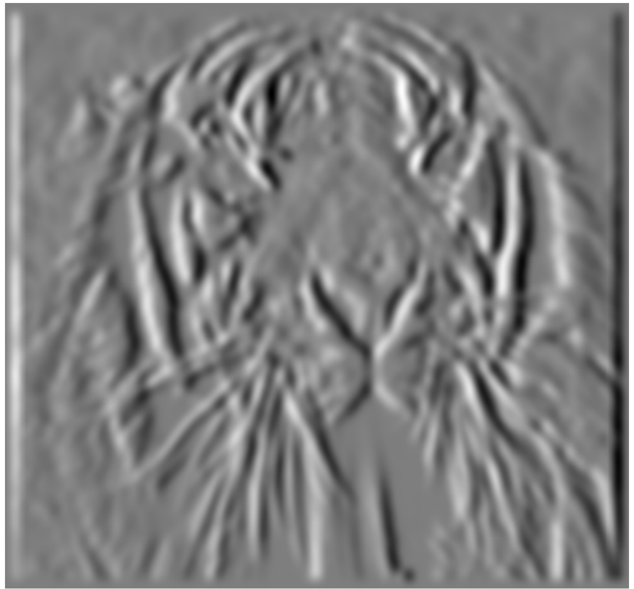
$$S_x = \frac{\partial(G_\sigma * I)}{\partial x}$$

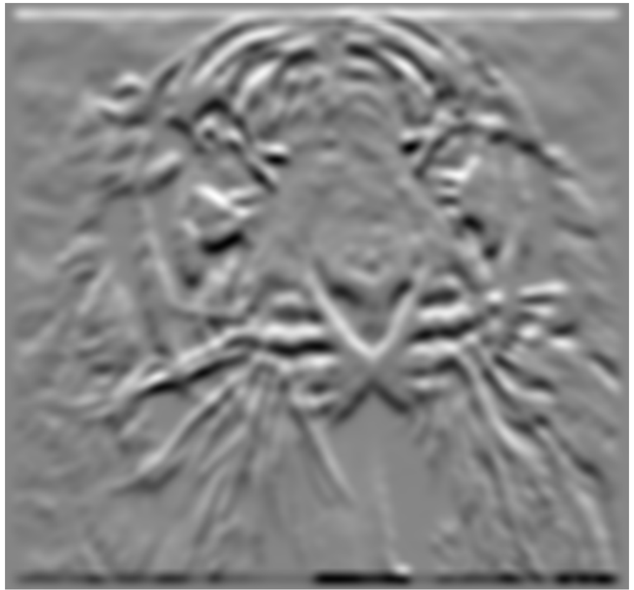
$$F_x = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$S_y = \frac{\partial(G_\sigma * I)}{\partial y}$$

$$F_y = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$|\nabla S| = \sqrt{S_x^2 + S_y^2}$$





2D edge detection | other approx. of derivative filters

- Prewitt:

$$F_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad F_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

- Sobel:

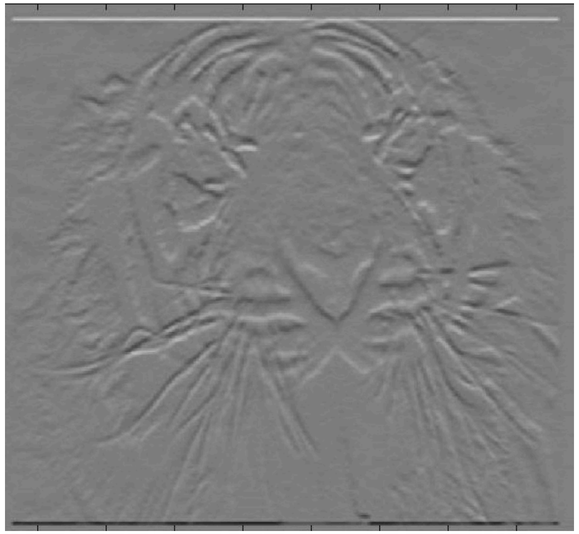
$$F_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad F_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- Roberts:

$$F_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad F_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Sample Matlab code

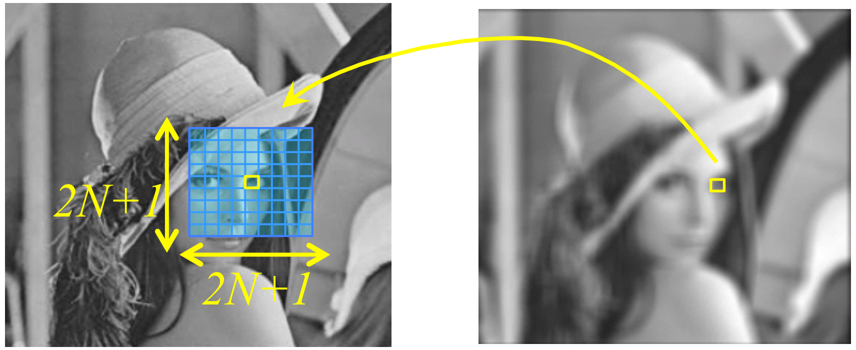
```
>> im = imread('lion.jpg')
>> Fy = fspecial('sobel');
>> outim = imfilter(double(im), Fy);
>> imagesc(outim);
>> colormap gray;
```



Key points on smoothing + derivative masks

Smoothing masks

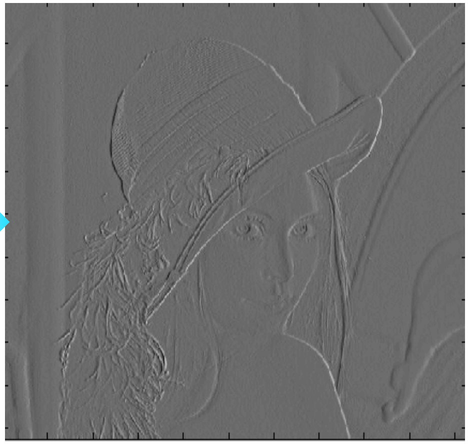
- Values positive
- Always **sum to 1** → constant regions same as input
- Amount of **smoothing proportional to mask size**



Derivative masks

- Opposite signs used to get high response in regions of high contrast
- Always **sum to 0** → no response in constant regions
- High absolute value at points of high contrast

$$F_x = \begin{pmatrix} 1 & -1 \end{pmatrix}$$



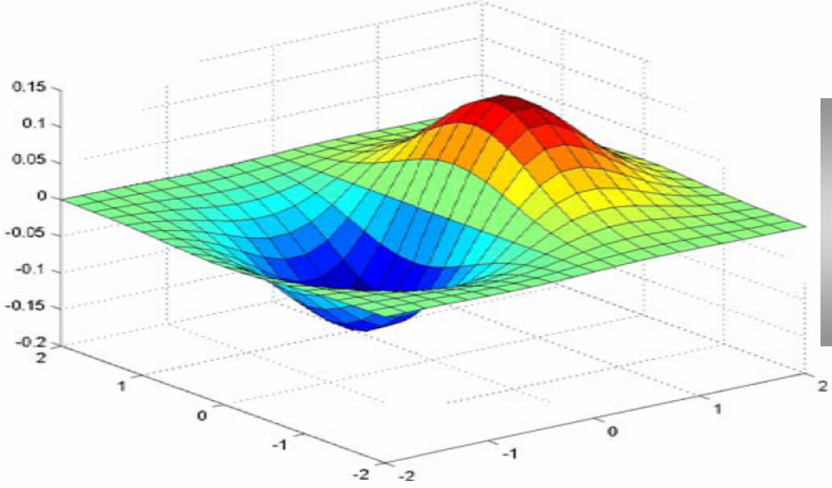
2D edge detection | derivative of gaussian filter

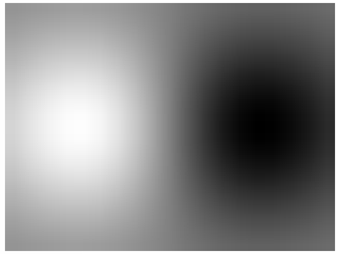
$$\nabla S = \nabla * G * I = \begin{pmatrix} (F_x * G) * I \\ (F_y * G) * I \end{pmatrix}$$

$$F_x = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

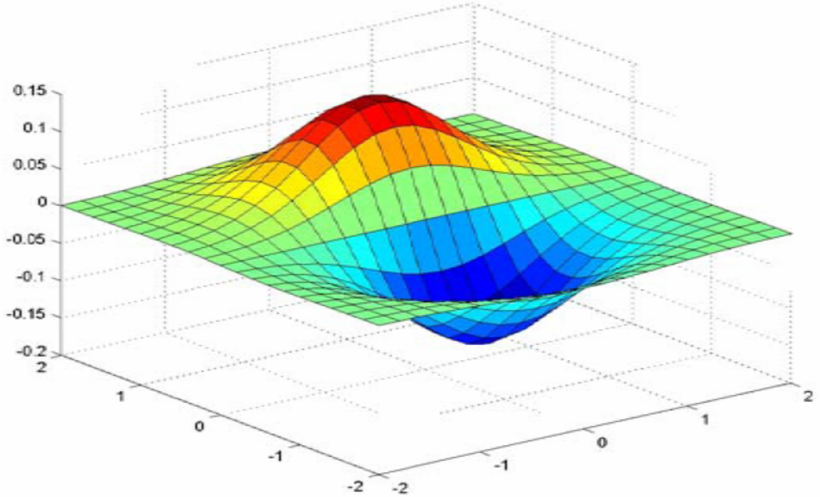
$$F_y = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

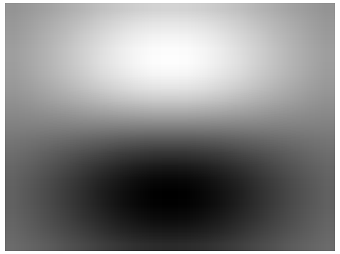
$$G = \begin{pmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{pmatrix}$$





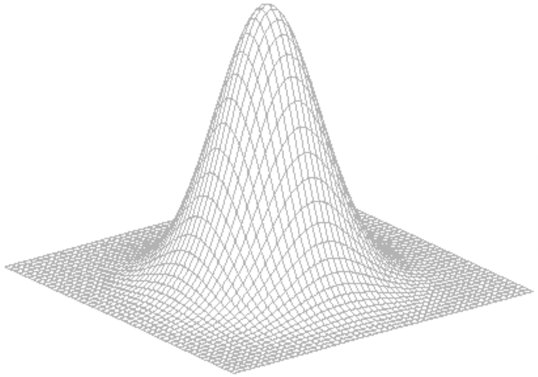
x-direction





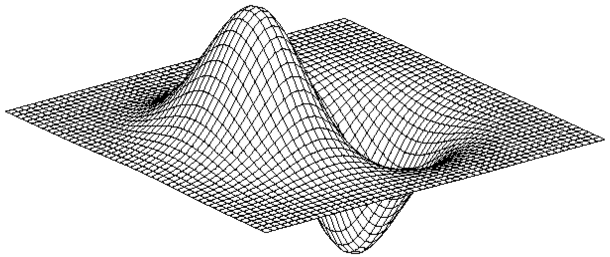
y-direction

2D edge detection | popular edge detection filters



Gaussian

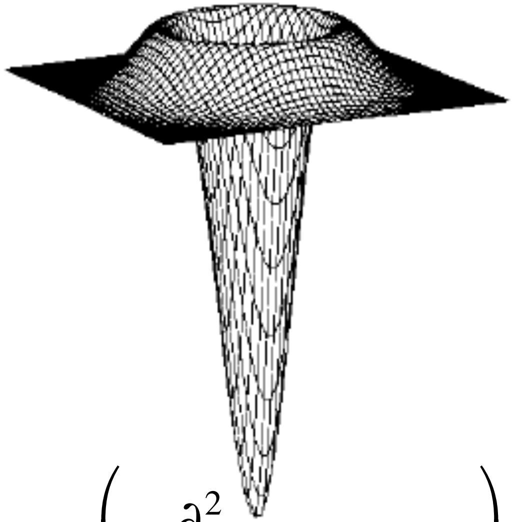
$$G_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



derivative of Gaussian

$$\nabla G_\sigma(u,v) = \begin{pmatrix} \frac{\partial}{\partial u} G_\sigma(u,v) \\ \frac{\partial}{\partial v} G_\sigma(u,v) \end{pmatrix}$$

Laplacian of Gaussian

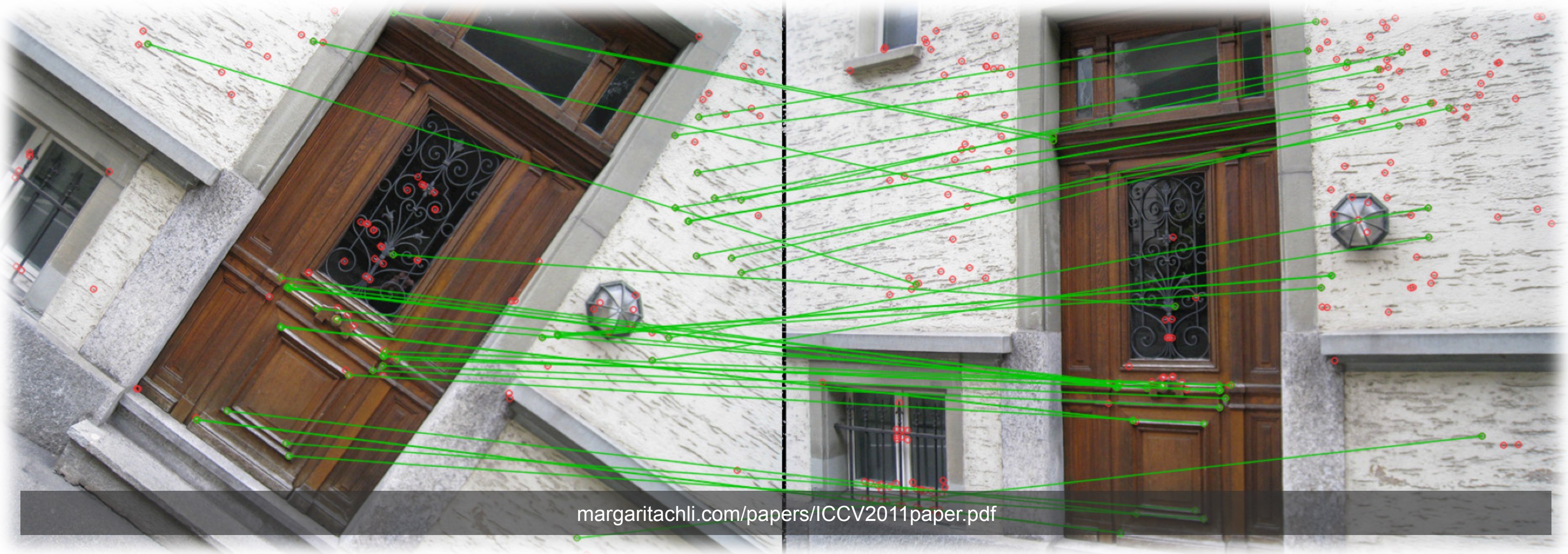


$$\nabla^2 G_\sigma(u,v) = \begin{pmatrix} \frac{\partial^2}{\partial u^2} G_\sigma(u,v) \\ \frac{\partial^2}{\partial v^2} G_\sigma(u,v) \end{pmatrix}$$

Point Features

Image Feature Extraction:

- Edges
- **Points: Shi-Tomasi & Harris corners**
  SIFT features
  - **and more recent algorithms from the state of the art…**



Point features | applications

Point features are widely used in:

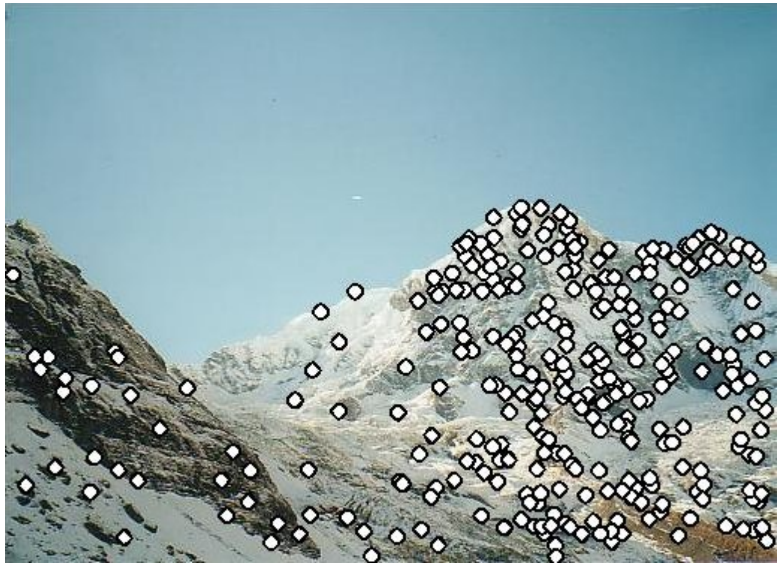
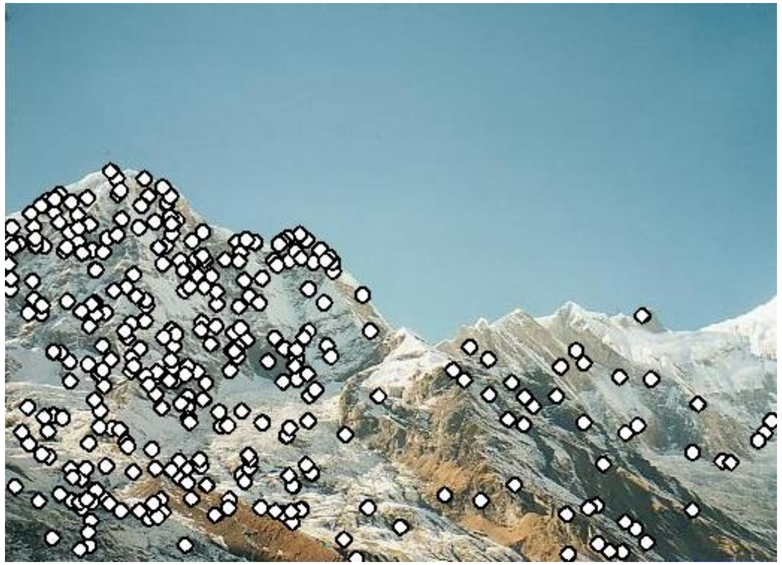
- Robot navigation
- Object recognition
- 3D reconstruction
- Motion tracking
- Indexing and database retrieval ⇨ Google Images
- …
- Image stitching: this panorama was generated using **AUTOSTITCH** (freeware)

Point features | how to build a panorama?

- We need to match (align) images

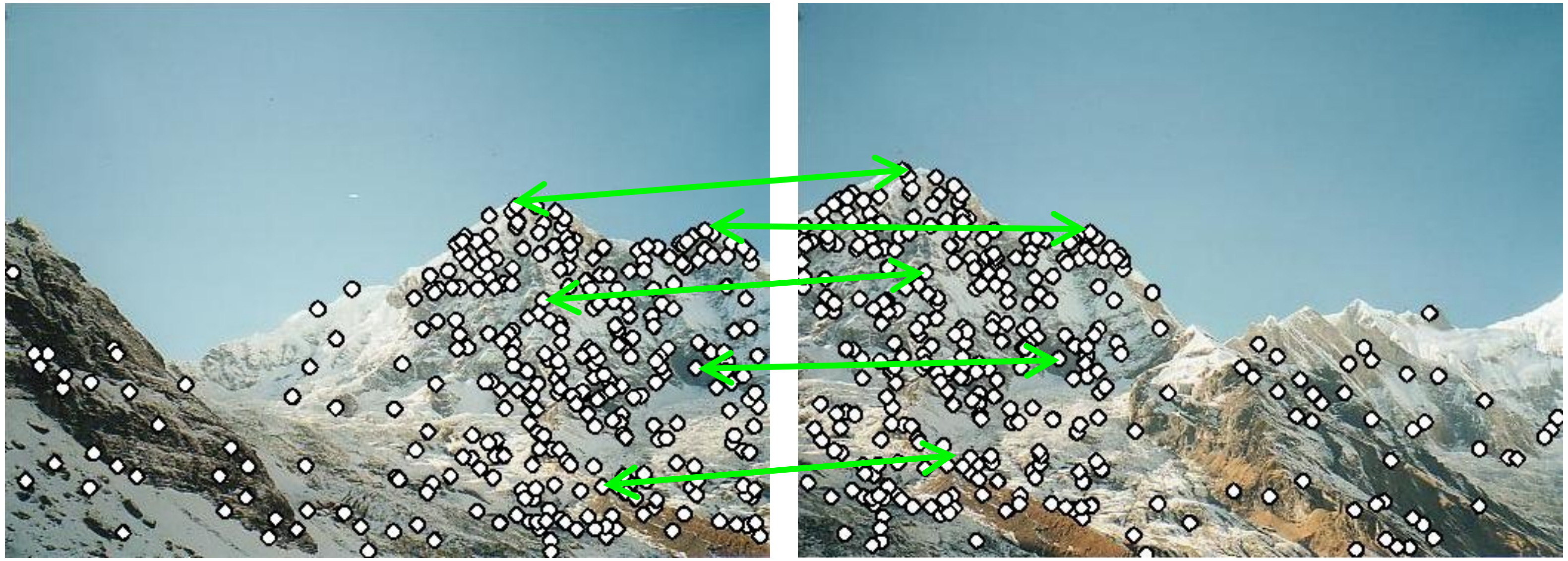
Point features | how to build a panorama?

- Detect feature points in both images

Point features | how to build a panorama?

- Detect feature points in both images
- Find corresponding pairs



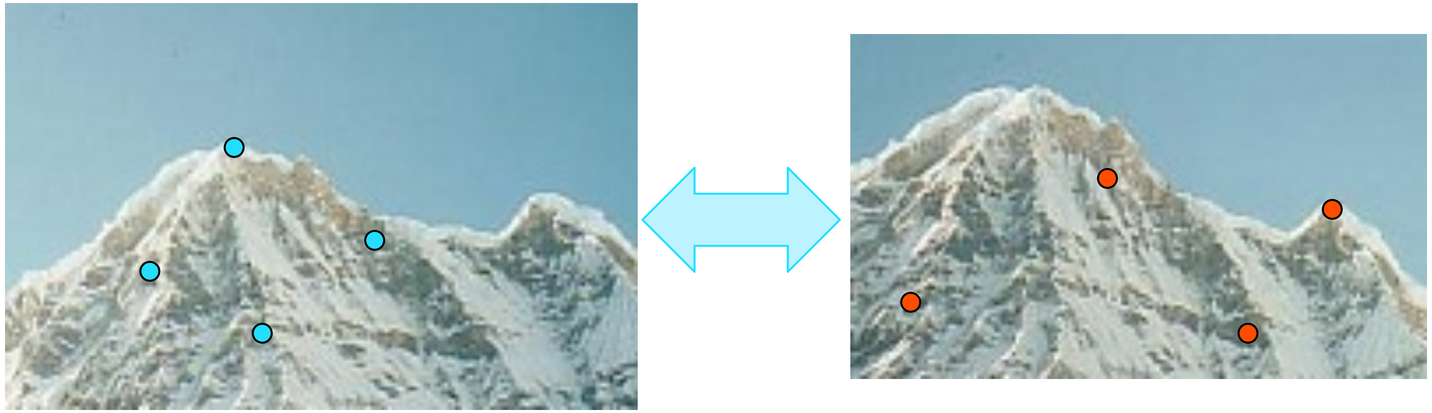
Point features | how to build a panorama?

- Detect feature points in both images
- Find corresponding pairs
- Use these pairs to align images

Point features | feature extraction

Problem 1:

- Detect the **same** points **independently** in both images, if they are in the field of view



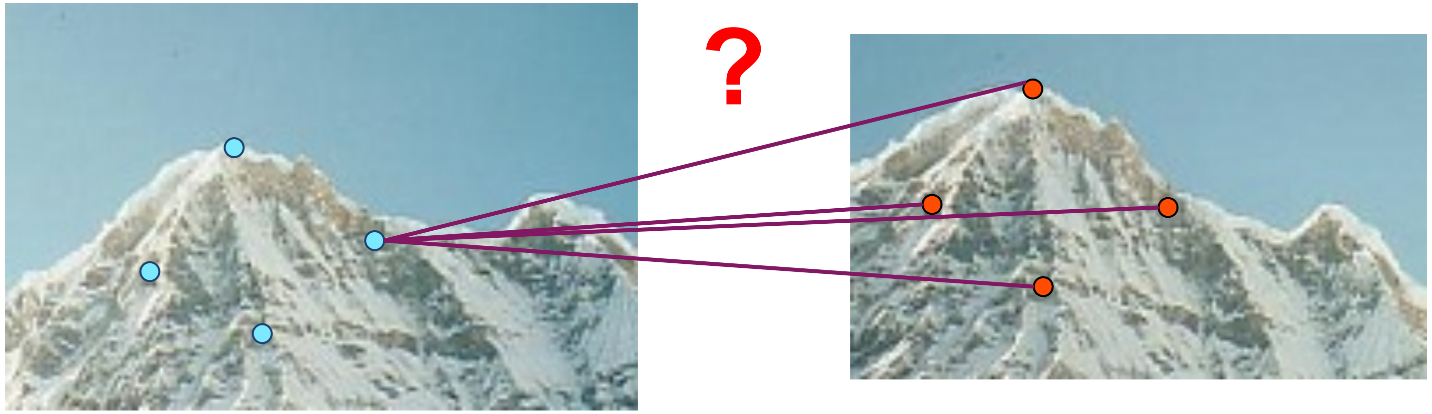
no chance to match!

We need a **repeatable** feature detector

Point features | feature matching

Problem 2:

- For each point, identify its correct correspondence in the other image(s)



We need a **reliable** and **distinctive** feature descriptor

Point features | what is a distinctive feature?

- Some patches can be localized or matched with higher accuracy than others

Image 1

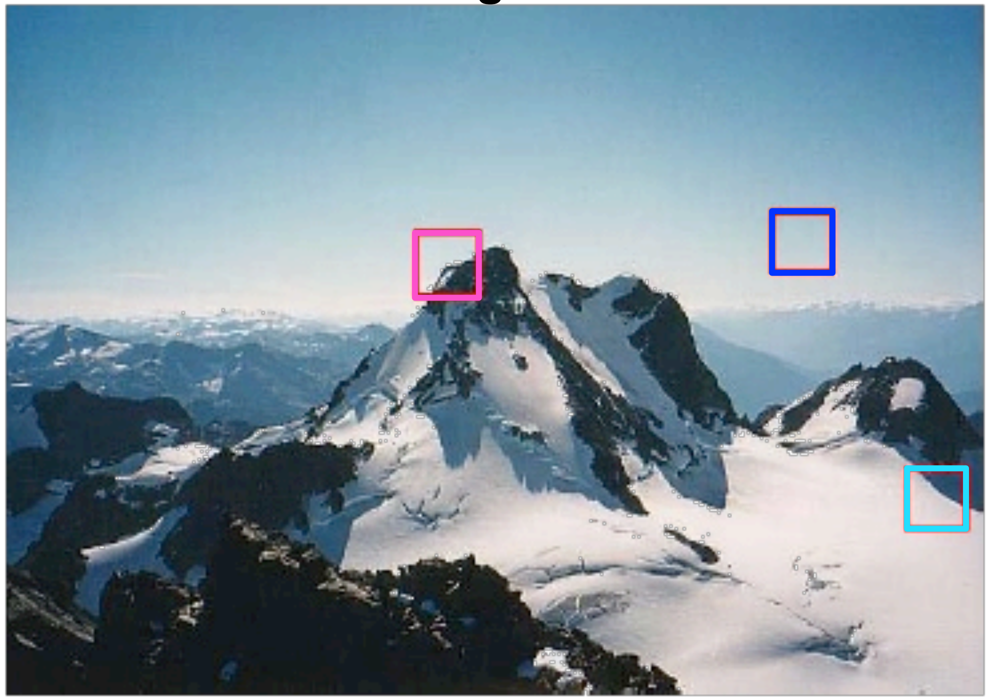
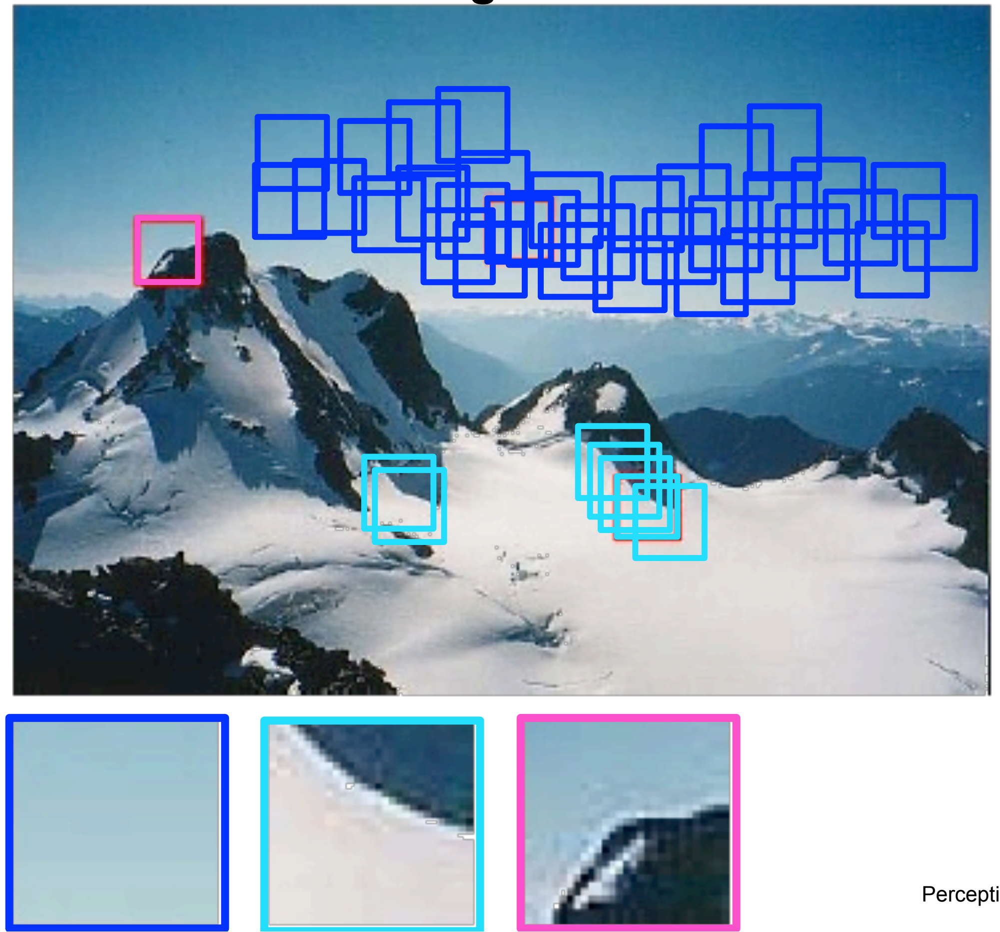


Image 2



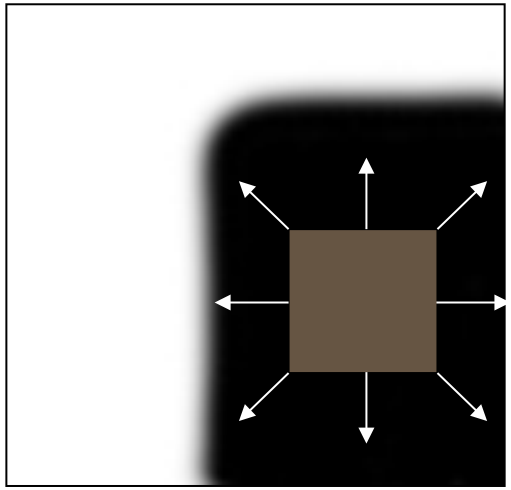
Corner detection

Corner detection | identifying corners

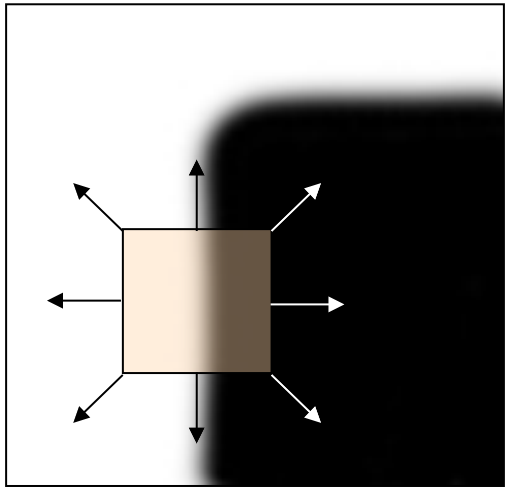
- **Key property:** in the region around a corner, image gradient has **two or more** dominant directions
- Corners are **repeatable** and **distinctive**

Corner detection | identifying corners

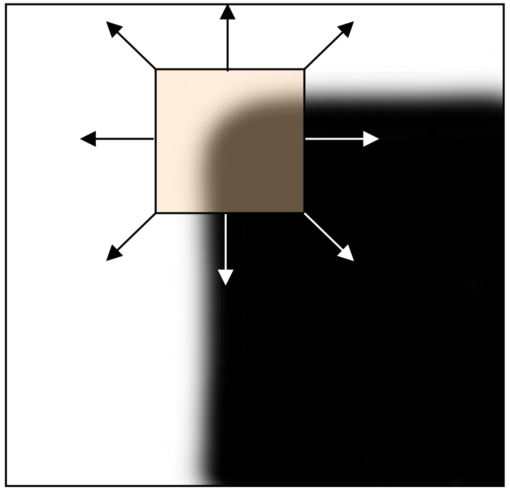
- How do we identify corners?
- Shifting a window in **any direction** should give a **large change** of intensity in at least 2 directions



"flat" region:
no intensity change



"edge":
no change along the edge direction



"corner":
significant change in at least 2 directions

Corner detection | how do we implement this?

- Two image patches of size P one centered at (x, y) and one centered at $(x+\Delta x, y+\Delta y)$
- The Sum of Squared Differences between them is:

$$SSD(\Delta x, \Delta y) = \sum_{x,y \in P} \left(I(x,y) - I(x+\Delta x, y+\Delta y) \right)^2$$

- Let $I_x = \frac{\partial I(x,y)}{\partial x}$ and $I_y = \frac{\partial I(x,y)}{\partial y}$. Approximating with a **1st order Taylor expansion**:

$$I(x+\Delta x, y+\Delta y) \approx I(x,y) + I_x(x,y)\Delta x + I_y(x,y)\Delta y$$

- This produces the approximation

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} \left(I_x(x,y)\Delta x + I_y(x,y)\Delta y \right)^2$$

Corner detection | how do we implement this?

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} \left(I_x(x,y)\Delta x + I_y(x,y)\Delta y \right)^2$$

- This can be written in a matrix form as

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\Rightarrow SSD(\Delta x, \Delta y) \approx \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

"Second moment matrix"

Corner detection | interpreting matrix M

- Since M is symmetric $\Rightarrow M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

- The Harris (and the "Shi-Tomasi") detector analyses the **eigenvalues,** λ_1 and λ_2, to decide if we are in presence of a corner ⇨ i.e. look for large intensity changes in at least 2 directions

Corner detection | eigen decomposition

- The **eigenvectors** $\boldsymbol{v}$ and **eigenvalues** λ of a square matrix A satisfy:

$$A\boldsymbol{v}=\lambda\boldsymbol{v}$$

- Then is an eigenvector $\boldsymbol{v}$ of A and λ is the corresponding eigenvalue.
- The eigenvalues are found by solving: $\det(A-\lambda I)=0$
- In this case, $A=M$ is a 2 x 2 matrix, so:

$$\det\begin{bmatrix} m_{11}-\lambda & m_{12} \\ m_{21} & m_{22}-\lambda \end{bmatrix}=0$$

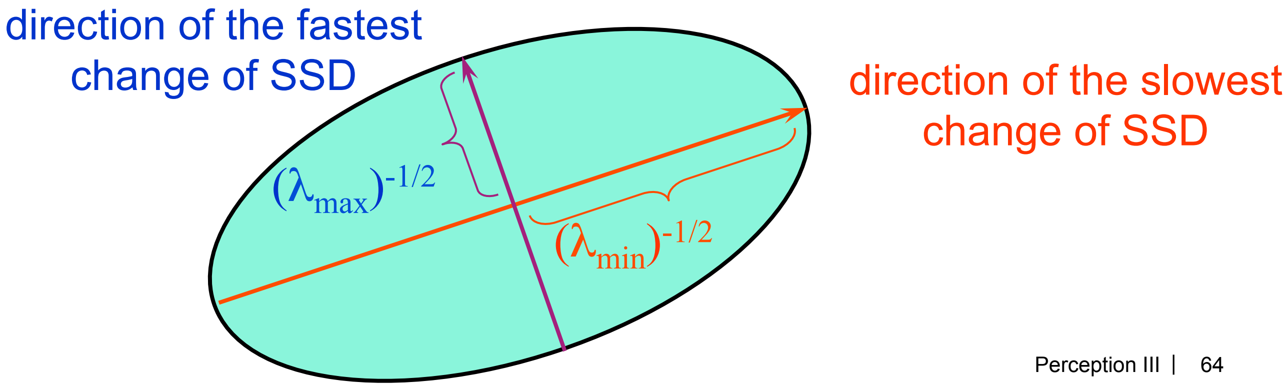
$$\lambda_{1,2}=\frac{1}{2}\left[(m_{11}+m_{22})\pm\sqrt{4m_{12}m_{21}+(m_{11}-m_{22})^2}\right]=0$$

- For each λ we can find its corresponding $\boldsymbol{v}$ (forming the columns in R) by

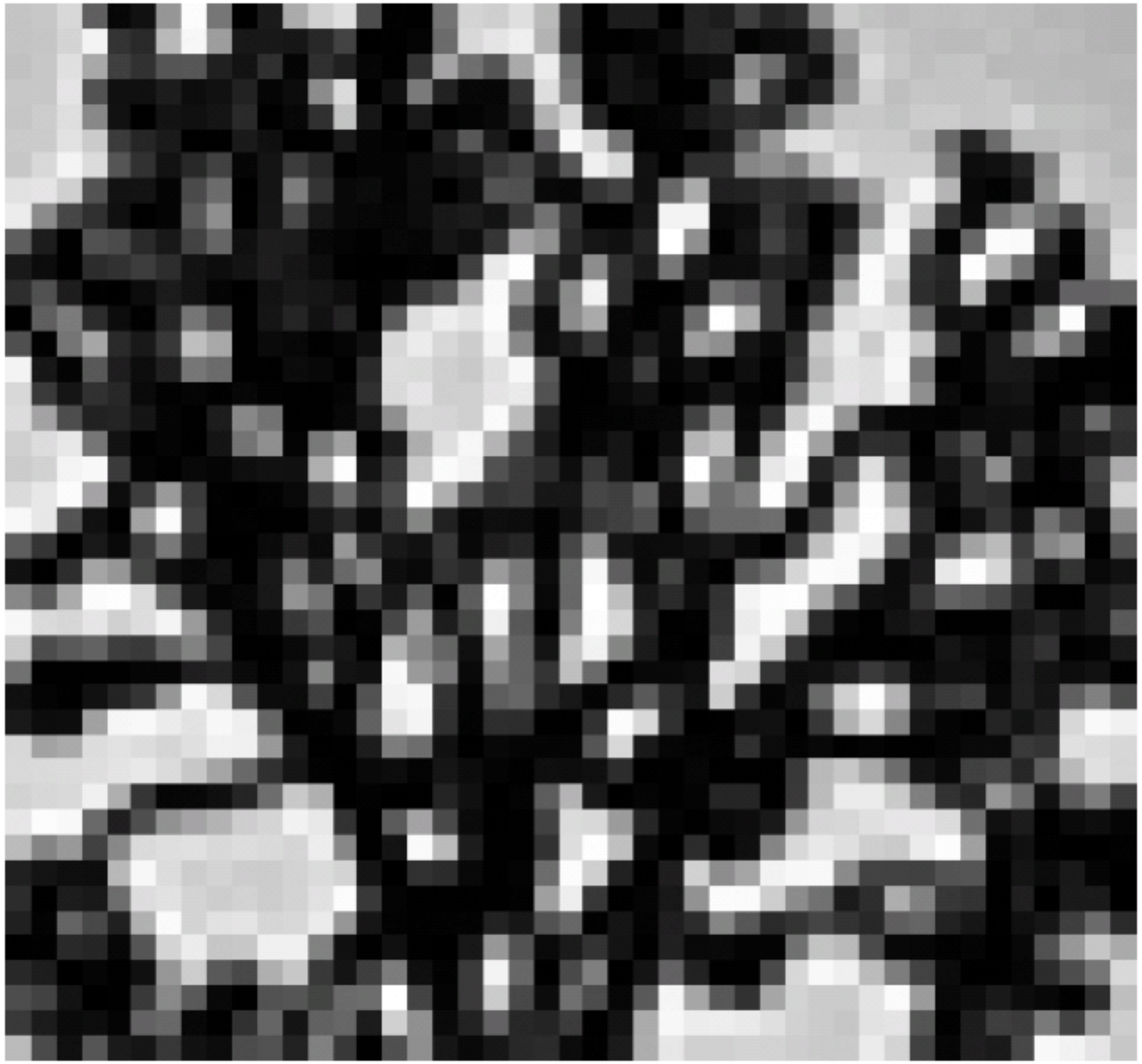
$$\begin{bmatrix} m_{11}-\lambda & m_{12} \\ m_{21} & m_{22}-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$$

Corner detection | interpreting matrix M

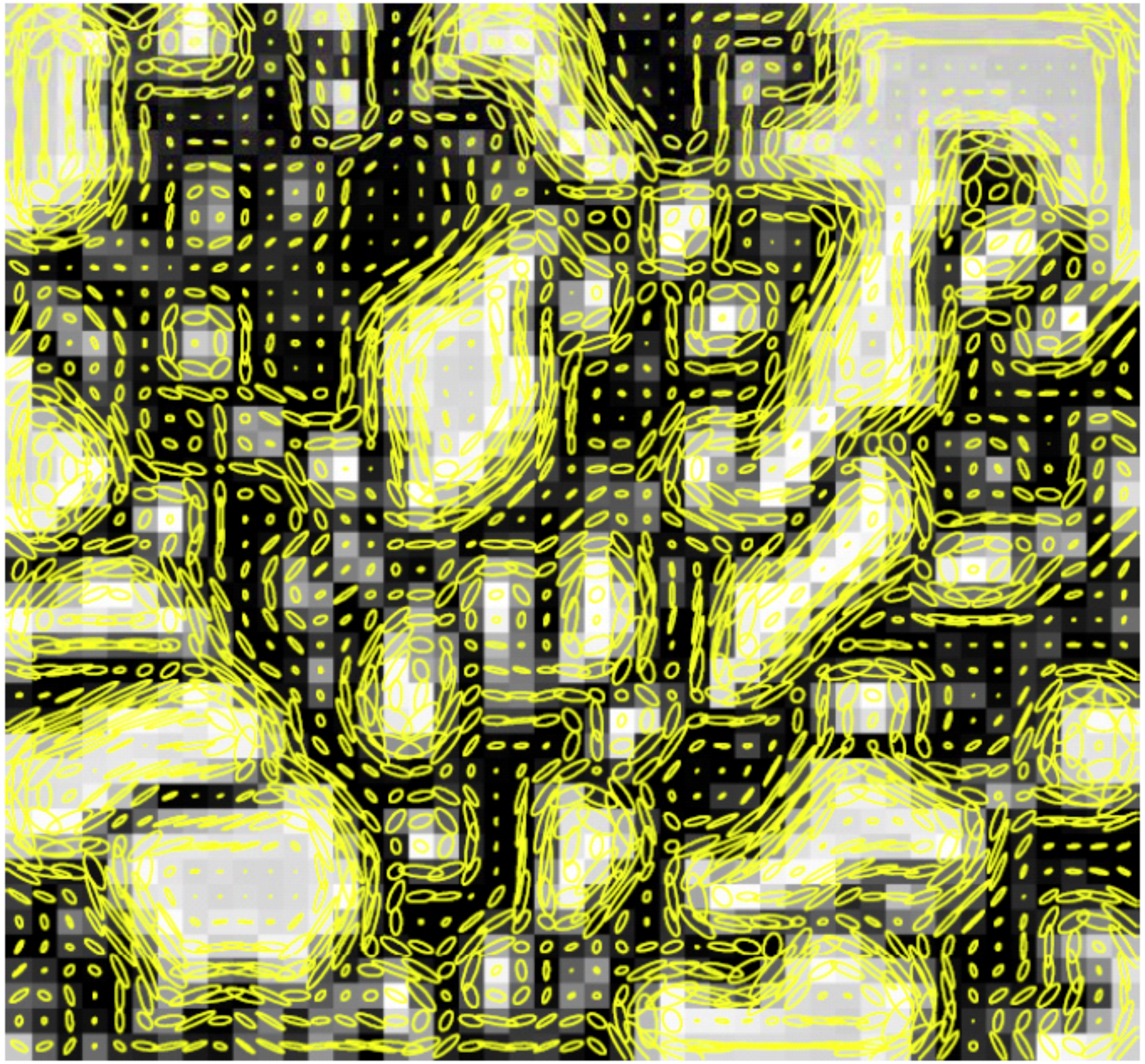
- Since M is symmetric $\Rightarrow M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$
- The Harris (and the "Shi-Tomasi") detector analyses the **eigenvalues,** λ_1 and λ_2 , to decide if we are in presence of a corner ⇨ i.e. look for large intensity changes in at least 2 directions
- We can visualize $\begin{bmatrix} \Delta x & \Delta y \end{bmatrix} M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = const$ as an **ellipse** with axis-lengths determined by λ_1 and λ_2 and the axes' orientations determined by R (i.e. the **eigenvectors** of M)
- The (two) eigenvectors identify the orthogonal directions of largest and smallest changes of SSD



Corner detection | visualization of 2nd moment matrices



Corner detection | visualization of 2nd moment matrices

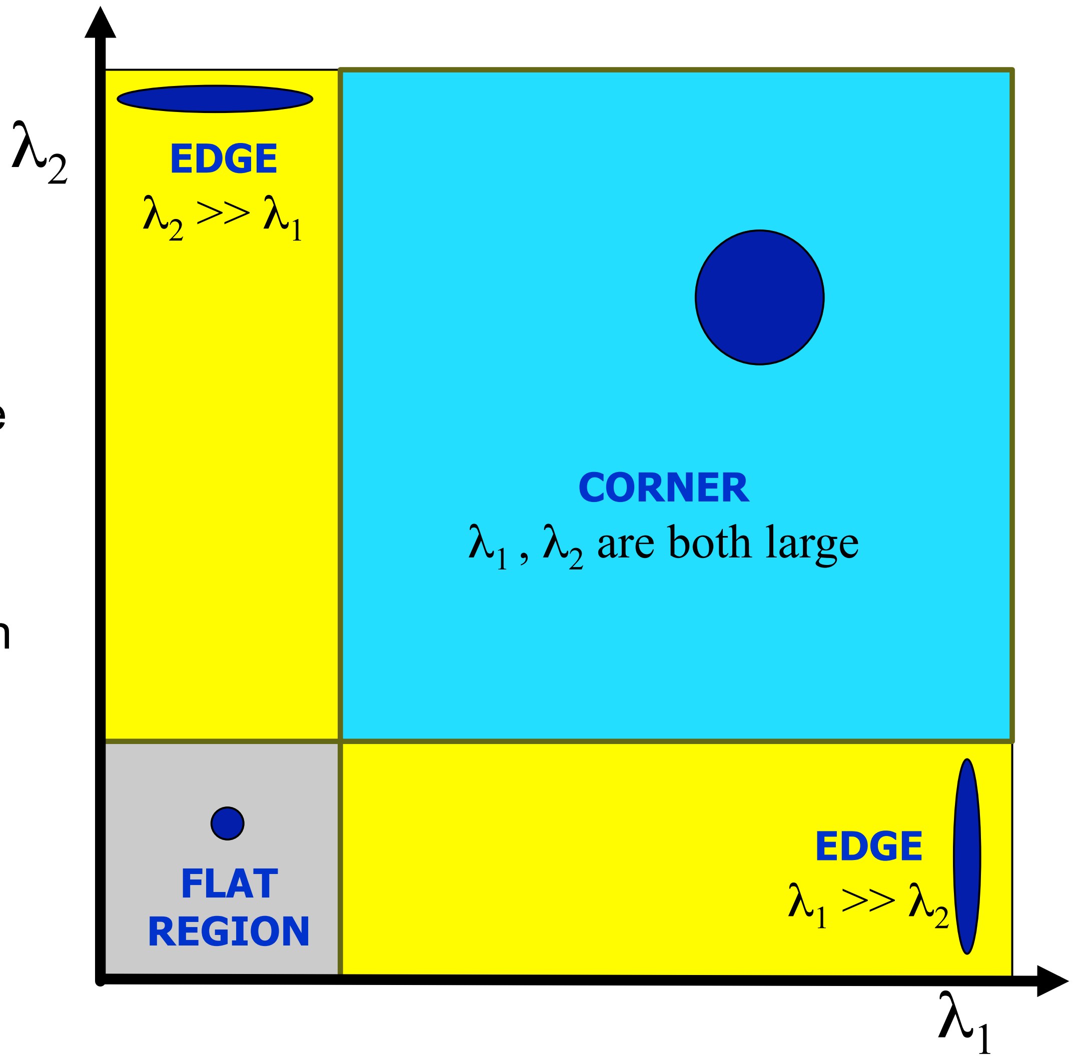


Corner detection | interpreting the eigenvalues

Does patch P describe a corner or not? $M = R^{-1}\begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}R$

- **No structure:** $\lambda_1 \approx \lambda_2 \approx 0$
  SSD is almost constant in all directions,
  so it's a **flat** region
- **1D structure:** $\lambda_1 >> \lambda_2$ is large (or vice versa)
  SSD has a large variation only in one direction, which is the one perpendicular to the **edge**.
- **2D structure:** λ_1, λ_2 are both large
  SSD has large variations in all directions and then we are in presence of a **corner**.
- **Shi-Tomasi** [1] cornerness criterion:

$$C_{SHI\text{-}TOMASI} = min(\lambda_1,\lambda_2) > thresh.$$



[1] J. Shi and C. Tomasi. "Good Features to Track,". CVPR 1994

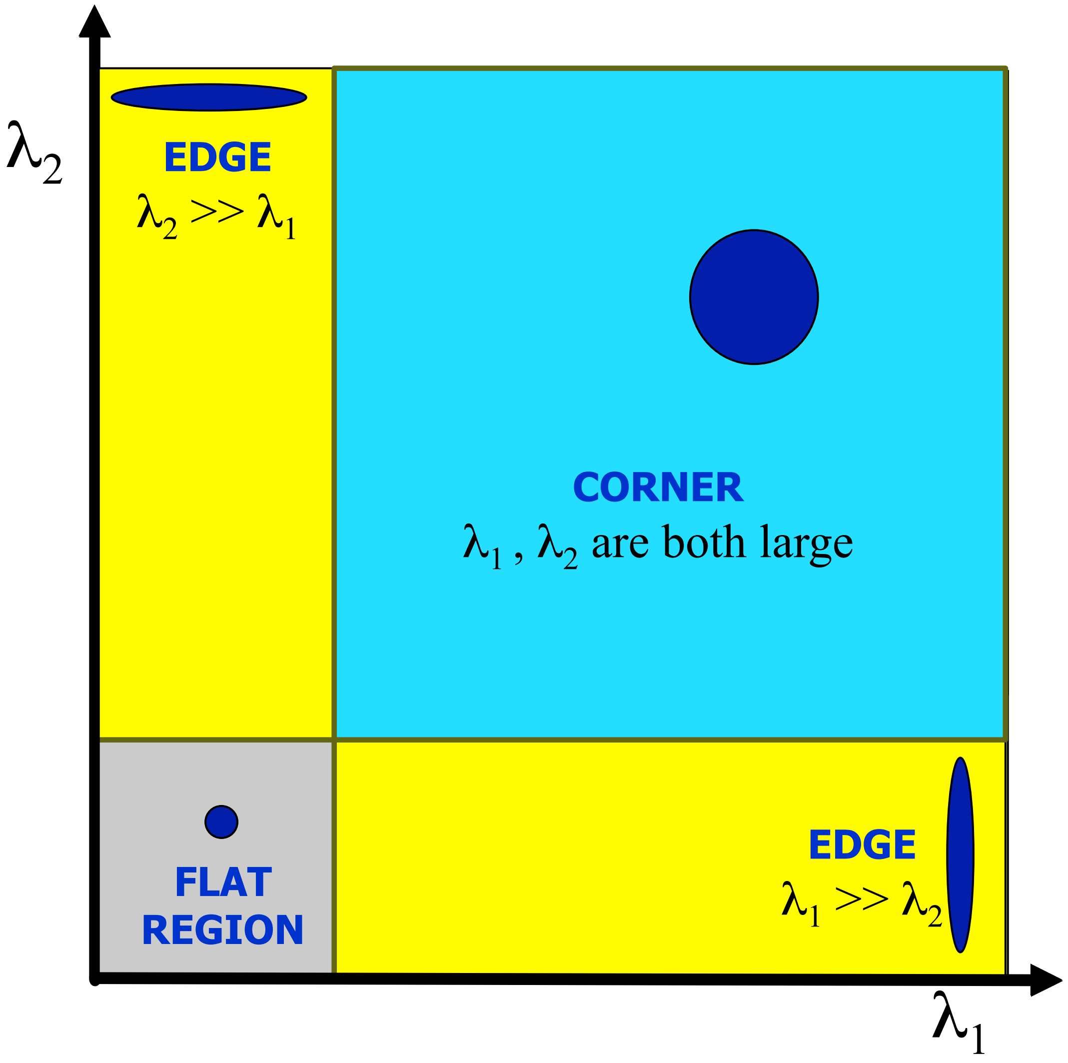
Corner detection | corner response function

- Computing λ_1 and λ_2 is expensive
  ⇨ Harris & Stephens suggested using a "**cornerness function**" instead:

$$C_{HARRIS} = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2 = \det(M) - \kappa \cdot trace^2(M)$$

  where $\kappa = const. \in [0.04, 0.15]$

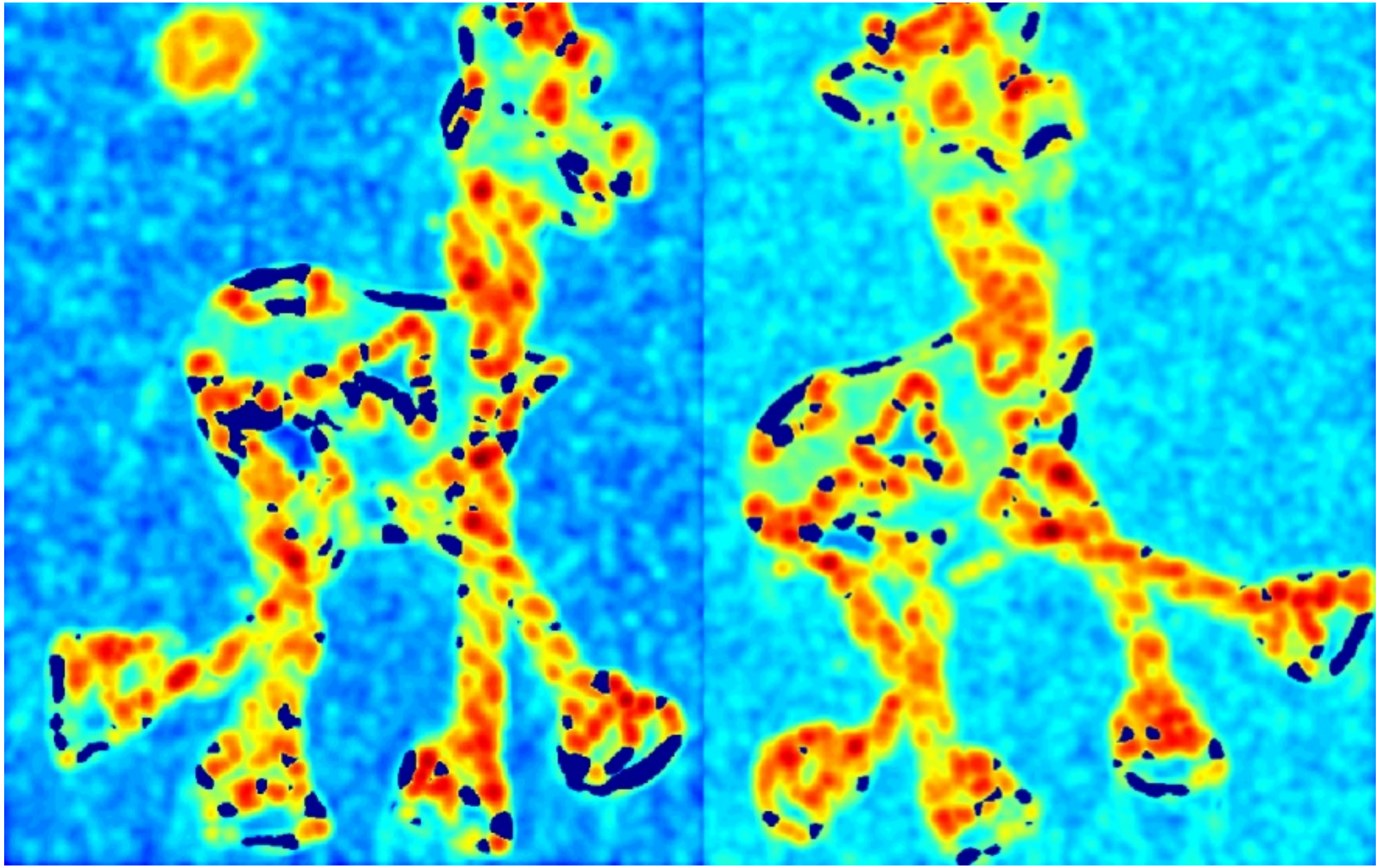
- **Harris** cornerness criterion [2]
- Last step of Harris corner detector: extract local maxima of the cornerness function



[2] C.Harris and M.Stephens. "A Combined Corner and Edge Detector.", *Proceedings of the Alvey Vision Conference, 1988*

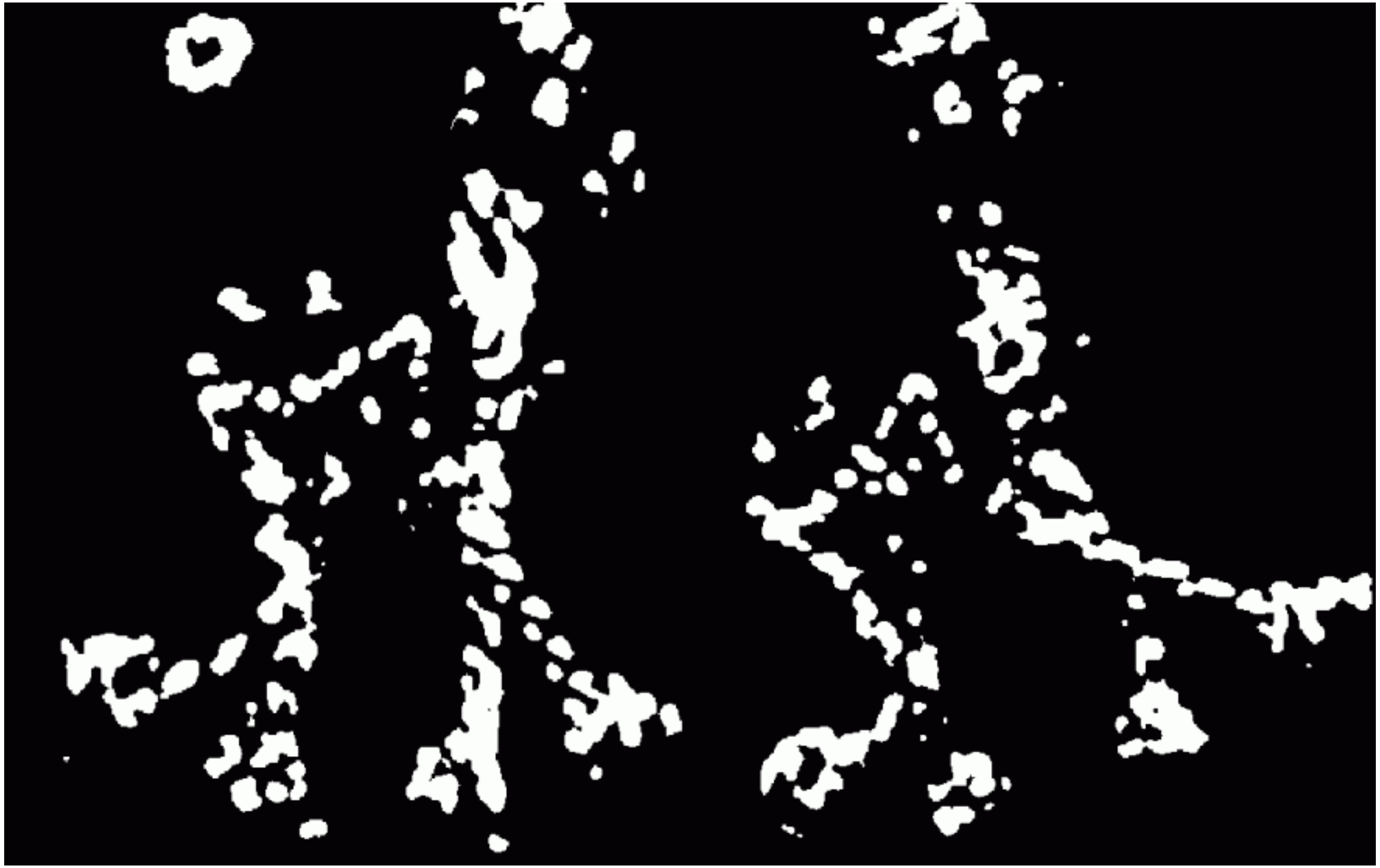
Harris corners | workflow

Harris corners | workflow



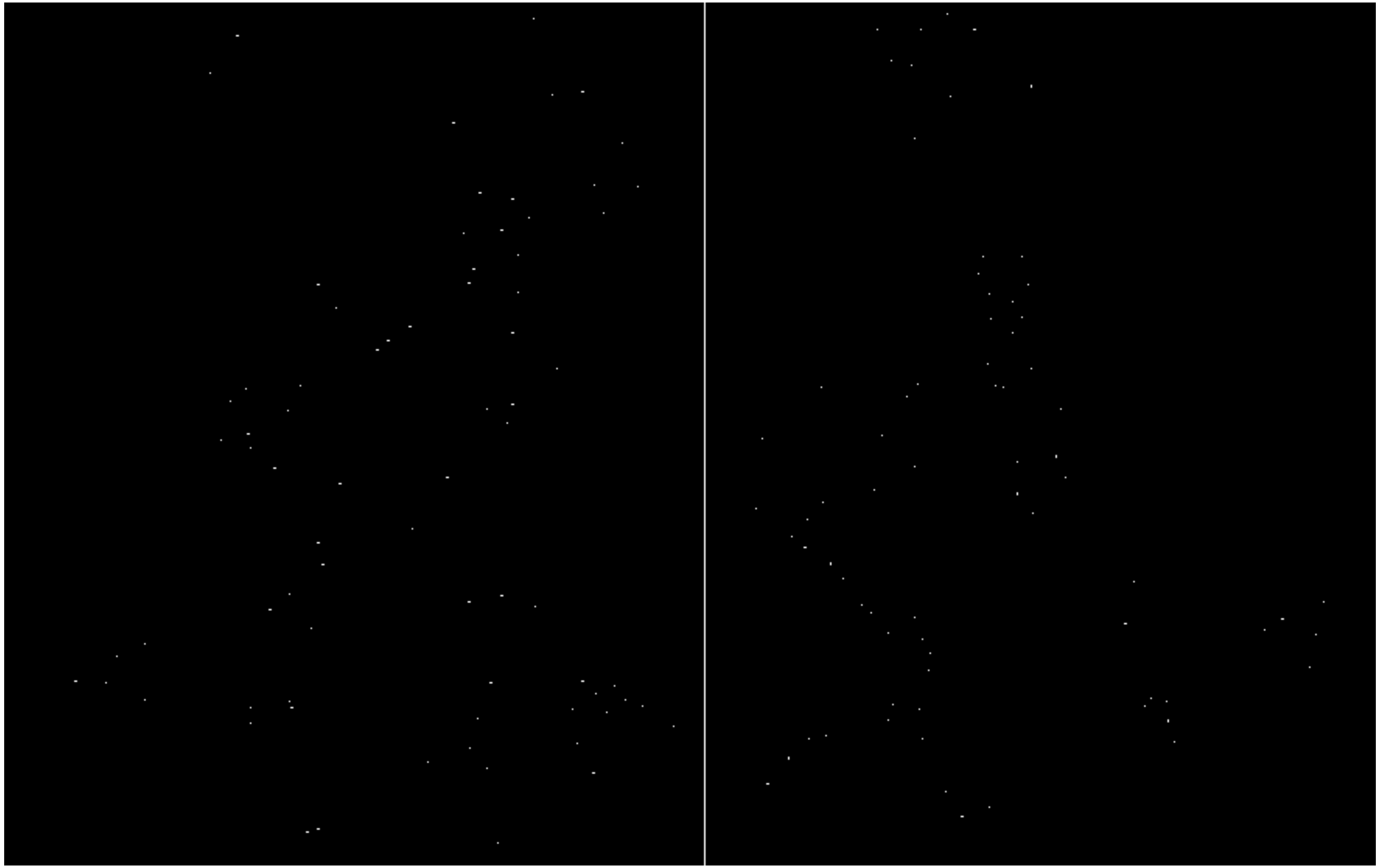
- Compute corner response C

Harris corners | workflow



- Find points with large corner response: $C > threshold$

Harris corners | workflow

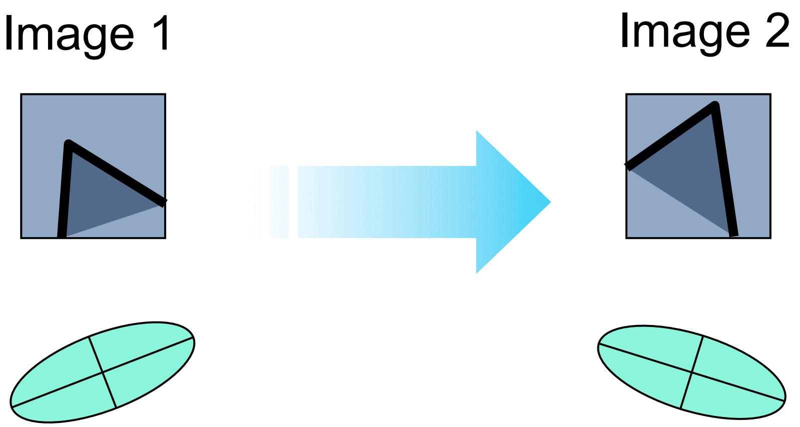


- Take only the points of local maxima of thresholded C

Harris corners | workflow

Harris corners | properties

- Rotation invariance?

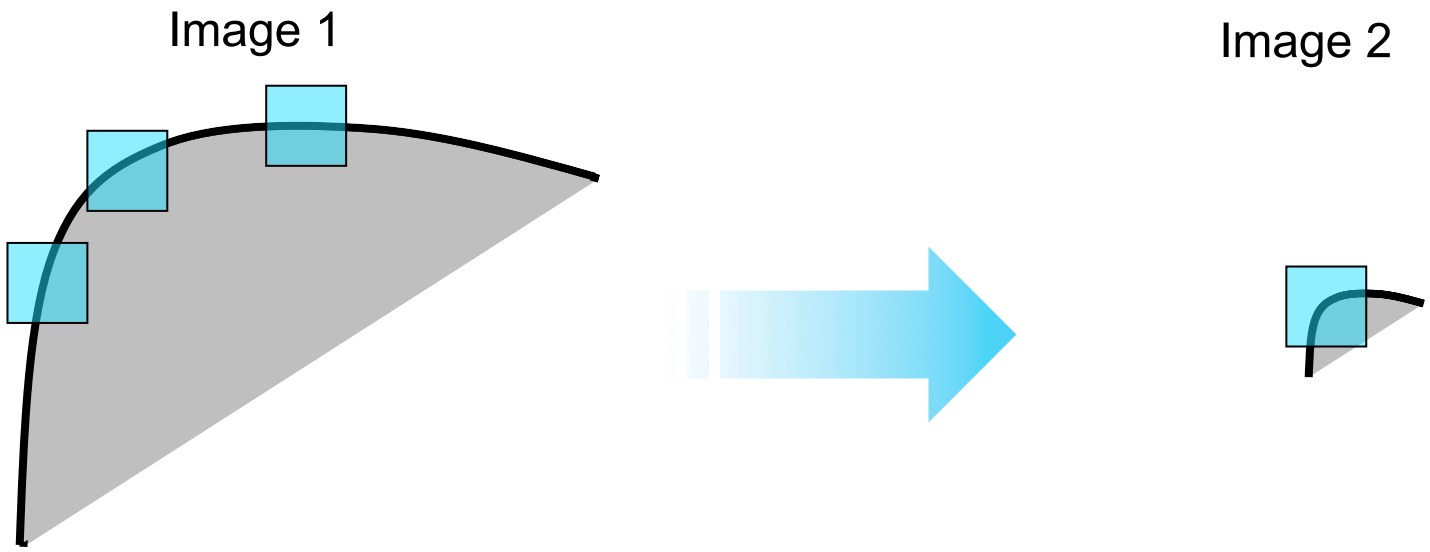


Ellipse rotates, but its shape (i.e. eigenvalues) remains the same

Harris corners are **invariant to image rotation**

Harris corners | properties

- Scale invariance?



All points will be classified as **edges**

Corner!

Harris corners are **not invariant to scale change**

Harris corners | properties' summary

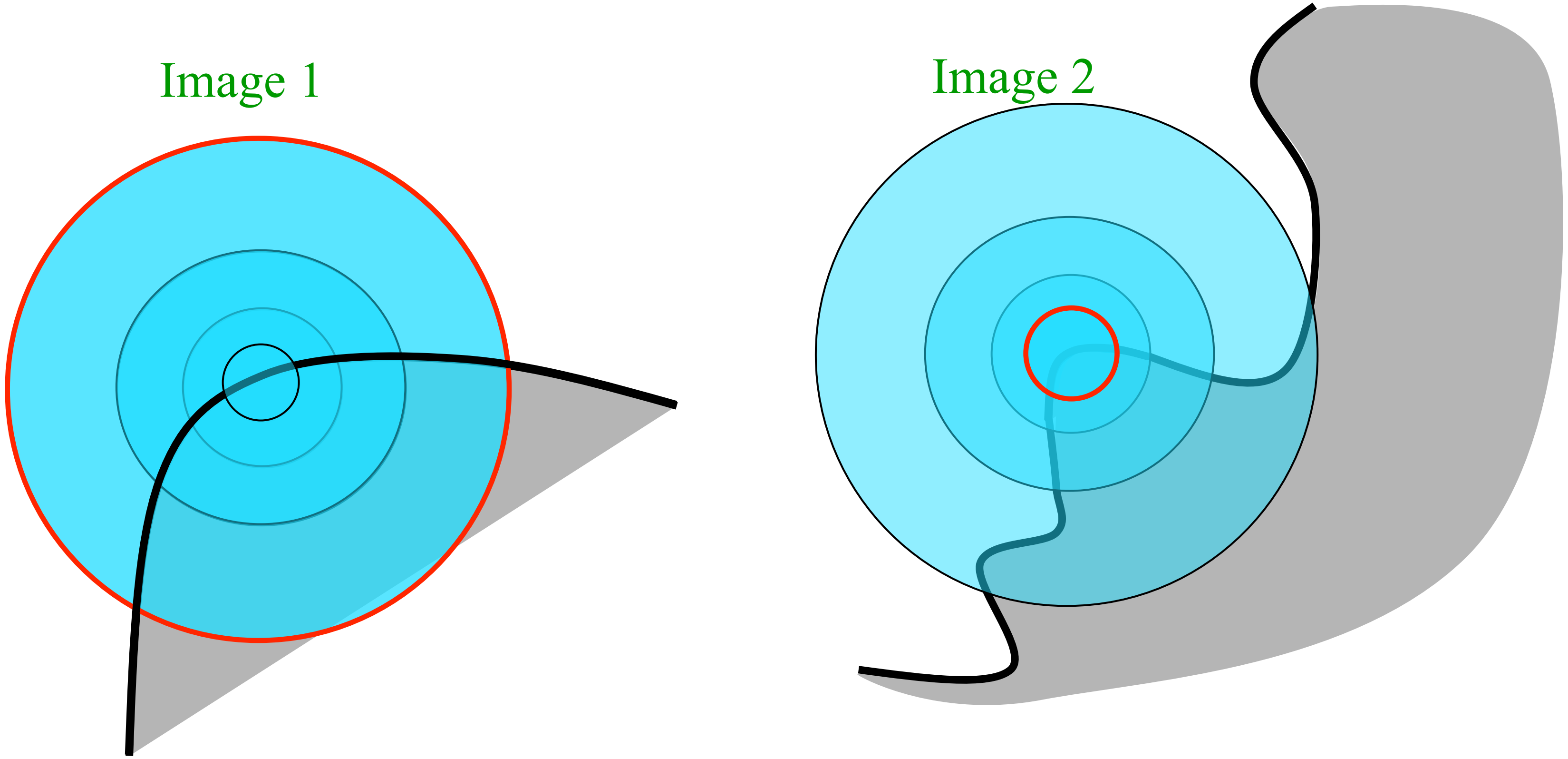
- Harris detector: probably the most widely used and known corner detector

- The detection is invariant to
  - Rotation
  - Linear intensity changes
  - **note:** to make the matching invariant to these we need suitable descriptor & matching

- The detection is NOT invariant to
  - Scale changes
  - Geometric affine changes: an image transformation, which distorts the neighborhood of the corner, can distort its '*cornerness*' response

Scale-invariant feature detection

- Consider regions (e.g. discs) of different sizes around a point
- **Aim**: corresponding regions look the same in image space, when the appropriate scale-change is applied



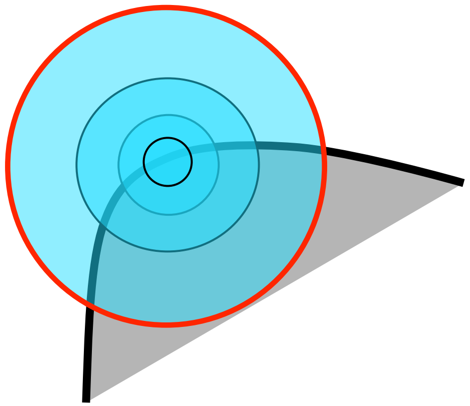
- Choose corresponding regions (discs) **independently** in each image

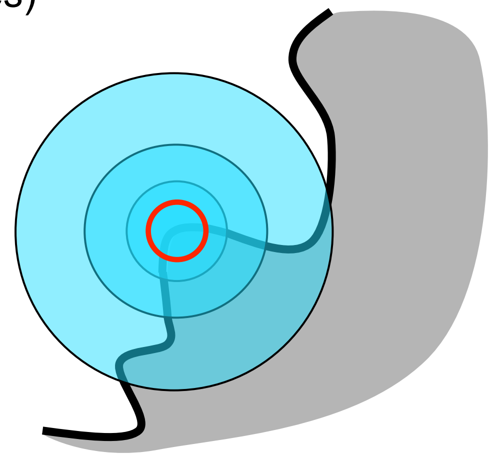
Scale-invariant feature detection

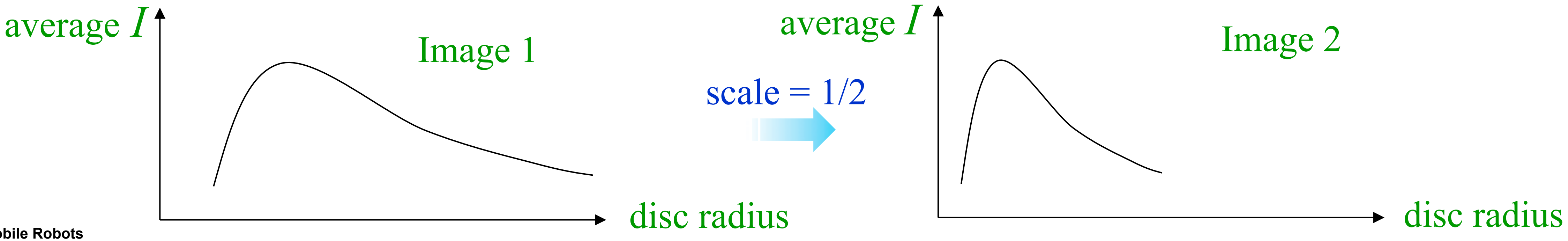
- Approach: design a function to apply on the region (disc) , which is “scale invariant”

(i.e. remains constant for corresponding regions, even if they are at different scales)

<u>example</u>: **average image intensity** over corresponding regions (at different scales) should remain constant



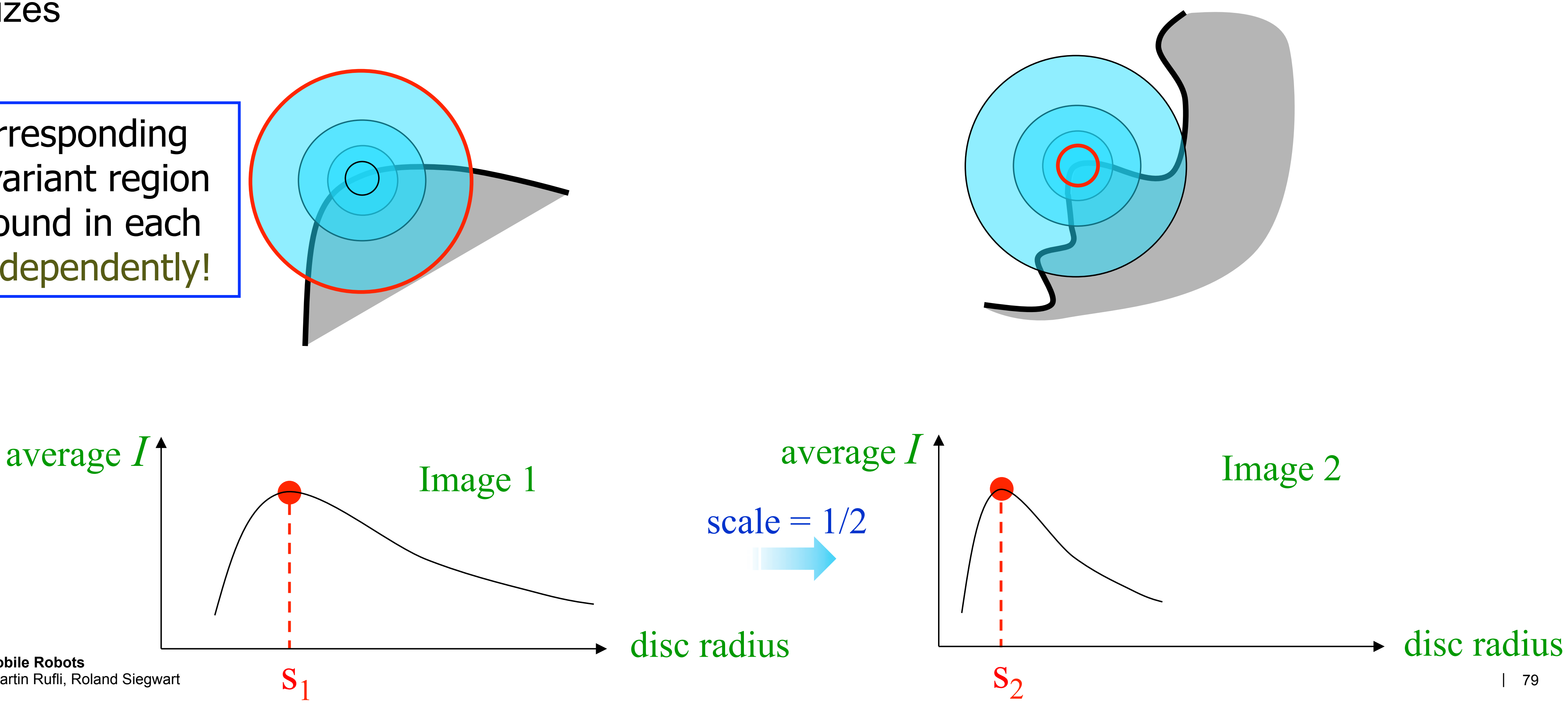




Scale-invariant feature detection

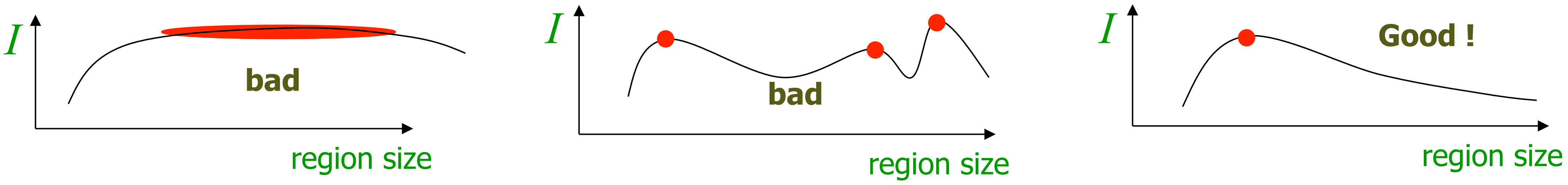
- Identify the local maximum in each response ⇨ these occur at corresponding region sizes

The corresponding scale-invariant region size is found in each image independently!



Scale-invariant feature detection

- A "good" function for scale detection has one clear, sharp peak



- Sharp, local intensity changes in an image, are good regions to monitor for identifying relative scale in usual images.

⇨ look for blobs or corners (i.e. sharp intensity discontinuities)

Scale-invariant feature detection | LoG scale detector

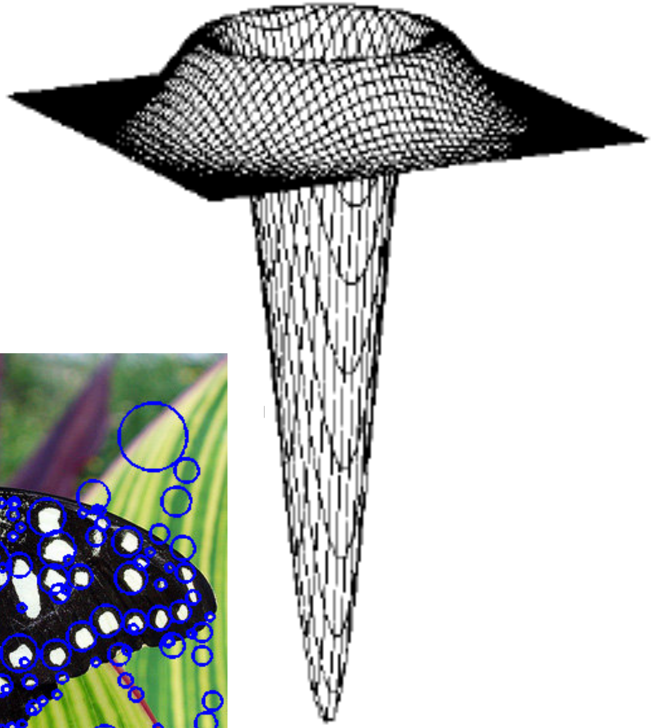
- Functions of determining scale: convolve image with kernel to identify sharp intensity discontinuities

$$f = \text{Kernel} * \text{Image}$$

- Detected scale corresponds to **local maxima or minima** of the convolved image region

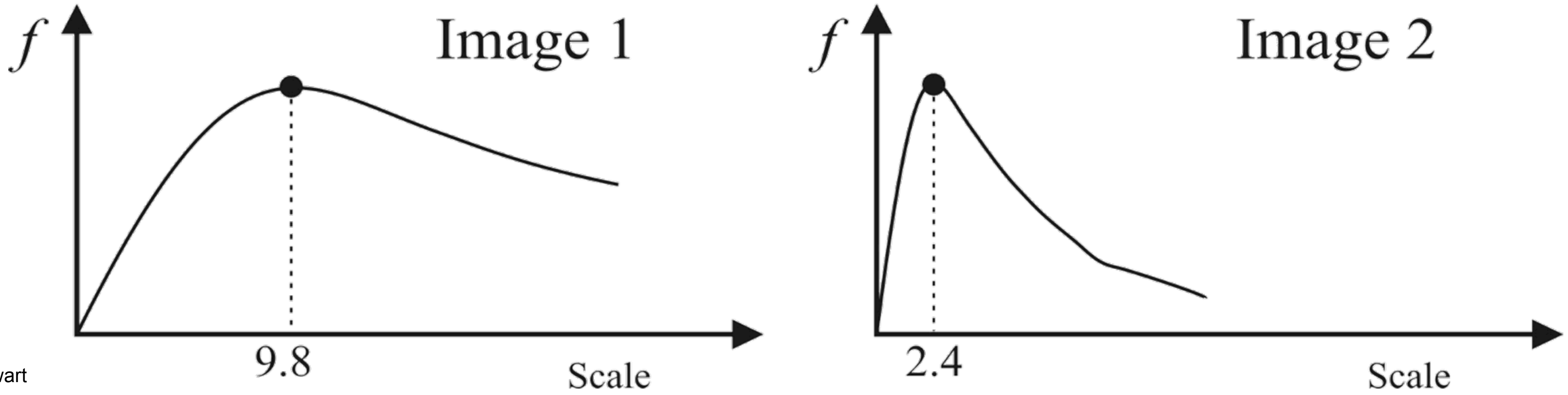
$$LoG = \nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$

Laplacian of Gaussian



Scale-invariant feature detection | LoG scale detector

- Response of LoG for corresponding regions:



Scale-invariant feature detection | DoG scale detector

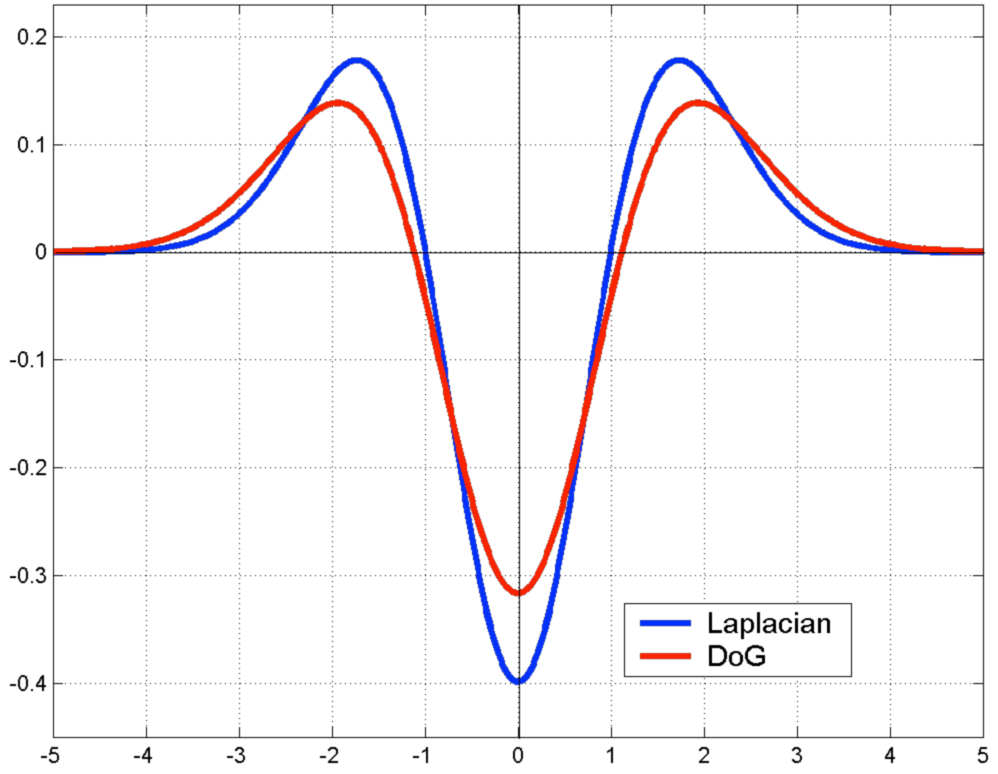
- Approximation to the LoG kernel for efficiency:

  Difference of Gaussians (DoG) kernel:

$$DoG = G_{k\sigma}(x, y) - G_{\sigma}(x, y)$$

- Used in the **SIFT** feature detector [Lowe et al., IJCV 2004]

- The **SURF** feature detector [Bay et al, CVIU 2008] implements the DoG kernel using a linear combination of rectangular functions

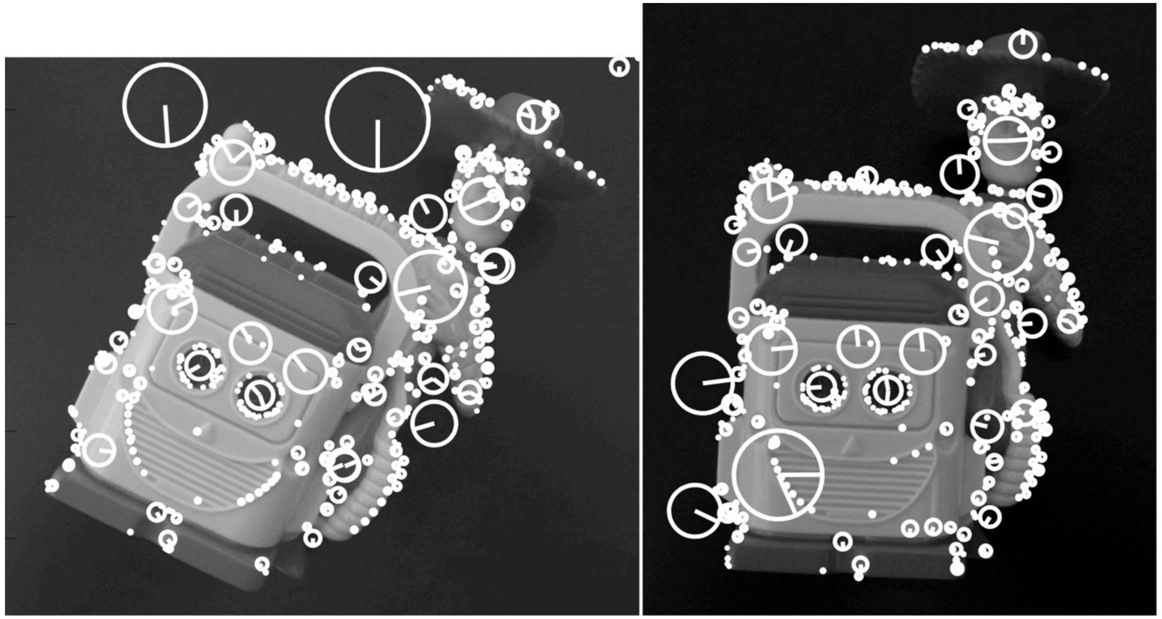


SIFT features [Lowe et al., IJCV 2004]

- **SIFT** = Scale Invariant Feature Transform
an approach for detecting and describing regions of interest in an image
- SIFT features are reasonably **invariant** to changes in:
rotation, scaling, changes in viewpoint, illumination
- SIFT **detector** uses **DoG kernel**, SIFT **descriptor** is based on **gradient orientations**
- Very powerful in capturing + describing **distinctive** structure, but also **computationally demanding**

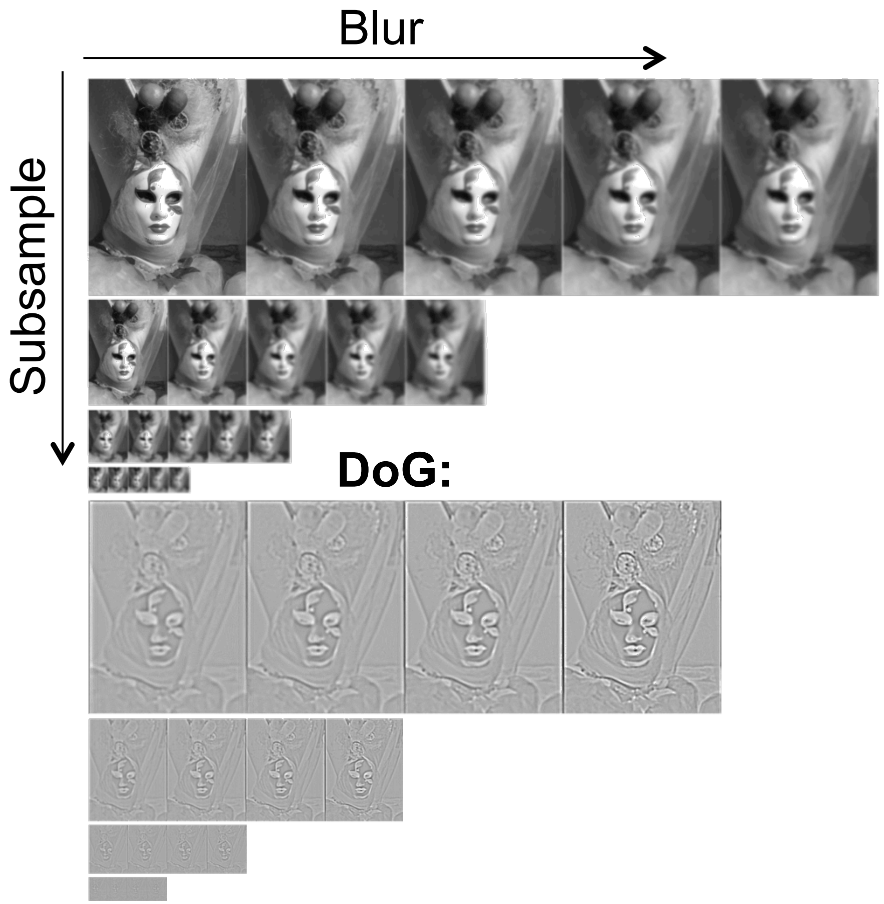
Main SIFT stages:

1. Extract keypoints + scale
2. Assign keypoint orientation
3. Generate keypoint descriptor

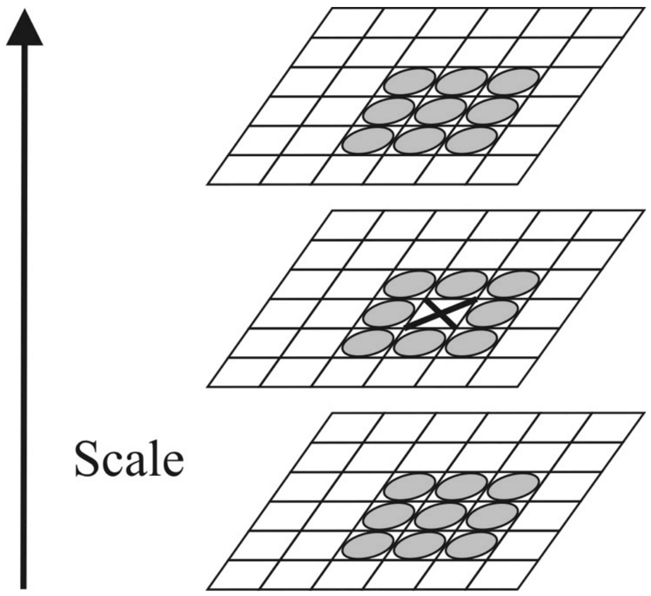


SIFT features | detector (keypoint location + scale)

1. Scale-space pyramid: subsample and blur original image



3. Keypoints: local extrema in the DoG pyramid



SIFT features | keypoint orientation assignment

Define “orientation” of keypoint to achieve **rotation invariance**

- Sample intensities around the keypoint
- Compute a histogram of orientations of intensity gradients

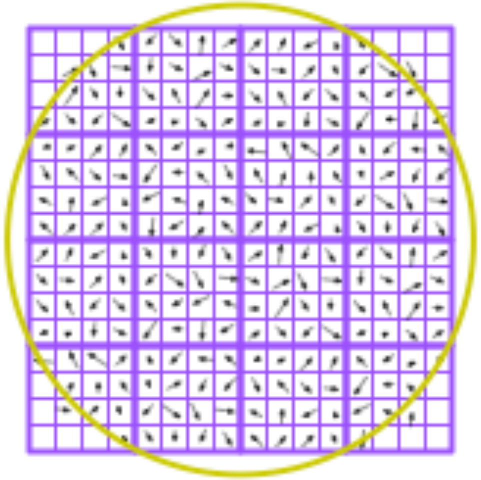
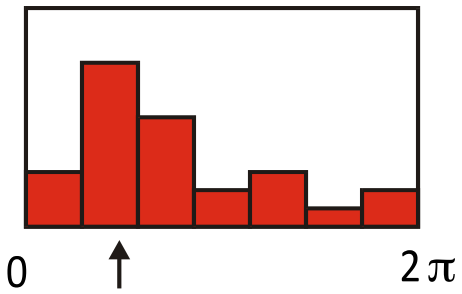


Image gradients

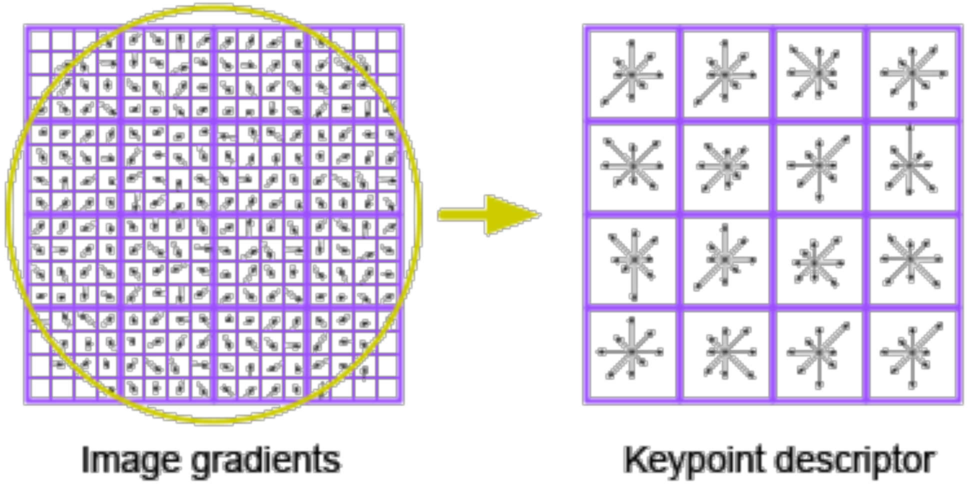
- Peaks in histogram: dominant orientations
- **Keypoint orientation = histogram peak**
- If there are multiple candidate peaks, construct a different keypoint for each such orientation



SIFT features | descriptor

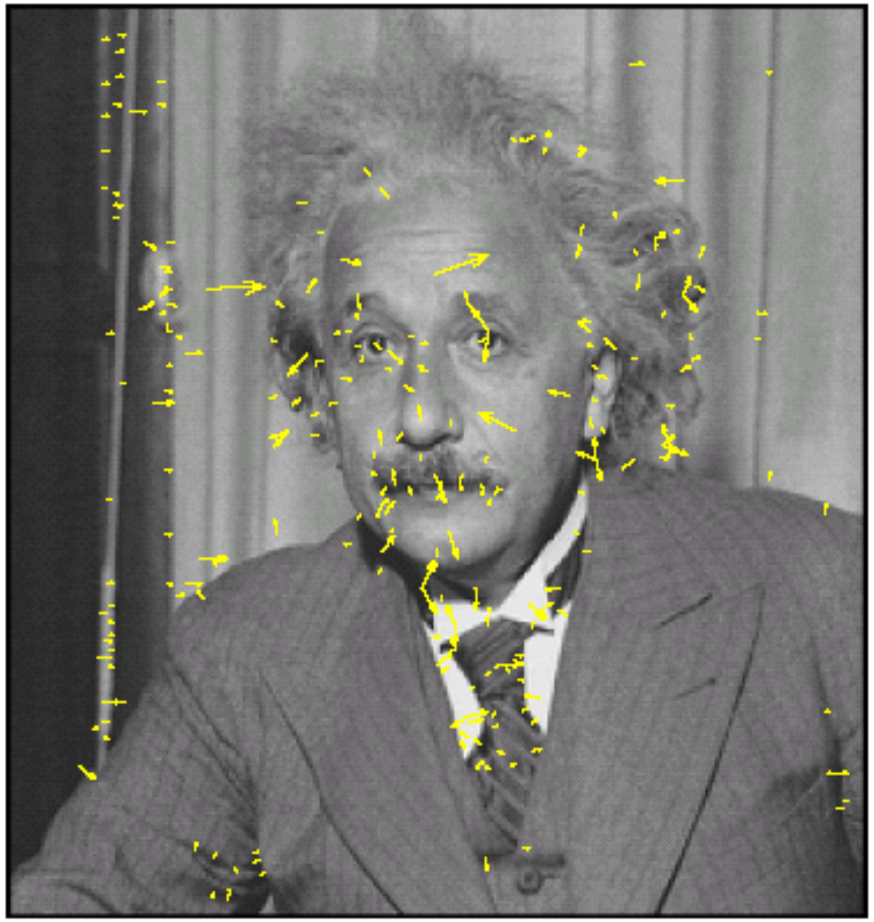
- **Descriptor** : "identity card" of keypoint
- Simplest descriptor: matrix of intensity values around a keypoint (image patch)
- Ideally, a descriptor should be
  - highly distinctive +
  - tolerant/invariant to common image transformations

- **SIFT descriptor**: 128-element vector
- Describe all gradient orientations **relative to the keypoint orientation**
- Divide keypoint neighborhood in **4×4** regions and compute orientation histograms along **8** directions
- SIFT descriptor: concatenation of all **4×4×8 (=128)** values
- Descriptor Matching: L_2-distance (i.e. SSD) between these descriptor vectors

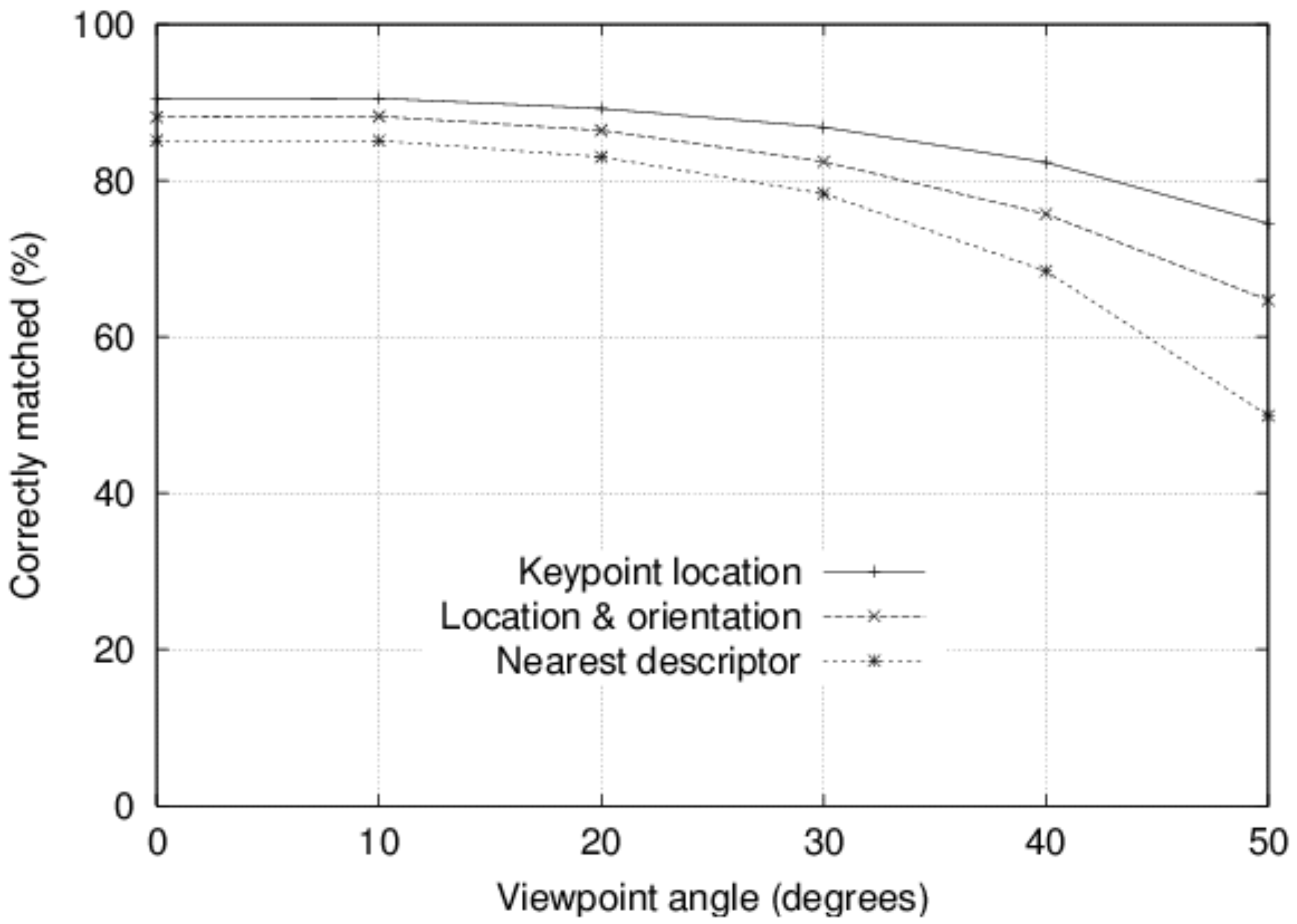


SIFT features

- Final SIFT keypoints with detected orientation & scale

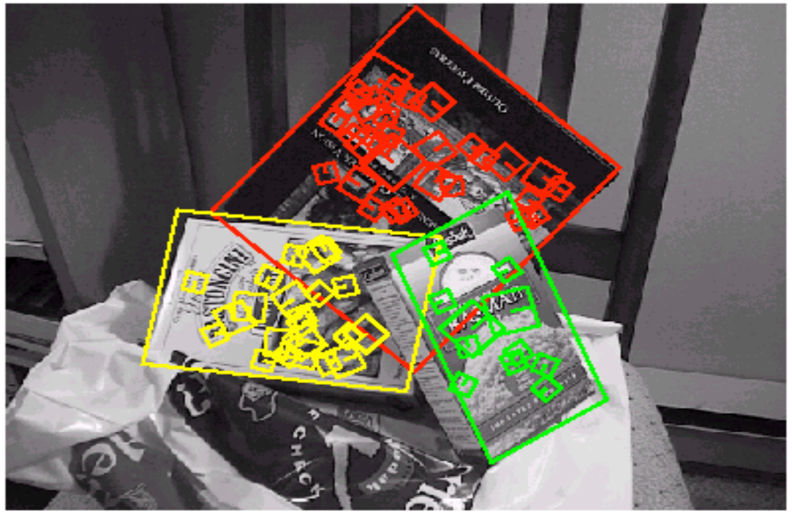


SIFT features | features' stability to viewpoint change



SIFT features | use in planar recognition

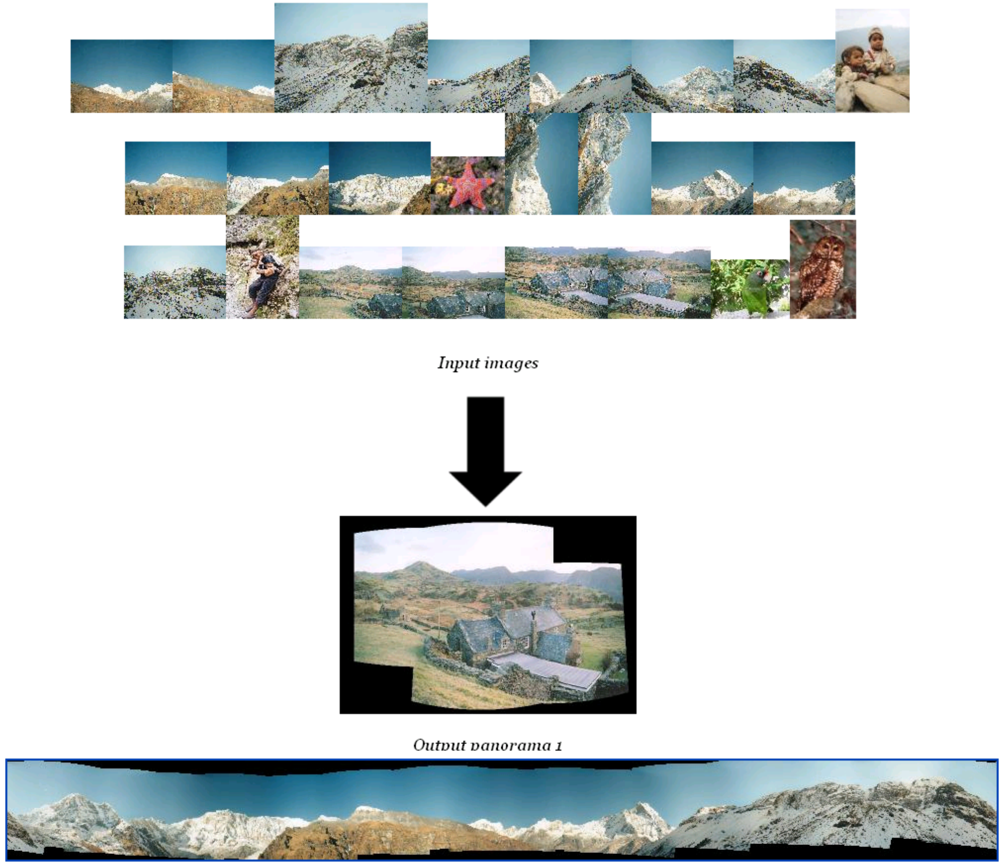
- **Planar** surfaces can be reliably recognized at a rotation of **60°** away from the camera
- Only 3 points are needed for recognition
- But objects need to have enough **texture**
- Recognition under occlusion



SIFT features | code and demos

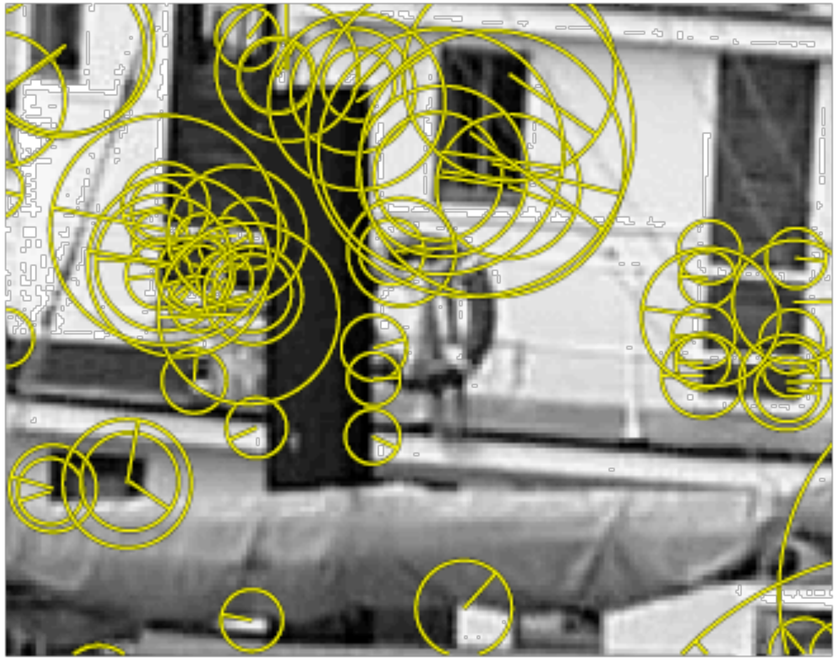
- **SIFT feature detector code:**
  for Matlab & C code to run with compiled binaries for Win and Linux (freeware)
  http://www.cs.ubc.ca/~lowe/keypoints/

- Make your own panorama with **AUTOSTITCH** (freeware):
  http://matthewalunbrown.com/autostitch/autostitch.html



More recent features from SOTA

- …suitable for Robotics applications



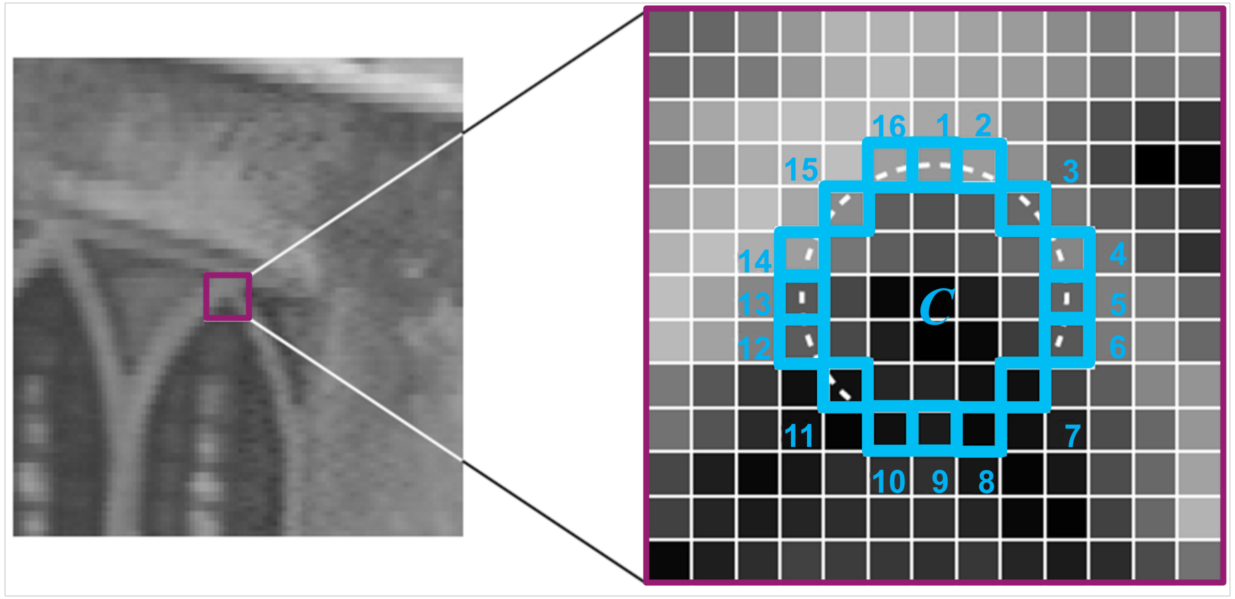
(a) Boat image 1

(b) Boat image 2

FAST corner detector [Rosten et al., PAMI 2010]

- **FAST**: **F**eatures from **A**ccelerated **S**egment **T**est
- Studies intensity of pixels on circle around candidate pixel C

- C is a FAST corner **if** a set of N contiguous pixels on circle are:
  - all brighter than `intensity_of(C)+theshold`, or
  - all darker than `intensity_of(C)+theshold`



- Typical FAST mask: test for **12** contiguous pixels in a **16**-pixel circle
- **Very fast detector** – in the order of 100 Mega-pixels/second

BRIEF descriptor [Calonder et. al, ECCV 2010]

- **BRIEF : B**inary **R**obust **I**ndependent **E**lementary **F**eatures
- Goal: high speed (in description and matching)

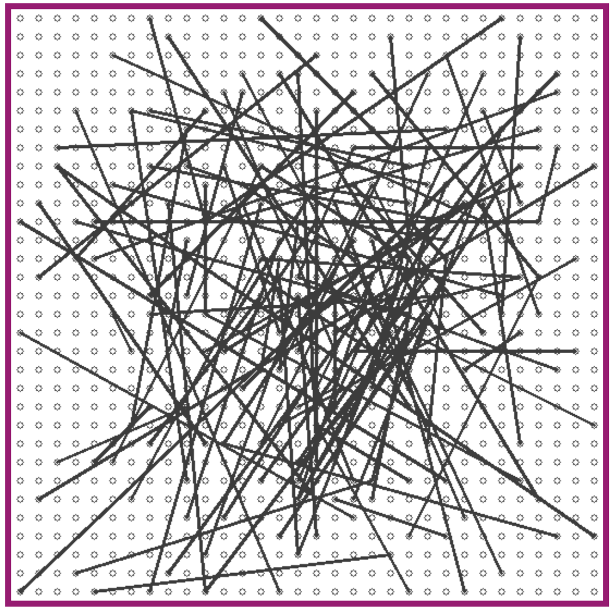
- **Binary** descriptor formation:
  - Smooth image
  - **for each** detected keypoint (e.g. FAST),

    sample all intensity pairs (I_1, I_2) (typically 256 pairs) according to pattern around the keypoint

    for each intensity pair **p**

    - **if** $I_1 < I_2$ **then** ***set*** bit **p** of descriptor to **1**
    - **else** ***set*** bit **p** of descriptor to **0**

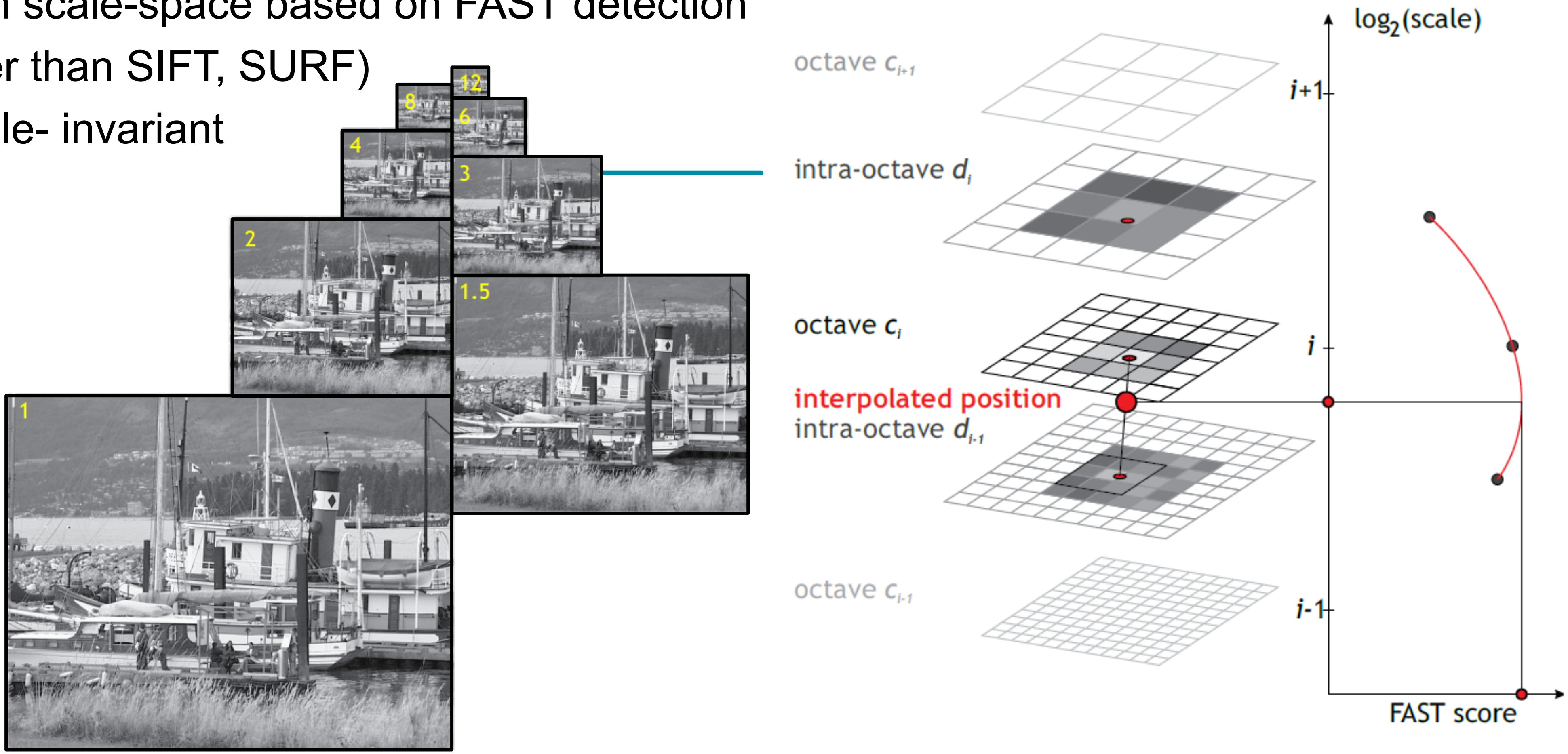
- Not scale/rotation invariant (extensions exist…)
- Allows **very fast** Hamming Distance matching: counting the no. different bits in the descriptors tested



Pattern for intensity pair samples – generated randomly

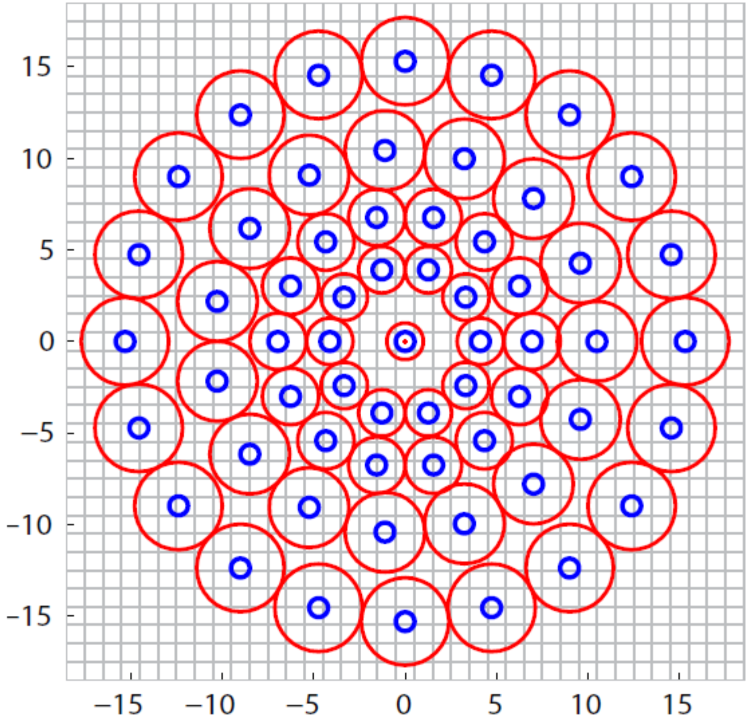
BRISK features [Leutenegger, Chli, Siegwart, ICCV, 2011] | detector

- **BRISK**: **B**inary **R**obust **I**nvariant **S**calable **K**eypoints
- Detects corners in scale-space based on FAST detection
- High-speed (faster than SIFT, SURF)
- Rotation- and scale- invariant



BRISK features | descriptor

- **Binary**, formed by pairwise intensity comparisons (like BRIEF)
- **Pattern** defines intensity comparisons in the keypoint neighborhood
- Red circles: size of the smoothing kernel applied
- Blue circles: smoothed pixel value used
- Compare short- and long-distance pairs for orientation assignment & descriptor formation
- Detection and descriptor speed: ≈10 times faster than SURF (and even faster than SIFT)
- Slower than BRIEF, but scale- and rotation- invariant



BRISK sampling pattern

BRISK feature | in action

Open-source code for FAST, BRIEF, BRISK and many more, available at the OpenCV library