Predictive Control with Model Uncertainty Estimation using Gaussian Process Regression

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Gaussian process (GP) regression has been widely used in supervised machine learning for its flexibility and inherent ability to describe uncertainty in the prediction. In the context of control, it is seeing increasing use for modeling of nonlinear dynamical systems from data, as it allows for direct assessment of the residual model uncertainty. In practice, a nominal linear model is often available, while more complex nonlinearities can be challenging and time-intensive to model from first principles. We present a model predictive control (MPC) approach that integrates a nominal linear system with an additive nonlinear part of the dynamics modeled as a GP. The resulting nonlinear stochastic control problem specifically takes into account the model uncertainties associated with the GP, and thus enables cautious control of the system.

Employing GP dynamics in MPC imposes the challenge of propagating the state probability distribution over the prediction horizon. The posterior distribution of a GP with uncertain inputs is generally intractable, but can be approximated by estimating its first and second moment and fitting a Gaussian distribution [1]. These moments can be either approximated by Taylor expansion of the posterior distribution functions or, in the special case of squared exponential kernels and zero mean function, computed exactly, which has recently been extended to multivariate and co-varying systems [2]. By successively employing these approximations at each prediction time step, predictive control with GP dynamics has been demonstrated in [3] and applied to multivariate systems in [4].

In this work, we show how the estimated state distributions can be exploited in a stochastic MPC formulation. Approximations of the distributions by either Taylor expansion or analytic computation are evaluated in their applicability to control problems and the necessity for a feedback mechanism during prediction is demonstrated. In the proposed control scheme, a pre-stabilizing state feedback control law is employed in order to limit the growth of uncertainty, resulting in a probability distribution also of the inputs. The resulting model allows for direct incorporation of chance constraints on states and inputs, as well as costs on the predicted uncertainty. Finally, we show that the formulation results in a deterministic nonlinear optimization problem, for which gradient information can by analytically obtained. We discuss computational aspects and efficient solution strategies and demonstrate the method for an example application.

References

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