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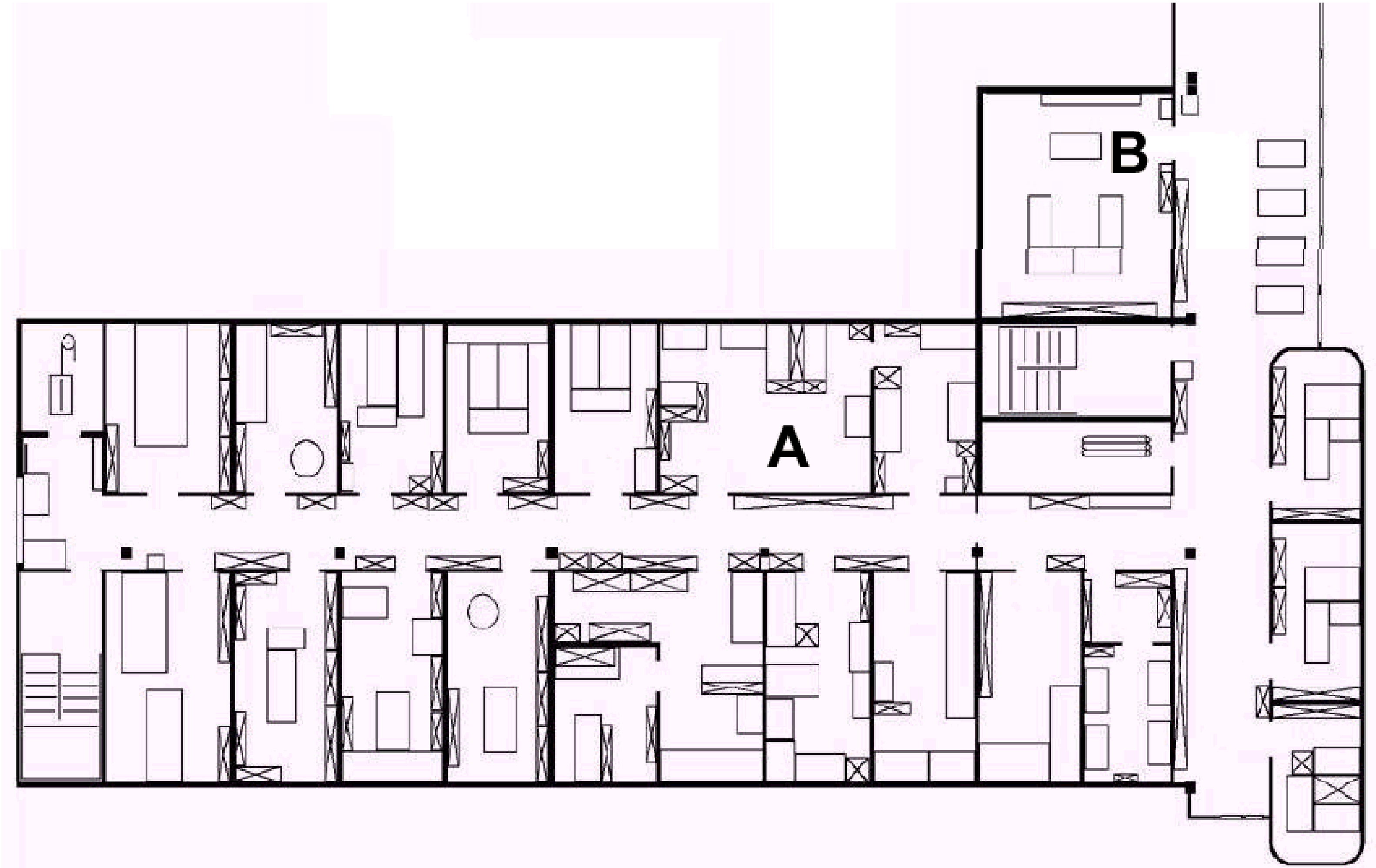
ETH Master Course: 151-0854-00L

Autonomous Mobile Robots

Localization I

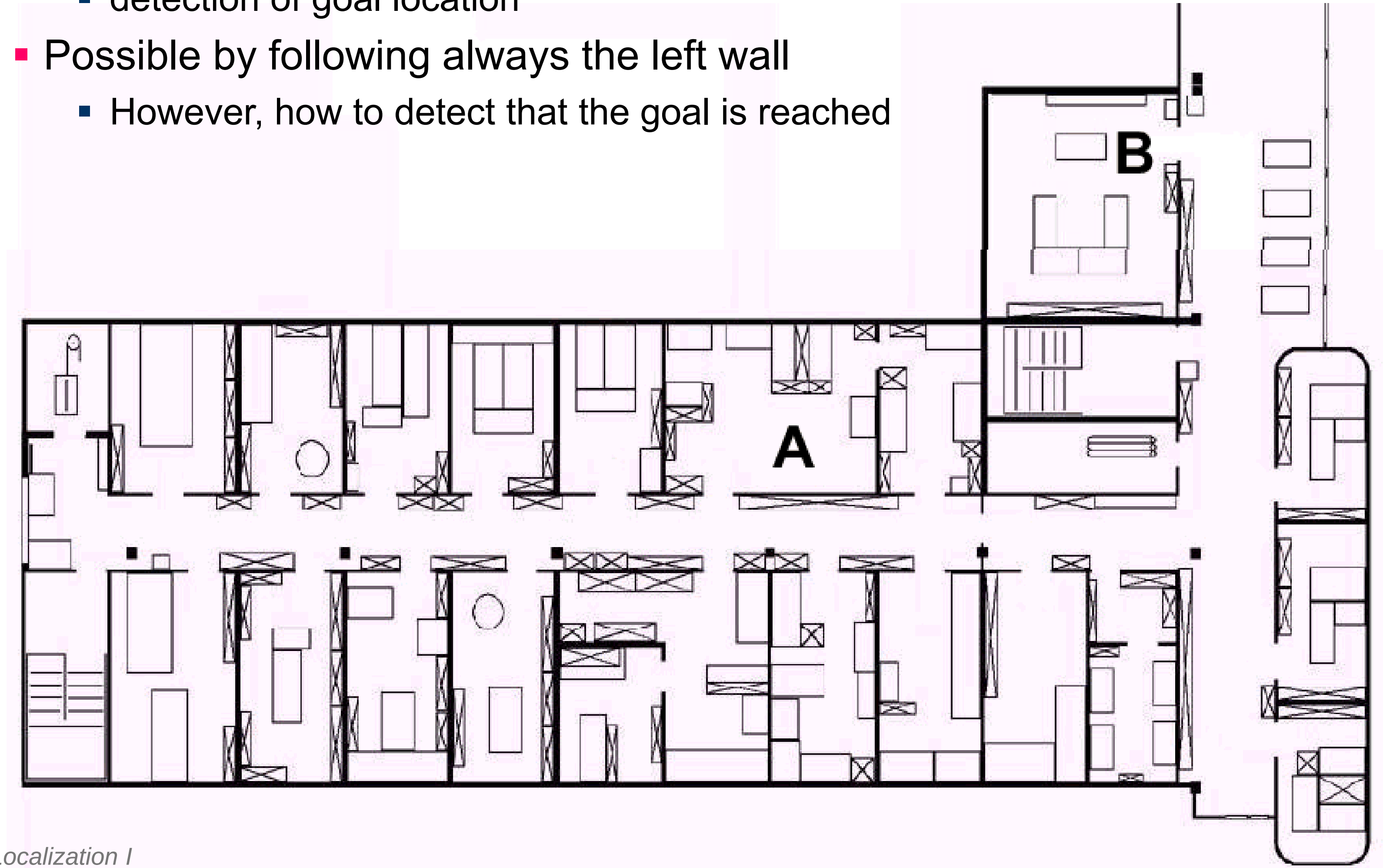
Introduction
Do we need to localize or not?

- To go from A to B, does the robot need to know where it is?



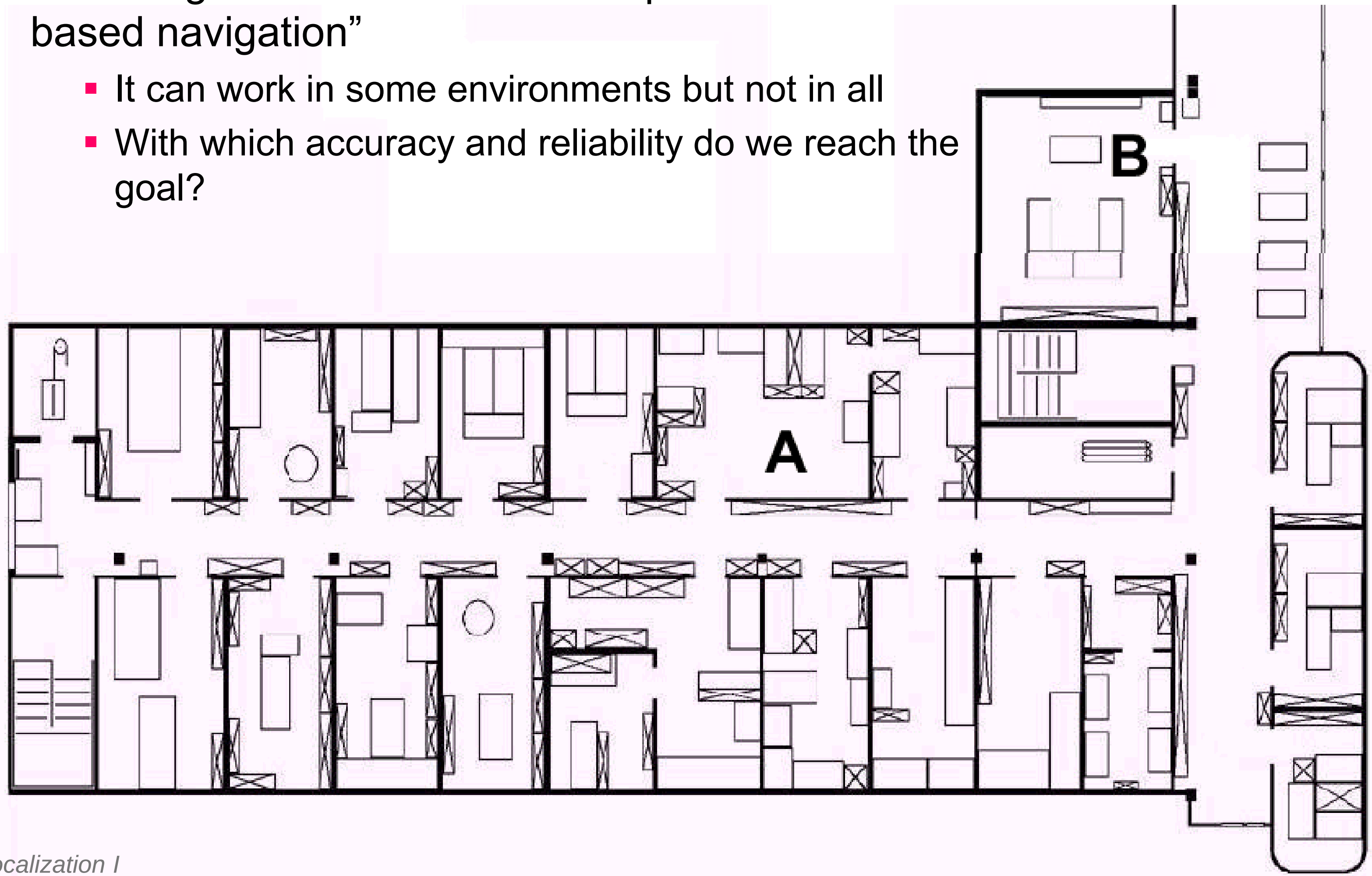
Introduction
Do we need to localize or not?

- How to navigate between A and B
  - navigation without hitting obstacles
  - detection of goal location
- Possible by following always the left wall
  - However, how to detect that the goal is reached



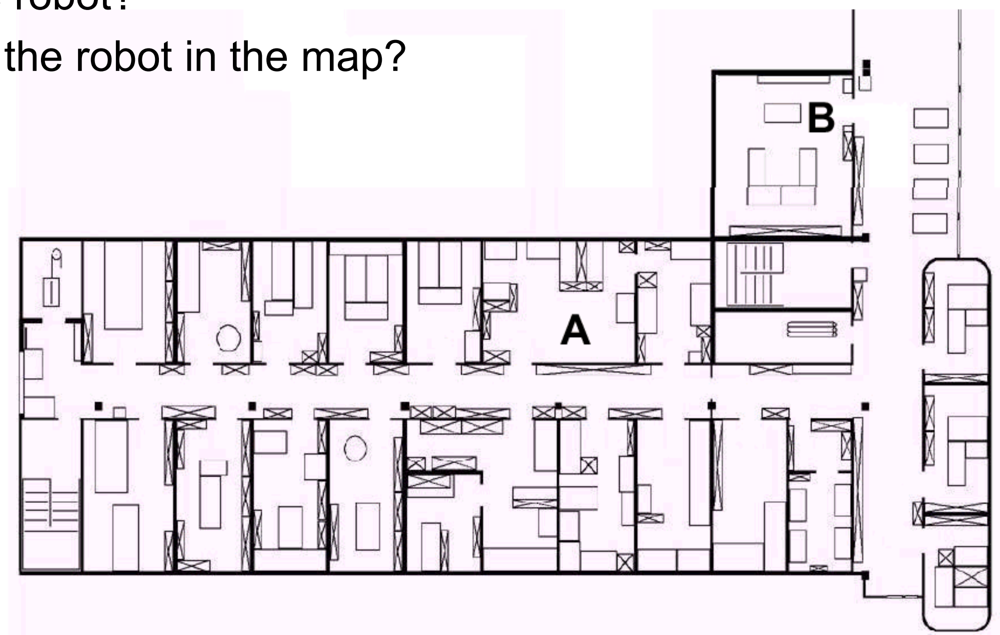
Introduction
Do we need to localize or not?

- Following the left wall is an example of "behavior based navigation"
  - It can work in some environments but not in all
  - With which accuracy and reliability do we reach the goal?



Introduction
Do we need to localize or not?

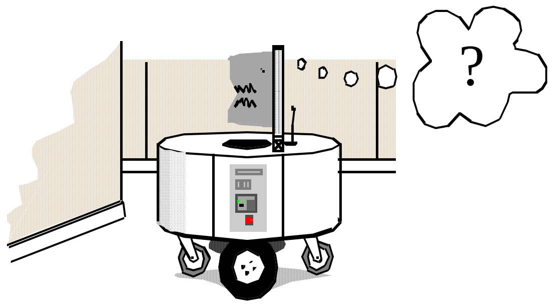
- As opposed to behavior based navigation is “map based navigation”
  - Assuming that the map is known, at every time step the robot has to know where it is. How?
    - If we know the start position, we can use wheel odometry or dead reckoning. Is this enough? What else can we use?
- But how do we represent the map for the robot?
- And how do we represent the position of the robot in the map?



Introduction
Definitions

- Global localization
  - The robot is not told its initial position
  - Its position must be estimated from scratch

- Position Tracking
  - A robot knows its initial position and “only” has to accommodate small errors in its odometry as it moves



Introduction
How to localize?

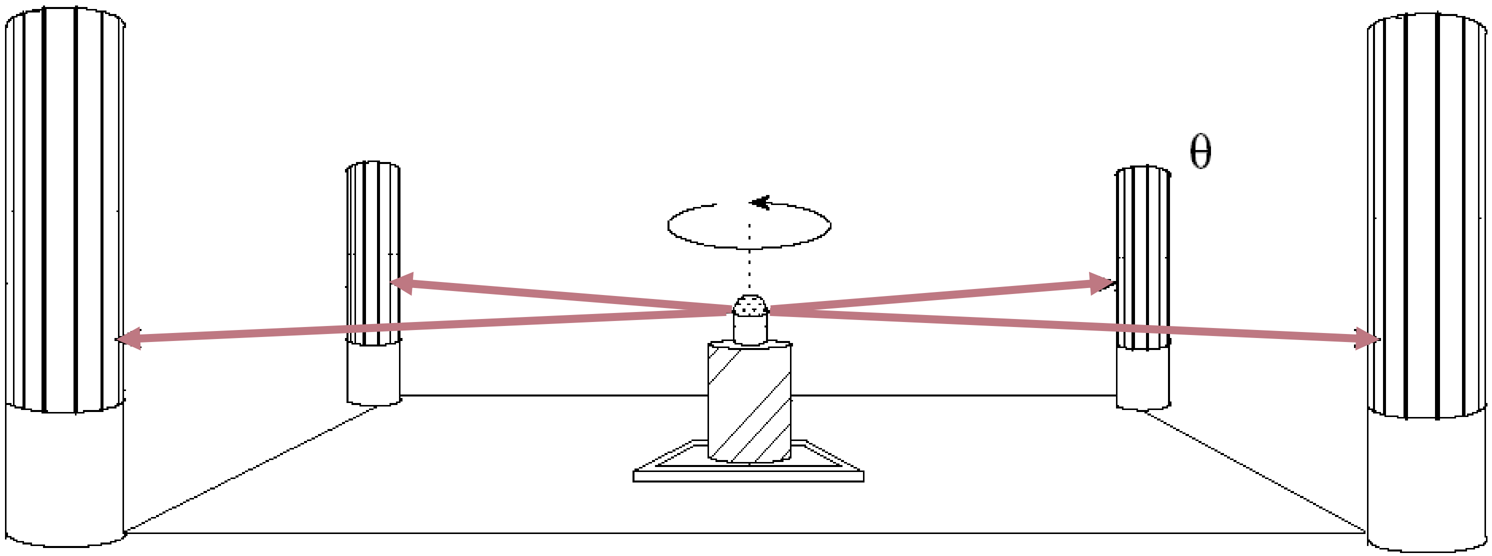
- Localization based on external sensors, beacons or landmarks

- Odometry

- Map Based Localization (without external sensors or artificial landmarks. Just use robot onboard sensors)
  - Example: Probabilistic Map Based Localization

Triangulation

- Ex 1: Poles with highly reflective surface and a laser for detecting them
- Ex 2: Coloured beacons and an omnidirectional camera for detecting them (example: RoboCup or autonomous robots in tennis fields)



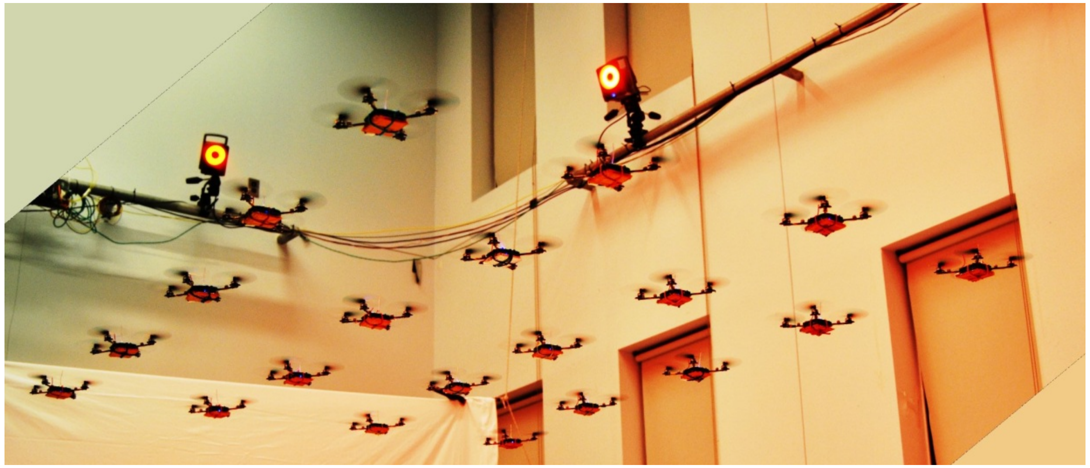
KIVA Systems, Boston (MA) (acquired by Amazon in 2011)

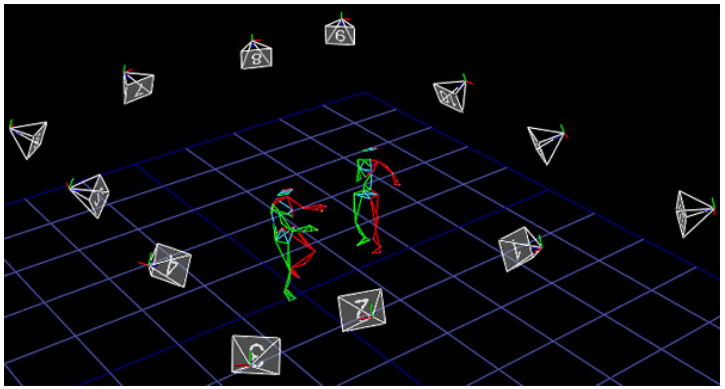
Unique marker with known absolute 2D position in the map

Prof. Raff D'Andrea, ETH

Introduction
Motion Capture Systems

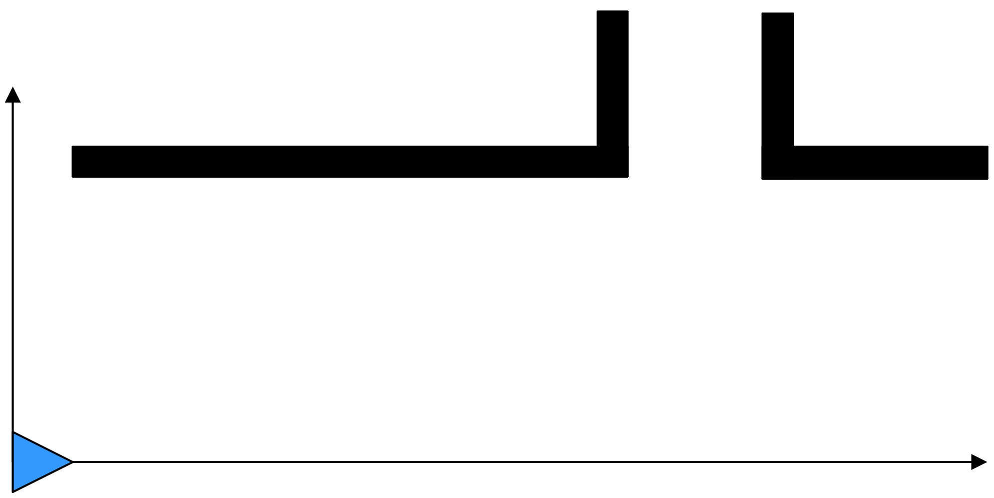
- High resolution (from VGA up to 16 Mpixels)
- Very high frame rate (several hundreds of Hz)
- Good for ground truth reference and multi-robot control strategies
- Popular brands:
  - VICON (10kCHF per camera),
  - OptiTrack (2kCHF per camera)





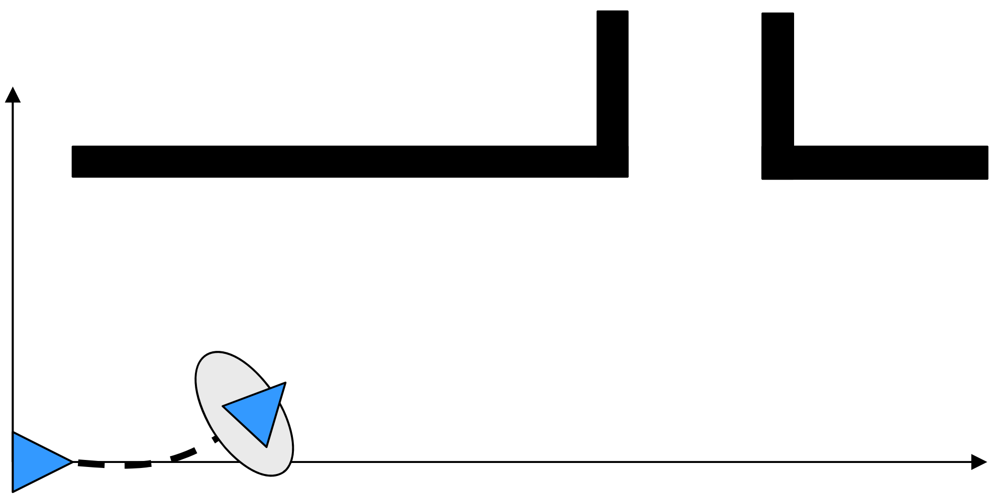
Introduction
Map-based localization

- Consider a mobile robot moving in a known environment.



Introduction
Map-based localization

- Consider a mobile robot moving in a known environment.
- As it starts to move, say from a precisely known location, it can keep track of its motion using odometry.



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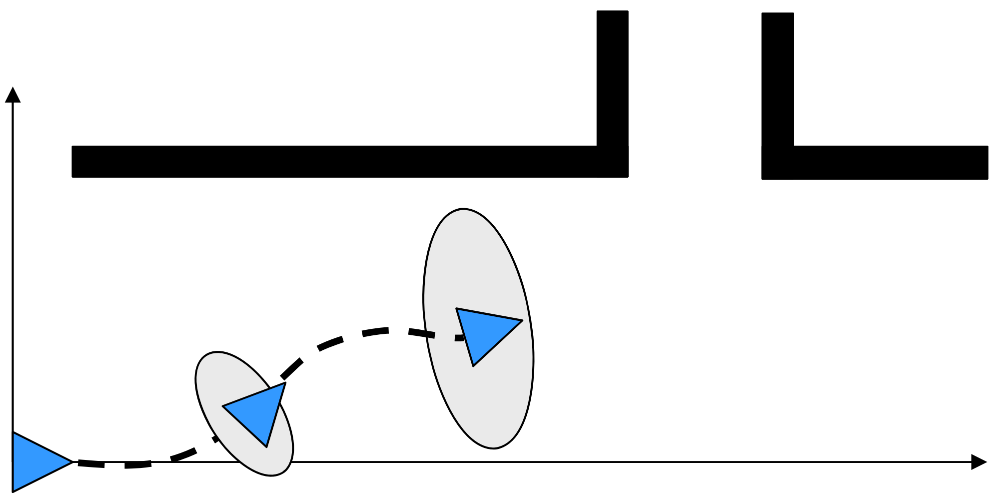

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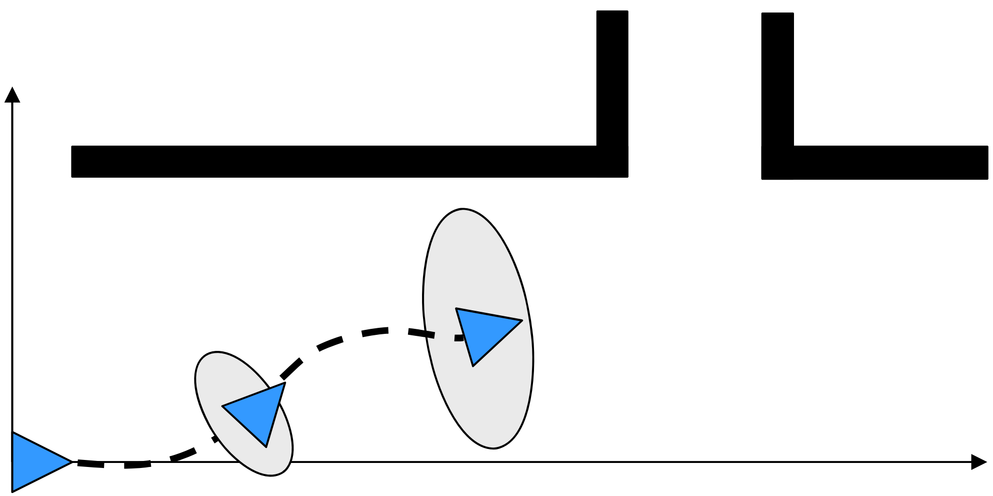
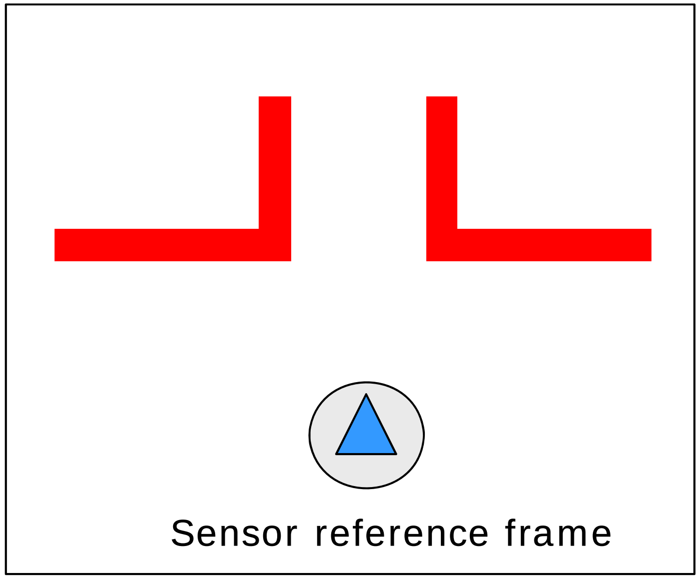
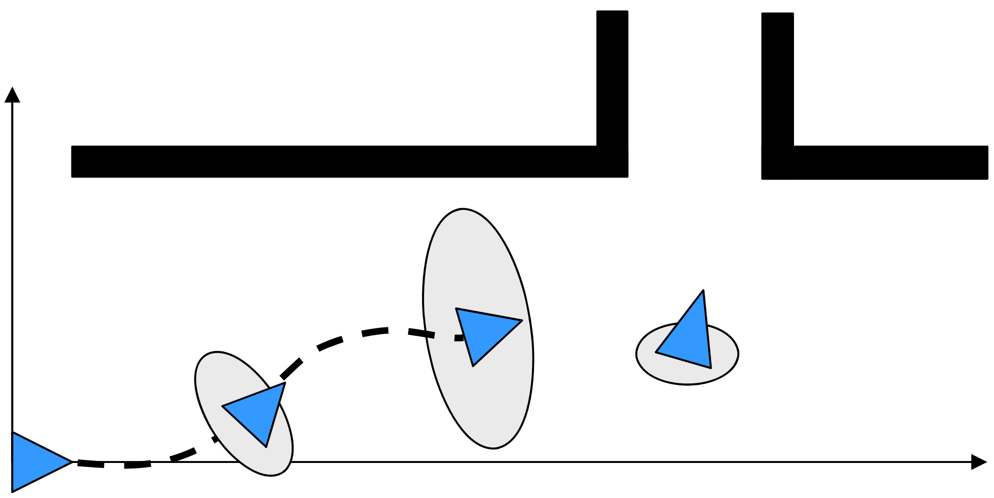
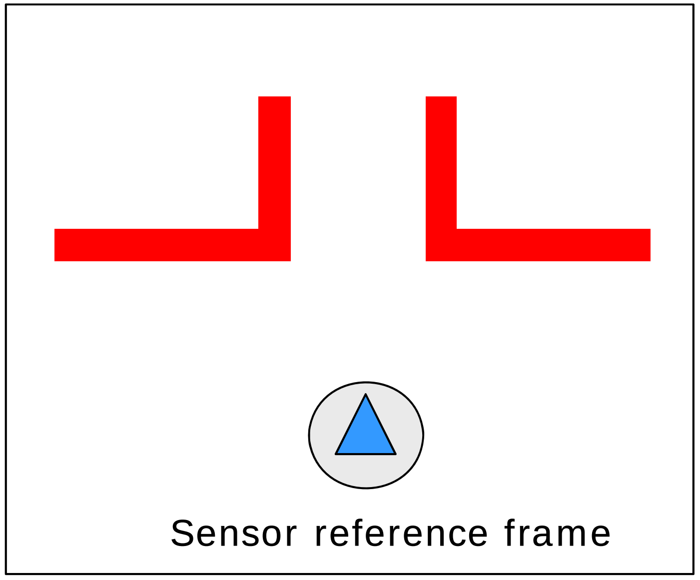




Introduction
Map-based localization

- Consider a mobile robot moving in a known environment.
- As it starts to move, say from a precisely known location, it can keep track of its motion using odometry.
- The robot makes an observation and updates its position and uncertainty





Ingredients
Probabilistic Map-based localization

▪ Probability theory	→ error propagation, sensor fusion
▪ Belief representation (map/position)	→ discrete / continuous
▪ Motion model	→ odometry model
▪ Sensing	→ measurement model

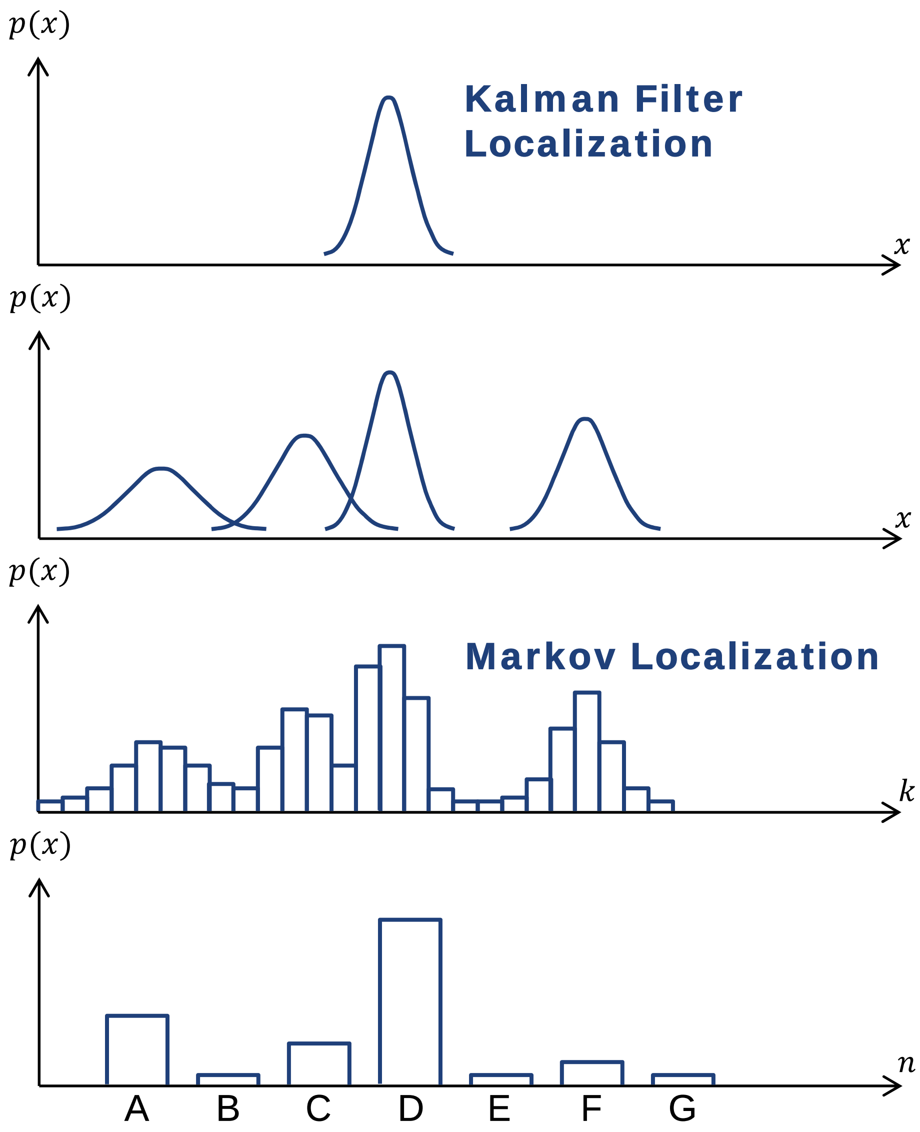
Probabilistic localization belief representation

a) Continuous map with single hypothesis probability distribution $p(x)$

b) Continuous map with multiple hypotheses probability distribution $p(x)$

c) Discretized metric map (grid k) with probability distribution $p(k)$

d) Discretized topological map (nodes n) with probability distribution $p(n)$



Belief Representation
Characteristics

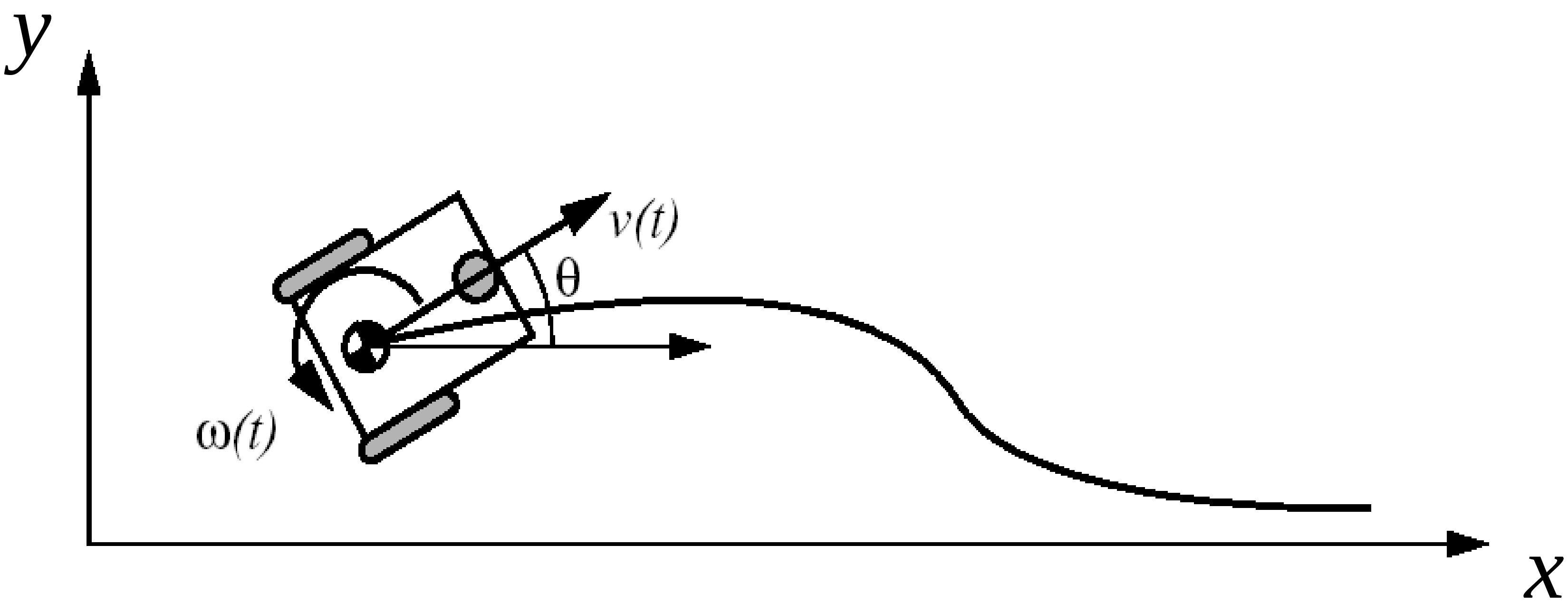
- Continuous
  - Precision bound by sensor data
  - Typically single hypothesis pose estimate
  - Lost when diverging (for single hypothesis)
  - Compact representation and typically reasonable in processing power.
- Discrete
  - Precision bound by resolution of discretisation
  - Typically multiple hypothesis pose estimate
  - Never lost (when diverges converges to another cell)
  - Important memory and processing power needed. (not the case for topological maps)

Odometry

- Definition
  - **Dead reckoning** (also **deduced reckoning** or **odometry**) is the process of calculating vehicle's current position by using a previously determined position and estimated speeds over the elapsed time
- Robot motion is recovered by integrating proprioceptive sensor velocities readings
  - Pros: Straightforward
  - Cons: Errors are integrated -> unbound
- Heading sensors (e.g., gyroscope) help to reduce the accumulated errors but drift remains

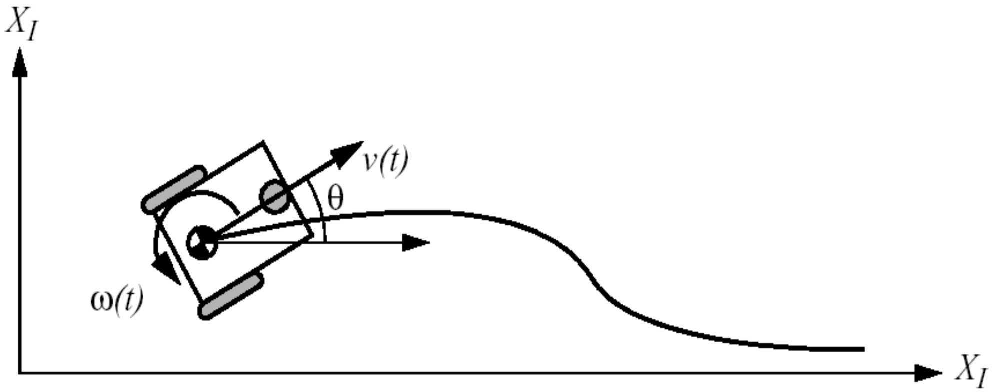
Odometry
The Differential Drive Robot (1)

$$x = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad \hat{x}_t = x_{t-1} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix} = f(x_{t-1}, u_t)$$



Odometry
Wheel Odometry

- Kinematics



$$\hat{x}_t = f(x_{t-1}, u_t) = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \Delta s \cos(\theta + \frac{\Delta\theta}{2}) \\ \Delta s \sin(\theta + \frac{\Delta\theta}{2}) \\ \Delta\theta \end{bmatrix}$$

→ This term comes from the application of the Instantaneous Center of Rotation

Can you demonstrate these equations?

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$\Delta\theta = \frac{\Delta s_r - \Delta s_l}{b}$$

Odometry
Odometric Error Propagation

- Error model

$$P_t = F_{x_{t-1}} \cdot \Sigma_{x_{t-1}} \cdot F_{x_{t-1}}^{\ \ T} + F_{\Delta S} \cdot \Sigma_{\Delta S} \cdot F_{\Delta S}^{\ \ T}$$

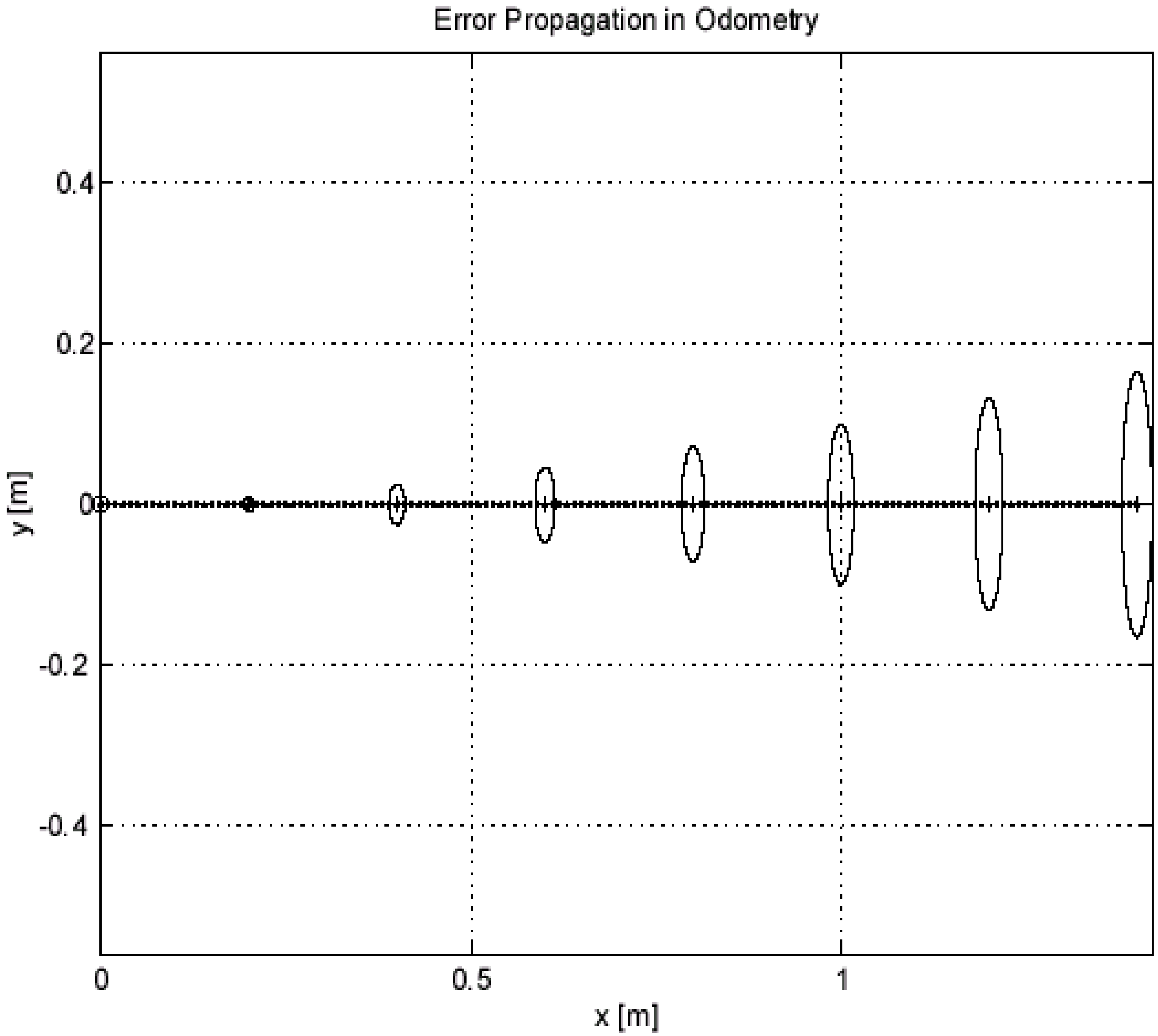
$$\Sigma_{\Delta S} = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix}$$

$$F_{x_{t-1}} = \nabla f_{x_{t-1}} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\Delta s \sin(\theta + \Delta\theta/2) \\ 0 & 1 & \Delta s \cos(\theta + \Delta\theta/2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_{\Delta S} = \begin{bmatrix} \frac{1}{2}\cos\left(\theta + \frac{\Delta\theta}{2}\right) - \frac{\Delta s}{2b}\sin\left(\theta + \frac{\Delta\theta}{2}\right) & \frac{1}{2}\cos\left(\theta + \frac{\Delta\theta}{2}\right) + \frac{\Delta s}{2b}\sin\left(\theta + \frac{\Delta\theta}{2}\right) \\ \frac{1}{2}\sin\left(\theta + \frac{\Delta\theta}{2}\right) + \frac{\Delta s}{2b}\cos\left(\theta + \frac{\Delta\theta}{2}\right) & \frac{1}{2}\sin\left(\theta + \frac{\Delta\theta}{2}\right) - \frac{\Delta s}{2b}\cos\left(\theta + \frac{\Delta\theta}{2}\right) \\ \frac{1}{b} & -\frac{1}{b} \end{bmatrix}$$

23 Odometry
Growth of Pose uncertainty for Straight Line Movement

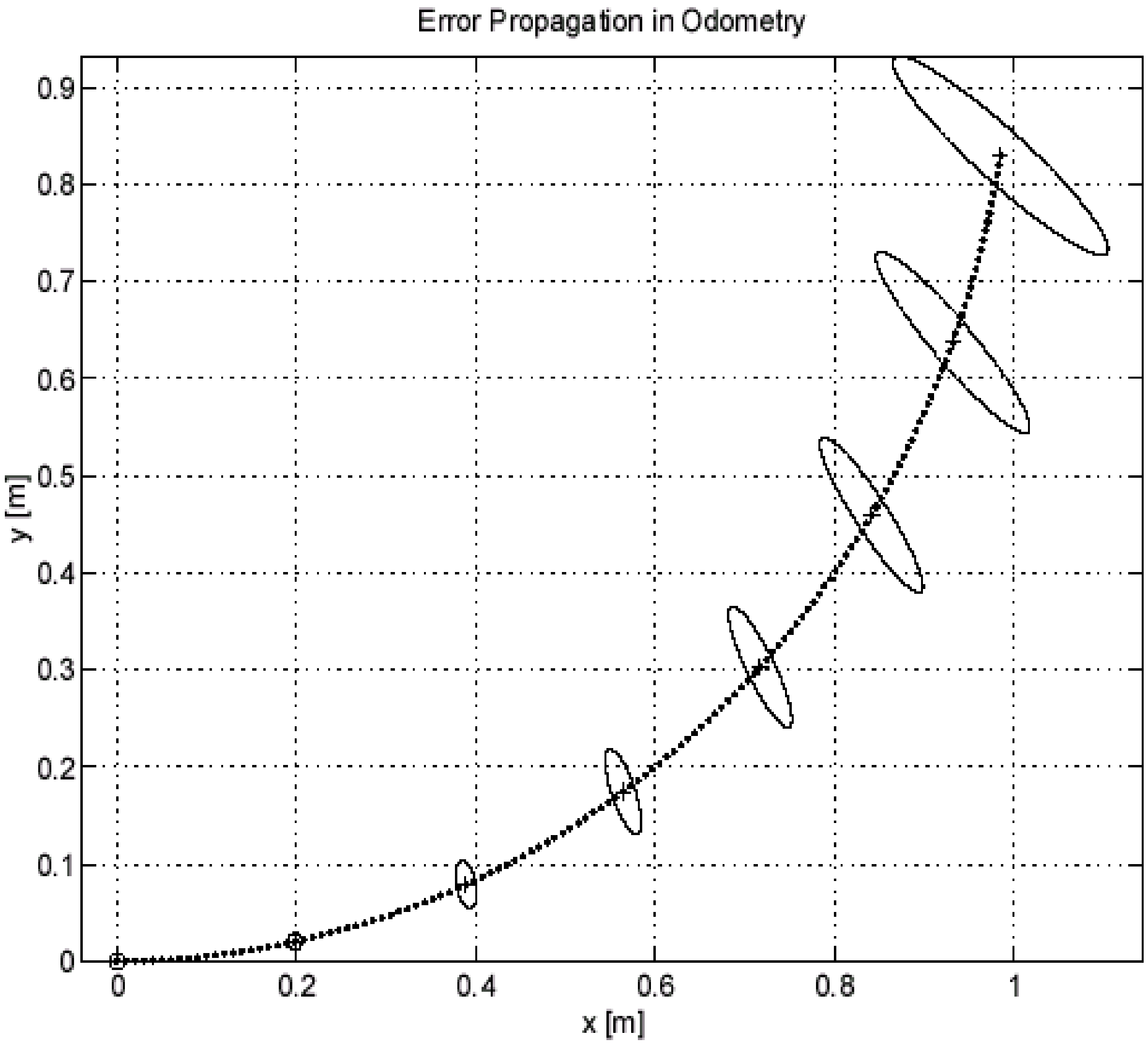
- Note: Errors perpendicular to the direction of movement are growing much faster!



Odometry

Growth of Pose uncertainty for Movement on a Circle

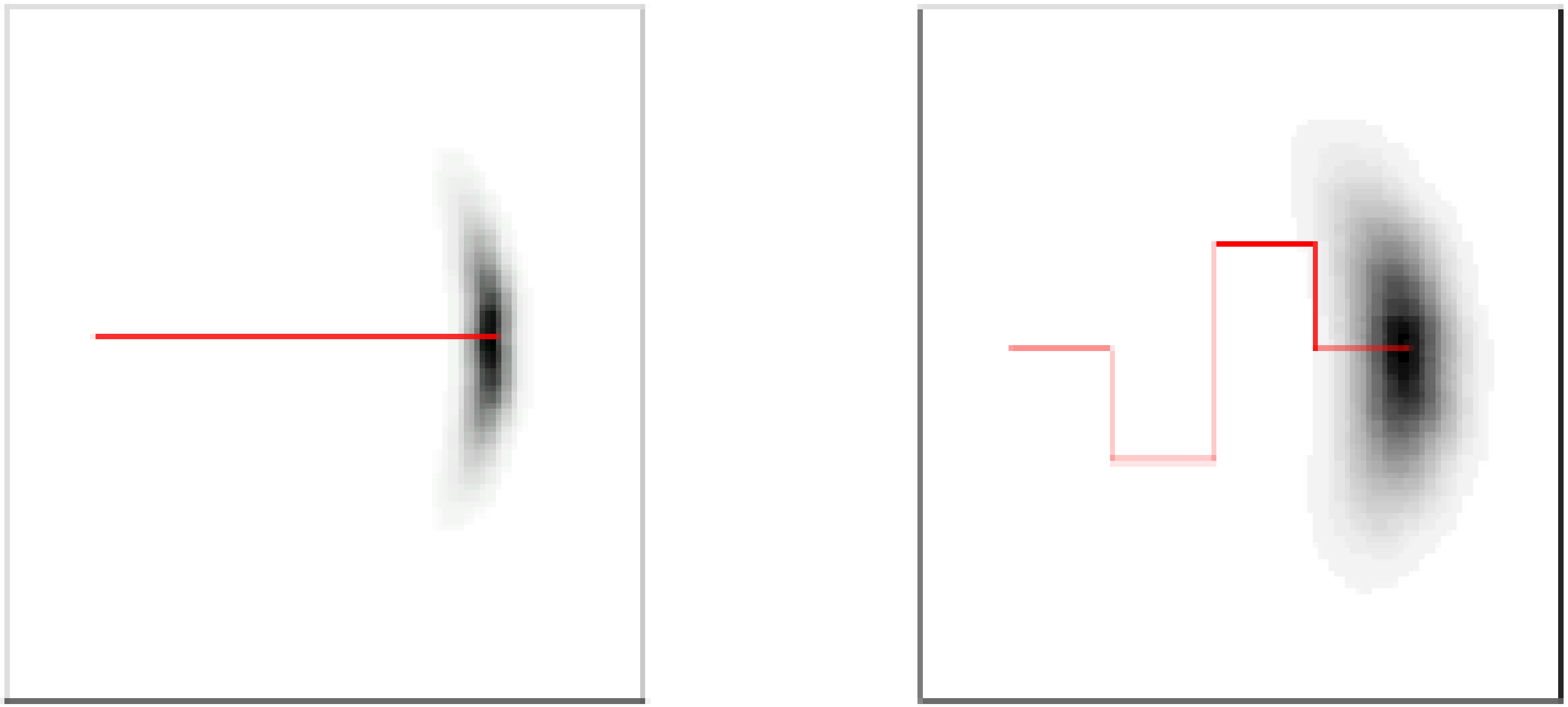
- Note: Errors ellipse does not remain perpendicular to the direction of movement!



Odometry
Example of non-Gaussian error model

- Note: Errors are not shaped like ellipses!

Courtesy AI Lab, Stanford



[Fox, Thrun, Burgard, Dellaert, 2000]

Odometry
Error sources

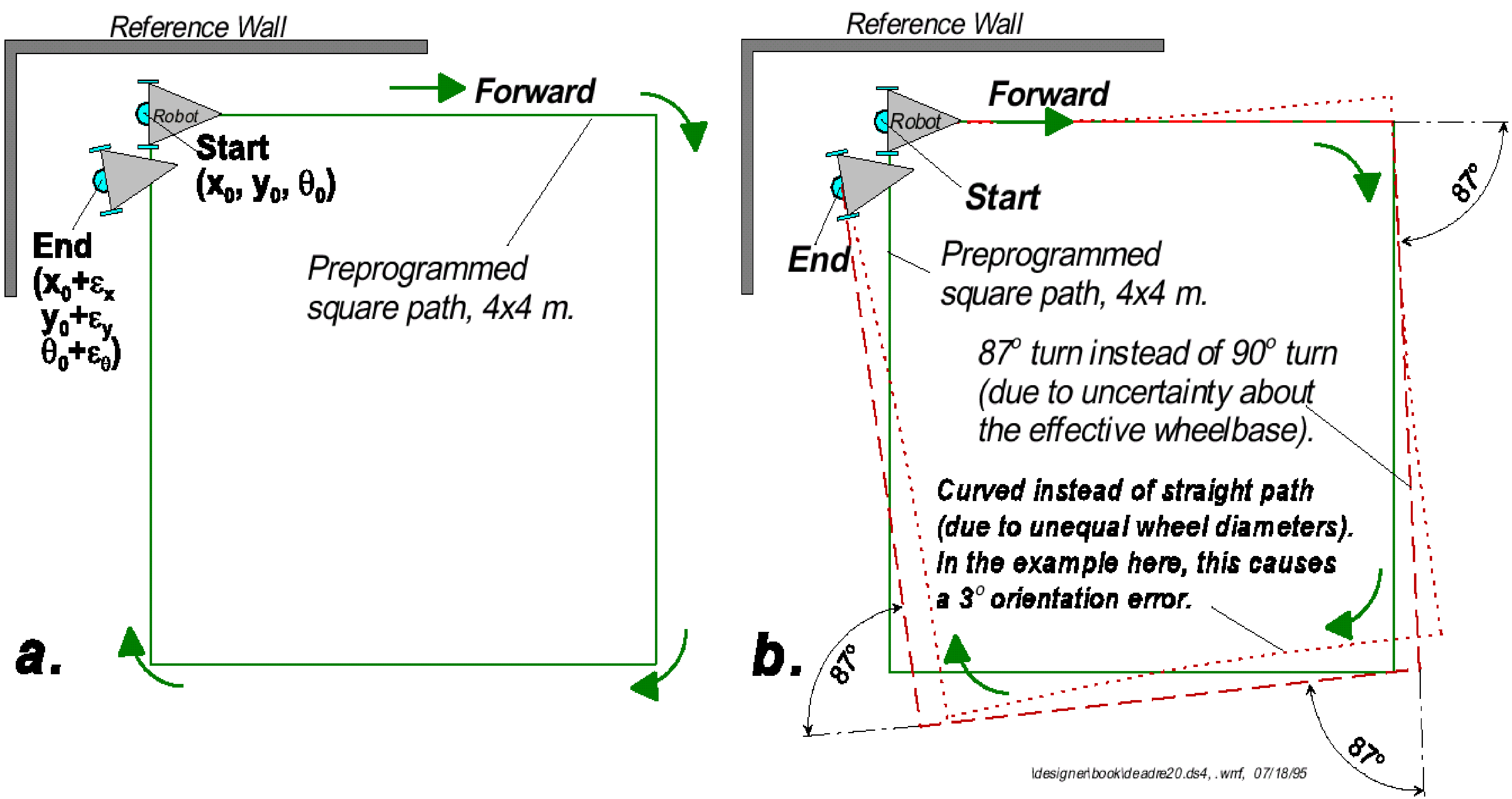
Deterministic (Systematic) ⟷ Non-Deterministic (Non-Systematic)

- Deterministic errors can be eliminated by proper calibration of the system.
- Non-Deterministic errors are random errors. They have to be described by error models and will always lead to uncertain position estimate.

- Major Error Sources in Odometry:
  - Limited resolution during integration (time increments, measurement resolution)
  - Misalignment of the wheels (deterministic)
  - Unequal wheel diameter (deterministic)
  - Variation in the contact point of the wheel (non deterministic)
  - Unequal floor contact (slippage, non planar …) (non deterministic)

Odometry
Calibration of systematic errors [Borenstein 1996]

- The unidirectional square path experiment



ETH Zürich

Odometry
Calibration of Errors II *(Borenstein [5])*

- The bi-directional square path experiment