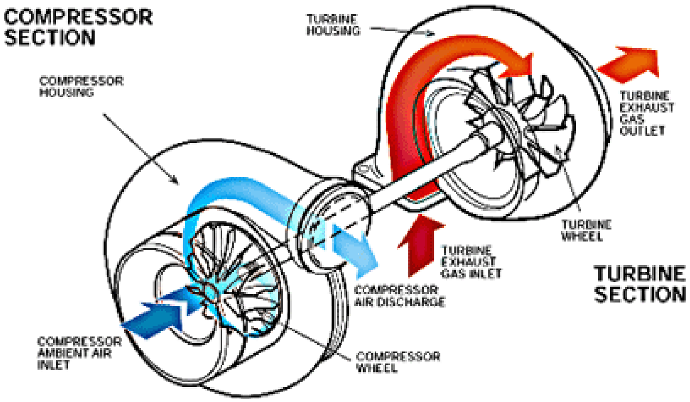
Lecture 8 – Turbocharger Modeling

System Modeling – Institute for Dynamic Systems and Control (IDSC)

Camillo Balerna
Dr. Guillaume Ducard

Lecture Overview

- **Turbocharger modeling**
  - Turbine & Compressor
  - Causality diagram
  - Inputs, outputs
  - Maps and operation



Source: https://auto.howstuffworks.com/turbo2.htm

- **Turbocharged internal combustion engines**
  - Engine power modeling
  - Naturally aspirated (NA) Vs turbocharged (TC) engines
  - Benefits of turbocharging
  - F1 electrified turbocharger

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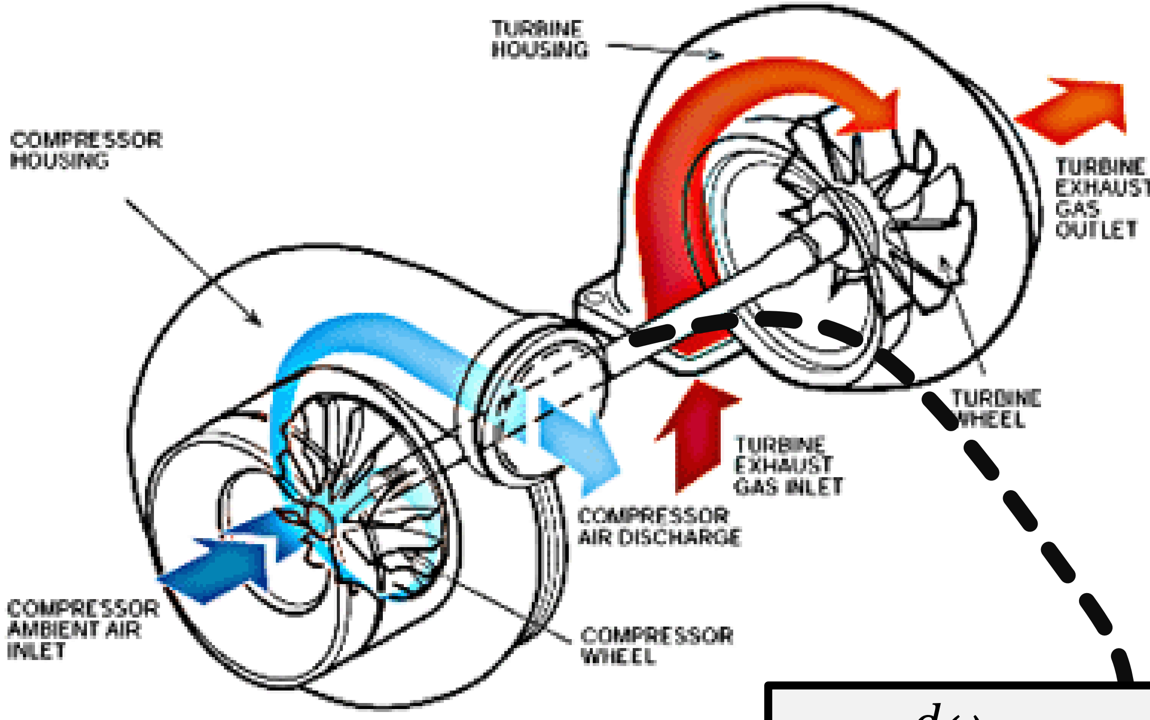
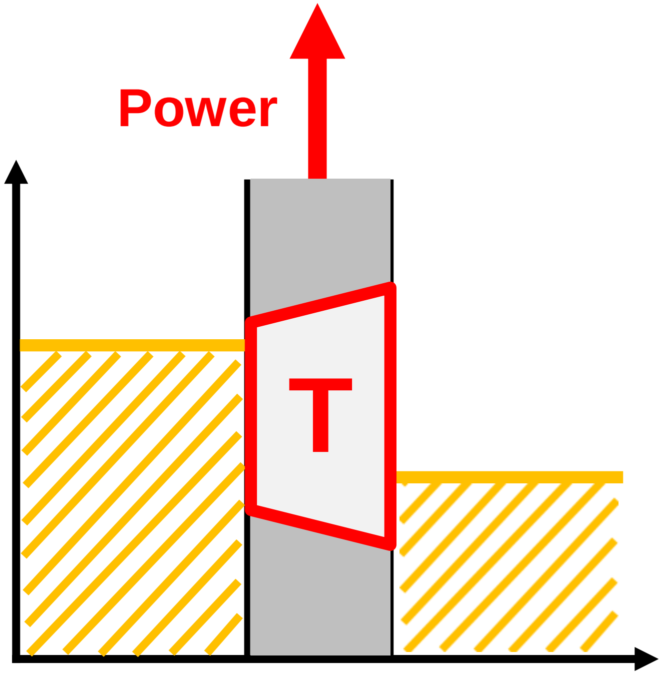
Turbocharger

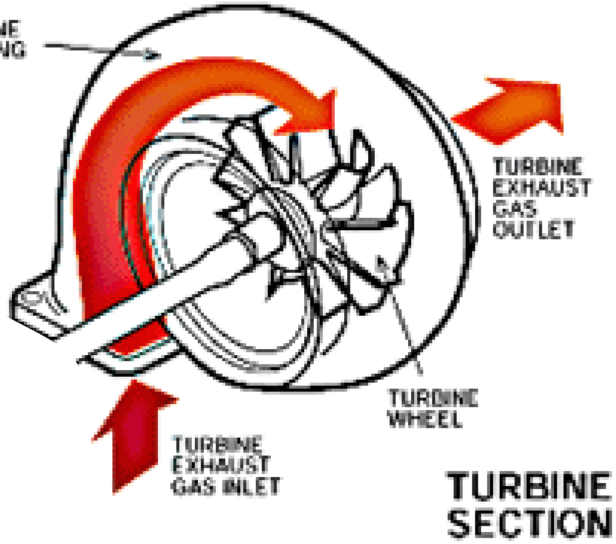


$$\Theta_{tc} \cdot \frac{d\omega_{tc}}{dt} = T_t - T_c + T_{ext}$$

Source: https://auto.howstuffworks.com/turbo2.htm

Turbine





Source: https://auto.howstuffworks.com/turbo2.htm

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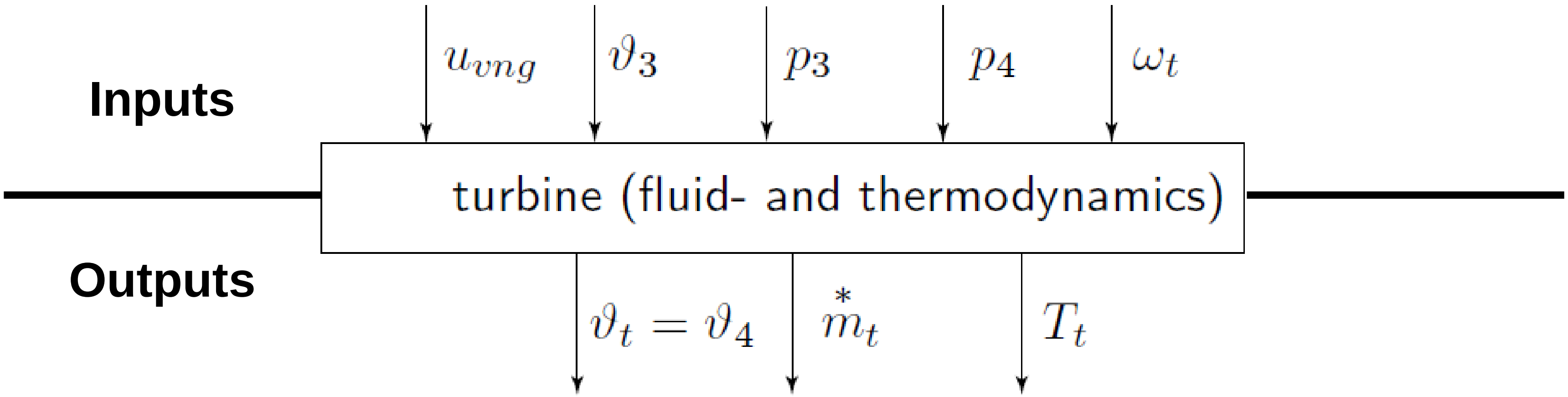
Turbine – Causality Diagram

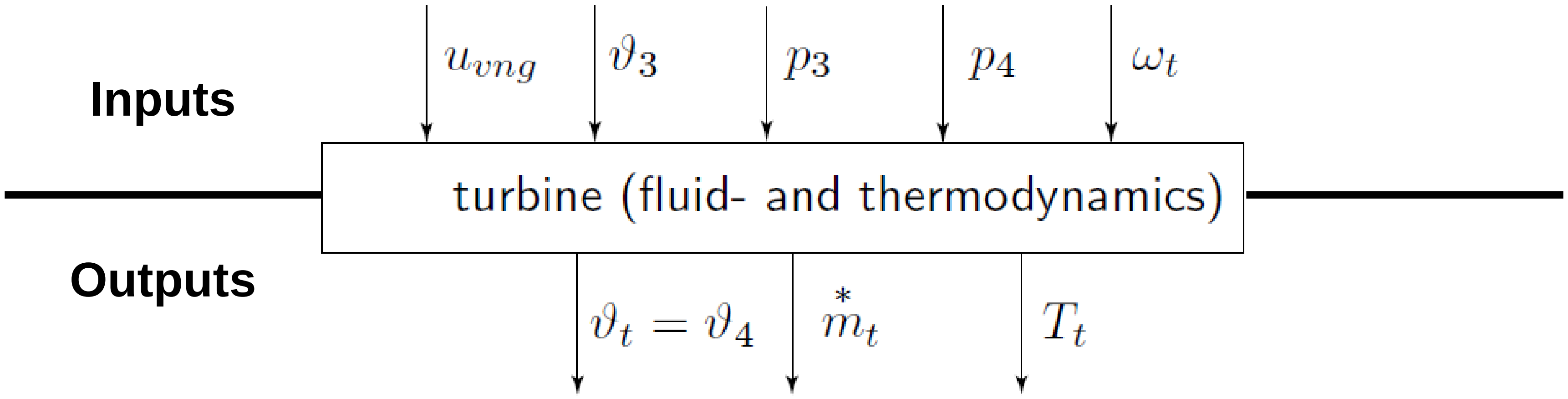


<u>Inputs</u>

- $\boldsymbol{\vartheta_3}$: Temperature before the turbine [K]
- $\boldsymbol{p_3}$: pressure before the turbine [Pa]
- $\boldsymbol{p_4}$: pressure after the turbine [Pa]
- $\boldsymbol{\omega_t}$: Turbine speed [rad/s]
- $\boldsymbol{u_{vng}}$: Variable nozzle geometry control input [−]

$$\Pi_t = \frac{p_3}{p_4} = \frac{p_{bef,t}}{p_{aft,t}}$$

Turbine – Causality Diagram



Outputs

- $\boldsymbol{\vartheta_4}$: Temperature of the flow exiting the turbine [K]
- $\boldsymbol{\dot{m}_t}$: Mass flow through the turbine [Kg/s]
- $\boldsymbol{T_t}$: Torque generated by the turbine [Nm]

Turbine – Outputs

- **Temperature** of the flow exiting the turbine

$$\vartheta_4 = \vartheta_3 \cdot \left[1 - \eta_t \cdot \left(1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right)\right]$$

$$\boldsymbol{\Pi_t = \frac{p_3}{p_4} = \frac{p_{bef,t}}{p_{aft,t}}}$$

- **Mass Flow** through the turbine

$$\dot{m}_t = \frac{p_3}{p_{ref,0}} \cdot \sqrt{\frac{\vartheta_{ref,0}}{\vartheta_3}} \cdot \dot{\mu}_t$$

- **Torque** produced by the turbine

$$T_t = \frac{P_\mathrm{t}}{\omega_t} = \frac{\dot{m}_\mathrm{t} \cdot c_p \cdot \vartheta_3}{\omega_t} \cdot \left[1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right] \cdot \eta_t$$

Turbine – Outputs derivation

Open system

$$\frac{dE}{dt} = \dot{H}_{in} - \dot{H}_{out} - \dot{W}_t + \dot{Q}$$

- Turbine does **not store energy** over time $\Rightarrow$ $\boldsymbol{\frac{dE}{dt} = 0}$
- Turbine is assumed to be **adiabatic** (no heat transfer) $\Rightarrow$ $\boldsymbol{\dot{Q} = 0}$

$$P_t = \dot{W}_t = \dot{H}_{in} - \dot{H}_{out} = \dot{m}_\mathrm{t} \cdot c_p \cdot (\vartheta_3 - \vartheta_4)$$

Isentropic relation

$$\frac{\vartheta_3}{\vartheta_{4,is}} = \left(\frac{p_3}{p_4}\right)^{\frac{(\kappa-1)}{\kappa}} = \Pi_t^{\frac{(\kappa-1)}{\kappa}}$$

Isentropic efficiency

$$\eta_t = \frac{\vartheta_3 - \vartheta_4}{\vartheta_3 - \vartheta_{4,is}}$$

Turbine exit temperature

$$\vartheta_4 = \vartheta_3 \cdot \left[1 - \eta_t \cdot \left(1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right)\right]$$

Turbine power produced

$$P_t = \dot{m}_\mathrm{t} \cdot c_p \cdot \vartheta_3 \cdot \left[1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right] \cdot \eta_t$$

Turbine – Outputs

- **Temperature** of the flow exiting the turbine

$$\vartheta_t = \vartheta_3 \cdot \left[1 - \eta_t \cdot \left(1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right)\right]$$

$$\boldsymbol{\Pi_t = \frac{p_3}{p_4} = \frac{p_{bef,t}}{p_{aft,t}}}$$

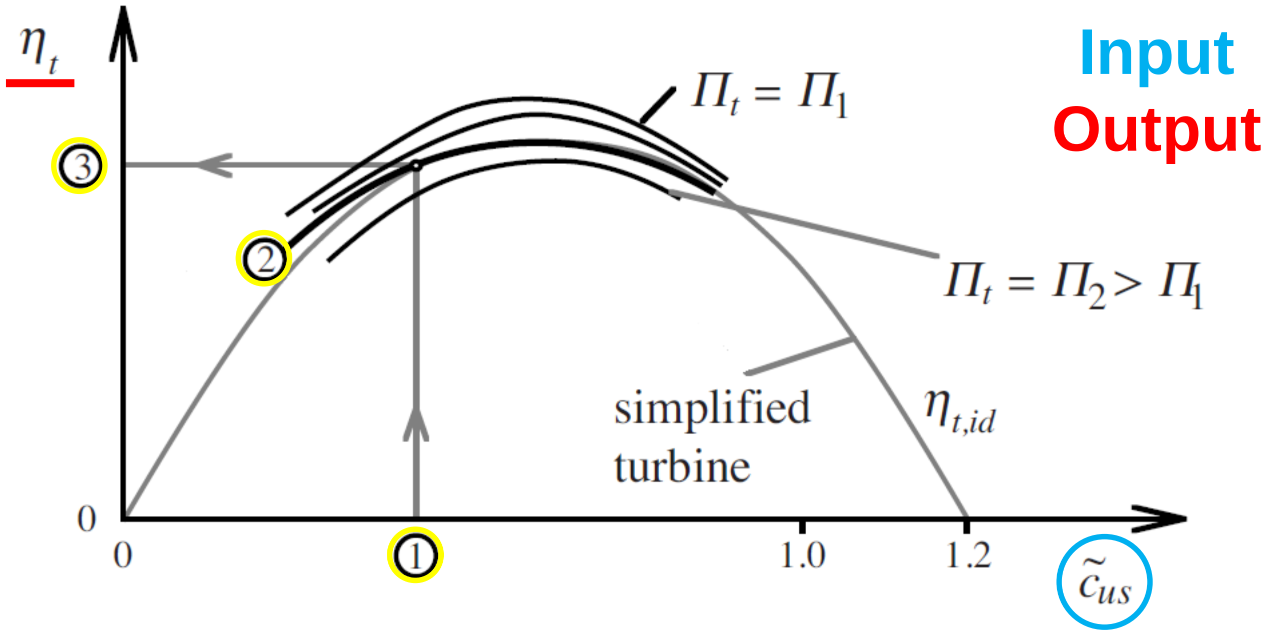
- **Mass Flow** through the turbine

$$\dot{m}_t = \frac{p_3}{p_{ref,0}} \cdot \sqrt{\frac{\vartheta_{ref,0}}{\vartheta_3}} \cdot \dot{\mu}_t$$

- **Torque** produced by the turbine

$$T_t = \frac{P_\mathrm{t}}{\omega_t} = \frac{\dot{m}_\mathrm{t} \cdot c_p \cdot \vartheta_3}{\omega_t} \cdot \left[1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right] \cdot \eta_t$$

Turbine – Efficiency Map

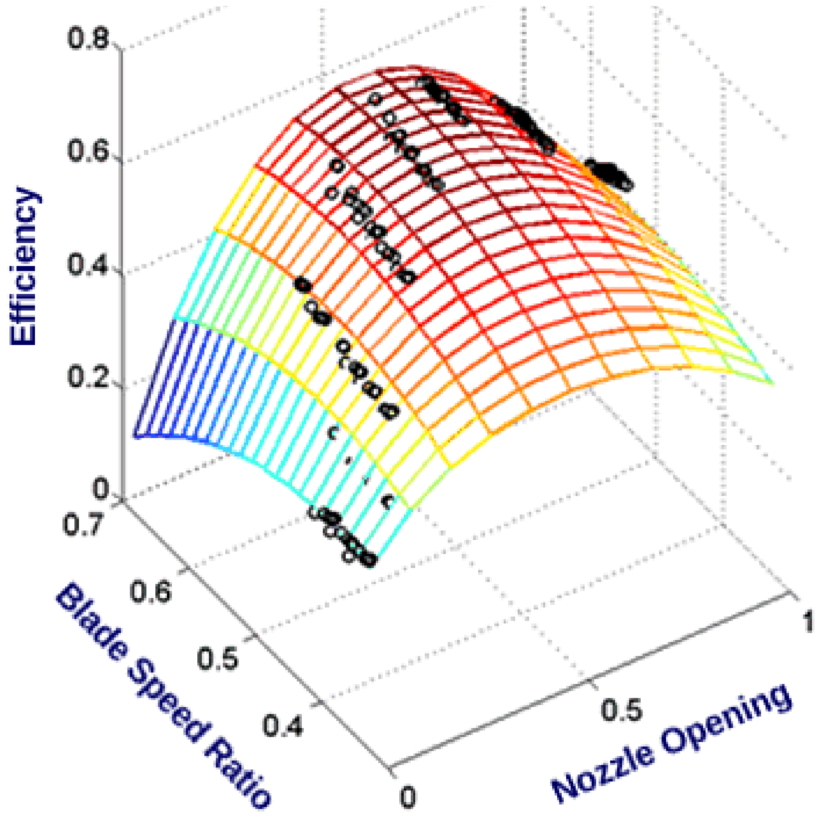


$$c_{us} = \sqrt{2 \cdot c_p \cdot \vartheta_3 \cdot \left[1 - \Pi_t^{(1-\kappa)/\kappa}\right]} \longrightarrow \boxed{\tilde{c}_{us} = \frac{r_t \cdot \omega_t}{c_{us}}}$$

- Since the turbine efficiency mainly depends on the angle of incidence of the inflowing gas, the turbine blade speed ratio $\tilde{c}_{us}$ is used as variable.

Turbine – Efficiency Map

Variable Geometry



Source: https://www.dieselnet.com/tech/air_turbo_vgt.php

Turbine – Outputs

- **Temperature** of the flow exiting the turbine

$$\vartheta_t = \vartheta_3 \cdot \left[1 - \eta_t \cdot \left(1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right)\right]$$

$$\boldsymbol{\Pi_t = \frac{p_3}{p_4} = \frac{p_{bef,t}}{p_{aft,t}}}$$

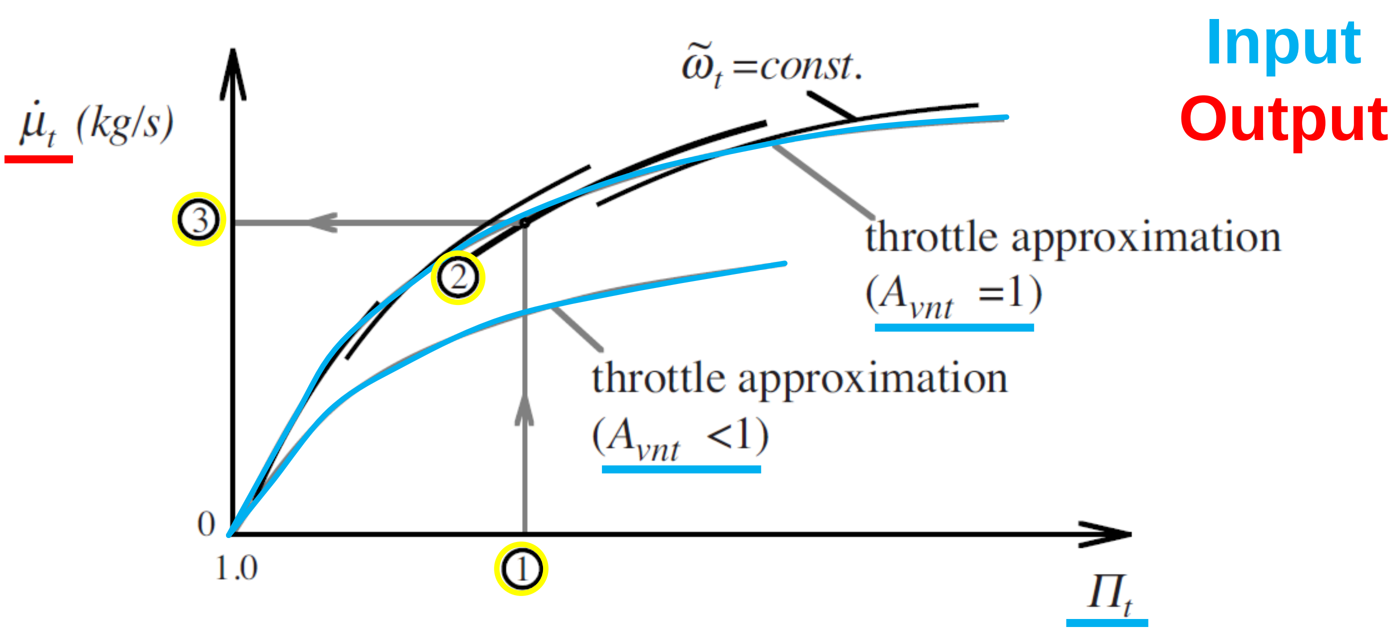
- **Mass Flow** through the turbine

$$\dot{m}_t = \frac{p_3}{p_{ref,0}} \cdot \sqrt{\frac{\vartheta_{ref,0}}{\vartheta_3}} \cdot \dot{\mu}_t$$

- **Torque** produced by the turbine

$$T_t = \frac{P_t}{\omega_t} = \frac{\dot{m}_t \cdot c_p \cdot \vartheta_3}{\omega_t} \cdot \left[1 - \Pi_t^{\frac{1-\kappa}{\kappa}}\right] \cdot \eta_t$$

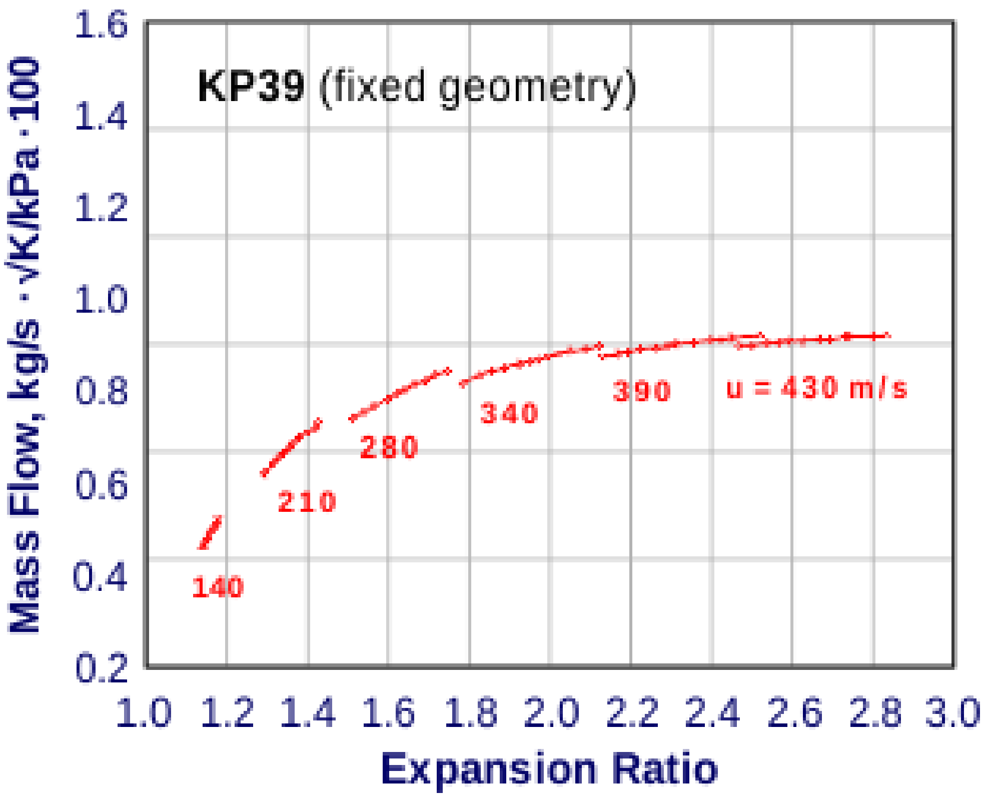
Turbine – Mass Flow Map



- For control purposes, the mass flow behaviour of fluid-dynamic turbines can be modeled quite well as orifice → compressible flow through a valve.
- If the turbine is a Variable Nozzle Turbine (VNT) or Variable Geometry Turbine (VGT), the mass flow and its maximum value depend on the nozzle position (as it is for the compressible flow through a valve).

Turbine – Mass Flow Map

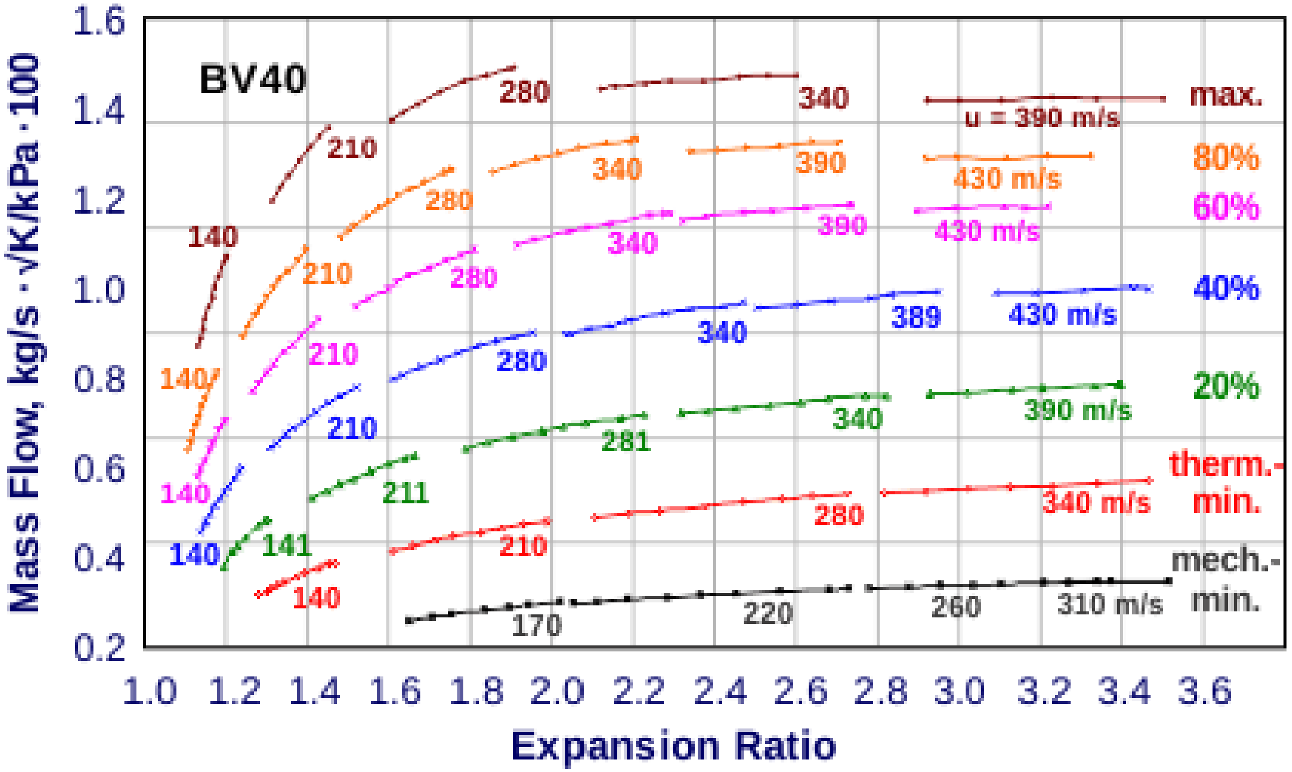
Fixed Geometry



Source: https://www.dieselnet.com/tech/air_turbo_vgt.php

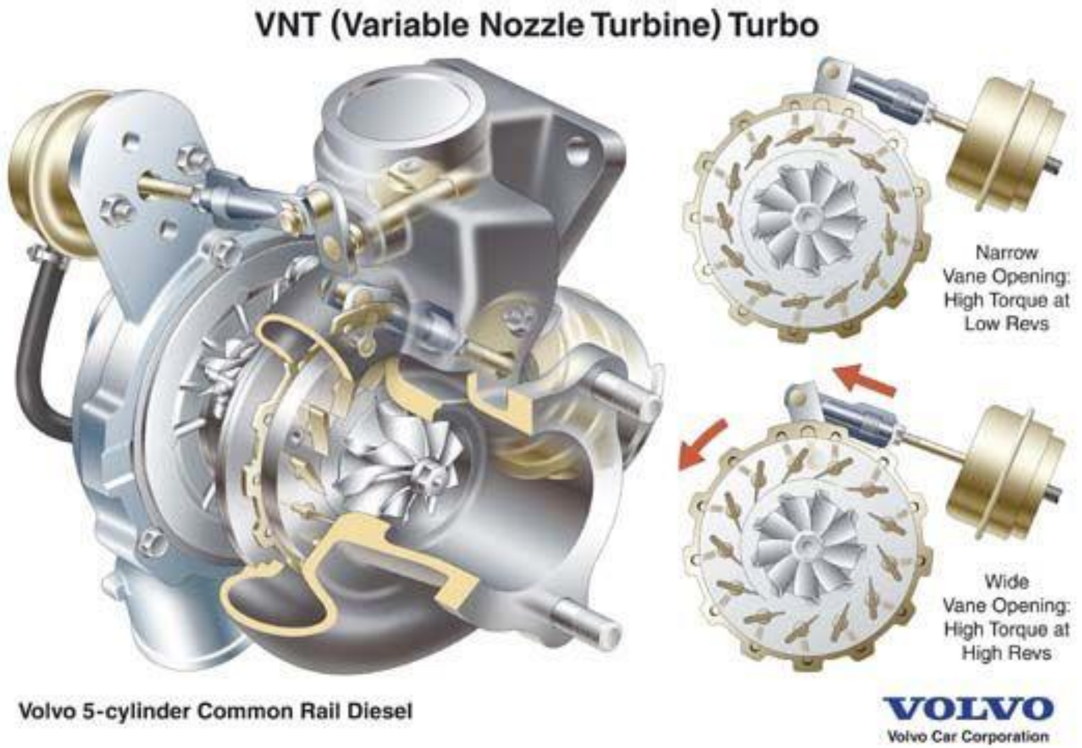
Turbine – Mass Flow Map

Variable Geometry



Source: https://www.dieselnet.com/tech/air_turbo_vgt.php

Turbine – Variable Geometry Turbine (VGT)



At Low Engine Speed
→ low mass flow
→ low pressure
→ low turbine power

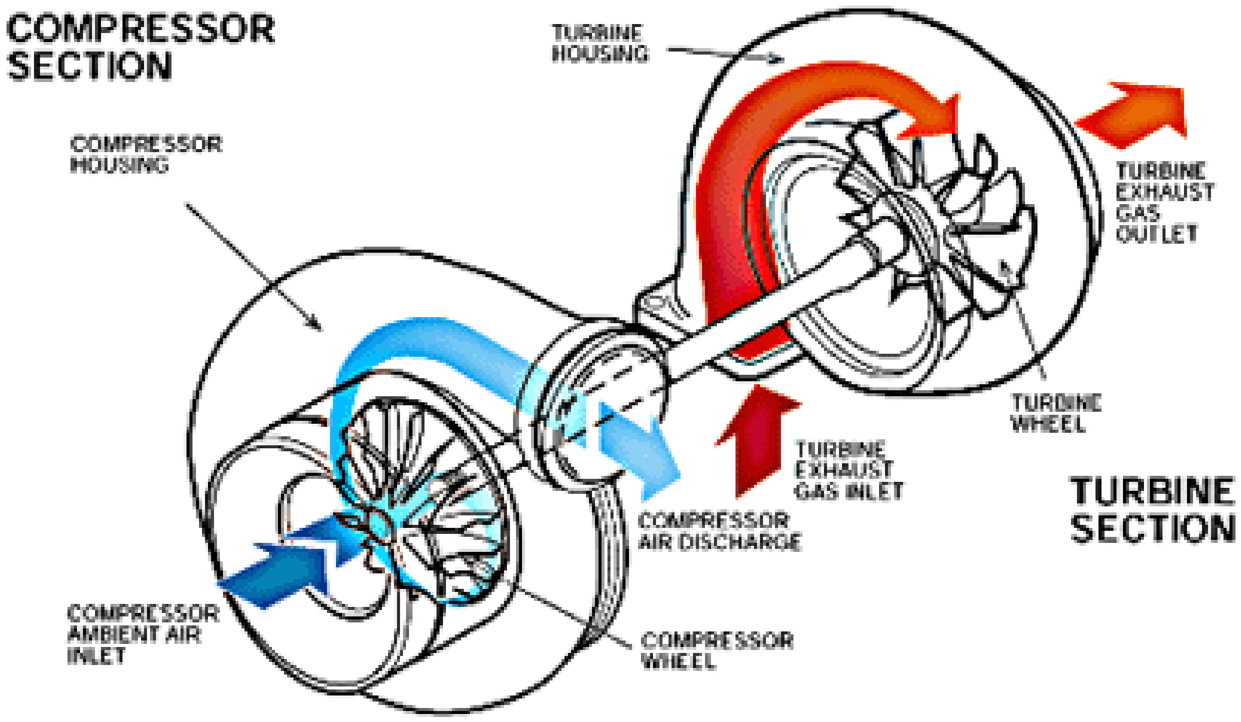
Narrow inlet area
→ Better incidence
→ Increased efficiency
→ Increased power

At High Engine Speed
→ high mass flow
→ high pressure

Wide inlet area
→ Avoid choke!

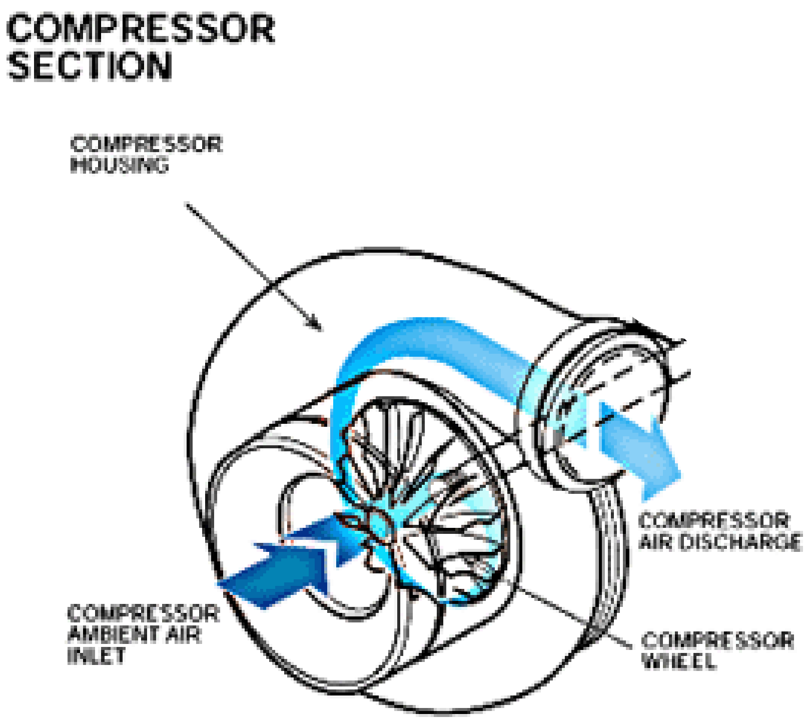
Source: https://www.intmarketing.org/en/automotive/113-variable-turbine-geometry.html

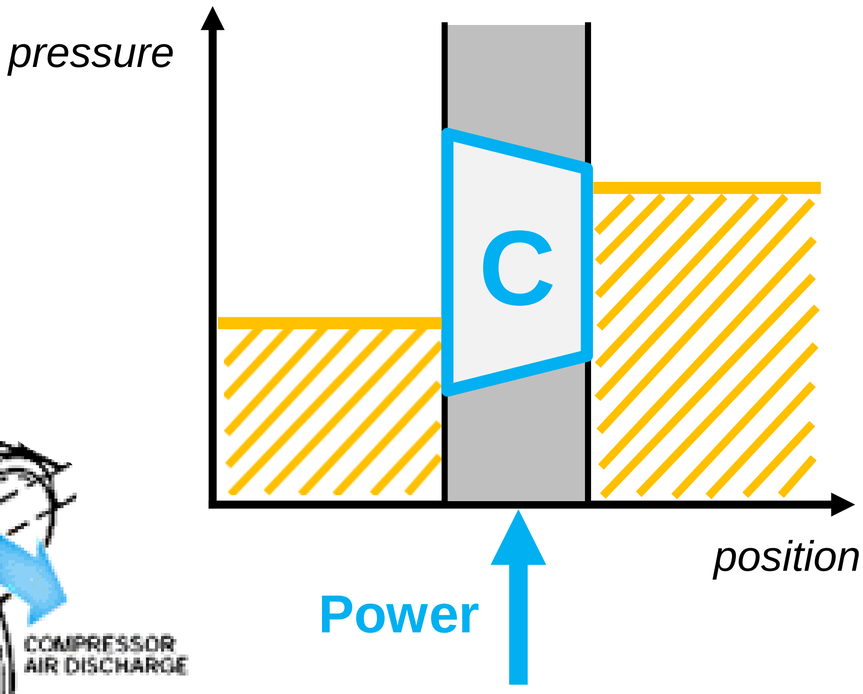
Compressor



Source: https://auto.howstuffworks.com/turbo2.htm

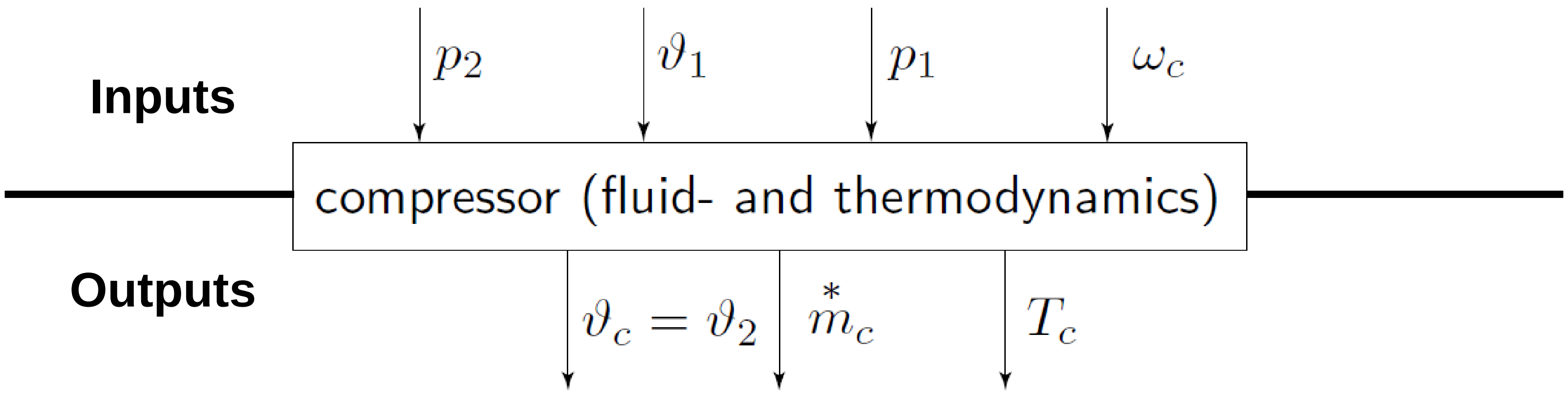
Compressor





Source: https://auto.howstuffworks.com/turbo2.htm

Compressor – Causality Diagram

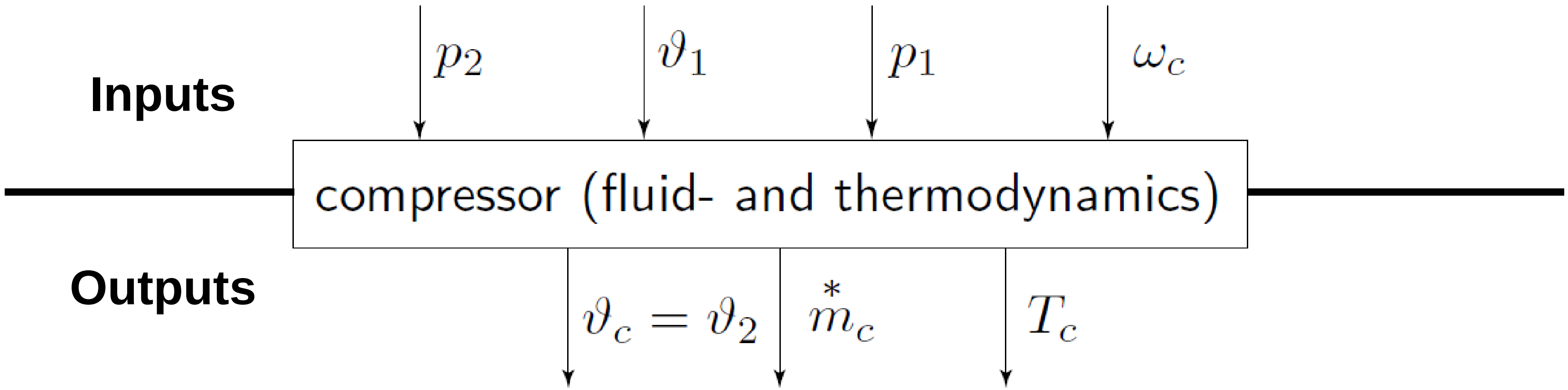


<u>Inputs</u>

- $\boldsymbol{p_1}$: Pressure before the compressor [Pa]
- $\boldsymbol{p_2}$: Pressure after the compressor [Pa]
- $\boldsymbol{\vartheta_1}$: Temperature before the compressor [K]
- $\boldsymbol{\omega_c}$: Compressor speed [rad/s]

$$\boldsymbol{\Pi_c = \frac{p_2}{p_1} = \frac{p_{aft,c}}{p_{bef,c}}}$$

Compressor – Causality Diagram



<u>Outputs</u>

- $\boldsymbol{\vartheta_c}$: Temperature of the flow exiting the compressor [K]
- $\boldsymbol{\dot{m}_c}$: Mass flow through the compressor [Kg/s]
- $\boldsymbol{T_c}$: Torque absorbed by the compressor [Nm]

Compressor – Outputs

- **Temperature** of the flow exiting the compressor

$$\vartheta_c = \vartheta_1 + \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{\vartheta_1}{\eta_c}$$

$$\boldsymbol{\Pi_c = \frac{p_2}{p_1} = \frac{p_{aft,c}}{p_{bef,c}}}$$

- **Mass Flow** through the compressor

$$\dot{m}_c = \frac{p_1}{p_{ref,0}} \cdot \sqrt{\frac{\vartheta_{ref,0}}{\vartheta_1}} \cdot \dot{\mu}_c$$

- **Torque** absorbed by the compressor

$$T_c = \frac{P_c}{\omega_c} = \frac{\dot{m}_c \cdot c_p \cdot \vartheta_1}{\omega_c} \cdot \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{1}{\eta_c}$$

Compressor – Outputs derivation

Open system

$$\frac{dE}{dt} = \dot{H}_{in} - \dot{H}_{out} - \dot{W}_c + \dot{Q}$$

- Compressor does **not store energy** over time $\Rightarrow \boldsymbol{\frac{dE}{dt} = 0}$
- Compressor is assumed to be **adiabatic** (no heat transfer) $\Rightarrow \boldsymbol{\dot{Q} = 0}$

$$P_c = -\dot{W}_c = \dot{H}_{out} - \dot{H}_{in} = \dot{m}_c \cdot c_p \cdot (\vartheta_2 - \vartheta_1)$$

Isentropic relation

$$\frac{\vartheta_{2,is}}{\vartheta_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\kappa-1)}{\kappa}} = \Pi_c^{\frac{(\kappa-1)}{\kappa}}$$

Isentropic efficiency

$$\eta_c = \frac{\vartheta_{2,is} - \vartheta_1}{\vartheta_2 - \vartheta_1}$$

Compressor exit temperature

$$\vartheta_2 = \vartheta_1 + \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{\vartheta_1}{\eta_c}$$

Compressor power absorbed

$$P_c = \dot{m}_c \cdot c_p \cdot \vartheta_1 \cdot \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{1}{\eta_c}$$

Compressor – Outputs

- **Temperature** of the flow exiting the compressor

$$\vartheta_c = \vartheta_1 + \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{\vartheta_1}{\eta_c}$$

$$\Pi_c = \frac{p_2}{p_1} = \frac{p_{aft,c}}{p_{bef,c}}$$

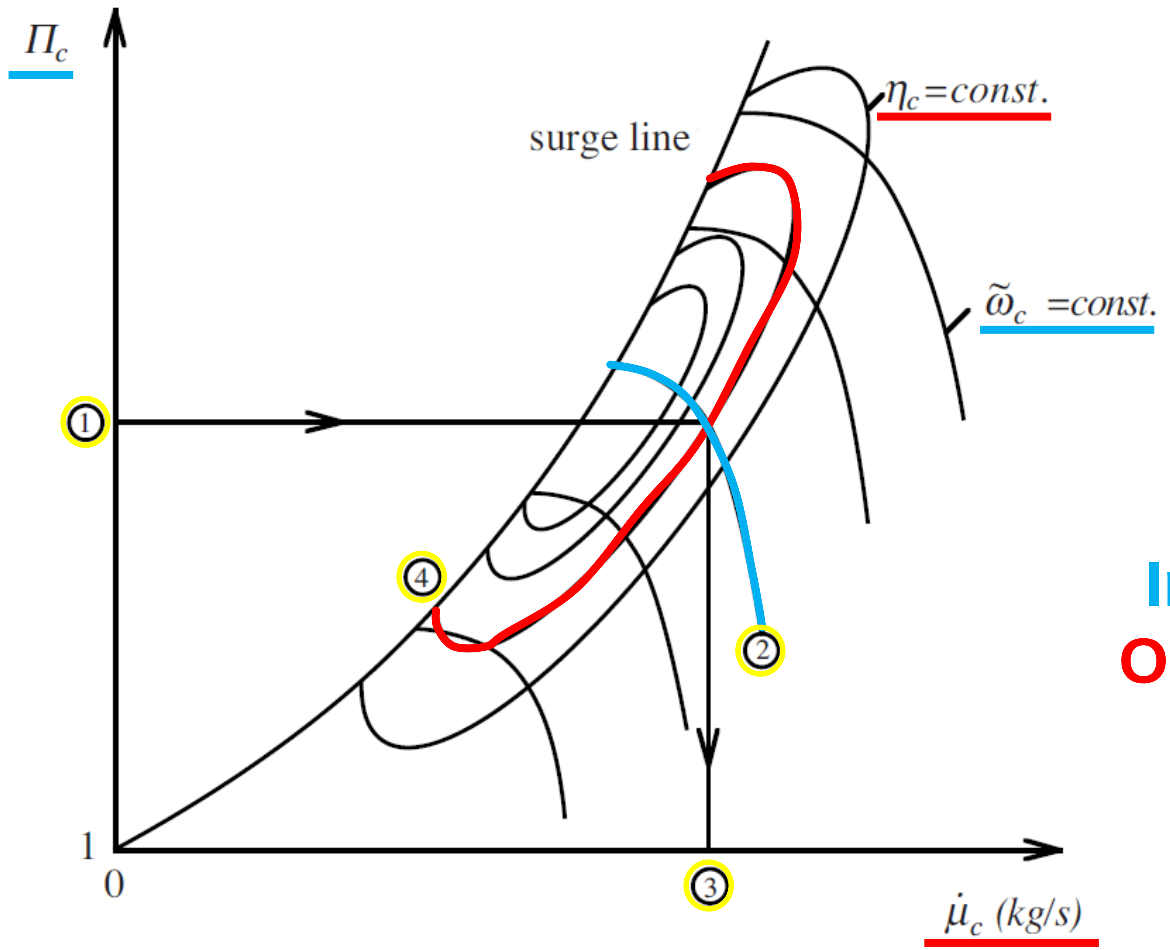
- **Mass Flow** through the compressor

$$\dot{m}_c = \frac{p_1}{p_{ref,0}} \cdot \sqrt{\frac{\vartheta_{ref,0}}{\vartheta_1}} \cdot \dot{\mu}_c$$

- **Torque** absorbed by the compressor

$$T_c = \frac{P_c}{\omega_c} = \frac{\dot{m}_c \cdot c_p \cdot \vartheta_1}{\omega_c} \cdot \left[\Pi_c^{\frac{\kappa-1}{\kappa}} - 1\right] \cdot \frac{1}{\eta_c}$$

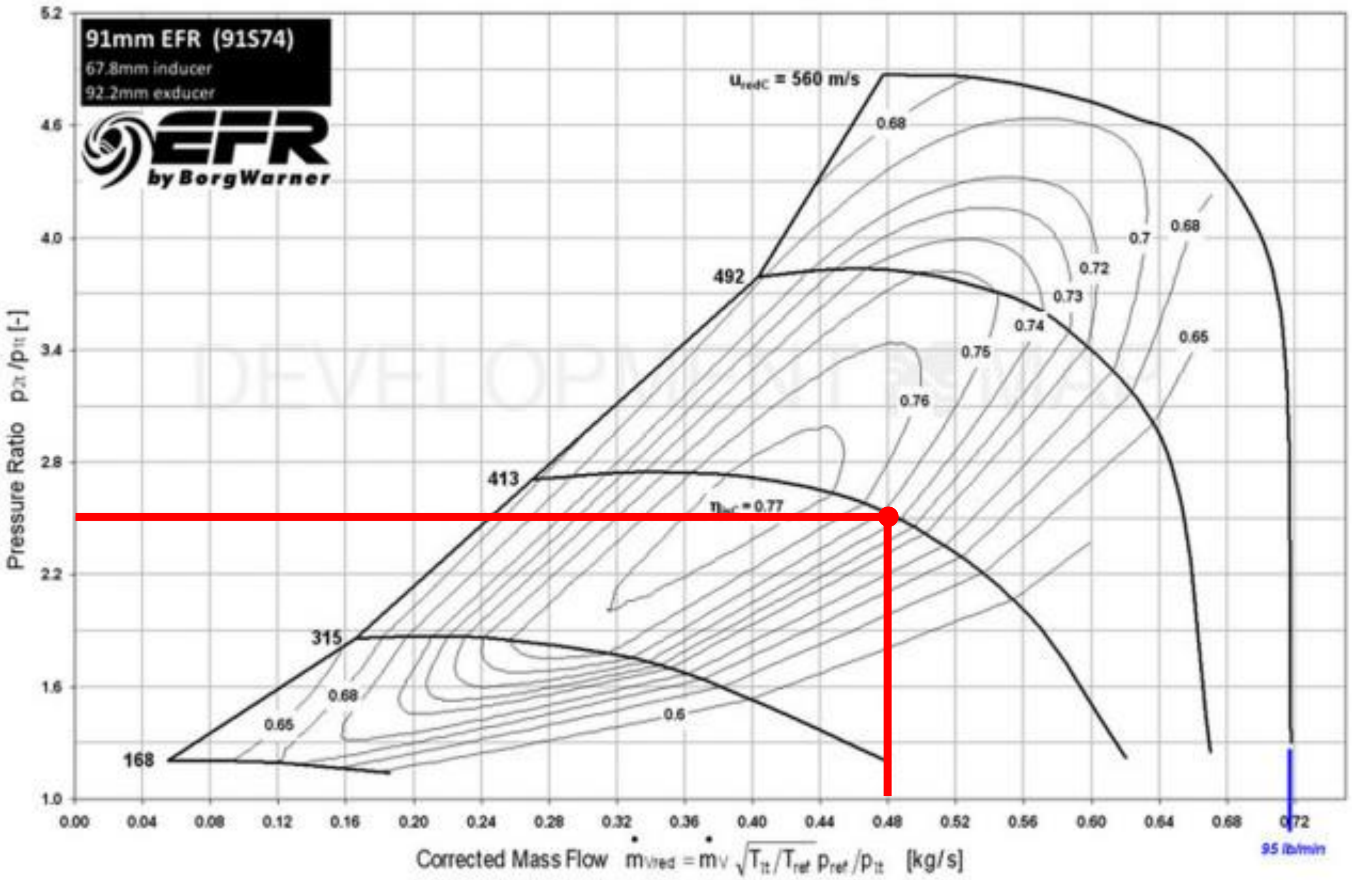
Compressor – Mass Flow & Efficiency Map



Input

Output

Compressor – Mass Flow & Efficiency Map

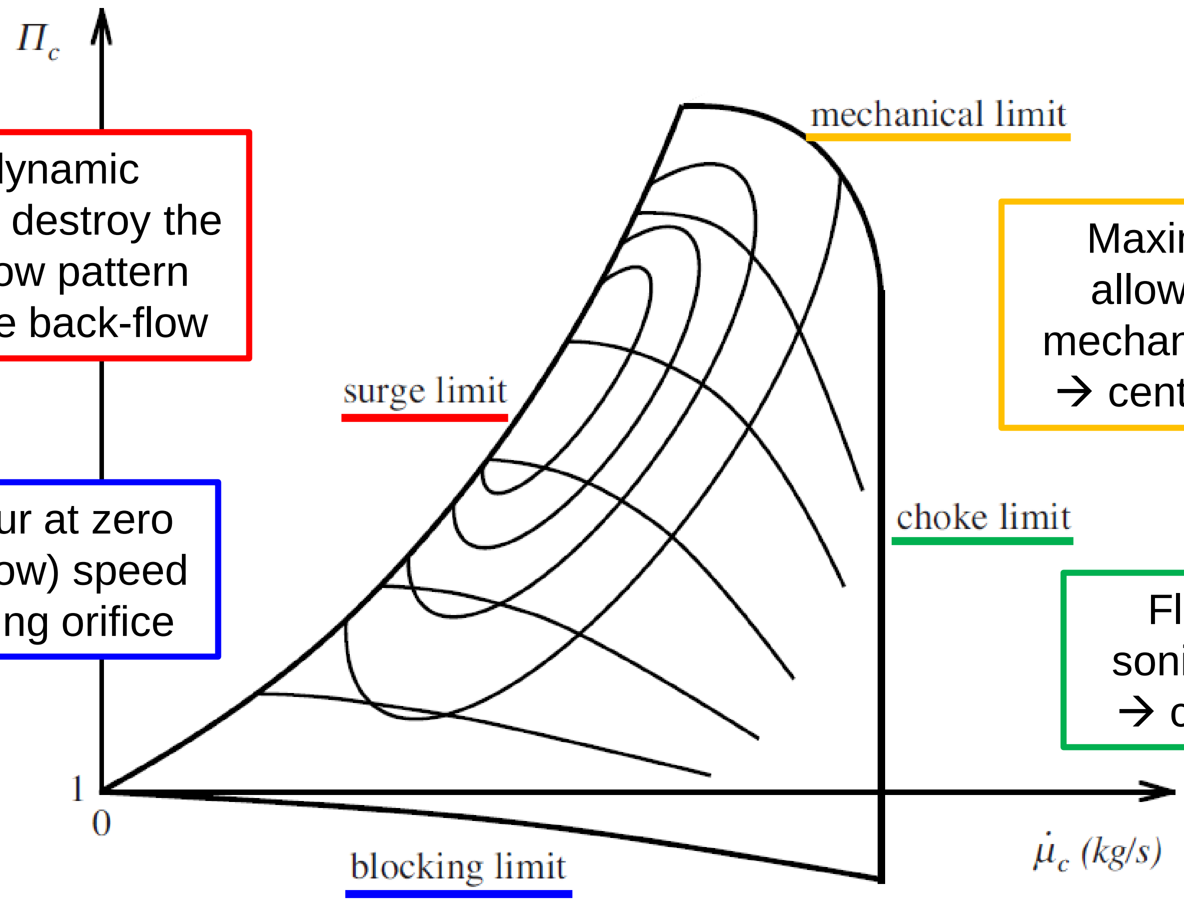


Source: http://www.enginelabs.com/engine-tech/power-adders/understanding-compressor-maps-sizing-a-turbocharger/

Compressor – Operational Limits

Fluid-dynamic instabilities destroy the regular flow pattern → possible back-flow

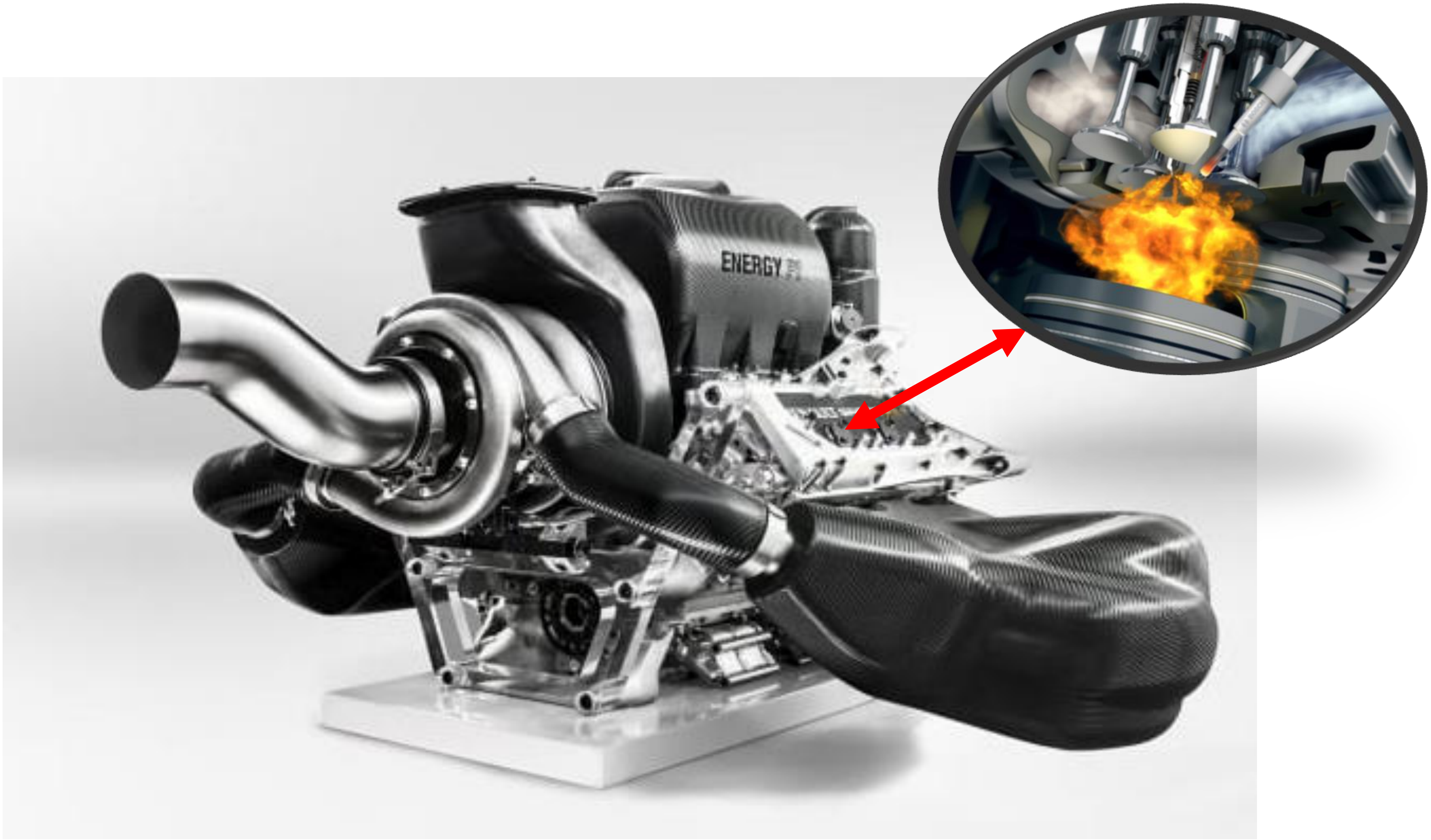
Behaviour at zero (or very low) speed → blocking orifice



Maximum speed allowed to avoid mechanical damages → centrifugal forces

Flow reaches sonics conditions → choked orifice

Formula 1 – Turbocharged Engine



Source: https://sport.sky.it/formula1/2017/03/21/formula-1--il-dizionario--power-unit-ed-elettronica.html

Internal Combustion Engine

Engine Power can be approximated as following:

$$P_{engine} = P_{comb,fuel} + P_{fric} + P_{pump}$$

Assume constant engine speed ω_e

- **Engine Power** coming from the **fuel combustion**:

$$P_{comb,fuel} = e_{comb} \cdot P_f = e_{comb} \cdot H_l \cdot \dot{m}_{fuel} \approx \underline{k_1 \cdot \dot{m}_{fuel}}$$

- **Engine Power** coming from the pistons **mechanical friction**:

$$P_{fric} \approx \underline{k_2}$$

- **Engine Power** coming from the **gas exchange**:

$$P_{pump} = (p_{intake} - p_{exhaust}) \cdot V_d \cdot \frac{\omega_e}{4\pi} \approx \underline{k_3 \cdot (p_{intake} - p_{exhaust})}$$

Internal Combustion Engine

- **Engine Power**, neglecting the friction and assuming $p_{intake} = p_{exhaust}$:

$$P_{engine} \approx k_1 \cdot \dot{m}_{fuel} + k_3 \cdot (p_{intake} - p_{exhaust}) \approx k_1 \cdot \dot{m}_{fuel}$$

- **Air to Fuel Ratio** is defined as following:

$$\lambda_{AF} = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \cdot \frac{1}{\sigma_0} \xrightarrow{\lambda_{AF} = 1} \dot{m}_{fuel} = \frac{\dot{m}_{air}}{\sigma_0}$$

- **Engine Air Mass Flow** is approximated as following:

$$\dot{m}_{air} = \frac{p_{intake}}{R_{air} \cdot \vartheta_{intake}} \cdot \frac{\omega_e}{4\pi} \cdot V_d \cdot \lambda_{vol} \approx k_4 \cdot p_{intake}$$

$$P_{engine} \propto \dot{m}_{fuel} \propto \dot{m}_{air} \propto p_{intake}$$

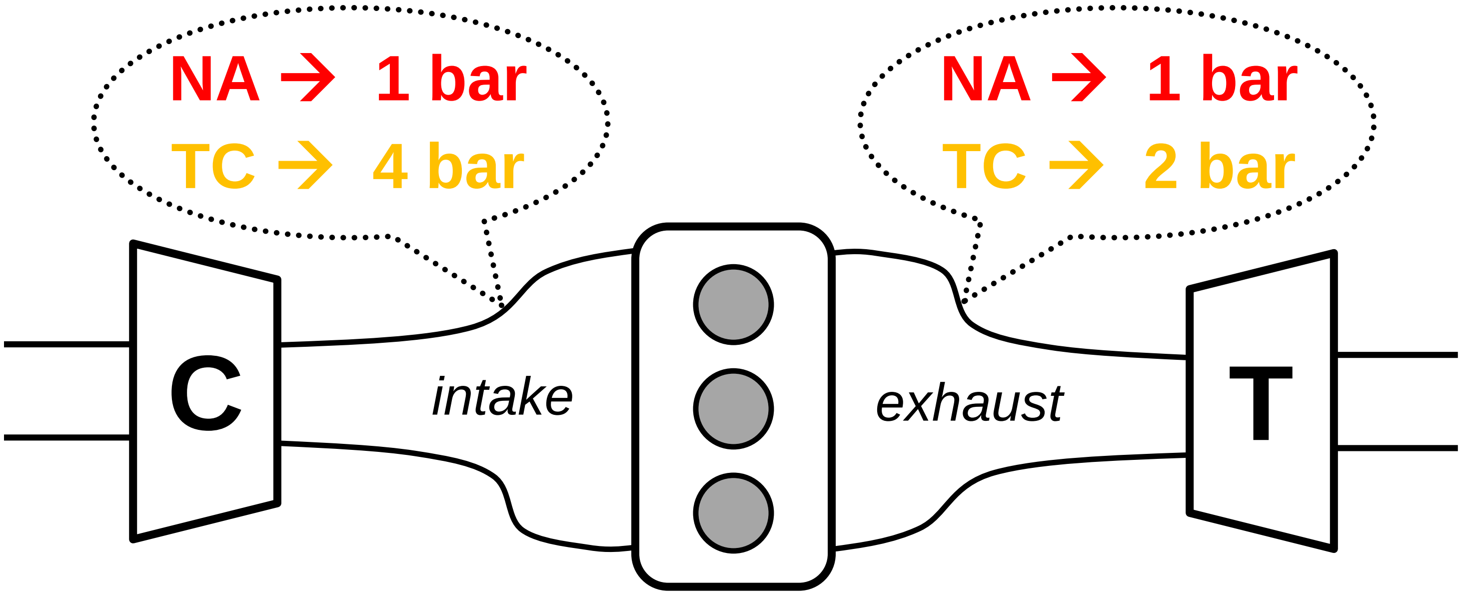
NA → 1 bar → 100 kW

TC → 4 bar → 400 kW

Turbocharged Engine

- **Engine Power**, neglecting the friction and for a specific $\dot{m}_{fuel}$:

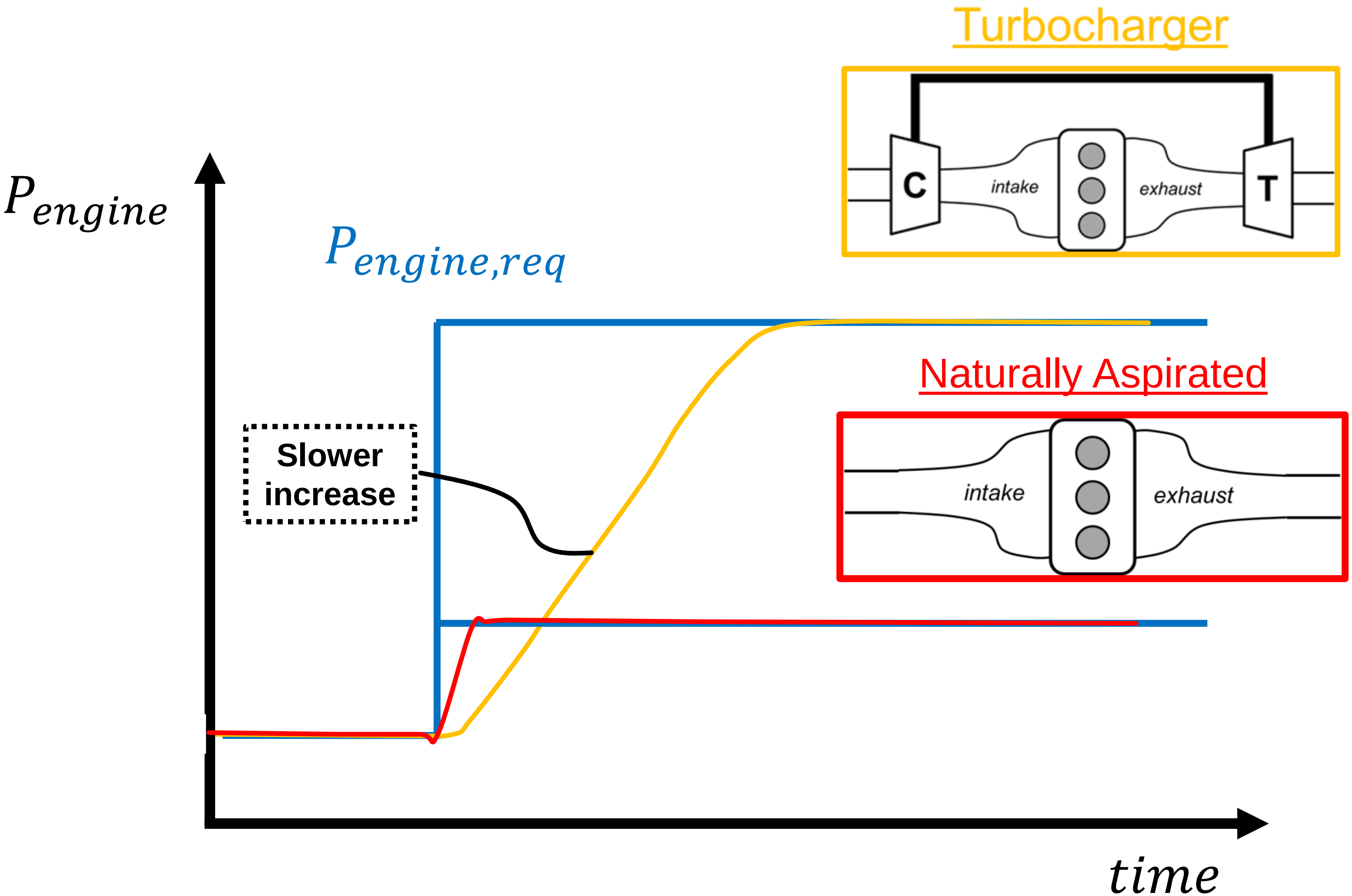
$$P_{engine} \approx k_1 \cdot \dot{m}_{fuel} + k_3 \cdot (p_{intake} - p_{exhaust})$$



$$\boldsymbol{\omega_e = 10'000\ rpm}$$
$$\boldsymbol{V_d = 1.6\ L}$$

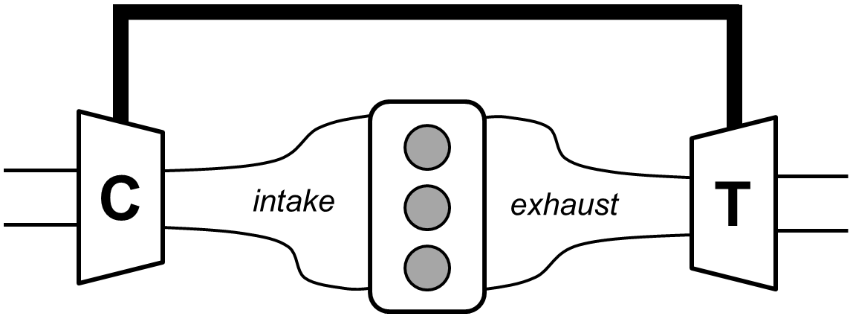
$$\boldsymbol{P_{pump,NA} \approx 0\ kW}$$
$$\boldsymbol{P_{pump,TC} \approx 26\ kW}$$

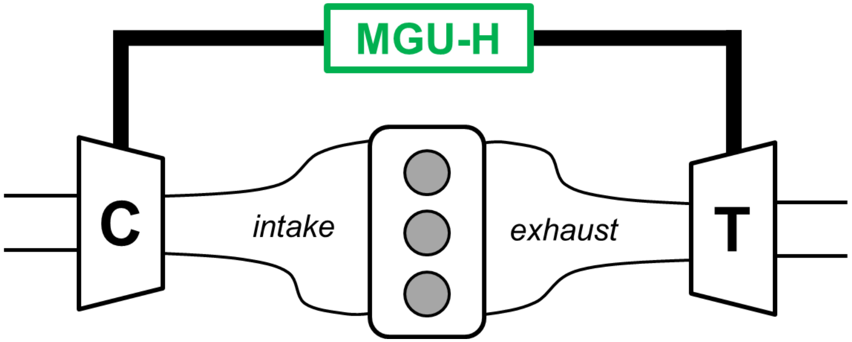
Engine Response



Formula 1 – Engine Response

How and how fast is the **engine power response** of a **conventional turbocharger** compared to an **electrified turbocharger** (e.g. F1) **?**





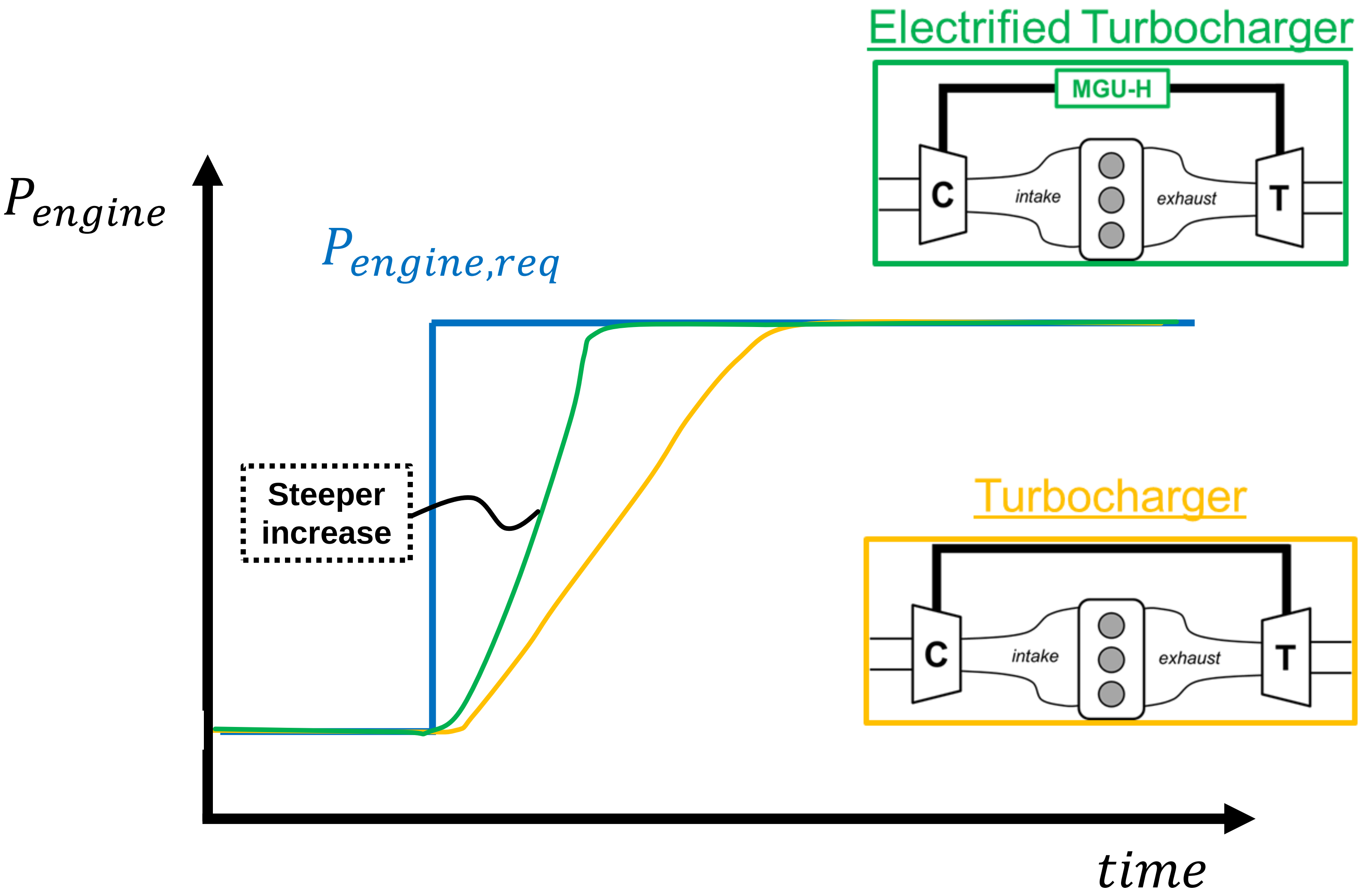
Turbocharger

1) Press throttle pedal
2) More fuel injected
3) Exhaust temperature increases
4) More power extracted by the turbine
5) Turbocharger speed increases
6) Compressor mass flow increases
7) More air → more fuel can be injected
8) More Engine power

Electrified Turbocharger

1) Press throttle pedal
2) More fuel injected & MGU-H positive
3) Turbocharger speed increases faster (→ MGU-H & exhaust temperature)
4) Compressor mass flow increases
5) More air → more fuel can be injected
6) More Engine power

Formula 1 – Engine Response



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Formula 1 Power Unit

Efficient control algorithms are designed for the hybrid electric propulsion system of the Formula 1 car, in order to achieve the fastest possible lap-time. (Presentations in English or German)

Mauro Salazar, maurosalazar@idsc.mavt.ethz.ch | Camillo Balerna, balernac@idsc.mavt.ethz.ch

http://www.idsc.ethz.ch/research-guzzella-onder/research-projects/Formula1.html