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Dynamic Model of a Multi-terminal HVDC Grid Based on a Simplified Converter Model

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Introduction

Many existing High Voltage Alternating Current (HVAC) bulk transmission systems, e.g., in Europe and North America, have reached their capability limits and a significant increase of transmission capacity is foreseen within the next decades. There are several reasons calling for an upgrade, where the most significant is the expected overall increase in consumption of electric energy. Another important driving force are the latest developments in High Voltage Direct Current (HVDC) technology such as higher power ratings of semiconductor devices and the introduction of Voltage Sources Converters (VSC) for power transmission, which are deemed to make HVDC networks a feasible option for future transmission systems. VSC have many advantages compared to the older Line-Commutated Converter (LCC), which have been used in commercial operation for over 50 years. The independent control of active and reactive power in VSC-Terminals is one property, which can be used to increase the power transfer capability and the stability of a grid.
Objectives of the semester thesis

The objective of this semester thesis is to expand an existing dynamic model[1] for VSC converter for a point-to-point connection. The new model should be capable to incorporate multi-terminal HVDC grids with a meshed topology. For this purpose the existing model should be studied first. The challenges for expanding this model to a multi-terminal grid have to be identified. Then, suitable solutions for the occurring problems have to be found. It time allows, the interaction between the HVDC system and a connected AC system can be studied. The system should be implemented into a Matlab simulation environment and different case studies should be conducted to show the working principles.

Task to be completed

- Literature study and short summary of relevant literature
- Study the existing model and understand the working principles
- Identify the problems, which arise when the model is expanded to multi-terminal systems
- Find suitable solutions for the problems
- Implement the new model into Matlab simulations
- Investigate several different case studies
- Write a final report and give a presentation about the thesis

References


Schedule

Start: 17.03.2014
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Abstract

This semester thesis deals with the dynamic modelling of a Voltage Source Converter based High Voltage Direct Current (VSC HVDC) system. The objective of the work is to expand an existing dynamic model of a two-terminal VSC HVDC system. The new model has to be capable to incorporate multi-terminal HVDC grids with a meshed topology. Different case studies should be simulated to show the working principles and to derive interpretations of the different observations.

A model is first derived for the different parts of the VSC HVDC system, namely an Alternating Current (AC) side model, a Direct Current (DC) side model and a simplified converter model. Pulse Width Modulation (PWM) is used for the VSC and makes rapid and independent control of the active and reactive power possible.

As the control structure of the VSC HVDC system plays a major role for the dynamic modelling, a well defined model of the control system needs to be derived as well. This model consists of certain control parameters, tuned according to some predefined properties of the dynamic response. Efficient parameter tuning is achieved by first deriving a linear control model. A substantial part of the work focuses on this tuning procedure, mainly done according to the performance of an ABB HVDC Light Model.

Once all parameters of the control system and the VSC HVDC system are identified and assigned to numerical values, a non-linear dynamic VSC HVDC model is proposed for a chosen Multi-Terminal Direct Current (MTDC) test grid. The MATLAB based modelling and simulation software Simulink is utilized for dynamic simulations of this non-linear VSC HVDC model. By applying a step input to the reference signal of different controllers, the dynamic response of certain system variables is studied and conclusions are drawn.
I would like to offer my special thanks to Roger Wiget, my research supervisor, for his enthusiastic encouragement and useful critiques of this research work. His willingness to give his time so generously has been very much appreciated. I would also like to acknowledge the help offered by Martin Andreasson, for providing valuable insights. Their constructive advice led to the successful completion of this project.
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<tr>
<td>ABB</td>
<td>Asea Brown Boveri</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>CSC</td>
<td>Current Source Converter</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>ENTSO-E</td>
<td>European Network of Transmission Operators for Electricity</td>
</tr>
<tr>
<td>FACTS</td>
<td>Flexible AC Transmission System</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current</td>
</tr>
<tr>
<td>IEM</td>
<td>International Electricity Market</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated Gate Bipolar Transistor</td>
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<tr>
<td>LT</td>
<td>Laplace Transformation</td>
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<tr>
<td>MMC</td>
<td>Multi-Level Converter</td>
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<td>Multi-Terminal Direct Current</td>
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<tr>
<td>QU</td>
<td>Reactive Power and DC Voltage</td>
</tr>
<tr>
<td>RES</td>
<td>Renewable Energy Sources</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
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<td>TSO</td>
<td>Transmission System Operator</td>
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<td>VSC</td>
<td>Voltage Source Converter</td>
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<td>Capacitance of DC capacitor at DC side of converter $l$ [F]</td>
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<td>$I_{dc,l}$</td>
<td>DC current at DC side of converter $l$ [A]</td>
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<td>Integral gain PI controller, present in reactive power controller</td>
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<td>$K_{i,id,l}$</td>
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<td>$K_{p,iq,l}$</td>
<td>Proportional gain PI controller, present in q-current controller</td>
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<td>$\omega$</td>
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<td>$\omega_n$</td>
<td>Natural frequency of second order transfer function [rad/s]</td>
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Chapter 1

Introduction

1.1 Motivation

In the previous decade in Europe, the International Electricity Market (IEM) was developed and targets easy international trade of electricity. However, national grids were originally interconnected only to aid in case of emergency conditions. Hence, they were never dimensioned with international trade in mind, resulting in an European network consisting of strong national grids and weakly interconnections. The scarcity of cross-border capacity impedes this aforementioned target of easy international electricity trading and creates a barrier for the development of a single IEM. It is recognised by the European Network of Transmission Operators for Electricity (ENTSO-E) that new technologies, such as VSC HVDC, are valuable investment options [1]. Since international trade causes unidentified or unscheduled variable flows, dealing with those flows and operating the grid becomes really challenging for the Transmission System Operator (TSO). Tie-lines can become dangerously high loaded. VSC based HVDC technology makes it possible to control the power in a DC line for point-to-point systems and the injected power at every terminal in MTDC systems.

Another ongoing trend, is the increase of infeed of Renewable Energy Sources (RES) [2]. RES can consist of remote sources of electrical power - often emissions-free sources like hydro or wind generation- and can be connected to load centres, hundreds or even thousands of kilometers away when HVDC is used. This is because HVDC systems can transmit more electrical power over longer distances than an AC transmission system of the same voltage range, resulting in fewer transmission lines, saving both money and land [3]. With the advent of the high power Insulated Gate Bipolar Transistor (IGBT), provided with a turn-off capability, a new era in HVDC technology has commenced, known as the VSC based HVDC technology. A new era in HVDC technology doesn’t mean a replacement of the already existing
CHAPTER 1. INTRODUCTION

Current Source Converter (CSC) based HVDC [4,5], using thyristor based converters instead of IGBT based converters. The new technology expanded the range of HVDC applications [6]. The independent control of active and reactive power and the fast reversion of power flow direction, especially critical for MTDC systems [7], make VSC HVDC transmission an attractive solution for connecting more RES to the network, as the controllability is increased and the system operating costs decreased. A higher penetration of RES and decommissioning of conventional power plants will cause higher power oscillations as the inertia in the system will be reduced. The installation of VSC based HVDC links for transient stability control or for power oscillation damping provides a solution [8].

The previous two trends show the increasing importance of VSC based HVDC transmission. When considering VSC HVDC for a transmission project, the power system engineer needs models of the various system components at every stage of the project. For the basic power system equipment such as transmission lines, transformers and generators with their controls, standard models already exist. Since VSC HVDC is a fairly new technology, no standard dynamic models for multi-terminal VSC based HVDC have been generally accepted yet. The group of dynamic models proposed in literature are mostly specific for one (or multiple but limited) particular cases and are not suitable for standard models. This semester thesis aims to propose a general dynamic model of a multi-terminal VSC based HVDC system, valid for every conceivable configuration of the DC grid.

1.2 Literature

Unlike CSC based HVDC, whose experience goes back to the 1950s, VSC based HVDC hasn’t obtained the same level of experience yet, since it is a fairly new technology. No standard models have been generally accepted yet so far, although research on the modelling is growing. Many steady-state models have been developed already as in [9], [10] and [11]. The group of dynamic VSC based HVDC models is rather heterogeneous and only proposed for fixed topologies. Also, very few dynamic models are explicitly developed for MTDC systems, since almost all HVDC systems until now are point-to-point transmissions, connecting or paralleling AC transmission systems. As the number of these point-to-point HVDC connections increases, it is becoming apparent that it would be beneficial to connect them directly, rather than through the broader AC grid, as they are currently. This is giving rise to plans for MTDC supergrids [12]. Consequently, the need for dynamic models of multi-terminal VSC based HVDC grids is growing.

In this semester thesis, the derivation of such a model is done by expanding
1.3. OBJECTIVES OF THE SEMESTER THESIS

the existing dynamic model of a VSC HVDC converter for a point-to-point connection. This model is described in [13] and will be utilized as a reference for the derivation. The paper [13] presents a modified dynamic VSC HVDC model of [14] and [15] which can be used for dynamic power system studies. The VSC HVDC two-level converter is modelled in the dq0 frame representation by applying a dq transformation. This is compliant with existing dynamic AC models such as generators and Flexible AC Transmission System (FACTS) devices. The model includes the dynamics of physical elements such as AC phase reactors, DC capacitors and DC line dynamics as well as the dynamics of the underlying converter control. Not only is the physical system modelled, also the tuning of the cascaded Proportional-Integral (PI) controllers is presented, based on a linearized control model. The tuning is done according to the performance of an ABB HVDC Light Model. The expansion of this model will be done in a similar way.

Before expanding the model, it is essential to have a clear understanding of the components and operating principles of a VSC HVDC system. More information about this is given in Chapter 2. More detailed literature is found in [17] and [18].

1.3 Objectives of the Semester Thesis

As VSC HVDC transmission is gaining more importance, the need for standard models, that can be readily implemented in power system simulation software, is also rising. This work aims to propose a general dynamic model of a multi-terminal HVDC grid, valid for every conceivable configuration of the DC grid. As a starting point, the existing dynamic model [13] of a VSC HVDC converter is chosen. The challenges for expanding this model to a multi-terminal grid are identified and suitable solutions of the occurring problems are proposed.

Firstly, models are defined for the different parts of the VSC HVDC system: AC side models, converter models and a DC side model. The same is done for the control systems after identifying the different control parameters. Efficient tuning of those control parameters of the non-linear dynamic model is achieved by first deriving a linear control model. Subsequently, a tuning procedure is proposed for the different controllers.

In the end, different case studies are conducted to show the working principles by simulating the different non-linear VSC HVDC models in the MATLAB based modelling and simulation software Simulink. Those case studies are mainly based on the data and performance of an ABB HVDC Light Model [13,16].
Chapter 2

VSC HVDC Transmission

Before starting to model a VSC HVDC system, it is essential to have an idea of its components and operating principles. Firstly, the relevant components of a converter station or terminal and their importance for the dynamic model are discussed. In a next section, the operation principles of a VSC HVDC system are explained. It shows that active and reactive power can be independently controlled, of course within certain limits. The operation range of a VSC HVDC system is limited by the current through the converter, the DC voltage and the rating of the line [15]. Different VSC based HVDC topologies are discussed in the last part of this chapter.

2.1 VSC HVDC Components

Figure 2.1 represents a terminal of a symmetrically grounded bipolar DC system. On the AC side, the AC grid is connected to the converter through a transformer, a shunt connected filter and a phase reactor. Those elements are three-phase elements but are depicted by their single-phase representation. DC lines and the DC capacitors are found on the DC side of the converter. From one converter, multiple DC lines can leave to other terminals.

Figure 2.1: Terminal of a symmetrically grounded bipolar DC system
CHAPTER 2. VSC HVDC TRANSMISSION

Not all electrical elements of a real terminal or converter station are represented. Elements, not important for dynamic modelling are omitted. For example on the AC side, since components as circuit breakers and disconnectors are assumed to be closed at all times, there is no need for modelling them.

2.1.1 Transformer

The transformer is an important element for the HVDC transmission. Although, converter transformers exhibit higher losses due to the harmonic content, standard transformers can be used for VSC HVDC systems. This is thanks to the filter, located between converter and transformer. In the following, the transformer is represented simply by its leakage reactance $X_{t,l}$.

2.1.2 Phase Reactors

The phase reactor is a large inductive element with small resistance. Its purpose is to control the complex current. By doing so, it can control the active and reactive power, injected in the DC grid. The phase reactor’s size determines the dynamic behaviour of the converter’s AC side. Hence, the phase reactor is an important element in modelling VSC HVDC systems.

2.1.3 Filter

The filter prevents harmonics from entering the AC system. It filters high frequency components to prevent the transformer from exposure to high frequency stress. As mentioned in Section 2.1.1, this allows using standard transformers. Assuming a model valid for only small deviations of the fundamental frequency, the AC filter is discarded in the dynamic model.

2.1.4 Valves

The valves are important elements of the HVDC system. Their main purpose is to switch voltages, much higher than the voltage of their switching elements. Therefore, those elements need to be connected in series. As switching elements, mostly IGBTs are chosen. For Root Mean Square (RMS) modelling, the valves don’t need to be modelled. The switching frequency is so high compared to the time order of the dynamics, that it can be assumed infinitely fast.

2.1.5 DC Capacitors

The DC capacitors are the most important elements in the DC circuit as they act as energy storage, which is vital for the operation of VSC HVDC
systems. Their size determines the dynamic behaviour of the DC circuit. Hence, they are important for the dynamic modelling.

2.1.6 DC grid

The DC grid consists of DC cables or overhead lines, connecting the different terminals of the system. The topologies of this DC grid is discussed in Section 2.3. The HVDC conductors, be it cables or overhead lines, are modelled by a $\pi$ circuit, sufficiently accurate for short to middle-long distances [19]. For long conductors, chained $\pi$ sections are a good approximation.

2.2 Operating Principles

The most basic three-phase VSC using IGBTs is the six-pulse bridge, depicted in Figure 2.2. There are different ways to operate the two level bridge in this figure. Operating in PWM reduces the harmonics, but also introduces flexibility: the voltage ratio factor can be controlled. That is why currently, most installations that have been built up till now, are two- or three-level schemes operated in PWM. However, the Modular Multi-Level Converter (MMC) is now becoming the most common type of voltage-source converter for HVDC. PWM makes it possible to regulate the voltage amplitude and the voltage phase angle of $u_c = U_c \angle \delta_c$.

![Three-phase six-pulse VSC bridge](image)

Figure 2.2: Three-phase six-pulse VSC bridge

2.2.1 Active and Reactive Power Control

The principles of power control are the same for every converter. The series reactance $X$ -the phase reactor, including the effect of the transformer- at the AC side of the converter is an essential element for power control. As mentioned in Section 2.1.3, the AC filter is discarded and doesn’t change the operating principles.
CHAPTER 2. VSC HVDC TRANSMISSION

Active power $P$ and reactive power $Q$ injected in the grid can be expressed by following equations:

\[ P = \frac{U_s U_c}{X} \sin(\delta_c) \]  
\[ Q = -\frac{U_s^2}{X} + \frac{U_s U_c}{X} \cos(\delta_c) \]

$u_s = U_s \angle 0$ is the AC system voltage phasor.

Changing the phase angle $\delta_c$ has a large influence on the active power and negligible influence on the reactive power, since $\delta_c$ is rather small. By varying this phase angle with respect to the angle of the system voltage, the active power can be controlled. Changing the voltage magnitude $U_c$ has a large influence on the reactive power and negligible influence on the active power. By varying this voltage magnitude, the reactive power can be controlled. The latter phenomenon is known as the $P\theta - QU$ Decoupling [19]. Each converter has thus two degrees of freedom: the angle and magnitude of the converter voltage, which can be used to independently control active and reactive power.

2.2.2 Converter Coordination

A DC network is created by connecting different terminals with DC lines. Since there is no power storage in the long term in the DC network, existing of $n$ terminals, following power balance is satisfied:

\[ \sum_{i=1}^{n} P_i = P_{loss} \]  

One terminal has to set its active power to compensate for the losses $P_{loss}$ in the DC network, by which the degrees of freedom are reduced with one, to $2n - 1$. This is done by controlling for one of the terminals the DC voltage instead of the active power. The converter of this terminal is called the ‘slack’ converter. It compensates just as the slack node in an AC grid, for losses in the DC network.

2.2.3 DC Voltage Control

The most important element of the DC circuit is the capacitor. The DC voltage control is explained for a two-terminal system, where converter 1 is Active and Reactive Power (PQ) controlled and converter 2, Reactive Power and DC Voltage (QU) controlled. The reasoning applies equally well to multi-terminal systems.
2.3. **VSC HVDC TOPOLOGIES**

When the active power infeed is increased in converter 1, both capacitors charge (Figure 2.3).

![Diagram showing DC voltage control step 1](image)

Figure 2.3: DC voltage control step 1: an increased active power infeed causes a charging of both capacitors

The DC voltage controller in converter 2 detects the voltage increase and a discharge of the capacitor will result until the set voltage $U_{dc2}$ is reached again (Figure 2.4). Also the other capacitor will discharge but the voltage $U_{dc1}$ in steady state will be higher than the initial value as the power balance has to be satisfied.

![Diagram showing DC voltage control step 2](image)

Figure 2.4: DC voltage control step 2: the DC voltage controller responds to an increased DC voltage

### 2.3 VSC HVDC Topologies

#### 2.3.1 Point-to-Point VSC HVDC Transmission

Allmost all HVDC systems until now are point-to-point transmissions, connecting or paralleling AC transmission systems. The system is formed by a rectifier connected to the AC system (e.g. wind farm), a HVDC link (e.g. DC cable) and an inverter connected to another AC grid (e.g. land based grid). The usual principle of operation is that the rectifier controls the injected/extracted active power flowing over the VSC HVDC link, beside the infeed of reactive power at its terminal, while the inverter controls beside the infeed of reactive power, the DC voltage at its terminal.
CHAPTER 2. VSC HVDC TRANSMISSION

2.3.2 Multi-Terminal VSC HVDC Transmission

As the number of these point-to-point HVDC connections increases, it is becoming apparent that it would be beneficial to connect them directly, rather than through the broader AC grid, as they are currently. This would result in multi-terminal VSC based HVDC transmission grids. They have advantages over two-terminal HVDC in many aspects such as control flexibility, reliability and economy. Unlike point-to-point VSC HVDC transmission, multi-terminal VSC HVDC systems are composed of more than two terminals, which are connected to a common HVDC grid. It is trivial to define what a DC grid exactly is, as there are many multi-terminal HVDC configurations. Three main topologies can be distinguished, see [20]. The focus of this semester thesis is on the topology shown in Figure 2.5.

![Figure 2.5: Meshed DC system with • as a representation of one terminal](image)

This topology proposes a meshed DC system, with a number of connections between the AC and DC system. Connections within the DC system without converters are possible. This results in redundancy in the DC system. Within this type of topology, there are still a lot of different configurations [21].

One way of operation for Voltage Source Converters, is that one converter controls the DC voltage while the others regulate their power transfer. All converters also control the reactive power at its AC terminal. The focus of this work will be on this way of operation. From a control issue, there are several important open questions. Therefore, the interaction between the existing AC system and the new supergrid needs to be investigated, for instance towards security. Another challenge are HVDC switchgears, since the absence of cyclic moments of current zero in a DC system inherently makes DC current switching more difficult than for AC systems.
Chapter 3

VSC HVDC Physical Model

Resulting from the chosen topology in Section 2.3.2, every terminal consists of only one VSC. Apparently, every terminal of the system has a similar structure, consisting of three clear distinguishable parts: the AC side, the DC side and the converter. Physical models for aforementioned parts are derived in this chapter. By combining them, a model for every single terminal can be derived and consequently for the whole VSC HVDC system.

3.1 Physical Model Overview

The physical model of one terminal $l$ of a generalised, symmetrically grounded bipolar DC system is depicted in Figure 3.1. A till B represents the AC side of the terminal, with A representing the AC system bus, whereas B till C represents the converter of the terminal. The DC side of this terminal starts at C.

![Figure 3.1: Physical model of one terminal $l$ of a generalised, symmetrically grounded bipolar DC system](image-url)
3.2 AC Side Model

A step down transformer and a phase reactor connect the AC grid with the VSC. The ideal step down transformer can be represented by an inductance $X_{t,l}$ as discussed in Section 2.1.1, whereas the phase reactor is characterized by an impedance, consisting of a small resistance $R_{r,l}$ and a more significant inductive reactance $X_{r,l}$. Following equations are derived from Figure 3.1:

$$
[u_{t,l}]_{abc} - [u_{c,l}]_{abc} = \frac{X_{r,l}}{\omega} \frac{d[i_{l}]_{abc}}{dt} + R_{r,l}[i_{l}]_{abc}
$$

(3.1)

$$
[u_{s,l}]_{abc} - [u_{t,l}]_{abc} = \frac{X_{t,l}}{\omega} \frac{d[i_{l}]_{abc}}{dt}
$$

(3.2)

By applying the dq transformation of Appendix A to both equations, coordinates are transformed from the three-phase stationary coordinate system $[u]_{abc}$ to the dq rotating coordinate system $u^{dq}$. This transformation is important for the control systems, clarified in Chapter 4.

Eqs. (3.1-3.2) are transformed to the rotating dq reference frame, resulting in

$$
\frac{X_{r,l}}{\omega} \frac{d[i_{l}]^{d}}{dt} - X_{r,l}i_{l}^{q} = -R_{r,l}i_{l}^{d} - u_{r,l}^{d} + u_{t,l}^{d}
$$

(3.3)

$$
\frac{X_{r,l}}{\omega} \frac{d[i_{l}]^{q}}{dt} + X_{r,l}i_{l}^{d} = -R_{r,l}i_{l}^{q} - u_{r,l}^{q} + u_{t,l}^{q}
$$

(3.4)

for Eq. (3.1) and resulting in

$$
u_{t,l}^{d} - u_{s,l}^{d} = X_{t,l}i_{l}^{q} - \frac{X_{t,l}}{\omega} \frac{d[i_{l}]^{d}}{dt}
$$

(3.5)

$$
u_{s,l}^{q} - u_{t,l}^{q} = X_{t,l}i_{l}^{d} + \frac{X_{t,l}}{\omega} \frac{d[i_{l}]^{q}}{dt}
$$

(3.6)

for Eq. (3.2). By neglecting the current dynamics of the transformer in Eqs. (3.5-3.6) as in [13] and by aligning the q-axis of the rotating dq reference frame with the AC system voltage $u_{s,l}$ (hence, $u_{s,l}^{d} = 0$), above Eqs. (3.5-3.6) result in

$$
u_{t,l}^{d} = X_{t,l}i_{l}^{q}
$$

(3.7)

$$
u_{s,l}^{q} - u_{t,l}^{q} = X_{t,l}i_{l}^{d}
$$

(3.8)

The above mentioned alignment is possible due to a synchronization technique, known as Phase Locked Loop (PLL) [15]. More information about the used dq transformation and the calculation, using this transformation, can be found in Appendix A.
Adding Eqs. (3.7-3.8) to Eqs. (3.3-3.4) gives

\[
\frac{X_{r,l}}{\omega} \frac{d}{dt} i_l^d - (X_{r,l} + X_{t,l}) i_t^d = -R_{r,l} i_l^d - u_{c,l}^d \quad (3.9)
\]

\[
\frac{X_{r,l}}{\omega} \frac{d}{dt} i_l^q + (X_{r,l} + X_{t,l}) i_t^q = -R_{r,l} i_t^q - u_{c,l}^q + u_{s,l}^q \quad (3.10)
\]

The latter equations form the AC side model of one terminal \( l \).

### 3.3 DC Side Model

The DC line between terminal \( l \) and terminal \( m \), at the DC side of terminal \( l \) can be modelled as a \( \pi \)-equivalent with parameters \( R_{dc,lm} \), \( L_{dc,lm} \) and \( C_{line,lm} \). Also at the DC side, the converter is provided with a converter capacitance \( C_{conv,l} \). This is a very important element for the dynamic modelling of the VSC HVDC system. Those capacitors act as energy storage. The equivalent DC capacitance \( C_{dc,l} \), shown in Figure 3.1 is calculated as

\[
C_{dc,l} = C_{conv,l} + \sum_{m=1}^{n} \frac{1}{2} C_{line,lm} \quad (3.11)
\]

with \( n \), the number of terminals in the VSC HVDC system. For deriving the dynamic circuit equations, following agreement is proposed for current directions: the current directions in the DC lines are fixed such that the current from converter \( l \) to converter \( m \) is positive if \( l < m \). The generalised dynamic circuit equations are now

\[
C_{dc,l} \frac{dU_{dc,l}}{dt} = I_{dc,l} - \sum_{m=l+1}^{n} I_{line,lm}, \quad l = 1 \quad (3.12a)
\]

\[
C_{dc,l} \frac{dU_{dc,l}}{dt} = I_{dc,l} + \sum_{m=1}^{l-1} I_{line,ml} - \sum_{m=l+1}^{n} I_{line,lm}, \quad l = 2, ..., n - 1 \quad (3.12b)
\]

\[
C_{dc,l} \frac{dU_{dc,l}}{dt} = I_{dc,l} + \sum_{m=1}^{l-1} I_{line,ml}, \quad l = n \quad (3.12c)
\]

and

\[
L_{dc,lm} \frac{dI_{line,lm}}{dt} = U_{dc,l} - U_{dc,m} - R_{dc,lm} I_{line,lm}, \quad \forall m < n, \forall l < m \quad (3.13)
\]

with \( n \) the number of terminals in the HVDC system. Eq. (3.12) for every terminal together with Eq. (3.13) for every DC line in the grid form the DC side model.
CHAPTER 3. VSC HVDC PHYSICAL MODEL

3.4 Converter Model

The Voltage Source Converter is a highly controllable device, available in many different layouts. The converter used in this semester thesis is assumed to be a lossless two-level PWM converter, as shown in Figure 2.2. PWM is a method for generating an analog signal (AC side) using a digital source (DC side). By modulating the width of the pulses, an analog output voltage is produced. To reduce harmonics, the switching frequency is high, usually around 2kHz. From the control point of view, the output voltage $u_c$ is assumed to follow a voltage reference signal $u_{c,ref}$. Due to the switches, there will be a small time delay, equal to half of a switching cycle. As this is very small in comparison to the dynamics, we will neglect this time delay and assume the converter voltage to follow directly the converter voltage reference, resulting in $u_c = u_{c,ref}$.

The PWM converter forms the connection between the DC side and the AC side. Those two are coupled to each other by following power balance:

$$P_{ac,l} = \frac{3}{2}(u_c^d i_l^d + u_c^q i_l^q) = 2I_{dc,l}U_{dc,l} = P_{dc,l}$$  \hspace{1cm} (3.14)

The detailed calculation of this power balance can be found in Appendix B.
Chapter 4

Control Systems

Control systems are necessary, since we want to control accurately the transmitted power in the system. The introduction of the VSC made it possible to independently control the active and reactive power in a VSC based terminal. As the control structure of the VSC HVDC system plays a major role for the dynamic modelling, a well defined model of the control systems has to be chosen as well. The model of different controllers in a VSC HVDC system is derived in this chapter.

4.1 Control Systems Overview

The most widely used method for the control of three phase PWM converters is the vector control method. This method makes the independent control of reactive and active power possible. Vector control involves the simplified representation of three phase systems, known as the aforementioned dq transformation. One of the main advantages of this dq transformation is that vectors of three phase systems (ac currents and ac voltages) are now constant vectors in steady state. Hence, static errors in the control system can be avoided by using PI controllers. The structure of a PI controller is shown in Figure 4.1.

![Diagram of PI controller with saturation and anti-windup](image)

Figure 4.1: Scheme of PI controller with saturation and anti-windup [13]
Since active and reactive power can be independently controlled, almost all terminals in the system will be PQ controlled. Terminals of a VSC HVDC system are connected through a DC grid. Injected power is equal to the losses in the lines plus the extracted power. One terminal must set its active power to compensate for these unknown active power losses. In this particular terminal, the DC voltage will be controlled instead of the active power. This principle is similar to the slack bus concept of an AC grid and is already explained in Section 2.2.2. This terminal will be QU controlled.

4.1.1 PQ Controlled Terminal

Every PQ controlled terminal consists of two independent control schemes, namely a control scheme for the active power and a control scheme for the reactive power. Both control schemes have a slower outer control loop and a faster inner control loop. The inner loop controls the currents. The AC system voltage \( u_{q,s,l} \) is assumed to be dependent on the AC system and independent of the HVDC system. Consequently, the active power \( P_l \) and reactive power \( Q_l \) are controlled by controlling the currents \( i_{q,l} \) and \( i_{d,l} \), according to following equations:

\[
P_l = \frac{3}{2} u_{q,s,l} i_{q,l} \quad (4.1)
\]

\[
Q_l = \frac{3}{2} u_{q,s,l} i_{d,l} \quad (4.2)
\]

Apparently, the PQ controlled converter has two inputs, namely an active power reference \( P_{l,ref} \) and a reactive power reference \( Q_{l,ref} \). The outputs of the outer active and reactive power controller are used as references for the inner current controllers, namely \( i_{q,l,ref} \) and \( i_{d,l,ref} \). More details about those controllers and their physical models, necessary for the feedback loops, are given in further sections of this chapter. The control scheme of a PQ controlled terminal is depicted in Figure 4.2.

![Figure 4.2: Control scheme of PQ controlled terminal with the grey boxes the representation of the physical model, derived in Chapter 3](image)
4.1.2 QU Controlled Terminal

The QU controlled terminal also consists of two independent control schemes, namely a control scheme for the reactive power and a control scheme for the DC voltage at the converter’s DC side. The control scheme for the reactive power consists here as well of a slower outer control loop and a faster inner control loop, controlling the d-current. The reactive power $Q_l$ is controlled by controlling this d-current $i_{dl}$ according to Eq. (4.2). The control scheme of the DC voltage in this model consists of only one control loop.

Apparently, the QU controlled converter has two inputs, namely a DC voltage reference $U_{dc,l,ref}$ and a reactive power reference $Q_{l,ref}$. The output of the outer reactive power controller is used as a reference for the inner current controller, namely $i_{dl,ref}$. More details about those controllers and their physical models, necessary for the feedback loops, are given in further sections of this chapter. The control scheme of a QU controlled terminal is depicted in Figure 4.3.

![Control scheme of QU controlled terminal](image)

Figure 4.3: Control scheme of QU controlled terminal with the grey boxes the representation of the physical model, derived in Chapter 3

4.2 Inner Current Controller

The inner and faster control loop controls the current through the phase reactor. The system behaviour is governed by Eqs. (3.9-3.10). It is seen in these equations that the currents are strongly coupled because of the dq transformation. To independently control both currents $i_{dl}$ and $i_{ql}$, feed-forward decoupling terms need to be introduced:

$$i_{ql} (X_{rl} + X_{lt})$$

$$i_{dl} (X_{rl} + X_{lt})$$

This leads to two independent current controllers. The current control schemes are shown in Figure 4.4.
The output $u_{c,ref,d}$ of this controller will be the input of the converter, as seen on the figure. The dynamic equations (3.10) and (3.9) represent the physical model for respectively the q- and d-current controller. The converter dynamics are represented by the converter block. As mentioned in Section 2.1.4, modelling the valves is not necessary. Hence, the converter block is modelled as a gain with value 1 (hence, $u_{c,l} = u_{c,ref,l}$). The structure of the PI controllers in this control scheme is shown in Figure 4.1.

### 4.3 Active and Reactive Power Controller

An outer and slower control loop is used to control the active and reactive power. Unlike the current controller, no other control terms (i.e. feedforward decoupling terms) than the PI controller are introduced in this control scheme. This means that the active power controller and reactive power controller of Figures 4.2 and 4.3 can be replaced by the PI controller of Figure 4.1. The output of the controllers are the desired current references, entering the inner current controllers. Eqs. (4.1-4.2) represent the physical model for the active and reactive power controller. The injected/extracted active and reactive power at every terminal can be independently controlled. In the derived model, they are also independent of any other DC voltage, active power or reactive power controller in the system. In reality, when the converter dynamics are not neglected, a change in the DC voltage reference signal will cause some very small transients in the active and reactive power at all terminals.
4.4 DC Voltage Controller

One converter of a VSC HVDC grid has to be QU controlled so that the DC voltage at its terminal remains constant. The DC voltage controller consists of a single PI controller with the same structure as in Figure 4.1. The physical model for the DC voltage controller is represented by Eqs. (3.12-3.14). Contrary to the active and reactive power controllers, the DC voltage is not controlled independently from the other controllers. Changing the active or reactive power in one of the terminals will activate the DC voltage controller to keep the DC voltage constant. This is seen by Eq. (3.14), included in the physical model of the DC voltage controller. Since reactive power is not transported through the DC grid, changing the reactive power in one of the terminals will activate the DC voltage controller much less. This is demonstrated in Chapter 6.
Chapter 5

Linearized Controller Tuning

A well defined model of the control system is chosen in the previous chapter, however the control parameters of those control schemes still have to be tuned. This chapter deals with this tuning process.

5.1 General Tuning Procedure

Today, a lot of tuning procedures are known. The one applied in this project, is done by deriving first a linearized control model. Once the linearized model is identified, the control parameters are tuned according to predefined dynamic step response properties as peak time $t_p$, overshoot $M_p$, rise time $\tau$, etc. The tuning according to those properties, is moderated by transforming once more the linearized model. The applied transformation now is the Laplace Transformation (LT). The big disadvantage of this tuning procedure is the loss of the saturation non-linearity for the different PI controllers, since those PI controllers in the linearized model lose their saturation characteristic and their anti-windup of Figure 4.1. Hence, necessary caution is requested. A mismatch between the expected and the actual dynamic response of the non-linearized model is possible due to the activation of limiters and saturation. In such cases, the control parameters need to be tuned heuristically with the initial derived parameters as starting values.

5.2 Current Controller Tuning

The control scheme of the d-current and q-current controller was represented by Figure 4.4. Following control parameters are identified:

- $K_{p, id, l}$ Proportional gain PI controller, present in d-current controller
- $K_{i, id, l}$ Integral gain PI controller, present in d-current controller
- $K_{p, iq, l}$ Proportional gain PI controller, present in q-current controller
- $K_{i, iq, l}$ Integral gain PI controller, present in q-current controller
5.2.1 Linearization

The linearization of this model can be done in two steps, namely by linearizing the physical model, represented by the dynamic relation between \( i^d_l \) and \( u^d_{c,l} \) and between \( i^q_l \) and \( u^q_{c,l} \), and also by linearizing the control terms.

**Step 1:** The first step of the linearization results in a linearization of Eqs. (3.9-3.10):

\[
\begin{align*}
\frac{X_{r,l}}{\omega} \dot{i}^q_l - (X_{r,l} + X_{t,l}) \dot{i}^q_l &= -R_{r,l} \dot{i}^d_l - \dot{u}^d_{c,l} \\
\frac{X_{r,l}}{\omega} \dot{i}^q_l + (X_{r,l} + X_{t,l}) \dot{i}^d_l &= -R_{r,l} \dot{i}^q_l - \dot{u}^q_{c,l} + \dot{u}^q_{s,l}
\end{align*}
\]

where \( \dot{x}_l, \dot{i}^d_l, \dot{i}^q_l \) denote the linearized values in the form of \( \dot{x} = x - x_0 \). This nomenclature will be used from now on, with \( x_0 \) the steady state and \( \dot{x} \) the deviation from the steady state. Applying the LT to previous equations gives:

\[
\begin{align*}
\dot{i}^d_l &= + \left( \frac{X_{r,l} + X_{t,l}}{s \frac{X_{r,l}}{\omega} + R_{r,l}} \right) \dot{i}^q_l - \frac{1}{s \frac{X_{r,l}}{\omega} + R_{r,l}} \dot{u}^d_{c,l} \\
\dot{i}^q_l &= - \left( \frac{X_{r,l} + X_{t,l}}{s \frac{X_{r,l}}{\omega} + R_{r,l}} \right) \dot{i}^d_l - \frac{1}{s \frac{X_{r,l}}{\omega} + R_{r,l}} \dot{u}^q_{c,l} + \frac{1}{s \frac{X_{r,l}}{\omega} + R_{r,l}} \dot{u}^q_{s,l}
\end{align*}
\]

**Step 2:** The second step is the linearization of the equations, representing the dynamic behaviour of the different current controllers. Since saturation and anti-windup will be neglected for the linearized model, following equations need to be linearized:

\[
\begin{align*}
\dot{u}^d_{c,l} &= - \left[ K_{p,i_d,l} (i^d_{l,ref} - i^d_l) + K_{i,i_d,l} \int_0^t (i^d_{l,ref} - i^d_l) \, du \right] + (X_{r,l} + X_{t,l}) \dot{i}^q_l \\
\dot{u}^q_{c,l} &= - \left[ K_{p,i_q,l} (i^q_{l,ref} - i^q_l) + K_{i,i_q,l} \int_0^t (i^q_{l,ref} - i^q_l) \, du \right] - (X_{r,l} + X_{t,l}) \dot{i}^d_l + \dot{u}^q_{s,l}
\end{align*}
\]

Linearization and LT issue the following:

\[
\begin{align*}
\dot{u}^d_{c,l} &= - (K_{p,i_d,l} + \frac{K_{i,i_d,l}}{s}) i^d_{l,ref} + (K_{p,i_d,l} + \frac{K_{i,i_d,l}}{s}) \dot{i}^d_l + (X_{r,l} + X_{t,l}) \dot{i}^q_l \\
\dot{u}^q_{c,l} &= - (K_{p,i_q,l} + \frac{K_{i,i_q,l}}{s}) i^q_{l,ref} + (K_{p,i_q,l} + \frac{K_{i,i_q,l}}{s}) \dot{i}^q_l - (X_{r,l} + X_{t,l}) \dot{i}^d_l + \dot{u}^q_{s,l}
\end{align*}
\]
5.2.2 Further Tuning

The control parameters are tuned according to predefined dynamic step response properties. For the current controller, this dynamic step response is the response from $i_{q}^{l}$ to $i_{q}^{l,ref}$ and from $i_{d}^{l}$ to $i_{d}^{l,ref}$. Therefore, Eqs. (5.7-5.8) from Step 2 need to be added to Eqs. (5.3-5.4) from Step 1, resulting in following transfer functions:

$$\hat{i}_{d}^{l} = \frac{K_{p,d}^{l} (s + \frac{K_{i,d}^{l}}{K_{p,d}^{l}})}{s^2 \frac{X_{r}^{l}}{\omega} + s (R_{r}^{l} + K_{p,d}^{l}) + K_{i,d}^{l}}$$

(5.9)

$$\hat{i}_{q}^{l} = \frac{K_{p,q}^{l} (s + \frac{K_{i,q}^{l}}{K_{p,q}^{l}})}{s^2 \frac{X_{r}^{l}}{\omega} + s (R_{r}^{l} + K_{p,q}^{l}) + K_{i,q}^{l}}$$

(5.10)

The literature (e.g. [13]) assumes the responses to be based on a first order transfer function with a rise time given by $\tau_{id}$ for the d-current controller and $\tau_{iq}$ for the q-current controller. Transfer functions (5.9) and (5.10) only result in first order transfer functions if following equations are satisfied:

$$\frac{K_{i,id}^{l}}{K_{p,d}^{l}} = R_{r}^{l} \frac{\omega}{X_{r}^{l}}$$

(5.11)

$$\frac{K_{i,iq}^{l}}{K_{p,q}^{l}} = R_{r}^{l} \frac{\omega}{X_{r}^{l}}$$

(5.12)

Consequently, the first-order transfer functions are now

$$\hat{i}_{d}^{l} = \frac{1}{s \frac{X_{r}^{l}}{\omega K_{p,d}^{l}} + 1} = \frac{1}{s \tau_{id} + 1}$$

(5.13)

$$\hat{i}_{q}^{l} = \frac{1}{s \frac{X_{r}^{l}}{\omega K_{p,q}^{l}} + 1} = \frac{1}{s \tau_{iq} + 1}$$

(5.14)

Once the numerical values of those rise times are identified, the parameters for the PI controller can be easily calculated. The rise times are sufficiently small for a fast current controller.

5.3 Active Power Controller Tuning

The control scheme of the active power controller was represented by Figure 4.2. Following control parameters are identified:

- $K_{p,P}^{l}$ Proportional gain PI controller, present in active power controller
- $K_{i,P}^{l}$ Integral gain PI controller, present in active power controller
5.3.1 Linearization

The linearization of this model is again done in two steps, namely by first linearizing the physical model, represented by the dynamic relation between $P_l$ and $i_{ql}^q$, and also by linearizing the control terms.

**Step 1:** The first step of the linearization results in a linearization of Eq. (4.1). By additionally applying the LT, following equation results:

$$\hat{P}_l = \frac{3}{2} u_{s,l,0}^q \hat{\dot{i}}^q_{s,l} + \frac{3}{2} i_{l,0}^q \hat{\dot{i}}^q_{s,l}$$  \hspace{1cm} (5.15)

with $u_{s,l,0}^q$ and $i_{l,0}^q$ as steady state values. Assuming $\hat{\dot{u}}_{s,l}^q$ to be zero and including the q-current controller (Eq. (5.14)), transforms the previous equation into

$$\hat{P}_l = \frac{3}{2} u_{s,l,0}^q \frac{1}{\tau_{iq}s} + \frac{3}{2} i_{l,0}^q \frac{1}{\tau_{iq}s}$$  \hspace{1cm} (5.16)

**Step 2:** The second step is the linearization of the equations, representing the dynamic behaviour of the PI controller. Since saturation and anti-windup will be neglected for the linearized model, following equation needs to be linearized:

$$\hat{i}_{l,ref}^q = \left[ K_{p,P,l}(P_{l,ref} - P_l) + K_{i,P,l} \int_0^t (P_{l,ref} - P_l) du \right]$$  \hspace{1cm} (5.17)

Linearization and LT issue the following:

$$\hat{i}_{l,ref}^q = (K_{p,P,l} + \frac{K_{i,P,l}}{s})P_{l,ref} - (K_{p,P,l} + \frac{K_{i,P,l}}{s})P_l$$  \hspace{1cm} (5.18)

5.3.2 Further Tuning

The control parameters are tuned according to predefined dynamic step response properties. For the active power controller, this dynamic step response is the response from $P_l$ to $P_{l,ref}$. Therefore, Eq. (5.18) needs to be added to Eq. (5.16), resulting in following transfer function:

$$\frac{\hat{P}_l(s)}{\hat{P}_{l,ref}(s)} = \frac{\frac{3}{2} u_{s,l,0}^q (K_{p,P,l}s + K_{i,P,l})}{s^2 + \frac{3}{2} u_{s,l,0}^q K_{p,P,l}s + \frac{3}{2} K_{p,P,l} \tau_{iq}}$$  \hspace{1cm} (5.19)

The literature (e.g. [13]) assumes the responses to be based on a second order transfer function with a peak time given by $t_p$ and overshoot by $M_p$. Transfer function (5.19) can be written in following standard form:

$$\left( \frac{s}{\omega_n} \right)^2 + 2\zeta \left( \frac{s}{\omega_n} \right) + 1$$  \hspace{1cm} (5.20)
with $\zeta$ the damping ratio, $\omega_n$ the undamped natural frequency and $\alpha$ a scaling factor of the zero [22]. A coefficient comparison with the transfer function of (5.19) gives

$$\alpha = \frac{2K_{i,P,l}\tau_{iq}}{(\frac{3}{2}K_{p,P,l}u_{s,l,0}^q + 1)K_{p,P,l}}$$  (5.21)

$$\omega_n^2 = \frac{3\,u_{s,l,0}^qK_{i,P,l}}{2\tau_{iq}}$$  (5.22)

$$2\zeta\omega_n = \frac{3\,u_{s,l,0}^qK_{p,P,l} + 1}{\tau_{iq}}$$  (5.23)

The peak time $t_p$ and the overshoot $M_p$ are design parameters and can be chosen freely. Once those properties are defined, $\zeta$ and $\omega_n$ can be calculated according to following equations [22]:

$$\zeta = -\ln(M_p)\frac{1}{\sqrt{\ln(M_p)^2 + \pi^2}}$$  (5.24)

$$\omega_n = \frac{\pi}{t_p\sqrt{1 - \zeta^2}}$$  (5.25)

The parameters of the PI controller can now easily be calculated:

$$K_{p,P,l} = \frac{2\zeta\omega_n\tau_{iq} - 1}{\frac{3}{2}u_{s,l,0}^q}$$  (5.26)

$$K_{i,P,l} = \frac{2\omega_n^2\tau_{iq}}{3\,u_{s,l,0}^q}$$  (5.27)

If $\alpha$ is large, the zero of the transfer function Eq. (5.19) will be far removed from the poles and thus the zero will have little effect on the response. The major effect of the zero is the increase of the overshoot $M_p$, whereas it has very little influence on the settling time [22]. The zero has little effect if $\alpha \geq 3$. If the desired design parameters fulfill this constraint, then $K_{p,P,l}$ and $K_{i,P,l}$ can be chosen according to (5.26) and (5.27). Otherwise a numeric pole zero placement has to be performed to achieve the design requirements.

As stressed in the introduction, the necessary caution is needed for the linearized controller tuning as the saturation and the anti-windup of the PI controllers are neglected in the linearized model. A mismatch between the expected second order response (e.g. red curve of Fig. 6 in [13]) and the actual response of the non-linearized model is possible due to the activation of limiters and saturation. In such cases, the control parameters need to be tuned heuristically with the derived control parameters of the previous procedure as starting values.
5.4 Reactive Power Controller Tuning

The control scheme of the reactive power controller was represented by Figure 4.2 and 4.3. Following control parameters are identified:

- $K_{p,Q,I}$ Proportional gain of PI controller, present in reactive power controller
- $K_{i,Q,I}$ Integral gain of PI controller, present in reactive power controller

The sequel of the tuning procedure for the reactive power controller is assumed to be exactly the same as for the active power controller. The parameters of the PI controllers can be calculated by:

$$K_{p,Q,I} = \frac{2\zeta \omega_n \tau_{id} - 1}{\frac{2}{3} u_{s,l,0}^q}$$

$$K_{i,Q,I} = \frac{2 \omega_n^2 \tau_{id}}{3 u_{s,l,0}^q}$$

with $\zeta$ and $\omega_n$ as in Eqs. (5.24-5.25).

Again, the necessary caution is needed for the linearized controller tuning as the saturation and the anti-windup of the PI controllers are neglected in the linearized model. A mismatch between the expected and the actual response of the non-linearized model is possible due to the activation of limiters and saturation. In such cases, the control parameters need to be tuned heuristically with the derived control parameters of the previous procedure as starting values.

5.5 DC Voltage Controller Tuning

From now on, the QU controlled terminal is chosen as terminal $n$, with $n$ the number of terminals in a multi-terminal VSC HVDC system. The control scheme of the DC voltage controller of that terminal was represented by Figure 4.3. Following control parameters are identified:

- $K_{p,dc,n}$ Proportional gain PI controller, present in DC voltage controller
- $K_{i,dc,n}$ Integral gain PI controller, present in DC voltage controller

5.5.1 Linearization

The linearization of this model can again be done in two steps, namely by linearizing the physical model, represented by the dynamic relation between $U_{dc,n}$ and $I_{dc,n}$, and also by linearizing the control terms.
5.5. DC VOLTAGE CONTROLLER TUNING

**Step 1:** The first step of the linearization results in a linearization of following set of equations:

- Eq. (3.12) For every terminal of the VSC HVDC system
- Eq. (3.13) For every DC line of the VSC HVDC system
- Eq. (3.14) For every PQ controlled terminal of the VSC HVDC system

For a multi-terminal VSC HVDC system consisting of \( n \) terminals and \( z \) DC lines, the aforementioned set of equations (5.30) consists of \( n + z + (n-1) \) equations. Applying the Laplace Transformation to the linearized set of equations results for Eq. (3.12) in

\[
sC_{dc,l} \hat{U}_{dc,l} = \hat{I}_{dc,l} - \sum_{m=l+1}^{n} \hat{I}_{line,lm}, \quad l = 1 \tag{5.31a}
\]

\[
sC_{dc,l} \hat{U}_{dc,l} = \hat{I}_{dc,l} + \sum_{m=1}^{l-1} \hat{I}_{line,ml} - \sum_{m=l+1}^{n} \hat{I}_{line,lm}, \quad l = 2, \ldots, n-1 \tag{5.31b}
\]

\[
sC_{dc,l} \hat{U}_{dc,l} = \hat{I}_{dc,l} + \sum_{m=1}^{l-1} \hat{I}_{line,ml}, \quad l = n \tag{5.31c}
\]

while for Eq. (3.13) in

\[
sL_{dc,lm} \hat{I}_{line,lm} = \hat{U}_{dc,l} - \hat{U}_{dc,m} - R_{dc,lm} \hat{I}_{line,lm}, \quad \forall m < n, \forall l < m \tag{5.32}
\]

and for Eq. (3.14) in

\[
\hat{P}_{dc,l} = 2 I_{dc,l,0} \hat{U}_{dc,l} + 2 U_{dc,l,0} \hat{I}_{dc,l} \tag{5.33}
\]

with \( I_{dc,l,0} \) and \( U_{dc,l,0} \) as steady state values. For the simplification of the tuning procedure, \( \hat{P}_{dc,l} \) and \( I_{dc,l,0} \) are assumed to be zero. Consequently, the \( \hat{I}_{dc,l} \) in Eq. (5.31) is replaced by zero, if the terminal is a PQ controlled terminal. The aforementioned set of \( n + z + (n-1) \) equations (5.30) will be reduced to a set of \( n + z \) equations:

- Eq. (5.31) with \( \hat{I}_{dc,l} = 0 \) For every PQ controlled terminal of the VSC HVDC system.
- Eq. (5.31) For the QU controlled terminal of the VSC HVDC system.
- Eq. (3.13) For every DC line of the VSC HVDC system

This set of equations (5.34) can be written in a matrix equation of following form:

\[
Y = A \cdot Y + B \cdot \hat{I}_{dc,n} \tag{5.35}
\]
The procedure to derive the different matrices of Eq. (5.35) is step by step explained in the following:

\[
Y = [(n+z) \times 1] \quad \begin{align*}
Y_{i1} &= \hat{U}_{dc,i} \\
Y_{i1} &= \hat{I}_{\text{line},lm} \\
\end{align*} \quad \{i : i = 1,\ldots,n\} \quad \{i : i = n + 1,\ldots,n + z\} \quad \text{and } l < m, m < n
\]

\[
A = [(n+z) \times (n+z)] \quad A = \begin{bmatrix} A1 & A2 \\ A3 & A4 \end{bmatrix}
\]

\[
A1 = [n \times n] \quad A1_{ij} = 0 \quad \forall i, j
\]

\[
A2 = [n \times z] \quad \begin{align*}
A2_{ij} &= \frac{1}{sC_{dc,i}} \\
&\text{with } Y_{(n+j)1} = \hat{I}_{\text{line},lm} \\
A2_{ij} &= \frac{1}{sC_{dc,i}} \\
&\text{with } Y_{(n+j)1} = \hat{I}_{\text{line},lm} \\
A2_{ij} &= 0 \\
&\text{with } Y_{(n+j)1} = \hat{I}_{\text{line},lm} \\
\end{align*} \quad \{i, j : i = l\} \quad \{i, j : i = m\} \quad \{i, j : j \neq l \text{ and } i \neq m\}
\]

\[
A3 = [z \times n] \quad \begin{align*}
A3_{ij} &= \frac{1}{sL_{dc,lm}} \\
&\text{with } Y_{(n+i)1} = \hat{I}_{\text{line},lm} \\
A3_{ij} &= \frac{-1}{sL_{dc,lm}} \\
&\text{with } Y_{(n+i)1} = \hat{I}_{\text{line},lm} \\
A3_{ij} &= 0 \\
&\text{with } Y_{(n+i)1} = \hat{I}_{\text{line},lm} \\
\end{align*} \quad \{i, j : j = l\} \quad \{i, j : j = m\} \quad \{i, j : j \neq l \text{ and } j \neq m\}
\]

\[
A4 = [z \times z] \quad \begin{align*}
A4_{ij} &= 0 \\
A4_{ij} &= -\frac{R_{dc,lm}}{sL_{dc,lm}} \\
&\text{with } Y_{(n+i)1} = \hat{I}_{\text{line},lm} \\
\end{align*} \quad \{i, j : i \neq j\} \quad \{i, j : i = j\}
\]

\[
B = [(n+z) \times 1] \quad \begin{align*}
B_{i1} &= 0 \\
B_{i1} &= \frac{1}{sC_{dc,i}} \\
\end{align*} \quad \{i : i \neq n\} \quad \{i : i = n\}
\]

Following former procedure leads to the formulation of the matrix equation (5.35) for every configuration of a VSC HVDC system. The equation can be solved towards \( Y \) by

\[
Y = [I - A]^{-1} \cdot B \cdot \hat{I}_{\text{dc},n} \quad (5.36)
\]

\( Y_{n1} \) represents the linearized, Laplace Transformed dynamic relation between \( \hat{U}_{dc,n} \) and \( \hat{I}_{\text{dc},n} \) that needed to be derived in the first step. The order of this transfer function depends on the configuration of the VSC HVDC system:

\[
\frac{Y_{n1}}{I_{\text{dc},n}} = \frac{\hat{U}_{dc,n}}{\hat{I}_{\text{dc},n}} = \frac{((n+z) - 1)^{\text{th} \ order}}{(n+z)^{\text{th} \ order}} \quad (5.37)
\]
5.5. DC VOLTAGE CONTROLLER TUNING

Step 2: The second step is the linearization of the equations, representing the dynamic behaviour of the PI controller. Since saturation and anti-windup will be neglected for the linearized model, following equation needs to be linearized:

\[
I_{dc,n} = \left[ K_{p,dc,n}(U_{dc,n,ref} - U_{dc,n}) + K_{i,dc,n} \int_0^t (U_{dc,n,ref} - U_{dc,n})du \right] (5.38)
\]

Linearization and LT issue the following:

\[
\hat{I}_{dc,n} = \left( K_{p,dc,l} + \frac{K_{i,dc,l}}{s} \right) \hat{U}_{dc,n,ref} - \left( K_{p,dc,l} + \frac{K_{i,dc,l}}{s} \right) \hat{U}_{dc,n} (5.39)
\]

5.5.2 Further Tuning

The control parameters are tuned according to predefined dynamic step response properties. For the DC voltage controller, this dynamic step response is the response from \( U_{dc,n} \) to \( U_{dc,n,ref} \). Therefore, Eq. (5.39) from Step 2 needs to be added to Eq. (5.37) from Step 1, resulting in the transfer function \( \hat{U}_{dc,n} \hat{U}_{dc,n,ref} \). For this semester thesis, the transfer function is assumed to be based on a transfer function, whose peak time is given by \( t_p \) and overshoot by \( M_p \). Since the transfer function is clearly not a second order one, another tuning procedure than the one applied to the active and reactive power controller needs to be found.

A suggested tuning process is the the root locus design method [22]. By assuming the transfer function to be approximated by a second order transfer function, an approximated \( \zeta \) and \( \omega_n \) can be calculated, see Eqs. (5.24-5.25). The necessary caution is requested as the additional poles and zeros of the transfer function can have a strong influence on the dynamic response. The PI controller allows us to place a zero and select a gain. In general, the zero is placed close to the natural frequency \( \omega_n \) [22]. After this choice of zero, the root locus can be plotted and a suitable gain needs to be selected. To have an indication of which value of the gain to select, lines of constant damping and constant natural frequency are plotted. Again, these lines are only valid for second order systems and must be treated as rough approximations for higher order systems.

The obtained control parameters \( K_{p,dc,n} \) and \( K_{i,dc,n} \) can be already a good approximation. Nevertheless, because of the additional poles and zeros, the real dynamic response can differ from the expected dynamic response. In that case, further tuning of the parameters needs to be done heuristically by slightly changing the obtained control parameters. This should be done in such a way that the voltage response corresponds to the predefined dynamic properties peak time \( t_p \) and overshoot \( M_p \).
Chapter 6

Simulations

According to the latter, dynamic simulations of a multi-terminal VSC HVDC system for every DC grid configuration within the chosen topology (see Section 2.3.2), can be executed now by the use of Simulink. The physical models, as well as the models for the control systems are derived in Chapter 3 and Chapter 4. In this chapter, a test grid is proposed and appropriate simulations are done.

6.1 Data MTDC Test Grid

First of all, power system parameters (e.g. converter capacitances, line parameters, etc.) need to be known. This project work appeals to data of the ABB HVDC Light M1 converter [16], supplemented with frequently recurring data in literature for those parameters. The used test grid data is given in Table 6.1 and is used for every converter of the MTDC grid.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Voltage</td>
<td>80 kV</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>102 MW</td>
</tr>
<tr>
<td>$Q_{\text{max}}$</td>
<td>50.50 MVar</td>
</tr>
<tr>
<td>$X_{r,l}$</td>
<td>0.15 Ω</td>
</tr>
<tr>
<td>$X_{t,l}$</td>
<td>0.10 Ω</td>
</tr>
<tr>
<td>$C_{\text{conv,l}}$</td>
<td>62.7 μF</td>
</tr>
<tr>
<td>$R_{\text{dc,lm}}$</td>
<td>0.0799 Ω/km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Power</td>
<td>101 MVA</td>
</tr>
<tr>
<td>$P_{\text{min}}$</td>
<td>-98.7 MW</td>
</tr>
<tr>
<td>$Q_{\text{min}}$</td>
<td>-50.50 MVar</td>
</tr>
<tr>
<td>$R_{r,l}$</td>
<td>0.03 Ω</td>
</tr>
<tr>
<td>Nominal DC Voltage</td>
<td>±80 kV</td>
</tr>
<tr>
<td>$C_{\text{line,lm}}$</td>
<td>0.308 μF/km</td>
</tr>
<tr>
<td>$L_{\text{dc,lm}}$</td>
<td>0.14 mH/km</td>
</tr>
</tbody>
</table>

Secondly, some predefined dynamic step response properties (for instance peak time $t_p$, rise time $\tau$, overshoot $M_p$, etc.) to reference input signals need to be known for the tuning of the control parameters. The chosen properties are mainly based on [15] and presented in Table 6.2 for the different controllers.
Table 6.2: Dynamic response properties of the different controllers

<table>
<thead>
<tr>
<th>Controller</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>d-Current Controller</td>
<td>First-order transfer function</td>
</tr>
<tr>
<td></td>
<td>Rise time $\tau_{id}$</td>
</tr>
<tr>
<td>q-Current Controller</td>
<td>First-order transfer function</td>
</tr>
<tr>
<td></td>
<td>Rise time $\tau_{iq}$</td>
</tr>
<tr>
<td>Active Power Controller</td>
<td>Overshoot $M_{p,P}$</td>
</tr>
<tr>
<td></td>
<td>Peak time $t_{p,P}$</td>
</tr>
<tr>
<td>Reactive Power Controller</td>
<td>Overshoot $M_{p,Q}$</td>
</tr>
<tr>
<td></td>
<td>Peak time $t_{p,Q}$</td>
</tr>
<tr>
<td>DC Voltage Controller</td>
<td>Overshoot $M_{p,dc}$</td>
</tr>
<tr>
<td></td>
<td>Peak time $t_{p,dc}$</td>
</tr>
</tbody>
</table>

Finally, the configuration of the DC grid is needed. Those DC grids can be really complex. For this chapter, a simulation based on the configuration of Figure 6.1 is done. Aforementioned agreements clarify the choice of current directions (see Section 3.3) and the choice of terminal 4 as QU controlled terminal (see 5.5). Line parameters, such as $R_{dc,lm}$, $L_{dc,lm}$ and $C_{dc,lm}$ are for simplicity reasons chosen to be identical and a line length of 50 km is chosen. Since line properties are chosen to be identical, the notations $R_{dc}$, $L_{dc}$ and $C_{dc}$ will now represent the resistance, the inductance and the capacitance of each of the lines.

![Figure 6.1: Configuration of MTDC test grid](image)

Furthermore, the assumption is made that the amplitude of the AC system voltage phasor $u_{s,l}$ of every terminal $l$ of the MTDC system remains constant with a value of 80 kV. The transformer is assumed to be a 1:1 transformer. According to Appendix B and according to the aformentioned alignment of the q-axis of the rotating dq reference frame with the AC system voltage $u_{s,l}$, $u_{s,l}^q$ will have a constant value of 80kV, while $u_{s,l}^d = 0$. 
6.2 Dynamic Model

Applying the models, derived in Chapter 3 and Chapter 4, to the discussed configuration gives the dynamic model, depicted in Figure 6.2.

Figure 6.2: Dynamic model of the discussed MTDC test grid with the grey boxes the representation of the physical models, derived in Chapter 3.
The figure depicts the three PQ controlled terminals (terminals 1 − 3) with $P_{l,\text{ref}}$ and $Q_{l,\text{ref}}$ as references for the two independent control schemes of each PQ controlled terminal. Also the QU controlled terminal (terminal 4), with $Q_{4,\text{ref}}$ and $U_{dc,4,\text{ref}}$ as references for the two independent control schemes of that terminal, are depicted.

Since the physical model of the DC voltage controller is represented by the set of equations (5.30), the DC voltage controller will be dependent on both the active power control scheme as the reactive power control scheme of the PQ controlled terminals and also dependent on the reactive power control scheme of the QU controlled terminal. This is because $I_{dc,l}$ in Eq. (3.12) of each terminal (part of the aforementioned set of equations (5.30)) is coupled to the AC side of its converter by the power balance of Eq. (3.14):

$$P_{ac,l} = \frac{3}{2}(u_{dc,l}^d i_{l}^d + u_{dc,l}^q i_{l}^q) = 2I_{dc,l} U_{dc,l} = P_{dc,l}$$

A change in active power reference $P_{l,\text{ref}}$ in one of the PQ controlled terminals (terminals 1 − 3) causes a change in $u_{dc,l}^q$ and $i_{l}^q$ and consequently an activation of the DC voltage controller. A change in reactive power reference $Q_{l,\text{ref}}$ in one of the terminals (terminals 1 − 4) causes a change in $u_{dc,l}^d$ and $i_{l}^d$ and consequently again an activation of the DC voltage controller.

### 6.3 Linearized Controller Tuning

For the derived dynamic model, the only unknowns are the control parameters of the different PI controllers. Those control parameters need to be tuned according to the dynamic step response properties of Table 6.2. The linearized controller tuning procedure of Chapter 5 can be applied to the different active power and reactive power controllers and to the DC voltage controller.

#### 6.3.1 Current Controller Tuning

Implementing the data of Table 6.1 and Table 6.2 to Eqs. (5.11-5.14) results in following control parameters for the different current controllers.

<table>
<thead>
<tr>
<th>d-current Controller</th>
<th>q-current controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{p,ld,l}$</td>
<td>0.00955</td>
</tr>
<tr>
<td>$K_{p,lq,l}$</td>
<td>0.00955</td>
</tr>
<tr>
<td>$K_{i,ld,l}$</td>
<td>0.59998</td>
</tr>
<tr>
<td>$K_{i,lq,l}$</td>
<td>0.59998</td>
</tr>
</tbody>
</table>

Since Table 6.1 is used for every converter of the MTDC grid, those control parameters count for all current controllers of the system.
6.3. LINEARIZED CONTROLLER TUNING

6.3.2 Active and Reactive Power Controller Tuning

Implementing the data of Table 6.1 and Table 6.2 to Eqs. (5.24-5.29) results in following control parameters for the active power controllers and reactive power controller.

Table 6.4: Control parameters of active and reactive power controllers

<table>
<thead>
<tr>
<th></th>
<th>Active Power Controller</th>
<th>Reactive Power Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_{p,P,l})</td>
<td>8.4707e-6</td>
<td>8.4707e-6</td>
</tr>
<tr>
<td>(K_{i,P,l})</td>
<td>5.7230e-4</td>
<td>5.7230e-4</td>
</tr>
<tr>
<td>(K_{p,Q,l})</td>
<td>8.4707e-6</td>
<td>8.4707e-6</td>
</tr>
<tr>
<td>(K_{i,Q,l})</td>
<td>5.7230e-4</td>
<td>5.7230e-4</td>
</tr>
</tbody>
</table>

The influence of the zero of the transfer functions \(\frac{P_{l}}{P_{l,ref}}\) and \(\frac{Q_{l}}{Q_{l,ref}}\) needs to be evaluated. An evaluation can be done by calculating \(\alpha\), according to Eq. (5.21). A value of \(\alpha\) higher than 3 refers to a very little zero-effect. Since the calculated \(\alpha\) has a value of 3.3505 in this case, no further tuning of the control parameters is needed. Since Table 6.1 is used for every converter of the MTDC grid, those control parameters count for all active power and reactive power controllers of the system.

6.3.3 DC Voltage Controller Tuning

Implementing data of Table 6.1 and 6.2 to Step 1 of the linearization procedure explained in Section 5.5.1 results in following matrices of Eq. (5.36):

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & \frac{-1}{sC_{dc}} & \frac{-1}{sC_{dc}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{sC_{dc}} & \frac{-1}{sC_{dc}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{sL_{dc}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{sL_{dc}} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{sL_{dc}} & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & \frac{-R_{dc}}{sL_{dc}} & 0 & 0 \\
0 & \frac{1}{sL_{dc}} & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & \frac{-R_{dc}}{sL_{dc}} & 0 & 0 \\
0 & 0 & \frac{1}{sL_{dc}} & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & 0 & \frac{-R_{dc}}{sL_{dc}} \\
0 & 0 & \frac{1}{sL_{dc}} & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & 0 & 0 & \frac{-R_{dc}}{sL_{dc}} \\
0 & 0 & 0 & 0 & \frac{-1}{sL_{dc}} & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix},
\]

\[
Y = \begin{bmatrix}
\hat{U}_{dc1} \\
\hat{U}_{dc2} \\
\hat{U}_{dc3} \\
\hat{U}_{dc4} \\
\hat{I}_{c12} \\
\hat{I}_{c13} \\
\hat{I}_{c23} \\
\hat{I}_{c24} \\
\hat{I}_{c34} \\
\end{bmatrix}
\]
The transfer function, defined in Eq. (5.37) will look as follows:

\[
\begin{align*}
\hat{U}_{dc,4} = & \frac{L_{dc}^2C_{dc}^2s^4 + 2R_{dc}L_{dc}C_{dc}^2s^3 + (R_{dc}^2C_{dc}^2 + 4L_{dc}C_{dc})s^2 + 4R_{dc}C_{dc}s + 2}{L_{dc}^2C_{dc}^3s^5 + 2R_{dc}L_{dc}C_{dc}^3s^4 + (R_{dc}^2C_{dc}^3 + 6C_{dc}^2L_{dc})s^3 + 6R_{dc}C_{dc}^2s^2 + 8C_{dc}s}
\end{align*}
\]

The order of this transfer function is lower compared to the order of Eq. (5.37). This is due to the assumption of identical line parameters. Some poles and zeros are cancelled out because of this assumption.

Adding Eq. (5.39) from Step 2 to the latter equation (6.1) results in following transfer function:

\[
\frac{\hat{U}_{dc,4}}{U_{dc,4,ref}} = \frac{K_{p,dc,4}\hat{G}(s)}{1 + K_{p,dc,4}\hat{G}(s)}
\]

with

\[
\hat{G}(s) = \frac{s + K_{i,dc,4}}{K_{p,dc,4}\hat{U}_{dc,4}}
\]

The control scheme of the DC voltage controller at terminal 4 is depicted in the overall dynamic model, see Figure 6.2. The term \(K_{p,dc,4}\hat{G}(s)\) represents the open loop transfer function of the linearized control scheme.

The provisional unknowns of the transfer function in Eq. (6.2) are the control parameters \(K_{i,dc,4}\) and \(K_{p,dc,4}\). These parameters need to be tuned according to the dynamic step response properties of Table 6.2. Since the transfer function is clearly not a second order one, another tuning procedure than the one applied to the active and reactive power controller needs to be found. Therefore, the root locus design method is applied first for an initial indication of the control parameters.

**Root Locus Design Method**

This method assumes the transfer function to be approximated by a second order transfer function. Consequently, their approximated damping ratio \(\zeta\) and natural frequency \(\omega_n\) for an overshoot of 20% and a peak time of 100 ms (see Table 6.2), are calculated by Eqs. (5.24-5.25):

\[
\zeta = 0.4559 \quad \omega_n = 35.0835
\]

The PI controller allows us to place a zero and select a gain for the transfer function of Eq. (6.2). In general, the zero \(\frac{K_{i,dc,4}}{K_{p,dc,4}}\) is placed close to the natural frequency \(\omega_n\). For the selection of a suitable gain \(K_{p,dc,4}\), the root
locus of Eq. (6.3) is plotted in Figure 6.3. Root locus analysis is a graphical method for examining how the roots of a system change with variation of a gain within a feedback system.

Figure 6.3: Root locus of Eq. (6.3) with (b) a close-up of (a)
CHAPTER 6. SIMULATIONS

The selection of a suitable gain $K_{p,dc,4}$ can be facilitated by also plotting the lines of constant damping $\zeta$ and lines of constant natural frequency $\omega_n$. These lines are only valid for second order systems and must be treated as rough approximations for higher order systems. On the root locus plot, the two dotted lines at about a 60 degree angle indicate pole locations with $\zeta = 0.456$ (overshoot of 20%). In between these lines, the poles will have $\zeta > 0.456$ (overshoot lower than 20%) and outside of the lines $\zeta < 0.456$ (overshoot higher than 20%). The semicircle indicates pole locations with a natural frequency $\omega_n = 35.1$ (peak time of 100 ms). Inside the circle, $\omega_n < 35.1$ (peak time more than 100 ms) and outside the circle $\omega_n > 35.1$ (peak time less than 100ms). Based on the root locus and the choice of the zero, following initial control parameters are selected:

$$K_{p,dc,4} = 0.008$$
$$K_{i,dc,4} = 0.281$$

The control parameters can now be evaluated according to the dynamic step response parameters. This can be graphically done by plotting the time response from $\hat{U}_{dc,4}$ to a step input $\hat{U}_{dc,ref,4}$ of 10 kV for the initial control parameters of Eq. (6.5), see Figure 6.4. Apparently, the predefined peak time and overshoot are not perfectly satisfied. This is due to the influence of additional zeros and poles. The figure shows an overshoot of about 30% for a peak time of a little less than 100ms.

![DC voltage response terminal 4](image)

Figure 6.4: The time response from $\hat{U}_{dc,4}$, represented by the blue line, to a step input $\hat{U}_{dc,ref,4}$, represented by the dashed red line, for the initial control parameters of Eq. (6.5)
By now slightly heuristically changing the parameters $K_{r,dc,4}$ and $K_{p,dc,4}$, the predefined dynamic step response properties can be approximated more accurate. The adapted control parameters are now given by

<table>
<thead>
<tr>
<th>DC Voltage Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{p,dc,4}$</td>
</tr>
<tr>
<td>$K_{i,dc,4}$</td>
</tr>
</tbody>
</table>

Again, the control parameters can now be graphically evaluated by plotting the time response from $\hat{U}_{dc,4}$ to a step input $\hat{U}_{dc,ref,4}$ of 10 kV for the adapted control parameters, see Figure 6.5. The overshoot of 20% and peak time of 100ms are clearly approached much better now.

![DC voltage response terminal 4](image)

Figure 6.5: The time response from $\hat{U}_{dc,4}$, represented by the blue line, to a step input $\hat{U}_{dc,ref,4}$, represented by the dashed red line, for the adapted control parameters of Table 6.5

### 6.4 Dynamic Simulations

Since all control parameters are identified now, the overall dynamic model of Figure 6.2 is defined and simulations can be done in Simulink. In the following, different reference input signals are applied to the controllers and the dynamic responses, as well as some steady-state values of certain variables are plotted and studied.
6.4.1 Active Power Step Input Signal

The dynamic and steady-state responses of the system variables towards a change in reference input for the active power controller are important ones to be studied, since in operation, those references will frequently change. Since active power can be transported across the DC grid, a change in reference input signal will affect a lot of variables of the VSC based HVDC system. In this simulation, the reference input signal of terminal 3 is a step function with an initial value of 0 MW and a final value of 65 MW. Regarding the maximum possible active power $P_{\text{max}}$ injected at a terminal in the DC grid, namely 102 MW, this change in reference is a substantial change.

Dynamics

First of all, the time response from $P_3$ to $P_{3,\text{ref}}$ is plotted in Figure 6.6 to verify the predefined dynamic step response properties of Table 6.2. The dynamic response approximates the predefined overshoot of 14% and peak time of 100ms, quite well. However, the influence of the zero in the transfer function $\hat{P}_3/\hat{P}_{\text{ref,3}}$, important for the tuning procedure, is slightly visible. It depends on the wanted accuracy of the model, if the control parameters should be tuned further. This possible further tuning is done heuristically.

The change in reference signal won’t affect the reactive power at the terminals but will affect a.o. the active power flows, the DC voltages $U_{\text{dc,l}}$ of every terminal and the active power $P_4$, extracted from the grid. The dynamics of those variables are given by Figure 6.7, Figure 6.8 and Figure 6.9.
For the power flow plot, $P_{lm}$ means the power flow from terminal $l$ to terminal $m$. The power flows in the lines have a similar dynamic response as the controlled active power $P_3$. While the overshoot of the highest loaded lines reaches almost 30%, the peak times of the dynamic responses in the different lines are maximum 100ms. Steady state is almost reached after around 200ms. These results for the active power flows in the lines don’t seem to cause significant problems. Only maybe in high-loaded HVDC grids, this overshoot might need certain caution.
The DC voltages at all terminals have a similar dynamic response. The voltages reach a peak value of around 30% of its initial value, which seems to be rather high in comparison to the change in steady state values. The change in steady state is only around 1%. This is of course due to the very low losses in the lines. The peak time is around 100ms, while steady state is almost reached after 200ms.

![Active Power extracted from DC grid at terminal 4](image)

Figure 6.9: The time response from the active power $P_4$ extracted from the DC grid at terminal 4 to a step input $P_{3,\text{ref}}$ of 65 MW

The dynamics of the active power extracted from the DC grid at the QU controlled terminal look similar to the dynamic response of $P_3$. However, the overshoot reaches almost 30% (i.e. double the overshoot of $P_3$), while the peak time is slightly delayed, compared to the dynamic response of $P_3$.

**Steady-state**

Following steady-state values are found for the power flows in the lines:

<table>
<thead>
<tr>
<th>$P_{12}$</th>
<th>$P_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1099 MW</td>
<td>-8.0808 MW</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>$P_{31}$</td>
</tr>
<tr>
<td>-8.1099 MW</td>
<td>8.1210 MW</td>
</tr>
<tr>
<td>$P_{23}$</td>
<td>$P_{32}$</td>
</tr>
<tr>
<td>-16.1728 MW</td>
<td>16.2532 MW</td>
</tr>
<tr>
<td>$P_{24}$</td>
<td>$P_{42}$</td>
</tr>
<tr>
<td>24.2536 MW</td>
<td>-24.0727 MW</td>
</tr>
<tr>
<td>$P_{34}$</td>
<td>$P_{43}$</td>
</tr>
<tr>
<td>40.6258 MW</td>
<td>-40.1233 MW</td>
</tr>
</tbody>
</table>

Since the path of lowest resistance between the injection terminal (terminal 3) and the extraction terminal (QU controlled terminal 4) is the DC line between terminal 3 and terminal 4, the power flow is as expected, the highest
in this line. The path of second lowest resistance consists of the line between terminal 3 and 2 and the line between terminal 2 and 4. This is verified again by the steady-state values. The difference between the numerical values of $P_{lm}$ and $P_{ml}$ refers to the active power losses in the line.

The steady-state values of the DC voltages at each terminal are

\[
\begin{align*}
U_{dc,1} &= 80.8014 \text{ kV} \\
U_{dc,2} &= 80.6011 \text{ kV} \\
U_{dc,3} &= 81.0018 \text{ kV} \\
U_{dc,4} &= 80.0000 \text{ kV}
\end{align*}
\]

Due to the low resistive losses in the lines, the steady state values of the DC voltages at every terminal are increased with maximum 1.3%. It is logical that the highest increase of DC voltage is at the terminal, where active power is injected.

The active power $P_4$ extracted from the DC grid at terminal 4, and the total active power loss $P_{loss}$ across the grid are

\[
\begin{align*}
P_4 &= 64.1943 \text{ MW} \\
P_{loss} &= 0.8057 \text{ MW}
\end{align*}
\]

This numerical value of $P_{loss}$ illustrates the aforementioned low resistive losses in the lines.

### 6.4.2 Reactive Power Step Input Signal

Regarding the dynamic model in Figure 6.2, a change in reference input signal $Q_{l,ref}$ for the reactive power controller of any terminal will affect a lot of variables of the DC grid. In this simulation, the reference input signal $Q_{3,ref}$ of terminal 3 is a step function with an initial value of 0 MVar and a final value of 40 MVar. Regarding the maximum possible reactive power $Q_{max}$ injected at a terminal in the DC grid, namely 50.50 MVar, this change in reference is a substantial change.

In Figure 6.10, the time response from $Q_3$ to $Q_{ref,3}$ is plotted to verify the predefined step response properties of Table 6.2. The change in reference signal will affect o.a. the active power flows, the DC voltages $U_{dc,l}$ at each terminal and the active power $P_4$, injected in the grid. The dynamics of those variables are given by Figure 6.11, Figure 6.12 and Figure 6.13. A steady-state analysis of those variables is not done in this section.
CHAPTER 6. SIMULATIONS

Figure 6.10: The time response from $Q_{dc,3}$, represented by the blue line, to a step input $Q_{dc,ref,3}$, represented by the dashed red line.

The dynamic response from $Q_{dc,3}$ to $Q_{dc,ref,3}$ approximates the predefined overshoot of 14% and peak time of 100ms, quite well. However, the influence of the zero in the transfer function $\frac{Q_4}{Q_{dc,ref}}$, important for the tuning procedure, is slightly visible. It depends on the wanted accuracy of the model, if the control parameters should be tuned further. This possible further tuning is done heuristically.

Figure 6.11: The time response from the power flows in the lines to a step input $Q_{dc,3,ref}$ of 40 MVar.
Again, the power flows in the lines have a similar dynamic response as the controlled reactive power $Q_3$. The transients of the power flows are of the order kW, instead of MW. Consequently, one can conclude that a change in the reference signal $Q_{3,\text{ref}}$ barely affects the active power flow in the lines.

![DC voltages at terminals](image)

Figure 6.12: The time response from the DC voltages at each terminal to a step input $Q_{3,\text{ref}}$ of 40 MVar

For the different terminals, the change in DC voltages is of the order of V, instead of kV. Again, one can conclude that a change in the reference signal $Q_{3,\text{ref}}$ barely affects the DC voltages at the different terminals.

![Active Power injected in DC grid at terminal 4](image)

Figure 6.13: The time response from the injected active power $P_4$ to a step input $Q_{3,\text{ref}}$ of 40 MVar
Also the order of the injected active power at the QU controlled terminal (terminal 4) is kW, instead of MW, indicating a weak influence of the change in reference signal $Q_{3,\text{ref}}$ on the injected active power at terminal 4. Previous results lead us to conclude that a change in the reactive power reference signal very weakly affect the DC grid variables. The influences are only local at the AC side of its terminal.

### 6.4.3 DC Voltage Step Input Signal

A change in reference input signal for the DC voltage controller will affect a lot of variables of the VSC based HVDC system, among them the DC voltages at every terminal, the power flows in the lines and the active power $P_4$ injected in the DC grid at terminal 4. In this simulation, the reference input signal is a step function with an initial value of 80 kV and a final value of 120 kV. The time response of the DC voltage at each terminal is plotted in Figure 6.14.

![DC voltages at terminals](image)

Figure 6.14: The time response from the DC voltages at each terminals to a step input $U_{dc,4,\text{ref}}$, represented by the dashed red line.

The DC voltages in all terminals have more or less identical dynamics. They all approach really accurate the predefined overshoot of 20% and the predefined peak time of 100ms.

The dynamics of the power flows $P_{12}$, $P_{31}$, $P_{32}$, $P_{24}$ and $P_{34}$ are plotted in Figure 6.15, while the time response of the active power $P_4$ injected in the grid at terminal 4 is plotted in Figure 6.16.
A step input of the reference signal $U_{dc,4,ref}$ causes a very fast and significant change in the power flows of the different lines. When the DC voltages at all terminals reach their steady-state, those transients of the power flows also extinguish. The settling time is around 200ms.

Again, a fast and significant change in injected active power is seen. Eventually, this transient extinguishes after around 200ms.
Chapter 7

Conclusions

A dynamic model of a VSC based HVDC system is derived in this semester thesis, starting from the basic mathematical equations. The model consists of AC side equations, DC side equations and converter (AC-DC coupling) equations. Also the control systems are included in the model. The DC side equations have been generalised to multi-terminal HVDC systems for the topology, proposing a meshed DC system with every terminal consisting of one converter and with the possibility of connections within the DC system. Reduced order models are derived formally by neglecting a.o. the converter dynamics, since the switching frequency is around 2 kHz. Several benefits should ensue from this simplification, such as less equations. Saturation and anti-windup are included in the dynamic model of the PI controllers.

Every terminal of the multi-terminal HVDC system is PQ controlled except for one, which is QU controlled. The control parameters are tuned according to some predefined properties of the dynamic responses. Therefore, a linear control model is derived. Tuning rules have been established and tested through simulations. The big disadvantage of this tuning procedure is the lost of the saturation non-linearity for the different PI controllers, since those PI controllers in the linearized model lose their saturation characteristic and their anti-windup. Hence, the real dynamic response can differ from the expected dynamic response when limiters and saturation are activated in the non-linear dynamic model. However, in such cases, the derived control parameters can be used as a starting point for further heuristical tuning. For a more efficient tuning method, a non-linear controller tuning method should be applied.

Simulations are done for a proposed MTDC test grid, whose data is mainly based on an ABB HVDC Light Model. The predefined dynamic step response properties used for control parameter tuning, are chosen based on literature. The simulation results illustrate the fast system response and
the influence of a change in the control references on different system variables. Fast variations in the active power and DC voltage reference signals will cause significant, but at first sight not problematic, transients in the DC voltages and power flows of the whole system, whereas a change in the reactive power input signal only very weakly affects other system variables.
Appendix A

dq transformation

A dq transformation is a mathematical transformation that rotates the reference frame of three-phase systems in an effort to simplify the analysis of three-phase circuits. In the case of a symmetrical three-phase system, application of the transformation reduces the three AC quantities to two DC quantities. Coordinates are transformed from the three-phase stationary coordinate system to the dq0 rotating coordinate system according to following equation:

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$  \hspace{1cm} (A.1)

The project makes use of this dq transformation. More dq transformations can be found in literature. Those are mostly really similar to the one in Eq. (A.1). Another frequently used dq transformation for example is Park’s transformation, illustrated by:

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$  \hspace{1cm} (A.2)
Appendix B

Power Balance

The three-phase converter voltage coordinates \([u_{c,l}]_{abc}\) with voltage amplitude \(U_{c,l}\) and phase angle \(\theta\) at the AC side of converter \(l\) are given by:

\[
\begin{align*}
    u_{c,l,a} &= U_{c,l} \cos(\omega t - \theta) \\
    u_{c,l,b} &= U_{c,l} \cos(\omega t - \theta - \frac{2\pi}{3}) \\
    u_{c,l,c} &= U_{c,l} \cos(\omega t - \theta + \frac{2\pi}{3})
\end{align*}
\] (B.1)

The three-phase phase reactor current coordinates \([i_l]_{abc}\) with current amplitude \(I_l\) and phase angle \(\theta'\) at the AC side of converter \(l\) are given by:

\[
\begin{align*}
    i_{l,a} &= I_l \cos(\omega t - \theta') \\
    i_{l,b} &= I_l \cos(\omega t - \theta' - \frac{2\pi}{3}) \\
    i_{l,c} &= I_l \cos(\omega t - \theta' + \frac{2\pi}{3})
\end{align*}
\] (B.2)

Applying the dq transformation of Appendix-A to the three phase symmetric voltage \([u_{c,l}]_{abc}\) of Eq. (B.1) gives:

\[
\begin{align*}
    u^d_{c,l} &= U_{c,l} \cos(\theta) \\
    u^q_{c,l} &= -U_{c,l} \sin(\theta) \\
    u^0_{c,l} &= 0
\end{align*}
\] (B.3)

Applying the dq transformation of Appendix-A to the three phase symmetric current \([i_l]_{abc}\) of Eq. (B.2) gives:

\[
\begin{align*}
    i^d_l &= I_l \cos(\theta') \\
    i^q_l &= -I_l \sin(\theta') \\
    i^0_l &= 0
\end{align*}
\] (B.4)
The power at the AC side of the converter \( l \) is:

\[
P_{ac,l} = u_{c,l_a} i_{a} + u_{c,l_b} i_{b} + u_{c,l_c} i_{c}
= \frac{3}{2} U_{c,l} I_l \cos(\theta' - \theta)
= \frac{3}{2} I_l \cos(\theta') U_{c,l} \cos(\theta) + \frac{3}{2} I_l \sin(\theta') U_{c,l} \sin(\theta)
\] (B.5)

Implementing the dq-voltages and dq-currents in above power equation gives:

\[
P_{ac,l} = \frac{3}{2} (u_{c,l_d} i_{d} + u_{c,l_q} i_{q})
\] (B.6)

The power at the DC side of the converter \( l \) is:

\[
P_{dc,l} = 2 U_{dc,l} I_{dc,l}
\] (B.7)

Resulting from the power balance of the converter \( l \):

\[
P_{ac,l} = P_{dc,l}
= \frac{3}{2} (u_{c,l_d} i_{d} + u_{c,l_q} i_{q}) = 2 I_{dc,l} U_{dc,l}
\] (B.8)
References


REFERENCES


