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151-0573-00 **System Modeling** (HS 2017) **Exercise 3**

Topic: Modeling of Mechanical Systems using Lagrange

Discussion: 10. & 12.10.2017

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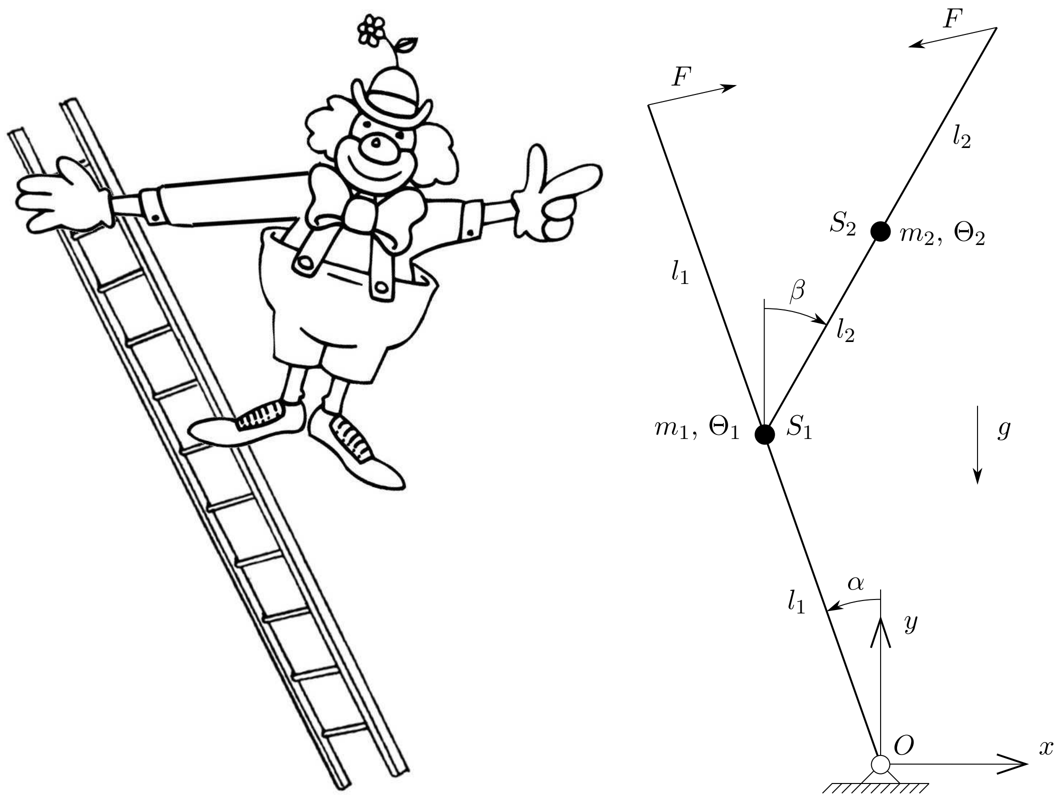
Problem 1 (Clown - HS10)

During a visit of the famous Circus Fantasticus you observe a clown, who balances on a ladder as depicted in Figure 1(a). The system can be modeled as follows: the ladder is a rigid body with length $2\,l_1$, mass m_1, center of gravity S_1, and moment of inertia Θ_1 with respect to its center of mass S_1. The clown is a rigid body with length $2\,l_2$, mass m_2, center of gravity S_2, and moment of inertia Θ_2 with respect to its center of mass S_2. The ladder is free to rotate about a frictionless pivot in the origin O. Furthermore, the clown is free to rotate about a frictionless pivot denoted as S_1. The force F, which is exerted by the clown's hand on the ladder, acts on the clown and the ladder as shown in Figure 1(b). Gravitational acceleration g acts in $-y$ direction. Furthermore, no friction is assumed in any element of the system.



(a) Clown balancing on ladder. (b) Mechanical Model.

Figure 1: Clown on Ladder.

For brevity, the time dependency of the variables α, β, F and q is not shown. Simplify the results as far as possible.

a) The angles α and β, depicted in Figure 1(b), are possible minimal coordinates for this system. Please give a further option of minimal coordinates.

b) Derive the potential energy of the ladder U_1 as a function of α and β.

c) Derive the potential energy of the clown U_2 as a function of α and β.

d) Derive the kinetic energy of the ladder T_1 as a function of α and β.

e) Derive the kinetic energy of the clown T_2 as a function of α and β.
(Hint: $\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = \cos(\alpha + \beta)$.)

f) Calculate the equations of motion by using Lagrange II with the minimal coordinates $q = (\alpha \;\; \beta)^{\mathrm{T}}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\beta}}\right) - \frac{\partial L}{\partial \beta} = Q_2$$

where

$$L = L(q) = \sum_{i=1,2} T_i(q) - U_i(q)$$

is the Lagrange function. **Hint:** You **do not** have to derive $Q_2 = Q_2(q, F)$.

g) From

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = Q_1$$

the following equation of motion results:

$$Q_1 = \left(l_1^2\,(m_1 + m_2) + \Theta_1\right)\ddot{\alpha} - m_2\,l_1\,l_2\left(\ddot{\beta}\cos(\alpha+\beta) - \dot{\beta}^2\sin(\alpha+\beta)\right) - (m_1 + m_2)\,l_1\,g\sin(\alpha) \quad .$$

Please calculate the inertia matrix $M(q)$ of the system.

h) Color the clown.

Problem 2 (Disk on the Hills)

A disk of radius r, mass m and moment of inertia Θ_C rolls without slipping on a circular profile of radius R. The center of the disk is connected to the vertical axis with a spring of stiffness k and zero unstreched length. The spring force *acts always in the horizontal direction*, while gravity acts in the vertical direction, as indicated in Figure 2. Denote with ψ the rotation angle of the disk about its center C. Denote with θ the angular position of the center of the disk with respect to the center of the circular profile. Denote with O the contact point between disk and circular profile.

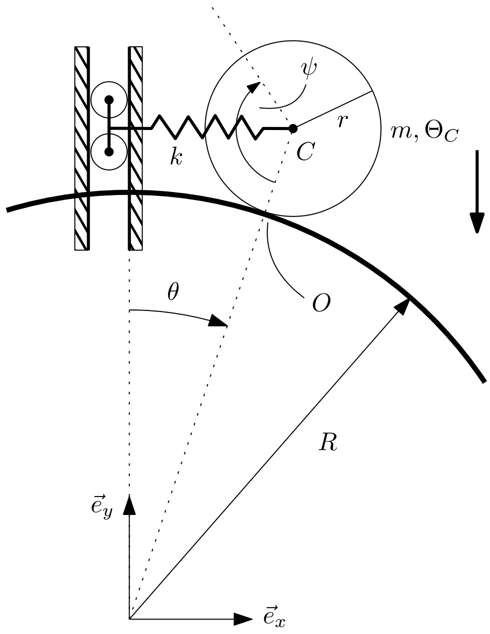


Figure 2: Disk on the hills

a) Find an expression which describes the no-slip condition. Are θ and ψ independent variables? How does this affect the analysis of the system?

b) Find the system equation of motion using the Lagrange Formalism, as presented in the lecture.
Hints:

- Use θ as generalized coordinate.
- The spring is unstreched when $\theta = 0$.
- No friction is assumed in any element of the system.