Topology Optimization
State-of-the-Art and Future Perspectives

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Topology Optimization in Aerospace

Bendsøe and Kikuchi (1988)

Design domain
FE-Discretization
Interpretation
Optimal material redistribution

Topology Optimization Applications

Autonomous industry (Fabian Duddeck)
Wind turbines (SUZLON and FE-Design GmbH)
Reconstructive surgery (Paulino/Sinn-Hanlon)
Micromachines (DTU Nanotech)

Acoustics
Small antennas
Extreme materials
Cloaking
Energy harvesting
Nano-photonics
Fluids
Structural colours
Applications in Architecture/Design

Discrete topopt formulation

\[
\begin{align*}
\min_{\rho} & : \Phi(\rho, U(\rho)) \\
\text{s.t.} & : \sum_{e=1}^{N} v_e \rho_e = v^T \rho \leq V^* \\
& : g_i(\rho, U(\rho)) \leq g_i^*, \quad i = 1, \ldots, M \\
& : \rho_e = \begin{cases} 0 \text{ (void)} \\ 1 \text{ (material)} \end{cases}, \quad e = 1, \ldots, N \\
& : K(\rho) U = F
\end{align*}
\]

0/1 Integer problem

- Combinations:
  - \( N=10, M=5 \Rightarrow 252 \)
  - \( N=20, M=10 \Rightarrow 185,000 \)
  - \( N=40, M=20 \Rightarrow 1.4 \times 10^9 \)
  - \( N=100, M=50 \Rightarrow 10^{20} \)

SIMP-approach


Objective function: \( \Phi(U(\rho)) \)
Equilibrium (FEM): \( K(\rho) U = F \)

Stiffness interpolation:

Sensitivity analysis – adjoint method

Augmented objective function: \( \Phi = \Phi(U(\rho)) + \lambda^T(KU - F) \)
Differentiate: \( \Phi' = \frac{\partial \Phi}{\partial U} U + \lambda^T(K'U + KU') \)
Collect \( U' \) terms: \( \lambda^T K + \frac{\partial \Phi}{\partial U} U' = 0 \Rightarrow \lambda^T (K + \frac{\partial \Phi}{\partial U}) = 0 \)
Adjoint problem: \( K^T \lambda = - \left( \frac{\partial \Phi}{\partial U} \right)^T \)
Final sensitivity: \( \Phi' = \lambda^T K'U \)
The Topology Optimization Process

- Initialize FEM
- Finite Element Analysis (Elastic, Thermal, Electrical, etc.)
- Sensitivity Analysis
  - Regularization (filtering)
  - Optimization (material redistribution)
- Sensitivity analysis by adjoint method
  \[ \frac{d\Phi}{d\rho_c} = \frac{\partial \Phi}{\partial \rho_c} + \lambda^T \left( \frac{\partial K}{\partial \rho_c} U + \frac{\partial F}{\partial \rho_c} \right) \]
  \[ K^T \lambda = -\frac{\partial \Phi}{\partial U} \]
- Mathematical Programming, Method of Moving Asymptotes (MMA) by Svanberg (1987)

Regularization by low-pass filtering

- Neighborhood:
  \[ N_e = \{ i \mid ||x_i - x_e|| \leq R \} \]
- Sensitivity filtering (Sigmund 1997, Sigmund&Maute 2012)
  \[ \overline{\partial \Phi} = \sum_{i \in N_e} H(x_i) \frac{\partial \Phi}{\partial \rho_c} \overline{\rho_c} \]
- Density filtering (Bruns&Tortorelli/Bourdin 2001)
  \[ E_c(\rho) = \overline{\rho_c} E_0, \quad \overline{\rho_c} = \sum_{i \in N_e} H(x_i) \rho_i \]
- PDE-based filtering (Lazarov&Sigmund 2011)
  \[ -\nu^2 \Delta \overline{\rho_c} + \overline{\rho_c} = \rho \]

The "TopOpt App"

- The "TopOpt App": AppStore (iOS)
  - Google Play (Android)
  - Web-version: www.topopt.dtu.dk

The "TopOpt3d App"

- The "TopOpt3d App": AppStore, iOS and PC – see www.topopt.dtu.dk

Stats: November 2015:
- Android: 4900, iOS: 9000, web: 9700
- iOS: 2600, web: 730

See www.topopt.dtu.dk for more information.
TopOpt Rhino plugin


Public Codes

99 Line basic Matlab (Including FE, grad’s, OC)
OS, A 99 line topology optimization code written in MATLAB, SMO, 2001, 22, 120-127

88 line advanced Matlab (+advanced filters)
Andreassen, E.; Clausen, A.; Schevenels, M.; Lazarov, B. & OS, Efficient topology optimization in MATLAB using 88 lines of code, SMO, 2011, 43, 1-16

On multigrid-CG for efficient topology optimization
Amir, O.; Aage, N. & Lazarov, B.S., Efficient topology optimization in MATLAB using 88 lines of code, SMO, 2011, 43, 1-16

Topology optimization using PETSc:
An easy-to-use, fully parallel, open-source topology optimization framework
Aage, N; Andreassen, E. & Lazarov, B.S., 2015, SMO, 51, 565-572

Freely downloadable from www.topopt.dtu.dk

Challenges and goals

Methods
• Manufacturing limitations/uncertainties
• Feature control – advanced geometry control
• Adaption to Additive Manufacturing (AM)
• Super large scale

Applications
• Extremal material design
• Non-linearities
• Multiphysics
• Wave propagation
• Multiscale

Length-scale control and robustness
**Compliant mechanism design**

Sensitivity filtering

Density filtering


\[
-\rho \Delta \rho + \rho = 0
\]

\[
\rho \rightarrow \tilde{\rho}(\rho) \rightarrow \hat{\rho} \tilde{\rho}(\rho)
\]

**Local geometry control**

**Erosion**
Sigmund (2007)
\[ \eta = 1 \]

"Volume preserving"
Xu et al (2010)
\[ \eta = 0.5 \]

**Dilation**
\[ \eta = 0 \]

**Robust formulation**

\[
\min_{\rho} \max_{\Phi} \left( \Phi(\tilde{\rho}(\rho)), \Phi(\tilde{\rho}^m(\rho)), \Phi(\tilde{\rho}^d(\rho)) \right)
\]

s.t.:
\[
K(\tilde{\rho}^e) U^e = F,
K(\tilde{\rho}^m) U^m = F,
K(\tilde{\rho}^d) U^d = F,
g = V(\tilde{\rho}^d)/V^* - 1 \leq 0
\]

\[ 0 \leq \rho \leq 1 \]

\[ \tilde{\rho}^e = \tilde{\rho}_{\eta=0.5+\Delta \eta}, \quad \tilde{\rho}^m = \tilde{\rho}_{\eta=0.5}, \quad \tilde{\rho}^d = \tilde{\rho}_{\eta=0.5-\Delta \eta} \]

Wang, Lazarov and Sigmund, SMO, 43, 767-784, (2011)
Robust topopt formulation

Uniform over/under etching

Over etched
Blue print
Under etched

Unique length scale control: c.f. Wang, Lazarov and Sigmund, SMO (2011), Qian and Sigmund, CMAME (2012)

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Robust electrostatic actuator design

Qian and Sigmund, CMAME (2012)

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Robust electrostatic actuator design

Qian and Sigmund, CMAME (2012)

Ultra high resolution TopOpt
(overcoming the Duplo problem)

Qian and Sigmund, CMAME (2012)

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Previous work in aircraft wing design

Stanford & Dunning, *Journal of Aircraft*, 2015, 52, 1298-1311:

"... the resulting structure typically bears no resemblance to traditional rib/spar networks, which may indicate one of two things. The first is that the appropriate physics, load cases, and/or constraint boundaries were not included in the optimization problem, and if they had been, the resulting topology would qualitatively approach a lattice of ribs and spars. The second is that the design problem was properly defined, and that the non-traditional topology may present an interesting new direction for efficient wing structures."

+100M design variables

The code:
- PETSc based – highly scalable
- Solver: F-GMRES with MG preconditioner.
- Open source (topopt.dtu.dk)
- Includes filters, MMA, IO.
- Comes with minimum compliance example

Aage; Andreassen & Lazarov, *SMO*, 2015, 51, 565-572
**NASA Common Research Model**

Geometry and pressure load data from NASA:

Meshing by structured slices:

~**1 billion** elements (1216 x 256 x 3456)...

... largest element size ~ 8 mm

**Results: 135 million elements**

Material design and non-linearities

Material with negative Poisson’s ratio

- FE on one cell with periodic B.C.
- Minimize Poisson’s ratio
- Constraint on bulk modulus and symmetry

Sigmund (1995)
3D Manufacturing and testing

Negative thermal expansion coefficient

3d negative thermal expansion

Finite deformations

\[ \alpha_{\text{red}} = 3.5 \]
\[ \alpha_{\text{blue}} = 1 \]
\[ E_{\text{red}} = 1 \]
\[ E_{\text{blue}} = 3.5 \]
\[ \nu^H = 0.18 \]
\[ E^H = 0.0016 \]
\[ \alpha^H = -5.4 \]

Produced by Erik Andreassen

Wang et al., CMAME, 2014, 276, 453-472
Clausen et al., Adv. Mater., 2015, 27(37), 5523-5527
Manufacturing using Direct Ink Writing

Design adapted to Direct Ink Writing

Uniform feature design using superellipses

Optimized designs for $\nu \in -0.8:0.2:0.8$
All designs printed row- and columnwise

Deformation pattern for $\nu = -0.8$

Complete set of realized designs

Numerics vs experiments
Parameterization for any $\nu \in [-0.8, 0.8]$


3D Poisson’s ratio -0.8

Small deformation:

Finite deformation:

Wang et al., 2016, to be submitted

Cooling fins for LED lamps

HYPERCOOL – Cool Danish Design

Thermofluidics
**Intuitive designs by industrial designer**

**Thermofluidic equations**

Incompressible Navier-Stokes equation for porous flow

\[
\mathbf{u} \cdot \nabla \mathbf{u} - Pr \nabla \cdot (\nabla \mathbf{u}) + \alpha \mathbf{u} + \nabla p = -Gr Pr^2 e g T
\]

\[
\nabla \cdot \mathbf{u} = 0
\]

Convection-diffusion equation

\[
\mathbf{u} \cdot \nabla T - \nabla \cdot (K \nabla T) = q
\]

**Optimization of fluid mixing**

**Natural convection cooler problem**

Andreasen; Gersborg & OS, *JNNM*, 2009, 61, 498-513
Conclusions

- TO is efficient in solving wide classes of engineering design problems
- Here mostly concentrated on solids – lots of application in fluids, thermofluidics, electromagnetics, nano-optics, etc.
- We are at the verge of being able to skip the post-processing step and send TO results directly to (additive) manufacturing
- Still several interesting challenges:
  - Large scale
  - Non-linearities
  - Multiphysics
  - Multiscale
  - Taking advantage of new manufacturing possibilities
Further reading

TopOpt background

- OS, On the usefulness of non-gradient approaches in topology optimization, SMo, 2011, 43, 589-596
- Schevenels, Lazarov & OS, Robust TopOpt account. f. spat. varying man. err., CMAME, 2011, 200, 3613-3627

Codes

- Aage; Andreassen & Lazarov, B.S., TopOpt using PETSc: An easy-to-use, fully parallel, open-source topopt framework, SMo, 2015, SMo, 51, 565-572

Material Design

- Clausen; Wang; Jensen; OS & Lewis, Topology Optimized Architectures with Programmable Poisson’s Ratio over Large Deformations, Advanced Materials, 2015, 27, 5523-5527

Fluid Applications


See www.topopt.dtu.dk for more

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Probability-based topology optimization

Random geometry errors

\[ \min \frac{m_{\phi} + k\sigma_{\phi}}{\rho} \]
\[ \text{s.t. } K(\tilde{\rho}g(x,\theta),\tilde{\rho}(\rho))U(\theta) = F \]
\[ g = m_{V}/V^* - 1 \leq 0 \]
\[ 0 \leq \rho \leq 1 \]

\( m_{\phi} \): mean value of \( \phi \)
\( \sigma_{\phi} \): standard deviation of \( \phi \)
\( k \): weighting factor
\( m_{V} \): mean value of \( V \)

MC
Stochastic collocation
Stochastic perturbation

Schevenels, Lazarov & Sigmund, CMAME, 2011, 200, 3613-3627

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Localized random variations

Probabilistic design

\[ \phi = -1.03 \]
\[ m_{\phi} = -0.26 \]
\[ \sigma_{\phi} = 0.36 \]

\[ \phi = -0.94 \]
\[ m_{\phi} = -0.91 \]
\[ \sigma_{\phi} = 0.02 \]
Smooth boundaries